Multi-ion Magnetohydrodynamics

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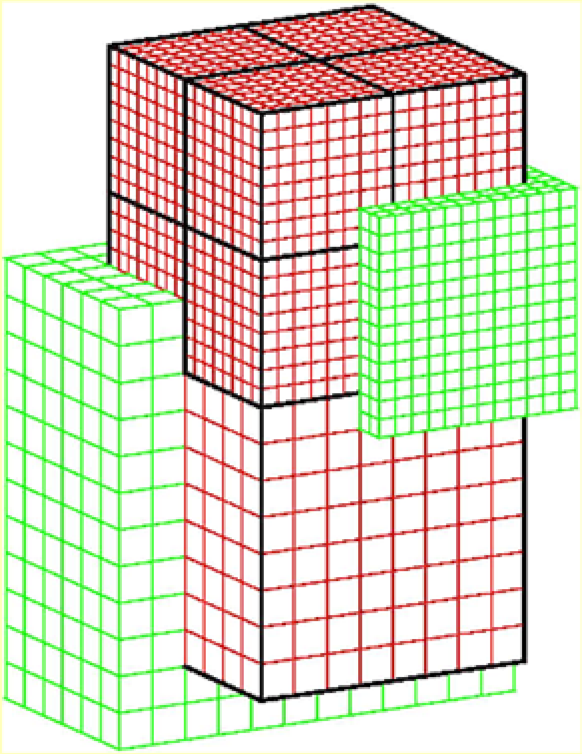
Outline

- BATS-R-US code
- Space Weather Modeling Framework
- Multi-fluid and multi-ion MHD
- Equations
- Algorithms
- Space physics applications
- Conclusions

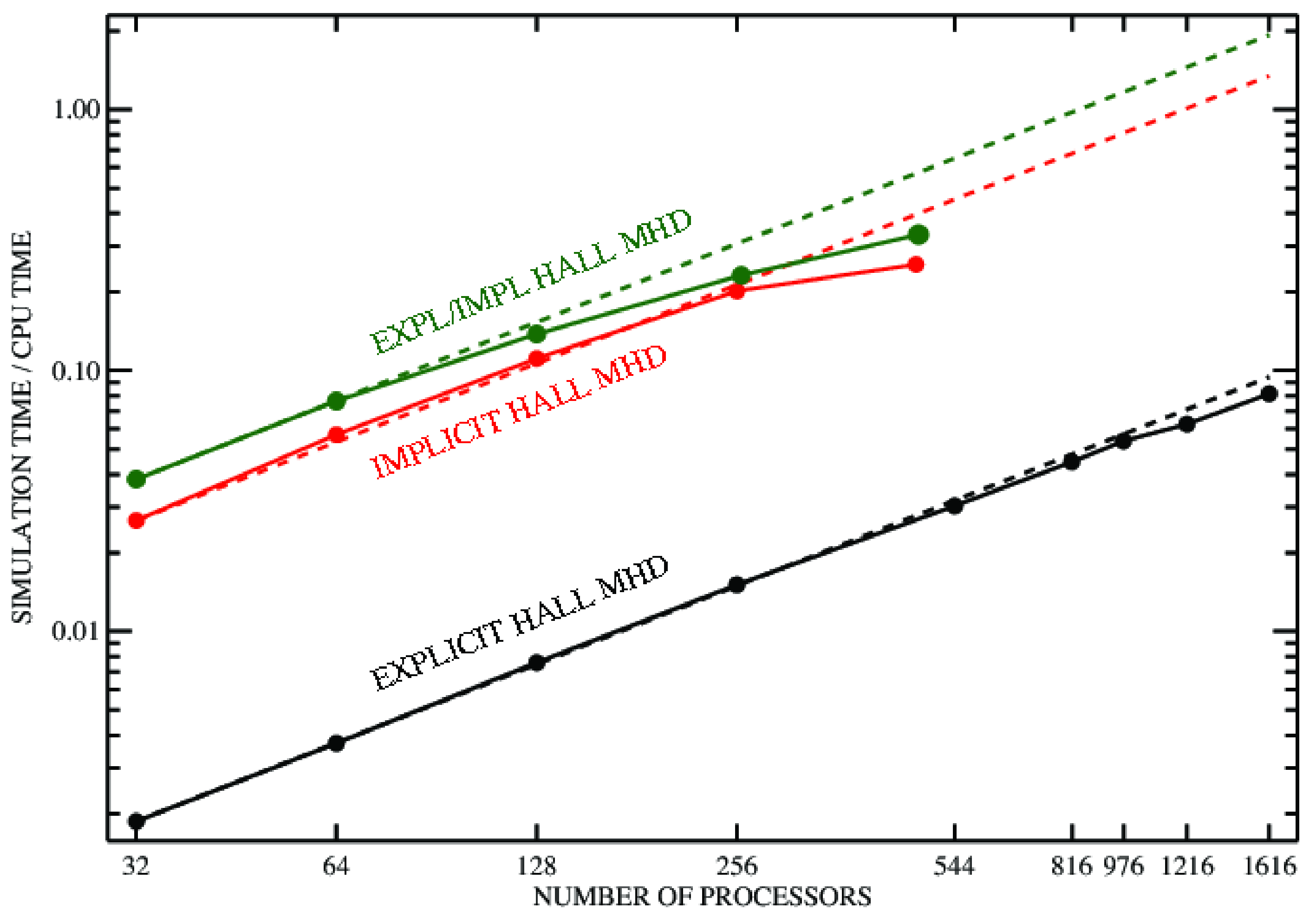
MHD Code: BATS-R-US

Block Adaptive Tree Solar-wind Roe Upwind Scheme



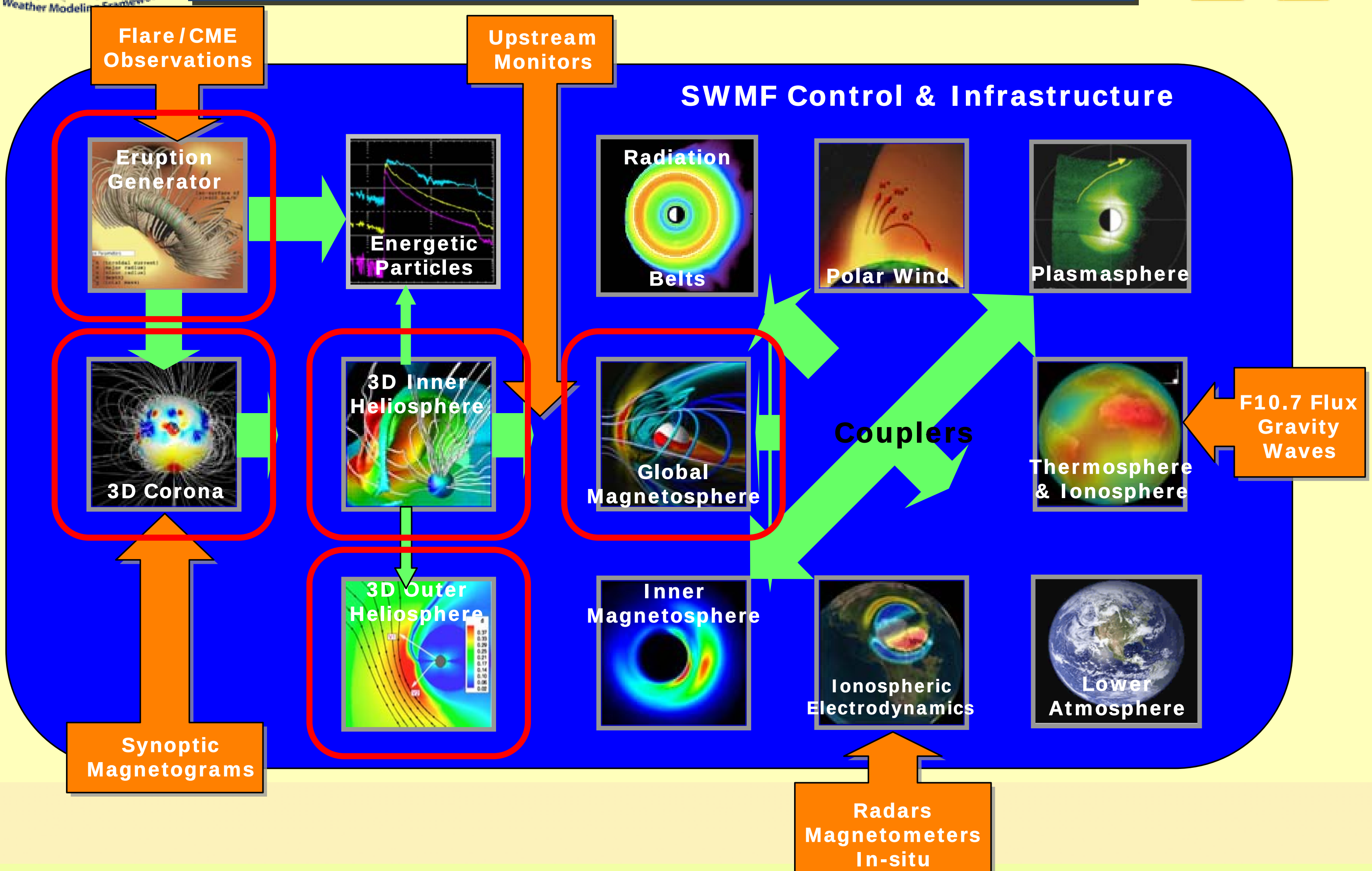
- Classical, semi-relativistic and Hall MHD
- Multi-species and multi-fluid MHD
- Radiation hydrodynamics with gray diffusion
- Multi-material, non-ideal equation of state
- Solar wind turbulence
- Conservative finite-volume discretization
- Parallel block-adaptive grid
- Cartesian and generalized coordinates
- Splitting the magnetic field into $B_0 + B_1$
- Divergence B control: 8-wave, CT, projection, parabolic/hyperbolic
- Shock-capturing TVD schemes: Rusanov, HLLE, AW, Roe, HLLD
- Explicit, point-implicit, semi-implicit, fully implicit time stepping
- 100,000+ lines of Fortran 90 code with MPI parallelization

Strong Parallel Scaling of BATS-R-US



Grid: 4804 blocks with 8x8x8 cells (2.5 million cells) ranging from 8 to 1/16 R_E. Simulations done on an SGI Altix machine.

BATS-R-US in the Space Weather Modeling Framework



The SWMF is freely available at http://csem.engin.umich.edu

Multi-Fluid MHD

- **Multi-Fluid MHD has many space physics applications**
  - ionospheric outflow, Earth magnetosphere, Martian ionosphere, outer heliosphere interaction with interstellar medium, etc.
- **BATS-R-US now contains a general multi-fluid solver with arbitrary number of ion and neutral fluids.**
- **Each fluid has separate densities, velocities and temperatures.**
- **One ion fluid + neutrals can be solved as MHD for ions, and HD for neutrals.**
- **Ions and neutrals are coupled by charge exchange and chemical reactions.**
- **Neutrals are coupled by collisions and chemical reactions.**
- **Coupling source terms can be evaluated point-implicitly.**

Multi-Ion MHD Derived

Momentum equations for ion fluids *s* with charge q_s and electrons with charge $-e$

$$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + I p_s) = +n_s q_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + S_{\rho_s \mathbf{u}_s}$$

$$\frac{\partial \rho_e \mathbf{u}_e}{\partial t} + \nabla \cdot (\rho_e \mathbf{u}_e \mathbf{u}_e + I p_e) = -n_e e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) + S_{\rho_e \mathbf{u}_e}$$

Express electric field from electron momentum equation neglecting small terms:

$$\mathbf{E} = -\mathbf{u}_e \times \mathbf{B} - \frac{1}{e n_e} \nabla p_e + \eta \mathbf{J}$$

Obtain electron density from charge neutrality and electron velocity from current:

$$n_e = \frac{1}{e} \sum_s n_s q_s$$

$$\mathbf{u}_e = -\frac{\mathbf{J}}{e n_e} + \mathbf{u}_+$$ where the charge averaged ion velocity is $$\mathbf{u}_+ = \frac{\sum_s n_s q_s \mathbf{u}_s}{e n_e}$$

The electron pressure p_e is either a fixed fraction of total ion pressure, or we solve

$$\frac{\partial p_e}{\partial t} + \nabla \cdot (p_e \mathbf{u}_e) = -(\gamma - 1) p_e \nabla \cdot \mathbf{u}_e + S_{p_e}$$

Multi-Ion MHD

For each ion fluid *s* we obtain (neglecting resistive terms):

Cannot be written in conservative form

Gyration of ions around each other. Can be stiff.

$$\frac{\partial \rho_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s) = S_{\rho_s}$$

$$\frac{\partial \rho_s \mathbf{u}_s}{\partial t} + \nabla \cdot (\rho_s \mathbf{u}_s \mathbf{u}_s + I p_s) = \frac{n_s q_s}{n_e e}(\mathbf{J} \times \mathbf{B} - \nabla p_e) + n_s q_s (\mathbf{u}_s - \mathbf{u}_+) \times \mathbf{B} + S_{\rho_s \mathbf{u}_s}$$

$$\frac{\partial p_s}{\partial t} + \nabla \cdot (p_s \mathbf{u}_s) = -(\gamma - 1) p_s \nabla \cdot \mathbf{u}_s + S_{p_s}$$

We can also solve for *hydro* energy density $e_s = \rho_s \mathbf{u}_s^2/2 + p_s/(\gamma - 1)$

$$\frac{\partial e_s}{\partial t} + \nabla \cdot [(e_s + p_s)\mathbf{u}_s] = \mathbf{u}_s \cdot \left[\frac{n_s q_s}{n_e e}(\mathbf{J} \times \mathbf{B} - \nabla p_e) + n_s q_s (\mathbf{u}_s - \mathbf{u}_+) \times \mathbf{B}\right] + S_{e_s}$$

Finally the induction equation with or without the Hall term becomes

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u}_e \times \mathbf{B}) = 0 \quad \text{or} \quad \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{u}_+ \times \mathbf{B}) = 0$$

Two-Stream Instability

- **Perpendicular ion velocities are coupled through the magnetic field**
- **Parallel ion velocities are not coupled by the multi-ion MHD equations.**
- Two-stream instability restricts the velocity differences parallel to **B**
  - We cannot resolve the two-stream instability
  - Use a simple ad-hoc friction source term in the momentum equations:

$$S_{\rho \mathbf{u}_s}^{friction} = \frac{1}{\tau_c} \sum_{q \neq s} \min(\rho_s, \rho_q)(\mathbf{u}_q - \mathbf{u}_s) \left(\frac{|\mathbf{u}_s - \mathbf{u}_q|}{u_c} \right)^{\alpha_c}$$

  - Using the minimum of the two densities makes the friction uniformly effective in regions of low and high densities.
  - τ_C is the time scale, u_C is the cut-off velocity, α_C is the cut-off exponent
  - Currently we use fixed parameters.
  - We will explore physics based parameter setting and formulas in the future.

Conservation

- **Multi-ion MHD equations cannot be written in conservation form.**
- **We would still like to maintain conservation for the total ion fluid.**
- **Density and the hydro part of the energy equations are in conservation form.**
- **Possible scheme for conserving total ion momentum:**
  - Solve for total ion fluid momentum using the following total ion pressure tensor:

$$P = \sum_s I p_s + \rho_s(\mathbf{u}_s - \mathbf{u})(\mathbf{u}_s - \mathbf{u})$$

  - Use conservative equation for total ion momentum, and non-conservative (there is no other choice) for the individual ion momenta.
  - Distribute total momentum (if all ions move in the same direction) among the ion fluids proportionally to the individual solutions:

$$(\rho_s \mathbf{u}_s)^{n+1} = (\rho_s \mathbf{u}_s)^* \frac{(\rho \mathbf{u})^{n+1}}{\sum_q (\rho_q \mathbf{u}_q)^*}$$

  - For ion momentum components with mixed signs do the opposite:

$$(\rho \mathbf{u})^{n+1} = \sum_s (\rho_s \mathbf{u}_s)^*$$

Positivity

- **Positivity is difficult to maintain in empty regions where some of the fluids do not occur.**
- **In some problems we can identify effectively single-ion regions based on geometry and/or physical state.**
  - For example the solar wind has high Mach number.
- **In other problems we have to check after every time step if any of the fluids have very small density or pressure relative to the total.**
- **For *minor* fluids**
  - Density is set to a small fraction ($\sim 10^{-4}$) of the total ion density.
  - Velocity and temperature are set to the same as for the total ion fluid.
  - This is a physically meaningful state that can interact properly with the truly multifluid regions.

Stability

- **Naïve explicit scheme is unconditionally unstable.**
- **Fully implicit scheme can be slow due to many variables.**
- **We can combine explicit scheme with point-implicit source terms:**

$$(\rho_s \mathbf{u}_s)^{n+1} = (\rho_s \mathbf{u}_s)^n - \Delta t \nabla \cdot \mathbf{F}^n + \Delta t S^{n+1}_{\rho \mathbf{u}_s}$$

$$+\Delta t \left[\frac{q_s}{M_s} (\rho_s \mathbf{u}_s - \rho_s \mathbf{u}_+)^{n+1} \times \mathbf{B}^n + \frac{n_s^n q_s}{n_e^n e} (\mathbf{J}^n \times \mathbf{B}^n - \nabla p_e^n) \right]$$

where M_S is the mass of ion s.

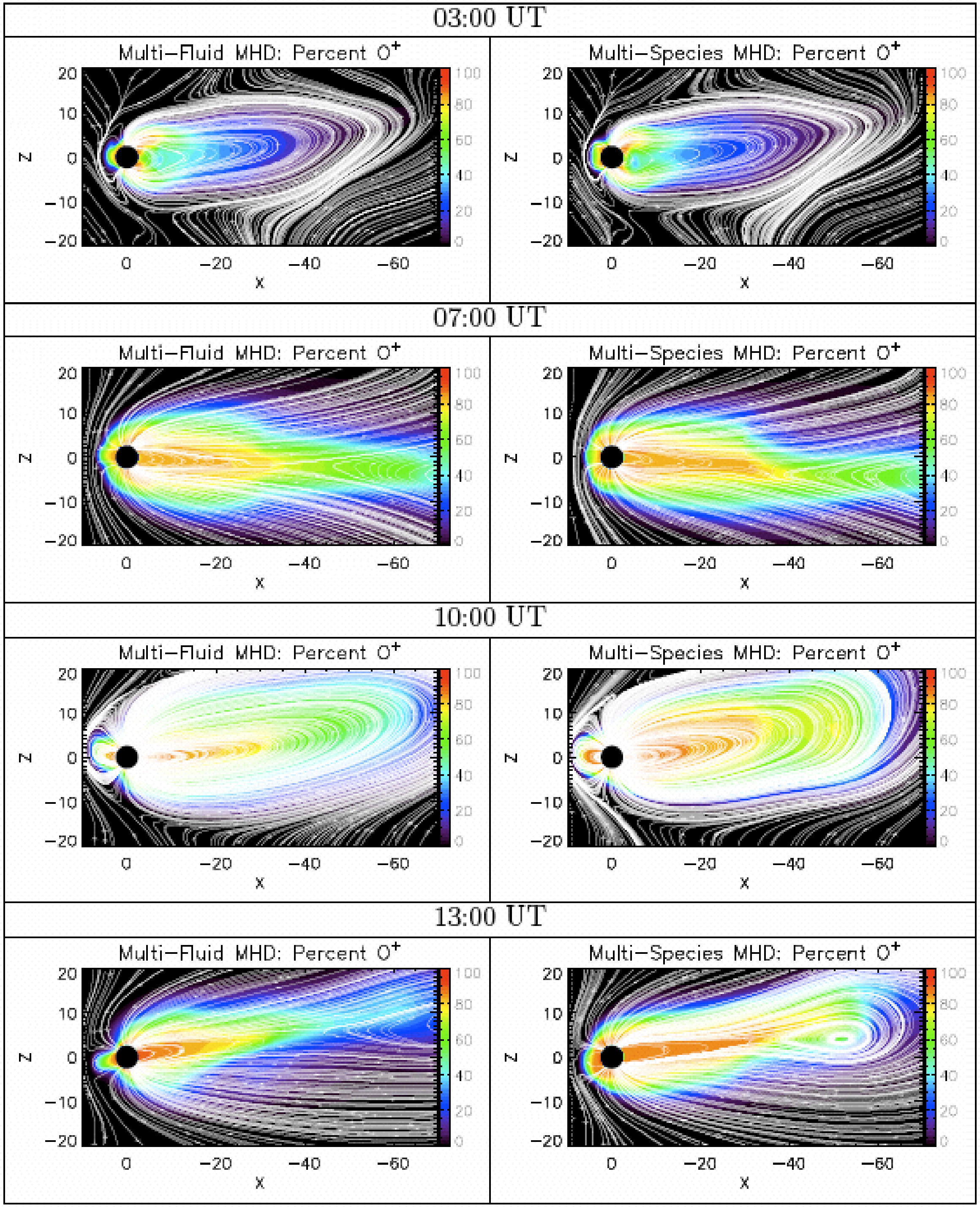
- The linear equations can be solved in every grid cell independently.
- The unknowns are the momenta of the ion fluids.
- The three spatial components are coupled by the artificial friction term.
- We use an analytic Jacobian matrix for sake of efficiency and accuracy.

Initial Results
(Glocer et al, submitted to JGR)

- **Modeling two magnetic storms**
  - May 4, 1998
  - March 31, 2001
- **Multi-fluid BATS-R-US running in the SWMF coupled with**
  - Polar Wind Outflow Model
  - Ridley Ionosphere-electrodynamics Model
  - Rice Convection Model (inner magnetosphere)
- **Comparison with**
  - single fluid model
  - global indexes (Dst, CPCP)
  - in situ satellite measurements

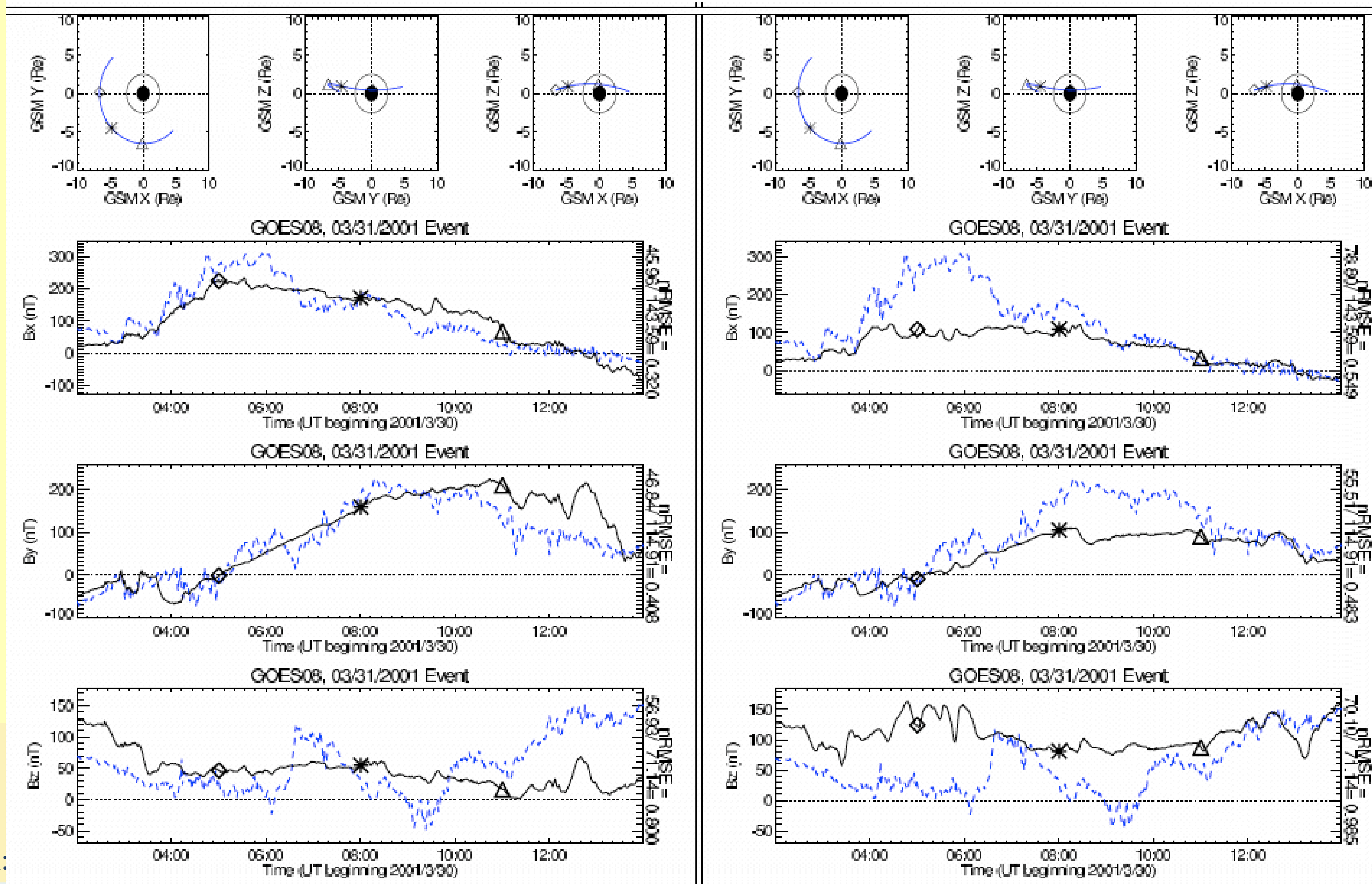
O^+/H^+ Ratio for March 31 Storm

- **Multi-Fluid vs. Multi-species**
  - Similar near Earth
  - Different further away

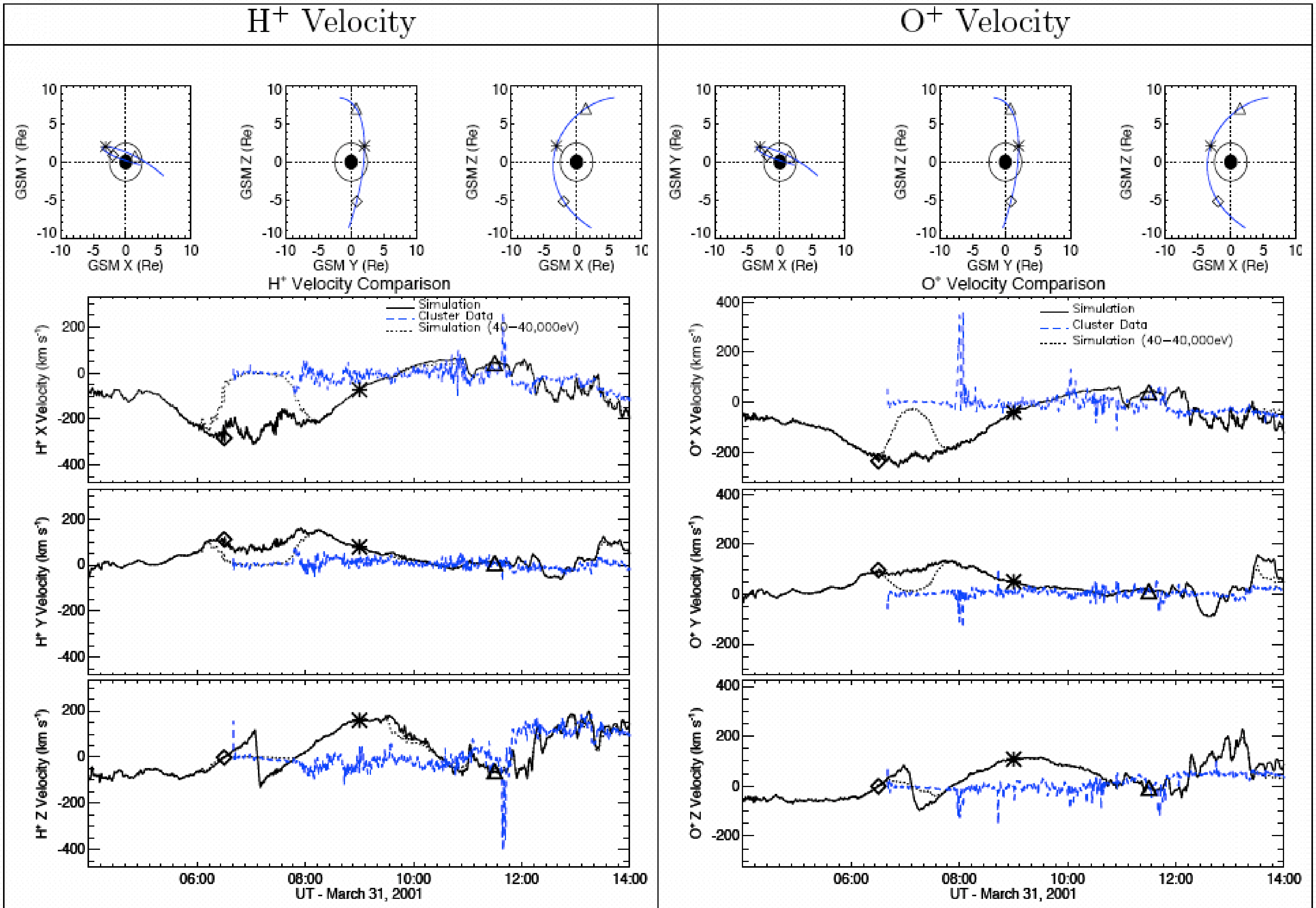


Magnetic Field vs Goes 8 Satellite

Multi-fluid MHD with O^+ outflow | Single-fluid MHD with no outflow

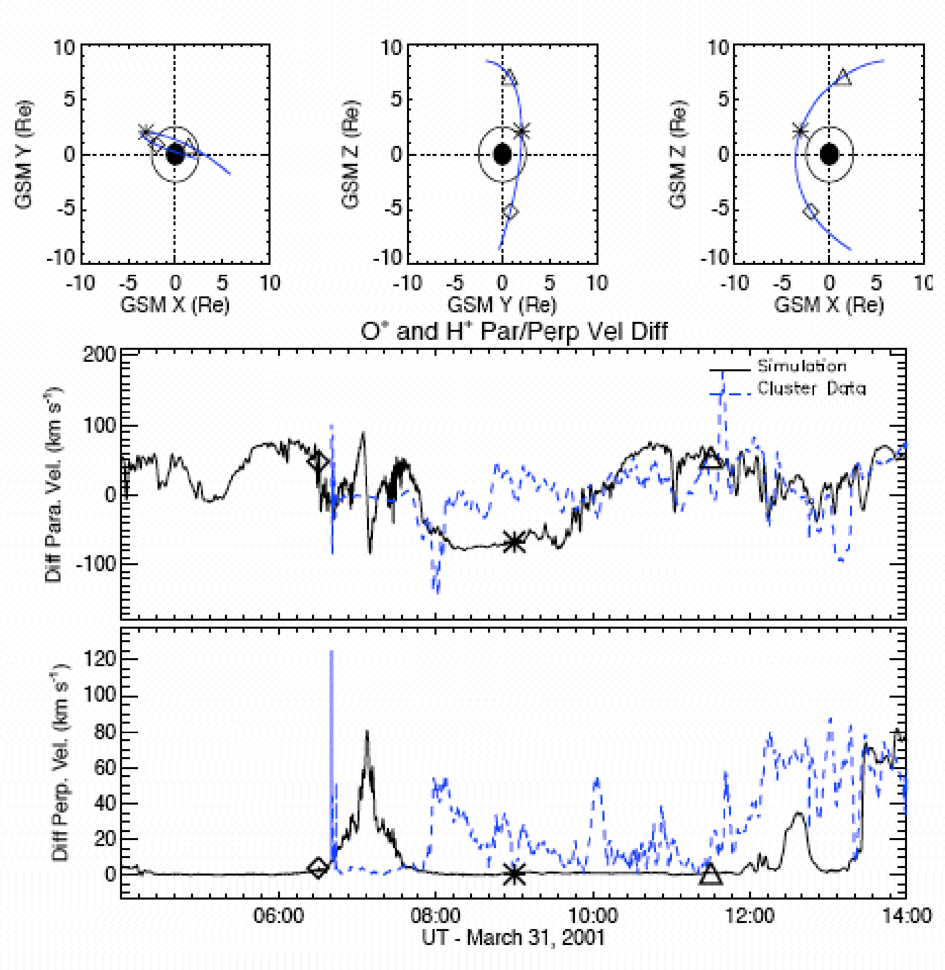


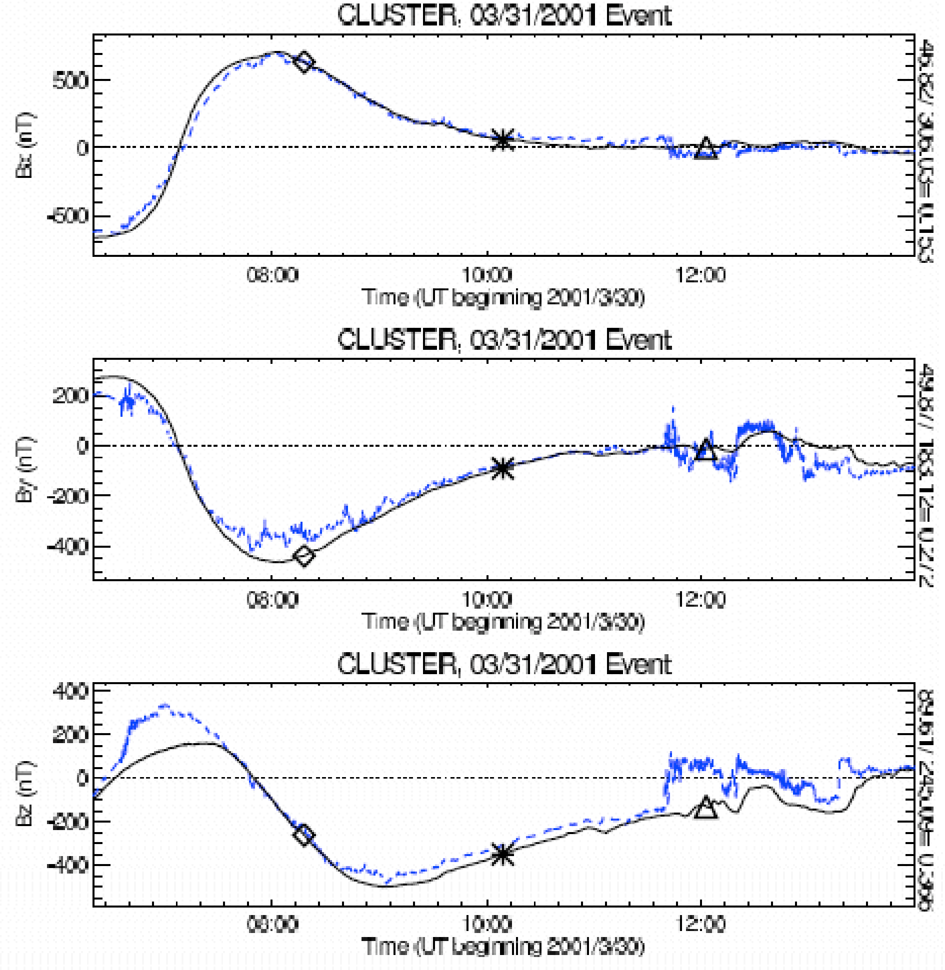
Velocities vs Cluster Satellite



Velocity Differences and Magnetic Field

O^+ and H^+ Par/Perp Velocity Diff	Magnetic Field





Conclusions

- We have implemented a general multi-fluid MHD solver in BATS-R-US.
- Issues of conservation, positivity and stability have been addressed.
- Two-stream instability is mimicked by an artificial friction term.
- Initial results in some space physics applications are promising although there is a lot of room for improvement in matching the observed data.
- Work in progress for the Mars ionosphere interaction with solar wind.