LES of Compressible Turbulence

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with thanks to

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Overview

1. LES in a nutshell
2. Subgrid scale closures
3. Turbulence energy spectra
4. Turbulence pressure

Resolved vs Subgrid Scales

Astrophysics: numerical resolution $\Delta \gg$ physical dissipation scale

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Finite volume codes:

- Interpret numerical solution as smoothed approximation
- Rigorous approach (Germano 1992, Schmidt et al. 2006): define filtered quantities $\rho = \langle \overset{\infty}{\rho} \rangle_\Delta$, $\mathbf{v} = \langle \overset{\infty}{\rho} \overset{\infty}{\mathbf{v}} \rangle_\Delta / \rho$, etc.
- Decomposition of hydrodynamical equations into balance laws for resolved quantities ρ, $\mathbf{v}$, etc., and higher-order moments, e. g., $\langle \overset{\infty}{\rho} \overset{\infty}{\mathbf{v}} \otimes \overset{\infty}{\mathbf{v}} \rangle_\Delta$

Germano Decomposition of Hydrodynamical Equations

Example: momentum equation in the limit $\mathrm{Re} \to \infty$:

$$\frac{\partial}{\partial t}\rho\mathbf{v} + \boldsymbol{\nabla}\rho\mathbf{v}\otimes\mathbf{v} = -\boldsymbol{\nabla}P + \boldsymbol{\nabla}\cdot\boldsymbol{\tau}_{\mathrm{sgs}} + \mathbf{F}$$

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- Turbulence stress tensor $\boldsymbol{\tau}_{\mathrm{sgs}} = -\langle \overset{\infty}{\rho}\overset{\infty}{\mathbf{v}} \otimes \overset{\infty}{\mathbf{v}}\rangle_{\Delta} + \rho\mathbf{v}\otimes\mathbf{v}$
- Trace-free part $\boldsymbol{\tau}^{*}_{\mathrm{sgs}}$ transports energy to subgrid scales - coupling to unresolved kinetic energy dissipation
- Diagonal part adds pressure: $\nabla P \to \nabla(P - \frac{1}{3}\mathrm{tr}\,\boldsymbol{\tau}_{\mathrm{sgs}})$

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$\langle\overset{\infty}{\rho}\overset{\infty}{\mathbf{v}}\otimes\overset{\infty}{\mathbf{v}}\rangle_\Delta$ not computable $\to$ closure for $\boldsymbol{\tau}_{\mathrm{sgs}}$

Subgrid Scale Model

SGS turbulence energy model (Schumann 1975, Schmidt et al. 2006): balance law for $K_{\rm sgs} = \rho k_{\rm sgs} := -\frac{1}{2}\mathrm{tr}\,\boldsymbol{\tau}_{\rm sgs}$

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\cdot\boldsymbol{\nabla}\right)\rho k_{\rm sgs} - \boldsymbol{\nabla}\cdot\left(\rho C_\kappa \Delta k_{\rm sgs}^{1/2}\boldsymbol{\nabla} k_{\rm sgs}\right) = (\tau^*_{ij})_{\rm sgs} S_{ij} - \frac{2}{3}\rho k_{\rm sgs} d - \rho C_\epsilon \frac{k_{\rm sgs}^{3/2}}{\Delta}$$

- advection-diffusion-production-dissipation equation for $K_{\rm sgs}$

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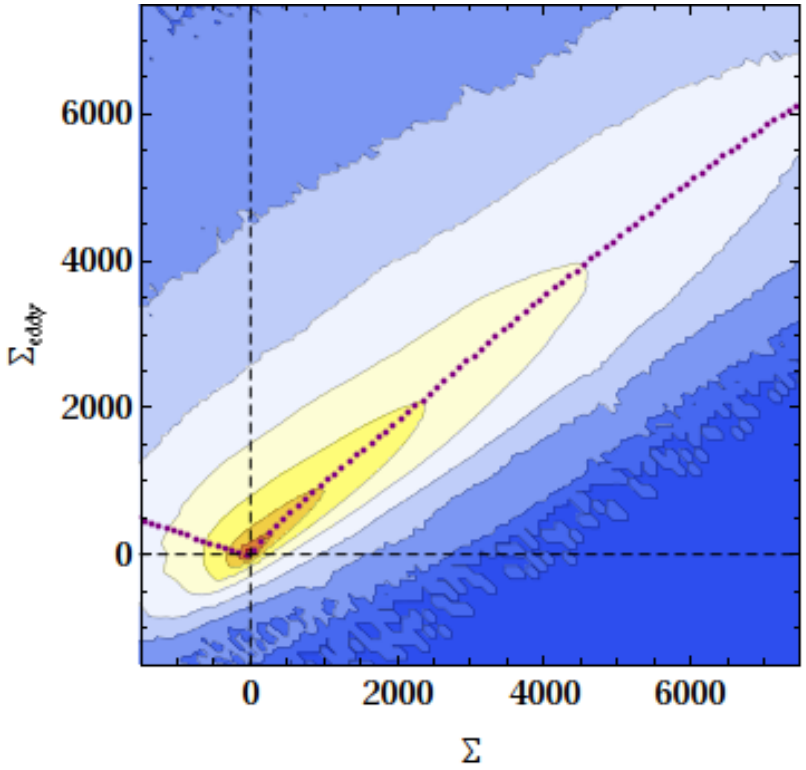
- advection-diffusion-production-dissipation equation for K_{sgs}
- closure for trace-free part $\boldsymbol{\tau}^*_{\mathrm{sgs}}$ in terms of $\boldsymbol{\nabla}\otimes\mathbf{v}$
- incompressible turbulence: eddy viscosity closure $\boldsymbol{\tau}^*_{\mathrm{sgs}} = 2\rho\nu_{\mathrm{sgs}}\mathbf{S}^*$, where $\nu_{\mathrm{sgs}} = C_\nu \Delta k_{\mathrm{sgs}}^{1/2}$ and $\mathbf{S}^*$ is the trace-free symmetric part of $\boldsymbol{\nabla}\otimes\mathbf{v}$

A Priori Tests

- Gaussian filter $\langle\ \rangle_\ell$ of length $\ell = 32\Delta$ applied to 1024^3 data from Federrath et al. 2008 (forced supersonic turbulence)
- Cheating: $\overset{\infty}{\mathbf{v}} \to \mathbf{v}$, etc. in the limit $\Delta/L \to 0$ (ILES)
- Calculate $\langle \rho \mathbf{v} \rangle_\ell$, $\langle \rho \mathbf{v} \otimes \mathbf{v} \rangle_\ell$, etc. in the inertial subrange
- Find closures for known turbulence stress $\boldsymbol{\tau}_\ell$ (supersonic turbulence: Woodward et al. 2002/06)

Eddy-Viscosity vs Nonlinear Closure

Correlation diagrams for turbulence energy flux $\Sigma_\ell = (\tau_{ij})_\ell S_{ij}$



$$(\tau_{ij})_\ell = 2\rho C_\nu \Delta k_\ell^{1/2} S_{ij}^* - \tfrac{1}{3}\rho k_\ell \delta_{ij}$$

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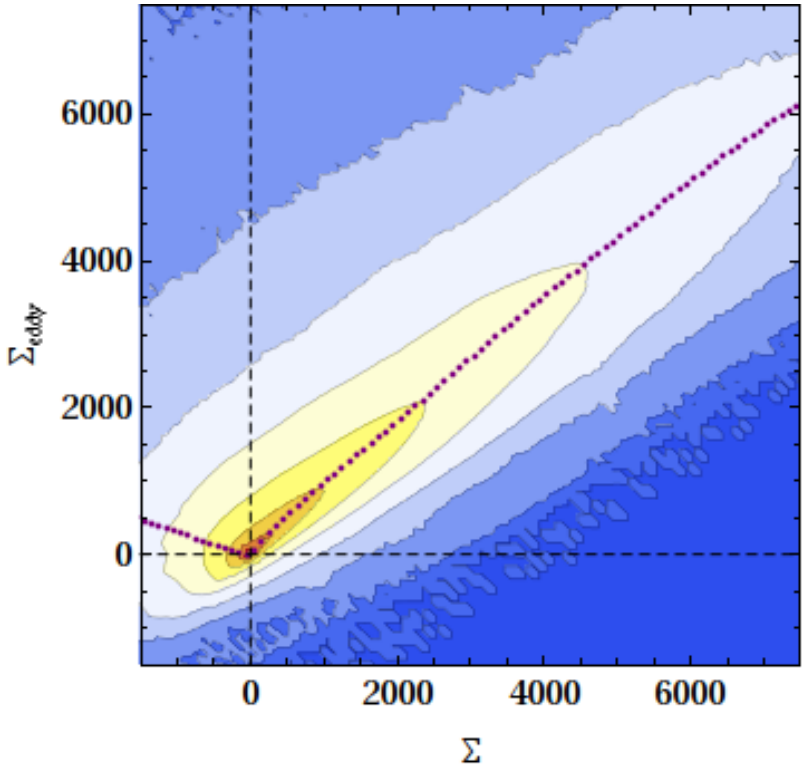

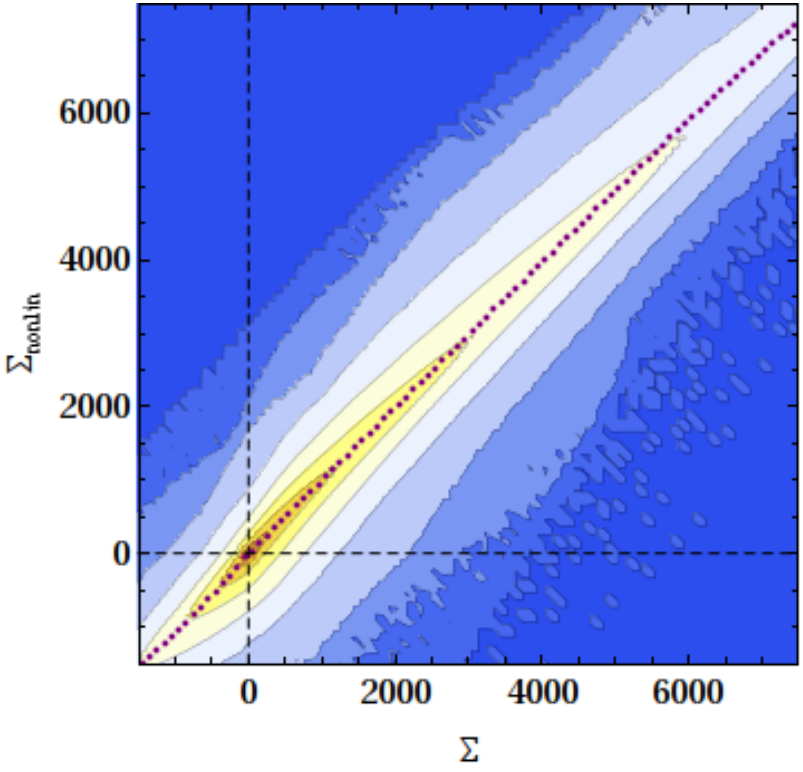

$$(\tau_{ij})_\ell = 2\rho C_\nu \Delta k_\ell^{1/2} S^*_{ij} - \tfrac{1}{3}\rho k_\ell \delta_{ij}$$

$$(\tau_{ij})_\ell = -\tfrac{1}{2}\rho k_\ell \left[C_2 \frac{2 v_{i,k} v_{k,j}}{|\boldsymbol{\nabla} \otimes \mathbf{v}|^2} + \frac{1-C_2}{3}\delta_{ij} \right]$$

Now let's compare plain PPM simulation to LES with non-linear closure for τ_{sgs} on 256^3 grids using Enzo 1.5...

Turbulence Energy Spectra: Plain PPM

Integrate kinetic energy over sperhical shells in Fourier space:

$$\mathcal{E}(k) = \oint \frac{1}{2} |\hat{\mathbf{v}}(k)|^2 k^2 \, \mathrm{d}\Omega_k$$

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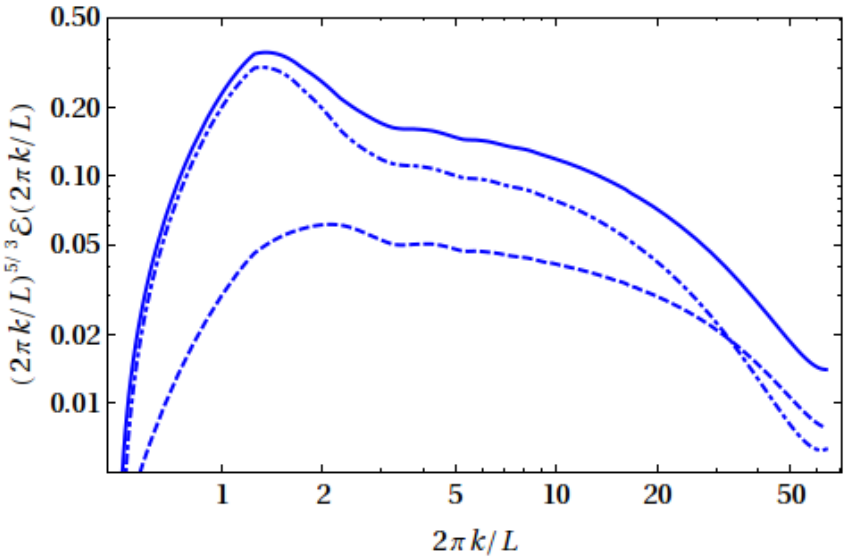

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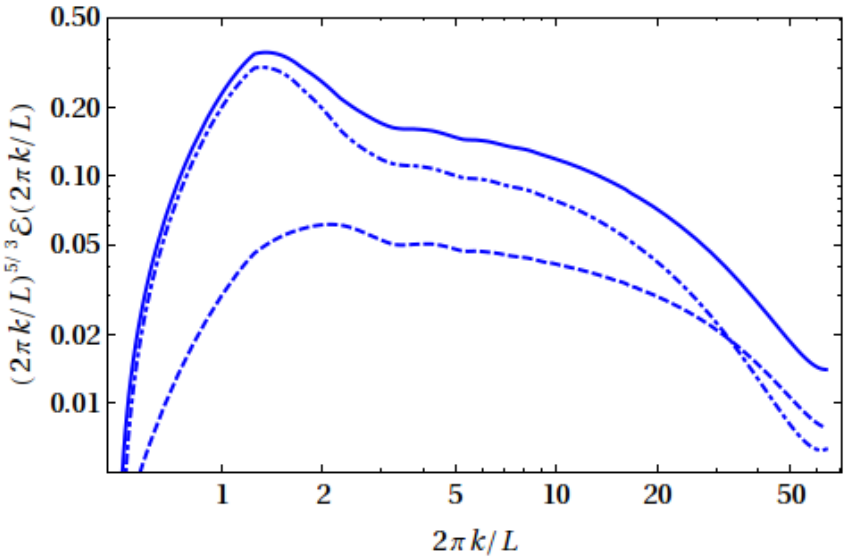

- Solenoidal forcing, $\mathcal{M}_{\mathrm{rms}} \approx 5.5$
- Steeper than Kolmogorov spectrum
- Too shallow compared to 1024^3 grid simulation (Federrath et al. 2009) $\rightarrow$ bottleneck effect flattens spectrum

Turbulence Energy Spectra: Plain PPM

Consider mass-weighted velocity $\tilde{\mathbf{v}} = \rho^{1/3}\mathbf{v}$
(rate of dissipation $\propto \rho v^3 = \tilde{v}^3$, Lighthill 1955):

$$E(k) = \oint \frac{1}{2} |\hat{\tilde{\mathbf{v}}}^2(k)| k^2 \, \mathrm{d}\Omega_k$$

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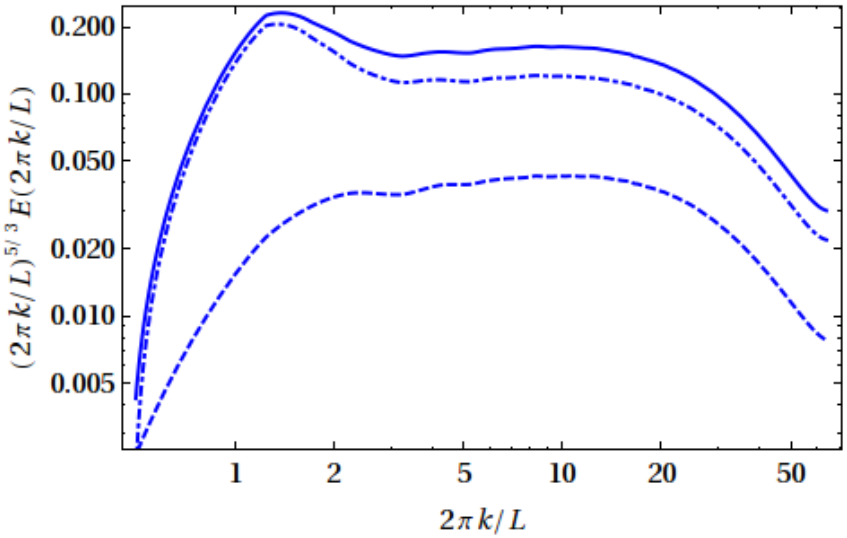

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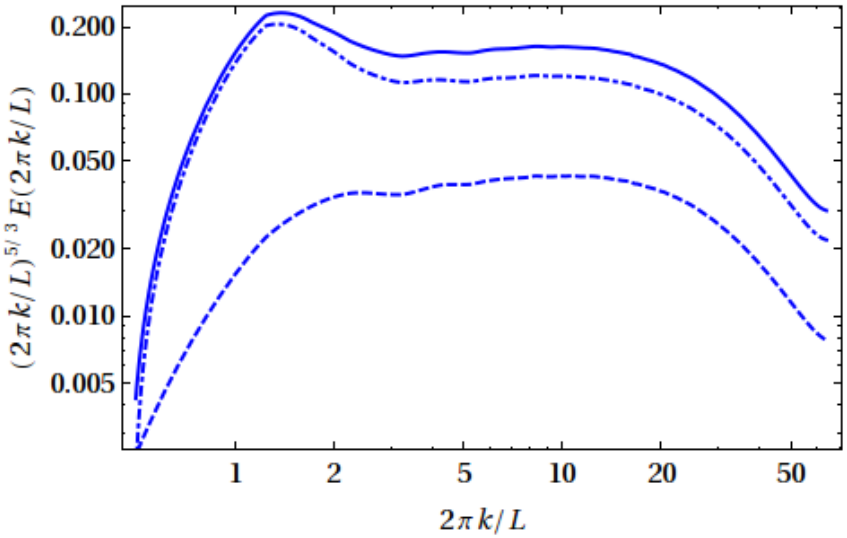
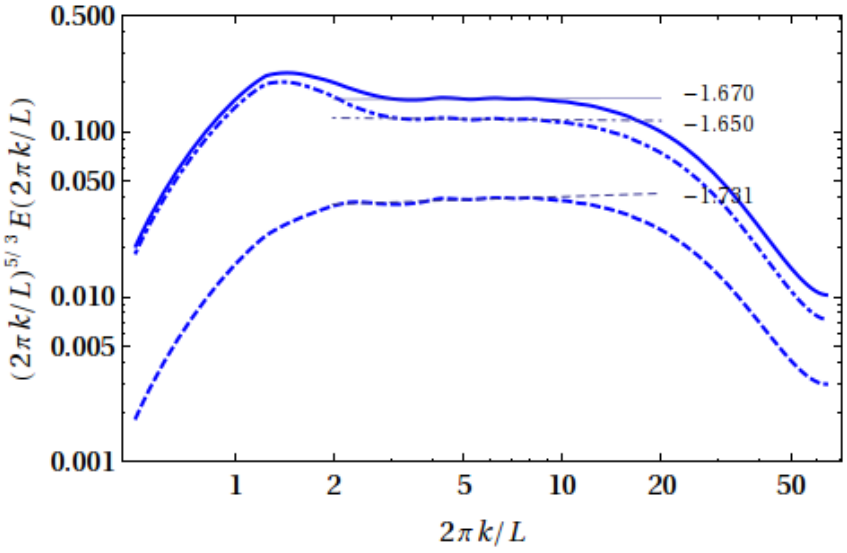


- $E(k)$ is less stiff than $\mathcal{E}(k)$
- Kritsuk et al. 2007: $E(k) \propto k^{-5/3}$ for supersonic turbulence
- At least 1024^3 grid required to see inertial subrange

Turbulence Energy Spectra: LES

solenoidal forcing



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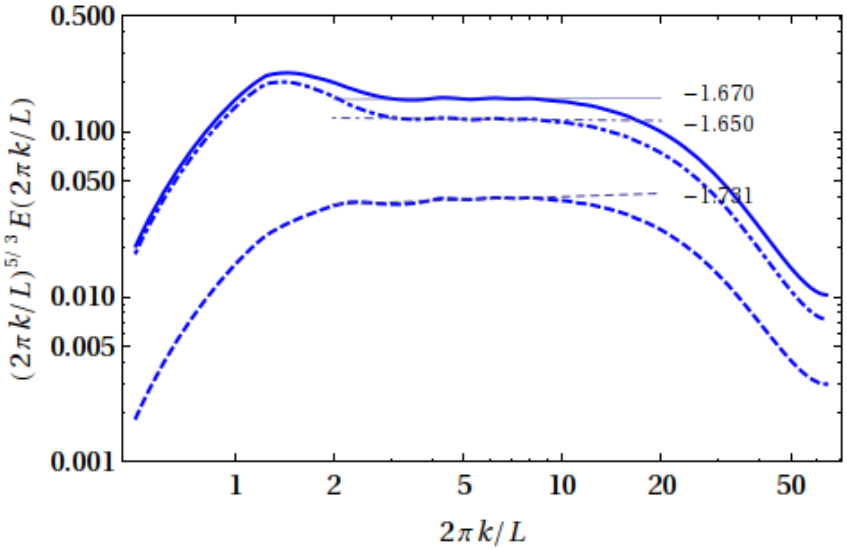
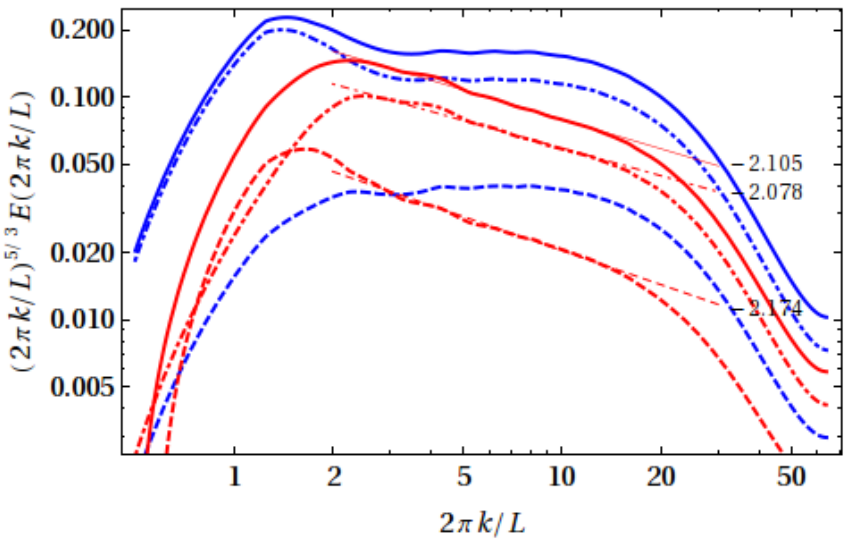


compressive forcing



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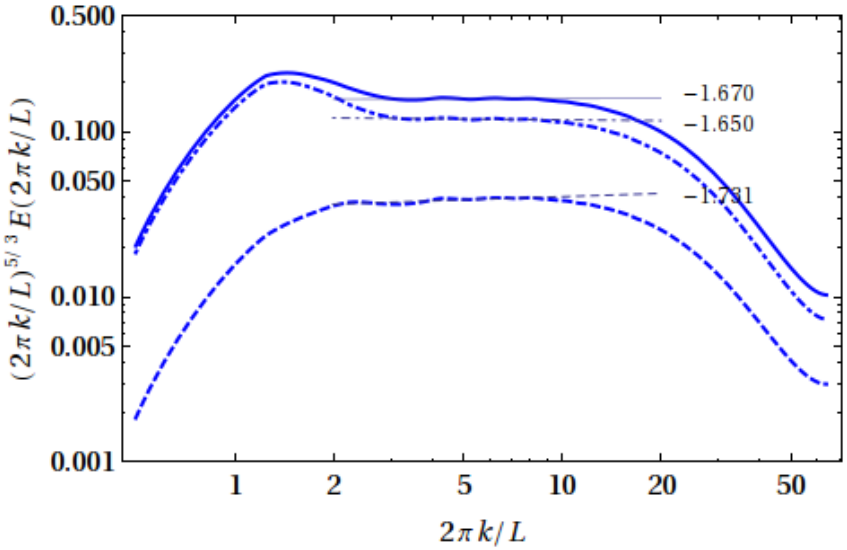

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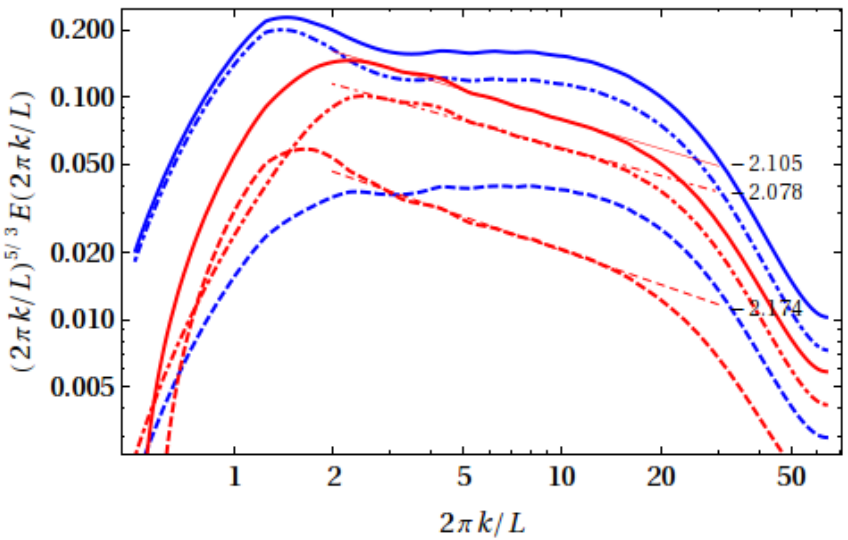

forcing	1024^3 (100000 CPU-h)	256^3 LES (1000 CPU-h)
solenoidal	1.64	1.67
compressive	2.10	2.11

Universal Scaling of Supersonic Turbulence?

Slopes of turbulence energy spectra $E(k)$ differ by about 0.4 for solenoidal/compressive forcing

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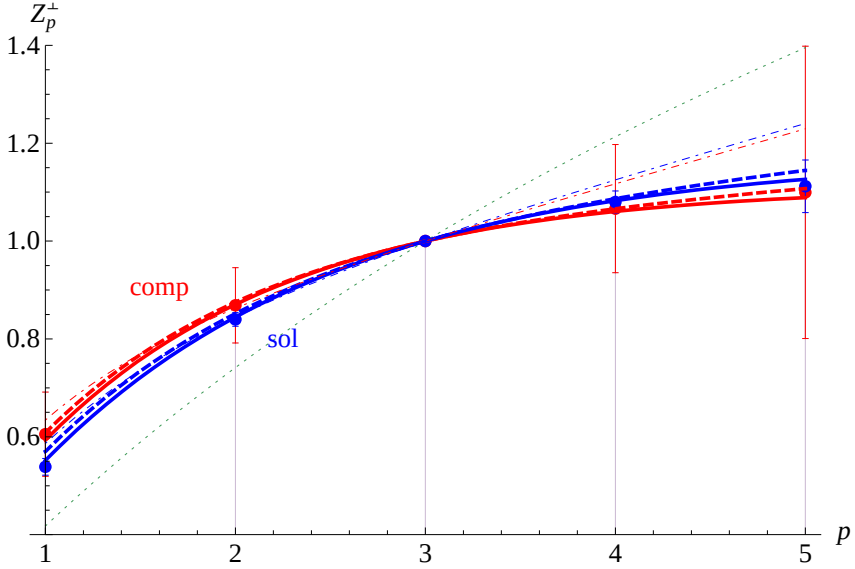
Slopes of turbulence energy spectra $E(k)$ differ by about 0.4 for solenoidal/compressive forcing

- Computation of structure functions $\tilde{S}_p(l) := \langle(\delta\tilde{v})^p(l)\rangle = \langle(\delta\rho^{1/3}v)^p(l)\rangle$ from 1024^3 data
- Relative scaling exponents $Z_p = \zeta_p/\zeta_3$ are almost equal (Schmidt, Federrath & Klessen 2008)



The Scale-Dependent Equation of State

- Microscopic equation of state for isothermal gas: $P \propto \rho$

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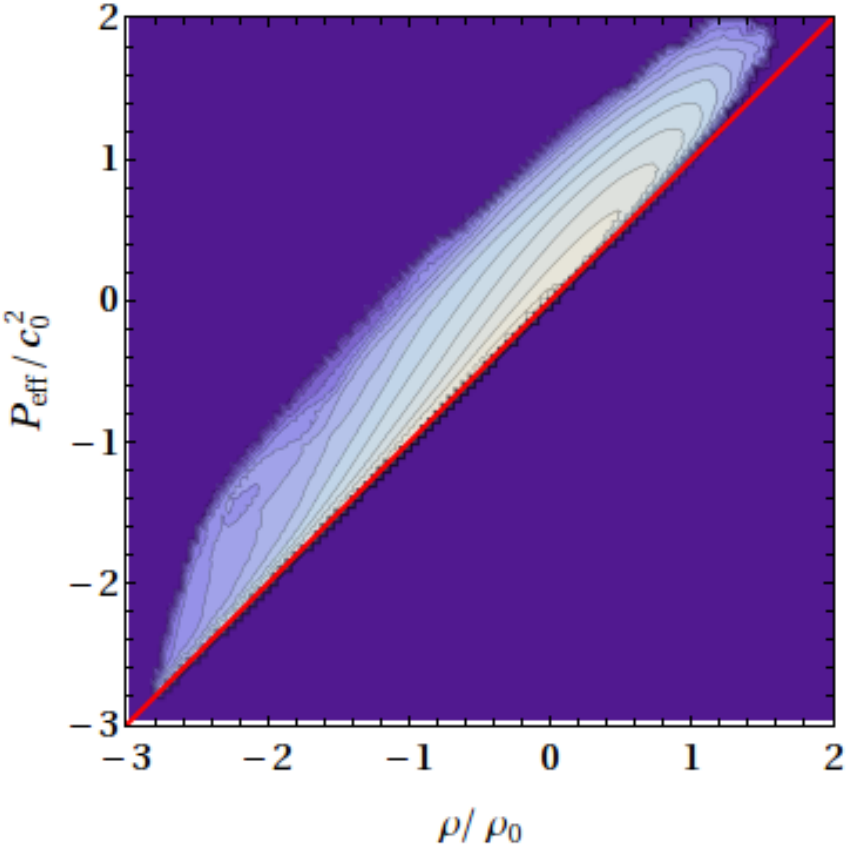
- Inertial-range velocity fluctuations produce turbulence pressure on top of thermal pressure (e. g., Bonazzola et al. 1987)
- Effective equation of state depends on length scale ℓ if $\delta v(\ell) \gtrsim c_\mathrm{s}$ ($\ell \gtrsim$ sonic length ℓ_s)
- In the framework of the Germano decomposition:

$$P_\mathrm{eff} = P + \frac{2}{3} K_\ell$$

where $K_\ell = -\frac{1}{2}\mathrm{tr}\,\boldsymbol{\tau}_\ell$ is the fraction of turbulence energy on scales $< \ell$ (if $\ell = \Delta$, then $K_\ell = K_\mathrm{sgs}$).

Pressure-Density Diagrams



1024^3 data filtered on length scale 16Δ

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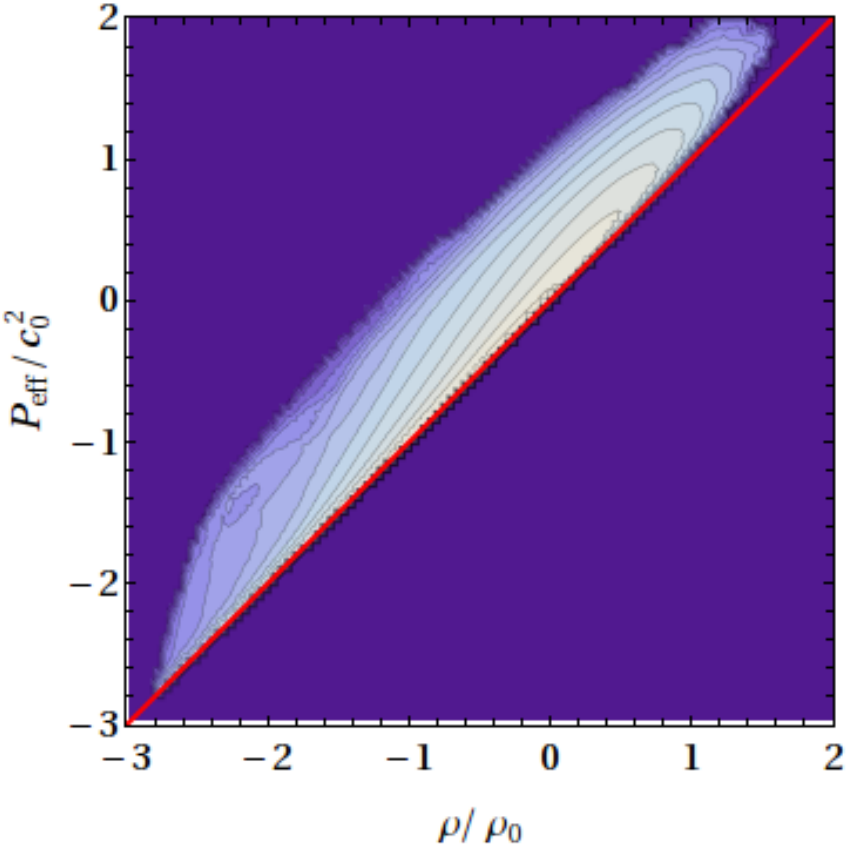

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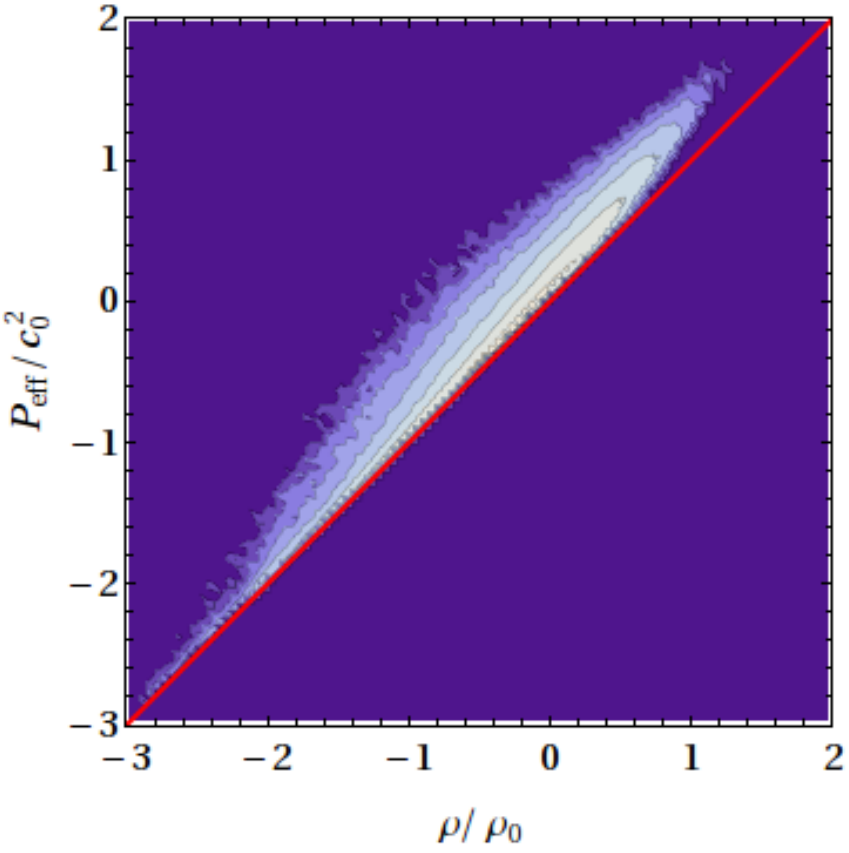
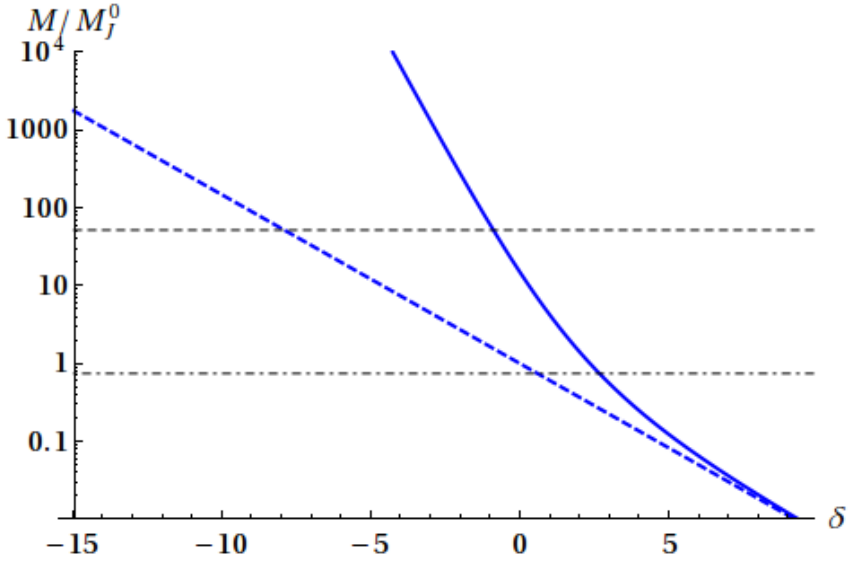


64^3 LES: $P_{\rm eff} = P + \frac{2}{3} K_{\rm sgs}$

Distribution of Gravitationally Unstable Cores (CMF)

Evaluation of mass spectrum according to Hennebelle & Chabrier 2008 using density pdfs of Federrath, Klessen & Schmidt 2008



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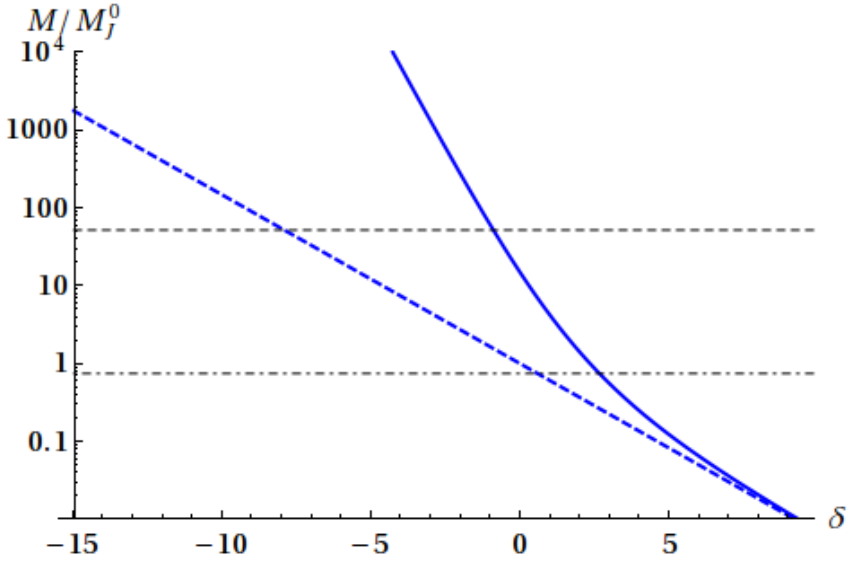

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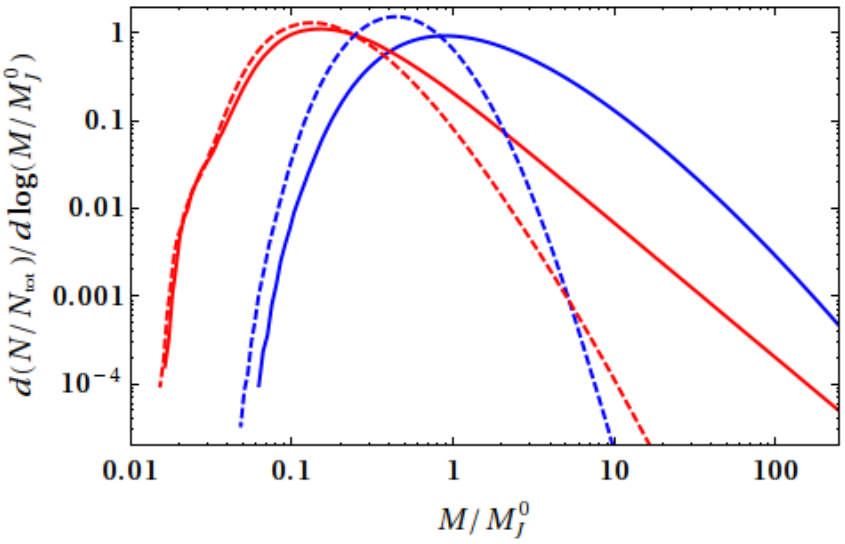

high-mass tails flatten compared to purely thermal support

Resume

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Adaptively Refined Large Eddy Simulations

- Disk galaxies: Hupp, Schmidt & Niemeyer in prep.
- Galaxy clusters: Maier, Iapichino, Schmidt & Niemeyer subm.