Relativistic particle transport in tangled magnetic fields

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Outline

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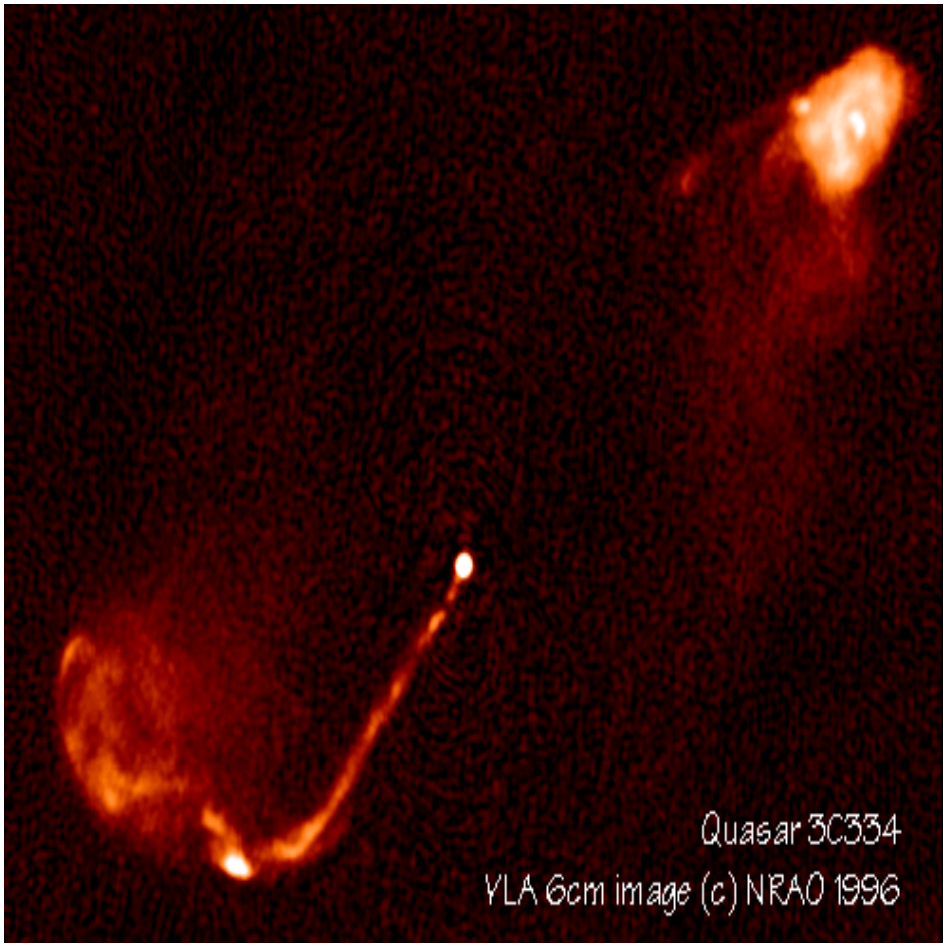
Motivation



3C334 VLA 6cm VLA

- Low and high frequency emission coincident
- Significant transverse extent in X-ray emitting particles
- Diffusion processes required in excess of classical efficiencies

Typical Parameters ($\rho \equiv r_{\rm g}/l_{\rm c}$)

γ	$\nu_{\rm c}$ GHz	$r_{\rm g}$ pc	ρ
10^3	0.028	5.5×10^{-8}	5.5×10^{-11}
10^5	280.0	5.5×10^{-6}	5.5×10^{-9}
10^7	2.8×10^6	5.5×10^{-4}	5.5×10^{-7}

Sample parameters relevant to radio galaxies, for an assumed magnetic field strength of 1 nT and correlation length 1 kpc

Quasilinear Theory

$\mathbf{B} = B_0\hat{\mathbf{z}} + \mathbf{B}_1$ where $b \equiv \left[\frac{<B_1^2>}{<B_0^2>}\right]^{1/2} \ll 1$

Two components to particle transport:

- Field line wandering => $\frac{<\Delta^2 x>}{2\Delta s} = D_M = \frac{b^2\lambda_{\parallel}^{corr}}{4}$
- Scattering => $\kappa_{\parallel} = \kappa_B/\epsilon \quad \kappa_{\perp} = \epsilon\kappa_B/(1+\epsilon)$

Transport regimes:

- Ballistic => $D_{\perp} \rightarrow \kappa_{\perp} + vD_M$
- Collisional ($\Lambda \lesssim 1$) => $D_{\perp} \rightarrow \kappa_{\perp}$ and $D_{\parallel} \rightarrow \kappa_{\parallel}$
- Compound ($\Lambda \gg 1$) => subdiffusive followed by diffusive

$$\kappa_B \equiv \frac{\gamma v^2 mc}{3eB}$$

$$\epsilon = \nu_{coll}/\omega_g$$

$$\Lambda \equiv \frac{b^2\lambda_{\parallel}^{corr}}{\sqrt{2}\epsilon\lambda_{\perp}^{corr}}$$

$$D_{\perp} \equiv \frac{<\Delta^2 x>}{2\Delta t}$$

$$D_{\parallel} \equiv \frac{<\Delta^2 z>}{2\Delta t}$$

See Duffy et al. A&A 1995

Outline

Particle path integration

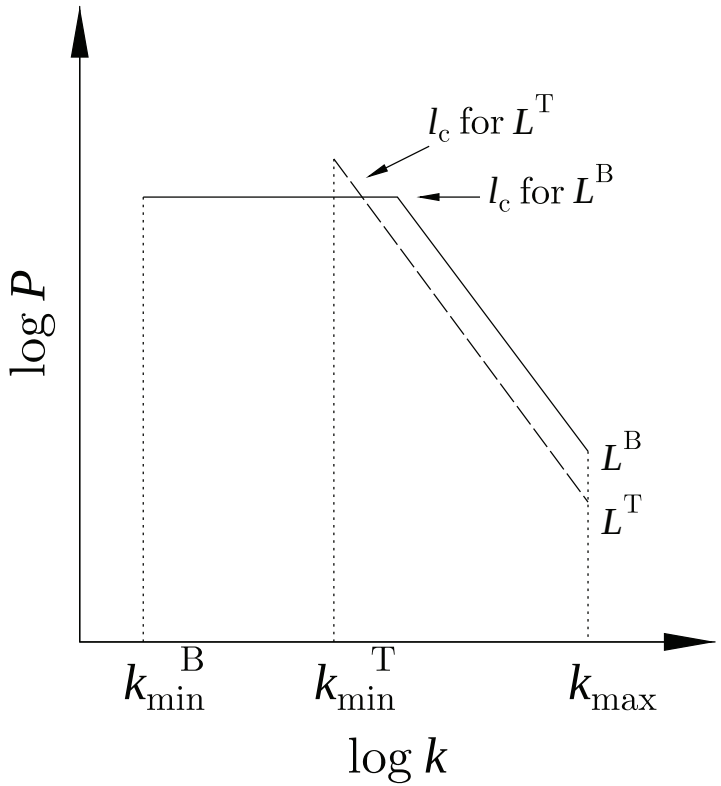
- Static **B** => energy conserved.
- Integration of $\gamma m_0 \frac{\mathrm{d}\mathbf{v}}{\mathrm{dt}} = Ze\left(\frac{\mathbf{v}}{c} \times \mathbf{B}\right)$ is problematic for high γ as the change in energy due to roundoff errors scales as γ^3.
- In terms of particle's four-velocity and proper-time, the Lorentz covariant equation of motion of a particle of charge e and rest mass m in a static magnetic field $\boldsymbol{B}(\boldsymbol{r})$ is

$$\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}\tau} = \frac{e}{m}\boldsymbol{u} \times \boldsymbol{B} \quad \boldsymbol{u} = \frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}\tau}$$

- Particle paths are obtained by integrating a Burlisch-Stoer routine.

B-field power-law models

- Random field with vanishing mean $\boldsymbol{B} = \int \mathrm{d}^3\boldsymbol{k}\, \boldsymbol{A}_{\boldsymbol{k}} \exp(\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{r})$
- $\nabla\cdot\boldsymbol{B} = 0$ requires $\boldsymbol{k}\cdot\boldsymbol{A}_{\boldsymbol{k}} = 0$
- $\boldsymbol{A}_{\boldsymbol{k}}$ statistically independent, field homogeneous and Gaussian $\langle \boldsymbol{A}_{\boldsymbol{k}}\cdot\boldsymbol{A}^*_{\boldsymbol{k}'}\rangle = P(\boldsymbol{k})\delta(\boldsymbol{k}-\boldsymbol{k}')$



B-field representation

$$A^2(k_n) = \sigma^2 G(k_n) \left[\sum_{n=1}^{N_m} G(k_n) \right]^{-1} \qquad G(k_n) = \frac{\Delta V_n}{1+(k_n \mathrm{L}_c)^{11/3}}$$

σ^2 = variance, L_c=correlation length

$$\boldsymbol{B}_1(\boldsymbol{r}) = \sum_{n=1}^{M} A_n \exp(\boldsymbol{k}_n \cdot \boldsymbol{r} + \beta_n) \hat{\boldsymbol{\xi}}_n$$

($\hat{\boldsymbol{\xi}} \cdot \hat{\boldsymbol{k}} = 0$ ensures solenoidal field)

B-field representation

Field can be represented numerically in one of two ways:

- Discrete mesh of dimension N interpolated to particle positions eg. Casse et al. PhysRevD 2002

$$\lambda_{min} = 2\Delta x \quad \lambda_{max} = N\Delta x \quad => \lambda_{max}/\lambda_{min} = 2/N$$

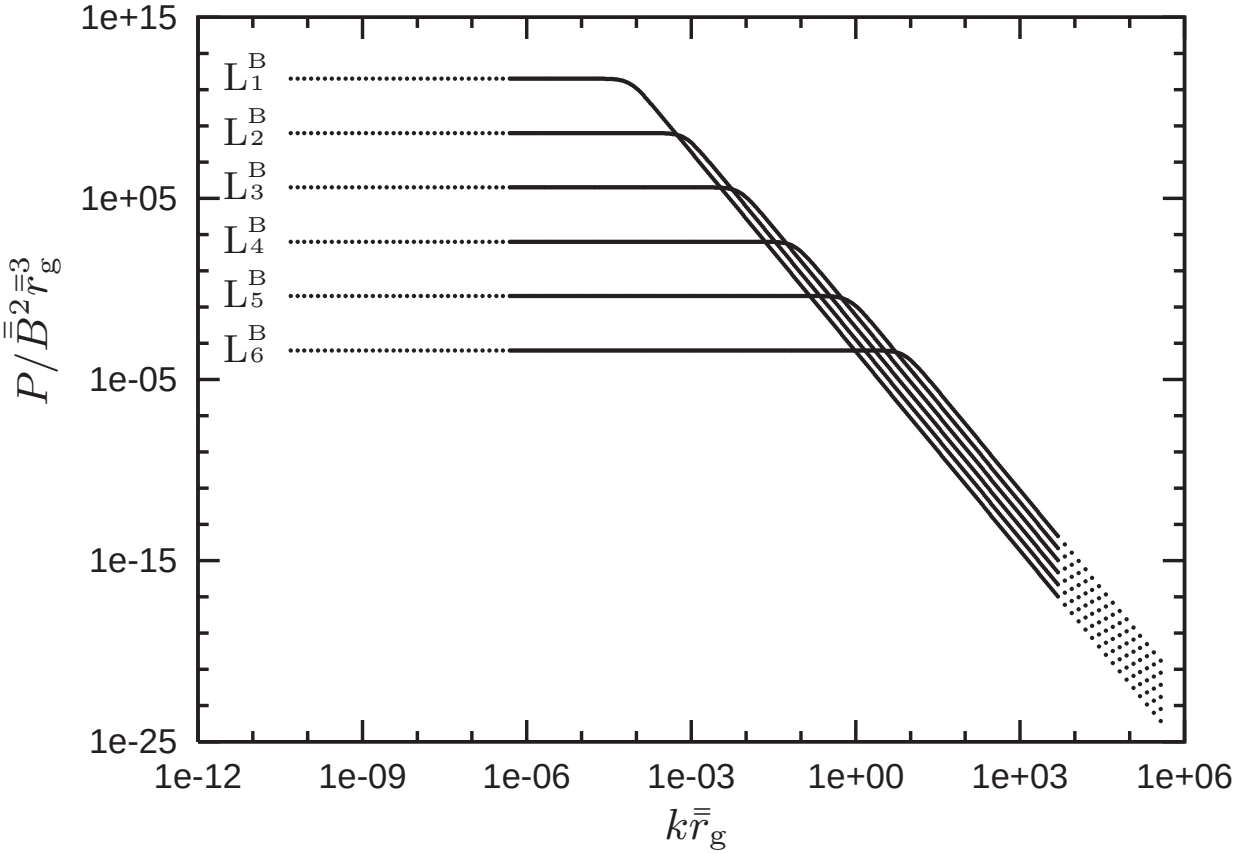
Typical feasible dynamic range only two orders of magnitude

(Recall $\rho \sim 10^{-11}$!)

- Continuous functional form eg. Giacalone & Jokipii ApJ 1999

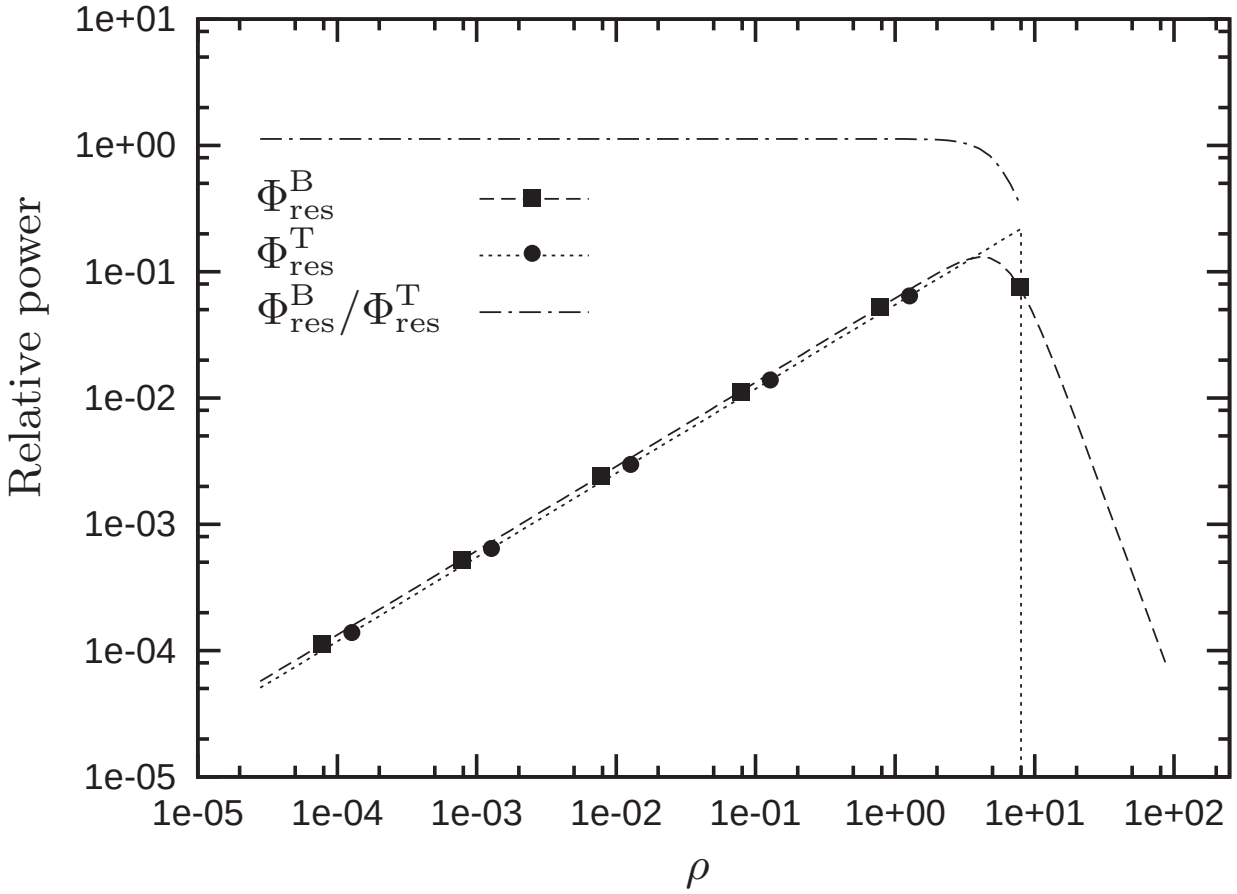
No limit on dynamic range, but computationally more demanding and requires that field is sufficiently resolved.

Field characteristics



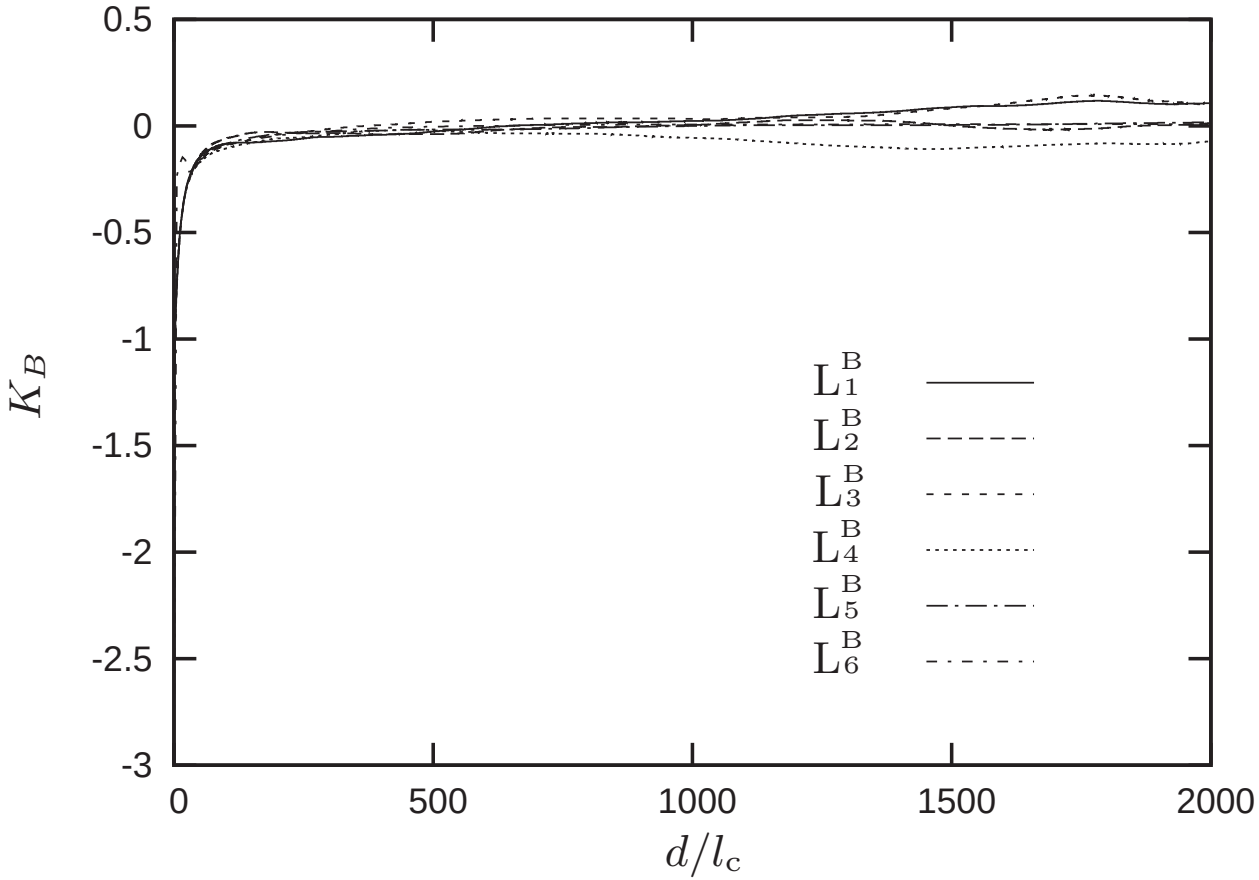
Broken power-law modes

Field characteristics



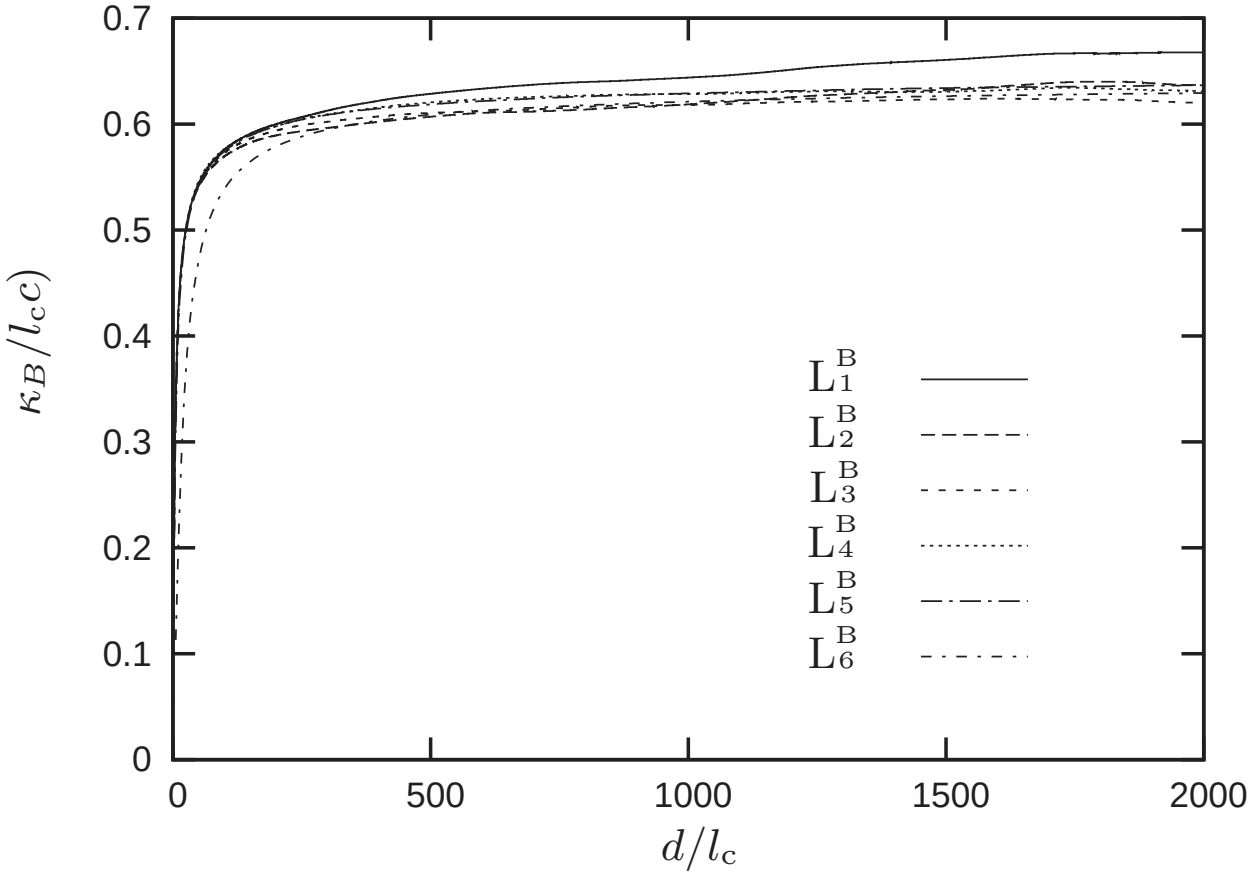
Resonant scale relative power: $\Phi_{\rm res} = \dfrac{k_{\rm res}^3 P(k_{\rm res})}{3\int {\rm d}k\, k^2 P(k)}$

Field characteristics



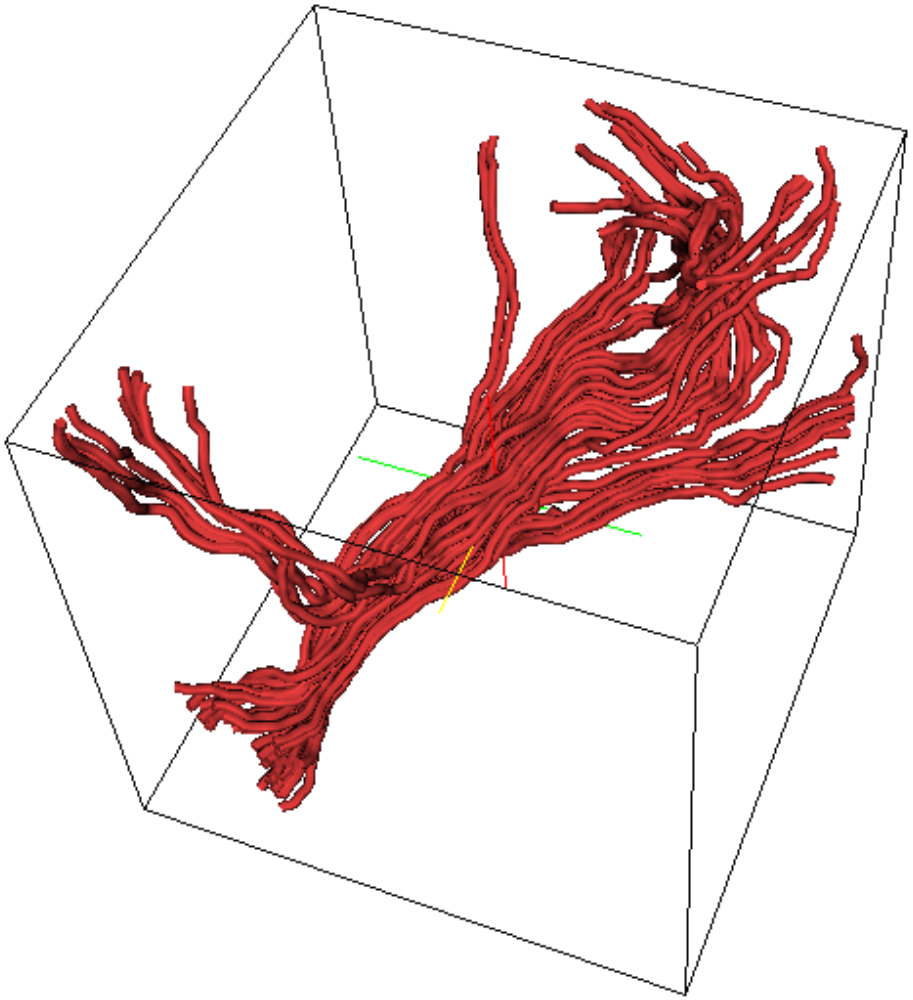
Running kurtosis: $K(t) \equiv \frac{1}{3}\left(\frac{\langle\Delta_x^4\rangle}{\langle\Delta_x^2\rangle^2} + \frac{\langle\Delta_y^4\rangle}{\langle\Delta_y^2\rangle^2} + \frac{\langle\Delta_z^4\rangle}{\langle\Delta_z^2\rangle^2}\right) - 3$

Field characteristics



Running diffusion: $\kappa(t) \equiv \frac{\langle \Delta_x^2 \rangle + \langle \Delta_y^2 \rangle + \langle \Delta_z^2 \rangle}{6t}$

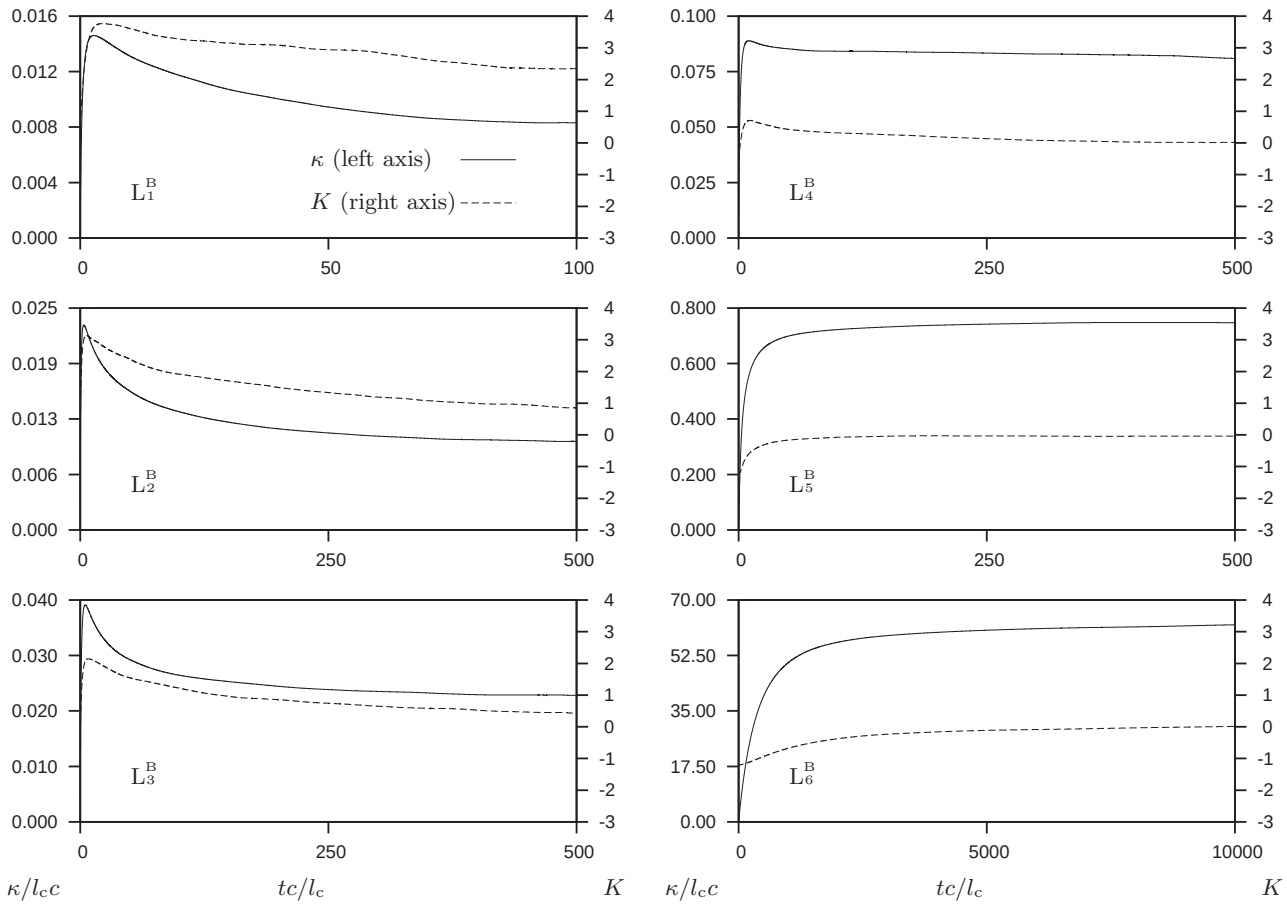
Field characteristics



Field line bundle for $(2L_c)^3$ (with initial bundle radius $0.009L_c$).

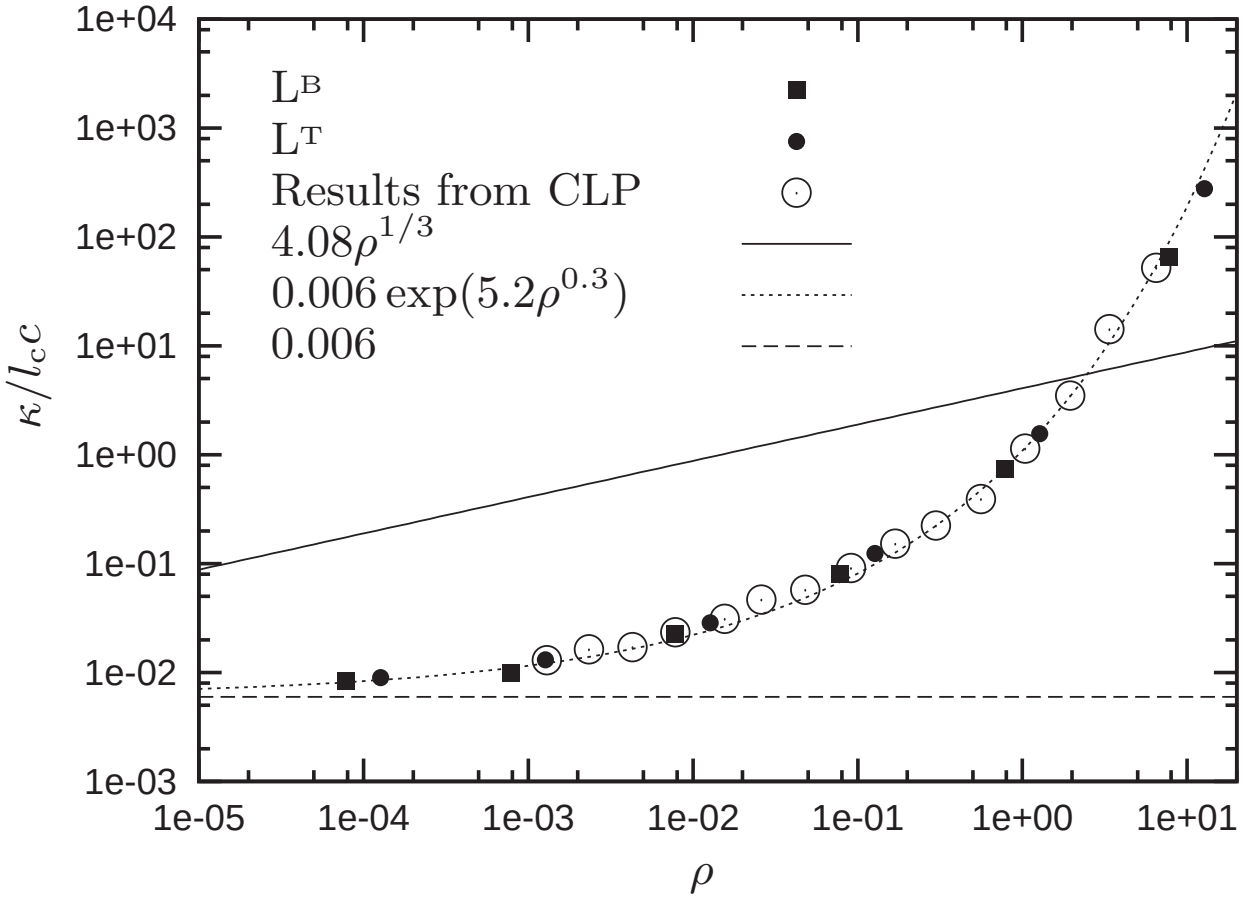
Outline

Particle Transport



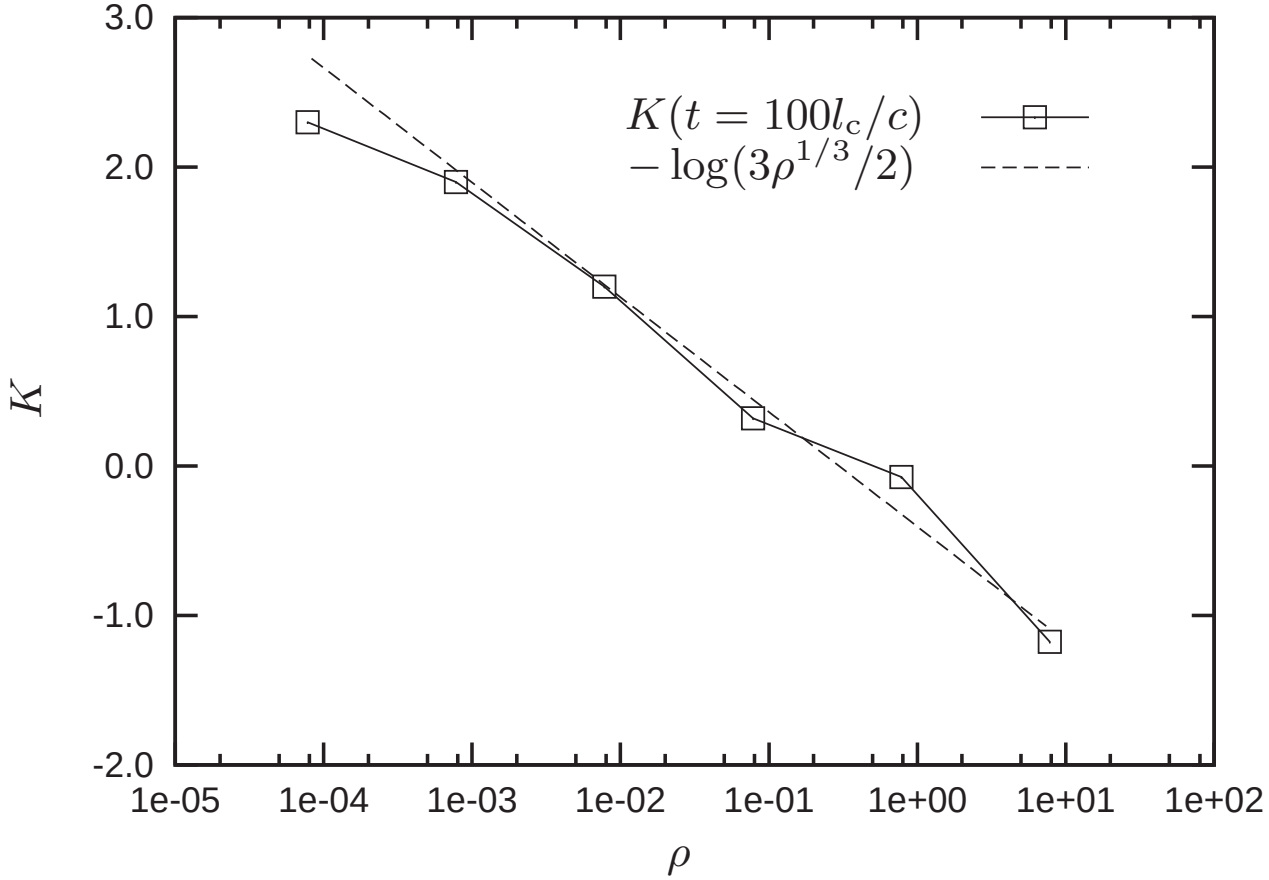
Running diffusion and kurtosis coefficients

Particle Transport



Comparison with diffusion coefficients from various sources

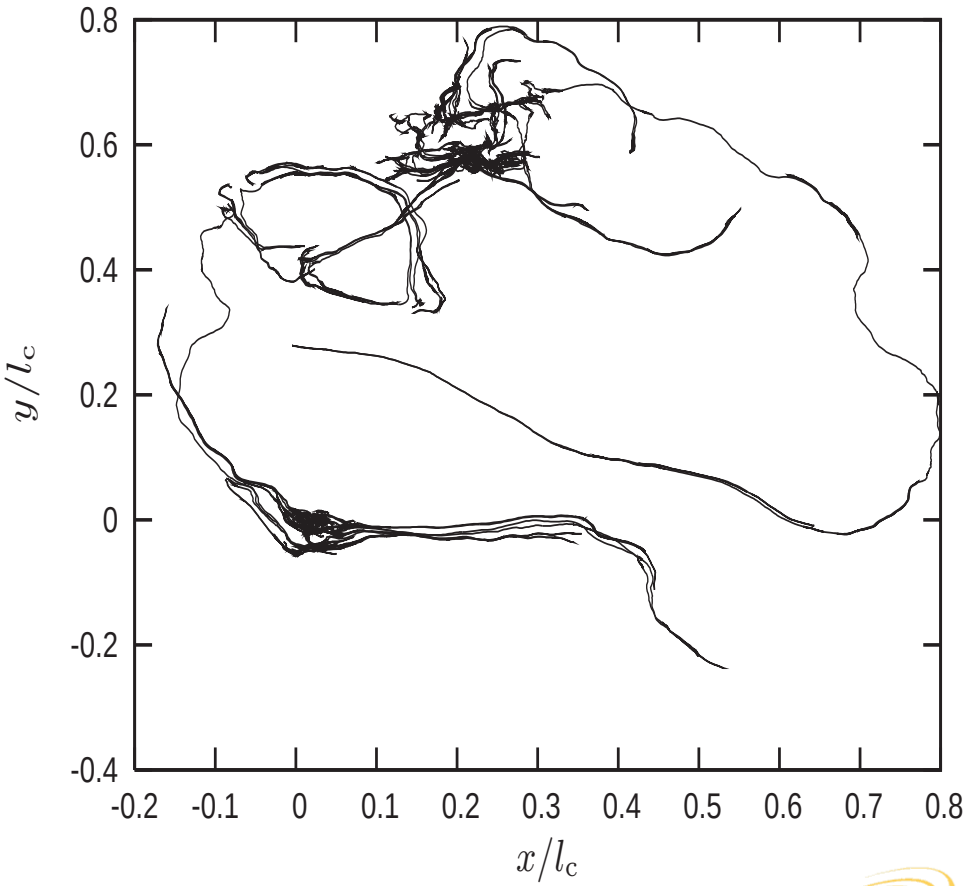
Particle Transport



Kurtosis at $t = 100 l_c/c$

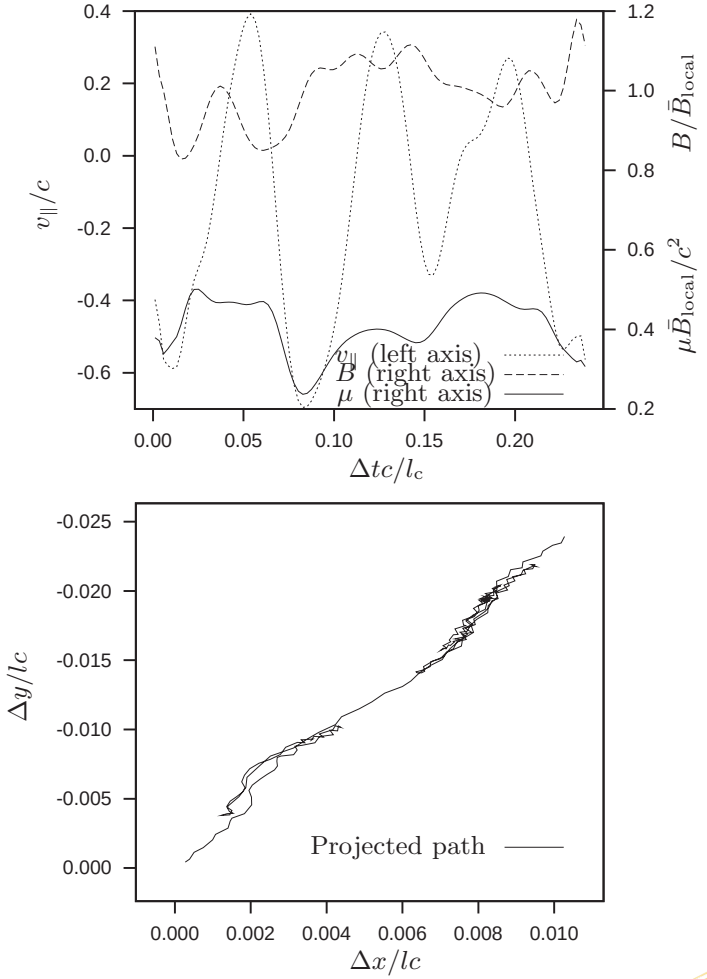
Understanding Subdiffusion

- Böhm diffusion:
  step $\sim \bar{\bar{r}}_{\mathrm{g}}$ in time $\sim 3\bar{\bar{r}}_{\mathrm{g}}/c$
- Field line diffusion:
  step $\sim \mathrm{L}_c$ in time $\sim \mathrm{L}_c/c$
- Ratio is $\sim \rho/3$:
  FLD should dominate at low ρ
- Only true for $\rho \sim 1$ => retracing
- Hop $\sim \bar{\bar{r}}_{\mathrm{g}}$ => more pronounced sub-diffusion at lower ρ



Understanding subdiffusion

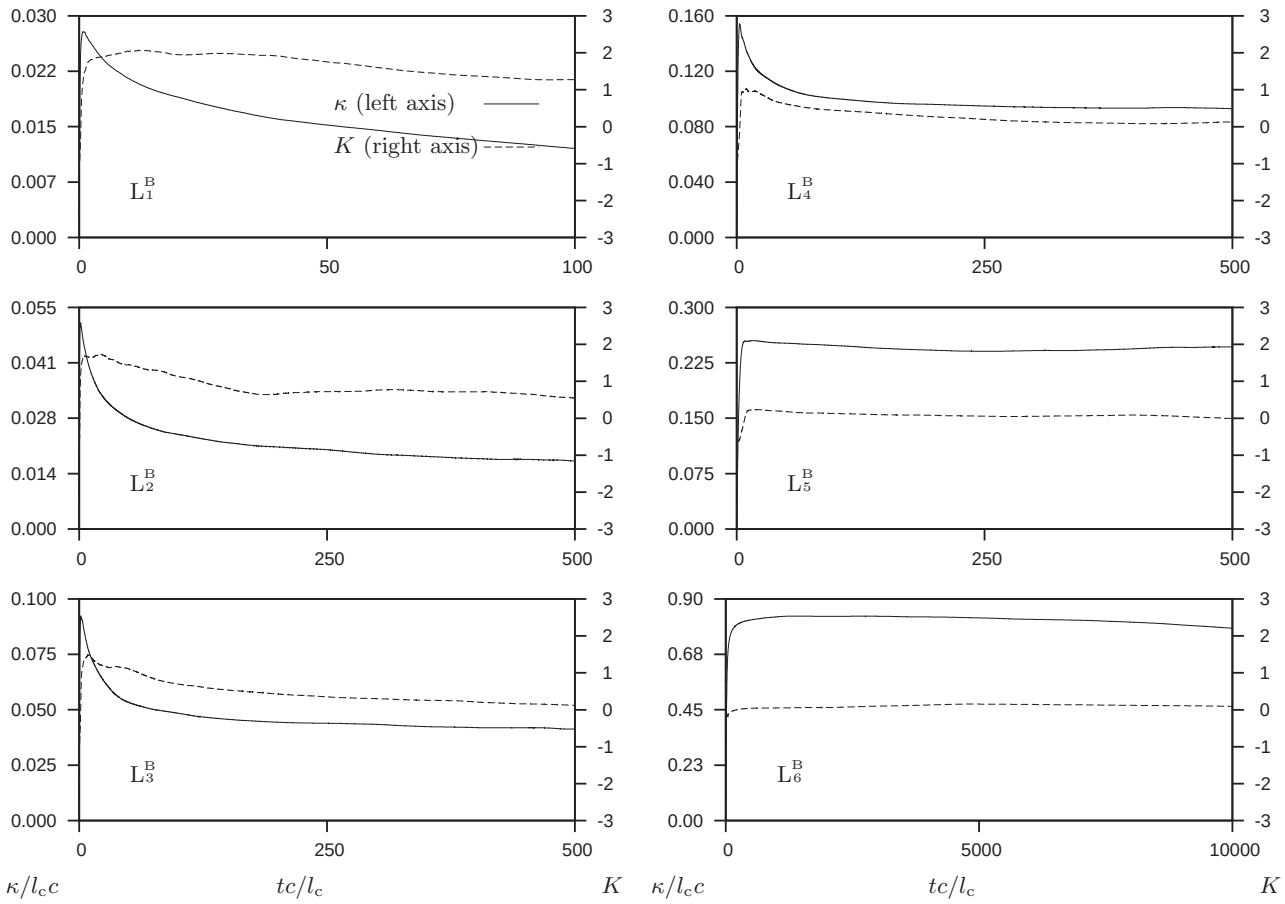
- Scatter θ through $\sim \pi/2$ for reversals
- Resonant scattering?
- Magnetic bottle?
- 4/6 reversals at maxima in B
- Variation in μ in agreement with variation in B



Field-line tracing model

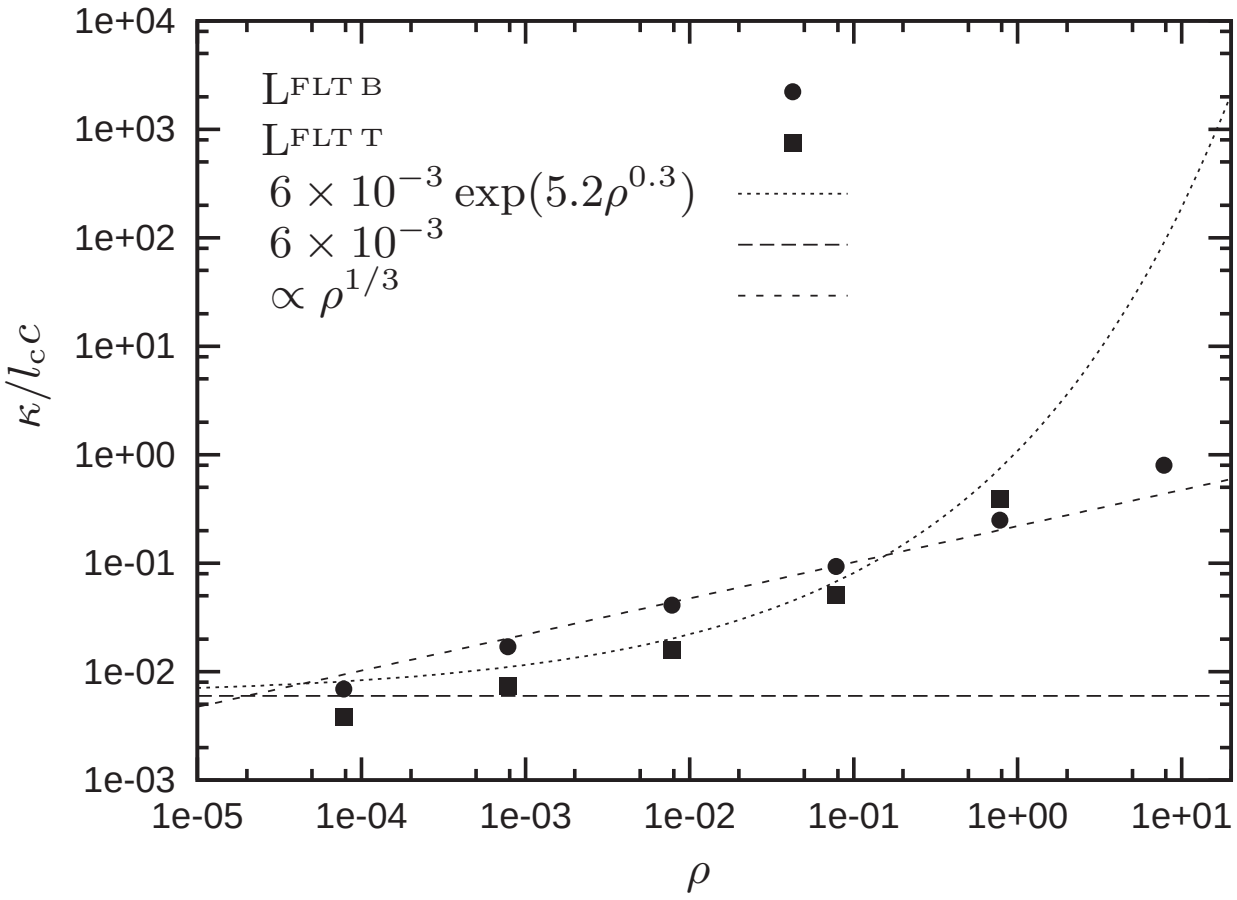
- Mean-free path s_{mf}
- Diffusion coefficient $D = s_{\mathrm{mf}} c$
- QLT: $D \simeq A \rho^{1/3} \mathrm{L}_c c \qquad (A = 4.08)$
- $s_{\mathrm{mf}} \simeq A \rho^{1/3} \mathrm{L}_c$
- Assume in time $t_{\mathrm{mf}} = s_{\mathrm{mf}}/c$ gyrocentre shifts across field lines by distance $\sim \bar{\bar{r}}_{\mathrm{g}}$ with equal probability of moving in either direction
- $s_{\mathrm{mf}} > \mathrm{L}_c$: kurtosis small
- $s_{\mathrm{mf}} \ll \mathrm{L}_c$: diffusion significantly non-classical

Field-line tracing model



Running diffusion and kurtosis

Field-line tracing model



Asymptotic diffusion coefficients

Summary

- At low-rigidities spatial distribution is not well described by Gaussian statistics during a prolonged initial subdiffusive phase.
- Field line re-tracing appears to be a dominant process in supressing diffusion.
- Simple random walk model appears to support this.
- Our results suggest that $\kappa \propto l_c c$ as $\rho \to 0$.
- Non-resonanant process becomes dominant (?)