IMPLICIT MULTI-D SIMULATIONS OF STELLAR INTERIORS

CONVECTION IN PULSATING STARS

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OUTLINE

- Astrophysical motivation
- Code description
- Test cases
- Stellar models (preliminary results)
- Conclusion

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ASTROPHYSICAL MOTIVATION
IMPLICIT APPROACH

- Explicit schemes are prone to the well known CFL restriction on the time step
- First order term : e.g. advection $\Delta t_{CFL} \sim \frac{h}{v}$
- Second order term : e.g. diffusion $\Delta t_{CFL} \sim \frac{h^2}{k}$
- Two approaches
  - splitting : explicit advection + implicit diffusion
  - "all in one" : all implicit (advection + diffusion) ←
- Applications :
  - Convection in stellar envelope
  - Early phase of stellar evolution (link between the hydro phase of gravitational collapse and the hydrostatic phase of stellar evolution), rotation, late phase of stellar evolution (pre-SN stage)

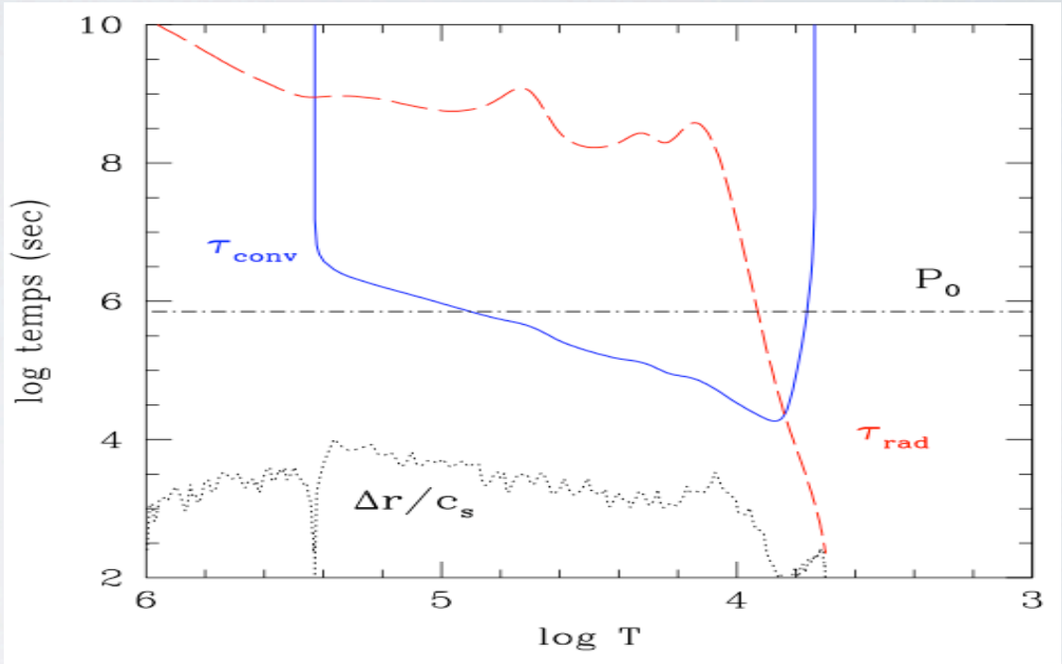
ASTROPHYSICAL MOTIVATION
CONVECTION-PULSATION INTERACTION

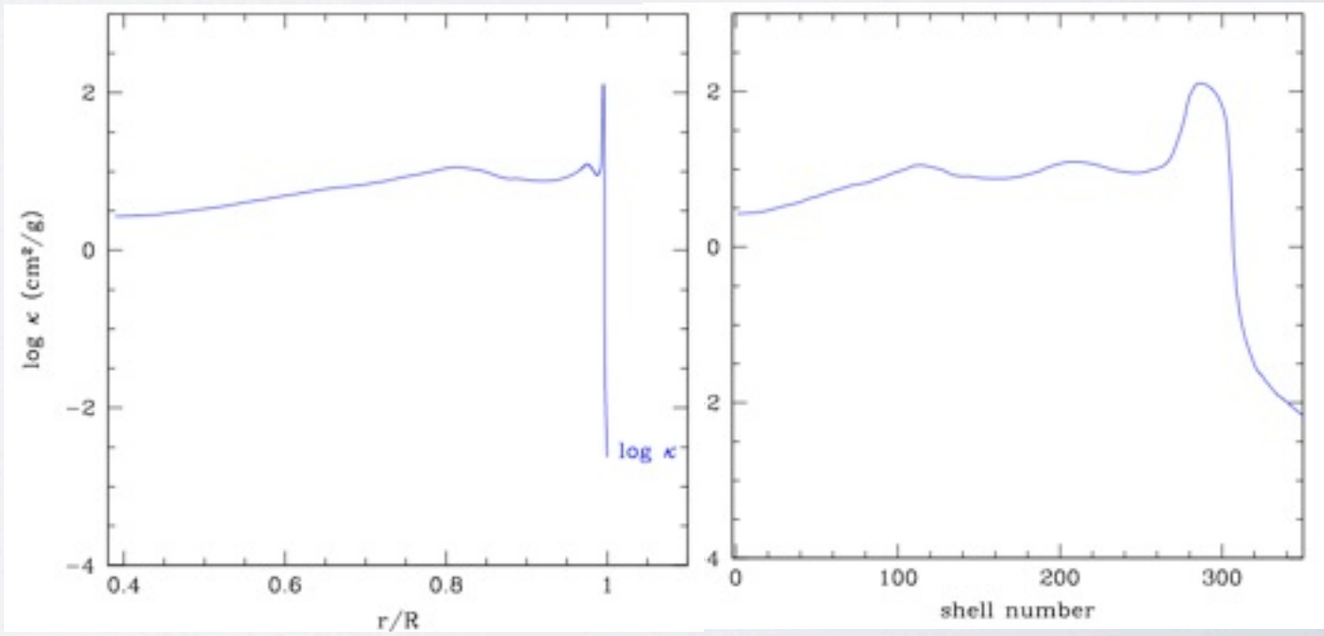
- Cepheid stars :
  - Stars located in the instability strip of the HR diagram
  - Unstable radial pressure mode => kappa mechanism
- Problem : toward low T_{eff}, linear stability analysis predicts unstable modes, in contradiction with observations
- Hypothesis : strong convection in cool stars damps the oscillation

➡ Multi-D models are required

ASTROPHYSICAL MOTIVATION
CEPHEIDS

- Characteristic timescales in the pulsating envelope :
  - Pulsation period ~ 10^6 s
  - Growth rate ~ 10^9 s
  - Convection ~ 10^4 s
- Fine spatial resolution is required
  - Stable explicit time-step : $\lesssim 10^2$ s





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CODE DESCRIPTION
HYDRODYNAMIC EQUATIONS

- Navier-Stokes equations
  - Artificial viscosity
- Thermal diffusion :
  - Diffusion approximation : $k_r = \frac{4acT^3}{3\kappa\rho}$
- Realistic EOS & opacities :
  - Tabulated for stellar interior

$$\frac{\partial}{\partial t}\rho + \vec{\nabla}.(\rho\vec{v}) = 0$$

$$\frac{\partial}{\partial t}\rho\vec{v} + \vec{\nabla}.(\rho\overline{\overline{vv}} - \overline{\overline{\tau}})) = -\vec{\nabla}P + \rho\vec{g}$$

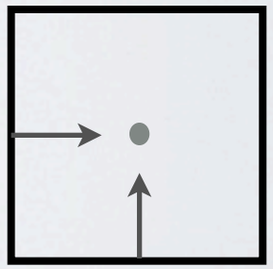
$$\frac{\partial}{\partial t}\rho e + \vec{\nabla}.(\rho e\vec{v}) = \epsilon_{\text{visc}} - P\vec{\nabla}.\vec{v} - \vec{\nabla}.\vec{F}_r$$

with $\vec{F}_r = -k_r\vec{\nabla}T$

$$P = P(\rho, e) \quad T = T(\rho, e) \quad \kappa = \kappa(\rho, T)$$

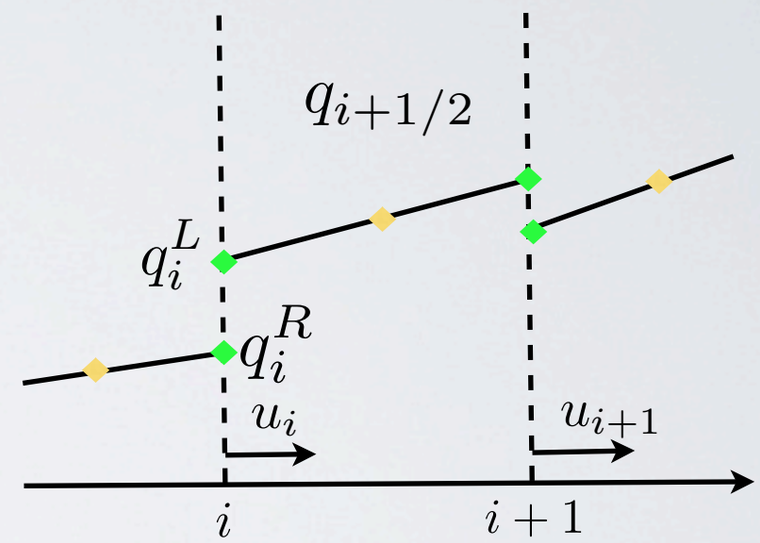
CODE DESCRIPTION
SPATIAL DISCRETIZATION

- 2.5D :
  - Spherical geometry with axial symmetry (r, θ)
- Finite volume discretization
- Staggered grid
  - Scalar values (ρ, e) are defined at cell center
  - Vector components are defined at the corresponding cell boundaries



- Extension to 3D planned in the future

CODE DESCRIPTION
SPATIAL & TEMPORAL DISCRETIZATION



- Flux are computed at cell interfaces
  - Values at interface are computed with an upwind interpolation scheme (Van Leer 2^d order)

- Time marching scheme :
  - β =1/2 : Crank-Nicholson (2^d order)

$$\Delta q + < R(q) >= 0$$

with $< R >= \beta R^{n+1} + (1-\beta) R^n$

CODE DESCRIPTION
IMPLICIT TIME MARCHING SCHEME

- Implicit scheme : going from time step n to time step n+1 involves the resolution of a nonlinear system of equations
  - Newton-Raphson iteration
- Linear system inversion :
  - Direct method (LU decomposition) with MUMPS
    (MUltifrontal Massively Parallel sparse direct Solver)

- $U^{(0)} = U^n$
- $\delta U^{(k)} = -J^{-1} \times D(U^{(k)})$

  $U^{(k+1)} = U^{(k)} + \delta U^{(k)}$

  until $\max(\frac{\delta U^{(k)}}{U^{(k)}}) \leq \epsilon$
- $U^{n+1} = U^{(k)}$

$$J = \frac{\partial D}{\partial U}$$

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- Stellar models
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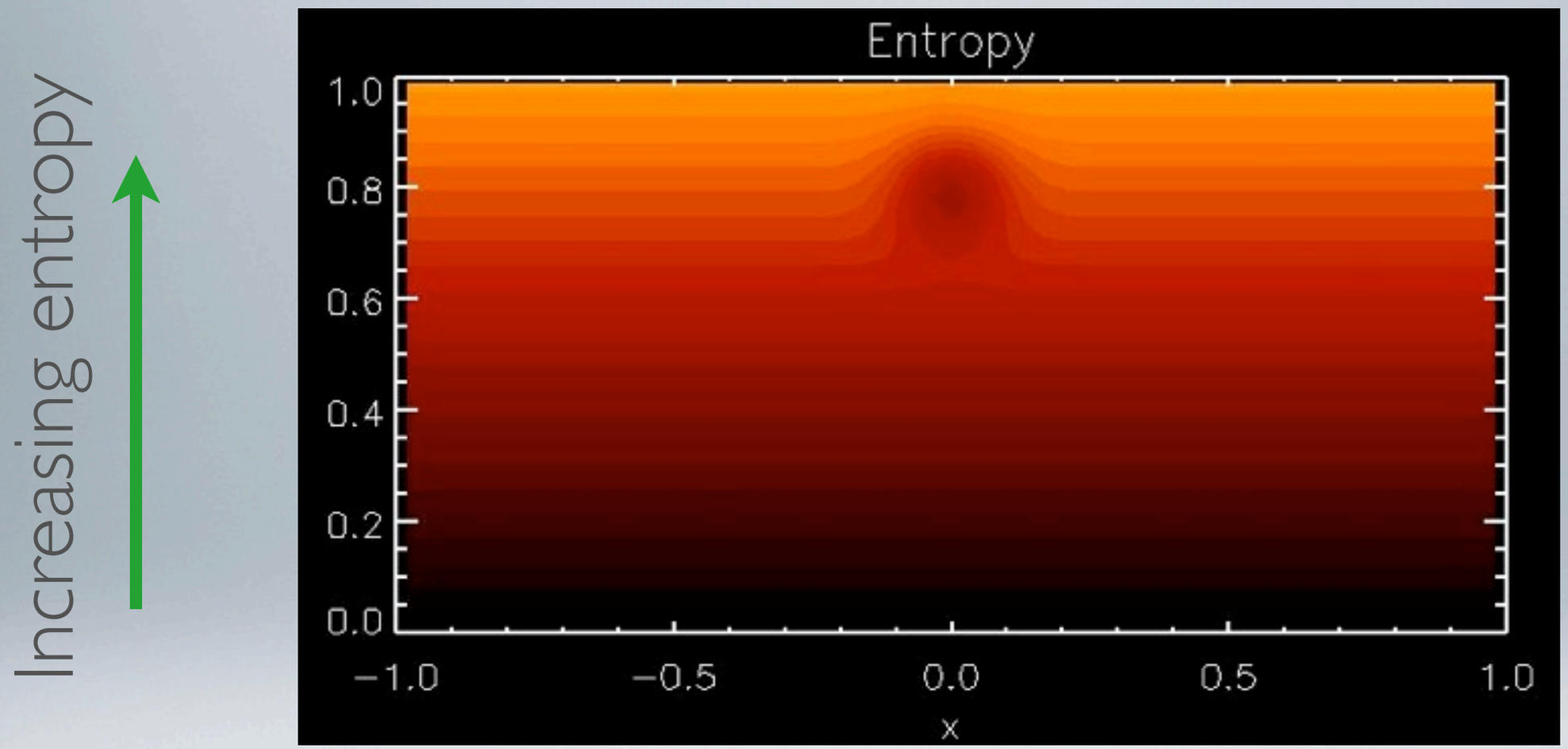
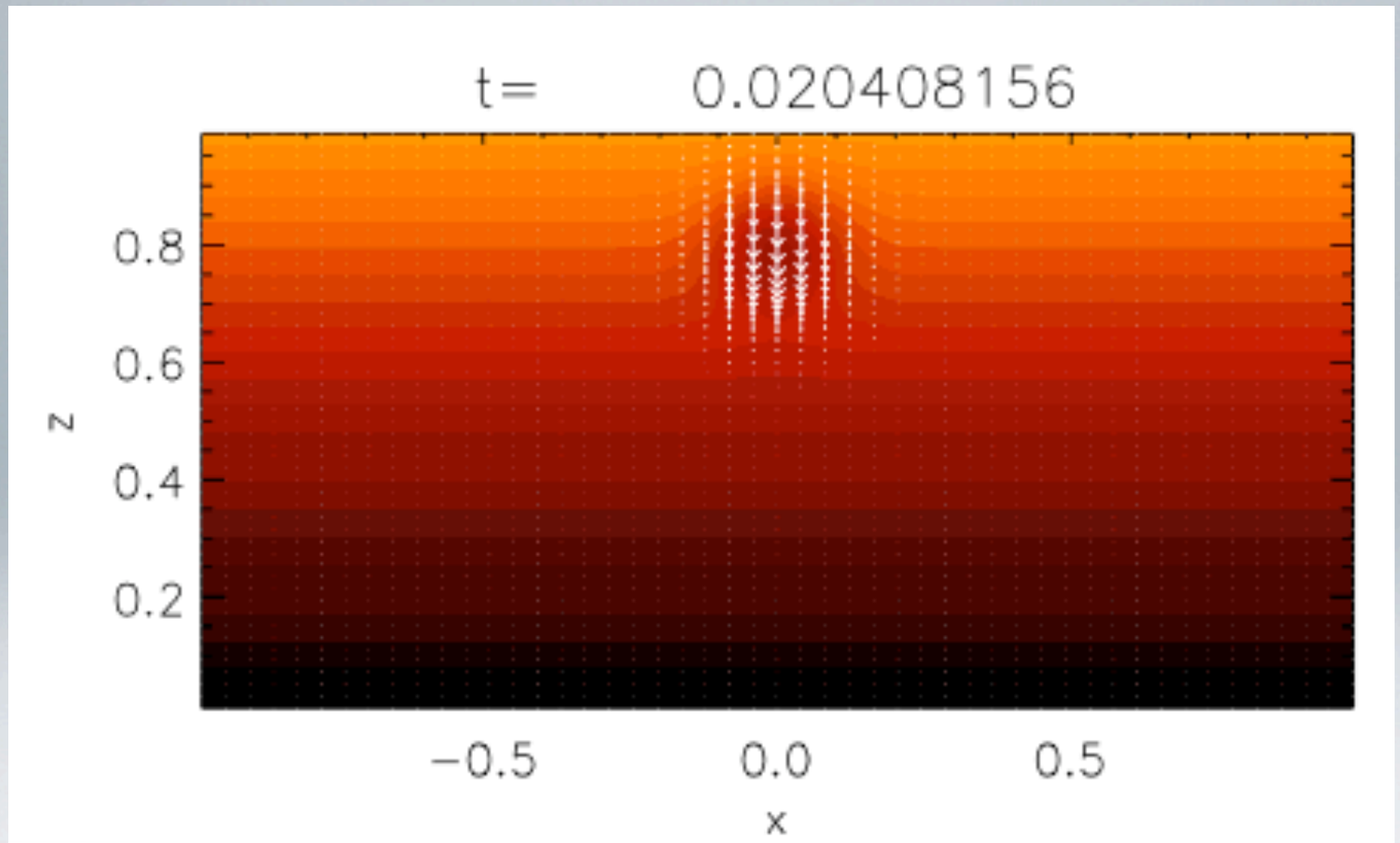
TEST CASES

OSCILLATING ENTROPY BUBBLE



- Setup (see Dintrans & Brandenburg 2004) :
  - Cartesian domain : [-1,1]x[0,1]
  - Stable stratified isothermal atmosphere ($g=1$, $c_s=1$)
  - **No thermal diffusion**, constant viscosity $\nu=5\times10^{-4}$
  - Entropy perturbation at z=0.8 (in hydrostatic equilibrium)

TEST CASES
OSCILLATING ENTROPY BUBBLE



Resolution : 50^2
Time : 50

B.C. : $u_z = 0$ at z=0,1
periodic conditions at x=-1,1

OSCILLATING ENTROPY BUBBLE ANALYSIS

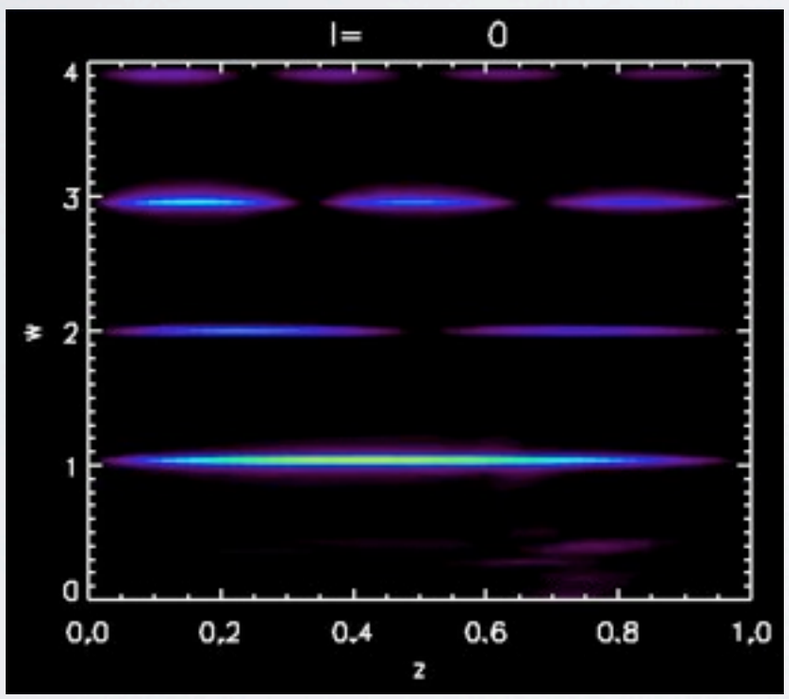
- Bubble oscillations excites gravity modes
- Non-exact hydrostatic equilibrium excites sound waves
- The problem is fully analytic, eigenmodes (n,l) and eigenfrequencies are known
  - Vertical sound waves (l=0): $\omega_n = (n+1)\pi \quad P \leq 2$
  - Gravity modes : $\omega_{l,n} \leq N \sim 0.82 \quad P \geq 7.6$
- Two physical processes with separate time scales => good lab for testing different choices of **Δ**t

ENTROPY BUBBLE, 50X50, T=50

ACOUSTIC MODES (ℓ =0)

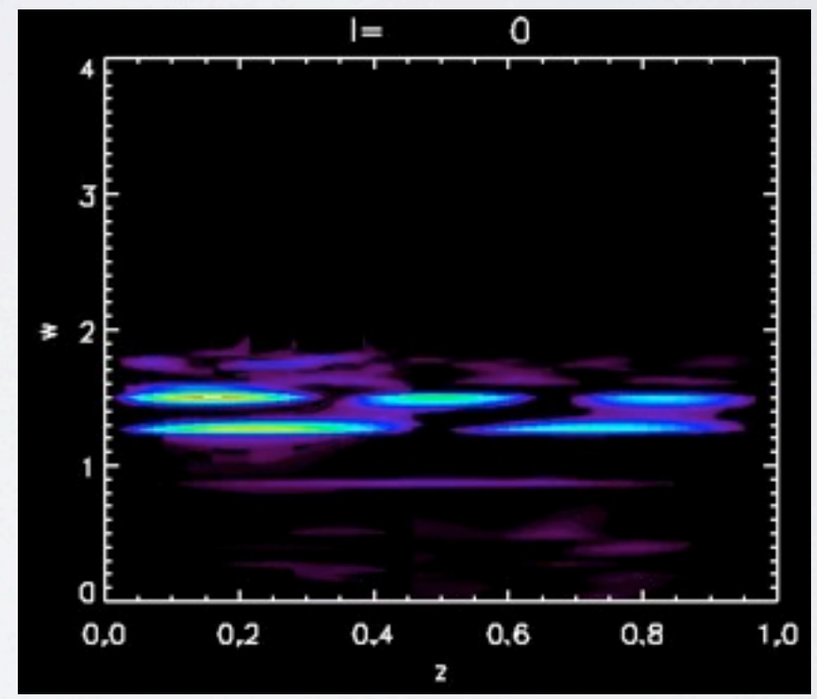
$$\rho u_z(t,x,z) \longrightarrow \rho \hat{u}_z(\omega,l,z)$$ Radial sound waves have l=0

$\Delta\omega$=0.125 (FFT resolution) $\omega_m=\pi/\Delta t$ (Nyquist frequency)



Δt=0.04 cfl=1 $\omega_m=24\pi$

CORRECT



Δt=0.5 cfl=12 $\omega_m=2\pi$

INCORRECT

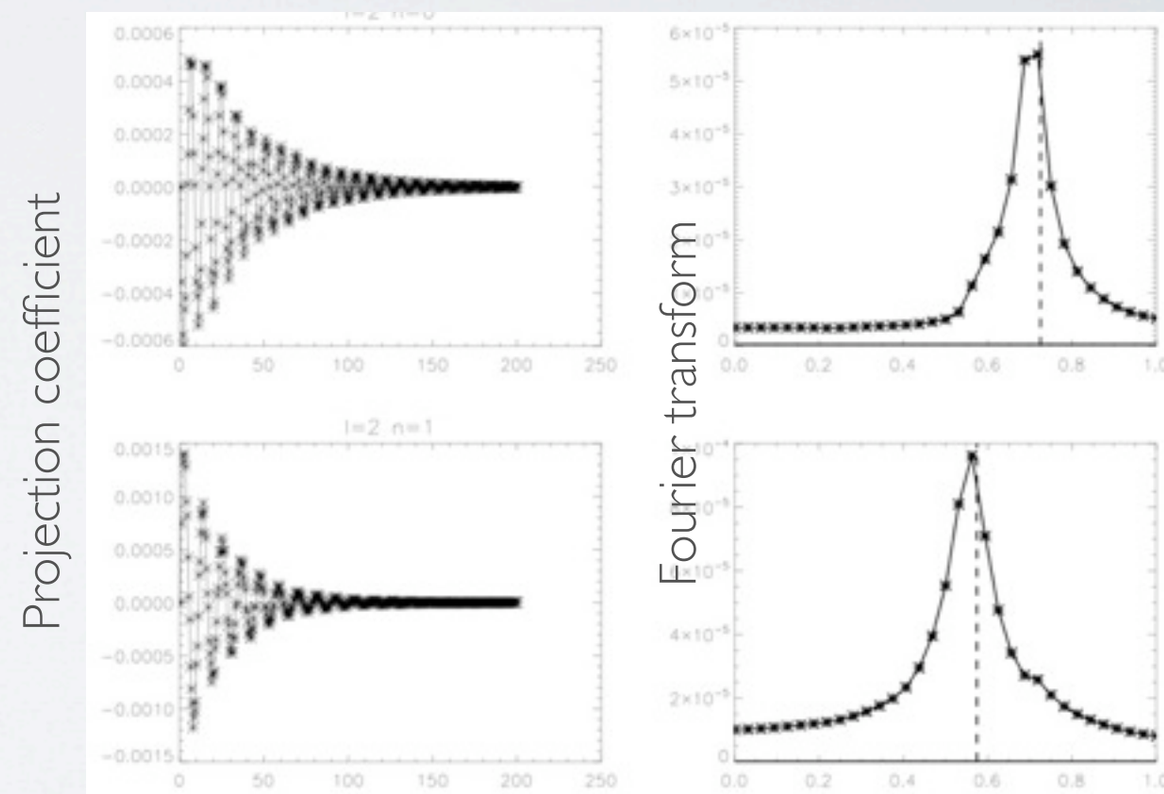
ENTROPY BUBBLE 150X150, T=200
GRAVITY MODES

$\Delta t=0.5$ cfl=37 $\Delta\omega=0.0314$ $\omega_m=2\pi$

Method : projection of the velocity field on the anelastic eigenvectors ($\Psi_{l,n}$) (with pulsation $\omega_{l,n}$)
(see Dintrans & Brandenburg 2004)

l=1, n=0

l=2, n=0



l=1, n=1

l=2, n=1

OSCILLATING ENTROPY BUBBLE SUMMARY

- The code reproduces correctly the waves content of the problem
- Time-step should be tuned on the physical process one is interested to
- Here since sound waves are "pollution", one can use larger time step than the characteristic sound wave timescale
  - But loss of accuracy ...

TEST CASES
RAYLEIGH-BENARD CONVECTION

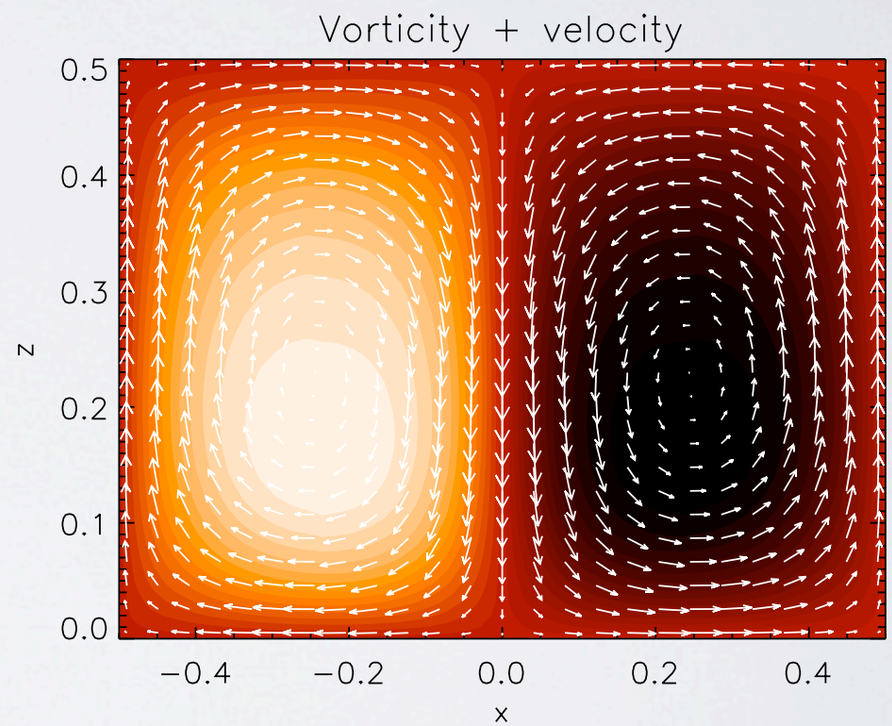
- Setup :
  - Cartesian domain : [-0.5,0.5]×[0,0.5]
  - Stratified atmosphere (g=1) : polytropic state with index m $\rho \propto T^m$ (m>-1)
    - Unstable for $m \leq 3/2$
  - Entropy perturbation at z=0.25 to drive the convective instability
  - Viscosity $\nu = 5\times10^{-4}$, conductivity $k = 10^{-2}$, Ra ~ $10^{4\text{-}5}$
  - Flux is imposed at the lower/upper boundaries, horizontal : periodic conditions

RAYLEIGH-BENARD CONVECTION

- One has : $\nabla_{\rm rad} = 1/(m+1)$
  $\nabla_{\rm ad} = 1 - 1/\gamma = 0.4$
- One expects $\frac{F_{\rm rad}}{F_{\rm tot}} = \frac{\nabla_{\rm ad}}{\nabla_{\rm rad}} = 0.4(m+1)$

  (see Brandenburg et al. 2004)

  - m=1 : F_{conv}/F_{tot} = 20 %
  - m=0 : F_{conv}/F_{tot} = 60 %
  - m=-0.9 : F_{conv}/F_{tot} = 96 %



50^2

RAYLEIGH-BENARD CONVECTION

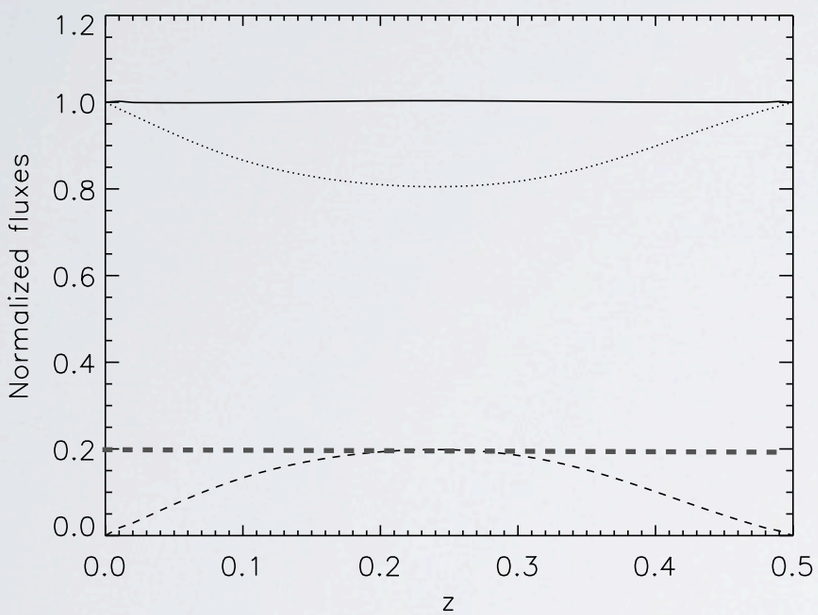
$F'_{\rm tot} = F'_{\rm rad} + F'_{\rm conv} + F'_{\rm kin} + F'_{\rm visc}$

$F'_{\rm conv}$

$F'_{\rm rad}$

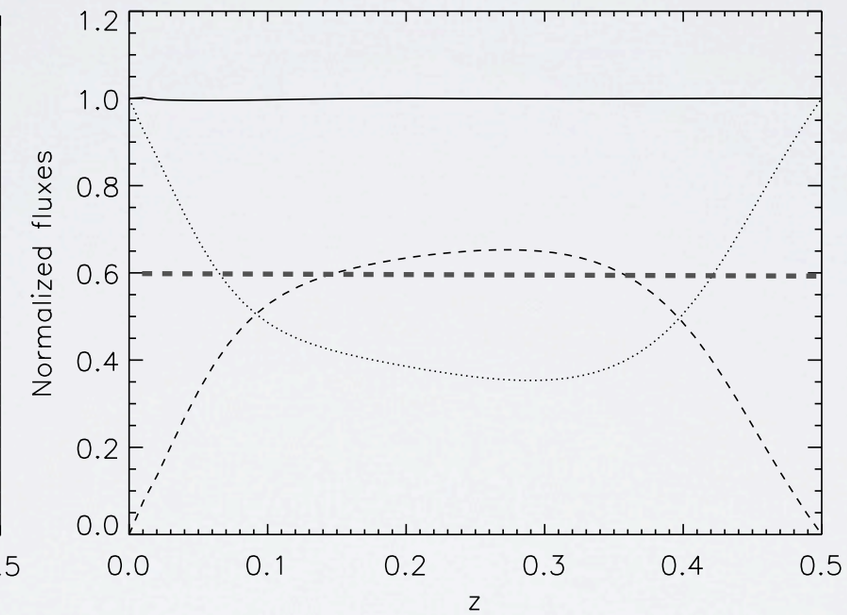
grid : 50^2

m=1



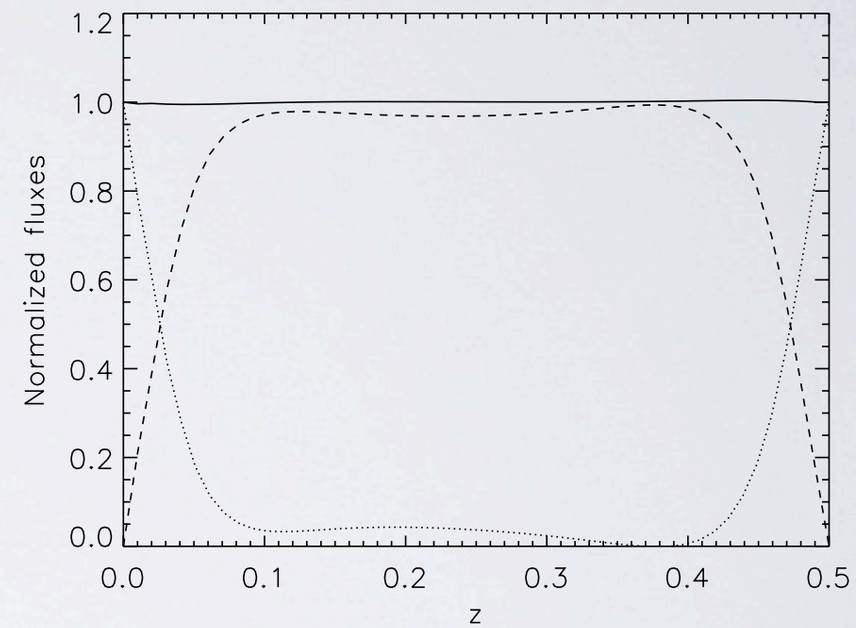
Δt=1.2, CFL=170

m=0



Δt=1.8, CFL=300

m=-0.9



Δt=0.25, CFL=80

- Relaxation toward a quasi-steady state : ~150 time units
- Expected F_{conv}/F_{tot} ratios are recovered

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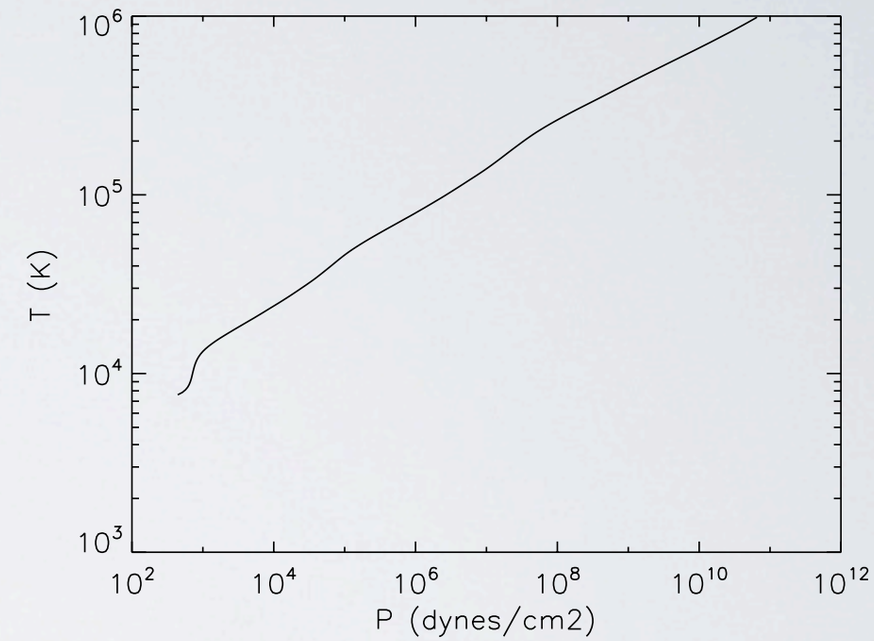
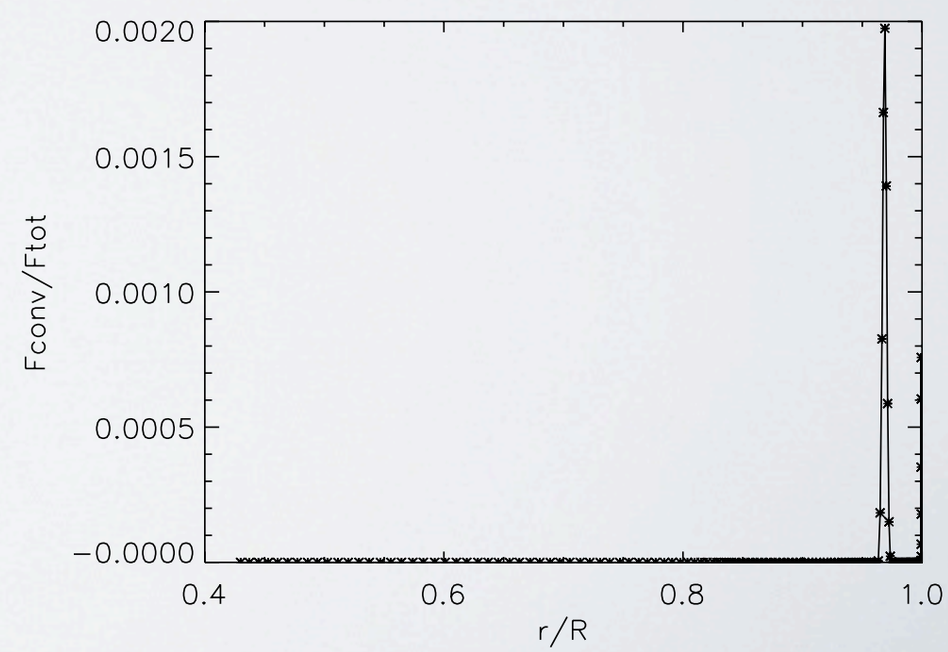
STELLAR MODELS
HOT STABLE MODEL

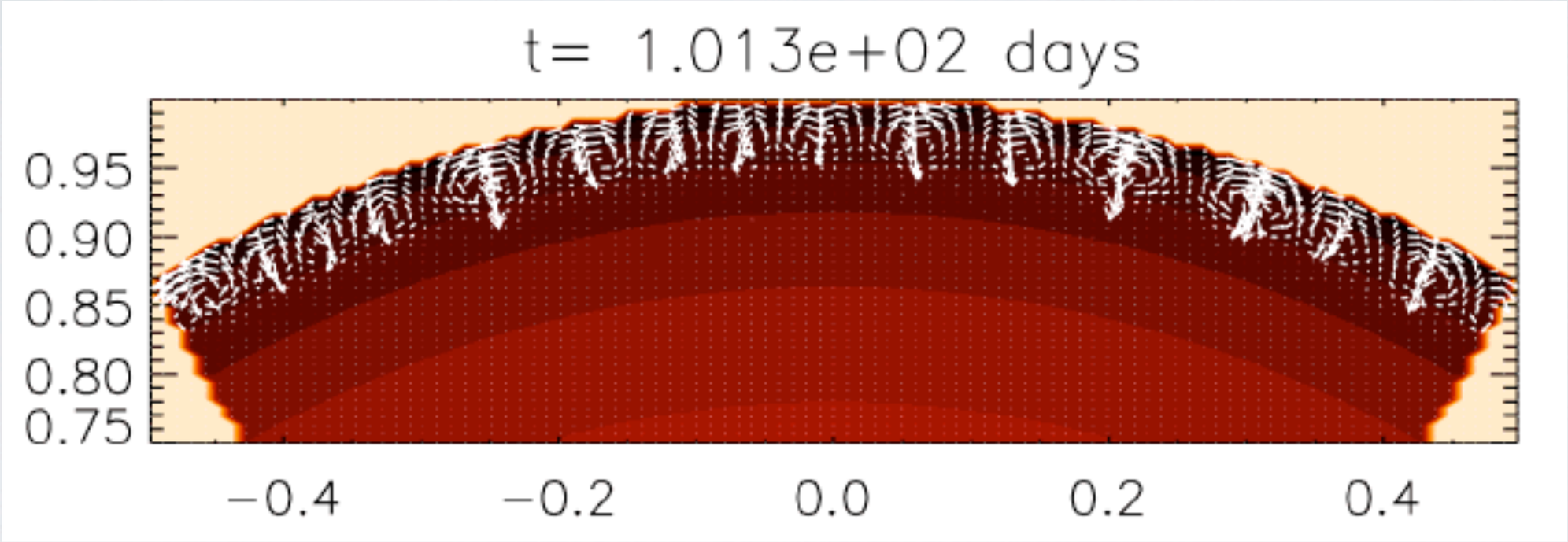
- Teff = 7500 K, M = 5 $M_\odot$, log($L/L_\odot$) = 3
  - Initial model extends to the photosphere
  - Initial grid is adapted
- Almost fully radiative model
- Strongly stratified : $\rho_{top} \sim 10^{-4}$ g cm^{-3} , $\rho_{bottom} \sim 10^{-10}$ g cm^{-3}





STELLAR MODELS
HOT STABLE MODEL

Grid : 150^2 domain : $[0.4,1]\times[\pi/3,2\pi/3]$



- $\Delta t_{CFL} \sim 30$ s, mean $\Delta t \sim 10^4$ s
- CFL: 80 => 1000, mean value ~400

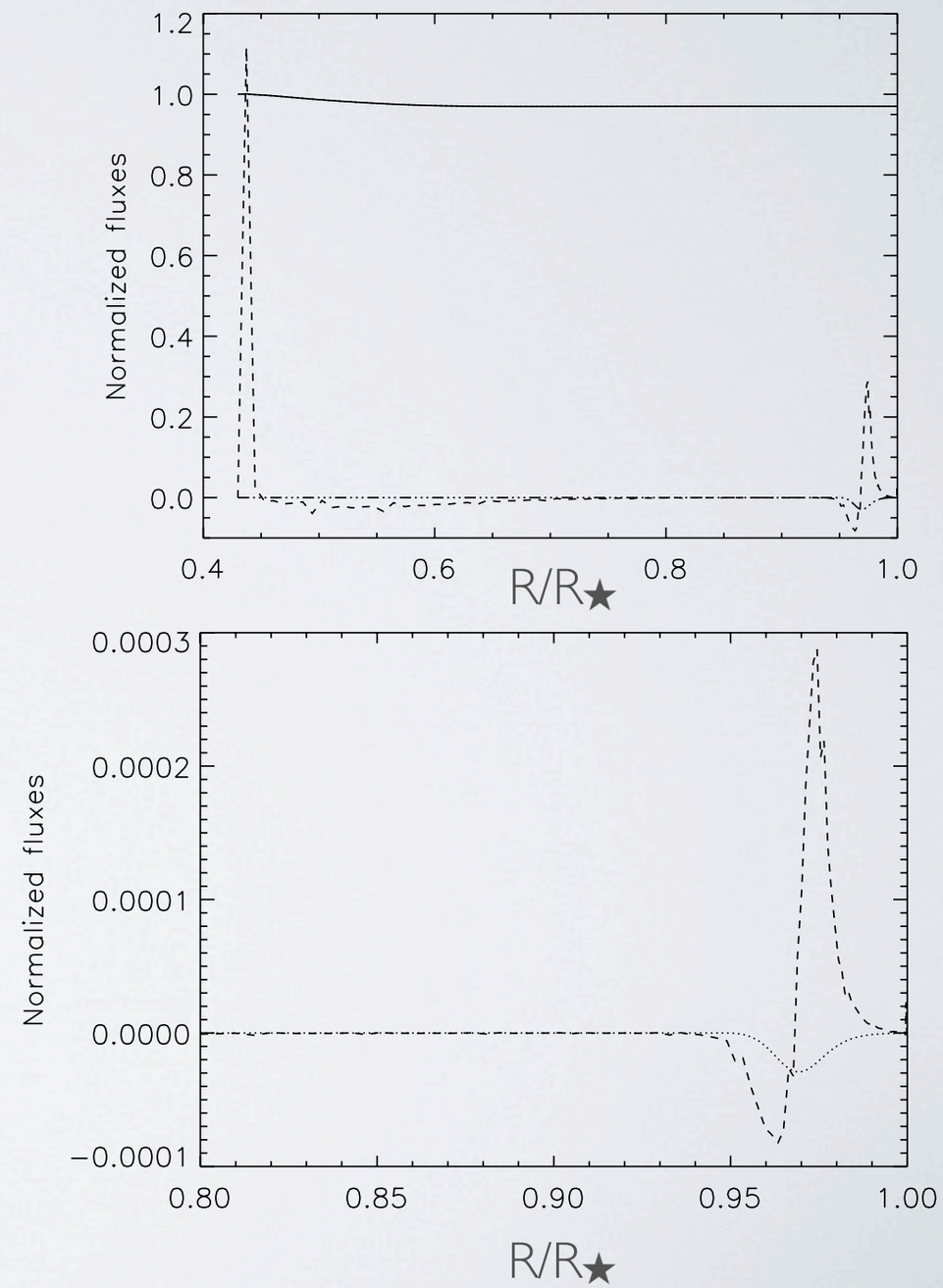
Boundary conditions :

$r=R_{out}$: stress-free, last cells radiate as σT^4

$r=R_{in}$: stress-free, entering flux is imposed

STELLAR MODELS
HOT MODEL

- Model not fully thermally relaxed : $\tau_{KH} \sim 10^5$ days
- V_{conv}/c_s : 0.02 (MLT : 0.05)
- Eddies turnover timescale : 10^5-10^7 s (MLT : 10^5 s)
- Eddies size : 5-6x10^{10} cm ~ 5 H_p (MLT : 1.7)
- Convective fluxes : 3x10^{-4} F_{tot} (MLT : 2x10^{-3})



STELLAR MODELS
SUMMARY

- First computations of stellar models are promising
- Influence of the time step ?
  - Should be tuned on the eddies turn-over time-scale ?
- Probable issues for the cepheid problem :
  - Relaxation of the initial model ?
  - How to start the convection ?
  - Pulsation : how to deal with the moving photosphere (outer boundary condition) ? Is a radially moving grid necessary ?

CONCLUSION

- Code has been validated on test cases
  - High CFL number can be used to catch physics working on larger time-scales
  - The code can handle strong convection (at least in a simple setup)
- Computation of stellar models just started
  - Numerical simulations of cepheids promise to be challenging
- Yet no reason to put our implicit approach into questions, but one should carefully check the accuracy of the results

THANKS