Three Dimensional Simulations of the Parker's Model of Solar Coronal Heating: Lundquist Number Scaling due to Random Photospheric Footpoint Motion

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Parker's model [Parker, *Astrophys. J.*, **174**, 499 (1972)] is one of the mostly discussed mechanisms for coronal heating and has generated much debate. We have recently obtained new scaling results in two dimensions (2D) version of this problem suggesting that the heating rate becomes independent of resistivity in a statistical steady state [Ng and Bhattacharjee, *Astrophys. J.*, **675**, 899 (2008)]. Our numerical work has now been extended to 3D by means of large-scale numerical simulations. Random photospheric footpoint motion is applied for a time much longer than the correlation time of the motion to obtain converged average coronal heating rates. Simulations are done for different values of the Lundquist number to determine scaling. In the large Lundquist number limit, we recover the case in which the heating rate is independent of the Lundquist number, predicted by previous analysis as well as 2D simulations. In the same limit the average magnetic energy built up by the random footpoint motion saturates at a constant level, due to the formation of strong current layers and subsequent disruption when the equilibrium becomes unstable. In this talk, we will present latest numerical results from large-scale 3D simulations, and discuss challenges in future developments.

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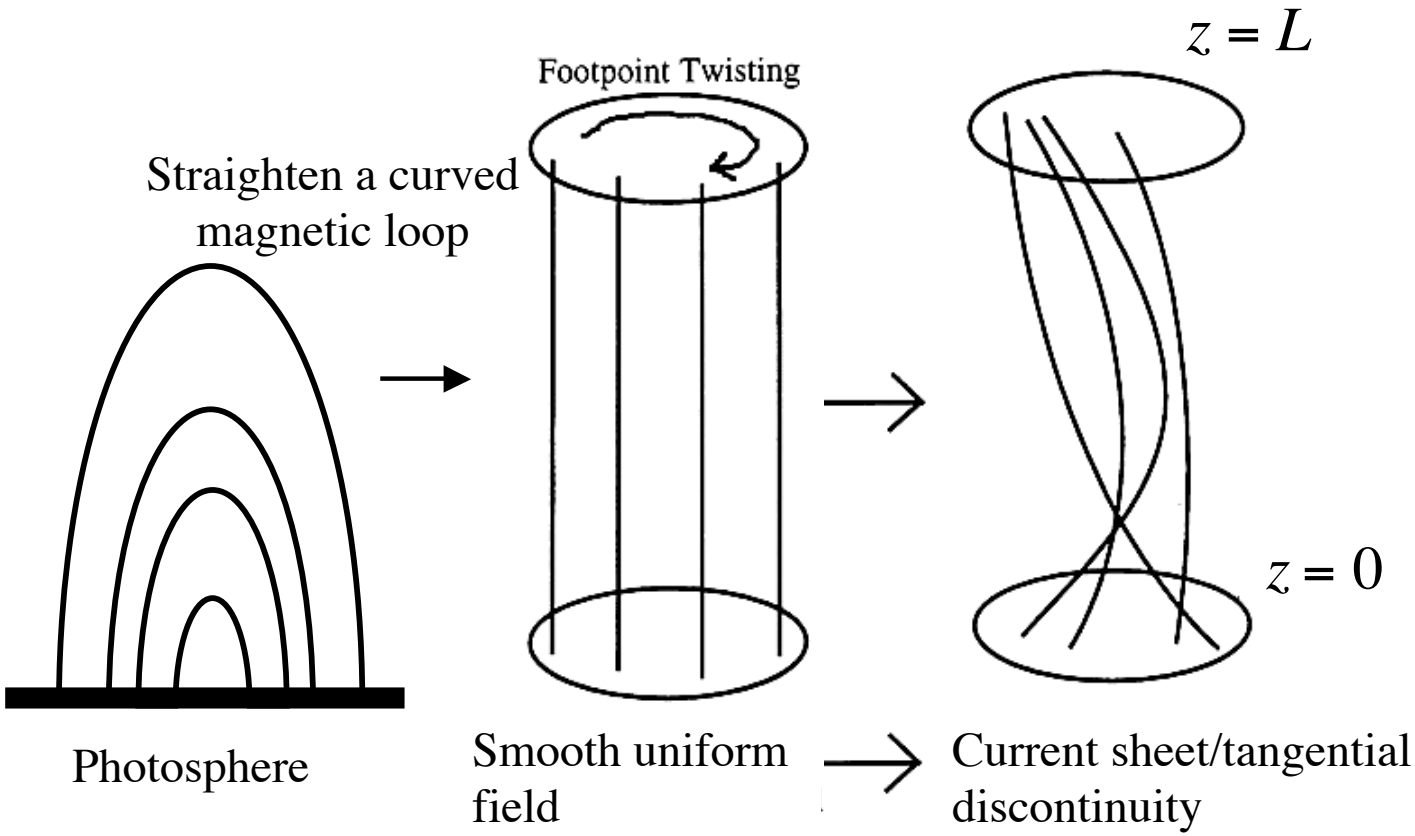
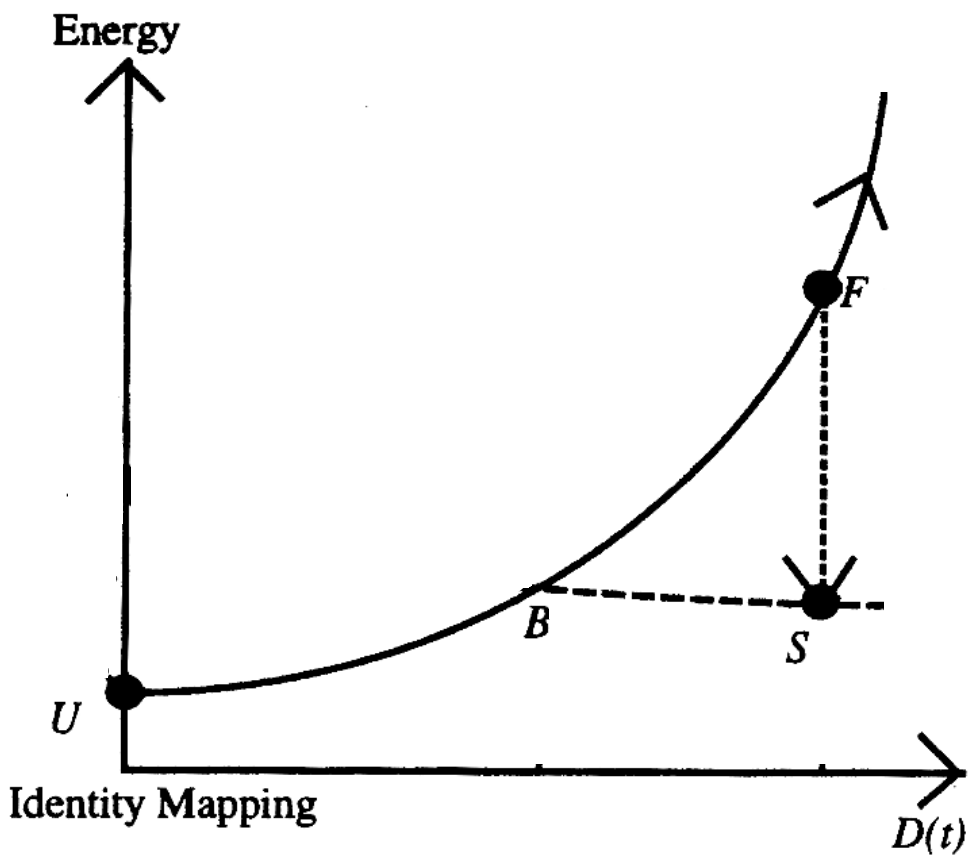
Parker's model of coronal heating through current sheet formation



A theorem on Parker's model

For any given footpoint mapping connected with the identity mapping, there is at most one smooth equilibrium [Ng & Bhattacharjee, Phys. Plasmas 1998].

An unstable but smooth equilibrium cannot relax to a second smooth equilibrium, hence must have current sheets.



Tectonics model of coronal heating

- Recent observations show that there is a magnetic carpet covering the solar surface with stronger magnetic field and are replaced in 10-40 hours.
- Recently a tectonics model of coronal heating in the magnetic carpet has been proposed [*Priest, Heyvaerts and Title*, 2002].
- Heating is provided by dissipation and reconnection via current sheets at separatrix surfaces between neighboring cells due to near-discontinuous footpoint motion.

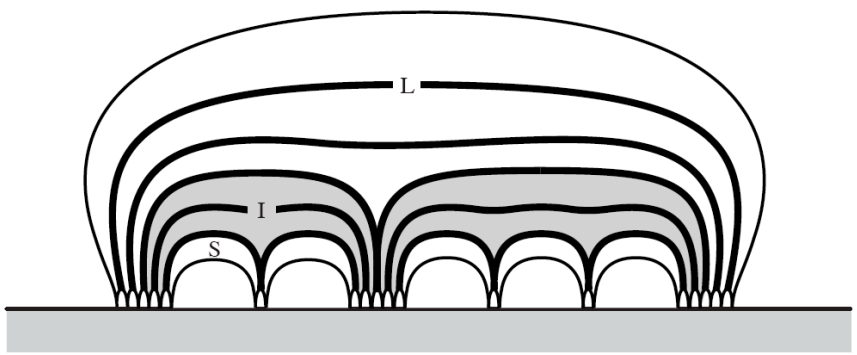


FIG. 3.—Schematic representation of the tectonics of the solar corona, showing separatrix current sheets (*thick curves*) along the boundaries of small loops (S) and both within and on the boundaries of intermediate (I) and large (L) loops. Each loop consists of one or several elementary flux tubes, each of which is bounded by a current sheet and linked to a discrete source in the photosphere.

[*Priest, Heyvaerts and Title*, 2002]

Tectonics model of coronal heating

In arguing that heating by reconnection is more important than Ohmic dissipation, *Priest, Heyvaerts and Title* [2002] invoke a calculation of the average Ohmic heating rate per unit solar surface:

$$\langle H \rangle = \frac{\Phi^2 v_{\rm phot}^2 \tau_{\rm coh}}{15\sqrt{\pi}\mu L w^4} \left(\frac{\eta \tau_{\rm coh}}{4w^2} \right)^{1/2}$$

It is obviously too small for coronal heating due to the $\eta^{1/2}$ dependence on resistivity (note that w^2/η is of the order of the resistive time scale τ_r, which is usually much larger than the coherence time $\tau_{\rm coh}$).

We would like to study the dependence on resistivity using another calculation, which will show that $\langle H \rangle$ is roughly independent on η, when $\tau_{\rm coh}$ is much smaller than τ_r.

Reduced MHD equations

$$\frac{\partial \Omega}{\partial t} + [\phi, \Omega] = \frac{\partial J}{\partial z} + [A, J] + \nu \nabla_\perp^2 \Omega$$

$$\frac{\partial A}{\partial t} + [\phi, A] = \frac{\partial \phi}{\partial z} + \eta \nabla_\perp^2 A$$

$\mathbf{B} = \hat{\mathbf{z}} + \mathbf{B}_\perp = \hat{\mathbf{z}} + \nabla_\perp A \times \hat{\mathbf{z}}$ --- magnetic field,

$\mathbf{v} = \nabla_\perp \phi \times \hat{\mathbf{z}}$ --- fluid velocity,

$\Omega = -\nabla_\perp^2 \phi$ --- vorticity, $\quad J = -\nabla_\perp^2 A$ --- current density,

η --- resistivity, ν --- viscosity,

$[\phi, A] \equiv \phi_y A_x - \phi_x A_y$

Reduced MHD equations -- 2D [Ng & Bhattacharjee 2008]

$$\frac{\partial \phi}{\partial t} = \frac{\partial A}{\partial z} + \nu \frac{\partial^2 \phi}{\partial x^2}$$

$$\frac{\partial A}{\partial t} = \frac{\partial \phi}{\partial z} + \eta \frac{\partial^2 A}{\partial x^2}$$

Only one transverse coordinate (x) so that nonlinear terms are identically zero. Can be used if nonlinear dynamics such as instability and reconnection is excluded in consideration.

Boundary conditions

Periodic boundary condition in x:

$$\phi(x,z,t) = \sum_{n=-\infty}^{\infty} \phi_n(z,t) e^{i2n\pi x}$$

Line-tied boundary condition in $z = 0$ and $z = L$:

$$\phi_n(0,t) = 0$$

$$\phi_n(L,t) = \phi_{nL}(t)$$

For a random footpoint driving, $\phi_{nL}(t)$ has a coherence time τ_{coh} .

Constant drive --- exact solution

Constant $\phi_{nL}(t)$, or $\tau_{\text{coh}} \to \infty$, in the saturation state:

$$\phi_n(z,t) = \phi_{nL} \frac{\sinh\left(\sqrt{\eta\nu}\, k_n^2 z\right)}{\sinh\left(\sqrt{\eta\nu}\, k_n^2 L\right)}$$

$$A_n(z,t) = \phi_{nL} \sqrt{\frac{\nu}{\eta}} \frac{\cosh\left(\sqrt{\eta\nu}\, k_n^2 z\right)}{\sinh\left(\sqrt{\eta\nu}\, k_n^2 L\right)}$$

where $k_n = 2n\pi$.

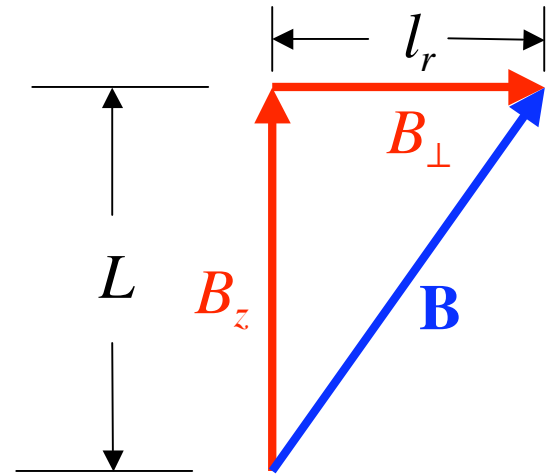
For small resistivity, $\sqrt{\eta\nu}\, k_n^2 L << 1$,

$$A_n(z,t) \xrightarrow{\eta \to 0} \frac{\phi_{nL}}{\eta k_n^2 L}$$

$$W_d = \eta \int J^2 d^3x \to \frac{1}{\eta L} \sum_{n=-\infty}^{\infty} \left|\phi_{nL}\right|^2$$

Constant drive --- physical picture

$$\frac{B_\perp}{B_z} \sim \frac{v_L \tau_r}{L} \equiv \frac{l_r}{L} = \frac{v_L w^2}{L\eta} >> 1$$



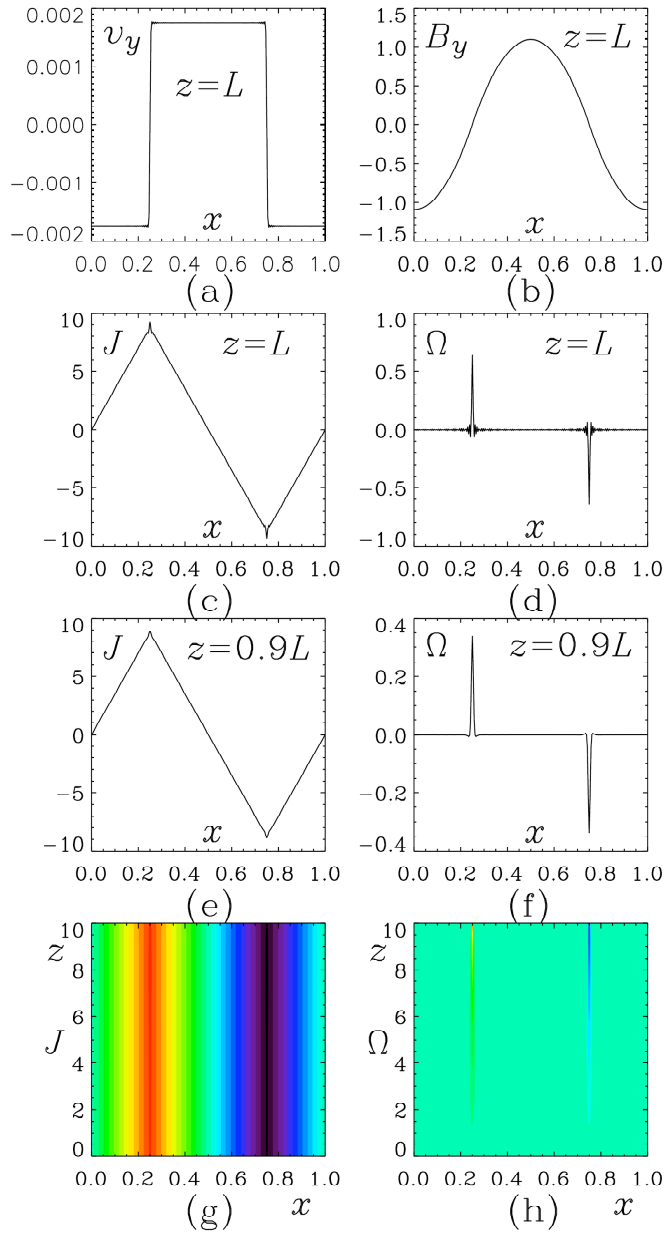
where l_r is the distance a photospheric footpoint move in a resistive time $\tau_r \sim w^2/\eta$.

Unphysically large heating rate:

$$\langle H \rangle \sim \frac{W_d}{w^2} \sim B_z^2 v_L \frac{l_r}{L} \propto \eta^{-1}$$

Serve as a reference case for the theory and simulation.

Constant drive --- simulation



Saturated state for a run with constant boundary flow for $\eta = 5\times10^{-6}$, $\nu = 10^{-5}$, $L = 10$. (a) Boundary flow velocity $v_y(x,L)$. (b) Transverse magnetic field $B_y(x,L)$. (c) Current density $J(x,L)$. (d) Vorticity $\Omega(x,L)$. (e) Current density $J(x,0.9L)$. (f) Vorticity $\Omega(x,0.9L)$. (g) Contour plot of the current density $J(x,z)$ using a rainbow color scale: black/dark purple for most negative contours and red for most positive contours. (h) Contour plot of the vorticity $\Omega(x,y)$.

Random drive

$$\phi_L(x,t) = \phi_0(t)\sum_{n=1}^{N}\frac{(-1)^n}{(2n-1)^2}\sin\left[(2n-1)2\pi x\right]$$

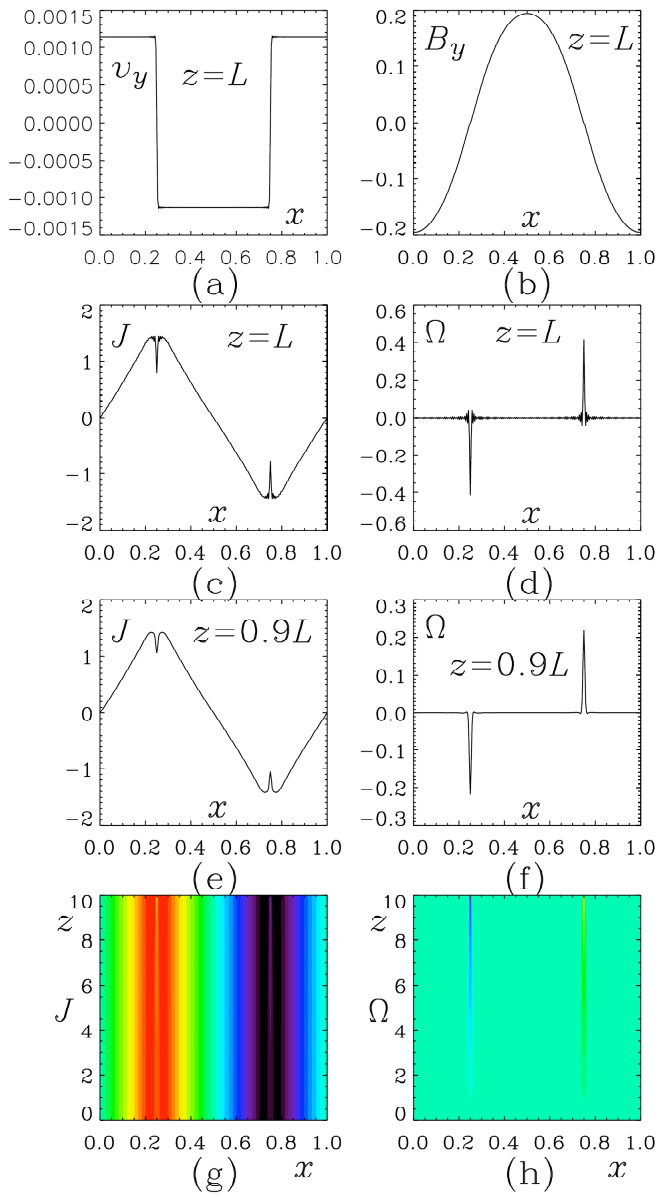
$$\phi_0(t) = \overline{\phi}_0\cos[\theta(t)]$$

$$\theta(t+\Delta t) = \theta(t) + \pi\sqrt{\Delta t/\tau_{\text{coh}}}\,\text{rand}(-1,1)$$

where $\overline{\phi}_0$ is small enough such that

$$\left|v_y\right| = \left|-\partial\phi/\partial x\right| << 1$$

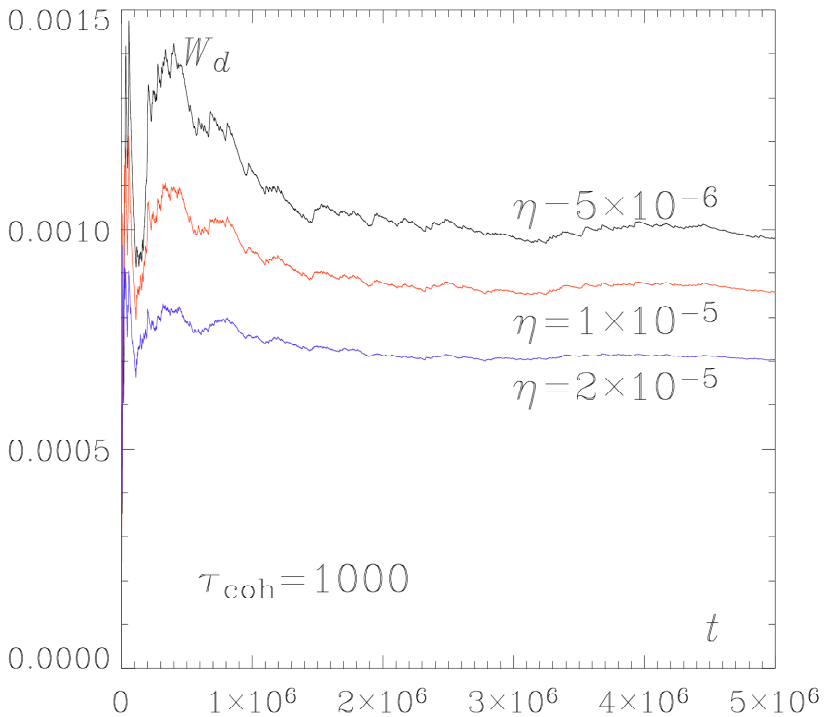
Random drive --- typical fields



Plots corresponding to the above figure for a run using random boundary flow with coherence time $\tau_{\text{coh}} = 1000$, at a time in a statistical state when the solution fluctuates around a certain average level. Other parameters are the same.

Random drive --- heating rate

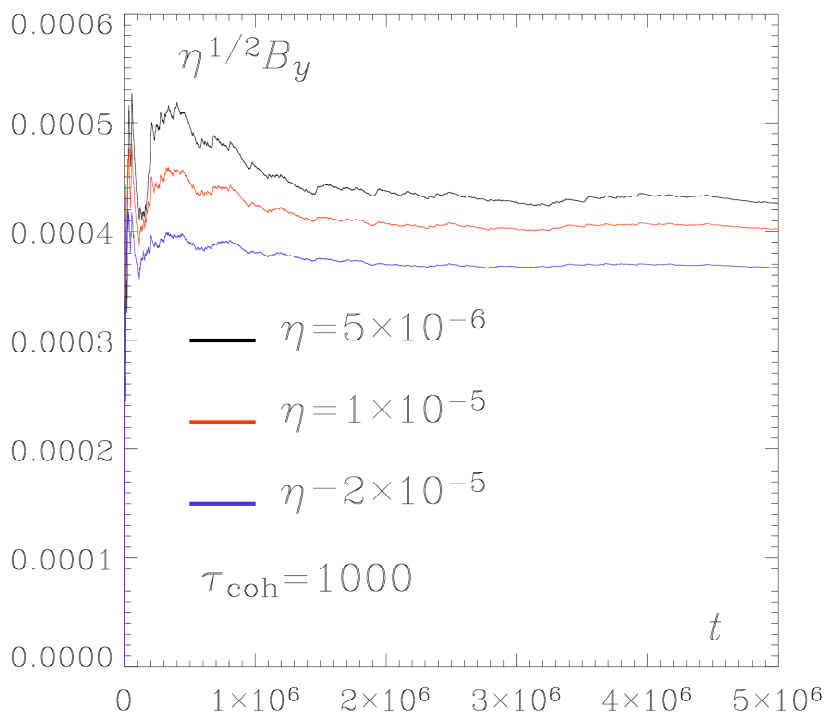
$$\overline{W}_d(t) \equiv \frac{1}{t}\int_0^t W_d(t')dt' = \frac{1}{t}\int_0^t \int \left[\eta J^2(\mathbf{x},t') + \nu\Omega^2(\mathbf{x},t')\right] d^3x dt'$$



The average energy dissipation rate $\overline{W}_d$ as a function of time for the case shown in above. Also plotted are $\overline{W}_d$ for the case with $\eta = 10^{-5}$ (red trace), and with $\eta = 2\times10^{-5}$ (blue trance). Other parameters are the same as the first case.

Average heating rate decreases when η increases. Dependence less than $1/\eta$, but very different from $\eta^{1/2}$.

Random drive --- transverse field production



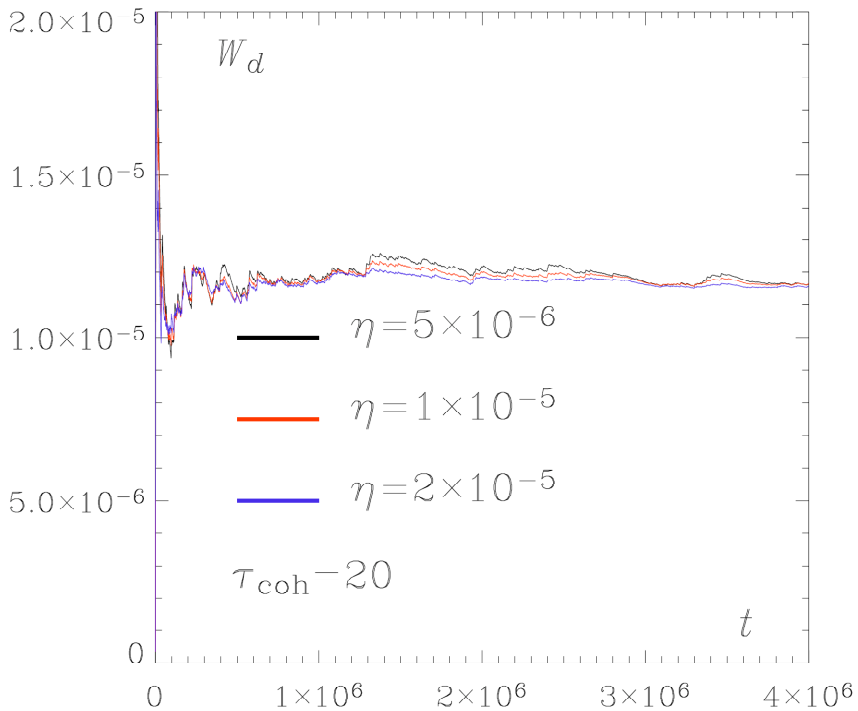
$$\overline{B}_y(t) \equiv \left[\frac{1}{t}\int_0^t \int B_y^2(\mathbf{x},t')d^2x dt'\right]^{1/2}$$

Root mean square $\overline{B}_y$ times $\eta^{1/2}$ as a function of time in the same runs as in the above figure.

$\overline{B}_y$ has roughly a $\eta^{-1/2}$ dependence.

Random drive --- heating rate/small τ_{coh}

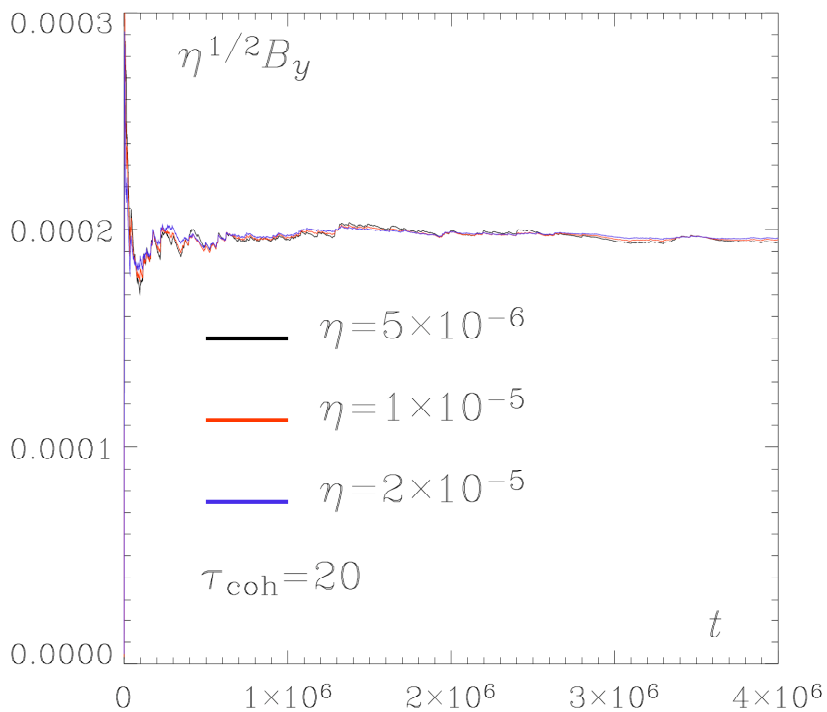
$$\overline{W}_d(t) \equiv \frac{1}{t}\int_0^t W_d(t')dt' = \frac{1}{t}\int_0^t \int \left[\eta J^2(\mathbf{x},t') + \nu\Omega^2(\mathbf{x},t')\right] d^3x dt'$$



Average energy dissipation rate $\overline{W}_d$ as a function of time for the case with $\tau_{\mathrm{coh}} = 20$, $\eta = 5\times10^{-6}$ (black trace). Also plotted are $\overline{W}_d$ for the case with $\eta = 10^{-5}$ (red trace), and with $\eta = 2\times10^{-5}$ (blue trace).

Average heating rate almost independent of η.

Random drive --- transverse B /small τ_{coh}



$$\overline{B}_y(t) \equiv \left[\frac{1}{t}\int_0^t \int B_y^2(\mathbf{x},t')d^2x dt'\right]^{1/2}$$

Root mean square $\overline{B}_y$ times $\eta^{1/2}$ as a function of time in the same run as in the above figure.

$\overline{B}_y$ has almost a $\eta^{-1/2}$ dependence.

Random drive --- physical picture

$$\frac{\overline{B}_y}{B_z} \sim \frac{l_c}{L} \sim \frac{v_L}{L}\sqrt{\tau_{\rm coh}\tau_r} \sim \frac{v_L}{L}\sqrt{\frac{\tau_{\rm coh}\overline{w}^2}{\eta}} >> 1$$

where $l_c = v_L\sqrt{\tau_r\tau_{\rm coh}}$ is the statistically expected distance moved by a footpoint in a random walk motion in a resistive time $\tau_r \sim w^2/\eta$.

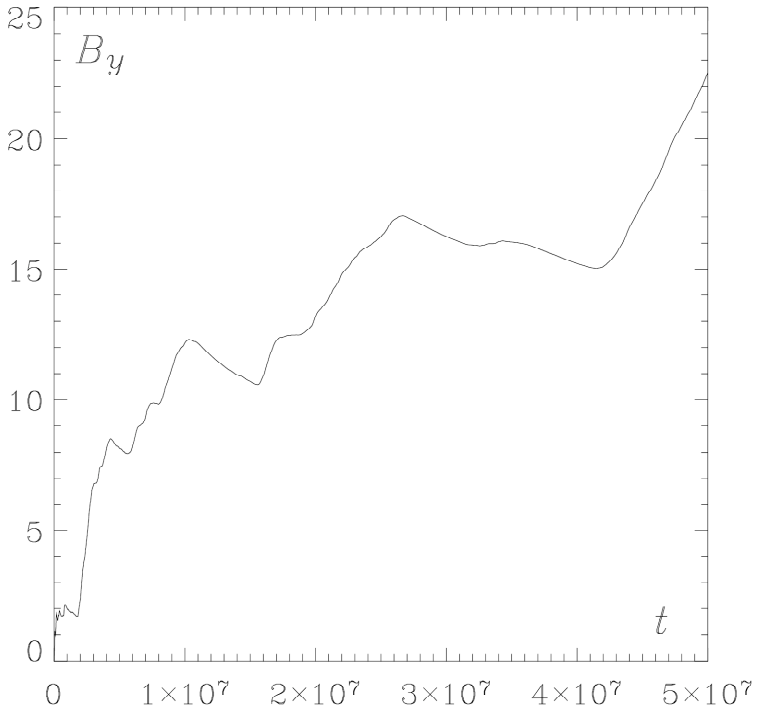
Heating rate

$$\overline{W}_d \sim \eta\int \overline{J}^2 d^3x \sim \eta\overline{B}_y^2(Lw^2)/w^2 \sim \frac{v_L^2}{L}B_z^2\tau_{\rm coh}w^2$$

so that $\langle H\rangle \sim \overline{W}_d/w^2$ is independent of η.

If $w \sim v_L\tau_{\rm coh}$, $\langle H\rangle \sim B_z^2 v_L w/L$, which is slightly smaller than what is required for coronal heating, unless $w/L \sim O(1)$.

Random drive --- very small η



Root mean square $\overline{B}_y$, as functions of time for the case with $\eta = 10^{-10}$, $\tau_{\text{coh}} = 600$.

$\overline{B}_y$ can get unphysically large for a small η.

Physically, the growth of B_y is limited by processes such as instabilities and reconnection.

Heating rate if growth of B_y is limited

If B_y is limited to $\overline{B}_y = fB_z$ in a time $t \sim \tau_E$

$$f \equiv \frac{\overline{B}_y}{B_z} \sim \frac{v_L}{L}\sqrt{\tau_{\text{coh}}\tau_E}$$

or

$$\tau_E \sim \left(\frac{fL}{v_L}\right)^2 \frac{1}{\tau_{\text{coh}}}$$

Heating rate:

$$\overline{W}_d \sim \frac{1}{\tau_E}\int \overline{B}_y^2 d^3x \sim \left(\frac{v_L}{fL}\right)^2 \tau_{\text{coh}}\left(fB_z\right)^2 Lw^2$$

$$\sim \frac{v_L^2}{L} B_z^2 \tau_{\text{coh}} w^2$$

is the same as before and is independent of η and f.

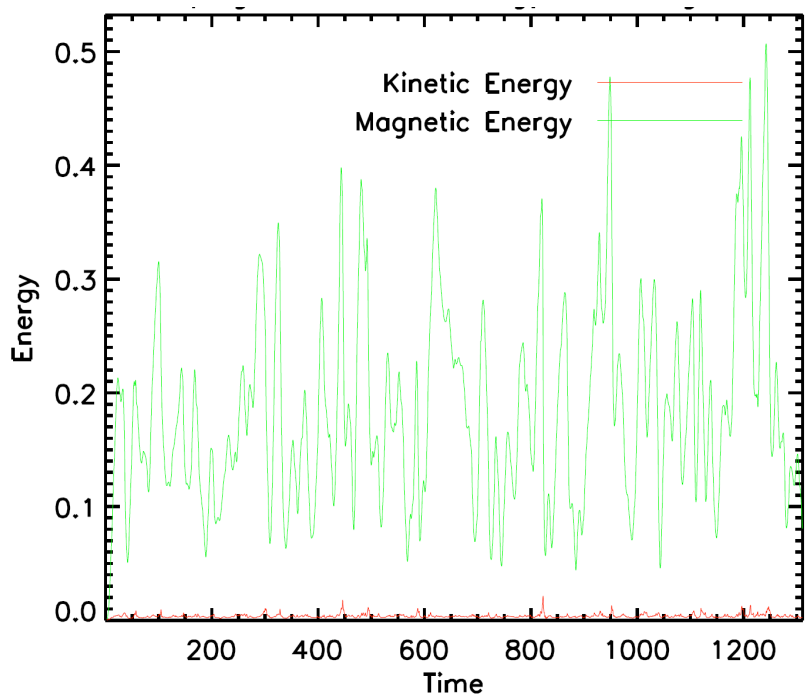
If $w \sim v_L\tau_{\text{coh}}$ with $w/L \sim O(1)$, $W_d \sim$ observed coronal heating rate.

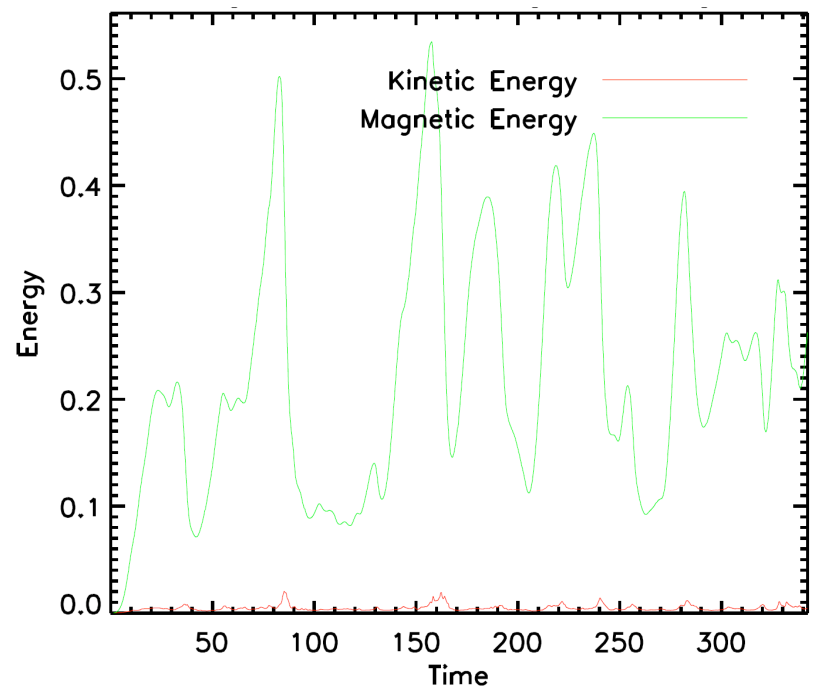
Random drive in 3D RMHD

- Method similar to Longcope (1992) or Longcope & Sudan (1994), but with higher resolutions. Their maximum resolution is 32x32x10, while we have run up to 512x512x32. Therefore we can study scaling laws down to a lower limit of η, and they shows different asympotic behaviours as obtained by their previous results.
- Rappazzo et al. (2007, 2008) has published simulations of 3D RMHD for the coronal heating problem also. However, they used a boundary photospheric motion that is constant in time, while we use random slow footpoint motion. Similar to our 2D calculations above, constant driving will produce stronger dissipation and thus they claimed to have found turbulence cascade in their simulations. We however don't see turbulence as a major factor within our simulations since the system is in quasi-equailibrium most of the time.
- The results of heating rate independent of resistivity have also been seen in full 3D MHD simulations [Galsgaard and Nordlund 1996, Gudiksen & Nordlund, 2002, 2005]. What we try to do here is to try to integrate a simplier model (RMHD) for a much longer time to get good time-average heating rate, and to establish scaling with resistivity.

Random drive in 3D RMHD

Magnetic energy limited by disruptions.



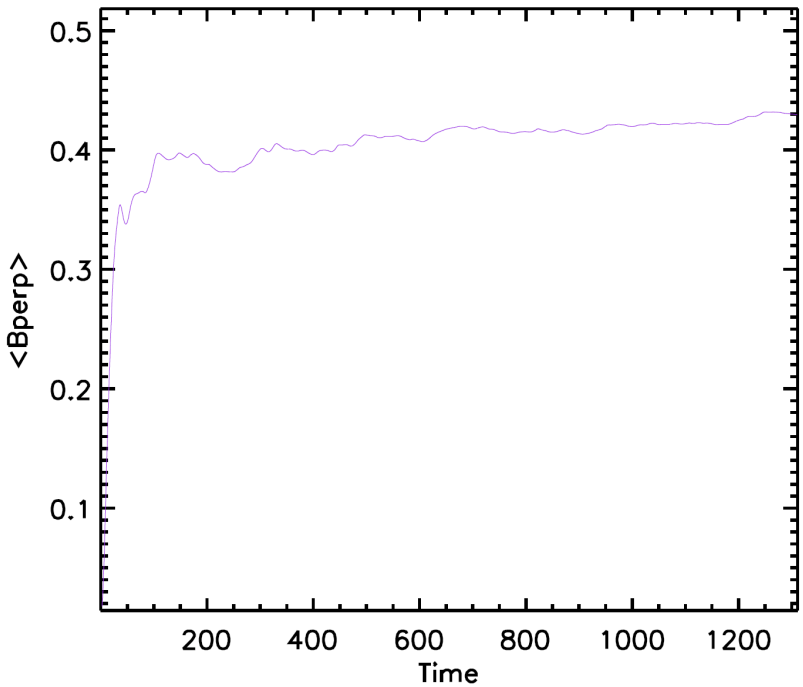


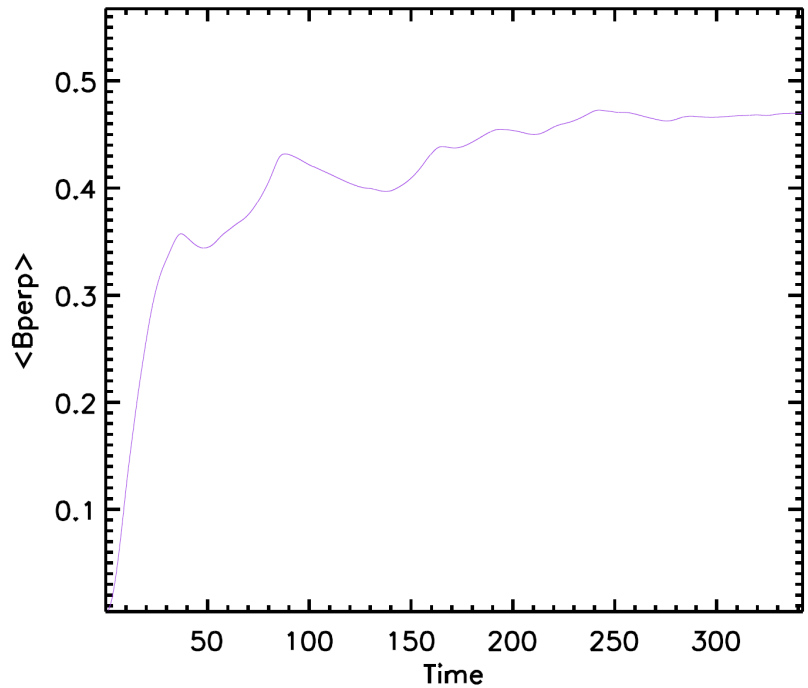
$\eta = \nu = 0.000625$ (64x64x16)

$\eta = 0.0003125$, $\nu = 0.000625$ (256x256x32)

Random drive in 3D RMHD

Average $\overline{B}_y$ saturated in time.



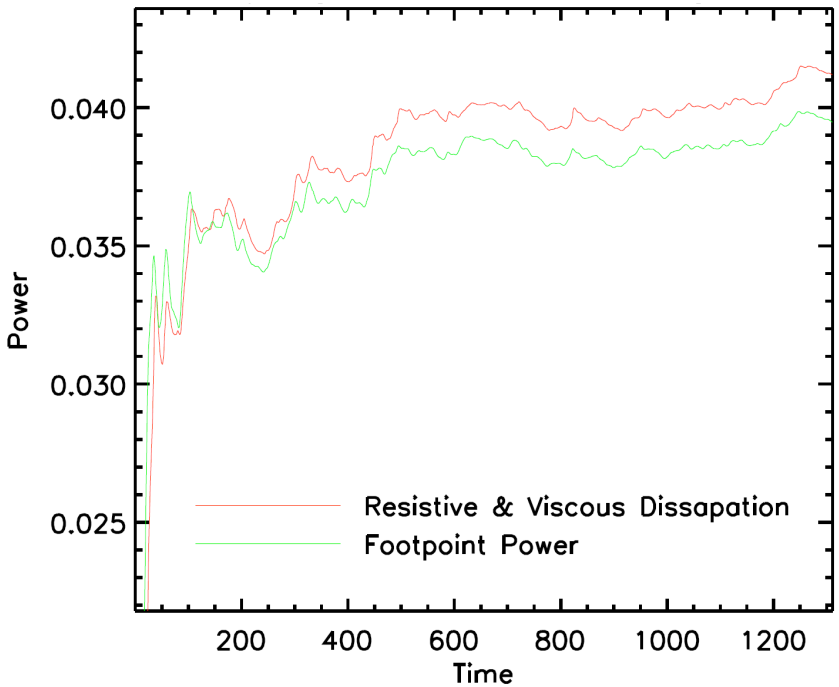


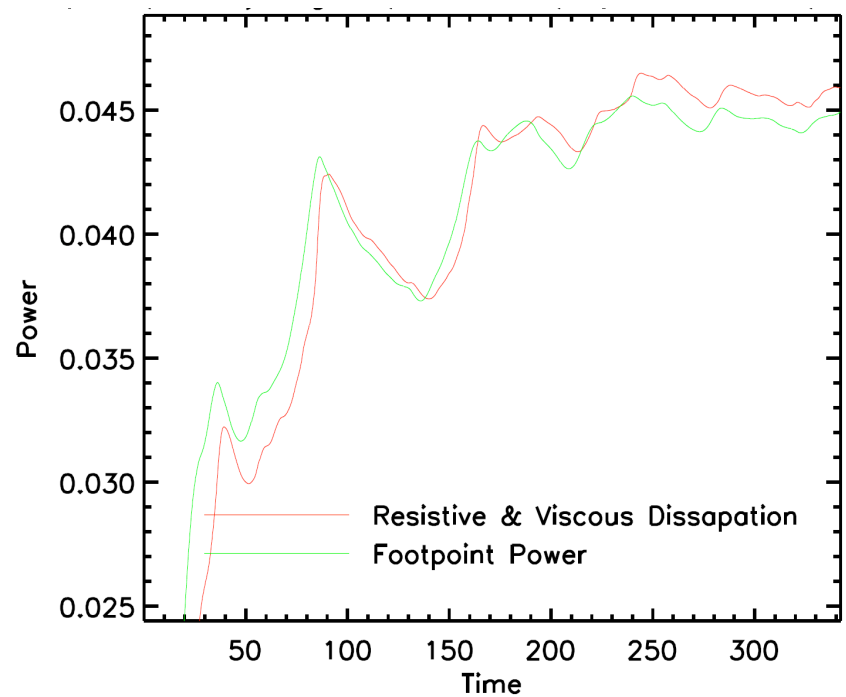
$\eta = \nu = 0.000625$ (64x64x16)

$\eta = 0.0003125$, $\nu = 0.000625$ (256x256x32)

Random drive in 3D RMHD

Energy dissipation rate saturated in time.



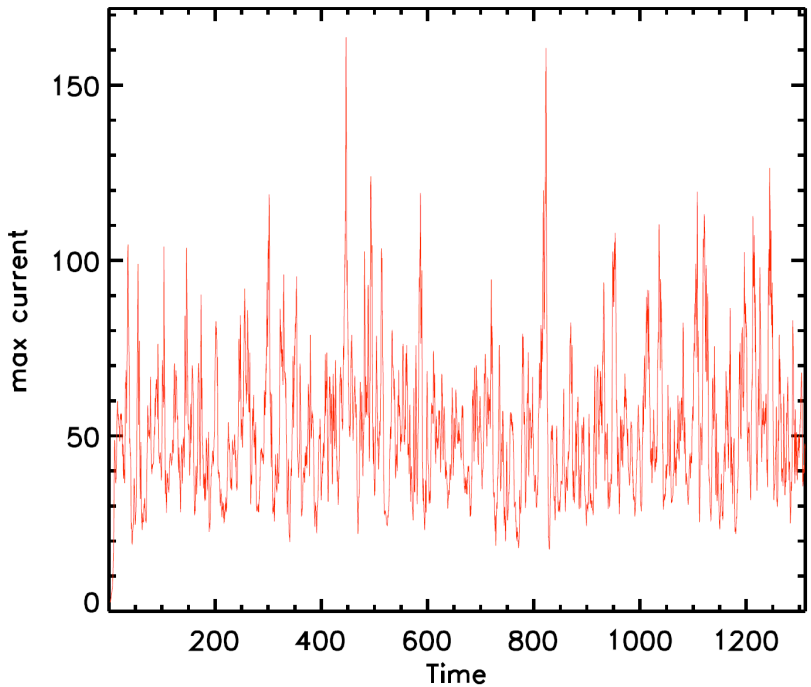


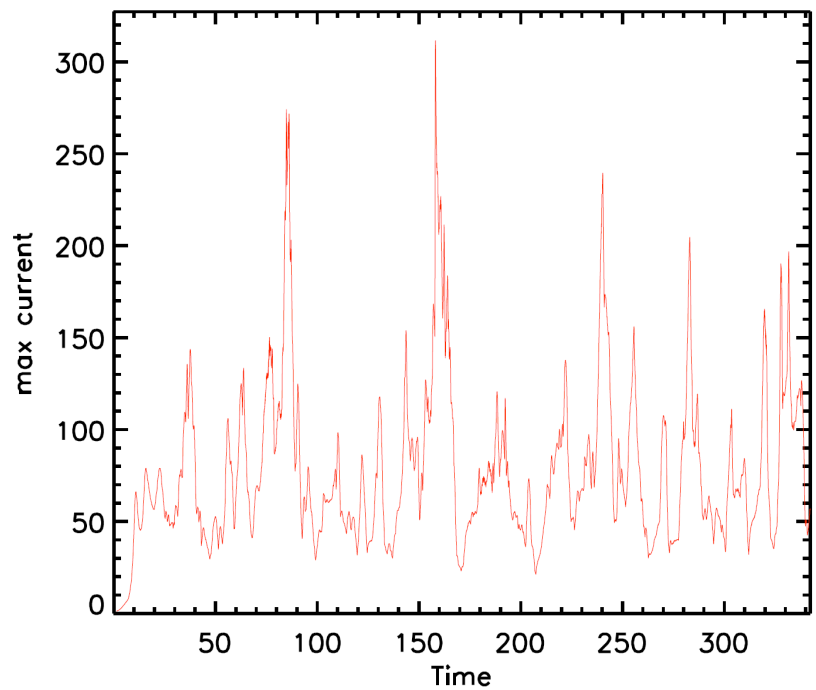
$\eta = \nu = 0.000625$ (64x64x16)

$\eta = 0.0003125$, $\nu = 0.000625$ (256x256x32)

Random drive in 3D RMHD

$J_{\max}$ larger for smaller η.



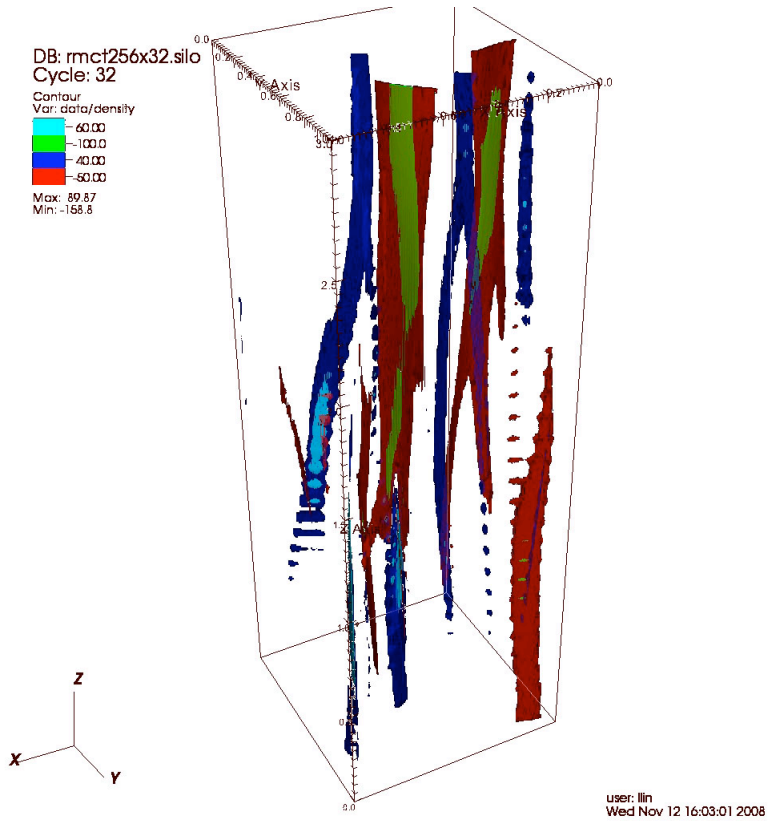


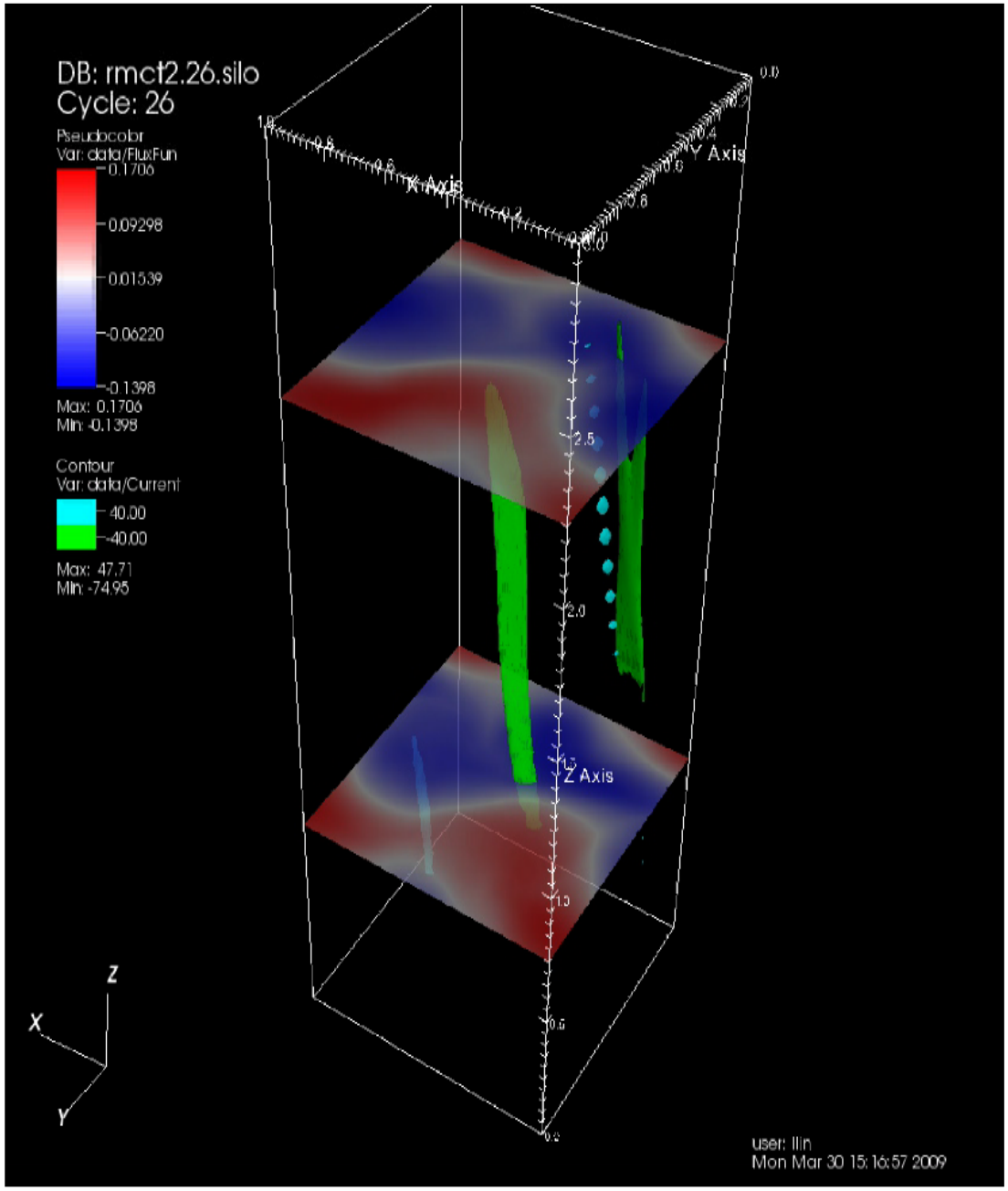
$\eta = \nu = 0.000625$ (64x64x16)

$\eta = 0.0003125$, $\nu = 0.000625$ (256x256x32)

Random drive in 3D RMHD

Formation of thin current layers.



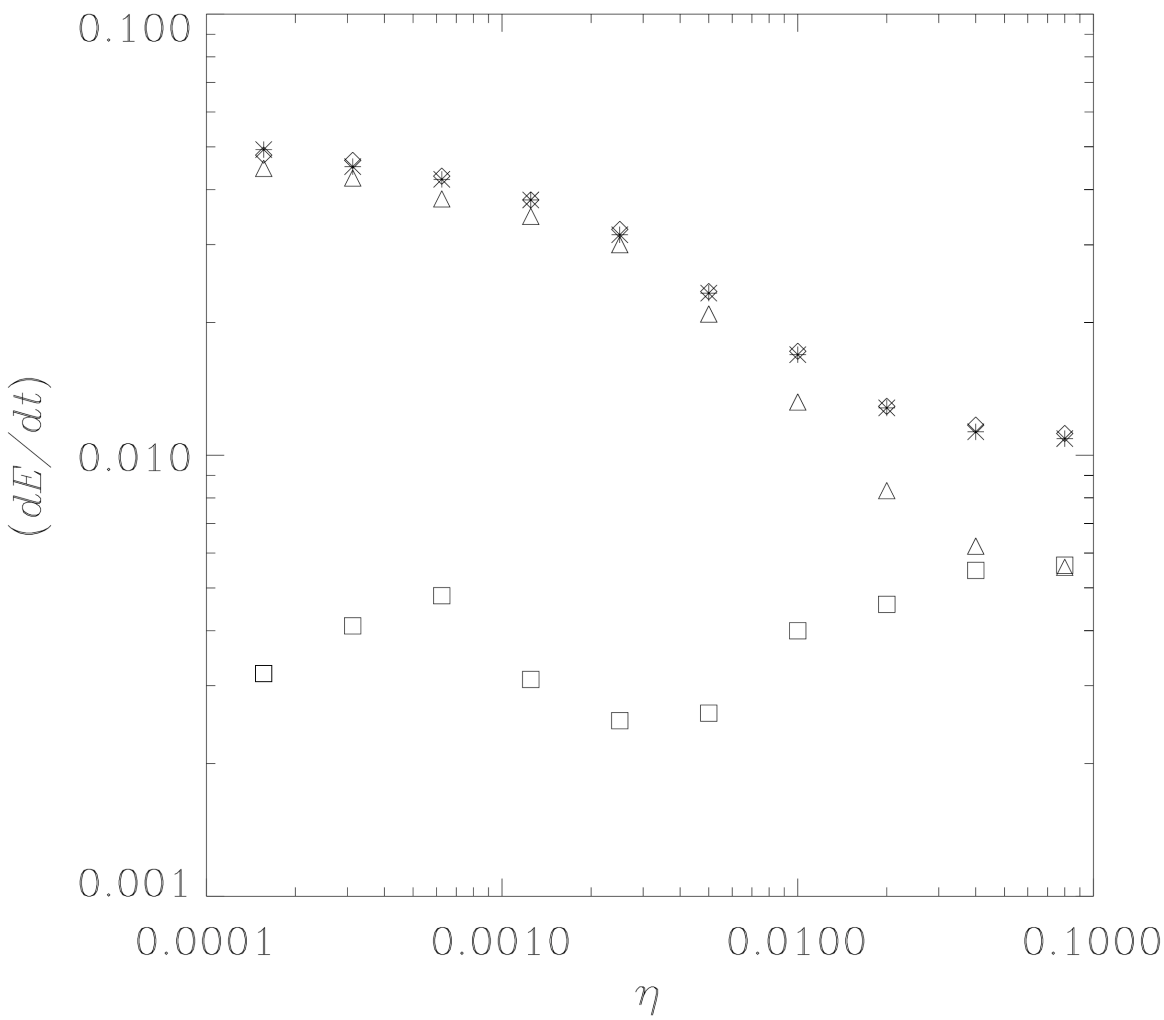


$\eta = \nu = 0.00015625$ (256x256x32)

current layers lead to energy disruptions

Random drive in 3D RMHD

Average energy dissipation rate saturated in η.



Δ : Ohmic dissipation

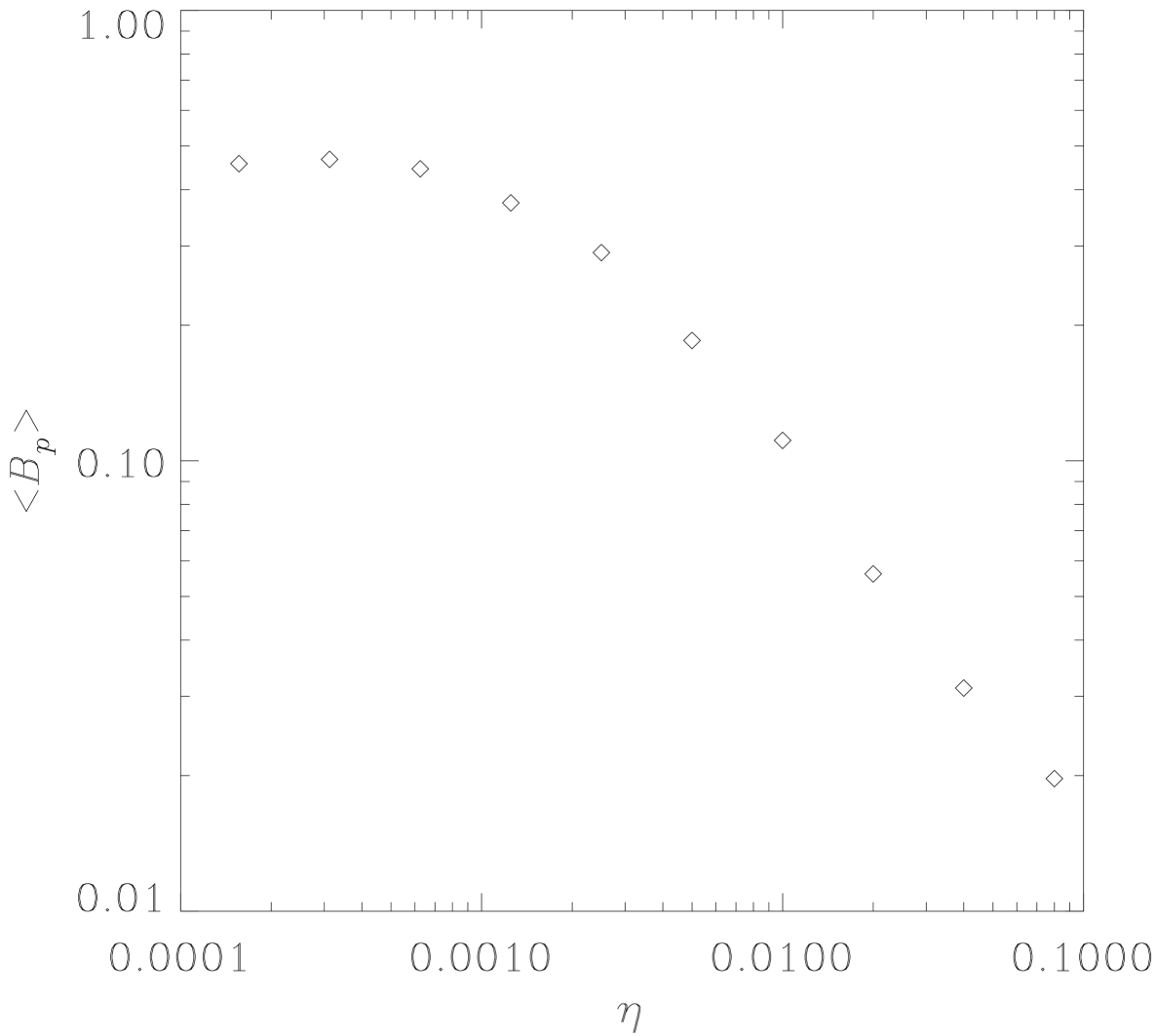
$\square$: viscous dissipation

$\ast$: total (Ohmic + viscous) dissipation

$\diamond$: Poynting flux (footpoint power)

Random drive in 3D RMHD

Average $\overline{B}_y$ saturated in η.

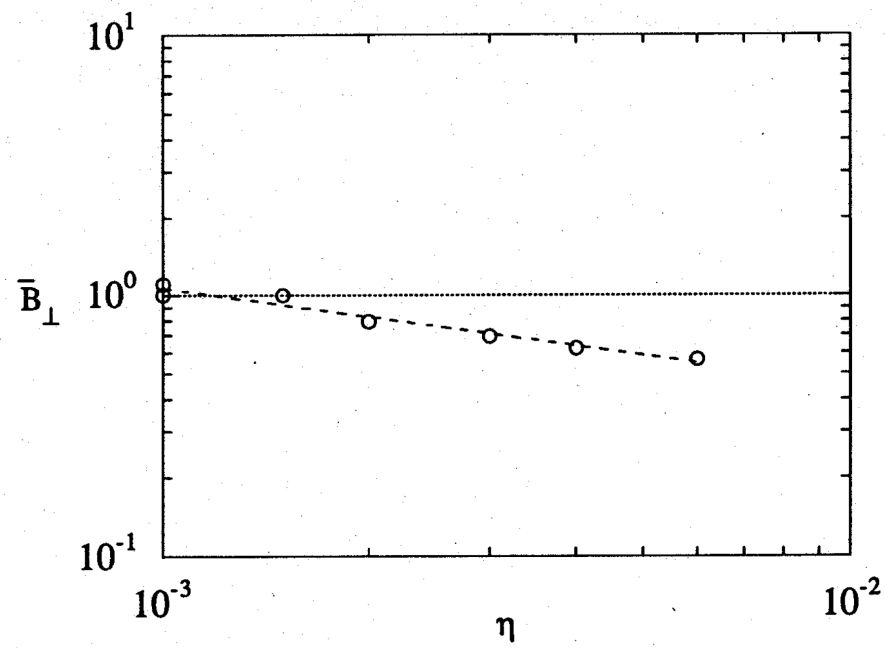


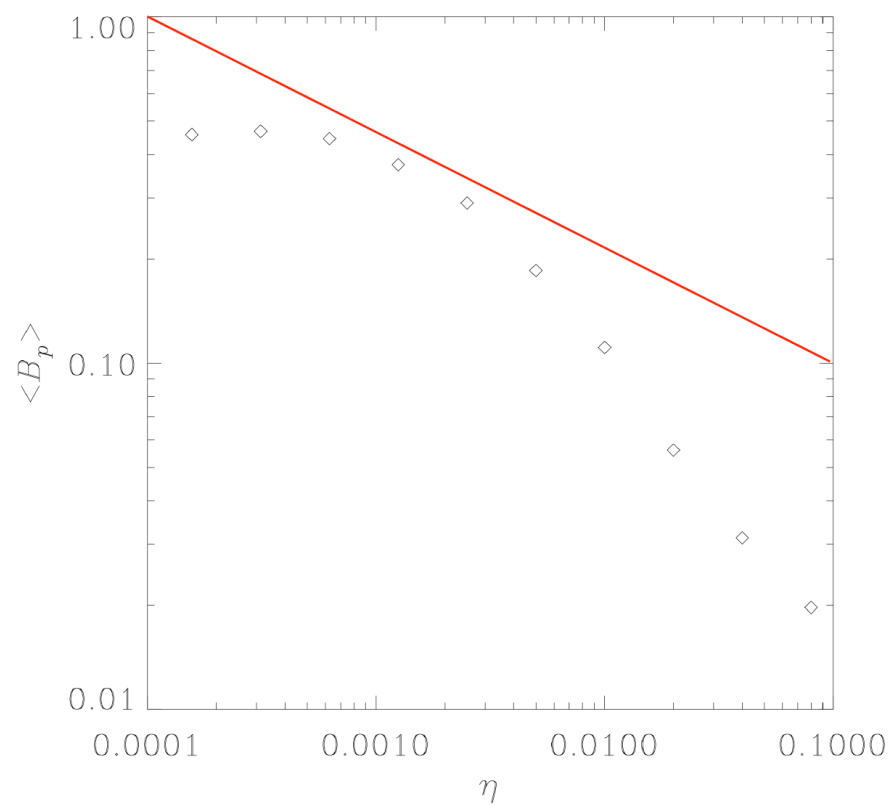
Note that $B_z = 1$.

Compare with previous scaling results

Longcope (1992) or Longcope & Sudan (1994) found:

$$\bar{B}_\perp \sim \{ l_F (N\tau_E)^{-1} \Delta^{-1/2} \}^{2/3} \, \eta^{-1/3}$$

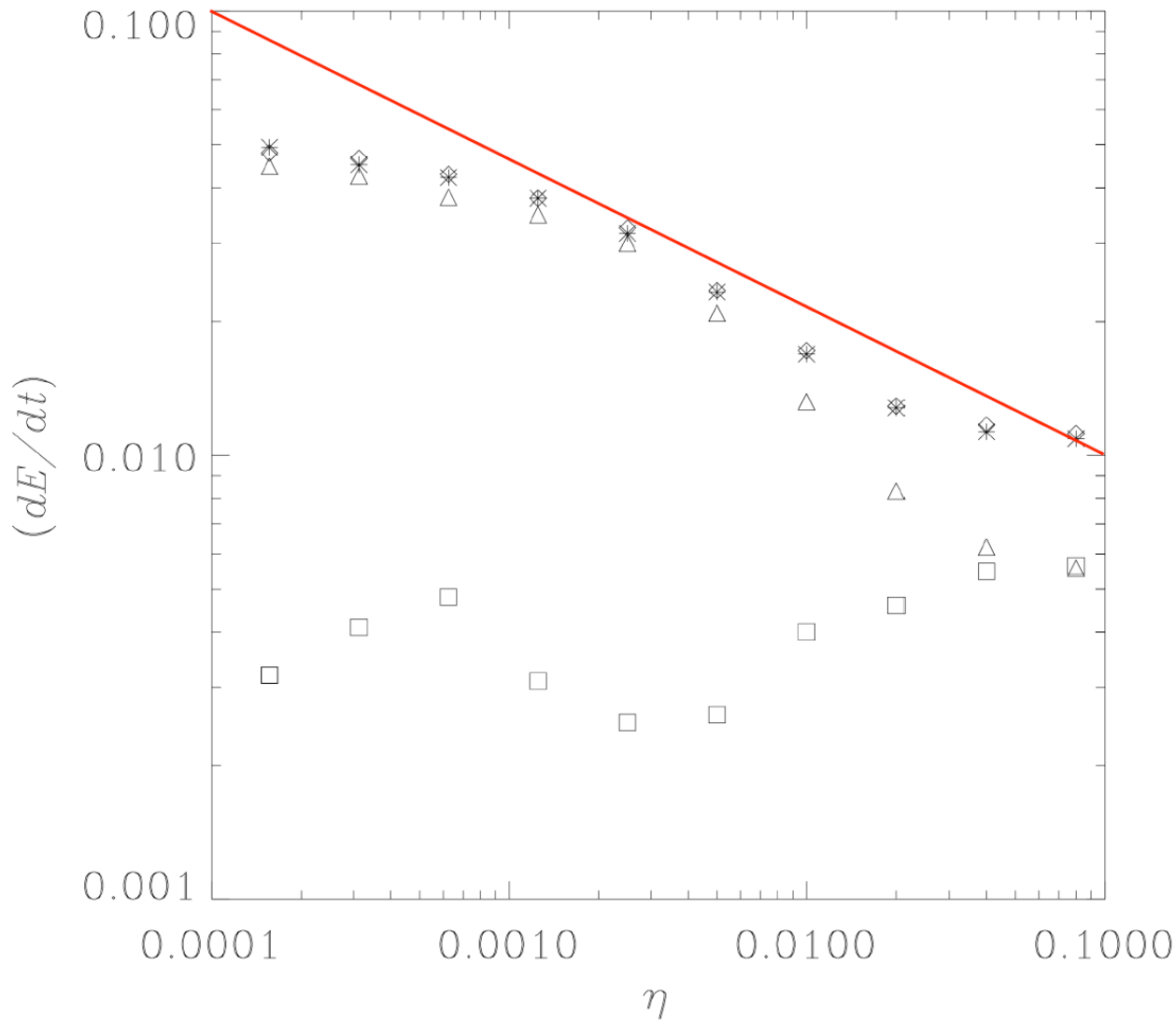




Compare with previous scaling results

Longcope (1992) or Longcope & Sudan (1994) found:

$$\langle P_F \rangle \sim v_F \overline{B}_\perp \propto \eta^{-1/3}$$



Conclusion

- A tectonics model of coronal heating in the magnetic carpet [*Priest, Heyvaerts and Title*, 2002] is considered using 2D and 3D RMHD simulations.
- It is shown numerically and by scaling analysis that for a <u>random</u> footpoint driving, the heating rate $<H>$ is independent of η. B_y is proportional to $1/\eta^{1/2}$ when τ_{coh} is much less than $\tau_{r.}$ in 2D if energy built up is not limited
- In realty, the growth of B_y would be limited by instabilities or reconnection. Thus, this is a process producing B_y for eventual dissipation by other processes. It is shown by scaling analysis that $<H>$ is independent of η as well as the saturation level.
- The saturation of $<H>$ and B_y in η, as well as the formation of current sheet is shown to be consistent with numerical simulations in 3D. These scaling behaviors are different from previous results.
- The main numerical challenge in this study is the need to obtain high-resolution results and to simulate for a very long time (long compared with the slow time-scale of the random boundary flow, which has to be much longer than the Alfvén time) to have good statistics for the scalings.