THE INTERACTION OF TURBULENCE WITH THE HELIOSPHERIC SHOCK

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Heliospheric observations



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- 2. T. R. Story, and G. P. Zank, JGR, A8, 17381 (1997); Zank & Mueller, JGR, 2003.
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Burlaga, Ness, and Acuna, ApJ, 2005

averages of *B*, λ , and δ from 2005 DOY 50–70 shown in Figure 5. Turbulence in the solar wind (Coleman 1968; Sari & Ness 1969) has a very small compressive component (Burlaga & Turner 1976). The turbulence has been treated as a quasi-two-dimensional and nearly incompressible fluid (Bruno & Carbone 2005; Zhou et al. 2004; Marsch 1992; Matthaeus et al. 1990; Zank & Matthaeus 1992). In the heliosheath, the opposite is observed; the fluctuations in the magnetic field strength *B* are large.

The distribution of the 48 s averages of *B* in interval B of Figure 2 (2005 DOY 50–70) is shown by the points in Figure 6*a*. A Gaussian fit to the points (Fig. 6*a*, solid curve, representing eq. [1]) and the 95% confidence band (shown by the dashed curves) indicate that *B* in this 20 day interval has a Gaussian distribution. The width of the *B* distribution gives $\sigma = 0.030 \pm 0.002 \text{ nT}$, demonstrating that *B* has a broader distribution than the components, for which $\sigma = (0.023 \pm 0.002) \text{ nT}$. This shows that the turbulence is primarily compressible.]

The turbulence in the heliosheath in interval B (Fig. 2) is isotropic, as shown by the distributions of $B_i - \langle B_i \rangle$, i = R, T, Ngiven by the points in Figure 6b. A single Gaussian fit to the points in all three of the distributions (Fig. 6b, solid curve) fits the distributions of the fluctuations of B_R , B_T , and B_N very well



Fig. 6.—(a) Squares: Distribution of the 48 s averages of the magnetic field strength measured in the sector from 2005 DOY 50–70. The solid curve, a Gaussian fit to the observations, shows that the fluctuations of the magnetic field strength are Gaussian. (b) Points: Distributions of the 48 s average components of the magnetic field minus their mean values observed from 2005 DOY 50–70. The solid curve is a Gaussian fit to the data, which illustrates that the fluctuations in **B** are isotropic.

- Clearly MHD turbulence in the inner heliosheath is different from that upstream of the HTS in the supersonic solar wind.
- Since the character of the turbulence is different immediately downstream of the HTS, it suggests that "processing" by the HTS plays an important role in modifying turbulent upstream fluctuations.
- There is little doubt that the subsonic, very hot plasma in the heliosheath will contribute to the distinctive turbulence observed by Voyager 1 but we need to understand the role of the HTS in providing the inner boundary or source for evolving turbulence in the heliosheath.
- Explore physics and implicit numerical problems of interaction of turbulence with shock waves basic physics not as well understood as one would like.
- Investigate the reaction of turbulence on shock structure.
- On the basis of a linear model, we examine the transmission, amplification, and generation of turbulence by the HTS.
- Put both together in simulations to examine assumptions and expectations

Structure of shock in response to upstream turbulence

Hypersonic approximation corresponds to weak shock approximation

$$\begin{split} &\frac{\partial u_{x1}}{\partial \tau} + \frac{f - \gamma g}{2} \frac{\partial u_{x1}}{\partial \xi} + \frac{\gamma - 1}{2} u_{x1} \frac{\partial u_{x1}}{\partial \xi} = \frac{\lambda}{2} \frac{\partial u_{y1}}{\partial \eta} + \frac{1}{2} \frac{\partial g}{\partial \tau} \\ &\frac{\partial u_{y1}}{\partial \xi} = \frac{\partial}{\partial \eta} (u_{x1} - g) \end{split}$$

$$\frac{\partial}{\partial\xi} \left(\frac{\partial u_{x1}}{\partial\tau} + \frac{f - \gamma g}{2} \frac{\partial u_{x1}}{\partial\xi} + \frac{\gamma - 1}{2} u_{x1} \frac{\partial u_{x1}}{\partial\xi} \right) = \frac{\lambda}{2} \frac{\partial^2}{\partial\eta^2} \left(u_{x1} - g \right) + \frac{\partial}{\partial\xi} \left(\frac{1}{2} \frac{\partial g}{\partial\tau} \right)$$

Two-dimensional Burger's equation



FIG. 2. Interaction between a fluctuating upstream state and a perpendicular MHD shock with $\gamma \approx 2$; (a) $f=0.12 \sin(4\pi y)$ and $h=0.15 \sin(4\pi y)$; (b) the initial condition; (c) the total pressure $p=p_1+B_{z_0}B_{z_1}/\mu$ profile in normalized units at time t=0.15; (d) the total pressure profile at time t=0.5.

Structural evolution of a perpendicular shock front, including "turbulent broadening" and "overshoots."



FIG. 4. Interaction between a fluctuating upstream state and a perpendicular MHD shock. Left panel: velocity vector of the initial condition. Right panel: velocity vector at t=0.5.



FIG. 5. Interaction between a fluctuating upstream state and a perpendicular MHD shock. Left panel: velocity magnitude of the initial condition. Right panel: velocity magnitude at t=0.5.

N-wave interacting with upstream sinusoidal fluctuations



FIG. 10. Interaction between a fluctuating upstream state and a perpendicular MHD *N*-wave: (a) $f=0.12 \sin(4\pi y)$ and $h=0.15 \sin(4\pi y)$; (b) the initial condition; (c) the total pressure $p=p_1+B_{z_0}B_{z_1}/\mu$ profile at time t=0.15; (d) the total pressure profile at time t=0.5.



FIG. 11. Normalized magnetic field fluctuation B_z : (a) the initial magnetic field fluctuation B_z ; (b) the magnetic field fluctuation B_z at time t=0.5.

MATHEMATICAL FORMULATION

- The interaction of upstream waves with a shock wave has been investigated in several papers, some of the most important of which are those by McKenzie & Westphal [4], Scholer & Belcher [5], Achterberg et al. [6], and Vainio & Schlickeiser [7], Ribner [8]
- With the exception of [4] and [6], these papers all focus on the reduced 2D problem of a fast shock interacting with an Alfvén wave when what is really needed is the interaction of an upstream fully turbulent spectrum of fluctuations interacting with a 3D shock wave.
- This problem is prohibitively difficult analytically for the full MHD problem but it can be solved hydrodynamically.
- Some numerical investigations: Balsara (supersonic interstellar turbulence), Jackson et al., Lele (hydrodynamics, aerodynamics), ...

MHD JUMP RELATIONS

 $\begin{bmatrix} B_n \end{bmatrix} = 0 \quad (1)$ $\left[\rho u_{n}\right] = 0 \quad (2)$ $\left[u_n \mathbf{B}_t - B_n \mathbf{u}_t\right] = 0 \quad (3)$ $\left| \rho u_n \left(\frac{1}{2} u^2 + w \right) + \frac{1}{4\pi} \left(B^2 u_n - \left(\mathbf{u} \cdot \mathbf{B} \right) B_n \right) \right| = 0 \quad (4)$ $\left| \rho u_n^2 + P + \frac{B^2}{8\pi} \right| = 0$ (5) $\left| \rho u_n \mathbf{u}_t - \frac{1}{4\pi} B_n \mathbf{B}_t \right| = 0 \quad (6)$ $w = \frac{\gamma}{\gamma - 1} \frac{P}{\rho}$

MATHEMATICAL FORMULATION - cont.

- In equations (1) (6), we use a single-fluid description for the solar wind plasma, essentially assuming that interstellar PUIs co-move with the solar wind flow. This is certainly a reasonable assumption. It is further straightforwardly estimated that the pressure contribution of PUIs in the far outer heliosphere far exceeds the thermal solar wind pressure (e.g., *Zank et al.*, [9]), making for a high plasma beta state ahead of the HTS. Observations of pressure balanced structures by *Burlaga et al.*, [10] indicate the dominance of the PUI pressure in the outer heliosphere.
- Thus, the total pressure P in the one-fluid model (1) (6) may be regarded as the PUI pressure.
- The adiabatic index γ must therefore reflect the PUI distribution. γ can vary between 5/3 and 2 for PUIs.
- For a bispherical PUI distribution, $\gamma = 5/3$ whereas for an unscattered ring-beam distribution, $\gamma = 2$. We will assume that because the scattering mfp is long, the PUI gas satisfies $\gamma = 2$.

MATHEMATICAL FORMULATION - cont.

• For a perpendicular shock wave, $u_n \neq 0$, $B_n = 0$, and the jump conditions are

$$\begin{bmatrix} \rho u_n \end{bmatrix} = 0 \quad \begin{bmatrix} \rho u_n \mathbf{u}_t \end{bmatrix} = 0 \quad \begin{bmatrix} \rho u_n^2 + P + \frac{B^2}{8\pi} \end{bmatrix} = 0$$
$$\begin{bmatrix} \frac{1}{2}u^2 + w + \frac{1}{4\pi}\frac{B^2}{\rho} \end{bmatrix} = 0 \quad \begin{bmatrix} B/\rho \end{bmatrix} = 0$$

• Observe that if we use $\gamma = 2$ and introduce $P^* = P + B^2/8\pi$ we immediately obtain

$$\begin{bmatrix} \rho u_n \end{bmatrix} = 0 \quad \left[\rho u_n \mathbf{u}_t \right] = 0 \quad \left[\rho u_n^2 + P^* \right] = 0 \quad (7)$$
$$\begin{bmatrix} \frac{1}{2} u^2 + \frac{2P^*}{\rho} \end{bmatrix} = 0 \quad \left[B/\rho \right] = 0 \quad (8)$$

• Structurally identical to the usual hydrodynamic jump conditions with a magnetic field contribution to the total pressure P* (i.e., P* is the sum of the PUI and magnetic pressures). "Sound speed" for system is fast mode speed for waves propagating perpendicularly to the magnetic field

$$V_{\perp}^{2} = 2P^{*}/\rho = 2P/\rho + 2B^{2}/(8\pi\rho) = C_{s}^{2} + V_{A}^{2}$$

HIGH-BETA PLASMA/HYDRO. MODEL

- Consider the interaction of fluctuations with oblique shock wave. Classical problem, beginning with Ribner [2] and McKenzie & Westphal [1].
- Reconsider using a somewhat more general formulation which has advantage of indicating how to develop a non-linear model.
- For simplicity, consider the interaction of entropy fluctuations with a shock wave.

- 1. J. F. McKenzie and K. O. Westphal, P&SS, 17, 1029 (1969)
- 2. H.S. Ribner, Report 1164 1957, pp. 199-215; Report 1233, pp 683-701

HIGH-BETA PLASMA/HYDRO. MODEL

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\Pi) = 0$$

$$\frac{\partial}{\partial t} (\rho u^2 + \rho \varepsilon) + \nabla \cdot (\rho \mathbf{u} (\frac{1}{2} u^2 + w)) = 0 \quad \varepsilon = \frac{1}{\gamma - 1} \frac{P}{\rho} \quad w = \frac{\gamma}{\gamma - 1} \frac{P}{\rho}$$

$$\Pi_{ij} = P \delta_{ij} + \rho u_i u_j$$
2D plane
$$\Pi = \begin{pmatrix} P + \rho u_x^2 & \rho u_x u_y \\ \rho u_x u_y & P + \rho u_y^2 \end{pmatrix}$$

$$\frac{\partial}{\partial t}(\rho u_x) + \frac{\partial}{\partial x}(\rho u_x^2 + P) + \frac{\partial}{\partial y}(\rho u_x u_y) = 0$$
$$\frac{\partial}{\partial t}(\rho u_x) + \frac{\partial}{\partial x}(\rho u_x u_y) + \frac{\partial}{\partial y}(\rho u_{xy}^2 + P) = 0$$

HIGH-BETA PLASMA/HYDRO. MODEL

• Dispersion relation for hydrodynamics:

 $\omega' \equiv \omega - \mathbf{u} \cdot \mathbf{k}$ $\omega'^{2} \left(\omega'^{2} - C_{s}^{2} k^{2} \right) = 0$ $\omega' = 0$ Entropy mode : $\delta \rho^{e} \neq 0$ $\delta \mathbf{u}^{e} = 0$ $\delta P = 0$ Vorticity mode: $\delta \rho = 0$ $\delta \mathbf{u}^{v} = \delta u^{v} \left(-\sin \phi^{v}, \cos \phi^{v} \right)$ $\delta P = 0$ $\omega'^{2} = C_{s}^{2} k^{2}$ (Fast) Acoustic mode: $\delta \mathbf{u}^{a} = \pm C_{s} \frac{\delta \rho}{\rho} \left(\cos \phi^{a}, \sin \phi^{a} \right)$ $\delta P^{a} = C_{s}^{2} \delta \rho^{a}$ $\delta \rho^{a} \neq 0$

NB REMINDER: Recall that for this case $P^* = P + B^2/8\pi$

MATHEMATICAL FORMULATION - cont.



MATHEMATICAL FORMULATION - cont.

• For a system of conservation laws



the Rankine-Hugoniot conditions are given by

$$\left[\mathbf{U}\right]\varphi_{t} + \sum_{j} \left[f_{j}\left(\mathbf{U}\right) \cdot \varphi_{x_{j}}\right] = 0$$

Non-steady R-H conditions with shock front deformation:

$$-\left[\rho\right]\xi_{t} + \left[\rho\left(u_{x}, u_{y}\right) \cdot \left(1, -\xi_{y}\right)\right] = 0$$

$$\Rightarrow -\left[\rho\right]\xi_{t} + \left[\rho\left(u_{x} - u_{y}\xi_{y}\right)\right] = 0 \quad (\text{mass flux})$$

$$-\left[\rho u_{x}\right]\xi_{t} + \left[\rho u_{x}\left(u_{x} - u_{y}\xi_{y}\right) + P\right] = 0 \quad (\text{normal momentum flux})$$

$$\varphi\left(x, y, t\right) = x - \xi\left(y, t\right) - \left[\frac{1}{2}\rho u^{2} + \rho\varepsilon\right]\xi_{t} + \left[\rho\left(u_{x} - u_{y}\xi_{y}\right)\left(\frac{1}{2}u^{2} + w\right)\right] = 0 \quad (\text{energy flux})$$

Yields the boundary condition: $[\mathbf{u}_t] = 0$

And now impose linearity i.e., $\xi_{y,t}^2 \ll 1$

$$\mathbf{u}_{t} = \mathbf{u} - u_{n} \left(1, -\xi_{y} \right) \simeq \left(u_{y} \xi_{y}, u_{y} + u_{x} \xi_{y} \right)$$

Five variable and 5 equations therefore well posed.

• Recover standard R-H conditions in steady frame i.e., $\xi_t = 0$



NB REMINDER: Recall that for this case $P^* = P + B^2/8\pi$



Consider simplest case of incident entropy wave:

$$\tau \equiv \frac{1}{\rho}$$

Upstream: $\delta P_1 = 0$ $\delta u_{x1} = \delta u_{y1} = 0$ $\delta \tau_1 = \delta \tau_1^e$

$$\begin{split} &\delta\tau_2 = \delta\tau_2^a + \delta\tau_2^e = \delta\tau_2 \left(P_2; P_1, \tau_1\right) \\ &\delta P_2 = \delta P_2^a = -C_{s2}^2 \frac{\delta\tau_2^a}{\tau_2^2} \\ &\delta \mathbf{u}_2 = \delta \mathbf{u}_2^a + \delta \mathbf{u}_2^v = \pm \frac{\tau_2}{C_{s2}} \left(\alpha_2^a, \beta_2^a\right) \delta\rho_2^a + \delta u_2^v \left(-\beta_2^v, \alpha_2^v\right) \\ &\left(\alpha_2^a, \beta_2^a\right) = \left(\cos\phi_2^a, \sin\phi_2^a\right) \quad \left(\alpha_2^v, \beta_2^v\right) = \left(\cos\phi_2^v, \sin\phi_2^v\right) \end{split}$$

Downstream: Excites acoustic and entropy-vorticity waves:

NB REMINDER: Recall that for this case $P^* = P + B^2/8\pi$

$$\begin{split} &\delta\tau_{2} = \left(\frac{\partial\tau_{2}}{\partial P_{2}}\right)_{H} \delta P_{2} + \left(\frac{\partial\tau_{2}}{\partial\tau_{1}}\right)_{H} \delta\tau_{1} + \left(\frac{\partial\tau_{2}}{\partial P_{1}}\right)_{H} \delta P_{1} \\ &= -\left(\frac{\partial\tau_{2}}{\partial P_{2}}\right)_{H} \frac{C_{s2}^{2}}{\tau_{2}^{2}} \delta\tau_{2}^{a} + \left(\frac{\partial\tau_{2}}{\partial\tau_{1}}\right)_{H} \delta\tau_{1} \end{split}$$

- Linear analysis of deformed shock front equations assuming perturbed shock front amplitude small. Very clear analysis of transmission and excitation of downstream fluctuations.
- Continuity of frequency and transverse wave number across deformed shock yields downstream propagation angles for transmitted and excited fluctuations.
- Can validate calculation numerically.
- Can impose upstream spectrum and calculate the downstream spectrum, ratio of compressive to incompressible fluctuations, distribution, anisotropy, amplification of incident turbulence.
- Note the generation of magnetic field at the shock.

VORTICITY-ENTROPY WAVE/SHOCK INTERACTION: Simulations

Simulate a two-dimensional interaction between a plane vorticity-entropy wave and a oblique shock wave.

Shock wave parameters: M - Mach number of upstream flow; α - angle between shock wave and upstream flow.

Disturbed upstream flow:

$$\begin{split} \rho &= \overline{\rho} + \overline{\rho} A_e \cos(k_x x + k_y y - k_t t) \\ u &= \overline{u} + U A_v \sin \psi_1 \cos(k_x x + k_y y - k_t t) \\ v &= \overline{v} - U A_v \cos \psi_1 \cos(k_x x + k_y y - k_t t) \\ p &= \overline{p} \end{split}$$

where

$$U = \sqrt{\overline{u}^2 + \overline{v}^2}, \quad k_t = k_x \overline{u} + k_y \overline{v}, \quad k_x = k \cos \psi_1, \quad k_y = k \sin \psi_1$$

k is the magnitude of the wavenumber vector ψ_1 denotes the angle between the wavenumber vector and *x* A_v and A_e are intensity of velocity and density upstream of the shock wave. Entropy wave $A_e = 0.025$, $\alpha = 90^\circ$, M = 2.9, k = 2, $\psi = 10^\circ$



Entropy wave $A_e = 0.025$, $\alpha = 90^\circ$, M = 2.9, k = 2, $\psi = 30^\circ$





Entropy wave $A_e = 0.025$, $\alpha = 90^\circ$, M = 2.9, k = 2, $\psi = 80^\circ$



Entropy wave
$$A_e = 0.025$$
, $\alpha = 90^\circ$, $\psi = 0^\circ$ vs. $k_x (k_y = 0)$



Dependence of post shock wave number against pre-shock wave number



Entropy wave
$$A_e = 0.25$$
, $\alpha = 90^\circ$, $\psi = 60^\circ$, $k_y = 10$



Density

Pressure

Entropy wave
$$A_e = 0.25$$
, $\alpha = 90^\circ$, $\psi = 60^\circ$, $k_v = 5$



Entropy wave $A_e = 0.25$, $\alpha = 90^\circ$, $\psi = 60^\circ$, $k_y = 5$ (continue)



ADPDIS3D loses regularity and symmetry of the flow

Entropy wave $A_e = 0.25$, $\alpha = 90^\circ$, $\psi = 60^\circ$, $k_y = 5$ (continue)



density







divergence of velocity



Entropy wave $A_e = 0.25$, $\alpha = 90^\circ$, $\psi = 60^\circ$, $k_y = 5$ (continue) 1D spectra (linear scales) 2 2.5 A 1.5 Α 0.5 0.5 20 30 wave number 10 20 30 40 10 40 wave number density x-velocity 2.5-1.8-1.6-2 1.4 1.2 1.5 A 1.0 Α 0.8 0.6 0.4 0.5 0.2-20 30 wave number 10 20 30 wave number 0 10 40 0 40 y-velocity pressure

Entropy wave $A_e = 0.25$, $\alpha = 90^\circ$, $\psi = 60^\circ$, $k_y = 5$ (continue)

1D spectra (linear scales)





Entropy waves with turbulent incident spectrum





Entropy waves with turbulent spectrum



LISM interaction with a randomly perturbed SW

We perturb the SW velocity: $U_R = \overline{U} + \varepsilon U'_R$, $U_{\theta} = \varepsilon U'_{\theta}$, $\varepsilon = 0.025$,

where ϵ measures the intensity of the input turbulence.

The initial spectrum is $E(k) \sim k^2$. Fluctuations are assumed to be isotropic.



The spectrum is chosen in such a way that the length scale of fluctuations lies between the grid size (0.5 AU) and 5 AU.

Time-variation of the density distribution



Time-variation of the velocity magnitude distribution















Shock Shape Calculated from Shock Speeds, Normals and Timing



Wind/Geotail saw this shock nearly simultaneously

Obs. at ACE is used to predict shock location at the time of Wind/Geotail observation

Neugebauer & Giacalone, 2007

Definition of 2-D Radius of Curvature



- S is the location of the shock determined from observed speed (v) and direction at ACE
- W is the location of another s/c that sees the shock a time Δt later

If the shock is planar, W would be on the vertical line - this was seldom the case. Can use simple geometry to get the radius of curvature

Distribution of shock radii of curvature



Neugebauer & Giacalone, 2007

CONCLUSIONS

- Need to understand the interaction of turbulence with shocks, motivated by Voyager observations downstream of the TS and models for particle acceleration at quasi-perpendicular shocks.
- Structure of shock modified smoothing, non-R-H jumps (i.e., non-steady)
- Incompressible upstream fluctuations generates compressible fluctuations downstream, amplified density fluctuations, and vortical flow.
- Begun to study effect of varying Mach number, obliquity, and incidence of entropy fluctuations.
- Development of turbulence and vortical structure downstream due to mode coupling
- Magnetic fields generated downstream implications for particle acceleration at perpendicular shocks interesting.
- Complications in numerically simulating interaction of fluctuations with shock wave are not trivial.