2D solar modelling

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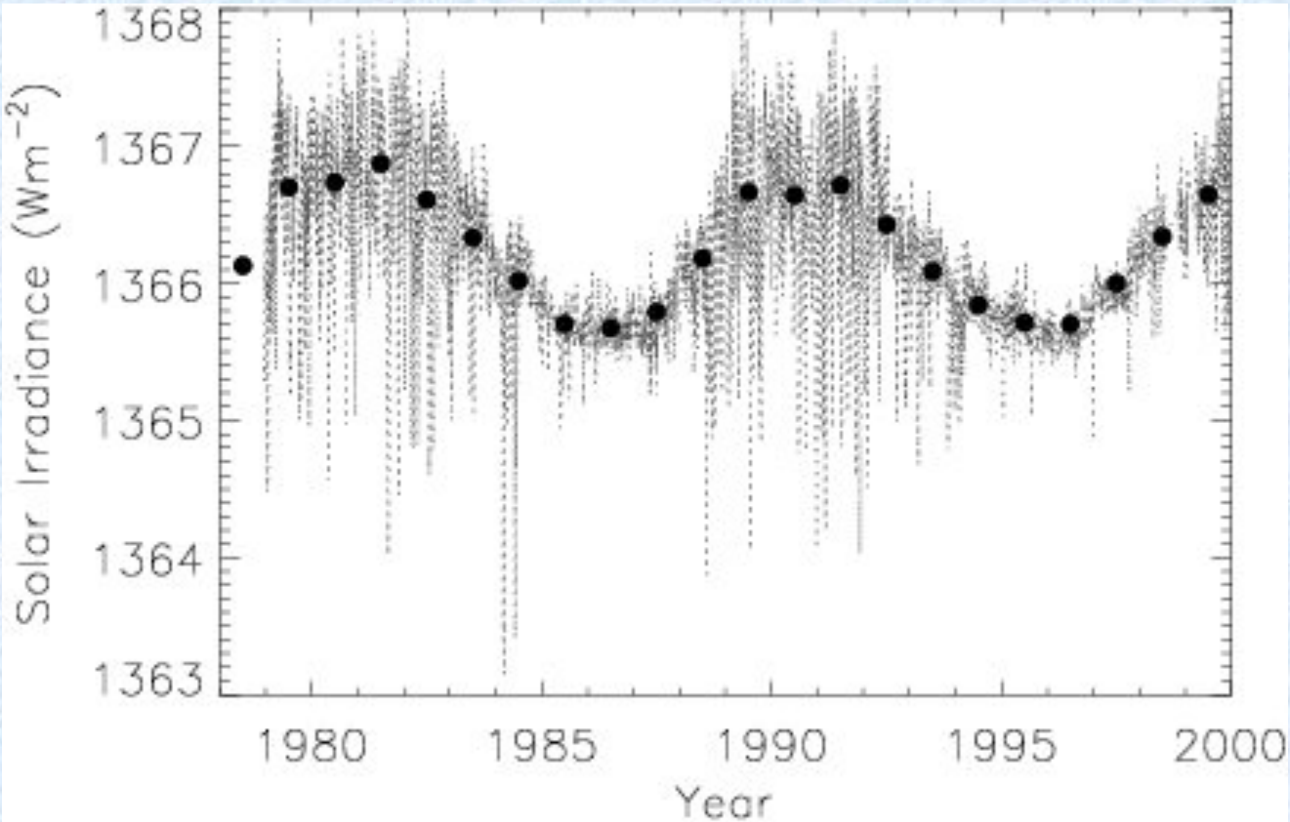
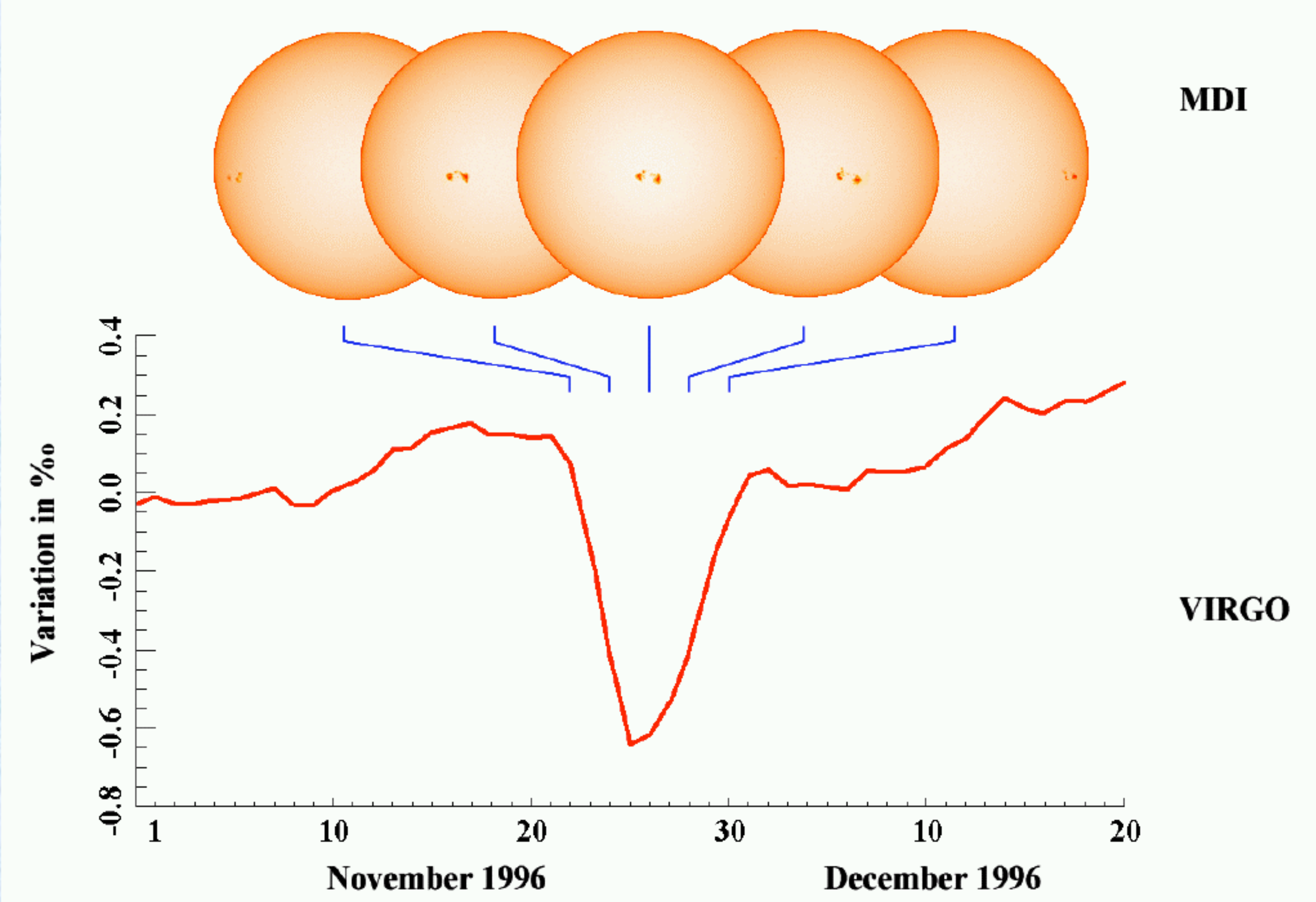
Motivation

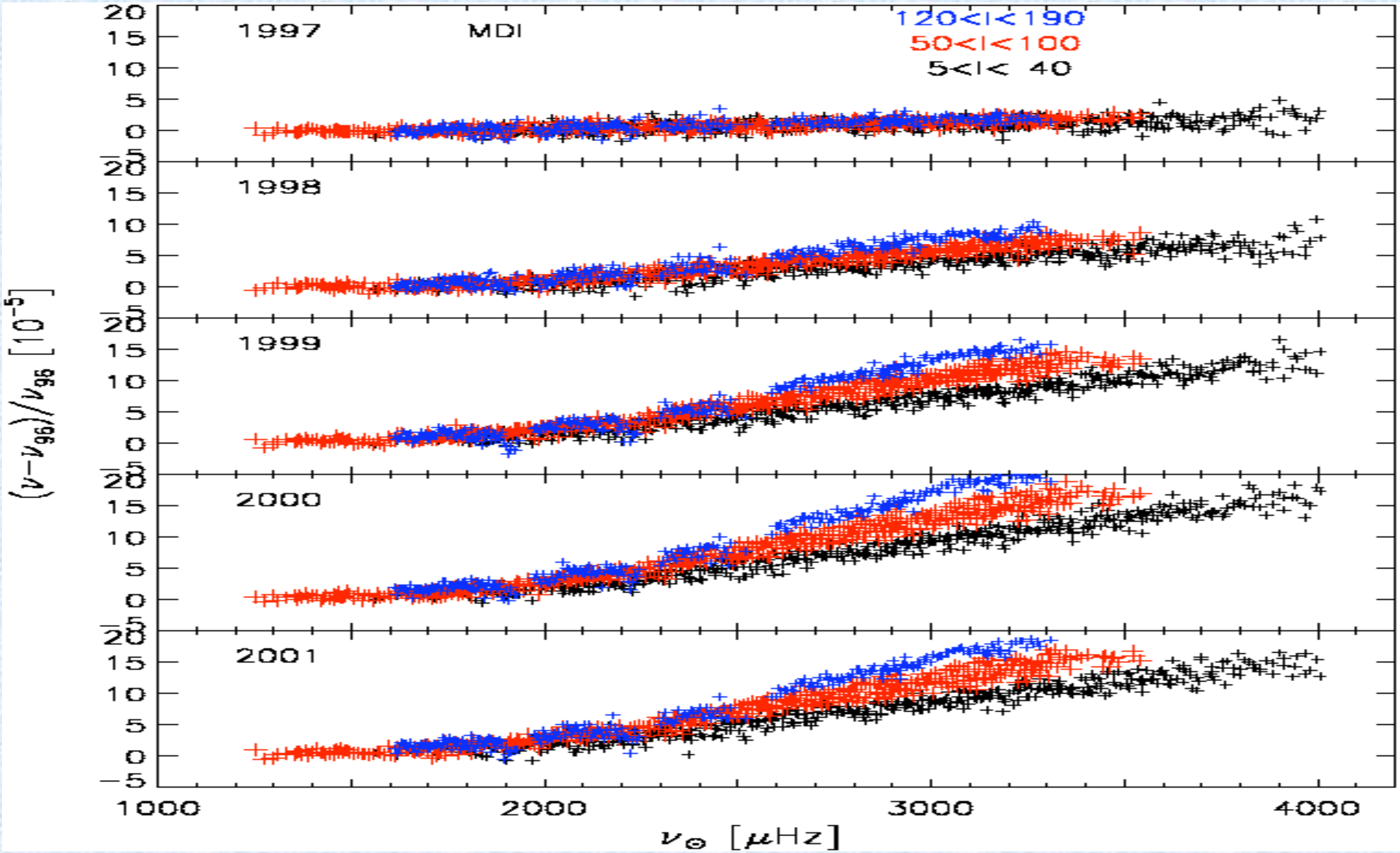
The total solar irradiance (TSI) is known to vary on two time-scales, associated to rotation (from minutes to months) and to the solar cycle (11 yr). The amplitude of both variations is approximately 0.1%





The short-timescale variations of TSI, are mostly related to the passage of active regions (spots and faculae) on the solar disk

The long-timescale variation of the TSI, that also involves the observed frequency spectrum, is still unknown



The cause of the long-timescale variations, in phase with the solar activity cycle, is still controversial

Effects of the surface features

Changes in the solar luminosity, due to structural changes

The structural changes involve the entire convection zone, so thay can have long timescale components for climate change

These long-timescale components are difficult to detect, due to the instrument degradation in energy flux-type measurements

Structural changes would be dominant for climate

A possible origin of the structural variations might be associated to the solar magnetic field

The solar dynamo models assume an initial poloidal configuration, which gradually develops a toroidal component in the tachocline layer, due to the differential rotation

A variable magnetic field modifies the pressure, the internal energy and the energy transfer, thus affecting the solar interior

The presence of a variable magnetic field would thus change all the global solar parameters: luminosity, radius and effective temperature

Simultaneous observations of all global parameters

physical model of the variations

Helioseismological validation

Radius is a powerful diagnostic of internal processes. Sofia et al. (1979) suggested that any change in the solar luminosity L must be accompanied by a change in the radius R: the key quantity is W=dlnR/dlnL

Ground-based measurements give results that are incompatible with each other. Partial results from SDS indicate that radius changes in opposite phase with the activity cycle

How we model the effects of a magnetic field in a stellar structure? (Lydon & Sofia 1995)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathrm{v}) = 0$$

Conservation of mass

$$\rho \frac{d\mathrm{v}}{dt} = -\nabla P - \rho \nabla \Phi + \frac{1}{4\pi}(\nabla \times B) \times B$$

Conservation of momentum

$$\rho T \frac{dS}{dt} = \rho \varepsilon - \nabla \cdot F$$

Conservation of energy

$$\nabla^2 \Phi = 4\pi G \rho$$

Poisson

$$F = -\frac{4acT^3}{3k\rho} \nabla T + F_{conv}$$

Transport of energy

Energy density

$$\chi_m = \frac{B^2}{8\pi\rho}$$

Magnetic pressure

$$P_m = \rho\chi_m$$

* Total pressure & the EOS

$$P_T = P_{gas} + \frac{1}{3}aT^4 + \rho\chi_m$$

$$\frac{d\rho}{\rho} = \alpha\frac{dP_T}{P_T} - \delta\frac{dT}{T} - \nu_m\frac{d\chi_m}{\chi_m}$$

* The variation of the entropy

$$dQ = dU + PdV + d\chi$$

$$dQ = c_P dT - (\delta/\rho)dP + \frac{P_T\delta\nu_m}{\rho\alpha\chi}d\chi$$

* The adiabatic gradient

$$\nabla'_{ad} = \left.\frac{d\ln T}{d\ln P_T}\right|_{ad} = \frac{\delta}{\rho c_P}\left(1 - \frac{\nu_m}{\alpha}\nabla_\chi\right) = \nabla_{ad}\left(1 - \frac{\nu_m}{\alpha}\nabla_\chi\right)$$

* The Schwarzschild criterium for radiative stability

$$D\rho = \rho_e - \rho_s = \left[(d\rho/dr)_e - (d\rho/dr)_s\right]\Delta r > 0$$

$$\left.\frac{d\ln\rho}{dr}\right|_e > \left.\frac{d\ln\rho}{dr}\right|_s$$

$$\alpha\left.\frac{d\ln P}{dr}\right|_e - \delta\left.\frac{d\ln T}{dr}\right|_e > \alpha\left.\frac{d\ln P}{dr}\right|_s - \delta\left.\frac{d\ln T}{dr}\right|_s$$

$$\left.\frac{d\ln T}{d\ln P}\right|_s < \left.\frac{d\ln T}{d\ln P}\right|_e \longrightarrow \nabla_{rad} < \nabla_{ad}$$

$$\nabla_{rad} < \nabla_{ad}\left(1 - \frac{\nu_m}{\alpha}\nabla_{\chi}\right)$$

* The convective flux

$$F_{\mathrm{conv}} = \rho \mathrm{v}_{\mathrm{conv}} dQ$$

* Convective velocities

$$f = -g(D\rho / \rho)$$

→ $$-\delta \frac{dT}{T}$$

→ $$-\delta \frac{dT}{T} - \nu_m \frac{d\chi}{\chi}$$

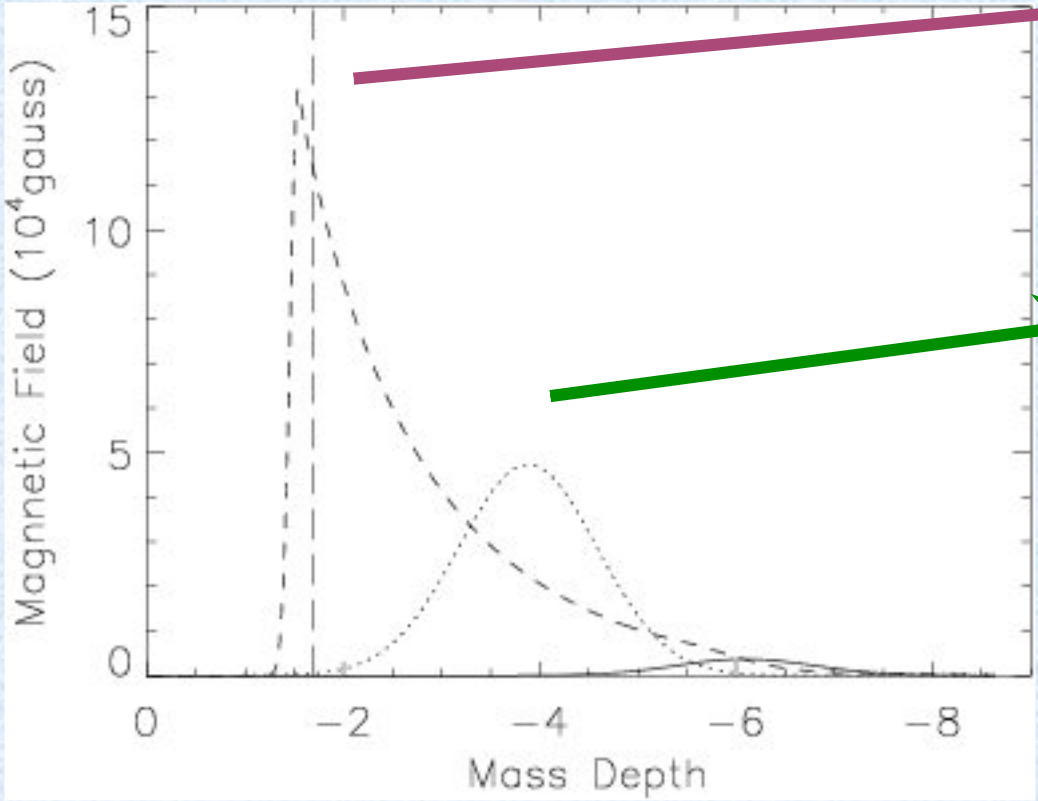
* Radiative losses

$$\left.\left(\frac{dQ}{dr}\right)\right|_e$$

→ {Radiation losses}

→ {Radiation losses} + dχ

Li & Sofia (2001) investigated the effects of a variable magnetic field on the structure and luminosity changes of a 1D solar model



centre

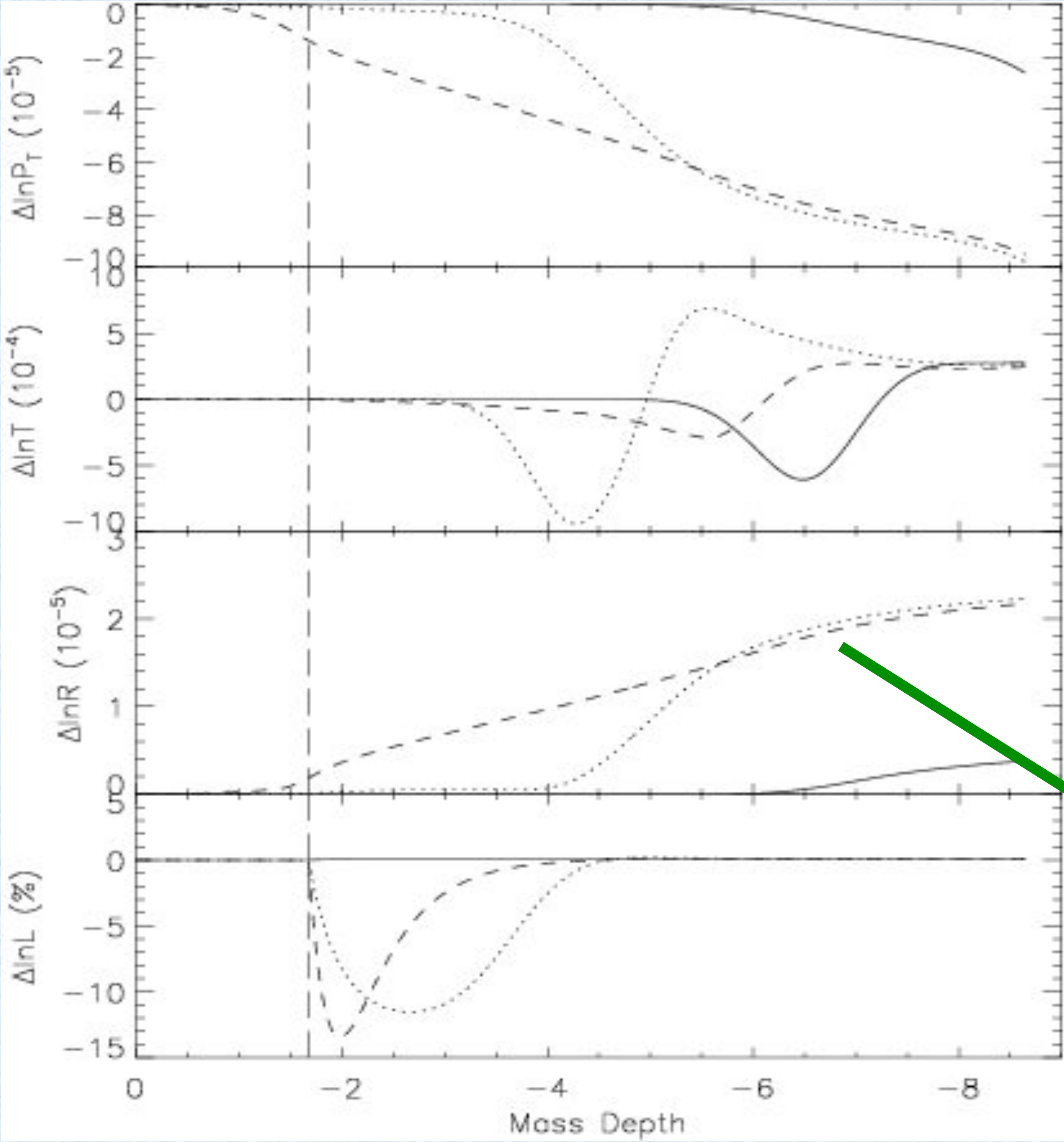
surface

Convective border

Variable Magnetic fields configurations determining a cyclic luminosity variation of 0.1%

$M_D = Log(1 - M_r/M_{sun})$

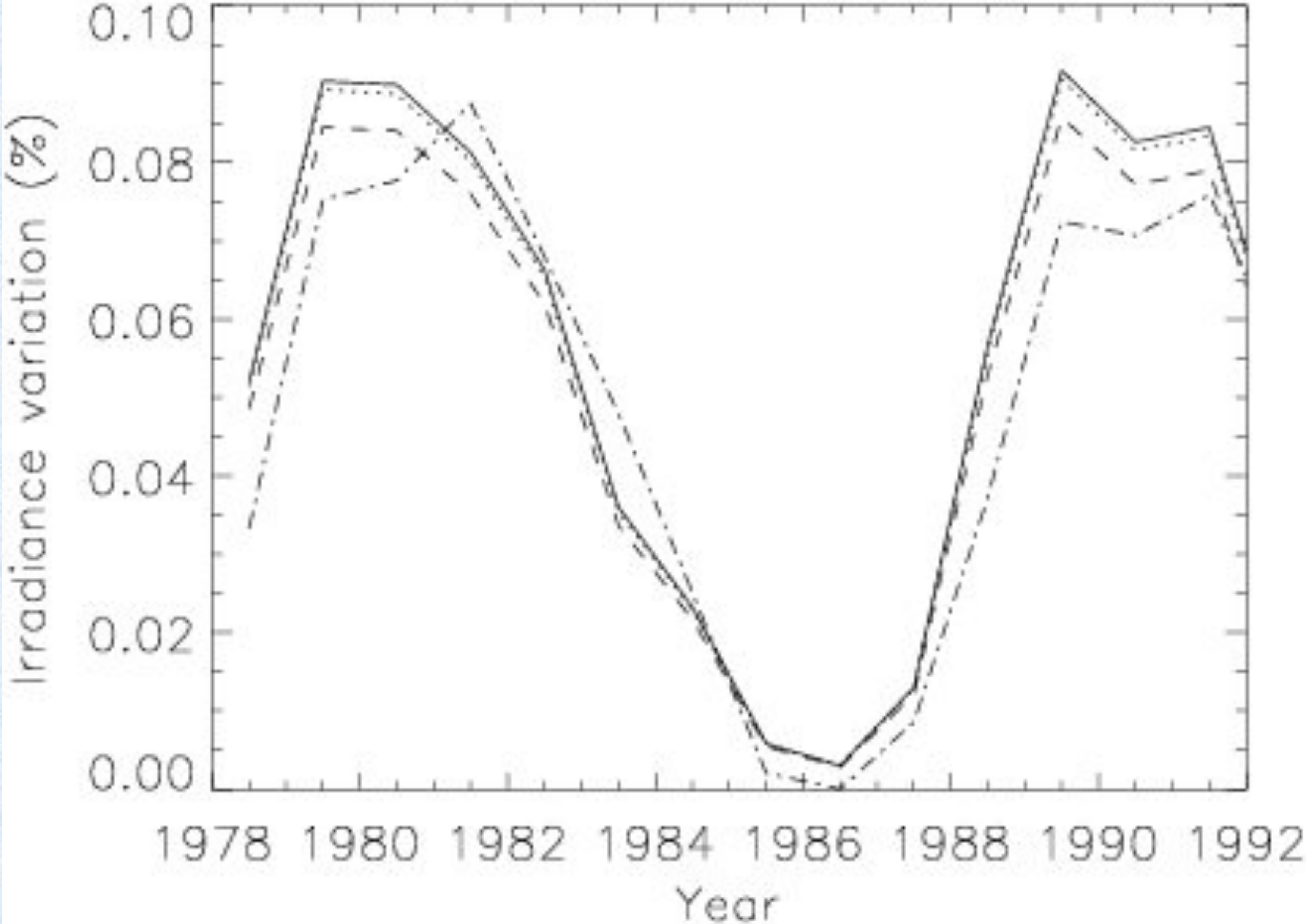
The deeper is the magnetic field, the larger is the magnitude required to produce a given luminosity change



centre surface

The specific properties of the effects are sensitive to the currently unknown details of the magnetic field (shape, magnitude, depth,...)

Deeper fields favour large variation of the radius


0.10
0.08
0.06
0.04
0.02
0.00
Irradiance variation (%)
1978
1980
1982
1984
1986
1988
1990
1992
Year

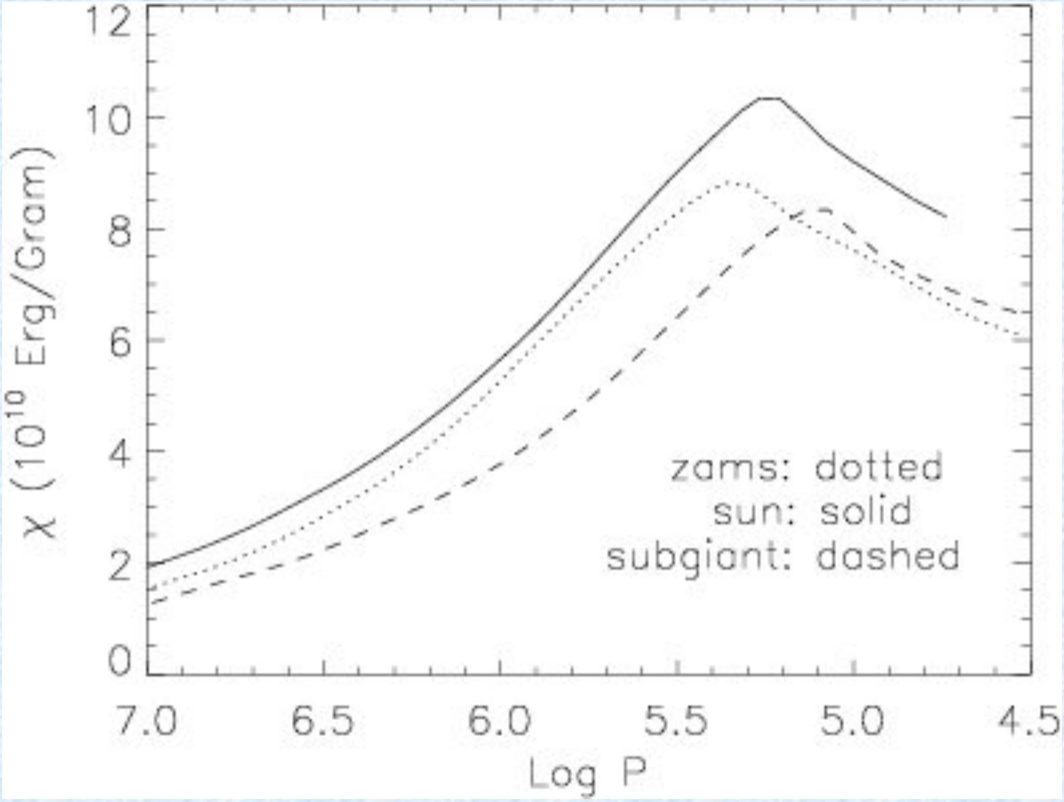
Turbulence

Li et al. (2002) studied the effects of turbulence on the solar structure and P-mode oscillation frequencies

The key quantity to include turbulence is the kinetic energy

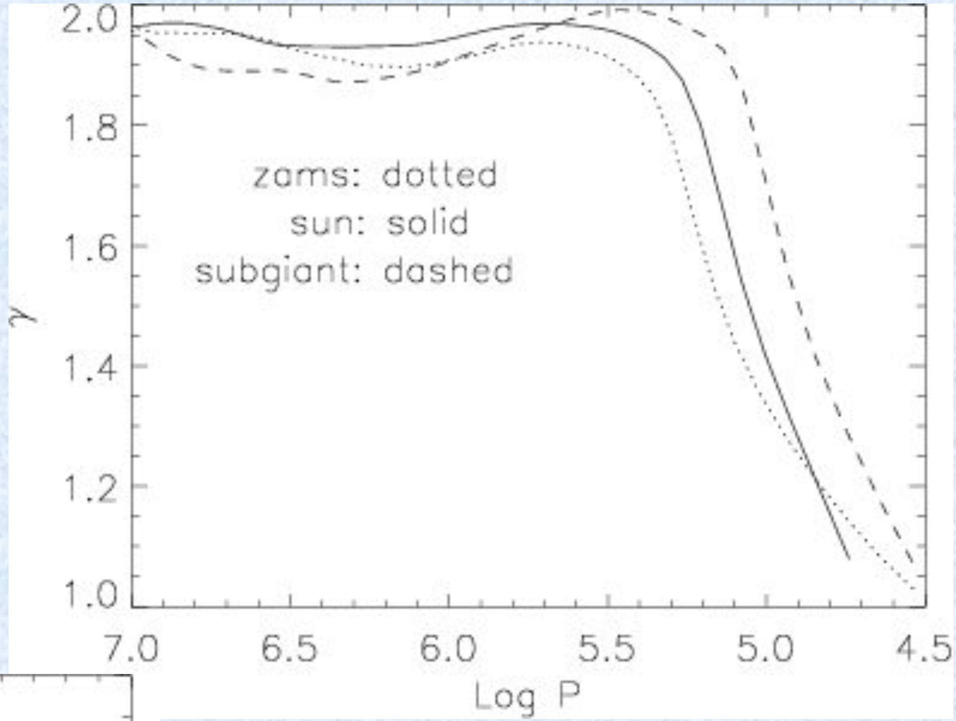
$$\chi_t = \frac{1}{2} v_i^2$$

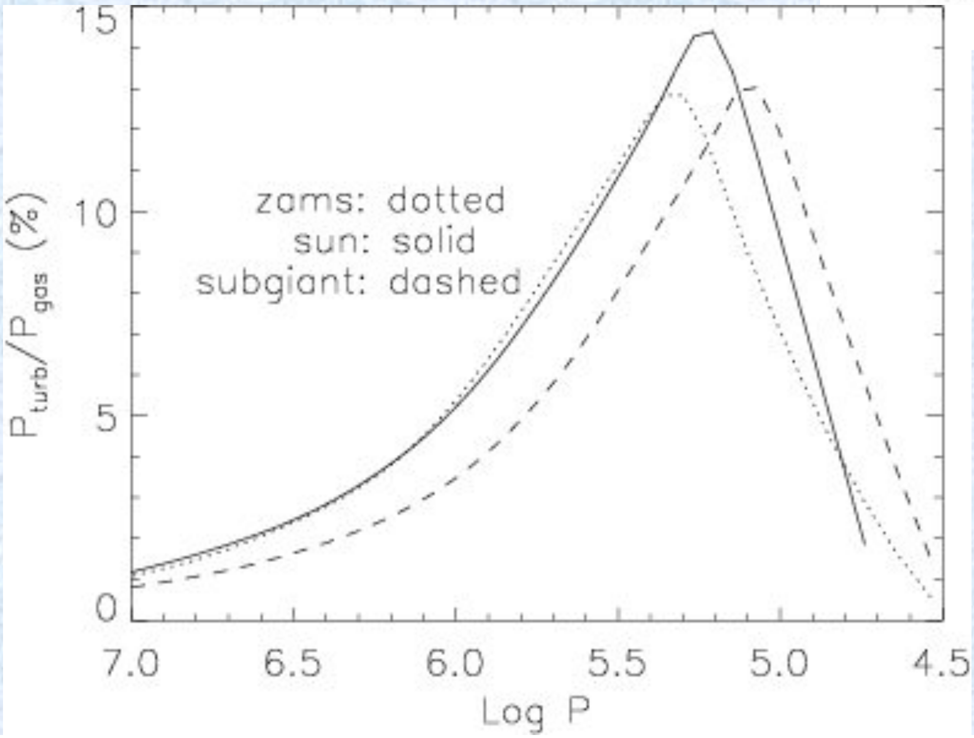
3D hydrodynamical simulations by Robinson et al. (2001)



A turbulent velocity field is characterized by the degree of anisotropy

$$\gamma = 1 + 2(v_z'' / v'')^2$$





$$P_t = \frac{1}{2}(\gamma - 1)\chi\rho$$

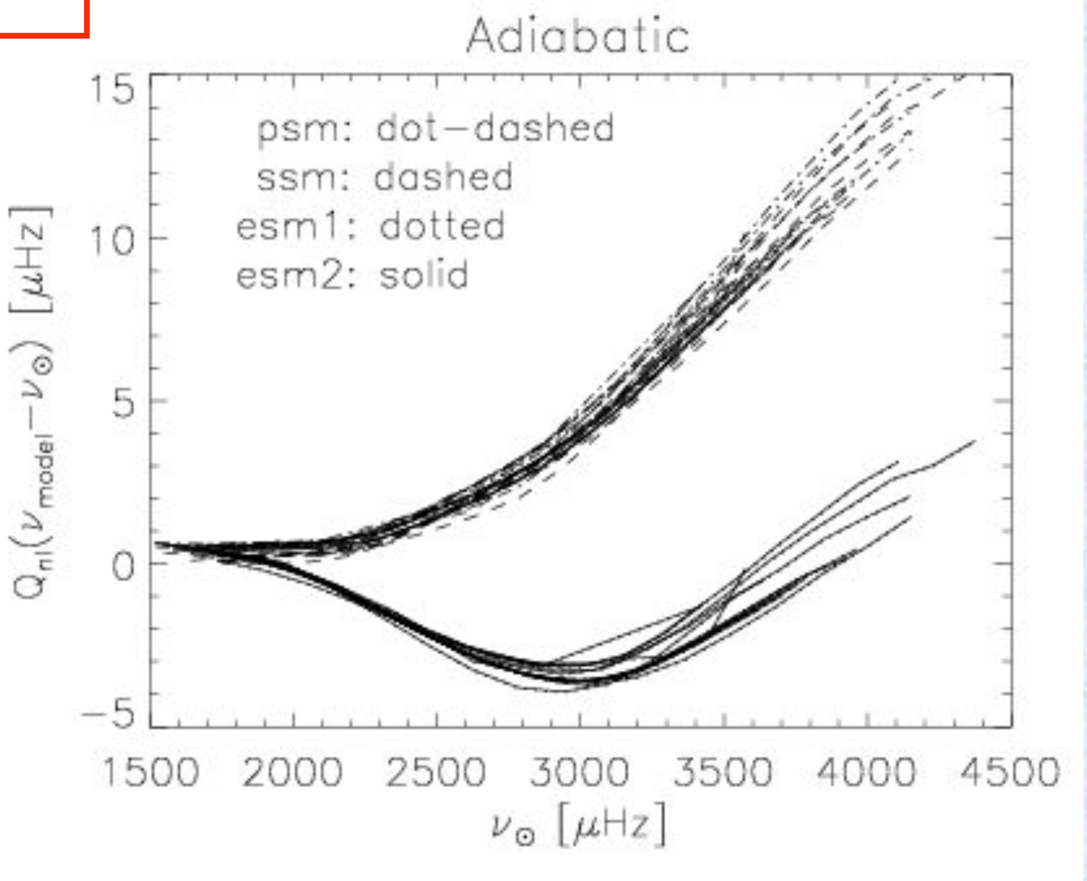
Isotropy: γ=5/3

$$P_T = P_{gas} + \frac{1}{3} a T^4 + (\gamma - 1)\chi\rho$$

$$\frac{d\rho}{\rho} = \alpha \frac{dP_T}{P_T} - \delta \frac{dT}{T} - \nu_t \frac{d\chi_t}{\chi_t} - \mu_t \frac{d\gamma}{\gamma}$$

$$\nabla_{rad} < \nabla_{ad}\left(1 - \frac{\nu_t}{\alpha}\nabla_{\chi_t} - \frac{\mu_t}{\alpha}\nabla_{\gamma}\right)$$

Inclusion of turbulence leads to a meaningful agreement between the observed and theoretical frequencies



Limitations of the 1D modelling

* The 1D treatment imposes unrealistic restrictions to the configuration of the dynamo field (only shellular fields possible!) and to the internal solar dynamics. The real Sun is multidimensional.

* In the 1D approach the energy flux can only reach the surface by penetrating the magnetic field, whereas we know that in case that a toroidal component is present the energy flow can also circumvemt the field

* Correct and complete interpretation of Picard data (particularly the measurement of the radius and the photospheric asimmetry) demands at least a 2D approach

Li et al. (2006) developed a fully 2D evolution code to describe the solar structure; their physical formulation was further refined and improved by Li et al. (2009)

The physical scheme used is based on the equipotential surfaces of Φ, that enters the Poisson equation

Two independent variables → Mass **m** inside an equipotential → Colatitude **θ**

This choice fixes a one-to-one relationship between **m** and **Φ**, and implicitly assumes azimuthal simmetry

Each equipotential surface is characterized by an average density ρ_m

$$\rho_m = \frac{1}{2r^2}\int_0^{\pi} r^2(r,\vartheta)\rho(r,\vartheta) sen\vartheta d\vartheta$$

2D effects

The gravitational acceleration is not purely radial

$$\overrightarrow{\nabla\Phi} = \left(\frac{\partial\Phi}{\partial r}, \frac{1}{r}\frac{\partial\Phi}{\partial\vartheta}\right) = \left(\frac{Gm}{r^2} + U, G\right)$$

$$G = H_\vartheta + T_\vartheta + R_\vartheta - \frac{P}{r\rho}\left(\frac{\partial \ln P}{\partial\vartheta}\right)$$

Deviation of the radial component of gravity from Gm/r^2

Colatitudinal gravitational acceleration

The energy flux also has a transverse component

$$\vec{F} = (F_r, F_\vartheta)$$

$$\frac{1}{4\pi}(\nabla\times B)\times B = -\nabla\left(\frac{B^2}{8\pi}\right) + \frac{1}{4\pi}(B\cdot\nabla)B = -\nabla P_m + H$$

pressure

tension

Full set of differential equations

$$\frac{\partial \ln r}{\partial \ln m} = \frac{m}{4\pi r^3 \rho_m}$$

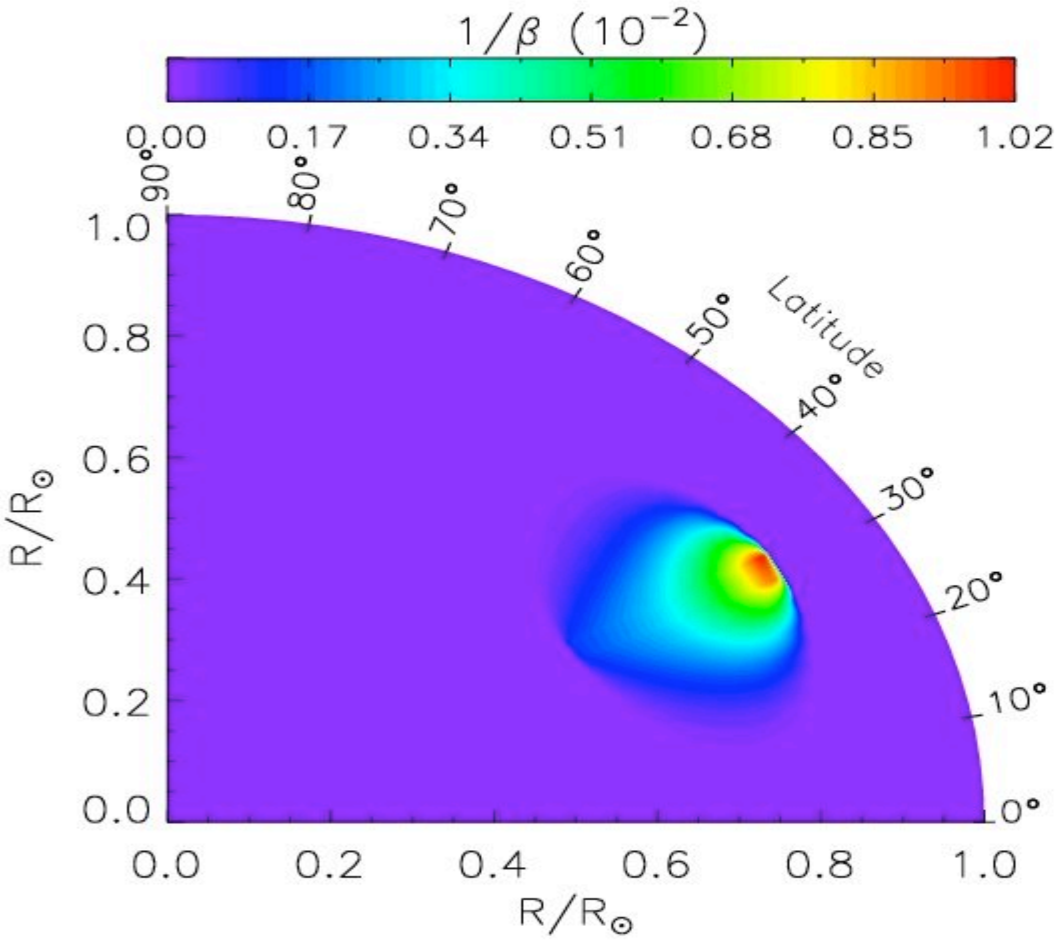
magnetic, turbulence, rotation

$$\frac{\partial \ln P}{\partial \ln m} = -\frac{m}{4\pi r^2 P}\frac{\rho}{\rho_m}\left(\frac{Gm}{r^2} + U - H_r - T_r - R_r\right)$$

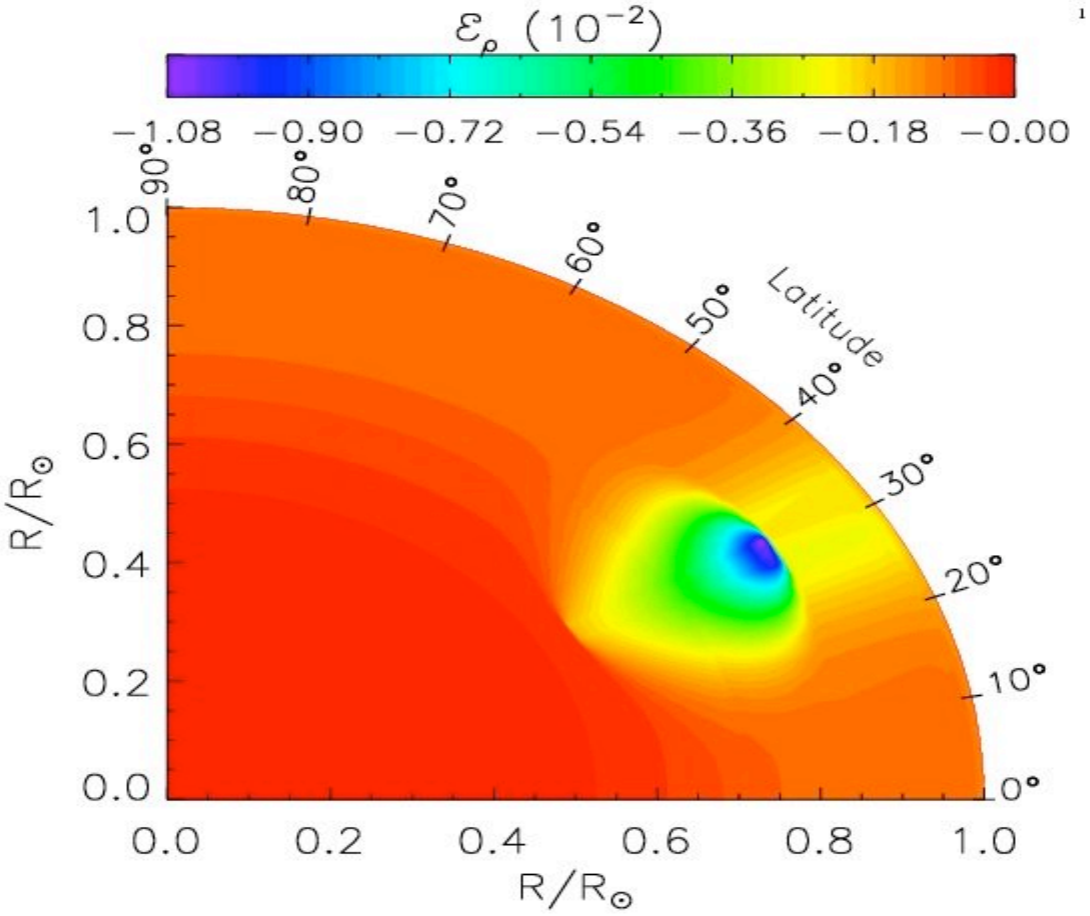
$$\frac{\partial \ln T}{\partial \ln m} = \frac{\partial \ln P}{\partial \ln m}\nabla$$

$$\frac{\partial L}{\partial \ln m} = \frac{m}{L_{sun}}\left(\varepsilon - T\frac{dS}{dt}\right)\frac{\rho}{\rho_m} - \frac{m}{L_{sun}}\frac{F_\vartheta \cot\vartheta}{r\rho_m} - \frac{m}{L_{sun}}\frac{1}{r\rho_m}\frac{\partial F_\vartheta}{\partial \vartheta}$$

$$\frac{\partial U}{\partial \ln m} = \frac{Gm}{r^2}\left(\frac{\rho}{\rho_m} - 1\right) - \frac{m}{4\pi r^3 \rho_m}\left(2U + G\cot\vartheta + \frac{\partial G}{\partial \vartheta}\right)$$

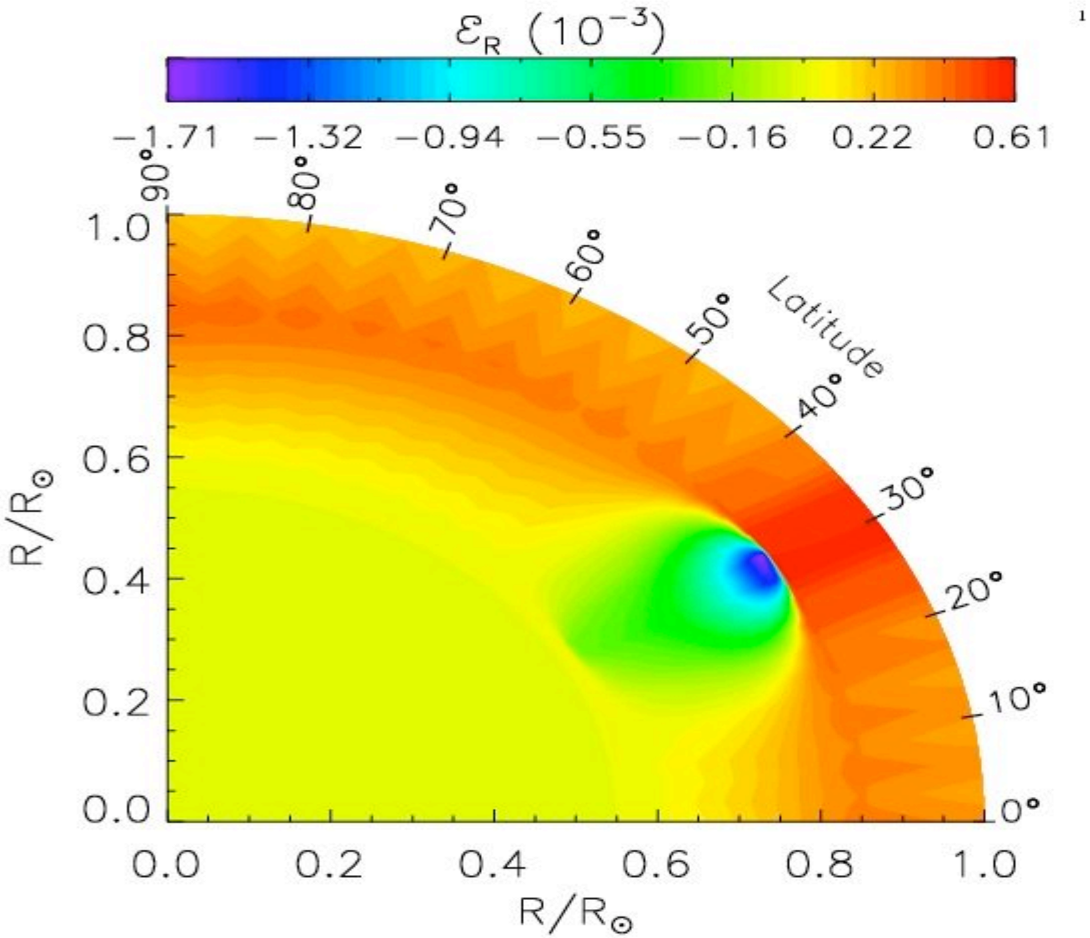


Magnetic pressure



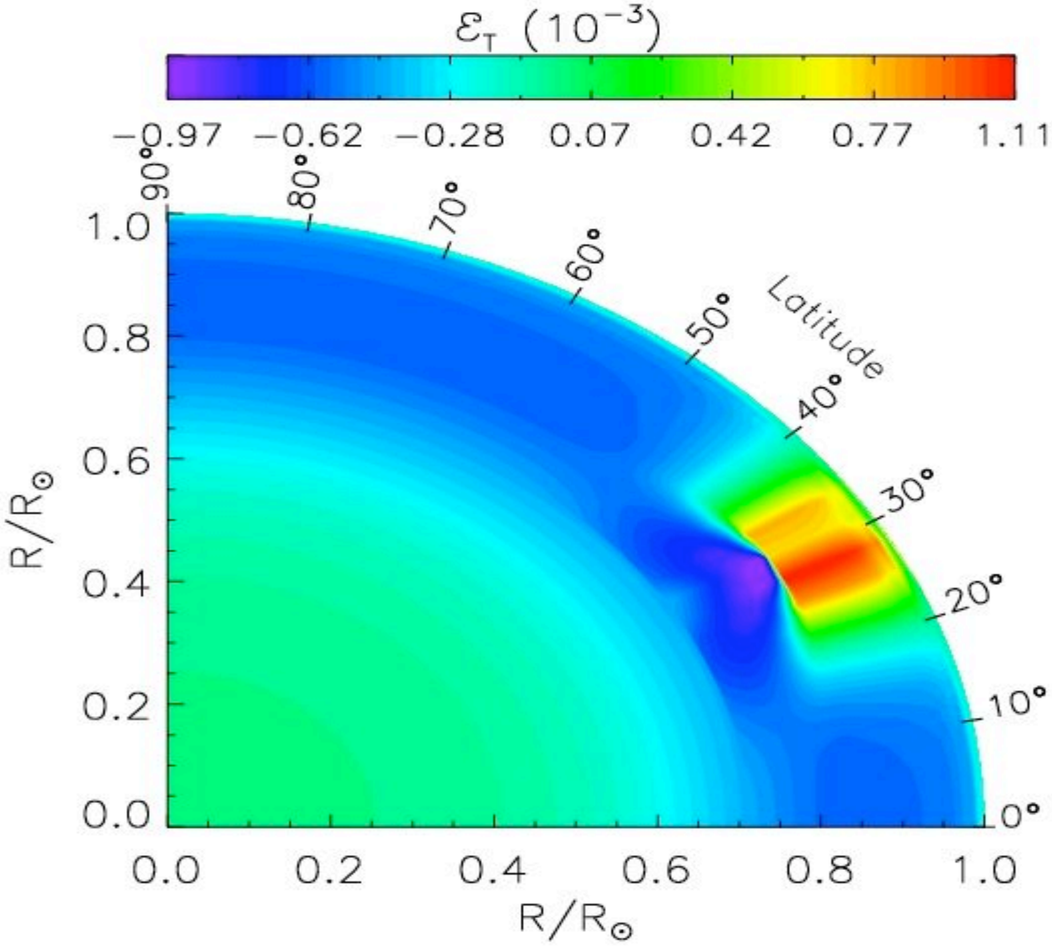
$\mathcal{E}_\rho$ (10^{-2})
−1.08 −0.90 −0.72 −0.54 −0.36 −0.18 −0.00
90° 80° 70° 60° 50° 40° 30° 20° 10° 0°
Latitude
1.0 0.8 0.6 0.4 0.2 0.0
R/R⊙
0.0 0.2 0.4 0.6 0.8 1.0
R/R⊙

Density

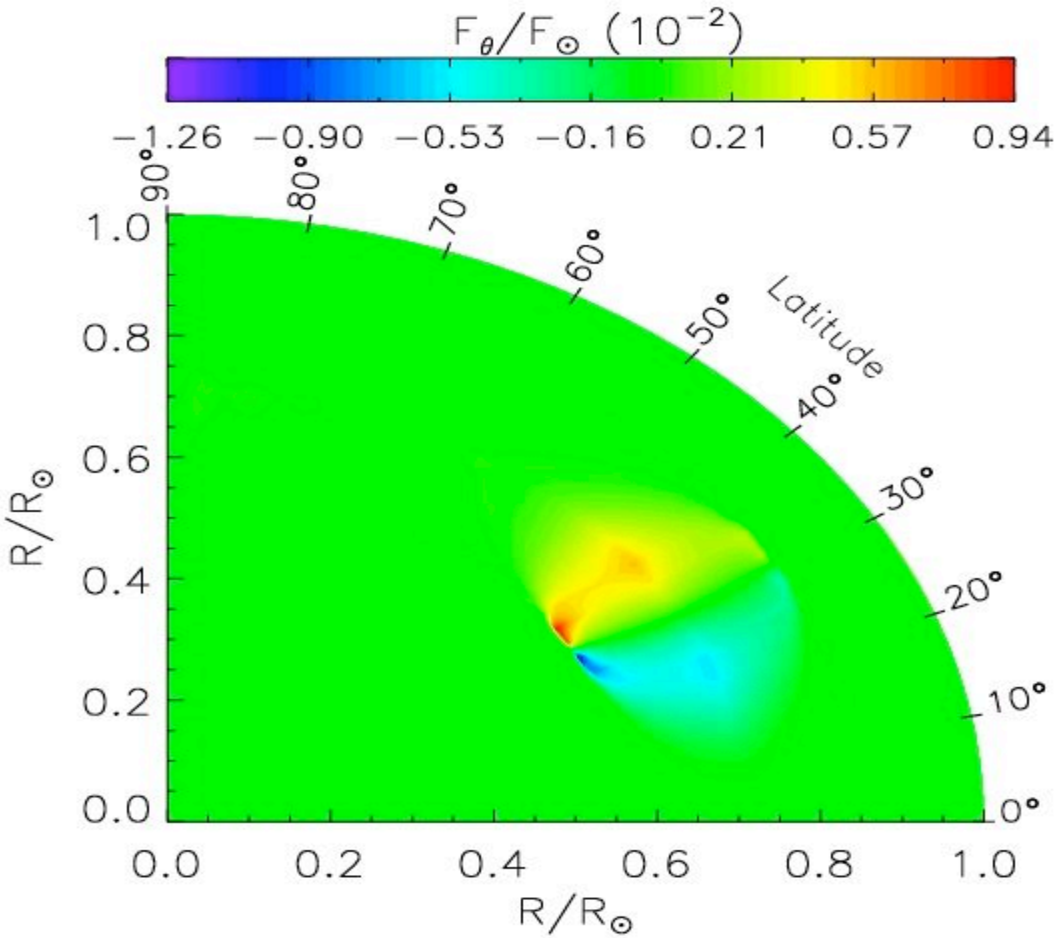


$\mathcal{E}_R$ (10^{-3})
−1.71 −1.32 −0.94 −0.55 −0.16 0.22 0.61
90° 80° 70° 60° 50° 40° 30° 20° 10° 0°
Latitude
1.0 0.8 0.6 0.4 0.2 0.0
R/R$_\odot$
0.0 0.2 0.4 0.6 0.8 1.0
R/R$_\odot$

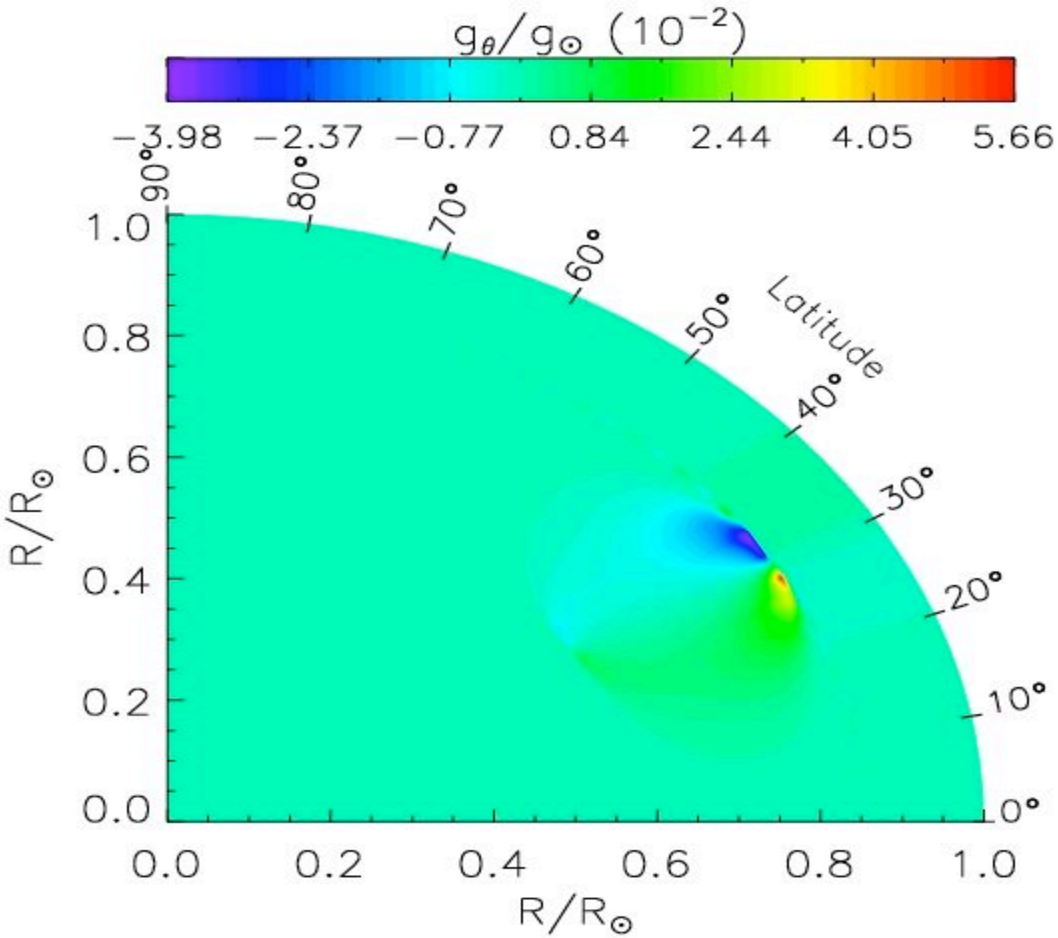
radius



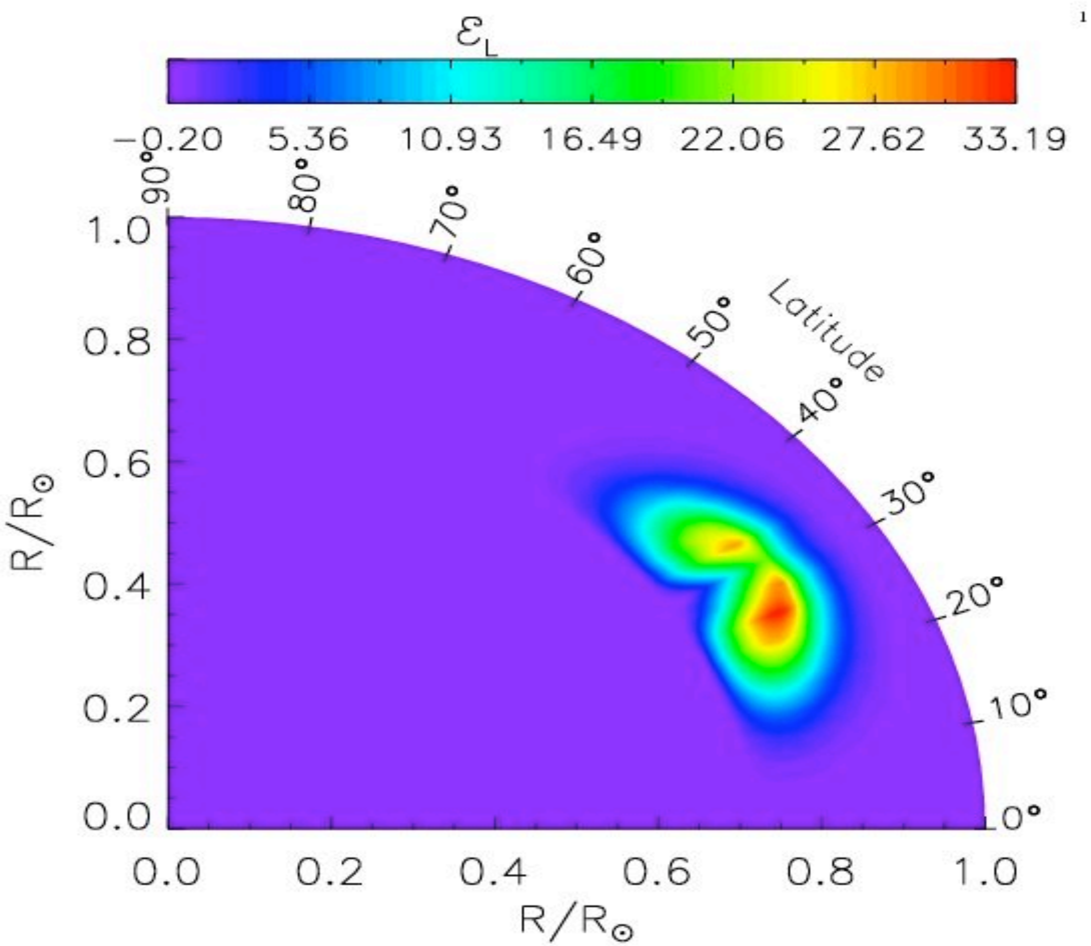
Temperature



Horizontal flux



Horizontal gravity



$\mathcal{E}_L$
−0.20 5.36 10.93 16.49 22.06 27.62 33.19
Latitude
90° 80° 70° 60° 50° 40° 30° 20° 10° 0°
1.0 0.8 0.6 0.4 0.2 0.0
R/R⊙
0.0 0.2 0.4 0.6 0.8 1.0
R/R⊙

Luminosity

Still to be done ...

Determine from PICARD data W=dlnR/dlnL

The limb profile will test the model atmosphere, and separate the effects of possible profile variations (both in latitude and time) from diameter changes.

We will allow us to separate internal variations (determined from photospheric temperature and diameter) from surface magnetic effects.

* Modelling turbulence-magnetic field interaction

* Include sophisticated atmosphere

* Determine the value of W