THE INTERACTION OF TURBULENCE WITH THE HELIOSPHERIC SHOCK

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Heliospheric observations

- Heliosheath expected to be turbulent [1], the result of upstream turbulence, enhancements, structures, etc. [2] being transmitted and interacting with the heliospheric termination shock (HTS).
- A turbulent heliosheath has been observed downstream of the HTS [3], but the character of the turbulence is significantly different from that of the solar wind.
- 48-second averages of the downstream magnetic field \( B(t) \) analyzed by [3] reveal an isotropic Gaussian distribution for each component. The distribution of 1-hour averages of the magnetic field \( B \) was also Gaussian in the heliosheath, unlike the log-normal distributions of \( B \) found in the supersonic solar wind. The Gaussian distribution indicates a scale invariance in the form of the magnetic field distribution.
- The second intriguing observation was that the turbulence was substantially compressible since the width of the 48-second averages of \( B \) is greater than that of the components.

averages of $B$, $\lambda$, and $\delta$ from 2005 DOY 50–70 shown in Figure 5. Turbulence in the solar wind (Coleman 1968; Sari & Ness 1969) has a very small compressive component (Burlaga & Turner 1976). The turbulence has been treated as a quasi-two-dimensional and nearly incompressible fluid (Bruno & Carbone 2005; Zhou et al. 2004; Marsch 1992; Mathaeus et al. 1990; Zank & Mathaeus 1992). In the heliosheath, the opposite is observed; the fluctuations in the magnetic field strength $B$ are large.

The distribution of the 48 s averages of $B$ in interval B of Figure 2 (2005 DOY 50–70) is shown by the points in Figure 6a. A Gaussian fit to the points (Fig. 6a, solid curve, representing eq. [1]) and the 95% confidence band (shown by the dashed curves) indicate that $B$ in this 20 day interval has a Gaussian distribution. The width of the $B$ distribution gives $\sigma = 0.030 \pm 0.002$ nT, demonstrating that $B$ has a broader distribution than the components, for which $\sigma = (0.023 \pm 0.002)$ nT. This shows that the turbulence is primarily compressible.

The turbulence in the heliosheath in interval B (Fig. 2) is isotropic, as shown by the distributions of $B_i - \langle B_i \rangle$, $i = R, T, N$ given by the points in Figure 6b. A single Gaussian fit to the points in all three of the distributions (Fig. 6b, solid curve) fits the distributions of the fluctuations of $B_R$, $B_T$, and $B_N$ very well.

![Figure 6](image-url)

**Fig. 6** — (a) Squares: Distribution of the 48 s averages of the magnetic field strength measured in the sector from 2005 DOY 50–70. The solid curve, a Gaussian fit to the observations, shows that the fluctuations of the magnetic field strength are Gaussian. (b) Filled: Distributions of the 48 s average components of the magnetic field minus their mean values observed from 2005 DOY 59–70. The solid curve is a Gaussian fit to the data, which illustrates that the fluctuations in $B$ are isotropic.
• Clearly MHD turbulence in the inner heliosheath is different from that upstream of the HTS in the supersonic solar wind.

• Since the character of the turbulence is different immediately downstream of the HTS, it suggests that “processing” by the HTS plays an important role in modifying turbulent upstream fluctuations.

• There is little doubt that the subsonic, very hot plasma in the heliosheath will contribute to the distinctive turbulence observed by Voyager 1 but we need to understand the role of the HTS in providing the inner boundary or source for evolving turbulence in the heliosheath.

• Explore physics and implicit numerical problems of interaction of turbulence with shock waves - basic physics not as well understood as one would like.

• Investigate the reaction of turbulence on shock structure.

• On the basis of a linear model, we examine the transmission, amplification, and generation of turbulence by the HTS.

• Put both together in simulations to examine assumptions and expectations
Structure of shock in response to upstream turbulence

Hypersonic approximation corresponds to weak shock approximation

\[
\frac{\partial u_{x1}}{\partial \tau} + \frac{f - \gamma g}{2} \frac{\partial u_{x1}}{\partial \xi} + \frac{\gamma - 1}{2} u_{x1} \frac{\partial u_{x1}}{\partial \xi} = \frac{\lambda}{2} \frac{\partial u_{y1}}{\partial \eta} + \frac{1}{2} \frac{\partial g}{\partial \tau}
\]

\[
\frac{\partial u_{y1}}{\partial \xi} = \frac{\partial}{\partial \eta} (u_{x1} - g)
\]

\[
\frac{\partial}{\partial \xi} \left( \frac{\partial u_{x1}}{\partial \tau} + \frac{f - \gamma g}{2} \frac{\partial u_{x1}}{\partial \xi} + \frac{\gamma - 1}{2} u_{x1} \frac{\partial u_{x1}}{\partial \xi} \right) = \frac{\lambda}{2} \frac{\partial^2}{\partial \eta^2} (u_{x1} - g) + \frac{\partial}{\partial \xi} \left( \frac{1}{2} \frac{\partial g}{\partial \tau} \right)
\]

Two-dimensional Burger's equation
Structural evolution of a perpendicular shock front, including "turbulent broadening" and "overshoots."
FIG. 4. Interaction between a fluctuating upstream state and a perpendicular MHD shock. Left panel: velocity vector of the initial condition. Right panel: velocity vector at $t=0.5$.

FIG. 5. Interaction between a fluctuating upstream state and a perpendicular MHD shock. Left panel: velocity magnitude of the initial condition. Right panel: velocity magnitude at $t=0.5$. 
FIG. 10. Interaction between a fluctuating upstream state and a perpendicular MHD N-wave: (a) \( f=0.12 \sin(4\pi y) \) and \( h=0.15 \sin(4\pi y) \); (b) the initial condition; (c) the total pressure \( p=p_1+B_x B_y/\mu \) profile at time \( t=0.15 \); (d) the total pressure profile at time \( t=0.5 \).
FIG. 11. Normalized magnetic field fluctuation $B_z$: (a) the initial magnetic field fluctuation $B_z$; (b) the magnetic field fluctuation $B_z$ at time $t=0.5$. 
MATHEMATICAL FORMULATION

- The interaction of upstream waves with a shock wave has been investigated in several papers, some of the most important of which are those by McKenzie & Westphal [4], Scholer & Belcher [5], Achterberg et al. [6], and Vainio & Schlickeiser [7], Ribner [8].

- With the exception of [4] and [6], these papers all focus on the reduced 2D problem of a fast shock interacting with an Alfvén wave when what is really needed is the interaction of an upstream fully turbulent spectrum of fluctuations interacting with a 3D shock wave.

- This problem is prohibitively difficult analytically for the full MHD problem but it can be solved hydrodynamically.

- Some numerical investigations: Balsara (supersonic interstellar turbulence), Jackson et al., Lele (hydrodynamics, aerodynamics), …
MHD JUMP RELATIONS

\[ [B_n] = 0 \] (1)
\[ [\rho u_n] = 0 \] (2)
\[ [u_n B_t - B_n u_t] = 0 \] (3)

\[ \rho u_n \left( \frac{1}{2} u^2 + w \right) + \frac{1}{4\pi} \left( B^2 u_n - (u \cdot B) B_n \right) = 0 \] (4)

\[ \left[ \rho u_n^2 + P + \frac{B^2}{8\pi} \right] = 0 \] (5)

\[ \rho u_n u_t - \frac{1}{4\pi} B_n B_t \] (6)

\[ w = \frac{\gamma P}{\gamma - 1 \rho} \]
MATHEMATICAL FORMULATION - cont.

- In equations (1) - (6), we use a single-fluid description for the solar wind plasma, essentially assuming that interstellar PUIs co-move with the solar wind flow. This is certainly a reasonable assumption. It is further straightforwardly estimated that the pressure contribution of PUIs in the far outer heliosphere far exceeds the thermal solar wind pressure (e.g., Zank et al., [9]), making for a high plasma beta state ahead of the HTS. Observations of pressure-balanced structures by Burlaga et al., [10] indicate the dominance of the PUI pressure in the outer heliosphere.

- Thus, the total pressure $P$ in the one-fluid model (1) - (6) may be regarded as the PUI pressure.

- The adiabatic index $\gamma$ must therefore reflect the PUI distribution. $\gamma$ can vary between 5/3 and 2 for PUIs.

- For a bispherical PUI distribution, $\gamma = 5/3$ whereas for an unscattered ring-beam distribution, $\gamma = 2$. We will assume that because the scattering mfp is long, the PUI gas satisfies $\gamma = 2$. 
MATHEMATICAL FORMULATION - cont.

- For a perpendicular shock wave, \( u_n \neq 0, \ B_n = 0 \), and the jump conditions are

\[
\begin{align*}
[\rho u_n] &= 0 \quad [\rho u_n u_t] = 0 \quad \left[ \rho u_n^2 + P + \frac{B^2}{8\pi} \right] = 0 \\
\left[ \frac{1}{2} u^2 + w + \frac{1}{4\pi} \frac{B^2}{\rho} \right] &= 0 \quad [B/\rho] = 0
\end{align*}
\]

- Observe that if we use \( \gamma = 2 \) and introduce \( P^* = P + \frac{B^2}{8\pi} \) we immediately obtain

\[
\begin{align*}
[\rho u_n] &= 0 \quad [\rho u_n u_t] = 0 \quad \left[ \rho u_n^2 + P^* \right] = 0 \quad (7) \\
\left[ \frac{1}{2} u^2 + \frac{2P^*}{\rho} \right] &= 0 \quad [B/\rho] = 0 \quad (8)
\end{align*}
\]

- Structurally identical to the usual hydrodynamic jump conditions with a magnetic field contribution to the total pressure \( P^* \) (i.e., \( P^* \) is the sum of the PUI and magnetic pressures). “Sound speed” for system is fast mode speed for waves propagating perpendicularly to the magnetic field

\[
V_{\perp}^2 = 2P^*/\rho = 2P/\rho + 2B^2/(8\pi\rho) = C_s^2 + V_A^2
\]
HIGH-BETA PLASMA/HYDRO. MODEL


• Reconsider using a somewhat more general formulation which has advantage of indicating how to develop a non-linear model.

• For simplicity, consider the interaction of entropy fluctuations with a shock wave.

HIGH-BETA PLASMA/HYDRO. MODEL

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \]

\[ \frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot (\Pi) = 0 \]

\[ \frac{\partial}{\partial t} (\rho u^2 + \rho \varepsilon) + \nabla \cdot \left( \rho \mathbf{u} \left( \frac{1}{2} u^2 + w \right) \right) = 0 \quad \varepsilon = \frac{1}{\gamma - 1} \frac{P}{\rho} \quad w = \frac{\gamma}{\gamma - 1} \frac{P}{\rho} \]

\[ \Pi_{ij} = P \delta_{ij} + \rho u_i u_j \]

2D plane \[ \Pi = \begin{pmatrix} P + \rho u_x^2 & \rho u_x u_y \\ \rho u_x u_y & P + \rho u_y^2 \end{pmatrix} \]

\[ \frac{\partial}{\partial t} (\rho u_x) + \frac{\partial}{\partial x} (\rho u_x^2 + P) + \frac{\partial}{\partial y} (\rho u_x u_y) = 0 \]

\[ \frac{\partial}{\partial t} (\rho u_y) + \frac{\partial}{\partial x} (\rho u_x u_y) + \frac{\partial}{\partial y} (\rho u_y^2 + P) = 0 \]
HIGH-BETA PLASMA/HYDRO. MODEL

- Dispersion relation for hydrodynamics:

\[
\omega' \equiv \omega - \mathbf{u} \cdot \mathbf{k} \\
\omega'^2 \left( \omega'^2 - C_s^2 k^2 \right) = 0 \\
\omega' = 0 \\
\text{Entropy mode:} \quad \delta \rho^e \neq 0 \quad \delta \mathbf{u}^e = 0 \quad \delta P = 0 \\
\text{Vorticity mode:} \quad \delta \rho = 0 \quad \delta \mathbf{u}^v = \delta u^v \left( -\sin \phi^v, \cos \phi^v \right) \quad \delta P = 0 \\
\omega'^2 = C_s^2 k^2 \\
\text{(Fast) Acoustic mode:} \quad \delta \mathbf{u}^a = \pm C_s \frac{\delta \rho}{\rho} \left( \cos \phi^a, \sin \phi^a \right) \quad \delta P^a = C_s^2 \delta \rho^a \quad \delta \rho^a \neq 0
\]

NB REMINDER: Recall that for this case \( P^* = P + B^2/8\pi \)
Let $\mathbf{n}$ denote the normal to the surface of discontinuity, defined by

$$\varphi(x, y, t) = x - \xi(y, t)$$

$$\Rightarrow \text{normal to surface given by}$$

$$\hat{\mathbf{n}} = \frac{(\varphi_t, \varphi_x)}{\sqrt{\varphi_x \cdot \varphi_x}} = \frac{(-\xi_t, 1, -\xi_y)}{\sqrt{1 + \xi_y^2}}$$

Suppose eventually that

$$\xi = \xi_0 \exp i(k_y y - \omega t)$$
MATHEMATICAL FORMULATION - cont.

- For a system of conservation laws

\[ \frac{\partial \mathbf{U}}{\partial t} + \sum_j \frac{\partial}{\partial x_j} f_j (\mathbf{U}) = 0 \]

the Rankine-Hugoniot conditions are given by

\[ [\mathbf{U}] \varphi_t + \sum_j \left[ f_j (\mathbf{U}) \cdot \varphi_{x_j} \right] = 0 \]
Non-steady R-H conditions with shock front deformation:

\[ -[\rho] \xi_t + \left[ \rho \left( u_x, u_y \right) \cdot (1, -\xi_y) \right] = 0 \]

\[ \Rightarrow -[\rho] \xi_t + \left[ \rho \left( u_x - u_y \xi_y \right) \right] = 0 \quad \text{(mass flux)} \]

\[ -[\rho u_x] \xi_t + \left[ \rho u_x \left( u_x - u_y \xi_y \right) + P \right] = 0 \quad \text{(normal momentum flux)} \]

\[ -\left[ \frac{1}{2} \rho u^2 + \rho \varepsilon \right] \xi_t + \left[ \rho \left( u_x - u_y \xi_y \right) \left( \frac{1}{2} u^2 + w \right) \right] = 0 \quad \text{(energy flux)} \]

Yields the boundary condition: \[ \left[ u_t \right] = 0 \]

And now impose linearity i.e., \[ \xi^2 \ll 1 \]

\[ u_t = u - u_n \left( 1, -\xi_y \right) \approx \left( u_y \xi_y, u_y + u_x \xi_y \right) \]

Five variable and 5 equations therefore well posed.
• Recover standard R-H conditions in steady frame i.e., \( \xi_t = 0 \)

\[
\begin{align*}
\left[ \rho u_n \right] &= 0 \\
\left[ \rho u_n^2 + P \right] &= 0 \\
\left[ u_n \right] &= 0 \\
\left[ \frac{1}{2} u_n^2 + w \right] &= 0
\end{align*}
\]

NB REMINDER: Recall that for this case \( P^* = P + B^2/8\pi \)
Consider simplest case of incident entropy wave:

\[ \tau = \frac{1}{\rho} \]

Upstream: \( \delta P_1 = 0 \quad \delta u_{x_1} = \delta u_{y_1} = 0 \quad \delta \tau_1 = \delta \tau_1^e \)

\[ \delta \tau_2 = \delta \tau_2^a + \delta \tau_2^e = \delta \tau_2(\mathcal{P}_2; \mathcal{R}_1, \tau_1) \]

\[ \delta P_2 = \delta \mathcal{P}_2^a = -C_{s_2}^2 \frac{\delta \tau_2^e}{\tau_2} \]

\[ \delta u_2 = \delta u_2^a + \delta u_2^e = \pm \frac{\tau_2}{C_{s_2}} (\alpha_2^a, \beta_2^a) \delta \mathcal{P}_2^a + \delta u_2^e(\beta_2^a, \alpha_2^a) \]

\[ (\alpha_2^a, \beta_2^a) = (\cos \phi_2^a, \sin \phi_2^a) \quad (\alpha_2^e, \beta_2^e) = (\cos \phi_2^e, \sin \phi_2^e) \]

\[ \delta \tau_2 = \left( \frac{\partial \tau_2}{\partial \mathcal{P}_2} \right)_H \delta \mathcal{P}_2 + \left( \frac{\partial \tau_2}{\partial \tau_1} \right)_H \delta \tau_1 + \left( \frac{\partial \tau_2}{\partial \mathcal{P}_1} \right)_H \delta \mathcal{P}_1 \]

\[ = -\left( \frac{\partial \tau_2}{\partial \mathcal{P}_2} \right)_H \frac{C_{s_2}^2}{\tau_2} \delta \tau_2^a + \left( \frac{\partial \tau_2}{\partial \tau_1} \right)_H \delta \tau_1 \]

**NB REMINDER:**
Recall that for this case \( \mathcal{P}^* = P + \frac{B^2}{8\pi} \)
• Linear analysis of deformed shock front equations assuming perturbed shock front amplitude small. Very clear analysis of transmission and excitation of downstream fluctuations.

• Continuity of frequency and transverse wave number across deformed shock yields downstream propagation angles for transmitted and excited fluctuations.

• Can validate calculation numerically.

• Can impose upstream spectrum and calculate the downstream spectrum, ratio of compressive to incompressible fluctuations, distribution, anisotropy, amplification of incident turbulence.

• Note the generation of magnetic field at the shock.
VORTICITY-ENTROPY WAVE/SHOCK INTERACTION: Simulations

Simulate a two-dimensional interaction between a plane vorticity-entropy wave and a oblique shock wave.

Shock wave parameters: M - Mach number of upstream flow; \( \alpha \) - angle between shock wave and upstream flow.

Disturbed upstream flow:

\[
\rho = \bar{\rho} + \bar{\rho} A_v \cos(k_x x + k_y y - k_t t)
\]

\[
u = \bar{u} + U A_v \sin \psi_1 \cos(k_x x + k_y y - k_t t)
\]

\[
v = \bar{v} - U A_v \cos \psi_1 \cos(k_x x + k_y y - k_t t)
\]

\[
p = \bar{p}
\]

where

\[
U = \sqrt{\bar{u}^2 + \bar{v}^2}, \quad k_t = k_x \bar{u} + k_y \bar{v}, \quad k_x = k \cos \psi_1, \quad k_y = k \sin \psi_1
\]

\( k \) is the magnitude of the wavenumber vector \( \psi_1 \) denotes the angle between the wavenumber vector and \( x \)

\( A_v \) and \( A_e \) are intensity of velocity and density upstream of the shock wave.
Entropy wave $A_e = 0.025$, $\alpha = 90^\circ$, $M = 2.9$, $k = 2$, $\psi = 10^\circ$

- Post shock 1D density spectrum
- Density fluctuations along $y=0.5$ and 5 main harmonics
Entropy wave $A_e = 0.025, \alpha = 90^\circ, M = 2.9, k = 2, \psi = 30^\circ$
Entropy wave $A_e = 0.025$, $\alpha = 90^\circ$, $M = 2.9$, $k = 2$, $\psi=80^\circ$

Density along $y=0.5$

Post shock 1D density spectrum

Density fluctuations along $y=0.5$ and 5 main harmonics
Entropy wave $A_e = 0.025$, $\alpha = 90^\circ$, $\psi = 0^\circ$ vs. $k_x (k_y = 0)$

Example: $k_x = 8$

Density fluctuations along $y = 0.5$ and 5 main harmonics

Pressure fluctuations along $y = 0.5$ and 5 main harmonics

Post shock 1D density spectrum

Post shock 1D pressure spectrum
Dependence of post shock wave number against pre-shock wave number

\[ k_e - \text{entropy wave frequency;} \]
\[ k_p - \text{acoustic wave frequency;} \]

\[ \Delta \rho_1 = 0.025 \]
\[ \Delta \rho_2 = 0.114 \pm 0.008 \]

\[ \Delta p_1 = 0 \]
\[ \Delta p_2 = 0.078 \]
Entropy wave $A_e = 0.25$, $\alpha = 90^\circ$, $\psi = 60^\circ$, $k_y = 10$

Density

Pressure
Entropy wave $A_e = 0.25$, $\alpha = 90^\circ$, $\psi=60^\circ$, $k_y=5$
Entropy wave $A_e = 0.25$, $\alpha = 90^\circ$, $\psi=60^\circ$, $k_y=5$ (continue)

ADPDIS3D loses regularity and symmetry of the flow
Entropy wave $A_e = 0.25$, $\alpha = 90^\circ$, $\psi = 60^\circ$, $k_y = 5$ (continue)
Entropy wave $A_e = 0.25$, $\alpha = 90^\circ$, $\psi = 60^\circ$, $k_y = 5$ (continue)

Parameters along $y = 0.5$

![Graph of density](image1)

![Graph of x-velocity](image2)

![Graph of pressure](image3)

![Graph of y-velocity](image4)
Entropy wave $A_e = 0.25$, $\alpha = 90^\circ$, $\psi=60^\circ$, $k_y=5$ (continue)

1D spectra (linear scales)

density

x-velocity

pressure

y-velocity
Entropy wave \( A_e = 0.25, \alpha = 90^\circ, \psi = 60^\circ, k_y = 5 \) (continue)

1D spectra (linear scales)

- Density
- X-velocity
- Pressure
- Y-velocity
Entropy waves with turbulent incident spectrum

pre shock 1D density spectrum

density along y=0.5

post shock 1D density spectrum

density fluctuations along y=0.5 and 5 main harmonics
Entropy waves with turbulent spectrum

- Pressure
- Density fluctuations
- Pressure (zoom)
- Vorticity (zoom)
LISM interaction with a randomly perturbed SW

We perturb the SW velocity: \[ U_R = \bar{U} + \varepsilon U'_R, \quad U_\theta = \varepsilon U'_\theta, \quad \varepsilon = 0.025, \]
where \( \varepsilon \) measures the intensity of the input turbulence.

The initial spectrum is \( E(k) \sim k^{-2} \). Fluctuations are assumed to be isotropic.

The spectrum is chosen in such a way that the length scale of fluctuations lies between the grid size (0.5 AU) and 5 AU.
Time-variation of the density distribution
Time-variation of the velocity magnitude distribution
Shock Shape Calculated from Shock Speeds, Normals and Timing

Wind/Geotail saw this shock nearly simultaneously

Obs. at ACE is used to predict shock location at the time of Wind/Geotail observation

Neugebauer & Giacalone, 2007
Definition of 2-D Radius of Curvature

S is the location of the shock determined from observed speed (v) and direction at ACE

W is the location of another s/c that sees the shock a time Δt later

If the shock is planar, W would be on the vertical line - this was seldom the case. Can use simple geometry to get the radius of curvature
Distribution of shock radii of curvature

Neugebauer & Giacalone, 2007
CONCLUSIONS

• Need to understand the interaction of turbulence with shocks, motivated by Voyager observations downstream of the TS and models for particle acceleration at quasi-perpendicular shocks.

• Structure of shock modified - smoothing, non-R-H jumps (i.e., non-steady)

• Incompressible upstream fluctuations generates compressible fluctuations downstream, amplified density fluctuations, and vortical flow.

• Begun to study effect of varying Mach number, obliquity, and incidence of entropy fluctuations.

• Development of turbulence and vortical structure downstream due to mode coupling

• Magnetic fields generated downstream - implications for particle acceleration at perpendicular shocks interesting.

• Complications in numerically simulating interaction of fluctuations with shock wave are not trivial.