Simulations of the magneto-rotational instability in core-collapse supernovae

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Progenitor and collapse

- Progenitor: a massive star ($\gtrsim 8 M_\odot$) after exhaustion of nuclear full (onion-shell structure)
- gravitational collapse of the core to a proto neutron star: ρ_{max} increases from $\sim 10^9$ g/cm^3 to $> \rho_{\text{nuc}} \sim 2 \times 10^{14}$ g/cm^3 within $\sim$ a free-fall time
- $e_{\text{core}} \sim 10^{53}$ erg relased, mostly in neutrinos
- collapse stops when nuclear density is reached
  $\Rightarrow$ formation of a shock wave
- the shock propagates outwards, but stalls due to energy loss in dissociation reactions

¿ How is the stalled shock wave revived?

Ingredients

multi-scale problem

- star: blue or red giant
- pre-collapse core: few 1000 km
- PNS: few 10 km
- stalled shock: few 100 km
- large (magnetic) Reynolds number
- many dynamical time scales

multi-physics problem

- multi-dimensional (GR)(M)HD
- turbulence
- nuclear equation of state
- neutrino transport (from optically thick to transparent), neutrino-matter interactions
- nuclear burning

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Exlosion mechanisms

How is the failed explosion revived?

Not a matter of energy ($e_{\mathrm{core}} \gg e_{\mathrm{env}}$), but of energy transfer.

- Spherical neutrino-driven explosion
- Standard model: neutrino heating aided by hydrodynamic instabilities
- Energy transfer by waves
- rotational mechanisms

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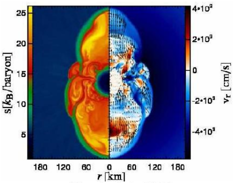

Marek et al, 2008

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Neutrino mechanism

- Neutrinos diffuse out of the PNS
- they heat the matter behind the shock.

$\Rightarrow$ explosions for cores in a limited mass range (Kitaura et al., 2006)

- compatible with standard pre-collapse evolution

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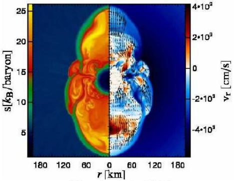

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Hydro instabilities

- Neutrino heating
- convection and standing accretion shock instability (Blondin et al., 2003, 2006; Foglizzo et al., 2007)

$\Rightarrow$ successful for $M \approx 11 \ldots 15 M_{\odot}$

- compatible with standard pre-collapse evolution

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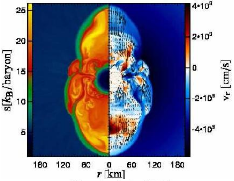

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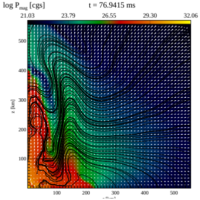
Waves

- accoustic (Burrows et al., 2006, 2007) or Alfvén waves (Suzuki et al., 2008) generated at the PNS
- waves dissipate near the shock
- successful?
- compatible with standard pre-collapse evolution?

Exlosion mechanisms



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Rotation

- tap into e_{rot} by magnetic fields (Thompson et al., 2004)
- successful?
- realistic?
  - rapid rotation: only certain stars
  - $|\vec{b}|$ sufficiently strong?

$\longrightarrow$ MRI? (Akiyama et al., 2003)

Field amplification in supernovae

Why magnetic fields?

- pulsar fields, magnetars
- asymmetric explosions: caused by large-scale fields?
- additional energy reservoir: rotation

But...

- strong (equipartition) fields needed
- typical pre-collapse fields are too weak

$\Rightarrow$
- special class of progenitors
- strong amplification

field amplification mechanisms

- compression: gravitational infall $\Rightarrow$ magnetic energy
- winding: differential rotation $\Rightarrow$ magnetic energy
- hydromagnetic instabilities: differential rotation, entropy/composion gradients $\Rightarrow$ magnetic energy

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General properties of the MRI

- local linear MHD instability of differentially rotating fluids
- weak initial magnetic field required
- run-away of angular-momentum transport along field lines
- instability criterion: negative Ω gradient
- growth time $\sim$ rotational period
- leads to MHD turbulence and efficient transport

Peculiarties of the MRI in SNe

Accretion discs

- Keplerian shear
- ⇒ Rayleigh-stable, MRI-unstable
- rapid growth
- MHD turbulence may provide viscosity required for accretion
- well-studied system, yet still many open questions

Supernovae

- differential rotation, thermal stratification
- ⇒ possibly: hydrodynamically unstable + MRI unstable
- growth: fast enough?
- saturation: strong enough?
- starting to receive interest

study the MRI in supernovae

- theoretical analysis of the instability criteria
- simulations of MRI-unstable systems

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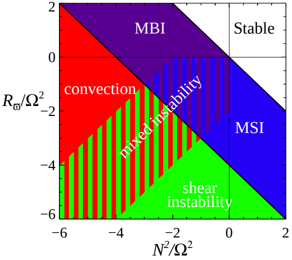
Regimes of the axisymmetric MRI

magneto-bouyant convection stabilised by rotation, but destabilised by the magnetic field

convective similar to hydrodynamic convection (Schwarzschild or Ledoux)



stable stabilised by positive entropy or Ω gradients

magneto-shear classical MRI, e.g., accretion discs

mixed interplay of many effects

shear regime Rayleigh unstable

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Physics and numerics

Simplified physics

- Full ideal MHD (rather than shearing box)
- simplified equation of state
- external gravity
- no neutrino transport

Code

- Eulerian, conservative
- high-order reconstruction (MP or WENO)
- MUSTA Riemann solver (Titarev & Toro, 2005)
- constraint transport

Models

- gas in hydrostatic equilibrium; uniform field or vanishing net flux
- axisymmetric and 3d simulations
- small (few kilometres) boxes resembling the equatorial region
- resolution between 0.625 and 40 metres
- shearing-disc boundary conditions (Klahr & Bodenheimer, 2003)

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Questions to be addressed

General problems

- dependence of the saturation on numerics: box size, resolution, boundaries
- dependence of the saturation on initial field strength and geometry

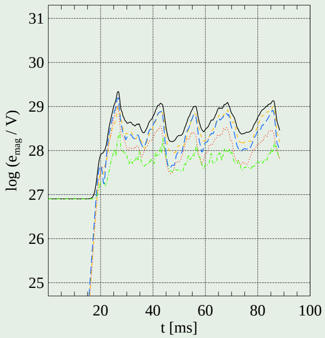
MRI in supernovae

- verify the regimes
- growth rates for typical SN conditions
- saturation level in SN cores

Many questions, but no final answers yet...

Dynamics

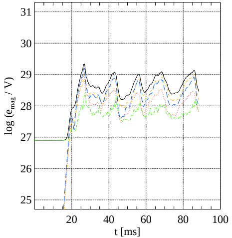
temporal evolution of the magnetic energy

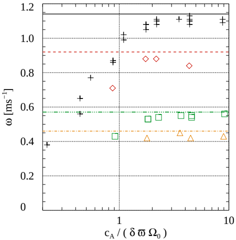


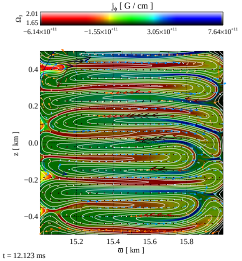
- confirm all (relevant) regimes of the linear analysis
- (de-)stabilisation by interplay of Ω and S gradients
- growth rates in agreement with linear analysis, i.e., a few milliseconds for rapidly rotating cores
- maximum field strength $\gtrsim 10^{15}$ G

Dynamics

- early phase: exponential growth of channel flows
- termination of growth and breakup of channels







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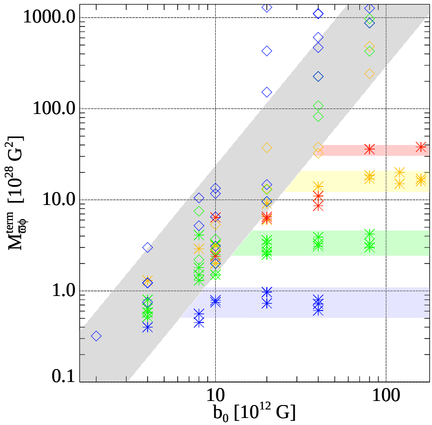
Scaling of the termination level

Termination ($\neq$ saturation)

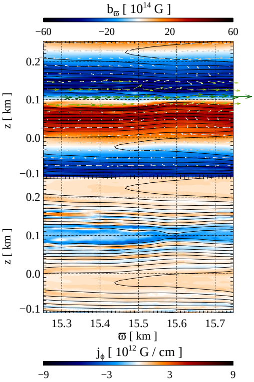
the Maxwell stress reached at the end of the growth of the MRI depends on (among other factors)

- the grid resolution: finer grid $\Rightarrow$ higher $M_{\varpi\phi}$
- the initial field: stronger b_0 $\Rightarrow$ higher $M_{\varpi\phi}$
- the rotational profile: slower $\Rightarrow$ higher $M_{\varpi\phi}$



Scaling of the termination level

- MRI growth terminates when channel flows are disrupted by resistive instabilities.
- Channels are generically unstable against secondary instabilities, here tearing modes (Goodman & Xu, 1994).
- MRI terminates approximately when the resistive instabilities grow faster than the MRI.



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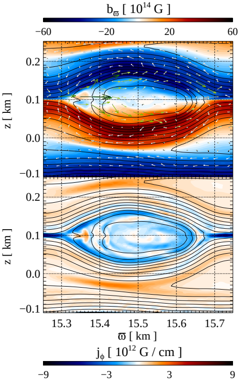

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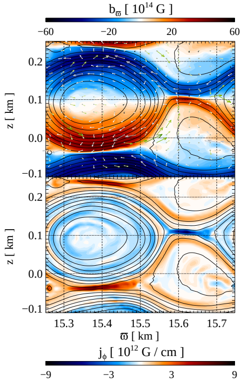

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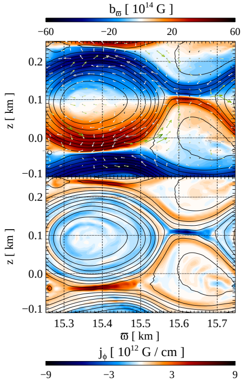

Scaling of the termination level

auxiliary simulations

simplified 2d models of channel flows to study resistive instabilities:

- wider channels $\Rightarrow$ tearing modes grow slower
- stronger fields $\Rightarrow$ tearing modes grow faster
- finer grids $\Rightarrow$ tearing modes slow slower

properties of channel flows

channel width determined from the MRI dispersion relation (fastest growing mode)

- weaker initial fields $\Rightarrow$ thinner
- slower rotation $\Rightarrow$ wider

field strength increases at MRI growth rate

growth rates of tearing modes & channel evolution

$\Rightarrow$ towards an explanation of the scaling of MRI termination

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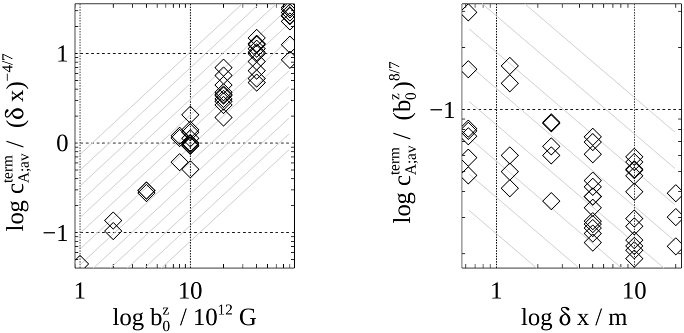

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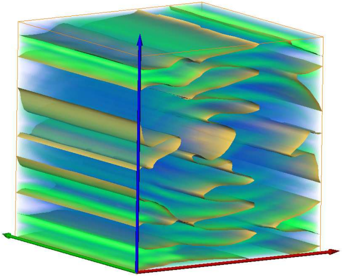
Saturation: turbulence and coherent flows

- Saturation: turbulent state
- efficient transport of angular momentum
- coherent flow and field patterns can be identified
- stable over several rotational periods
- example: average value of the toroidal field on slices $z =$ const. as a function of time (cf. Lesur & Ogilvie, 2008)



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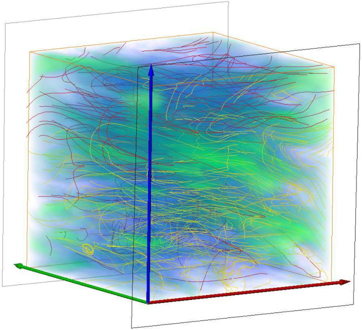

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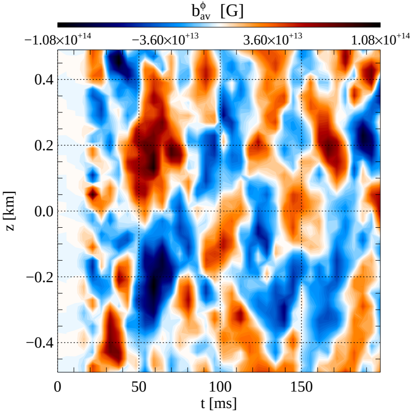

Summary

- Explosion mechanism of core-collapse supernovae involves a combination of neutrino transport, hydrodynamic instabilities, and possibly magnetic fields.
- Post-collapse cores are MRI-unstable.
- Starting to explore the MRI in core-collapse supernovae by
  - analysis of the dispersion relation
  - simplified simulations of model systems
- Results suggest possible importance of the MRI.

$\Rightarrow$ More accurate modelling required.