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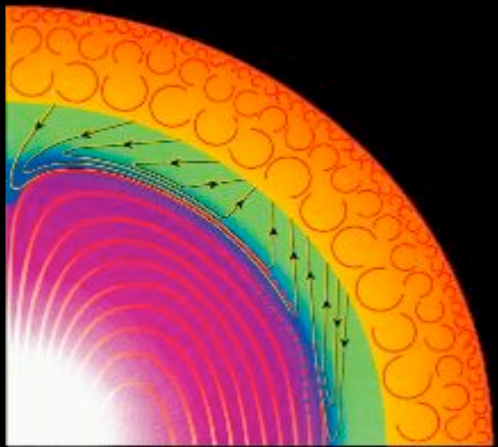
Surface Impact of the Radiative Dynamics : a First Step



1st PICARD Workshop, Institut d'Astrophysique de Paris, 8-9 March 2010

Outline

I. Context : a quick overview of the radiative dynamics

II. The fossil magnetic field model

III. Surface impact of the radiation zone dynamics :

1. Method
2. *Results :* large-scale magnetic field
3. *Results :* internal rotation

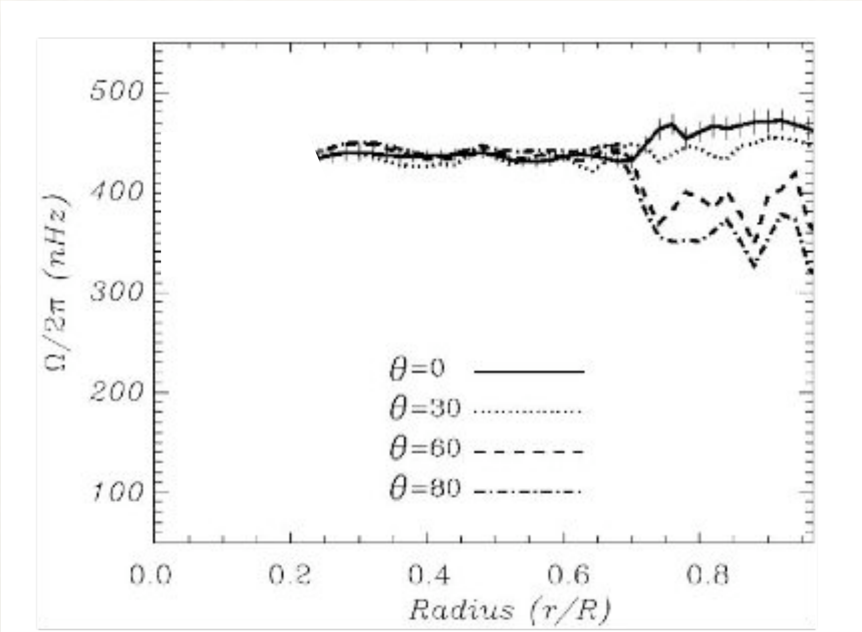
IV. Perspectives

I. Context

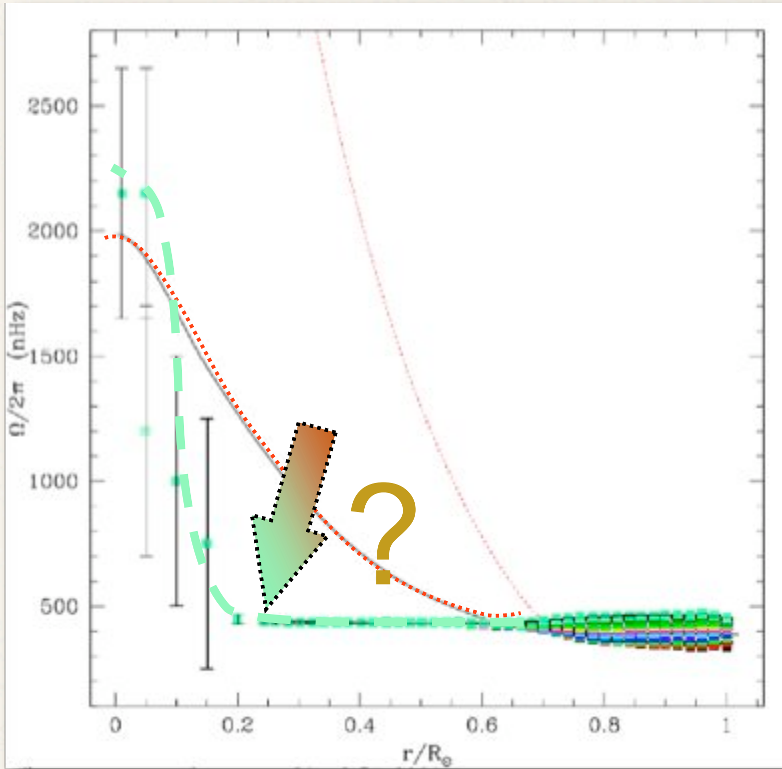
The radiation zone dynamics : Quick Overview

$98\%\ M_\odot$

The solar internal rotation



Eff-Darwich et al. (2008); Mathur et al. (2008)



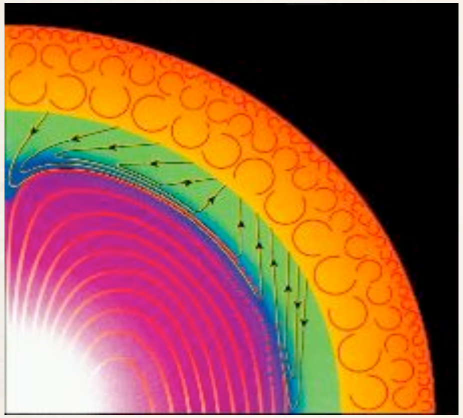
Garcia et al. (2007); Turck-Chièze et al. (2010)

- **Tachocline : seat of a strong transition between the two rotation regimes**
- **Angular momentum transport in the solar radiation zone**

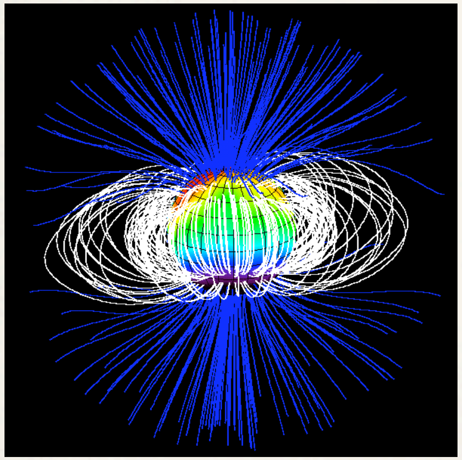
- **Internal waves ?** (Talon & Charbonnel, 2005 + *cf*. Stéphane Mathis talk)
- **Large scale magnetic field ?** (**Eggenberger et al., 2005**; Garaud, 2008)

The radiation zone dynamics : Quick Overview

The solar radiation zone : *ORIGINS* of a possible *"fossil"* magnetic field



Gough & McIntyre (1998)

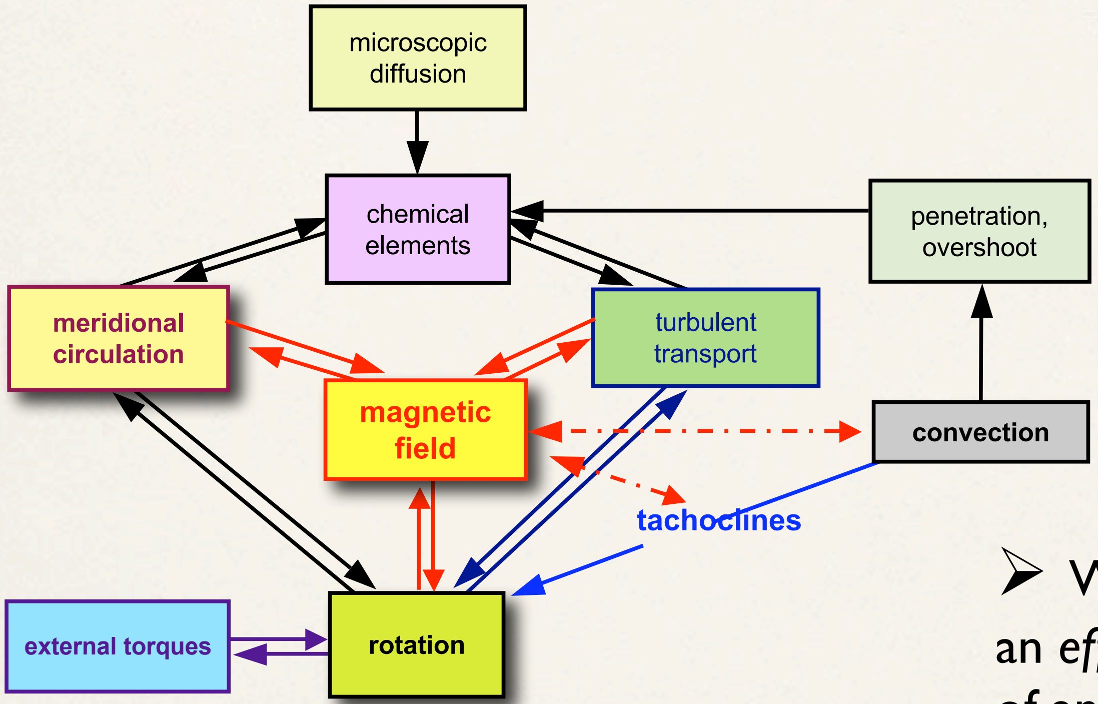


Convective Star V374 Peg;
Donati et al. (2006); Morin et al. (2008)

- **Dynamo field penetrating from the upper convection zone?** *Garaud (1999)*
- **Dynamo at work *within* the radiation zone?** *Braithwaite (2006)* *Zahn et al. (2007)*
- **Flux conservation of a seed magnetic field, of primordial origin ?**
- **Heritage of a dynamo having occured during early stages of stellar formation (totally convective) ?** *Browning et al. (2009)*

The radiation zone dynamics : Quick Overview

The secular MHD Transport :
coupling meridional circulation, differential rotation and magnetic field



Mathis & Zahn, 2004; 2005

➢ Will the magnetic field be an *efficient enough* agent of angular momentum tranport?

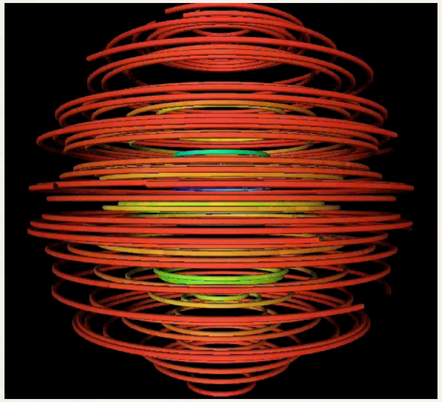
➢ How would it interact with the overlying dynamo field?

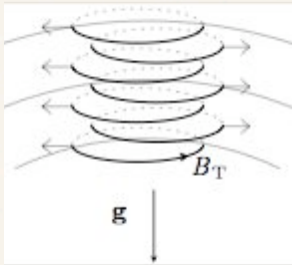
II. The fossil magnetic field model

On the track of *stable equilibrium* magnetic configurations

- Purely toroidal fields and purely poloidal ones are well known to be unstable (since works by Tayler, Markey, Wright in the 1970s)

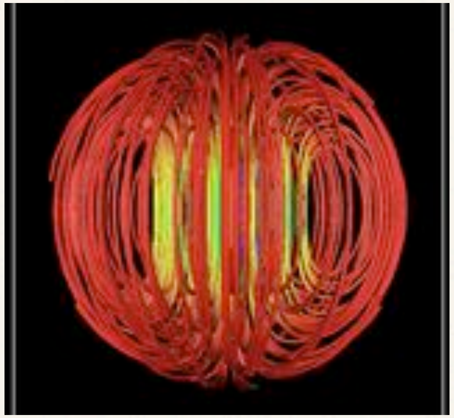
toroidal

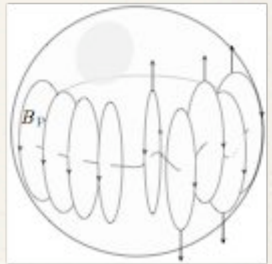




Spruit (1999); Brun (2007)

poloidal

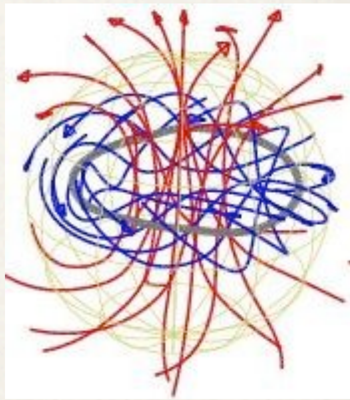




Braithwaite (2007)

- The only stable configuration known up to date : twisted field found in a numerical experiment -> *a quasi-axisymmetric MHS equilibrium is observed.*

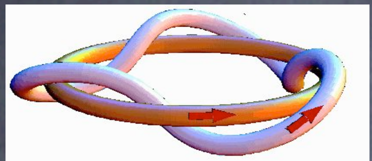
- We assume the field to be the result of a relaxation; it is in axisymmetric MHS equilibrium.



Braithwaite (2004; 2006)

The most likely to act over secular timescales

Minimum energy solution (for a given helicity):

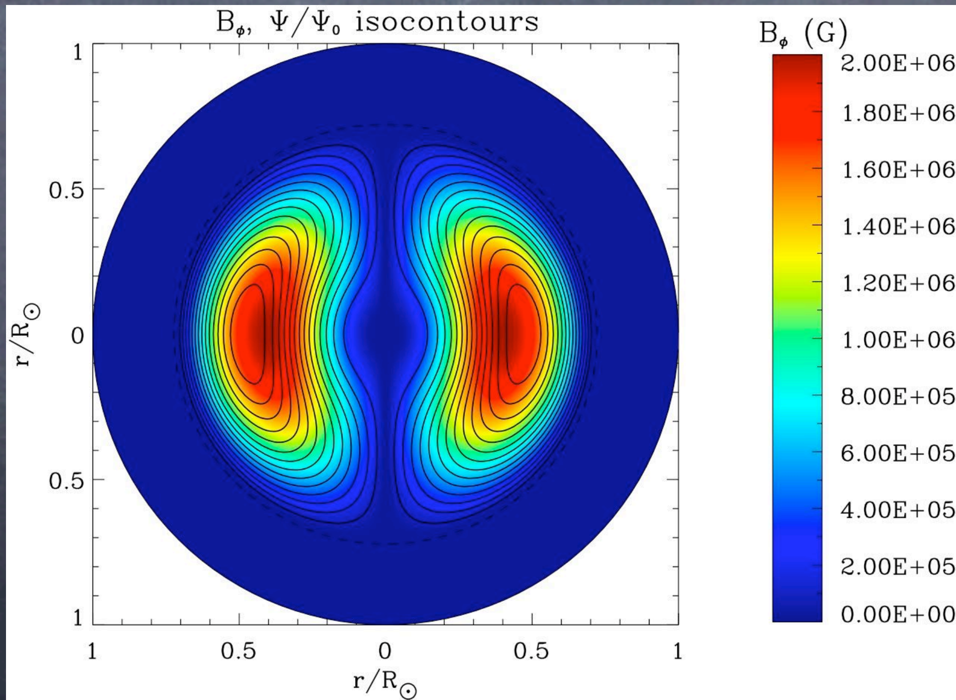


$$
\begin{aligned}
\Psi(r,\theta) &= \Psi_{\rm h} + \Psi_{\rm p} \\
&= \sin^2\theta \times \left\{ \sum_{l=0}^{\infty} K_1^l \frac{\lambda_1^{l,i}}{R_{\rm sup}}\, r\, j_{l+1}\left(\lambda_1^{l,i}\frac{r}{R_{\rm sup}}\right) C_l^{3/2}(\cos\theta) \right. \\
&\quad - \mu_0\, \beta_0 \frac{\lambda_1^{0,i}}{R_{\rm sup}}\, r\, j_1\left(\lambda_1^{0,i}\frac{r}{R_{\rm sup}}\right) \int_r^{R_{\rm sup}} \left[y_1\left(\lambda_1^{0,i}\frac{\xi}{R_{\rm sup}}\right) \bar{\rho}\, \xi^3 \right] {\rm d}\xi \\
&\quad \left. - \mu_0\, \beta_0 \frac{\lambda_1^{0,i}}{R_{\rm sup}}\, r\, y_1\left(\lambda_1^{0,i}\frac{r}{R_{\rm sup}}\right) \int_{R_{\rm inf}}^{r} \left[j_1\left(\lambda_1^{0,i}\frac{\xi}{R_{\rm sup}}\right) \bar{\rho}\, \xi^3 \right] {\rm d}\xi \right\}.
\end{aligned}
$$

→ force-free (first term)

→ non force-free (second and third terms)

Duez & Mathis, A&A, 2010, ***accepted***



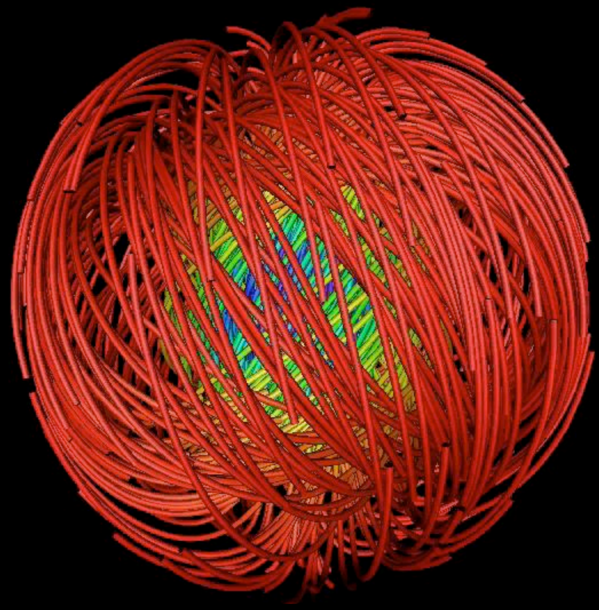
- «non force-free» (owing the stratification of the radiation zone).
- Here we assume a confined field
  Woltjer (1959); Dixon (1989)

A stable equilibrium

Braithwaite, Duez & Mathis, in prep.

3D MHD simulation : inputs

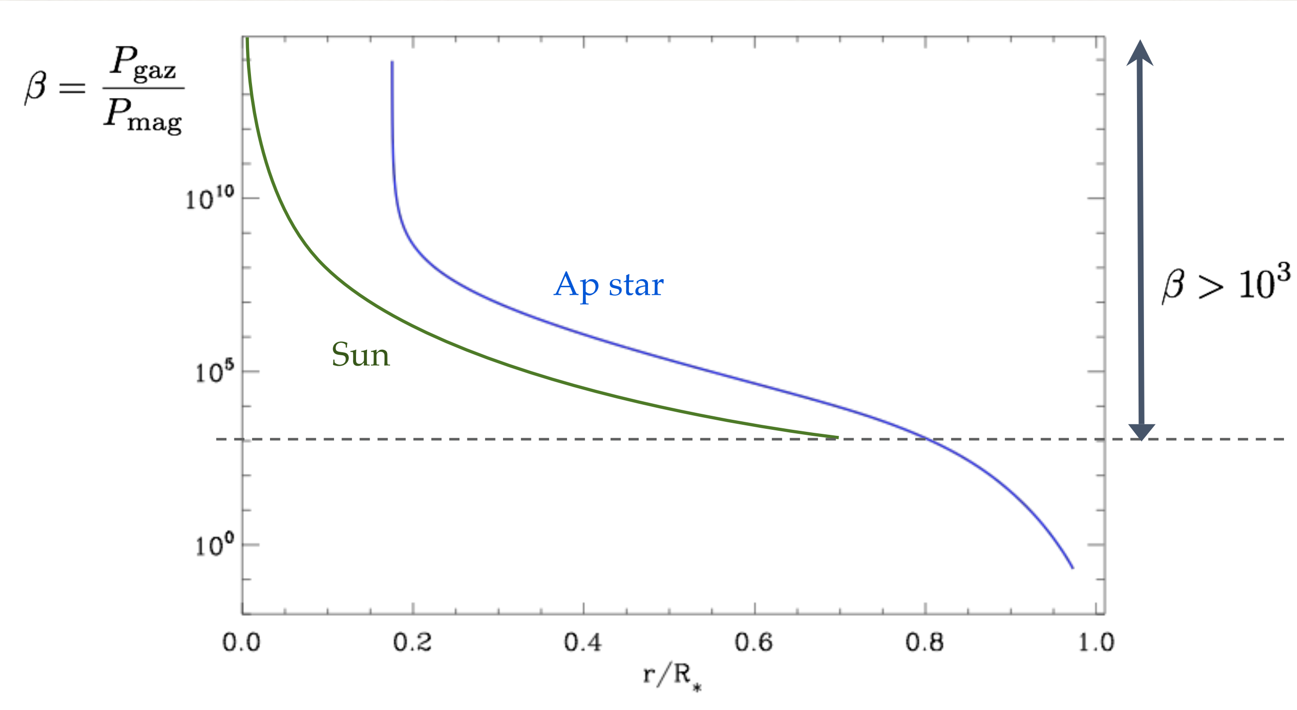
- Initial polytrope with index $n = 3$;
- Enhanced magnetic diffusivity (Spitzer, 1962) ;
- Quasi-potential boundary conditions ;
- White noise perturbations, 1% in density.



- **Good candidate as an initial condition for the secular magneto-rotational transport.**
- **As a *non force-free* field, it *influences* the stellar structure.**

III.
Surface impact of the radiation zone dynamics

A semi-analytical, static approach



Magnetic pressure weak in front of gaseous pressure

Perturbative treatment possible

Perturbations relatively to the sphere :

$$\Phi_{\rm grav}(r,\theta) = \Phi_0(r) + \Phi^{(1)}(r,\theta) = \Phi_0(r) + \sum_{l\geqslant 0} \widehat{\Phi}_l(r) P_l(\cos\theta)$$

$$P(r,\theta) = P_0(r) + P^{(1)}(r,\theta) = P_0(r) + \sum_{l\geqslant 0} \widehat{P}_l(r) P_l(\cos\theta)$$

$$\rho(r,\theta) = \rho_0(r) + \rho^{(1)}(r,\theta) = \rho_0(r) + \sum_{l\geqslant 0} \widehat{\rho}_l(r) P_l(\cos\theta)$$

$$T(r,\theta) = T_0(r) + T^{(1)}(r,\theta) = T_0(r) + \sum_{l\geq 0} \widehat{T}_l(r) P_l(\cos\theta)$$

1. Method

* Gravitational potential : $$\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}\left(r\widehat{\phi}_l\right) - \frac{l(l+1)}{r^2}\widehat{\phi}_l - \frac{4\pi G}{g_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}r}\widehat{\phi}_l = \frac{4\pi G}{g_0}\left[\mathcal{X}_{\mathcal{F}_{\mathcal{L}};l} + \frac{\mathrm{d}}{\mathrm{d}r}\left(r\mathcal{Y}_{\mathcal{F}_{\mathcal{L}};l}\right)\right].$$

* Density : $$\widehat{\rho}_l = \frac{1}{g_0}\left[\frac{\mathrm{d}\rho_0}{\mathrm{d}r}\widehat{\phi}_l + \mathcal{X}_{\mathcal{F}_{\mathcal{L}};l} + \frac{\mathrm{d}}{\mathrm{d}r}\left(r\mathcal{Y}_{\mathcal{F}_{\mathcal{L}};l}\right)\right].$$

* Pressure : $$\frac{\widehat{P}_l}{\rho_0} = -\widehat{\phi}_l - r\frac{\mathcal{Y}_{\mathcal{F}_{\mathcal{L}};l}}{\rho_0}$$

* Radius $$r_P(r,\theta) = r\left[1 + \sum_{l>0} c_l(r)\, P_l(\cos\theta)\right] \longrightarrow c_l = \frac{1}{r}\frac{1}{\mathrm{d}P_0/\mathrm{d}r}\left[\widehat{\phi}_l + r\frac{\mathcal{Y}_{\mathcal{F}_{\mathcal{L}};l}}{\rho_0}\right]$$

* Equation of state (Kippenhahn & Weigert, 1990)

$$\frac{\mathrm{d}\rho}{\rho} = \alpha_s\frac{\mathrm{d}P}{P} - \delta_s\frac{\mathrm{d}T}{T} + \varphi_s\frac{\mathrm{d}\mu_s}{\mu_s}$$

→ Temperature perturbations : $$\widehat{T}_l = \frac{T_0}{\delta_s}\left[\alpha_s\frac{\widehat{P}_l}{P_0} - \frac{\widehat{\rho}_l}{\rho_0} + \varphi_s\frac{\widehat{\mu}_{s;l}}{\mu_{s;0}}\right].$$

→ Luminosity perturbations :

$$L = L_0 + \widehat{L}_{\mathrm{tot}},\ \widehat{L}_{\mathrm{tot}}(r) = S_{\mathrm{nuc}}(r) + L_{\mathrm{Poynt}}(r) + S_{\mathrm{Ohm}}(r)$$

$$S_{\mathrm{nuc}}(r) = \int_\Omega \varepsilon^{(1)}\rho_0 \mathrm{d}\Omega = \int_0^{m(r)} \left\langle \varepsilon^{(1)}\right\rangle_\theta \mathrm{d}m = 4\pi\int_0^r \left\{\varepsilon_0\left[\lambda\frac{\widehat{\rho}_0}{\rho_0} + \nu\frac{\widehat{T}_0}{T_0}\right]\right\}\rho_0\, r'^2 \mathrm{d}r'.$$

$$L_{\mathrm{Poynt}}(r) = \int_0^r\int_\Omega \nabla\cdot\boldsymbol{S}(r',\theta')\,\mathrm{d}\Omega\, r'^2\mathrm{d}r'$$

$$S_{\mathrm{Ohm}}(r) = \int_0^r\int_\Omega Q_\Omega(r',\theta')\,\mathrm{d}\Omega\, r'^2\mathrm{d}r'$$

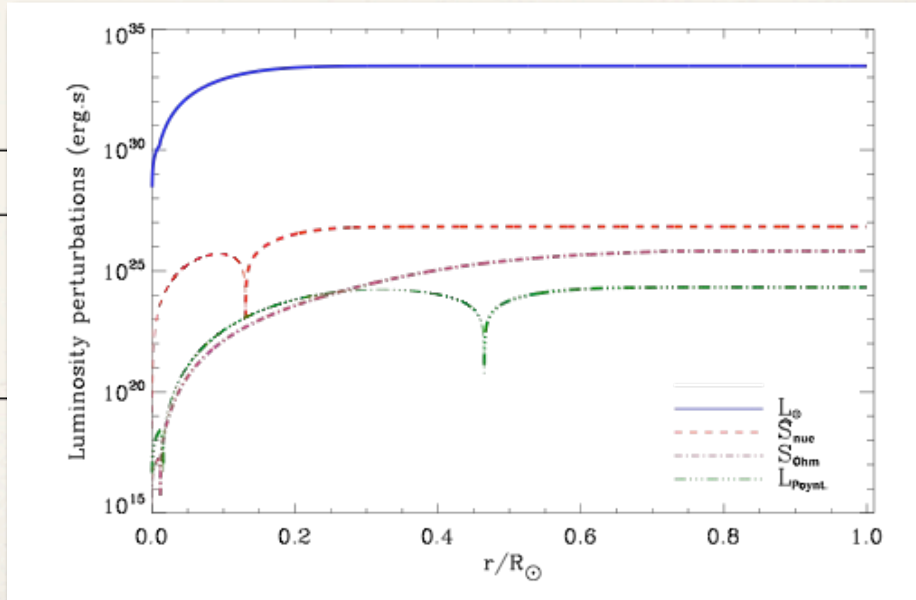
Duez, Mathis & Turck-Chièze, MNRAS, 2010, **402**,271-281

2. Results : large-scale magnetic field

→ **Surface perturbations (young Sun); B_0 = 2 MG**

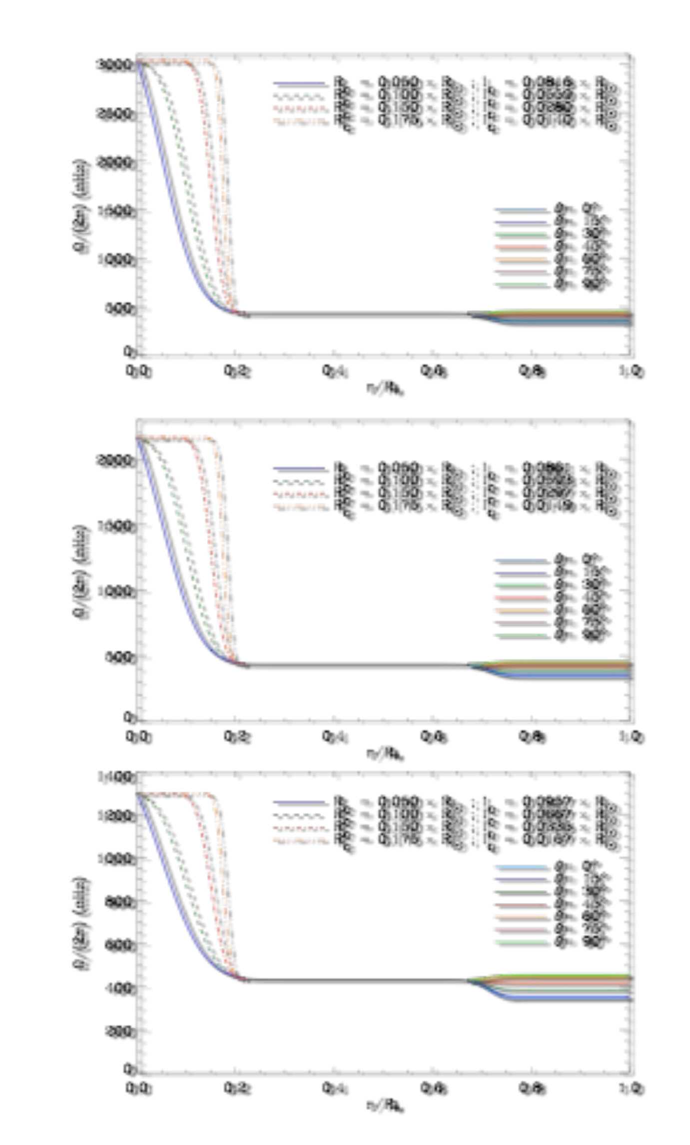
FO perturbation	Young Sun	SO perturbation	Young Sun
J_0	-1.68×10^{-6}	J_2	3.31×10^{-7}
$\tilde{\rho}_0$	4.57×10^{-3}	$\tilde{\rho}_2$	-9.04×10^{-4}
$\tilde{P}_0$	9.78×10^{-3}	$\tilde{P}_2$	-1.93×10^{-3}
$\tilde{T}_0$	5.21×10^{-3}	$\tilde{T}_2$	-1.03×10^{-3}
c_0	1.73×10^{-6}	c_2	-3.42×10^{-7}

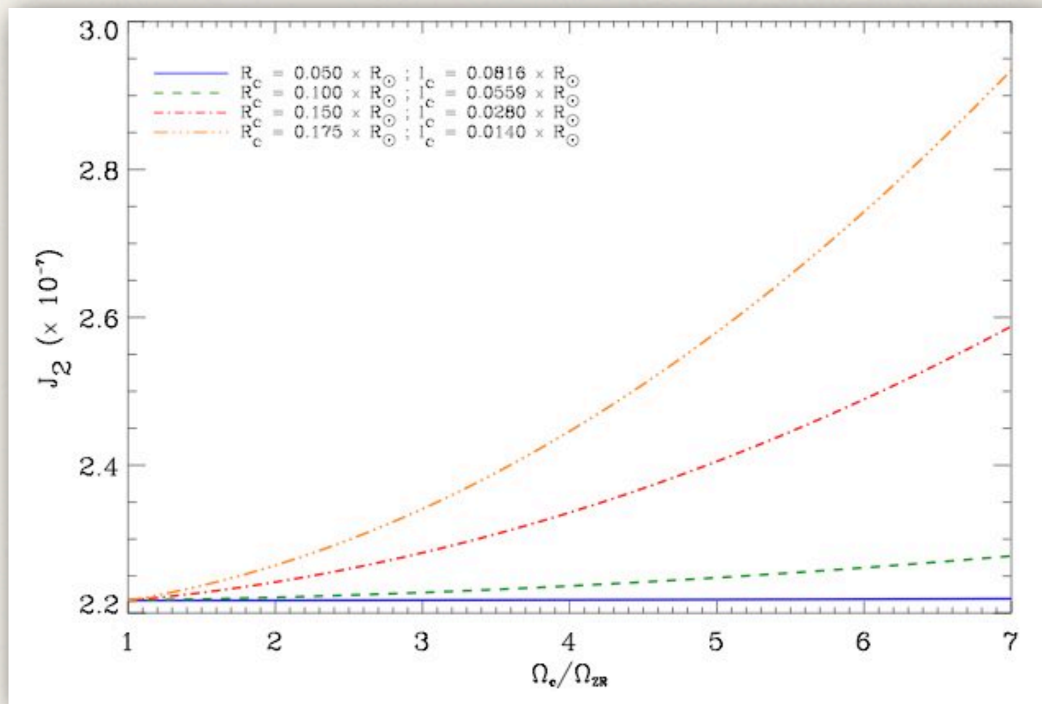
Perturbation to the luminosity	Young Sun
L_{Ohm}	6.59×10^{25}
L_{Poynt}	2.12×10^{24}
$\widehat{L}_{\mathrm{nuc}}$	6.73×10^{26}
L_{tot}	2.93×10^{33}



Duez, Mathis & Turck-Chièze, MNRAS, 2010, **402**,271-281

3. Comparison with rotation-induced perturbations





→ **Using the same method, we find a quadrupolar gravitational moment :**

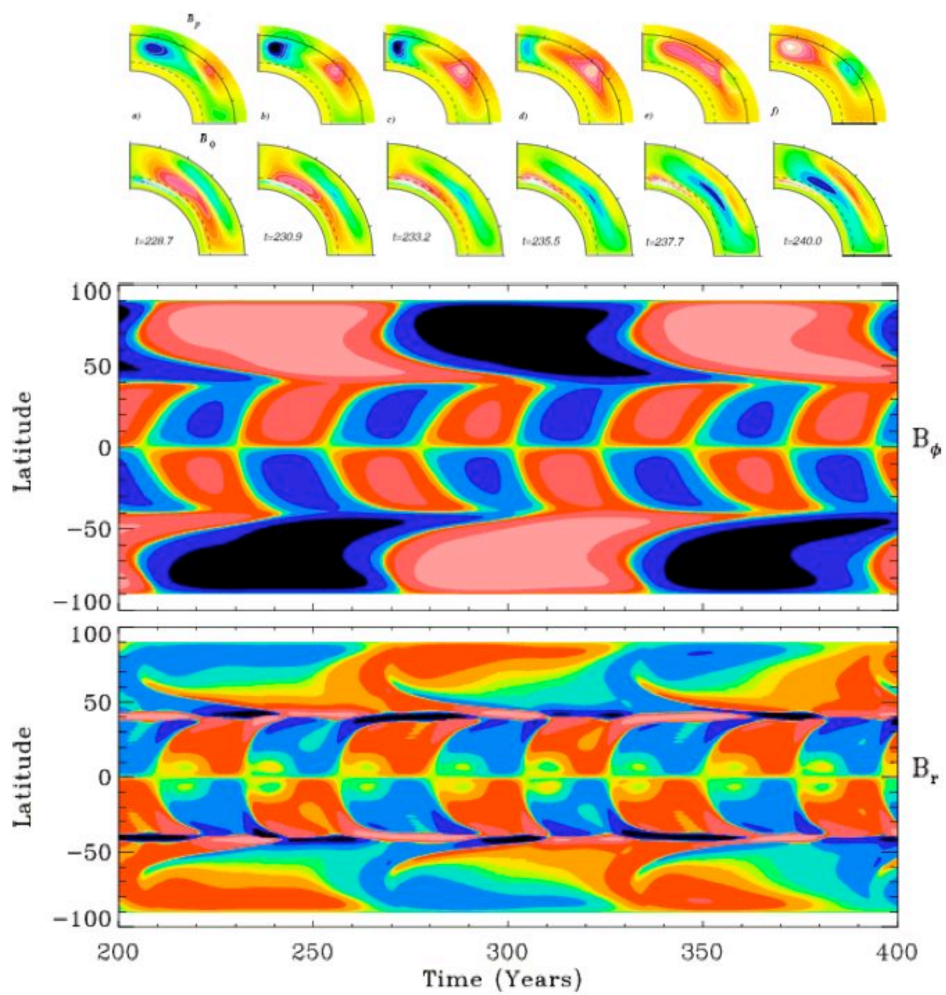
(In total agreement with Roxbugh, 2001)

$$2.21 \times 10^{-7} < J_2 < 2.94 \times 10^{-7}$$

→ **Will SODISM be able to bring some constraints ?**

IV. Perspectives

1. **Surface impact of the magnetic cyclic variability in the convection zone**



Lorentz force coefficients from the 2D STELEM Code

↓

Semi-analytical perturbative treatment

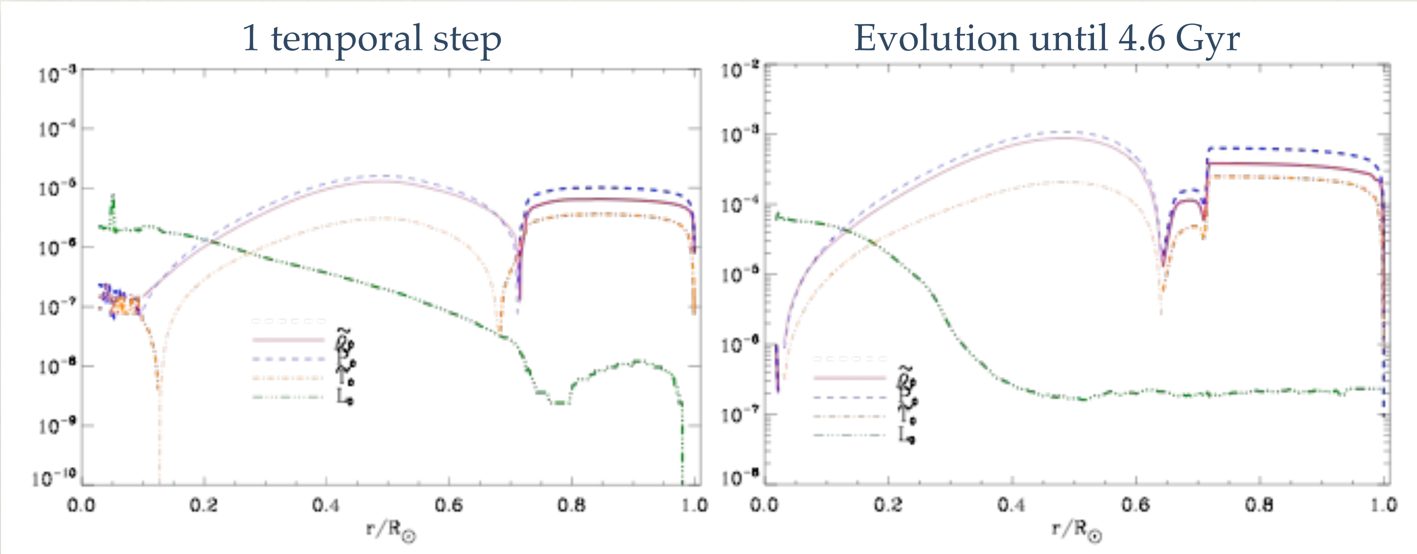
↓

Radius perturbations, J_l, …

2. Secular evolution, involving :

- **magnetism ;**
- **differential rotation ;**
- **meridional circulation ;**
- **shear-induced turbulence.**

A first step : implementation of a static magnetic field in the stellar evolution code CESAM



(Solar model parameters following Couvidat et al., 2003)

Perturbation	1 time step evolution	evolution until actual solar age
$\widehat{r}_0$	3.210×10^{-7}	5.790×10^{-7}
$\widehat{\rho}_0$	2.361×10^{-7}	3.080×10^{-7}
$\widehat{P}_0$	9.706×10^{-8}	-2.025×10^{-6}
$\widehat{T}_0$	-2.108×10^{-7}	-2.511×10^{-7}
$\widehat{L}_0$	5.254×10^{-11}	2.325×10^{-7}

Élément	Sans champ magnétique	Avec champ magnétique	Différence
^1H	$7.365359793668 \times 10^{-1}$	$7.365337409678 \times 10^{-1}$	$3,03909 \times 10^{-5}$
^2H	$1.840619143064 \times 10^{-17}$	$1.840604262519 \times 10^{-17}$	
^3He	$7.967990778229 \times 10^{-5}$	$7.968048038889 \times 10^{-5}$	
^4He	$2.459754994785 \times 10^{-1}$	$2.459776212934 \times 10^{-1}$	$8,626 \times 10^{-5}$

2. Secular evolution, involving :

- magnetism ;
- differential rotation ;
- meridional circulation ;
- shear-induced turbulence.

Induction equation : $[\mathcal{U}_\varphi(r,\theta) + \mathcal{U}_M(r,\theta)] \times \mathbf{B}$

$$\frac{\mathrm{d}\xi_0^l}{\mathrm{d}t} = \frac{1}{\mathcal{N}_l^0} r \mathcal{Z}_{\mathbf{Ad};l} + \eta_h r \Delta_l \left(\frac{\xi_0^l}{r}\right)$$

$$\frac{\mathrm{d}\chi_0^l}{\mathrm{d}t} + \partial_r(\dot{r})\chi_0^l = \frac{1}{\mathcal{N}_l^0}\left[\mathcal{X}_{\mathbf{Ad};l} + \partial_r\left(r\mathcal{Y}_{\mathbf{Ad};l}\right)\right] + \left[\partial_r\left(\eta_h \partial_r \chi_0^l\right) - \eta_v l(l+1)\frac{\chi_0^l}{r^2}\right]$$

Thermal wind equation :

$$\varphi\Lambda_l - \delta\Psi_l = \frac{r}{\overline{g}}\left[\mathcal{D}_l(\overline{\Omega},\Omega_l) + \frac{\mathcal{X}_{\mathcal{F}_\mathcal{L};l}}{r\overline{\rho}} + \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathcal{Y}_{\mathcal{F}_\mathcal{L};l}}{\overline{\rho}}\right)\right]$$

Temperature fluctuations

Mean molecular weight fluctuation

Lorentz force

Differential rotation

...

Angular momentum transport :

$$\begin{cases} \rho\frac{\mathrm{d}}{\mathrm{d}t}(r^2\overline{\Omega}) - \frac{1}{5r^2}\partial_r\left(\rho r^4\overline{\Omega}U_2\right) = \frac{1}{r^2}\partial_r\left(\rho\nu_v r^4\partial_r\overline{\Omega}\right) + \boxed{\overline{\Gamma}_{\vec{\mathcal{F}}_\mathcal{L}}(r)} \\ \rho\frac{\mathrm{d}}{\mathrm{d}t}\left(r^2\Omega_2\right) - 2\rho\overline{\Omega}r\left[2V_2 - \alpha U_2\right] = -10\rho\nu_h\Omega_2 + \Gamma_2 \end{cases}$$

$$\boxed{\Gamma_{\vec{\mathcal{F}}_\mathcal{L}}(r,\theta)} = r\sin\theta\int_0^{2\pi}\mathcal{F}_{\mathcal{L},\varphi}\frac{\mathrm{d}\varphi}{2\pi} = \sum_{l=0}^{\infty}\Gamma_l(r)\sin^2\theta\, P_l(\cos\theta)$$

Mathis & Zahn, 2005

Towards a complete(*) picture of the solar interior evolution along secular (climatic) timescales.

(*) Axisymmetric, spectral models : 2D

Thank you for your attention.