Magnetic buoyancy as an origin of solar variability?

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Workshop PICARD 8/03/10

In collaboration
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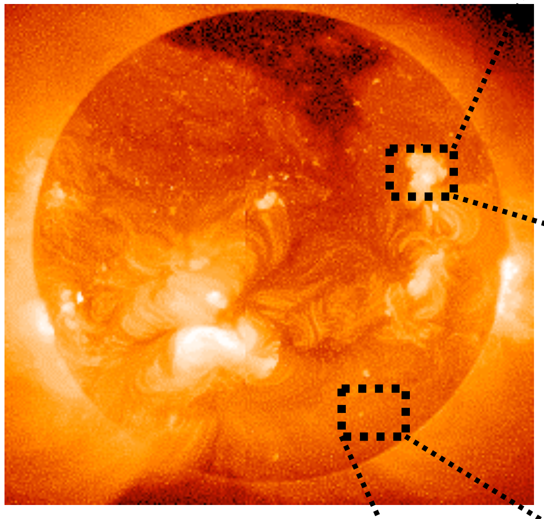
UNIVERSITY OF CAMBRIDGE

Astrophysical Fluid Dynamics
and Nonlinear Dynamics

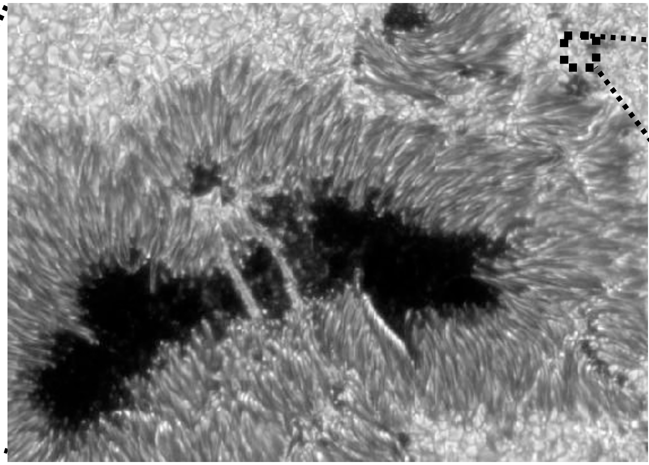
Irfu
cea
saclay

Scales of solar magnetism and convection



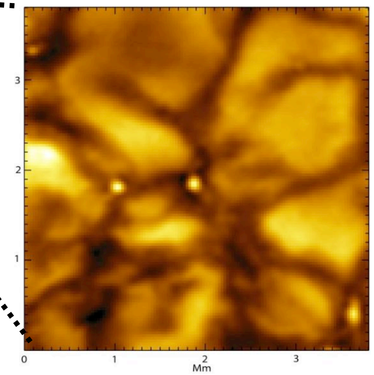
Large structures:
Eruptions,
Coronal holes
CMEs

200+ Mm :
Giant cells?
(10-20 days)

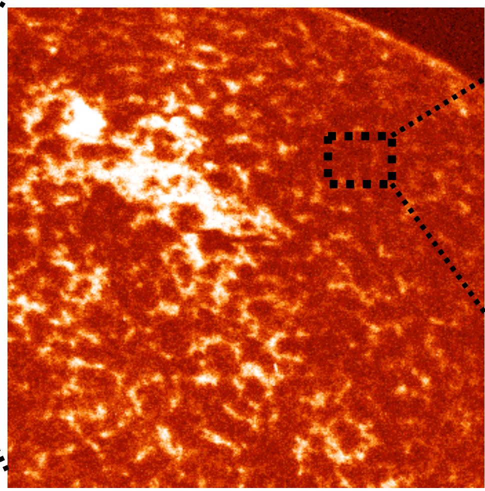


30-50 Mm:
Supergranulation
(20 hours)
Sunspots
(hours to days)

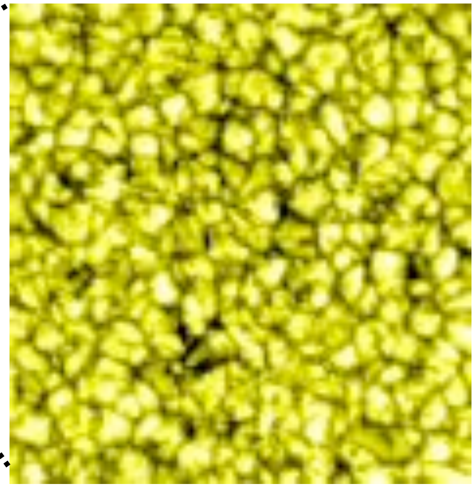
7-10 Mm:
Mesogranulation?
(2 hours)



Smaller stuctures:
Intergranular lanes
Magnetic bright points
Flux tubes < 100km

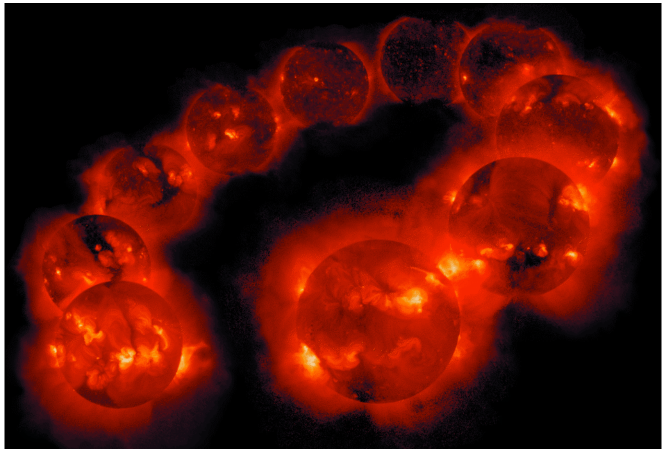


1-2 Mm:
Granulation
(5 minutes)

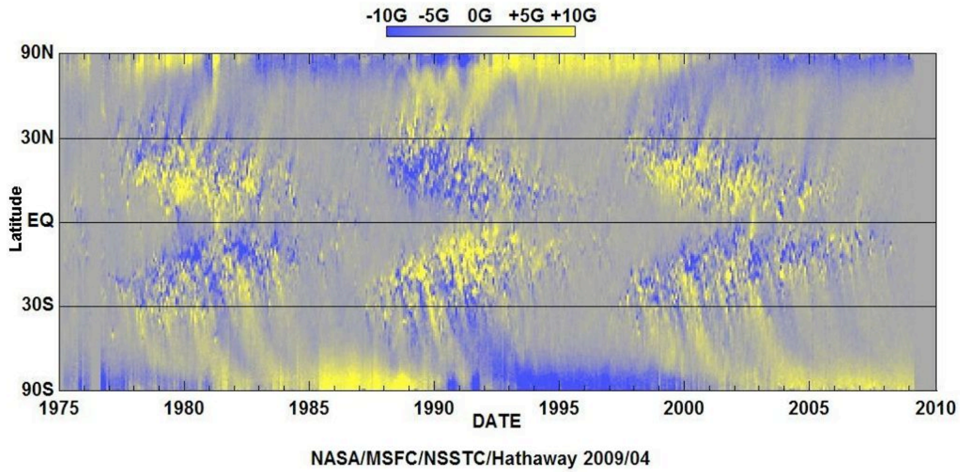


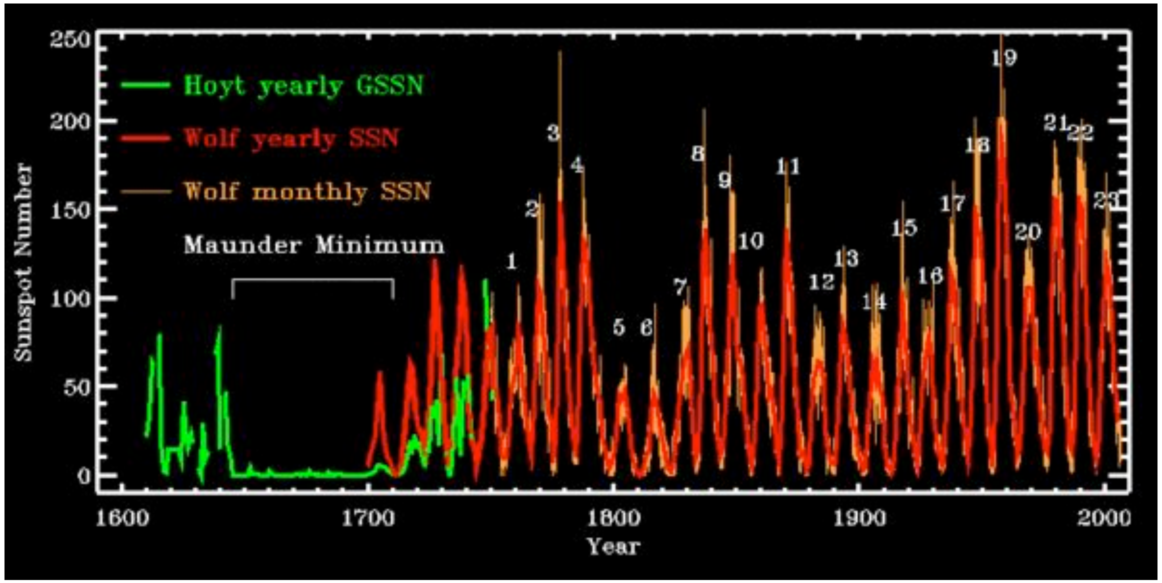
The variability of the 22-yr cycle

Yohkoh soft Xray images over a whole solar cycle (1991-2001)



Butterfly diagram (observed radial field) (D. Hathaway)





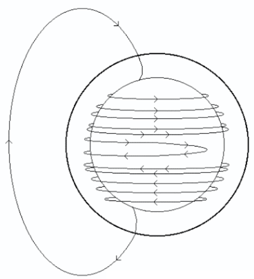
Varying sunspot number (Hoyt & Schatten 1998)

Theory: *the induction equation*

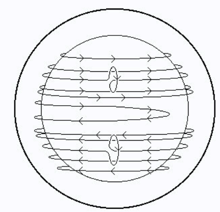
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B})$$

$$\frac{\partial \mathbf{B}}{\partial t} = \underbrace{(\mathbf{B} \cdot \nabla)\mathbf{u}}_{Shearing\,of\,B} - \underbrace{(\mathbf{u} \cdot \nabla)\mathbf{B}}_{Advection\,of\,B} - \underbrace{\mathbf{B}(\nabla \cdot \mathbf{u})}_{Compressibility} - \underbrace{\nabla \times (\eta_m \nabla \times \mathbf{B})}_{Magnetic\,diffusion}$$

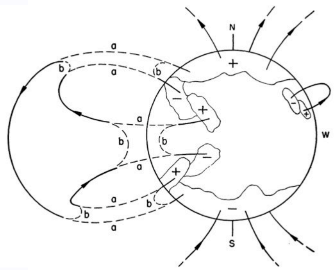
Source of magnetic field



✓Ω-effect



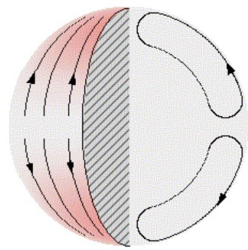
✓α-effect



✓ Babcock-Leighton source term

Transport of magnetic field

- ✓ **Large-scale flows (meridional circulation)**
- ✓ **Downward pumping by penetrative convection**



- ✓ **Transport from the base of the convection zone to the surface**

2D numerical simulations

Mean induction equation

Simplified description of physical processes

Fast and efficient tool
Parametric studies

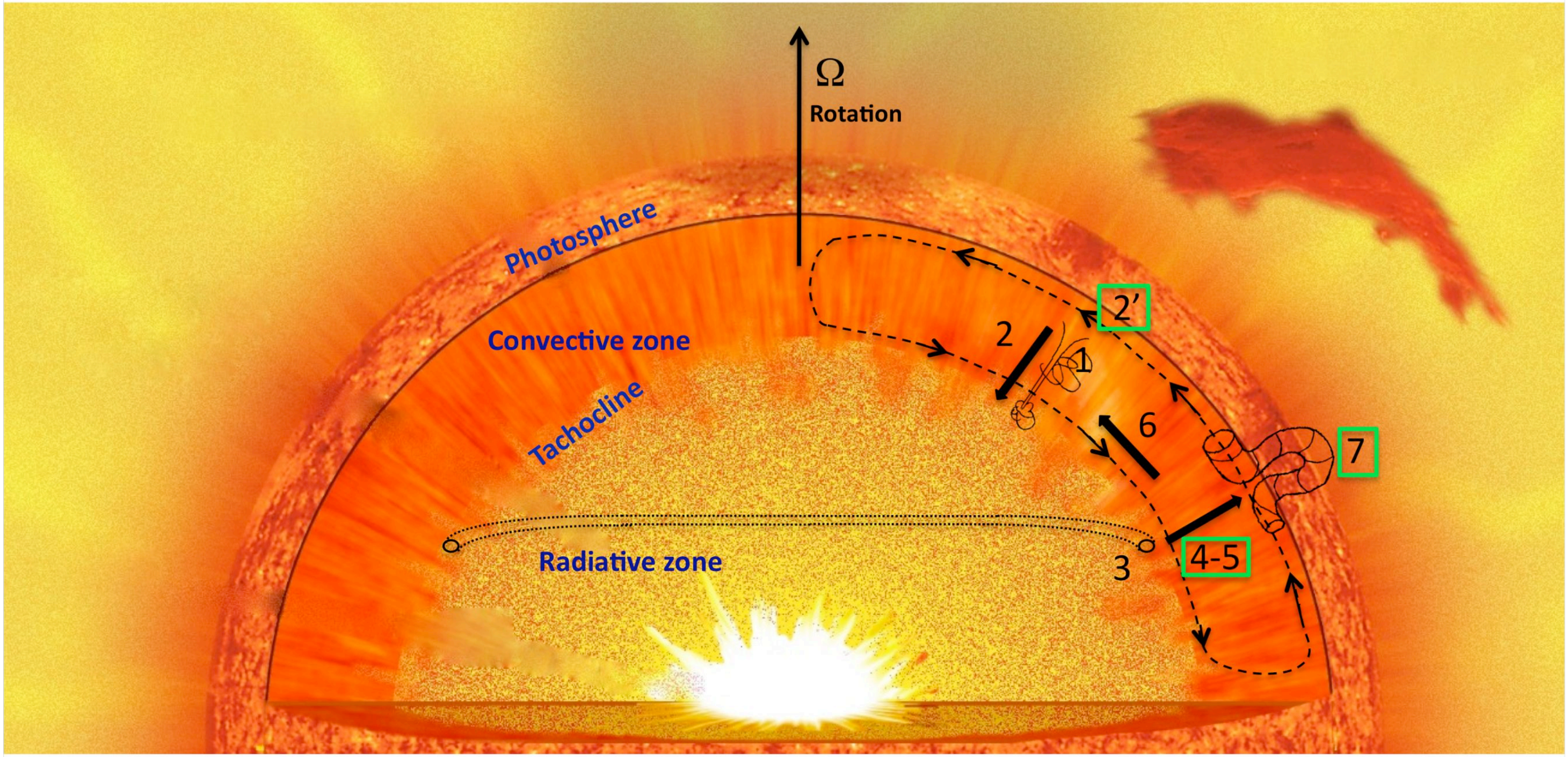
3D numerical simulations

MHD equations

Much more complex

Self-consistent simulations

Schematic theoretical view of the solar cycle



1: magnetic field generation, self-induction
2: pumping of mag. field
or
2': **transport by meridional flow**
3: stretching of field lines through Ω effect

4: Parker instability
5: **emergence+rotation**
6: recycling through α−effect or
7: **emergence of twisted bipolar structures at the surface**

Flux tube rise from the BCZ to the surface

Previous calculations:

Thin flux tubes: all the variables only vary along the tube axis (Spruit,1981)

2D simulations: cartesian or spherical (e.g. Emonet & Moreno-Insertis 1998)

3D simulations: with convection in cartesian geometry (Cline 2003, Fan et al 2003)
without convection in spherical geometry (Fan 2008)

→ **First 3D simulations in a fully convective rotating spherical shell**

Various isentropic cases:
- twisted/ not twisted
- varying the field strength

Various cases in the fully convective case:
- weak B/ strong B
- varying magnetic diffusivity

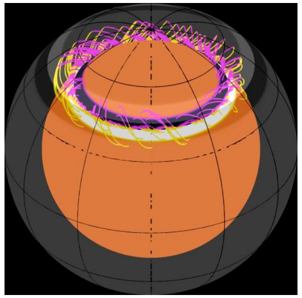
✓ Beq is approximately 6×10^4 G

✓ For the twisted case, the twist is set to a value above the 2D-threshold:

$$\sin\psi = 0.5 > \sin\psi_{\min} \approx 0.42$$

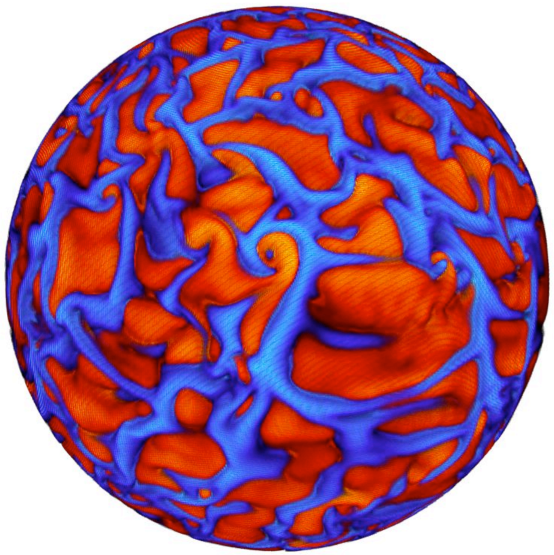
✓ Pressure and entropy equilibrium, density deficit in the tube (magnetic buoyancy)

$$\frac{\rho_{in}}{\rho_{ext}} = \left(1 - \frac{B^2}{8\pi P^g_{ext}}\right)^{1/\gamma}$$



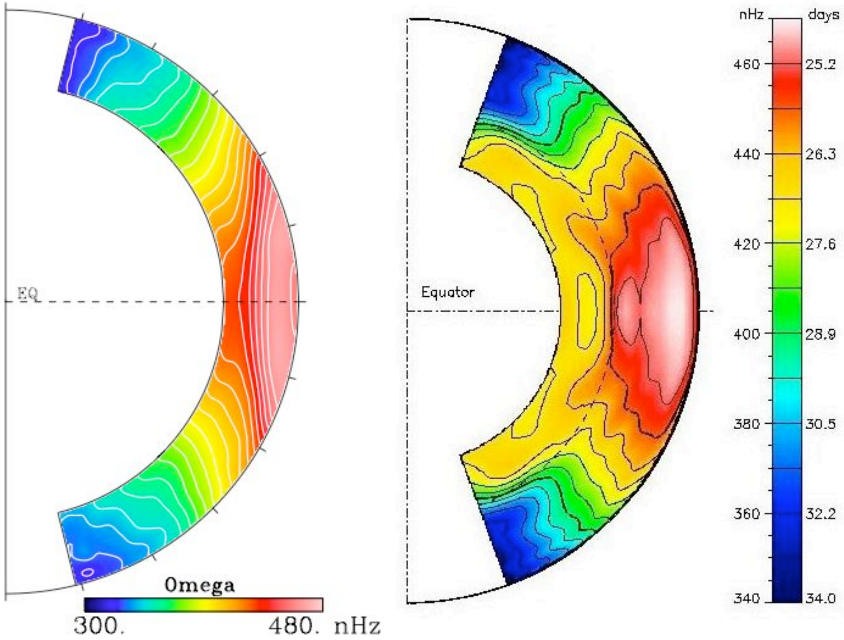
A realistic model of the solar convection zone

η=1.13 10^{12} cm^2 s^{-1}
ν=1.13 10^{12} cm^2 s^{-1} at mid-CZ, vary as $1/\bar{\rho}^{1/3}$ Re= 120, Pr=0.25, Pm=1
κ=4.53 10^{12} cm^2 s^{-1}



Radial velocity

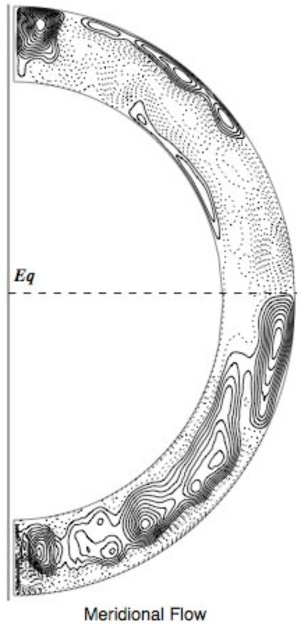
(volume rendering obtained with SDvision)



Differential rotation

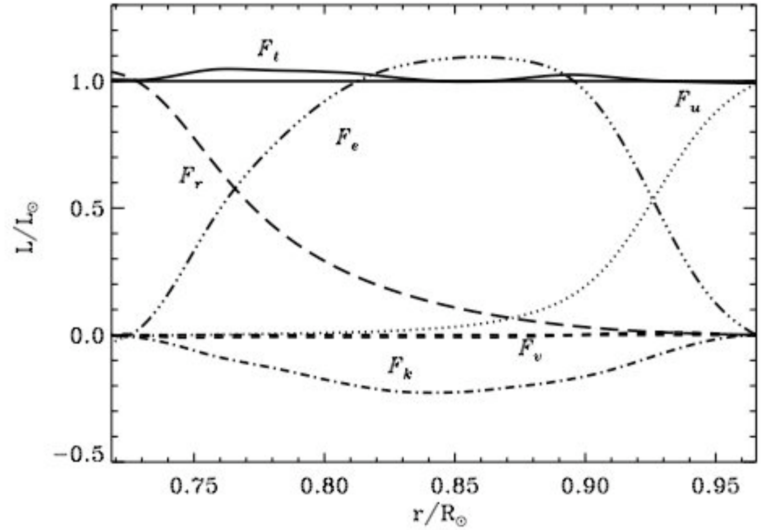
Miesch, Brun & Toomre 2006

$$S(r_{bot},\theta) = a_2\, Y_2^0 + a_4\, Y_4^0$$

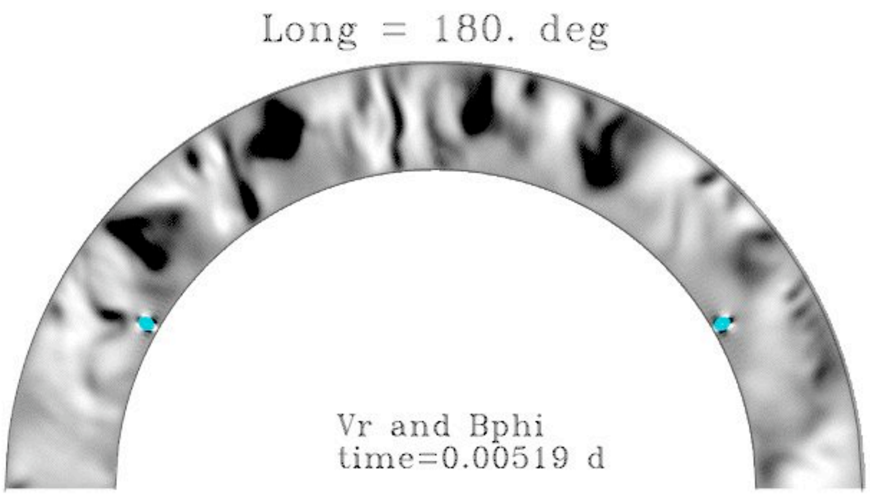


Meridional circulation

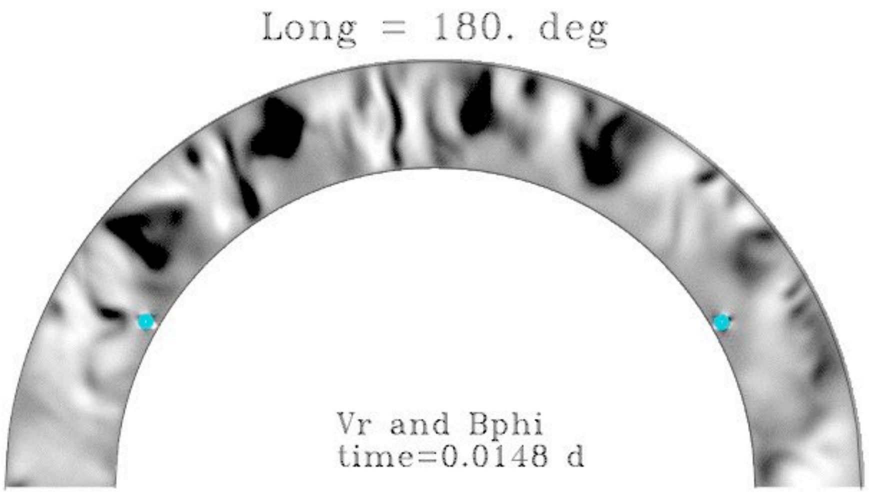
Convective cases



Influence of the initial magnetic intensity



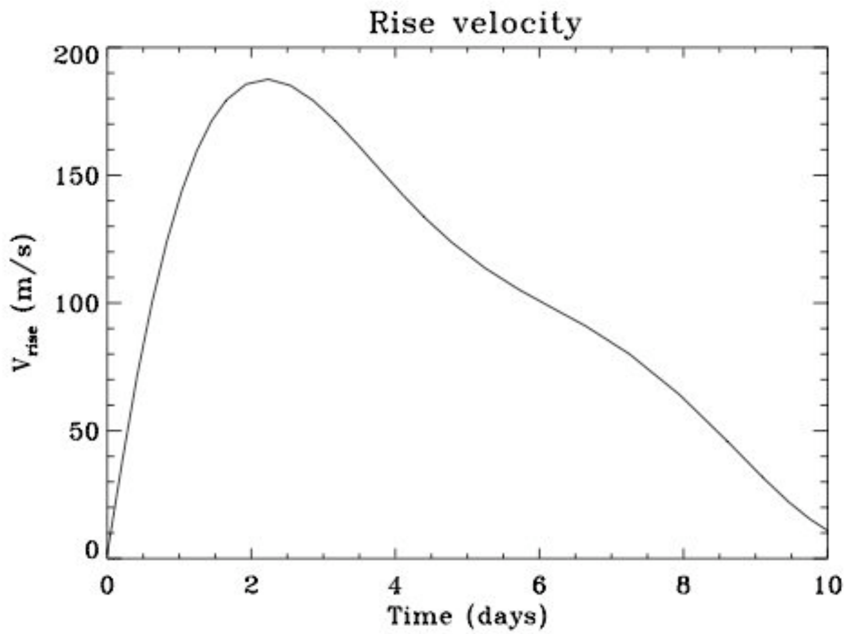
Strong B



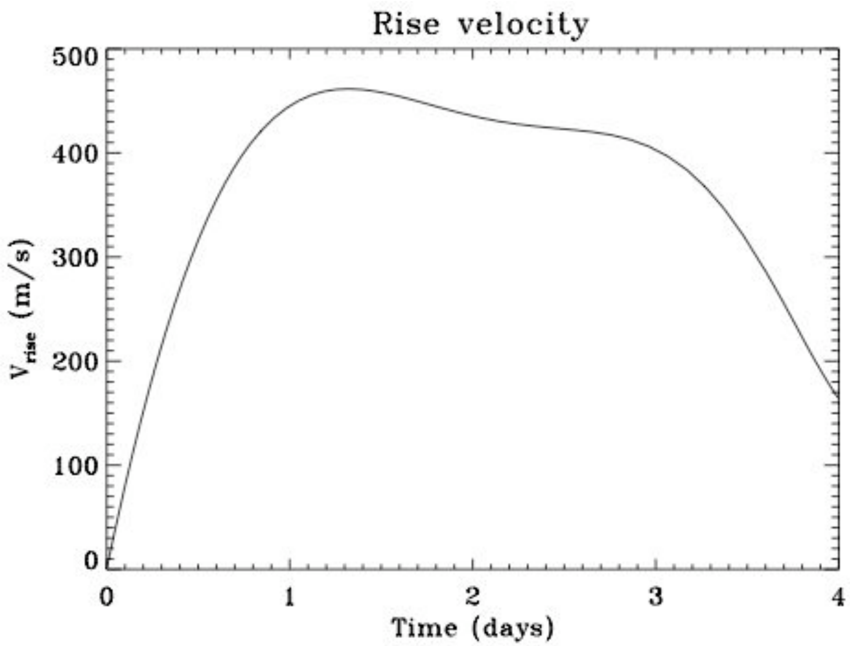
Weak B

Convective cases
rise velocity

Weak B



Strong B



The tube acceleration in the first phase is due to the buoyancy force which is directly linked to the density deficit in the tube.

Thus we have in the acceleration phase:

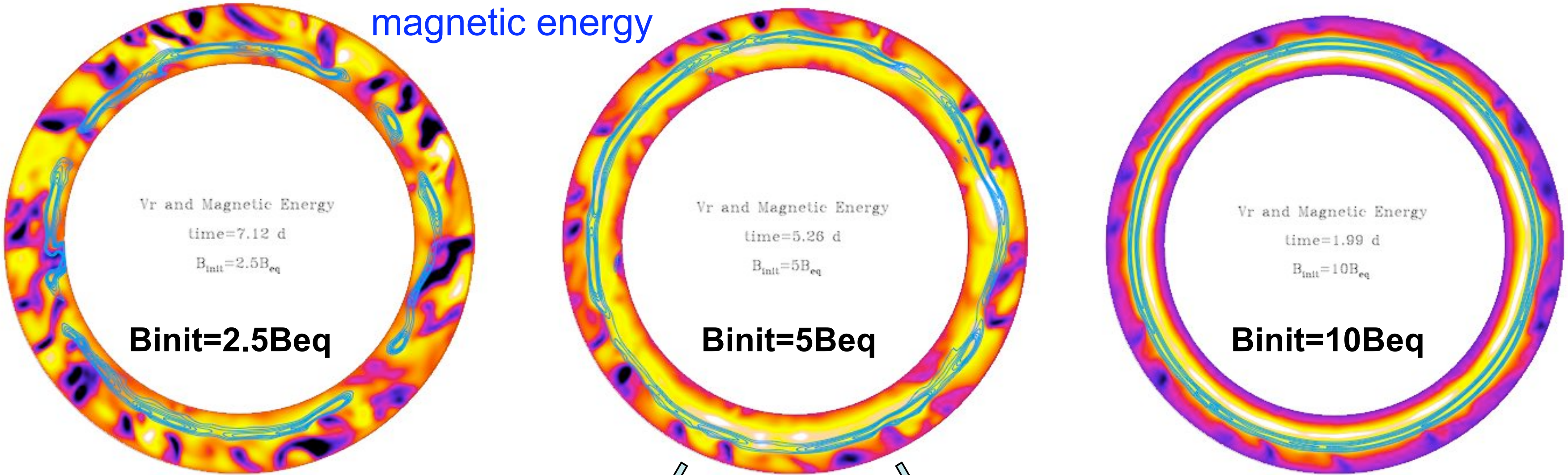
$$acceleration \;\; \alpha \;\; \Delta\rho \;\; \alpha \;\; B_0^2$$

Convective cases
Influence of the initial magnetic intensity

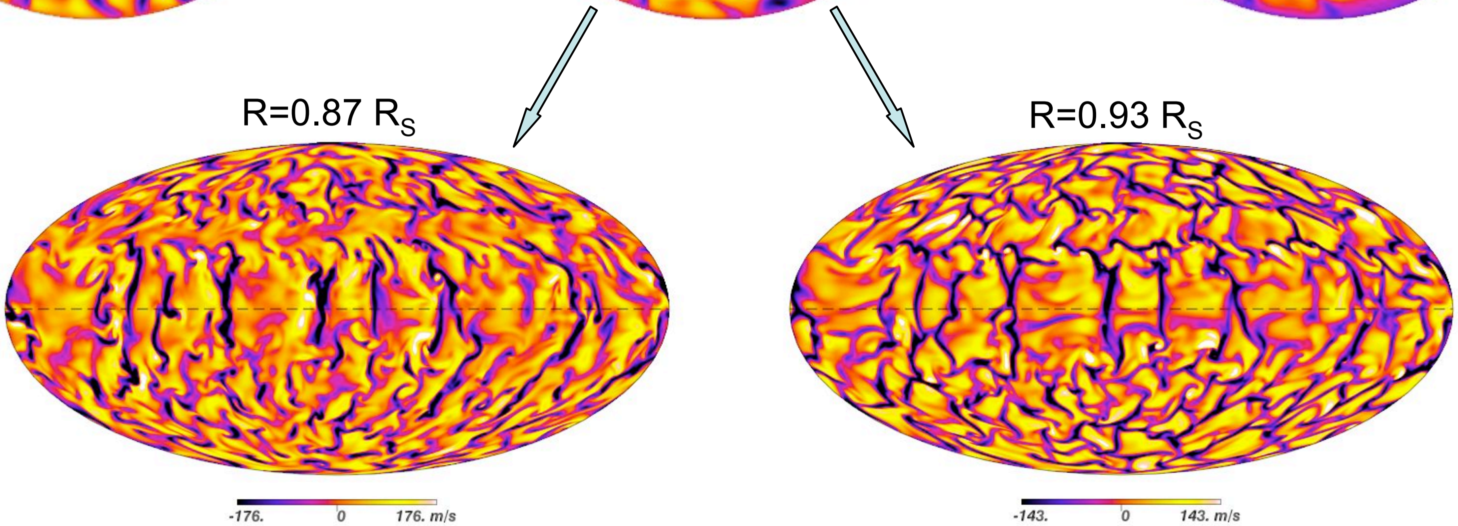
Jouve & Brun, 2009, ApJ, 701, 1300

If B is weak, the evolution of the flux tube is much more influenced by convective motions.

Radial velocity and magnetic energy



Radial velocity



Conclusions on 3D models

- We need a sufficient twist to be able to keep a coherent rising flux tube.

- The initial magnetic field strength controls the rise velocity of the flux tube.

- The initial magnetic field strength controls the trajectory of the flux tube (vertical vs radial rise).

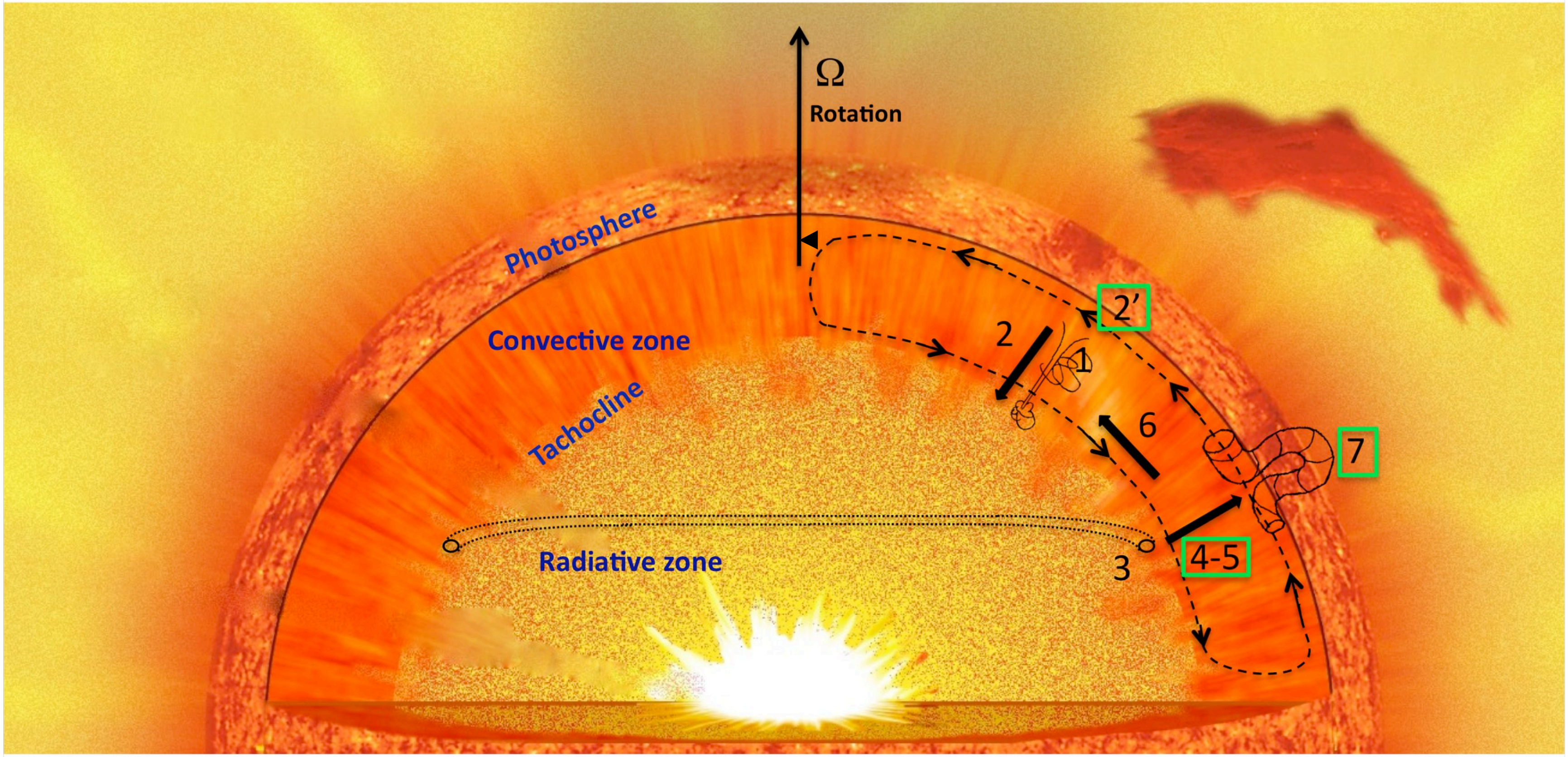
- To get magnetic flux emerging at specific longitudes as active regions, we need to keep a reasonably low field strength.

- Meridional flow seems to have a significant impact on the tube behaviour when the magnetic field intensity is sufficiently small.

- Differential rotation makes it harder for tubes to emerge at low latitudes.

→ **Idea: reintroducing some of these results in 2D mean-field models**

Schematic theoretical view of the solar cycle



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The STELEM code

(Emonet & Charbonneau 1998, Jouve & Brun 2007)

Mean-field induction equation

$$\frac{\partial \langle \mathbf{B} \rangle}{\partial t} = \nabla \times (\langle \mathbf{U} \rangle \times \langle \mathbf{B} \rangle) + \nabla \times (\alpha \langle \mathbf{B} \rangle) - \nabla \times [(\eta_m + \beta) \nabla \times \langle \mathbf{B} \rangle]$$

Poloidal/toroidal decomposition

$$\mathbf{B}(r,\theta,t) = \nabla \times (A_\phi(r,\theta,t)\hat{\mathbf{e}}_\phi) + B_\phi(r,\theta,t)\hat{\mathbf{e}}_\phi$$

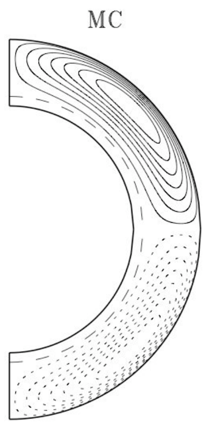
$$\mathbf{U}(r,\theta) = \mathbf{u_p}(r,\theta) + r \sin\theta\, \Omega(r,\theta)\hat{\mathbf{e}}_\phi$$

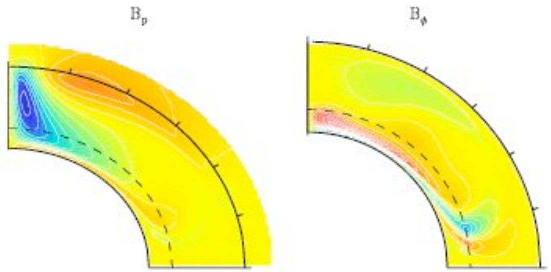
2 coupled adimensioned PDEs

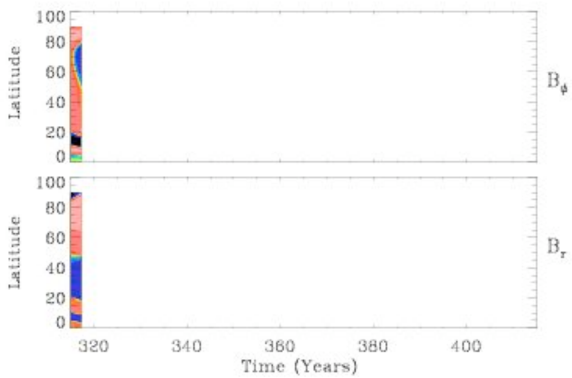
$$\frac{\partial A_\phi}{\partial t} = \frac{\eta}{\eta_t}(\nabla^2 - \frac{1}{\varpi^2})A_\phi - R_e \frac{\mathbf{u}_p}{\varpi} \cdot \nabla(\varpi A_\phi) + C_\alpha \alpha B_\phi + C_s S(r,\theta,B_\phi)$$

$$\frac{\partial B_\phi}{\partial t} = \frac{\eta}{\eta_t}(\nabla^2 - \frac{1}{\varpi^2})B_\phi + \frac{1}{\varpi}\frac{\partial(\varpi B_\phi)}{\partial r}\frac{\partial(\eta/\eta_t)}{\partial r} - R_e \varpi \mathbf{u}_p \cdot \nabla(\frac{B_\phi}{\varpi}) - R_e B_\phi \nabla \cdot \mathbf{u}_p + C_\Omega \varpi(\nabla \times (\varpi A_\phi \hat{\mathbf{e}}_\phi)) \cdot \nabla\Omega$$

Standard model:
single-celled
meridional circulation







Cyclic field

Butterfly diagram close to observations

Parameters:

v_0=6.4 m.s^{-1}
η_t=5.e10cm^2.s $^{-1}$
s_0=20 cm.s^{-1}
Ω_{eq}=460 nHz

The Babcock-Leighton source term

Standard

$$S(r,\theta,B_\phi) = \underbrace{\frac{1}{4}\left[1+\mathrm{erf}\left(\frac{r-r_2}{d_2}\right)\right]\left[1-\mathrm{erf}\left(\frac{r-R_\odot}{d_2}\right)\right]}_{\text{Confinement at the surface}} \times \underbrace{\left[1+\left(\frac{B_\phi(r_c,\theta,t)}{B_0}\right)^2\right]^{-1}}_{\text{Quenching}} \underbrace{\cos\theta\sin\theta}_{\text{« Ad hoc » latitudinal dependence}} \underbrace{B_\phi(r_c,\theta,t)}_{\text{Toroidal field at the base of the CZ}}$$

Confinement at the surface

Quenching

« Ad hoc » latitudinal dependence

Toroidal field at the base of the CZ

Modified

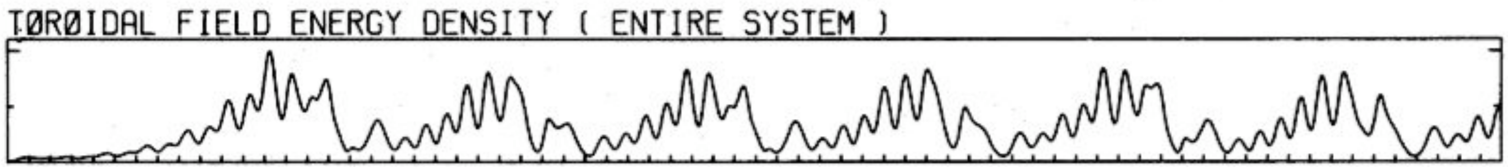
$$S(r,\theta,B_\phi) = \frac{1}{2}\left[1+\mathrm{erf}\left(\frac{r-r_2}{d_2}\right)\right]\left[1-\mathrm{erf}\left(\frac{r-R_\odot}{d_2}\right)\right] \times \left[1+\left(\frac{B_\varphi(r_c,\theta,t-\tau(B_\varphi))}{B_0}\right)^2\right]^{-1} \cos\theta \; B_\varphi(r_c,\theta,t-\tau(B_\varphi))$$

With $\tau(B_\varphi) \propto \dfrac{1}{B_\varphi^2}$

Jouve & Proctor, 2010, submitted

Previous work on delays and dynamo models

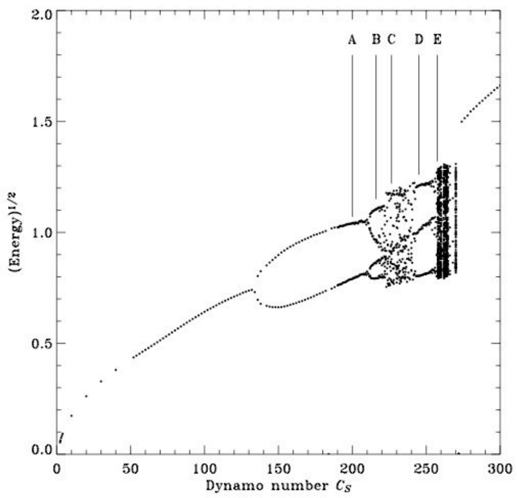
- Yoshimura 78 introduces delays in both equations and gets long-term modulation (but large delays compared to the cycle period: 30 yr delay to get a 80 yr modulation)



- Wilmot-Smith et al 05 consider delays in a flux-transport BL model but very simple (0D) and consider mainly the delay due to MC

$$\frac{dB_\phi(t)}{dt} = \frac{\omega}{L}A(t - T_0) - \frac{B_\phi(t)}{\tau},$$

$$\frac{dA(t)}{dt} = \alpha_0 f\left(B_\phi(t - T_1)\right)B_\phi(t - T_1) - \frac{A(t)}{\tau}.$$



- Charbonneau et al 05 increase s0 in their models and find a modulation of the cycle and a series of bifurcations leading to chaotic behaviour.
They argue it's linked to the time delay introduced by the meridional flow.

The time delay due to the buoyant rise of structures is never considered because thought to be too small to do anything to the 11-yr cycle. We seem to find that it's not true.

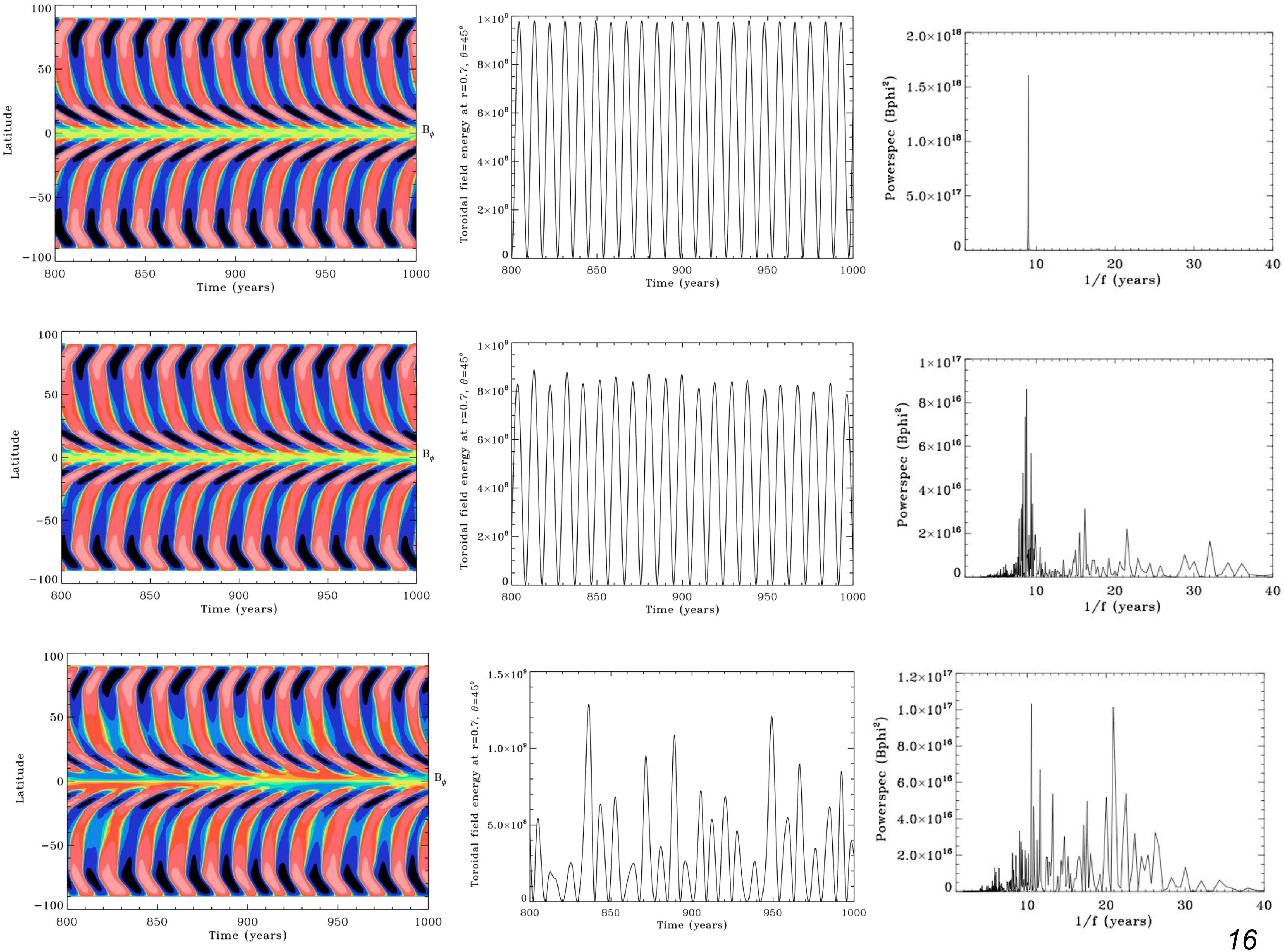
Modulation of the cycle?

Butterfly diagram | **Magnetic energy** | **Powerspectrum**

Standard (no delay)

14 days on 10^3G fields

14 days on $5x10^4$G fields



A reduced model of the rise of flux tubes without using explicit delays

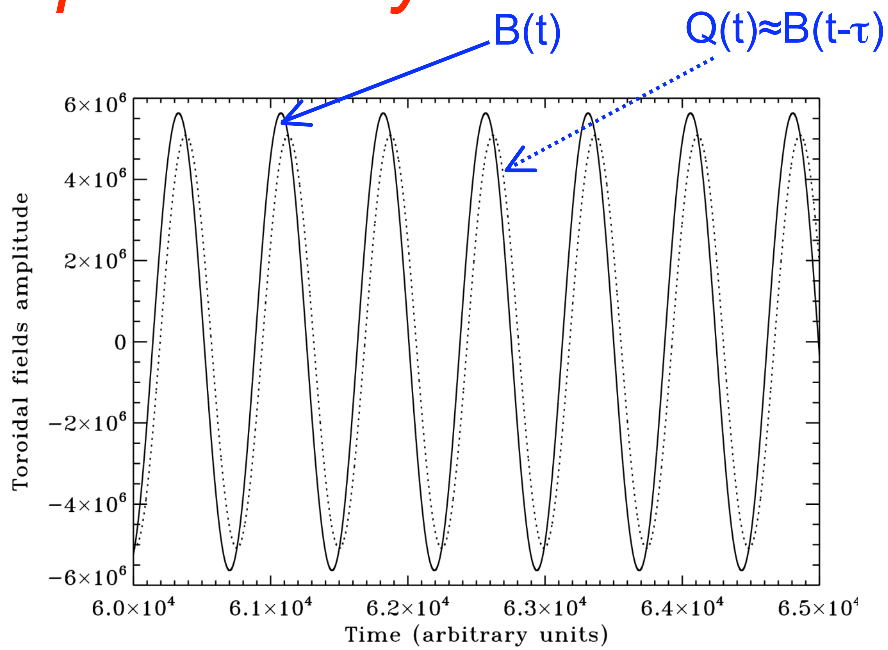
We consider the following set of equations, Q is here delayed by t compared to B:

$$\frac{dA}{dt} + ikv_p A = \frac{SQ}{1+\lambda|Q|^2} - \eta k^2 A$$

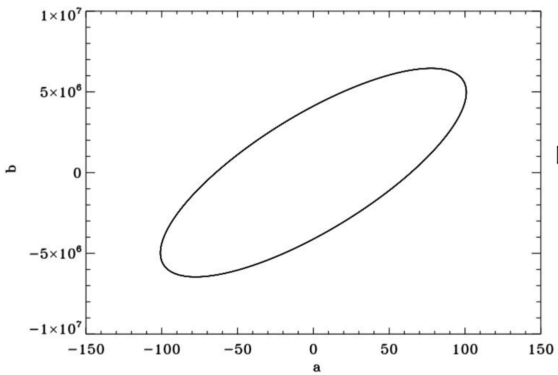
$$\frac{dB}{dt} + ikv_p B = ik\Omega A - \eta k^2 B$$

$$\frac{dQ}{dt} = \frac{1}{\tau}(B - Q)$$

with $\tau = \dfrac{\tau_0}{1+|B|^2}$

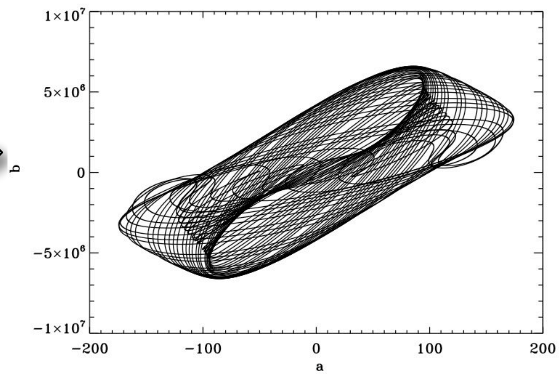


$t_0=0.1$



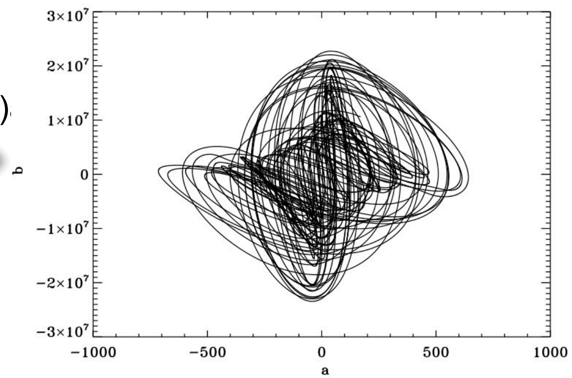
Bifur-
cation

$t_0=0.4$



Bifur-
cation(s)

$t_0=5$



Jouve & Proctor, 2010, submitted

From the periodic to the aperiodic solution…

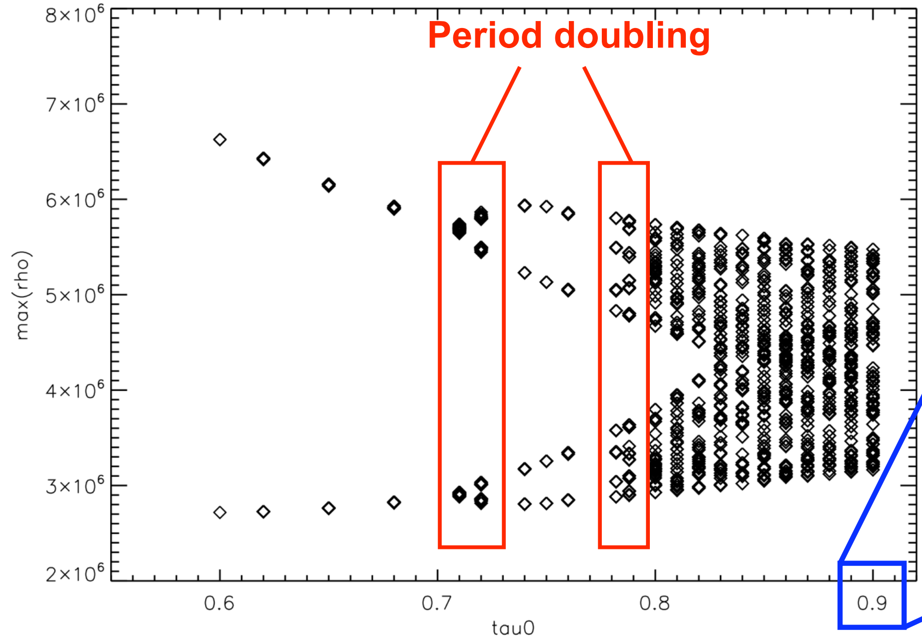
By noticing the existence of a symmetry in the equation (phase invariance), we can further reduce our system from 6 to 5 degrees of freedom.

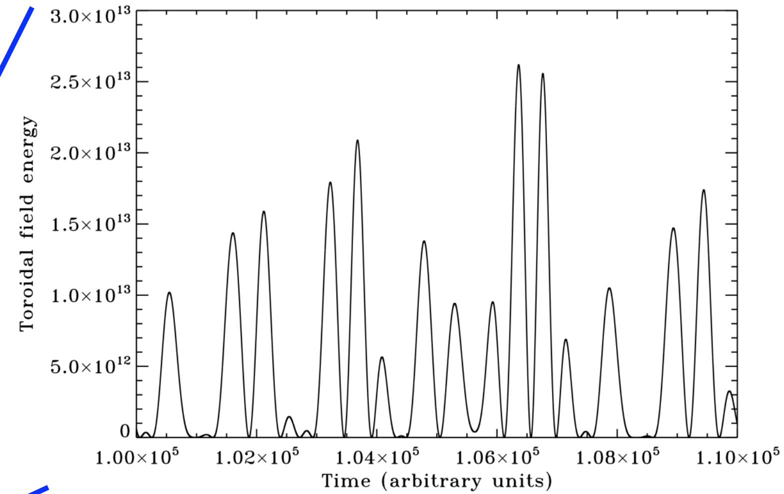
$$A(t) = \rho\, x\, \exp(i\theta)$$
$$B(t) = \rho \exp(i\theta)$$
$$Q(t) = \rho\, y\, \exp(i\theta)$$

with ρ, θ real and x, y complex

$$\frac{d\rho}{dt} = -\Omega\rho\Im(x) - \eta\rho$$
$$\frac{dx}{dt} = \frac{Sy}{1+\lambda\rho^2|y|^2} - i\Omega x^2$$
$$\frac{dy}{dt} = \frac{1+\rho^2}{\tau_0}(1-y) - (i\Omega x - iv_p - \eta)y$$

The route to aperiodic solutions (seen in the 2D model) is then very clear…





Aperiodic modulation of the cycle

Conclusions and perspectives on 2D models

Reintroducing results from 3D calculations in simple mean-field dynamo models help to have some insights on which process has which effect on the global solar cycle.

We find that even in a linear model, we do get some modulation of the cycle amplitude when time delays are introduced.

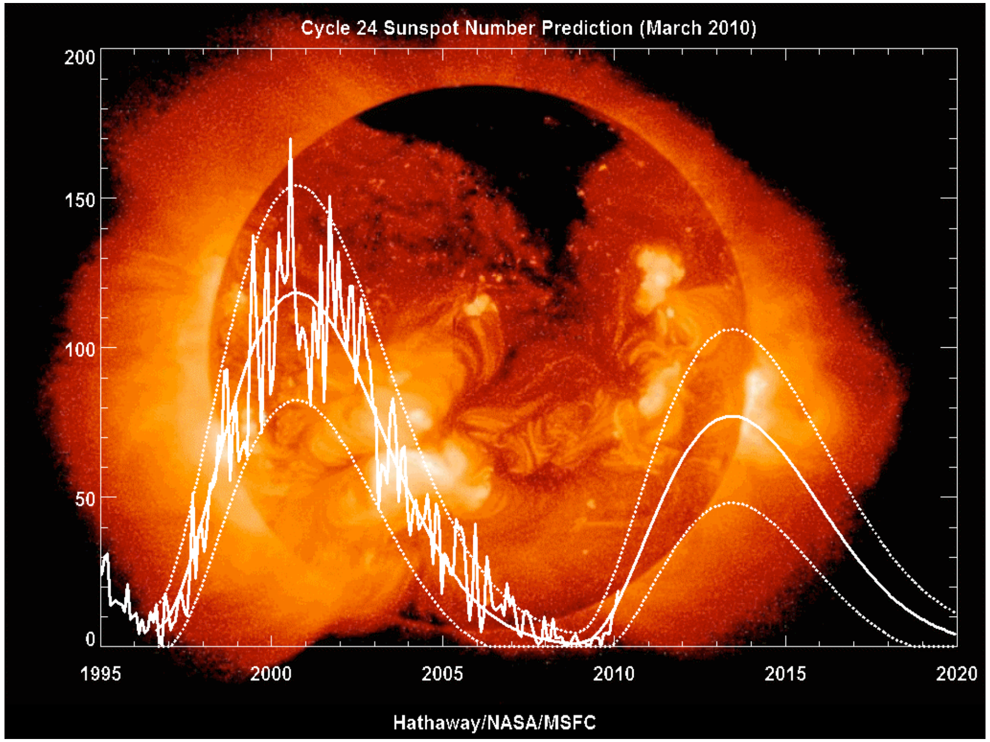
An analytical and numerical study on a reduced set of nonlinear odes exhibits a sequence of bifurcations, leading to aperiodic solutions similar to the complete 2D-model and more importantly to the actual solar cycle.

→ The magnetic-field dependent rise time of flux tubes are shown to be a potential source of modulation of the solar cycle!

Perspectives

• The origin of the solar variability still has to be investigated through various techniques (multi-D numerical simulations, reduced models, analytical studies).
• It may lead to better ways to predict the solar cycle.
• Data assimilation (used for years in meteorology on Earth) may be of great help (e.g. Dikpati & Gilman 2006, Kitiashvili & Kosovichev, 2008).
• Picard will provide the data we need to improve our models (in particular the spectral irradiance measurements with PREMOS).

Hathaway cycle prediction using geomagnetic indices



(http://solarscience.msfc.nasa.gov/predict.shtml)

Perspectives

- 3D evolution of magnetic flux tubes in a magnetized convection zone.
- Influence of a tachocline (m=1 Tayler instability?).
- Introducing an atmosphere.

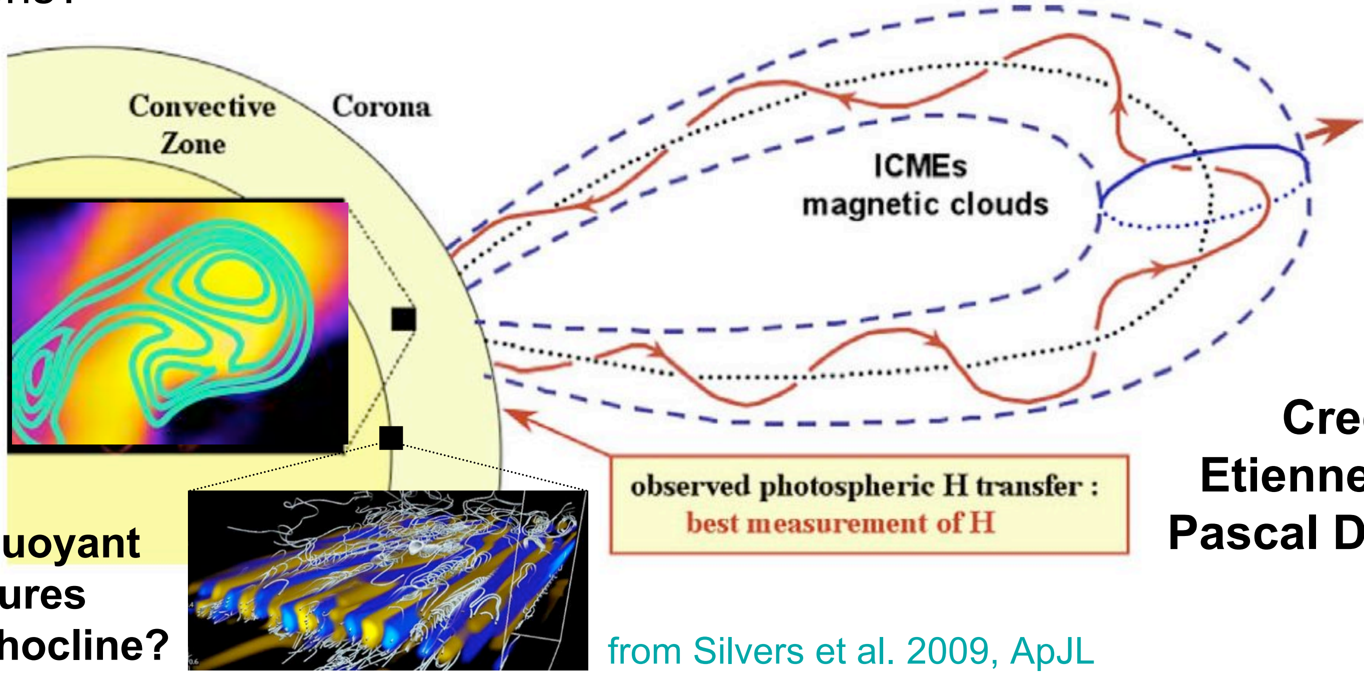
- **Magnetic helicity from the base of the CZ to the external Sun:**

  In the tachocline: origin of the twist of the field lines (buoyancy calculations).

  In the bulk of the CZ: Magnetic helicity conservation in flux tubes (modified induction equation, Snoopy code).

  At the surface: Which configuration in active regions? How does it compare to observations?

Twisted flux tube + convection



Helical buoyant structures in the tachocline?

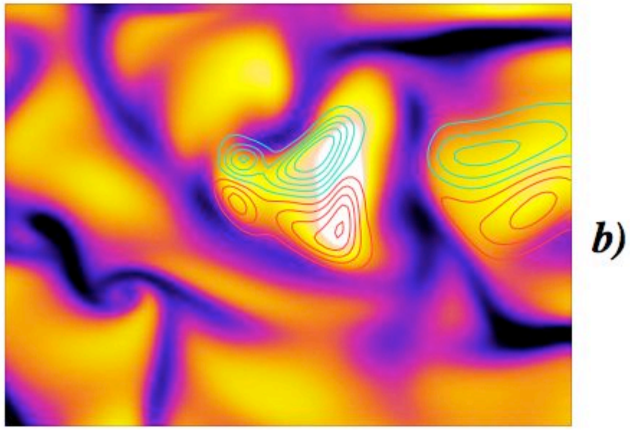
from Silvers et al. 2009, ApJL

Credit: Etienne Pariat Pascal Démoulin

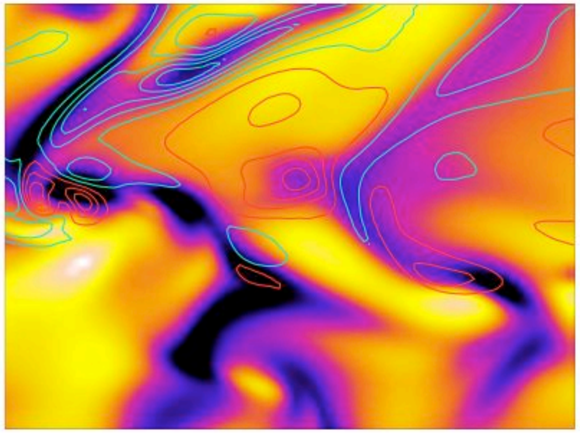
Structure of emerging regions

Zoom on an emerging region: radial magnetic field and radial velocity

Convective motions act to modify the orientation of active regions



t=7.97 d

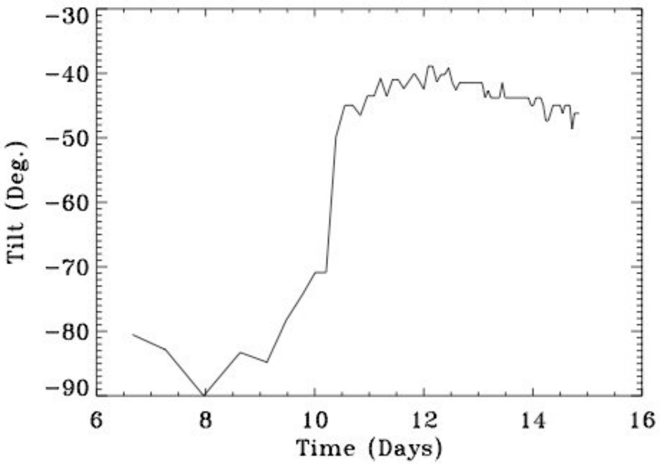


t=14 d

Kosovichev & Stenflo, 2008, ApJ

Evolution of the tilt angle

Simulations



Observations