Advantages and Limitations Of The Shell Model

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1 Strasbourg-Madrid-GSI Shell-Model collaboration
What do we mean by SHELL MODEL nowadays

- An approximation to the exact solution of the nuclear A-body problem
- Using effective interactions in restricted valence spaces (or regularized interactions in the No Core Shell Model description of very light nuclei)
- Where the Monopole hamiltonian is the (spherical) mean field
- and the Multipole hamiltonian is the “correlator”
The three pillars of the shell model

The Effective Interaction

Valence Spaces

Algorithms and Codes

Define a valence space
Shell Model Problem

- Define a valence space
- Derive an effective interaction

\[ \mathcal{H}\psi = E\psi \rightarrow \mathcal{H}_{\text{eff}}\psi_{\text{eff}} = E\psi_{\text{eff}} \]
Shell Model Problem

- Define a valence space
- Derive an effective interaction

\[ \mathcal{H} \Psi = E \Psi \rightarrow \mathcal{H}_{\text{eff}} \Psi_{\text{eff}} = E \Psi_{\text{eff}} \]

- Build and diagonalize the Hamiltonian matrix.
Shell Model Problem

- Define a valence space
- Derive an effective interaction

\[ \mathcal{H}\psi = E\psi \rightarrow \mathcal{H}_{\text{eff}}\psi_{\text{eff}} = E\psi_{\text{eff}} \]

- Build and diagonalize the Hamiltonian matrix.

In principle, all the spectroscopic properties are described simultaneously (Rotational band AND $\beta$ decay half-life).
Choice of the basis:

- m-scheme: \( |\Phi_\alpha\rangle = \prod_{i=nljm} a_i^\dagger |0\rangle = a_{i_1}^\dagger ... a_{i_A}^\dagger |0\rangle \)

Simple \( \mathcal{H}_{IJ} \) but Maximal size: \( D \sim \binom{d_{\pi}}{p} \cdot \binom{d_{\nu}}{n} \)

- coupled-scheme (J or JT):

\[
\left[ \begin{array}{c}
| (j_1)^{n_1} v_1 \gamma_1 x_1 \rangle \\
| (j_2)^{n_2} v_2 \gamma_2 x_2 \rangle \\
\vdots \\
| (j_k)^{n_k} v_k \gamma_k x_k \rangle \\
\end{array} \right]^{-2} \cdots \left[ \begin{array}{c}
| (j_k)^{n_k} v_k \gamma_k x_k \rangle \\
\end{array} \right]^{-k}
\]

Reduced dimensions BUT complicated and much more non zero terms
Diagonalization

- Lanczos Method: iterative process

\[ \mathcal{H}\psi_i = E_{i-1}\psi_{i-1} + E_{ii}\psi_i + E_{i+1}\psi_{i+1} \]

\[
\begin{pmatrix}
E_{11} & E_{12} & 0 & 0 & 0 \\
E_{12} & E_{22} & E_{23} & 0 & 0 \\
0 & E_{32} & E_{33} & E_{34} & 0 \\
0 & 0 & E_{43} & E_{44} & E_{45}
\end{pmatrix}
\]

- storage of “many” vectors to build the eigenvectors solved by increase of disk capacity
- Extension RAM memory (CPU time/ELAPSED time) → 1
- regular increase of CPU power
Giant Matrices

- **Giant matrices**: $H_{IJ}$ recalculated in the iterative process.
- Basic idea: Factorization $|I\rangle \equiv |i\alpha\rangle$
  - $|i\rangle$: proton state, $|\alpha\rangle$: neutron state
- $\text{dim}(i) \ll \text{dim}(I)$

Precalculations in each separate space

- **m scheme** (code Antoine): $H_{IJ} = V(K)$
- **coupled scheme** (code Nathan): $H_{IJ} = h_{ij}.h_{\alpha\beta}.W(K)$
  - $I=i+\alpha$, $J=j+\beta$, $K=r+\mu$

- **Recent improvements**: generalized factorization $i, \alpha$
  - ex: semi-magic nuclei $N=126$
  - $i \equiv 1i_{13}, 1h_9 \alpha \equiv 2f_{7/2}, 2f_{5/2}, 3p_{3/2}, 3p_{1/2}$
Actual Limitations

- **m scheme**
  
  Huge dimensions of the matrices ($10^9$-$10^{10}$)

  storage of Lanczos vectors on disk

  AMD Opteron 64bits 2.2 GHz / 8 Gb RAM

  $^{56}\text{Ni}$: $D=10^9$ 1h30/it.

- **Coupled scheme**
  
  Small dimensions of the matrices ($10^7$-$10^8$)

  Parallelization: each processor has the initial and a final vector

  $\Psi_i = \bigcup_m \Psi_{i,f}^m$

  $\Psi_{f}^{(m)} = \sum_n \mathcal{H}(m,n) \Psi_i^{(n)}$

  final vectors are added

  $\Psi_f = \sum_k \Psi_f^{(k)}$

  ImaBIO Cluster 24 nodes Xeon 2.8 GHz
  Nuc Theo Cluster 10 nodes Xeon 2.7 GHz

  $^{128}\text{Xe}$ GS: $D=70 \times 10^6$ ($D_{M=0}=10^{10}$)

  400 Gb precalculation storage

  $10^{14}$ non zero terms !!! 8h/it.(34 procs)

  $2^+$ out of reach

powered by \LaTeX

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Advantages and Limitations Of The Shell Model
Actual Limitations

• **m scheme**
  Huge dimensions of the matrices (10^9-10^{10})
  storage of Lanczos vectors on disk
  AMD Opteron 64bits 2.2 GHz / 8 Gb RAM

56Ni: D=10^9 1h30/it.

Very large cases:
splitting of the initial and final vectors

\[ \psi_{i,f} = \bigcup_m \psi_{i,f}^m \]
\[ \psi_f^m = \sum_n \mathcal{H}^{(m,n)} \psi_{i}^{(n)} \]

• **Coupled scheme**
  Small dimensions of the matrices (10^7-10^8)
  Parallelization: each processor has the initial and a final vector

\[ \psi_{f}^{(k)} = \mathcal{H}^{(k)} \psi_{i} \]

final vectors are added

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2^+ out of reach
The evolution of the Spherical mean field in the valence spaces. What is missing in the monopole hamiltonian derived from the realistic NN interactions, be it through a G-matrix, $V_{low-k}$ or other options?

The multipole hamiltonian does not seem to demand major changes with respect to the one derived from the realistic nucleon-nucleon potentials.

Do we really need three body forces? Would they be reducible to simple monopole forms?

Much more to come with Andres Zuker’s talk.
Ab initio calculations

Valence space: all states with excitation energy in the H.O. basis until \( N\hbar\omega \)

- \( N=10 \) for \( A\leq 8 \)
- \( N=8 \) for \( A\leq 16 \)

- Specific problem: number of \( |nljm\rangle \) states
  - 276 shells for protons or neutrons
  - 4600 states in the 22 \( \hbar\omega \) space (20 in fp shell)

- Lee-Suzuki unitary transformation to derive effective hamiltonian in the model space

- Use of modern realistic interactions CD-Bonn, Argonne, \( N^3\)LO, Idaho-A ...
Advantages and Limitations Of The Shell Model


**8Li and 8B quadrupole moments**

<table>
<thead>
<tr>
<th></th>
<th>N=0</th>
<th>N=2</th>
<th>N=4</th>
<th>N=6</th>
<th>N=8</th>
<th>N=10</th>
<th>N=12</th>
<th>exp.</th>
<th>QFMC</th>
</tr>
</thead>
<tbody>
<tr>
<td>8B</td>
<td>4.27</td>
<td>4.51</td>
<td>4.76</td>
<td>5.03</td>
<td>5.29</td>
<td>5.55</td>
<td>5.76</td>
<td>6.83(21)</td>
<td>8.2(3)</td>
</tr>
<tr>
<td>8Li</td>
<td>1.79</td>
<td>2.24</td>
<td>2.43</td>
<td>2.60</td>
<td>2.72</td>
<td>2.83</td>
<td>2.92</td>
<td>3.19(7)</td>
<td>3.0(2)</td>
</tr>
</tbody>
</table>

**Two first $\frac{3}{2}^-$ states in 11Li**

<table>
<thead>
<tr>
<th></th>
<th>N=2</th>
<th>N=4</th>
<th>N=6</th>
<th>N=8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E(\frac{3}{2})^{-}_1 - E(\frac{3}{2})^{-}_2$</td>
<td>22.5</td>
<td>20.4</td>
<td>15.2</td>
<td>12.8</td>
</tr>
<tr>
<td>n components</td>
<td>$\psi_1$</td>
<td>$\psi_2$</td>
<td>$\psi_1$</td>
<td>$\psi_2$</td>
</tr>
<tr>
<td>n=0</td>
<td>0.72</td>
<td>0.0</td>
<td>0.67</td>
<td>0.0</td>
</tr>
<tr>
<td>n=2</td>
<td>0.28</td>
<td>1.0</td>
<td>0.16</td>
<td>0.77</td>
</tr>
<tr>
<td>n=4</td>
<td>0.17</td>
<td>0.23</td>
<td>0.13</td>
<td>0.17</td>
</tr>
<tr>
<td>n=6</td>
<td>0.08</td>
<td>0.15</td>
<td>0.06</td>
<td>0.13</td>
</tr>
<tr>
<td>n=8</td>
<td>0.06</td>
<td>0.08</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Spectroscopy in $^{10}\text{B}$

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3N: status

P. Navratil, V. G. Gueorgiev, J. P. Vary, W. E. Ormand, and A. Nogga
<table>
<thead>
<tr>
<th>Nucleus/property</th>
<th>Expt.</th>
<th>NN+N NN</th>
<th>NN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^6\text{Li}$ :</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>E(1^+_1 0)\rangle$ [MeV]</td>
<td>31.995</td>
<td>32.63</td>
</tr>
<tr>
<td>$Q(1^+_1 0)\ [e \text{ fm}^2]$</td>
<td>-0.082(2)</td>
<td>-0.124</td>
<td>-0.052</td>
</tr>
<tr>
<td>$\mu(1^+_1 0)\ [\mu_N]$</td>
<td>+0.822</td>
<td>+0.836</td>
<td>+0.845</td>
</tr>
<tr>
<td>$E_x(3^+_1 0)$ [MeV]</td>
<td>2.186</td>
<td>2.471</td>
<td>2.874</td>
</tr>
<tr>
<td>B(E2;3^+_1 0 → 1^+_1 0)</td>
<td>10.69(84)</td>
<td>3.685</td>
<td>4.512</td>
</tr>
<tr>
<td>B(E2;2^+_1 0 → 1^+_1 0)</td>
<td>4.40(2.27)</td>
<td>3.847</td>
<td>4.624</td>
</tr>
<tr>
<td>B(M1;0^+_1 1 → 1^+_1 0)</td>
<td>15.43(32)</td>
<td>15.038</td>
<td>15.089</td>
</tr>
<tr>
<td>B(M1;2^+_1 1 → 1^+_1 0)</td>
<td>0.149(27)</td>
<td>0.075</td>
<td>0.031</td>
</tr>
<tr>
<td>$^{10}\text{B}$ :</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>E(3^+_1 0)\rangle$ [MeV]</td>
<td>64.751</td>
<td>64.78</td>
</tr>
<tr>
<td>$r_p\ [\text{fm}]$</td>
<td>2.30(12)</td>
<td>2.197</td>
<td>2.256</td>
</tr>
<tr>
<td>$Q(3^+_1 0)\ [e \text{ fm}^2]$</td>
<td>+8.472(56)</td>
<td>+6.327</td>
<td>+6.803</td>
</tr>
<tr>
<td>$\mu(3^+_1 0)\ [\mu_N]$</td>
<td>+1.801</td>
<td>+1.837</td>
<td>+1.853</td>
</tr>
<tr>
<td>$rms(Exp - Th)\ [\text{MeV}]$</td>
<td>-</td>
<td>0.823</td>
<td>1.482</td>
</tr>
<tr>
<td>B(E2;1^+_1 0 → 3^+_1 0)</td>
<td>4.13(6)</td>
<td>3.047</td>
<td>4.380</td>
</tr>
<tr>
<td>B(E2;1^+_2 0 → 3^+_1 0)</td>
<td>1.71(0.26)</td>
<td>0.504</td>
<td>0.082</td>
</tr>
<tr>
<td>B(GT;3^+_1 0 → 2^+_1 1)</td>
<td>0.083(3)</td>
<td>0.070</td>
<td>0.102</td>
</tr>
<tr>
<td>B(GT;3^+_1 0 → 2^+_2 1)</td>
<td>0.95(13)</td>
<td>1.222</td>
<td>1.487</td>
</tr>
<tr>
<td>$^{12}\text{C}$ :</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$</td>
<td>E(0^+_1 0)\rangle$ [MeV]</td>
<td>92.162</td>
<td>95.57</td>
</tr>
<tr>
<td>$r_p\ [\text{fm}]$</td>
<td>2.35(2)</td>
<td>2.172</td>
<td>2.229</td>
</tr>
<tr>
<td>$Q(2^+_1 0)\ [e \text{ fm}^2]$</td>
<td>+6(3)</td>
<td>+4.318</td>
<td>+4.931</td>
</tr>
<tr>
<td>$rms(Exp - Th)\ [\text{MeV}]$</td>
<td>-</td>
<td>1.058</td>
<td>1.318</td>
</tr>
<tr>
<td>B(E2;2^+_2 0 → 0^+_0)</td>
<td>7.59(42)</td>
<td>4.252</td>
<td>5.483</td>
</tr>
<tr>
<td>B(M1;1^+_1 0 → 0^+_0)</td>
<td>0.0145(21)</td>
<td>0.006</td>
<td>0.003</td>
</tr>
<tr>
<td>B(M1;1^+_2 1 → 0^+_0)</td>
<td>0.951(20)</td>
<td>0.913</td>
<td>0.353</td>
</tr>
<tr>
<td>B(E2;2^+_1 1 → 0^+_0)</td>
<td>0.65(13)</td>
<td>0.451</td>
<td>0.301</td>
</tr>
</tbody>
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<th>NN</th>
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</thead>
<tbody>
<tr>
<td>$^{11}$B : $</td>
<td>E_{3/2_1}^{1/2_2}</td>
<td>$ [MeV]</td>
<td>76.205</td>
</tr>
<tr>
<td>$r_p(3/2_1 - 1/2_2)$ [fm]</td>
<td>2.24(12)</td>
<td>2.127</td>
<td>2.196</td>
</tr>
<tr>
<td>$Q(3/2_1 - 1/2_2)$ [e fm$^2$]</td>
<td>+4.065(26)</td>
<td>+3.065</td>
<td>+2.989</td>
</tr>
<tr>
<td>$\mu(3/2_1 - 1/2_2)$ [$\mu_N$]</td>
<td>+2.689</td>
<td>+2.063</td>
<td>+2.597</td>
</tr>
<tr>
<td>$r_{ms}(Exp-Th)$ [MeV]</td>
<td>-</td>
<td>1.067</td>
<td>1.765</td>
</tr>
<tr>
<td>B(E2; $3/2_1 - 1/2_2 \rightarrow 1/2_1 - 1/2_2$)</td>
<td>2.6(4)</td>
<td>1.476</td>
<td>0.750</td>
</tr>
<tr>
<td>B(GT; $3/2_1 - 1/2_2 \rightarrow 3/2_1 - 1/2_2$)</td>
<td>0.345(8)</td>
<td>0.235</td>
<td>0.663</td>
</tr>
<tr>
<td>B(GT; $3/2_1 - 1/2_2 \rightarrow 2/2_1 - 1/2_2$)</td>
<td>0.440(22)</td>
<td>0.461</td>
<td>0.841</td>
</tr>
<tr>
<td>B(GT; $3/2_1 - 1/2_2 \rightarrow 5/2_1 - 1/2_2$)</td>
<td>0.526(27)</td>
<td>0.526</td>
<td>0.394</td>
</tr>
<tr>
<td>B(GT; $3/2_1 - 1/2_2 \rightarrow 3/2_1 - 2/2_2$)</td>
<td>0.461(23)</td>
<td>0.829</td>
<td>0.236</td>
</tr>
</tbody>
</table>

| $^{13}$C : $|E_{1/2_1}^{1/2_2}|$ [MeV] | 97.108 | 103.23 | 90.31 |
| $r_p(1/2_1 - 1/2_2)$ [fm] | 2.29(3) | 2.135 | 2.195 |
| $\mu(1/2_1 - 1/2_2)$ [$\mu_N$] | +0.702 | +0.394 | +0.862 |
| $r_{ms}(Exp-Th)$ [MeV] | - | 2.144 | 2.089 |
| B(E2; $3/2_1 - 1/2_2 \rightarrow 1/2_1 - 1/2_2$) | 6.4(15) | 2.659 | 4.584 |
| B(M1; $3/2_1 - 1/2_2 \rightarrow 1/2_1 - 1/2_2$) | 0.70(7) | 0.702 | 1.148 |
| B(GT; $1/2_1 - 1/2_2 \rightarrow 1/2_1 - 1/2_2$) | 0.20(2) | 0.095 | 0.328 |
| B(GT; $1/2_1 - 1/2_2 \rightarrow 3/2_1 - 1/2_2$) | 1.06(8) | 1.503 | 2.155 |
| B(GT; $1/2_1 - 1/2_2 \rightarrow 5/2_1 - 1/2_2$) | 0.16(1) | 0.733 | 0.263 |
| B(GT; $1/2_1 - 1/2_2 \rightarrow 3/2_1 - 2/2_2$) | 0.39(3) | 1.050 | 0.221 |
| B(GT; $1/2_1 - 1/2_2 \rightarrow 3/2_1 - 3/2_2$) | 0.19(2) | 0.400 | 0.151 |
| Total energy rms [MeV] | - | 1.314 | 1.671 |

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Valence space

The choice of the valence space:

- In light nuclei the harmonic oscillator closures determine the natural valence spaces:

$^4\text{He} \rightarrow ^{16}\text{O} \rightarrow ^{40}\text{Ca} \rightarrow ^{80}\text{Zr}$

- $p$ shell
  - Cohen/Kurath
- $sd$ shell
  - Brown/Wildenthal
- $pf$ shell
  - Deformed
Valence space

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  - $sd$ shell: Brown/Wildenthal
  - $pf$ shell: Deformed

- In heavier nuclei: $jj$ closures due to the spin-orbit term show up
  
  $N=28, 50, 82, 126$
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  - $pf$ shell

  Cohen/ Brown/ Deformed Kurath/ Wildenthal

- In heavier nuclei:
  - $jj$ closures due to the spin-orbit term show up $N=28, 50, 82, 126$

- the transition $HO \rightarrow jj$: occurs between $^{40}\text{Ca}$ and $^{100}\text{Sn}$ where the protagonism shifts from the $1f_{7/2}$ to the $1g_{9/2}$
A valence space can be adequate to describe some properties and completely wrong for others.

<table>
<thead>
<tr>
<th></th>
<th>(48 \text{Cr} )</th>
<th>((f_{\frac{7}{2}})^8)</th>
<th>((f_{\frac{7}{2}} p_{\frac{3}{2}})^8)</th>
<th>((fp)^8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Q(2^+)) (e.fm(^2))</td>
<td>0.0</td>
<td>-23.3</td>
<td>-23.8</td>
<td></td>
</tr>
<tr>
<td>(E(2^+)) (MeV)</td>
<td>0.63</td>
<td>0.44</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>(E(4^+)/E(2^+))</td>
<td>1.94</td>
<td>2.52</td>
<td>2.26</td>
<td></td>
</tr>
<tr>
<td>(BE2(2^+ \rightarrow 0^+)) (e(^2).fm(^4))</td>
<td>77</td>
<td>150</td>
<td>216</td>
<td></td>
</tr>
<tr>
<td>(B(GT))</td>
<td>0.90</td>
<td>0.95</td>
<td>3.88</td>
<td></td>
</tr>
</tbody>
</table>
Valence space

- For the quadrupole properties $f_7^2 p_{3/2}^3$ is a good space (Quasi-SU3 orbitals) whereas for magnetic and Gamow-Teller processes the presence of the spin orbit partners is compulsory.

- In the tin isotopes the natural valence space consists in a $^{100}$Sn core and valence orbits:
  \[
  (d_{5/2}^5 g_{7/2}^7 s_{1/2}^1 d_{3/2}^3 h_{11/2}^{11})^\nu
  \]

  However, numerous E1 transitions have been measured that are forbidden in this space because:
  \[
  (a_{11/2}^\dagger \tilde{a}_{j})^\lambda \neq 1,
  \]
  it is then necessary to incorporate the $1g_{9/2}$ orbit.

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Physics Goals

Precision Spectroscopy towards larger masses

Description of the nuclear correlations in the laboratory frame

Changing Magic Numbers far from Stability: The competing roles of spherical mean field and correlations

Double $\beta$ decay, the key to the nature of the neutrinos, the absolute scale of their masses and their hierarchy

No core shell model for light nuclei. Ab initio description of the low-lying intruder states and of the origin of the Gamow-Teller quenching

Nuclear Structure and Nuclear Astrophysics
The most popular “flaws” of the standard SM description

- Not all the regions of the nuclear chart are amenable to a SM description yet
- Quadrupole effective charges are needed (But their value is universal and rather well understood)
- Spin operators are quenched by another universal factor which relates to the regularization of the interaction (also known as short range correlation). Indeed, BMF approaches share this shortcoming
The most popular “flaws” of the standard SM description

- Not all the regions of the nuclear chart are amenable to a SM description yet
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Quadrupole excitations on CS nuclei

- 40 Ca
- 100 Sn

 Levels:
- N=2
- N=3
- N=4
- N=5
- N=6

Shell Model States:
- (gds)
- (fp)
- (sd)
SD band in $^{36}$Ar


good description in terms of $ph$ excitations
**SD band in $^{36}$Ar**


- good description in terms of $ph$ excitations
- Decay of the SD bands shows the mixing between npnh configurations
SD band in $^{36}$Ar


- good description in terms of $ph$ excitations
- Decay of the SD bands shows the mixing between npnh configurations
- Mixing should not destroy the agreement of 4p4h calculations
SD band in $^{36}$Ar


- Good description in terms of $ph$ excitations
- Decay of the SD bands shows the mixing between npnh configurations
- Mixing should not destroy the agreement of 4p4h calculations
- Complex mechanism and theoretical challenge
Decay out of the SD band in $^{36}$Ar

- Improvement upon 4p-4h calculations
- Backbending reproduced and correct moment of inertia

E. Caurier, F. Nowacki, and A. Poves
Decay out of the SD band in $^{36}$Ar

![Graph showing decay out of the SD band in $^{36}$Ar]

- good overall agreement between experiment and calculations
Decay out of the SD band in $^{36}$Ar

- Good overall agreement between experiment and calculations
- Reconstruction of 4p-4h results due to mixing with 6p-6h states (as deformed as 4p-4h ones)
Decay out of the SD band in $^{36}$Ar

- Good overall agreement between experiment and calculations.
- Reconstruction of 4p-4h results due to mixing with 6p-6h states (as deformed as 4p-4h ones).
- E. Caurier, F. Nowacki, and A. Poves
Decay out of the SD band in $^{36}$Ar

Out-band transitions in the SD band of $^{36}$Ar (B(E2)’s in e$^2$fm$^4$ and energies in keV)

<table>
<thead>
<tr>
<th>Transition</th>
<th>$E_\gamma$ (keV)</th>
<th>$B$(E2) (e$^2$fm$^4$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^+_{SD} \rightarrow 0^+_1$</td>
<td>4950</td>
<td>4.6(23)</td>
</tr>
<tr>
<td>$4^+_{SD} \rightarrow 2^+_1$</td>
<td>4166</td>
<td>2.5(4)</td>
</tr>
<tr>
<td>$4^+_{SD} \rightarrow 2^+_2$</td>
<td>1697</td>
<td>19.2(30)</td>
</tr>
<tr>
<td>$6^+_{SD} \rightarrow 4^+_1$</td>
<td>3552</td>
<td>5.3(8)</td>
</tr>
<tr>
<td>$10^+<em>1 \rightarrow 8^+</em>{SD}$</td>
<td>1975</td>
<td>43.6(74)</td>
</tr>
<tr>
<td>$12^+_{SD} \rightarrow 10^+_1$</td>
<td>3448</td>
<td>15.0(30)</td>
</tr>
</tbody>
</table>

E. Caurier, F. Nowacki, and A. Poves

The Superdeformed band of $^{40}$Ca

![Graph showing the Superdeformed band of $^{40}$Ca with experimental data (exp) and sdpf-full model predictions.](#)
Transition Quadrupole Moments

Transition quadrupole moment (e fm$^2$)

F. Nowacki

Advantages and Limitations Of The Shell Model
B(E2)’s in Tin isotopes

A. Banu et al.,

The most popular “flaws” of the standard SM description

- Not all the regions of the nuclear chart are amenable to a SM description yet
- Quadrupole effective charges are needed (But their value is universal and rather well understood)
- Spin operators are quenched by another universal factor which relates to the regularization of the interaction (also known as short range correlation). Indeed, BMF approaches share this shortcoming
Renormalization of the spin operator in the *pf*-shell

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>Uncorrelated</th>
<th>Correlated</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td><em>Unquenched</em></td>
<td><em>Q = 0.74</em></td>
</tr>
<tr>
<td>$^{51}$V</td>
<td>5.15</td>
<td>2.42</td>
<td>1.33</td>
</tr>
<tr>
<td>$^{54}$Fe</td>
<td>10.19</td>
<td>5.98</td>
<td>3.27</td>
</tr>
<tr>
<td>$^{55}$Mn</td>
<td>7.96</td>
<td>3.64</td>
<td>1.99</td>
</tr>
<tr>
<td>$^{56}$Fe</td>
<td>9.44</td>
<td>4.38</td>
<td>2.40</td>
</tr>
<tr>
<td>$^{58}$Ni</td>
<td>11.9</td>
<td>7.24</td>
<td>3.97</td>
</tr>
<tr>
<td>$^{59}$Co</td>
<td>8.52</td>
<td>3.98</td>
<td>2.18</td>
</tr>
<tr>
<td>$^{62}$Ni</td>
<td>7.83</td>
<td>3.65</td>
<td>2.00</td>
</tr>
</tbody>
</table>
Quenching of GT strength in the $pf$-shell
$^{48}\text{Ca}(p,n)^{48}\text{Sc}$ Strength Function

Advantages and Limitations Of The Shell Model
48\(^{\text{Ca}}\)(p,n)48\(^{\text{Sc}}\) Strength Function

9470 it.
Quenching of GT operator in the \( pf \)-shell

\[
|i\rangle = \alpha|0\hbar\omega\rangle + \sum_{n \neq 0} \beta_n|n\hbar\omega\rangle,
\]

\[
|f\rangle = \alpha'|0\hbar\omega\rangle + \sum_{n \neq 0} \beta'_n|n\hbar\omega\rangle,
\]

then

\[
\langle f \parallel T \parallel i \rangle^2 = \left( \alpha\alpha' T_0 + \sum_{n \neq 0} \beta_n\beta'_n T_n \right)^2,
\]

- \( n \neq 0 \) contributions negligible
- \( \alpha \approx \alpha' \)

projection of the physical wavefunction in the \( 0\hbar\omega \) space is \( Q \approx \alpha^2 \)

transition quenched by \( Q^2 \)
Quenching of M1 operator in the pf-shell

![Graph showing B(M1) values for different excitation energies.]

- **52** Cr

K. Langanke, G. Martinez-Pinedo, P. Von Neumann-Cosel, and A. Richter


KB3G interaction

**Advantages and Limitations Of The Shell Model**

F. Nowacki
Quenching of M1 operator in the \( pf \)-shell

\[ \text{KB3 interaction} \]

\[
\begin{array}{c}
\text{\( \alpha_{\text{eff}} \)} \\
\alpha_0
\end{array}
\]

\[
\begin{array}{c}
\text{\( 48\text{Ca} \)} \\
\text{\( 50\text{Ti} \)} \\
\text{\( 52\text{Cr} \)} \\
\text{\( 54\text{Fe} \)}
\end{array}
\]

Neumann-Cosel et al.
Correlations in nuclei

FIG. 3. Density difference between $^{206}$Pb and $^{205}$Tl. The experimental result of Cavendon et al. (1982) is given by the error bars; the prediction obtained using Hartree-Fock orbitals with adjusted occupation numbers is given by the curve. The systematic shift of 0.0008 fm$^{-3}$ at $r \leq 4$ fm is due to deficiencies of the calculation in predicting the core polarization effect.

Advantages and Limitations Of The Shell Model

50 \leq Z, N \leq 82 region
**β decay systematics**

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>(^{128}\text{Sn})</th>
<th>(^{130}\text{Sn})</th>
<th>(^{132}\text{Sb})</th>
<th>(^{132}\text{Te})</th>
<th>(^{133}\text{Te})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transition</td>
<td>(0^+ \rightarrow 1^+)</td>
<td>(0^+ \rightarrow 1^+)</td>
<td>(4^+ \rightarrow 3, 4, 5^+)</td>
<td>(0^+ \rightarrow 1^+)</td>
<td>(\frac{3}{2}^+ \rightarrow \frac{1}{2}, \frac{3}{2}, \frac{5}{2}^+)</td>
</tr>
<tr>
<td>(T_{1/2}) exp.</td>
<td>59.07m</td>
<td>3.72m</td>
<td>2.79m</td>
<td>3.2d</td>
<td>12.5m</td>
</tr>
<tr>
<td>(T_{1/2}) calc. (0.74)</td>
<td>32.21m</td>
<td>2.47m</td>
<td>1.56m</td>
<td>1.73d</td>
<td>6.42m</td>
</tr>
<tr>
<td>Renorm.</td>
<td>0.54</td>
<td>0.6</td>
<td>0.55</td>
<td>0.54</td>
<td>0.53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(^{134}\text{Te})</th>
<th>(^{135}\text{Xe})</th>
<th>(^{136}\text{Cs})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0^+ \rightarrow 1^+)</td>
<td>(\frac{3}{2}^+ \rightarrow \frac{1}{2}, \frac{3}{2}, \frac{5}{2}^+)</td>
<td>(5^+ \rightarrow 4, 5, 6^+)</td>
</tr>
<tr>
<td>(41.8m)</td>
<td>9.14h</td>
<td>13.16d</td>
</tr>
<tr>
<td>(29.19m)</td>
<td>7.07h</td>
<td>8.1d</td>
</tr>
<tr>
<td>0.62</td>
<td>0.63</td>
<td>0.57</td>
</tr>
</tbody>
</table>

Our valence space leads to a renormalization of the \(\sigma T\) operator of a factor 0.57
Integrated strength

Integrated B(GT) values for $^{130}_{78}$Te$_{52}$ (left) and $^{128}_{76}$Te$_{52}$ (right) calculated within the $(g_7d_5d_3s_1h_{11})$ space (experimental data are from (p,n) reactions).
$^{136}\text{Cs}$ half-life

Gamow-Teller strength function from the $5^+$ ground state of $^{136}\text{Cs}$:

$T_{1/2} \text{ th.} \quad 13 \text{ d}$

$T_{1/2} \text{ exp.} \quad 13.16 \text{ d}$
At the very neutron rich or very proton rich edges, the $T=0$ and $T=1$ channels of the effective nuclear interaction weight very differently than they do at the stability line. Therefore the effective single particle structure may suffer important changes, leading in some cases to the vanishing of established shell closures or to the appearance of new ones.
Neutron rich nuclei: N=20

Advantages and Limitations Of The Shell Model
Dipole response in Neon chain

- Neons bound from stability to n-rich

- Shell evolution ➔ Island of inversion

- Shape evolution ➔
  - SU3 Deformation (N=Z)
  - Sphericity (N=14)
  - Intruder Deformation (N=20)

- Full $sd$ diagonalization + full $1\hbar\omega$ excitations

- Exact removal of Center of Mass spurious components
Neons Dipole Strength

\[ B(E1) \text{ (e}^2 \cdot \text{fm}^2) \]

\[ ^{20}\text{Ne} \]
Neons Dipole Strength

Advantages and Limitations Of The Shell Model

![Graph showing Neons Dipole Strength](image-url)
Neons Dipole Strength

$B(E1) \ (e^2 \cdot fm^2)$

$^{24}\text{Ne}$

Energy (MeV)

powered by \LaTeX
Neons Dipole Strength

$^{26}\text{Ne}$

$B(E1) \ (e^2.f.m^2)$

Energy (MeV)

Advantages and Limitations Of The Shell Model
Neons Dipole Strength

$^{28}\text{Ne}$

$B(E1) \ (e^2 \cdot fm^2)$

Energy (MeV)

Advantages and Limitations Of The Shell Model
Neons Dipole Strength

![Graph showing B(E1) vs Energy (MeV) for 30Ne. The graph highlights the dipole strength distribution with a peak around 10 MeV.]
Neutron rich nuclei: N=20

The graph shows the energies in MeV for different neutron levels in nuclei with N=20. The levels are labeled as f5, p1, f7, p3, d3, s1, and d5. The nuclei are labeled as (1) 028, (2) Si34, (3) S36, and (4) Ca40.
$^{26}$Ne Pygmy Resonance

recently observed at RIKEN:
$E_{\text{exp}} = 9$ MeV
$E_{\text{SM}} = 8.5$ MeV
\( B(E1)_{\text{exp}} = 0.60 \pm 0.06 \, e^2 \cdot \text{fm}^2, \quad 5.9 \pm 1.0 \% \, S_{TRK} \)

\( B(E1)_{\text{SM}} = 0.42 \, e^2 \cdot \text{fm}^2, \quad 4.0 \% \, S_{TRK} \)
$^{26}$Ne Pygmy Resonance

Main contributions:

$\nu(2s_{1/2}^{-1} 2p_{3/2})$

$\nu(2s_{1/2}^{-1} 2p_{1/2})$
Thanks to: E. Caurier, K. Langanke, G. Martinez-Pinedo, J. Menedez, A. Poves, K. Sieja, O. Sorlin, A. Zuker