Four-dimensional (x,y,z + wave frequency) models for the solar environment.

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Outline

- MHD wave turbulence: coherent waves or incoherent wave packages?
- Discuss a coupled model for the wave turbulence in a solar wind. The distinctive feature of out approach is an account for a frequency spectrum of the turbulence.
- Result of integrated SEP-turbulence model driven by a realistic CME simulation
- We keep focused on the global model for the heliosphere (SC+IH+SEP+OH+Turbulence) implemented in the framework code (SWMF of the UofM)

What is the MHD Turbulence?

- The sources of the MHD turbulence in the heliosphere:
  - Bursts in the chromosphere / transient region (Hinode papers) – potential source responsible for heating, powering and accelerating the solar wind
  - Re-charge of the anisotropic flux of the pickup ions
  - Streaming instability at the diffuse shocks, accelerating the solar enrgetic particles
- A coupled model for the turbulence in a solar wind can hardly be based on the averaged integral characteristics only (such as the energy density). An account for a frequency spectrum of the turbulence is needed to quantify the absorption – generation mechanisms, most of them being resonant in nature.
- On the other hand, the approach basad on the Elzasser variables is, first, too detailed to be implemented in the global model, and, second, "too coherent".
  We employ "wave packages".

Wave action equation (a conservation law for a number of wave quanta):

$$\frac{\partial N_\sigma}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot \left(\frac{\partial \omega_\sigma}{\partial \mathbf{k}} N_\sigma \right) - \frac{\partial}{\partial \mathbf{k}} \cdot \left(\frac{\partial \omega_\sigma}{\partial \mathbf{x}} N_\sigma \right) = \gamma_\sigma N_\sigma$$

Witham method (G.B.Whitham, Linear and Non-linear waves, 1974) allows us to achieve full description of wave propagation and interaction with moving background (the wave stress tensor). Here we consider Galilean invariance, which results (1), in a Doppler formula:

$$\omega'_\sigma = \omega_\sigma - \mathbf{k} \cdot \mathbf{u}$$

And (2) simple and universal relationship between the wave action and their energy density:

$$I_\sigma = \mathbf{u} \cdot \frac{\partial L_w}{\partial \mathbf{u}} + \omega_\sigma \frac{\partial L_w}{\partial \omega_\sigma} - L_w$$

$$I_\sigma = \omega'_\sigma \frac{\partial L_w}{\partial \omega_\sigma} = \omega'_\sigma N_\sigma$$

Wave stress tensor

- Wave action equation expressed in terms of the energy density spectrum reads:

$$\frac{\partial I_\sigma}{\partial t} + \frac{\partial}{\partial \mathbf{x}} \cdot (\mathbf{V}_g I_\sigma) - \frac{\partial}{\partial \mathbf{k}} \cdot \left(\frac{\partial \omega_\sigma}{\partial \mathbf{x}} I_\sigma \right) -$$

$$- \frac{I_\sigma}{\omega'_\sigma} \left(\frac{\partial \omega'_\sigma}{\partial t} + (\mathbf{u} \cdot \nabla)\omega'_\sigma - \mathbf{V}'_{g\sigma} \cdot \nabla \mathbf{u} \cdot \mathbf{k} \right) = \gamma_\sigma I_\sigma$$

- Galilean invariance yields temporal derivative of frequency is proportional to spatial derivatives of background velocity. This allows us to introduce the following wave stress tensor:

$$\mathbf{P}_{w\sigma} = \frac{I_\sigma}{\omega'_\sigma} \left\{ \mathbf{V}'_{g\sigma} \otimes \mathbf{k} - \frac{\delta \left[\frac{\partial \omega'_\sigma}{\partial t} + (\mathbf{u} \cdot \nabla)\omega'_\sigma \right]}{\delta(\nabla \otimes \mathbf{u})} \right\}$$

- Coupled momentum equation for the background motion

$$\frac{\partial n\mathbf{p}}{\partial t} + \nabla \cdot \mathbf{F}_\mathbf{p} + \nabla \cdot \left(\sum_\sigma \int \mathbf{P}_{w\sigma} d^3\mathbf{k} \right) = 0$$

Transport equation for MHD turbulence.

- !!! Assume that the wave dispersion can be expressed in terms of the wave vector component parallel to the magnetic field !!!:

$$\omega' = \omega'(k_\parallel), \qquad \mathbf{V}'_{g\sigma} = \mathbf{b}\frac{\partial \omega'}{\partial k_\parallel}$$

- Unity vector of magnetic field evolves according to induction equation:

$$\frac{\partial \mathbf{b}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{b} = (\mathbf{b}\cdot\nabla)\mathbf{u} - \mathbf{b}\left[\mathbf{b}\cdot(\mathbf{b}\cdot\nabla)\mathbf{u}\right]$$

- Now the wave energy spectrum can be integrated by $k_\perp$. We use ω' as the phase coordinate

$$I_{\omega'\sigma} = \frac{1}{V'_{g\sigma}}\int dk_\perp I_\sigma, \quad E_w = \int d\omega' I_{\omega'\sigma}$$

The transport equation for waves:

$$\frac{\partial I_{\omega'\sigma}}{\partial t} + \nabla\cdot\left[(u + bV'_{g\sigma})I_{\omega'\sigma}\right] - \frac{\partial}{\partial \ln\omega'}\left(\text{"}\nabla\cdot u\text{"}\frac{I_{\omega'\sigma}}{2}\right) = 0, \quad P_w = \int \frac{I_{\omega'\sigma}}{2}\text{"}1\text{"}\,d\omega'$$

for high-frequency waves the “velocity divergence” modifies. Obtained equation is analogous to the Parker equation for cosmic rays.

High-frequency waves (whistlers and ion-cyclotron).

These waves produce anisotropic stress tensor which affects the anisotropy of background motion.

- For propagation along the magnetic field the Hall MHD gives the dispersion relation:

$$\omega' = \omega_{Bi}\left(x\sqrt{1+x^2/4} \pm x^2/2\right), \quad x = V_A k_{\|}/\omega_{Bi}$$

- After calculating the stress tensor described above (note that evolution of Alfven speed is found from ideal MHD):

$$P_w = \frac{I_{\omega'\sigma}}{2}\begin{pmatrix} 1 & & \\ & 1 & \\ & & 1 \end{pmatrix} \pm \frac{I_{\omega'\sigma}}{2}\,\frac{\omega}{2\omega_{Bi} \pm \omega}\begin{pmatrix} -1 & & \\ & -1 & \\ & & +1 \end{pmatrix}$$

- we find that pressure of “whistlers” is purely parallel, whereas that of ion-cyclotron waves is transverse.

Alfven waves, comparison with the WKB.

- For Alfven waves with a linear dispersion law,

$$\omega' = k_{\|} V_{A\sigma}, \quad V_{A\sigma} = \pm V_A$$

we can express $\partial V_A / \partial t$ from the MHD equations. Then we evaluate P and derive the transport equation as follows:

$$\frac{\partial I_{\omega'\sigma}}{\partial t} + \nabla \cdot \left[(\vec{u} + \vec{b} V_{A\sigma}) I_{\omega'\sigma} \right] - \nabla \cdot \vec{u} \frac{\partial}{\partial \ln \omega'} \left(\frac{I_{\omega'\sigma}}{2} \right) = \gamma I_{\omega'\sigma}, \quad \overleftrightarrow{P}_w = \int \frac{I_{\omega'\sigma}}{2} \overleftrightarrow{1} d\omega'$$

  - Cf the WKB equation (Dewar,1970) for $E_\sigma = \int d\omega' I_{\omega'\sigma}$:

$$\frac{\partial E_\sigma}{\partial t} + \nabla \cdot \left[(u + b V_{A\sigma}) E_\sigma \right] + \left(\nabla \cdot u \frac{E_\sigma}{2} \right) = \Gamma E_\sigma, \quad P_w = \frac{E_\sigma}{2} 1, \quad \gamma_w = \frac{3}{2}$$

- In expanding solar wind the advection in a phase space occurs in a ‘counter-cascade’ direction, thus, reducing the dissipation of turbulence. <u>As a result, in coronal holes (diverging flow) the solar wind might be mostly accelerated by waves, whereas in the helmet streamer (no flow) plasma is mostly heated.</u>

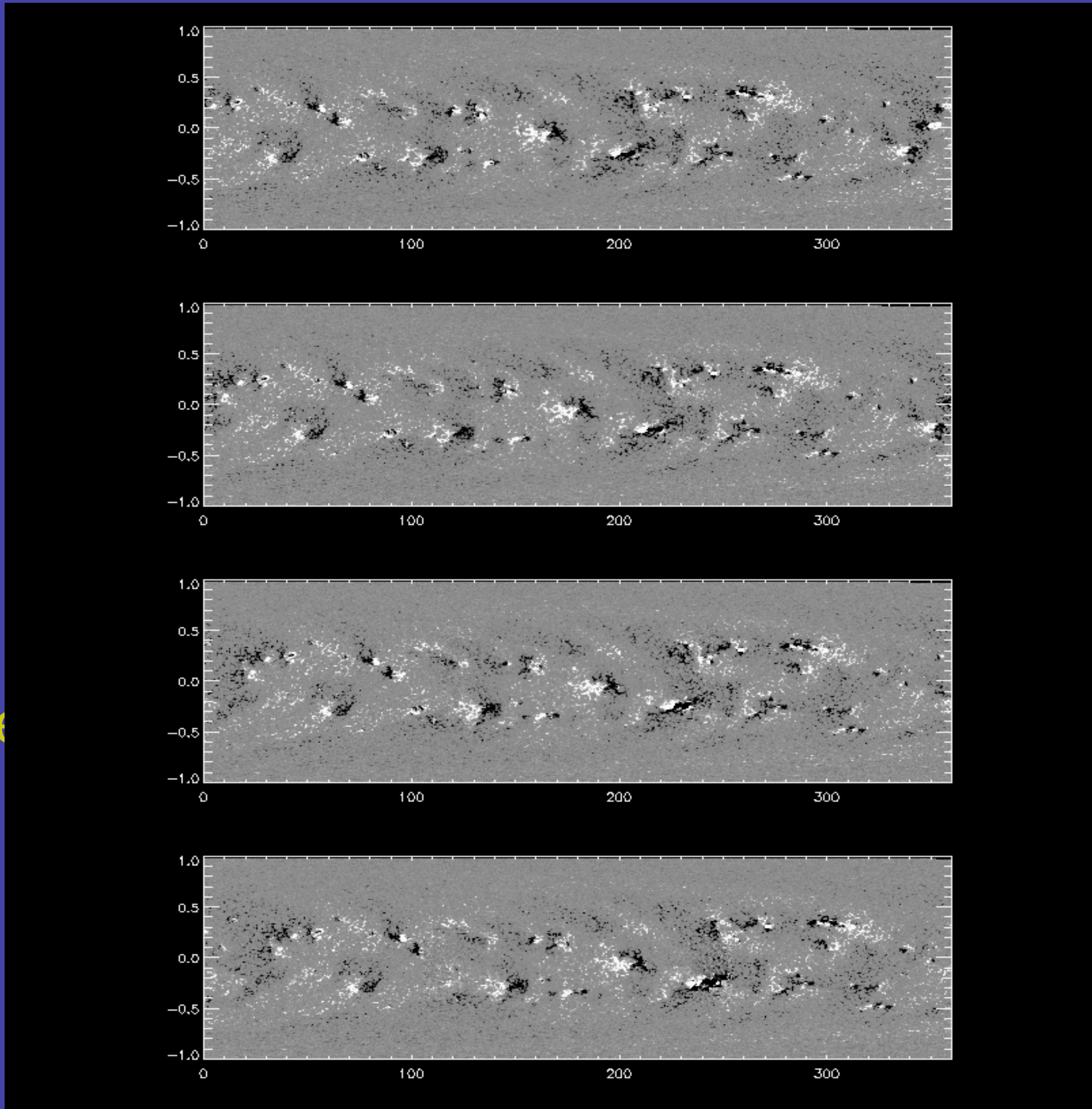
Turbulent solar wind

The advantages of the framework approach

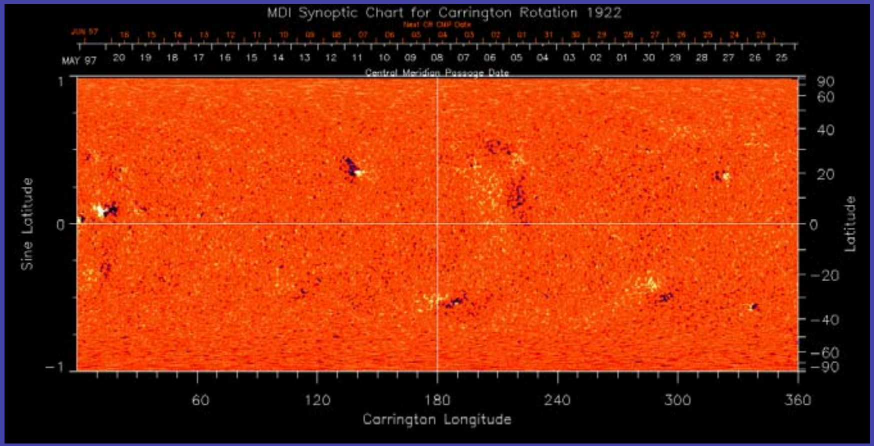
- Coupled Solar Corona (SC) and Inner Heliosphere (IH) modules can be tested and verified together:
  - Comparison with the satellite data at 1 AU: the source of the solar wind is in the SC, while the propagation towards 1AU occurs in IH
  - Synthetic images, like C3 simulated image: for the most of pixels the signal should be integrated over the line of sight throughout both SC and IH
- The SEP models in SWMF and the fact that they are in SWMF provide the following benefits:
  - Reliable description of the magnetic connectivity;
  - Consistent simulation of the CME-SEP events
  - Consistent model of a hydromagnetic turbulence
  - POTENTIAL forecast capability (relativistic electrons plus magnetic connectivity, >100 MeV protons escaped from the turbulent caldron).

What do we have now as the solar wind model?

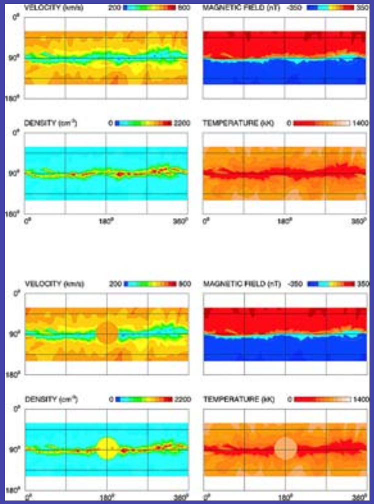
- To better resolve the active region, which was very close to the west limb at the start time for the CME (April,21, 2002, UT01:51) we incorporate the MDI magnetogram of April, 18 (the third from the top), at which the active region was at
- ~W50, S13



Solar Wind Model (O.Cohen et al, ApJL, 2006 - model, JGR, 2007-validation)

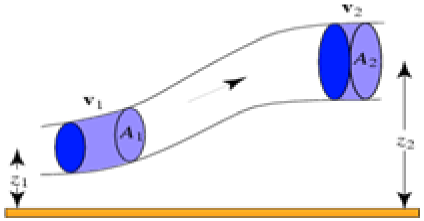


Magnetogram Data

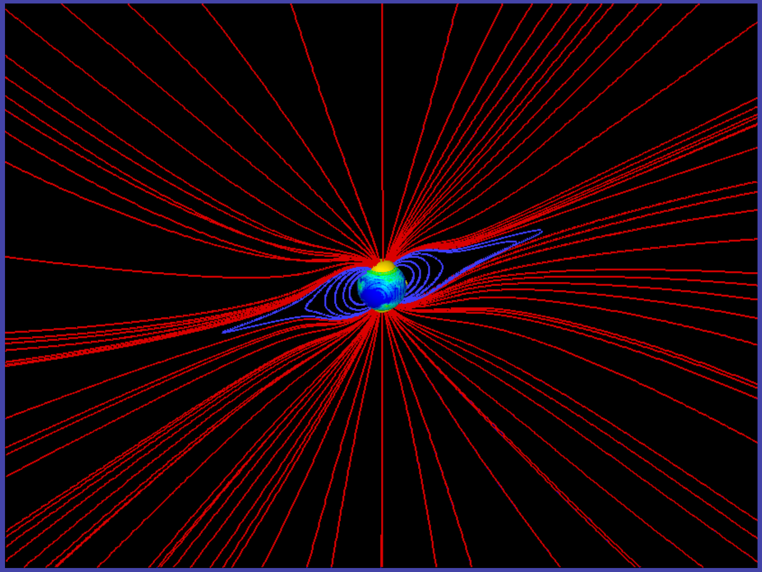


WSA

$$\int_{R_0}^{r} d\left(\frac{\gamma}{\gamma-1}\frac{p}{\rho}\right)+\frac{GM}{r}+\frac{u^2{}_{sw}}{2}=0$$



Bernoulli integral



Self-consistent MHD Solution

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- What we WANT TO have is an integrated model for the solar wind and for a frequency spectrum of the MHD turbulence
  - Recent observations from Hinode demonstrated that the MHD turbulence might be a source which powers, accelerates and heats the solar wind
  - Recent progress in supercomputers make it possible to use routinely 4D models (3 spatial +1 phase), if the dependence on the 4th coordinate is really important
  - The progress in computational technologies is also important and allows to maintain a good quality of numerical results on adaptive grids at realistic resolution (we benefit from TVD technologies at resolution interface and 7-wave Riemann solver for MHD)
  - Therefore, the coupled model with frequency-resolved spectrum of turbulence is doable

Proposal (PI R.Frazin)

- The coupled solar wind - turbulence model catches some significant effects, although at the time it can not compete with semi-empirical model.
- Being ‘the first principles based’, the model of turbulence in the close proximity of the Sun is too much dependent on the assumptions about wave-wave cascade.
- The proposed research should study the 3d distribution of the plasma density and those for electron and ion temperature. These observations will be used to constrain the model parameters: quantify the absorption mechanisms and the cascade rate:
  - The mechanism of the wave absorption on the Cherenkov resonance with electrons and that on the gyro-resonance with ions can be distinguished by measuring electron and ion temperatures separately
  - The faster is the cascade rate the more gradual is decrease in the solar wind temperature with the heliocentric distance

SEP - turbulence model

- The coupled model for SEP-turbulence should necessarily be formulated in terms of full 3D transport equation for the turbulence. There is no excuse for ‘truncating’ somehow the ‘Parker’ transport equation for turbulence as long as the Parker equation is used to describe the particle acceleration and transport

FLAMPA (Sokolov et al , Ap.J.Letters,2004)

- Consider as an example a diffusive kinetic equation (Parker,1966) governing the Diffuse-Shock-Acceleration Mechanism: THIS IS A 3D EQUATION

$$\frac{\partial f}{\partial t} + \mathbf{u} \cdot \nabla f = \nabla \cdot (\mathbf{D} \cdot \nabla f) + \frac{p}{3}\frac{\partial f}{\partial p}\nabla \cdot \mathbf{u}$$

- Here ***f*** is an isotropic part of the SEP distribution function, ***u*** is the bulk plasma velocity, ***D*** is the diffusion tensor.
  - Assume that the diffusion only occurs along the magnetic field line:

$$\mathbf{D} = D\mathbf{n}_B\mathbf{n}_B, \nabla \cdot (B\mathbf{n}_B) = 0$$

  - Assume the magnetic field to be frozen into the plasma
  - Introducing the Lagrangian coordinates along the magnetic field line, the kinetic equation can be written for each magnetic line separately:

$$\frac{df}{dt} = B\frac{\partial}{\partial s}\left(\frac{D}{B}\frac{\partial f}{\partial s}\right) - \frac{1}{3}\frac{d\ln\rho}{dt}\frac{\partial f}{\partial \ln p}, \quad ds^2 = d\mathbf{x}^2$$

Coupled SEP-Turbulence Model

In QL approximation, the growth rate of wave intensity is:

$$\Gamma = -\frac{4\pi^2 V_A}{k\left[I_+(k)+I_-(k)\right]}\int_{p_{res}(k)}^{\infty} dp\, p^2\, \frac{p p_{res}(k)}{m_i}\left[1-\frac{p^2_{res}(k)}{p^2}\right]\frac{\partial f_0}{\partial s}$$

Transport equation for waves can be also re-written in Lagrangian coordinates, using inverse rigidity of the resonant ions instead of the frequency, as a phase coordinate ($d_\sigma/dt=d/dt+V_{A\sigma}\partial/\partial s$)

$$\frac{D_\sigma Y_\sigma}{Dt}-\frac{\partial Y_\sigma}{\partial \ln h}\left[2\frac{D_\sigma}{Dt}\ln B-\frac{1}{2}\frac{D}{Dt}\ln\rho-\frac{1}{2}\frac{D_\sigma}{Dt}\ln\rho\right]=$$

$$=\left[\frac{1}{2}\frac{D}{Dt}\ln\rho+2\frac{D_\sigma}{Dt}ln B+\gamma\right]Y_\sigma$$

Spatial diffusion coefficient is:

$$D_{xx}=\frac{vB^2}{4\pi}\int_{k_r}^{\infty}\frac{dk\left(k^2-k_r^2\right)}{k^5\left[I_+(k)+I_-(k)\right]}$$

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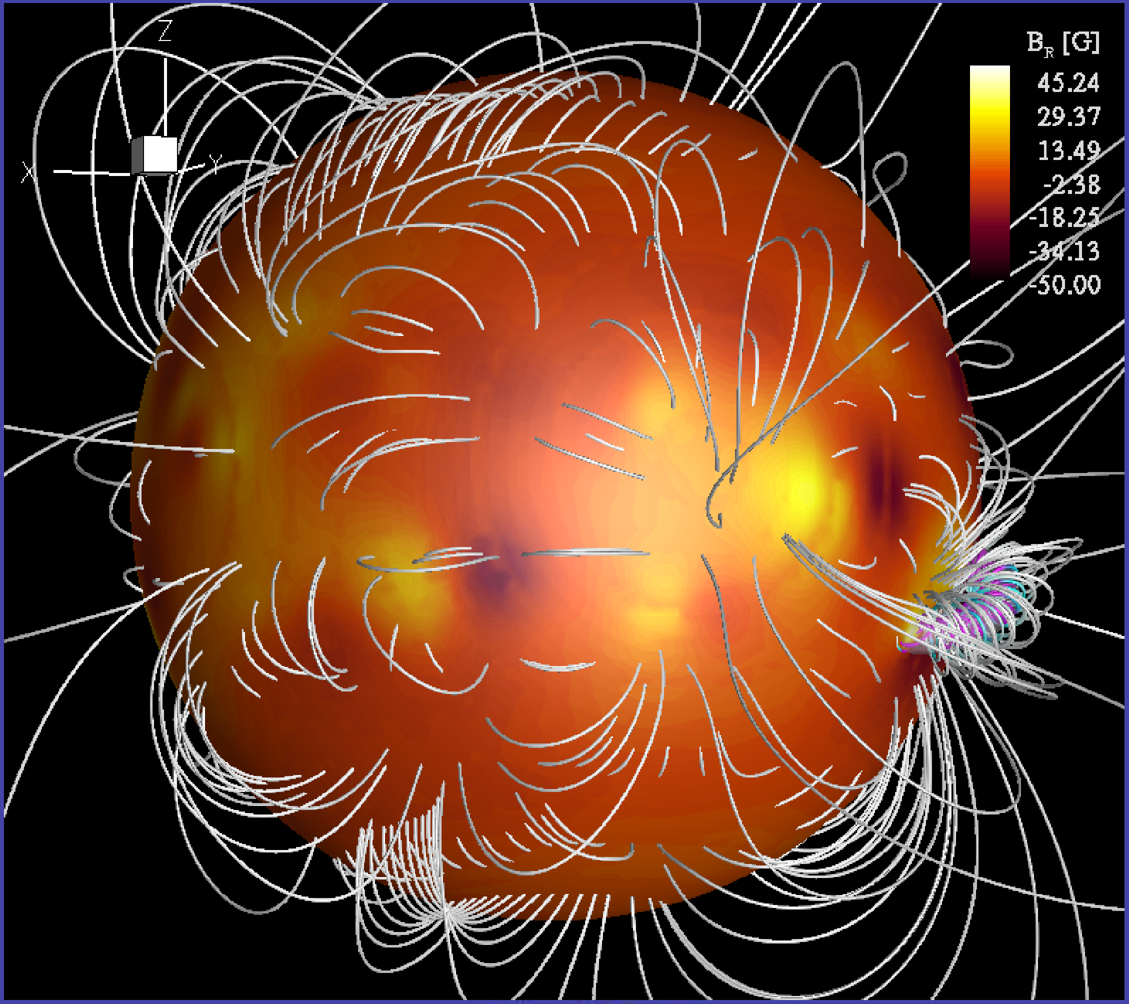
CME Dynamics of the April, 21, 2002 event

We Superimpose a Current Loop of a Finite R/a ratio

Image shows the superposition of a 3D force-free flux rope (Titov & Démoulin type). We subtract the idealized magnetic dipole field from the TD solution and add the background field taken from observations.

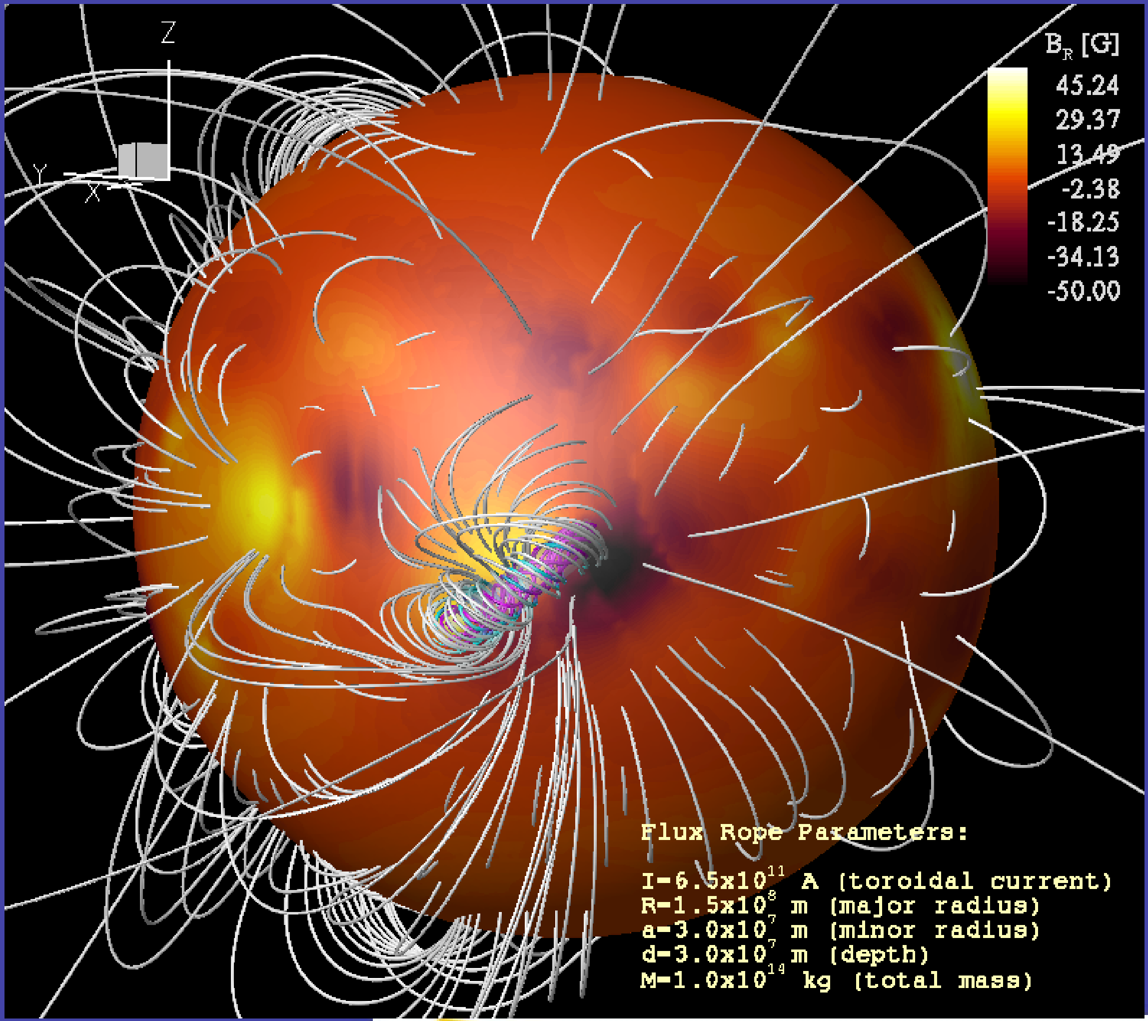
Flux rope in the ambient magnetic field would be exactly in a critical equilibrium, if the potential magnetic field in the active region is exactly equal to that of the magnetic dipole. Practically, some supercritical current is applied to accelerate the system evolution.



A view from another position

The flux rope orientation and size is chosen following

Cermedes&Bothman data for the Source Region geometry



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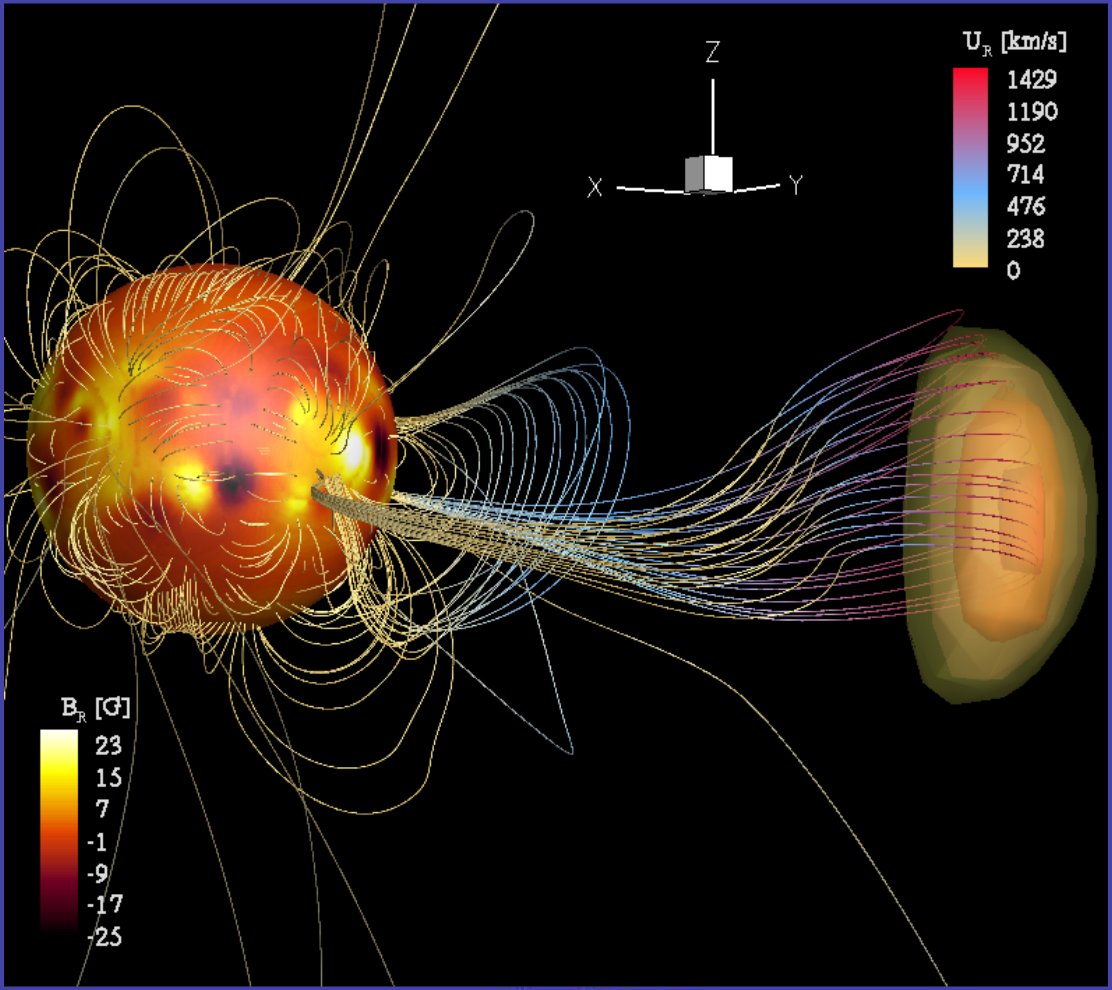
3D image of the CME

A view from the Earth position

Time=30 min after the CME.

Note: accelerated protons reached the Earth at this time.



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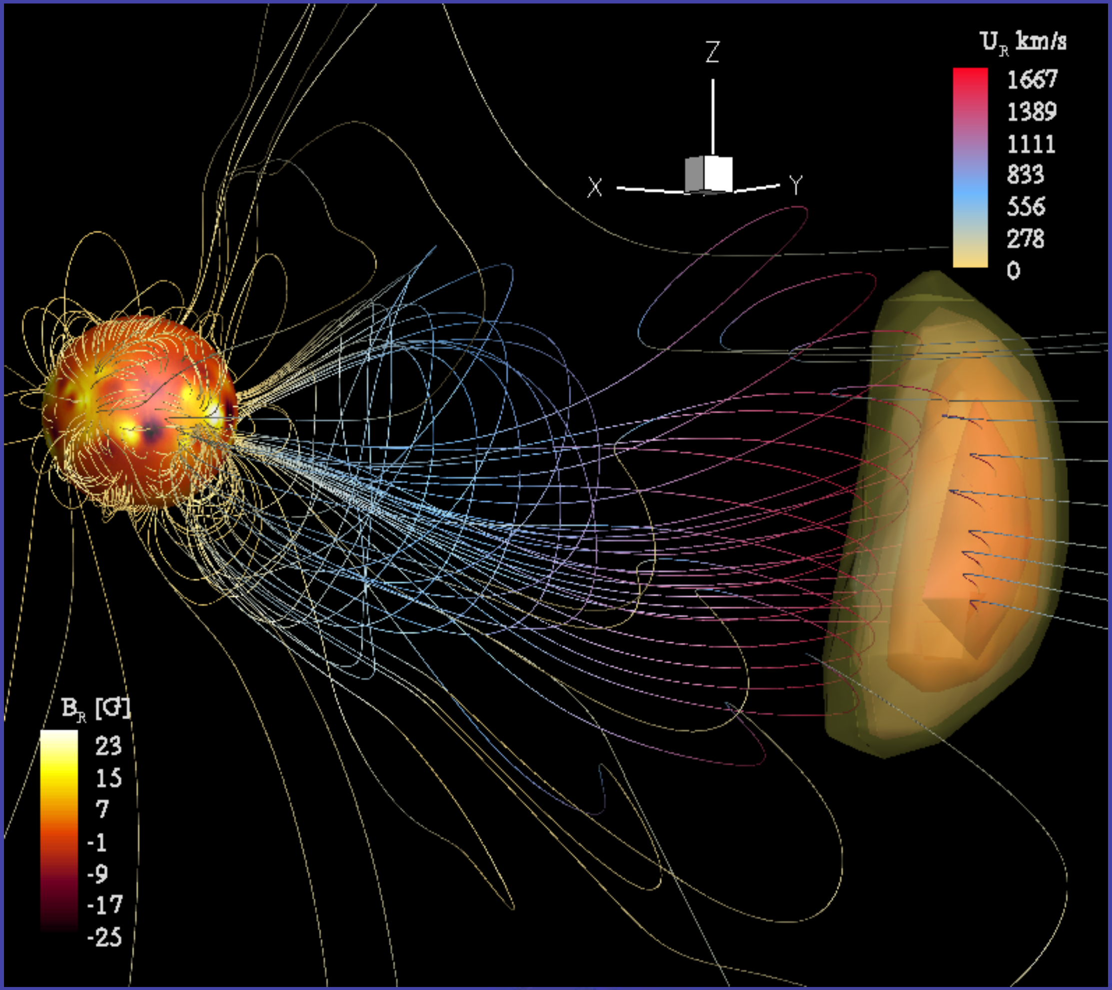
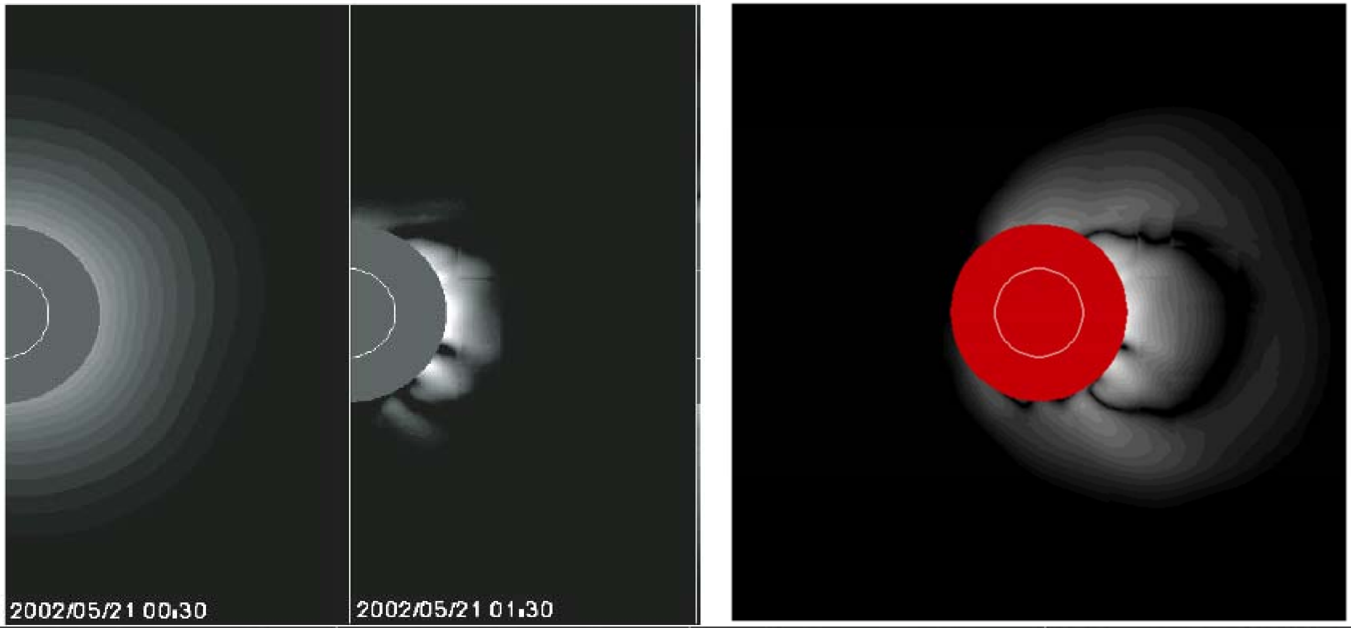
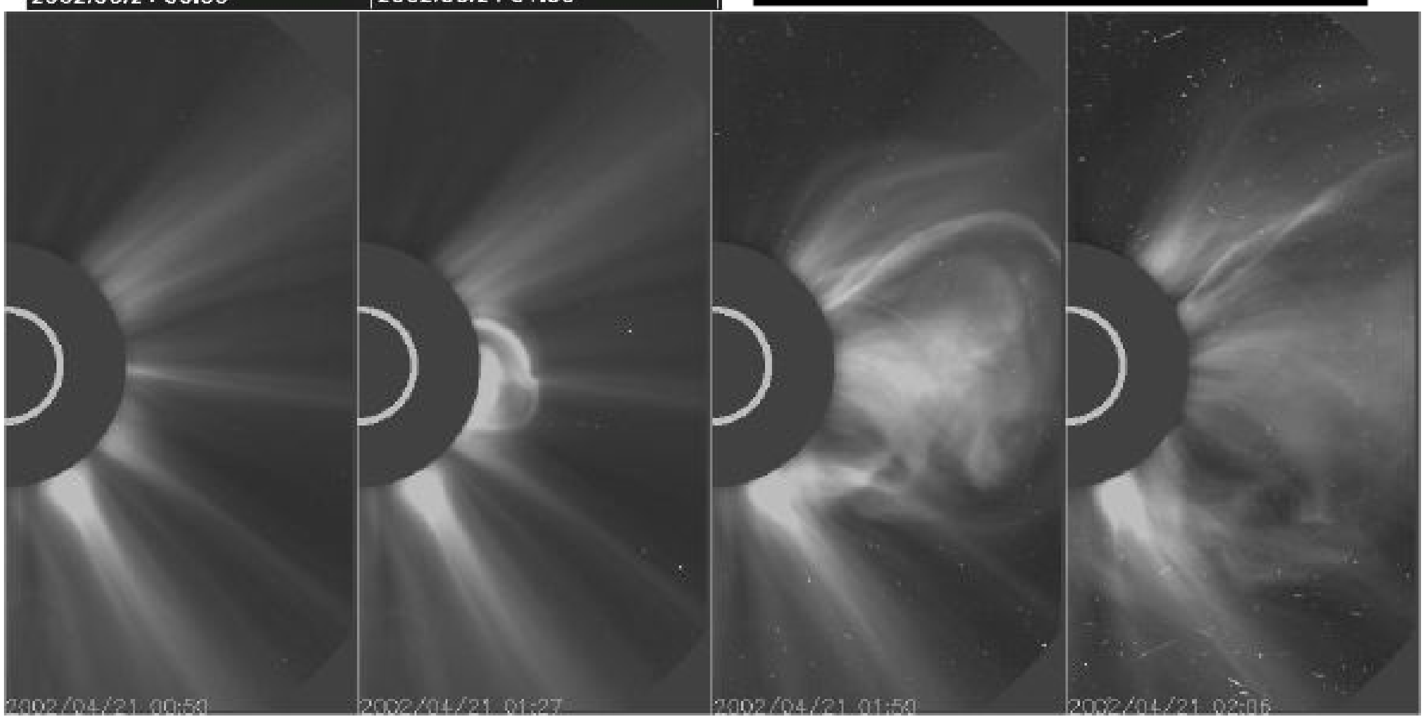
3D image of the CME

Time=60 min after the CME.



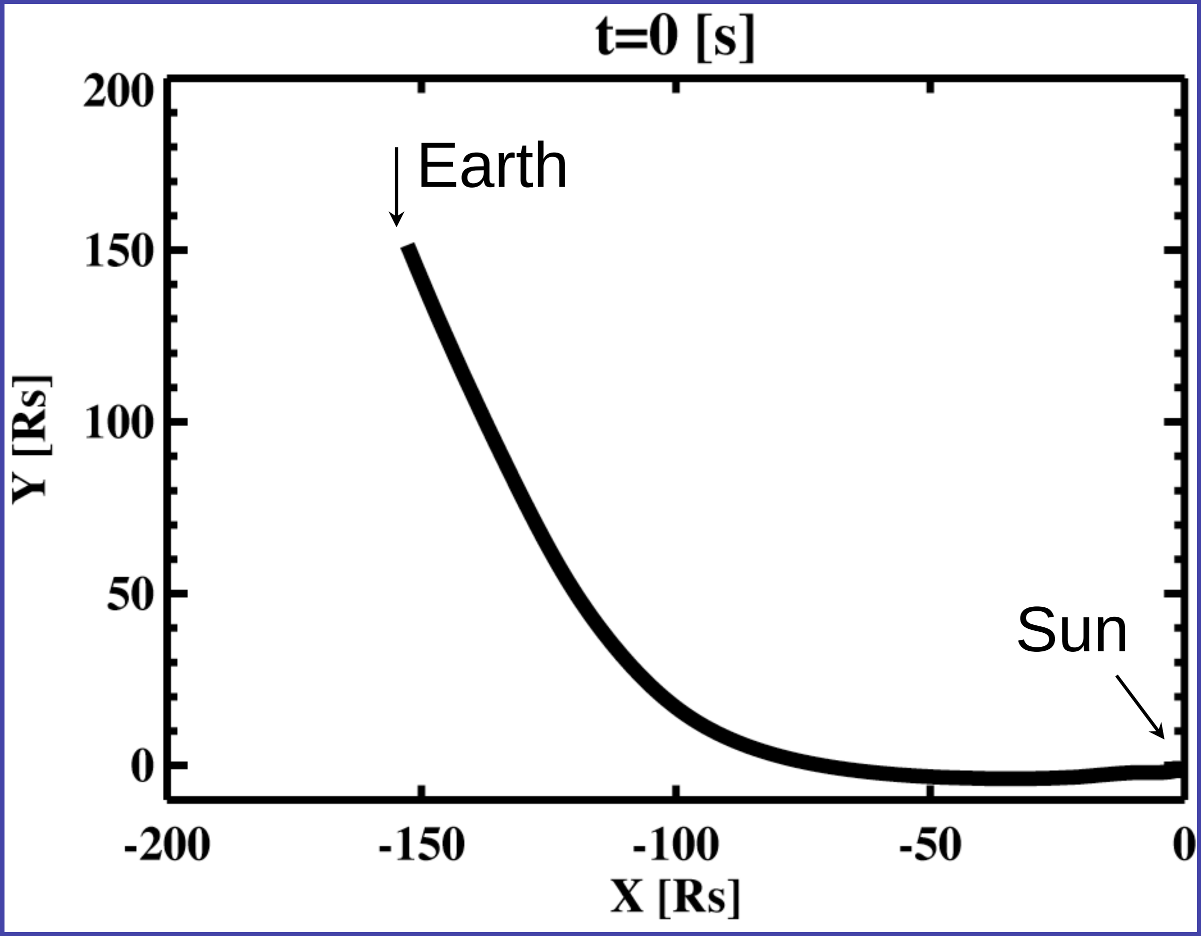
Comparison of the white light images





We extract the magnetic field line connecting the Earth position to the Sun

The line connects to the Sun near the west limb and the CME source! So the perfect magnetic connectivity with the SEP source occurred in the April, 21, 2002 event.



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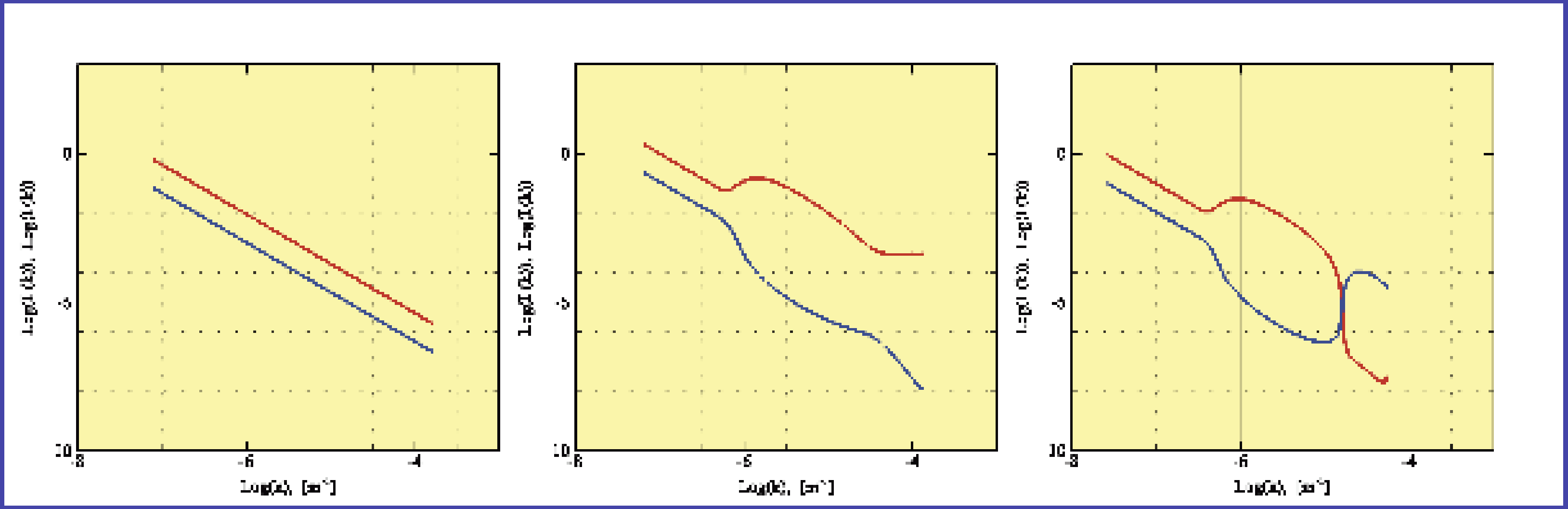
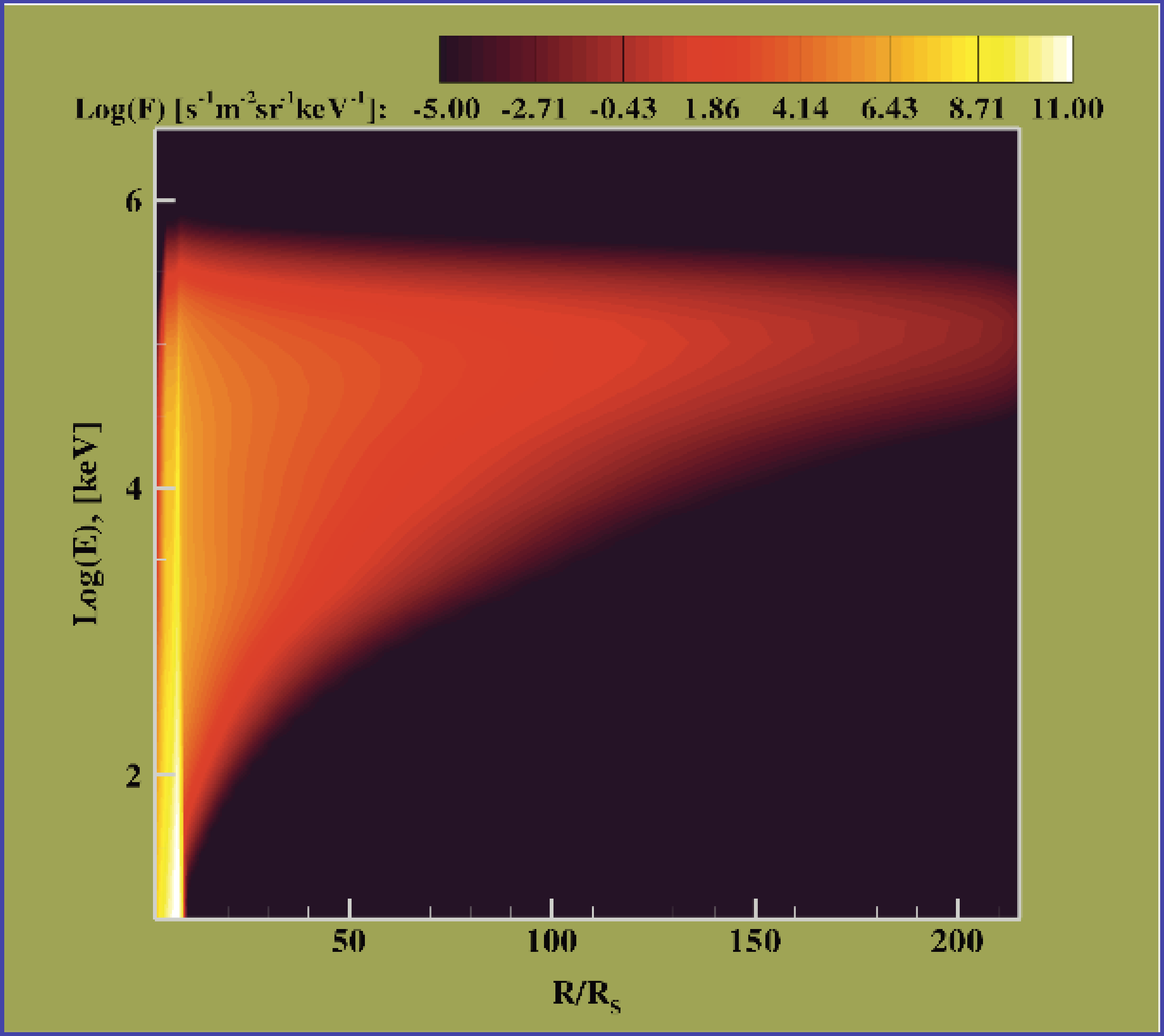
Self-Consistent Simulations with coupled turbulence

- Turbulence spectra (blue color is for waves propagating towards the Sun, red – for outward propagating waves), SW is the CME shock



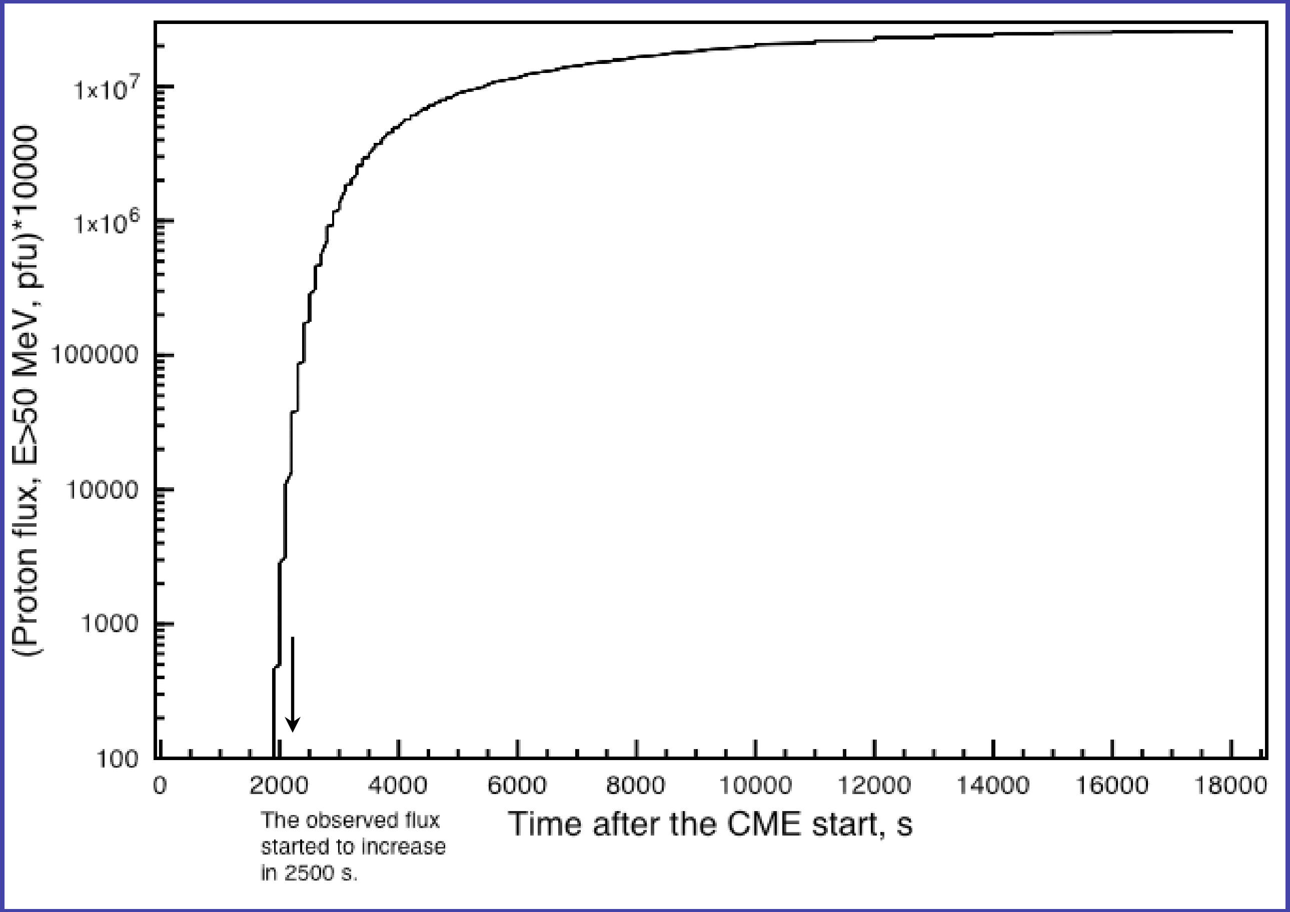
- Far away SW In front of SW Behind SW

Spatial profile of the accelerated particles



SEP flux at the Earth position.

Simulation results for the SEP onset time (30-40 min) and for an integral flux (2500 pfu) well match observation results.



Conclusion

- The coupled turbulence - solar wind model might be promising but at the time the result's quality needs to be improved and the model parameters needs to be constrained.
- We reported results of integrated SEP-turbulence model driven by a realistic CME simulation.