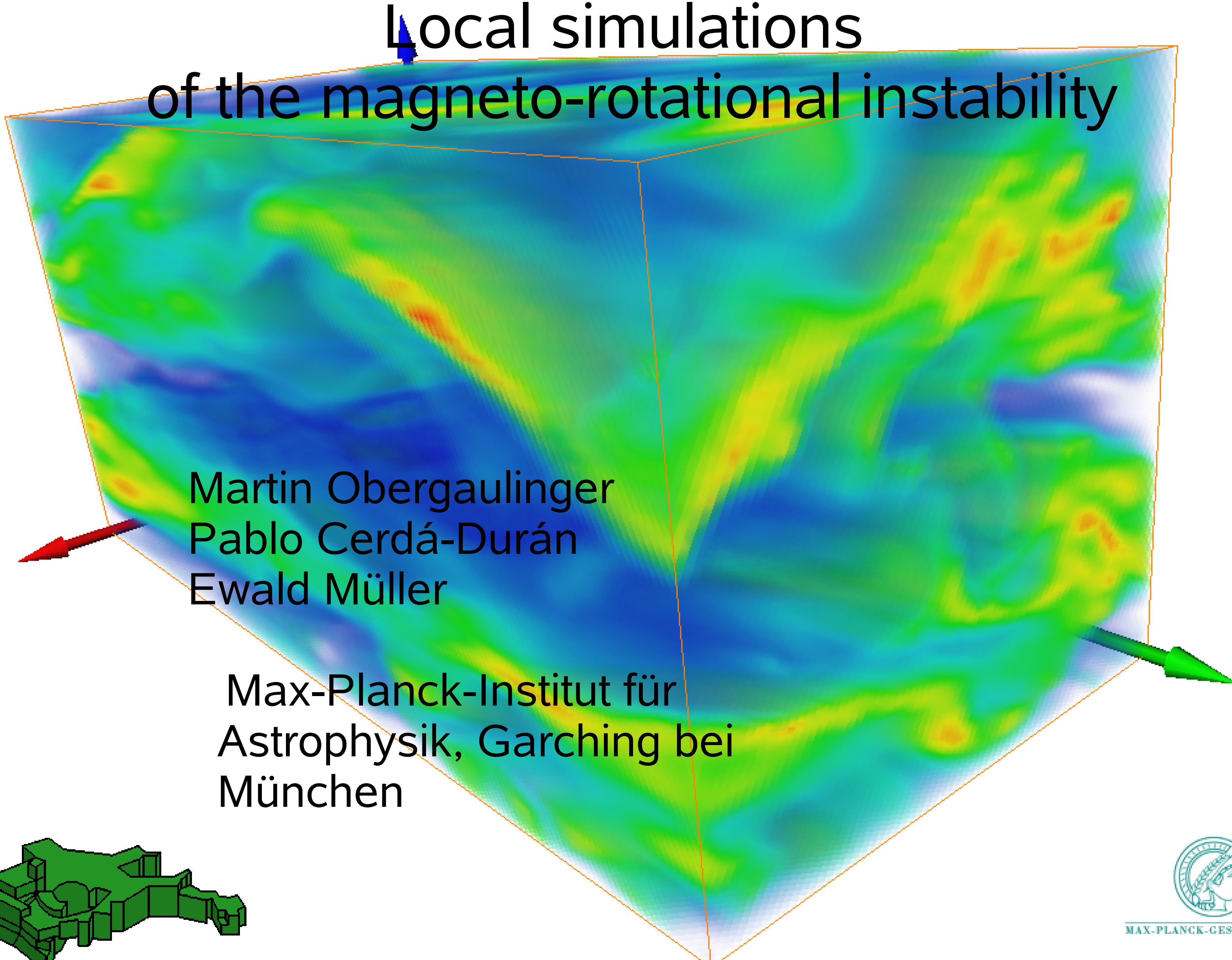
Local simulations of the magneto-rotational instability

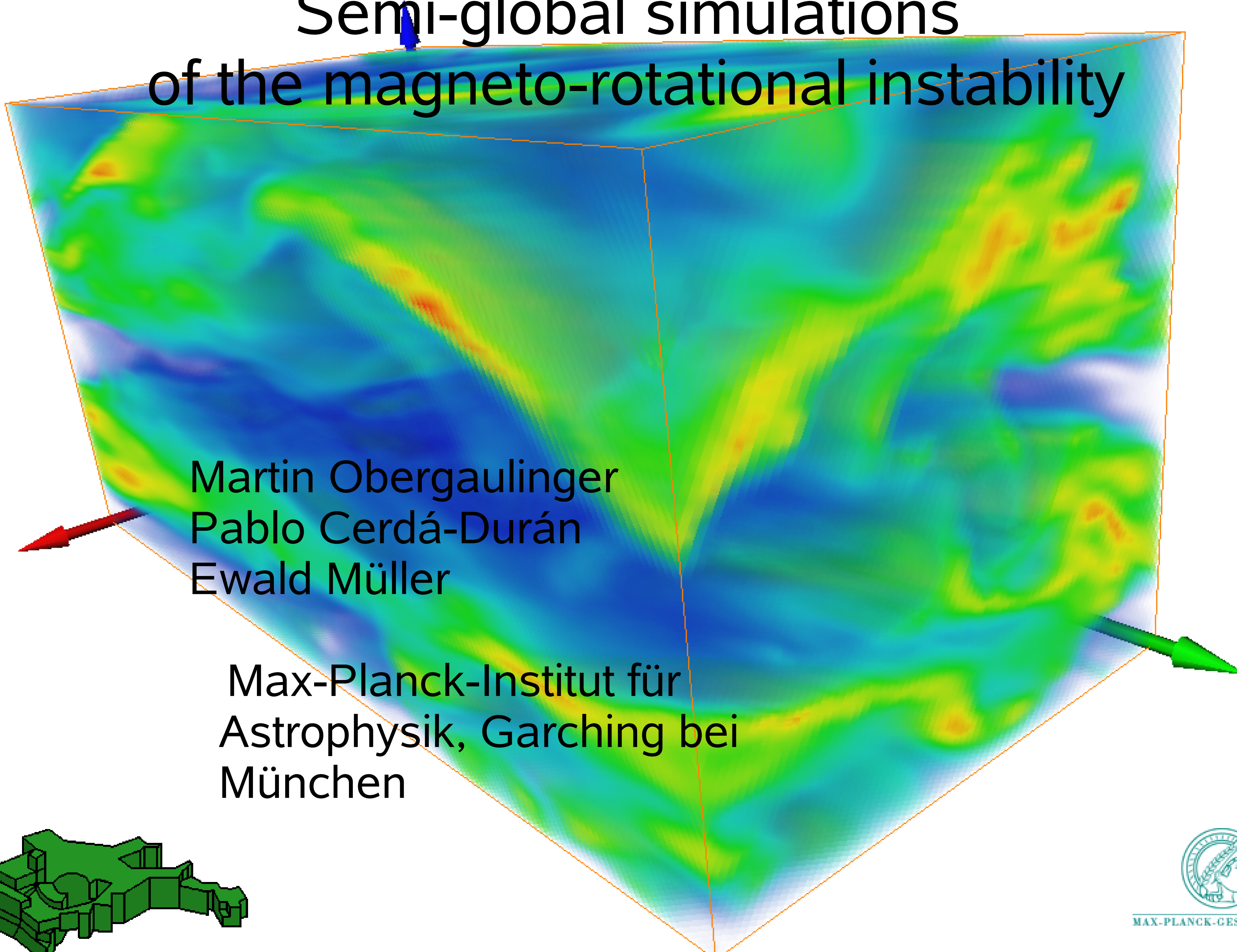


Martin Obergaulinger
Pablo Cerdá-Durán
Ewald Müller

Max-Planck-Institut für Astrophysik, Garching bei München

MAX-PLANCK-GESELLSCHAFT

Semi-global simulations of the magneto-rotational instability



Martin Obergaulinger
Pablo Cerdá-Durán
Ewald Müller

Max-Planck-Institut für Astrophysik, Garching bei München

MAX-PLANCK-GESELLSCHAFT

Overview

- Magnetic fields on SNe
- Physics of the MRI
- MRI simulations: global vs. local
- A set of local (semi-global) simulations

Effects of magnetic fields

- transport of angular momentum from the centre to the surrounding matter
  -> affecting structure and rotation of the PNS
- conversion of rotational into thermal energy
  -> imparting energy to gas at the threshold of explosion
- shaping the ejecta: jet-like outflows
- MHD explosions possible?

Effects of magnetic fields

LeBlanc & Wilson (1970)
Bisnovatyi-Kogan (1976)
Meier et al. (1976)
Müller & Hillebrandt (1979)
Symbalisti (1984)
Yamada & Sawai (2004)
Kotake et al. (2004*ff*)
Takiwaki et al. (2004*ff*)
Sawai et al. (2005*ff*)
Ardeljan et al. (2005)
Obergaulinger et al. (2006)
Cerdá-Durán et al. (2007)
Burrows et al. (2007), Dessart et al. (2007)
Endeve et al. (2008),
...

Effects of magnetic fields

- transport of angular momentum from the centre to the surrounding matter
  -> affecting structure and rotation of the PNS
- conversion of rotational into thermal energy
  -> imparting energy to gas at the threshold of explosion
- shaping the ejecta: jet-like outflows
- MHD explosions possible?

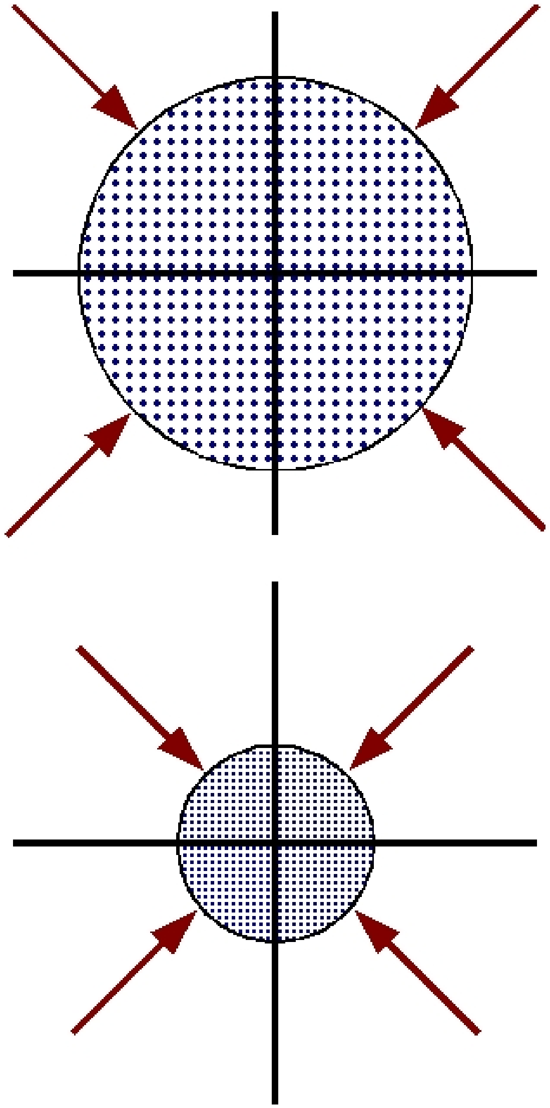
- However: strong fields are required
  $> 10^{14}$ G after bounce

Pre-collapse magnetic fields

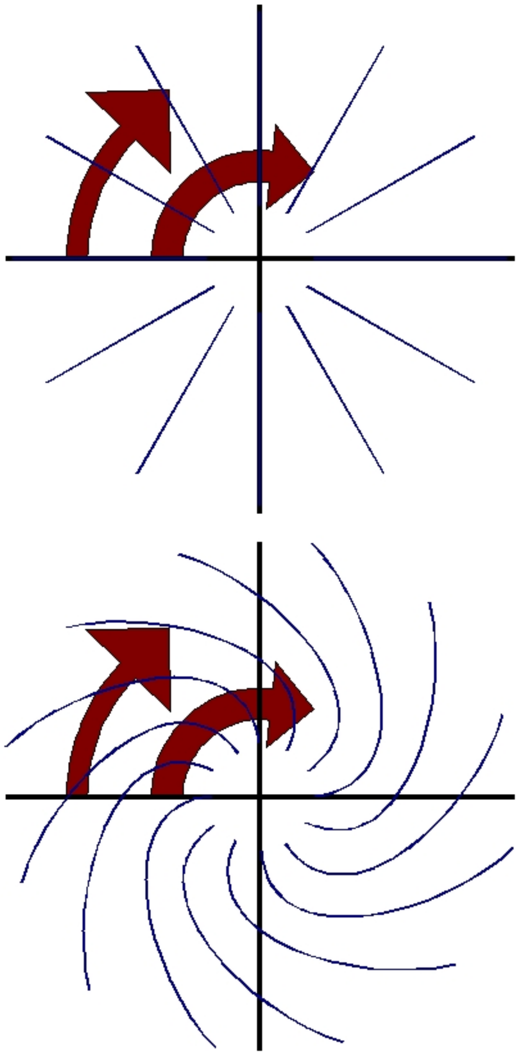
- recent stellar-evolution models take into account rotation and magnetic braking during pre-SN phases (e.g., *Tayler-Spruit dynamo;* Heger, Woosley, & Spruit 2005*ff*, Maeder & Meynet 2003*ff*)
- comparably weak fields, slow rotation
- typical values: 10^9 G toroidal, 10^6 G poloidal field
- corresponding post-collapse fields are dynamically negligible
- faster rotation, stronger fields for subclasses of SNe?

Field amplification mechanisms

- compressional amplification feeds off kinetic energy of infall
- $e_{mag} \sim - \boldsymbol{b}^2/2 \text{ div } \boldsymbol{v}$
- gives a factor of 100...1000 during collapse
- purely passive, irrespective of field strength and geometry



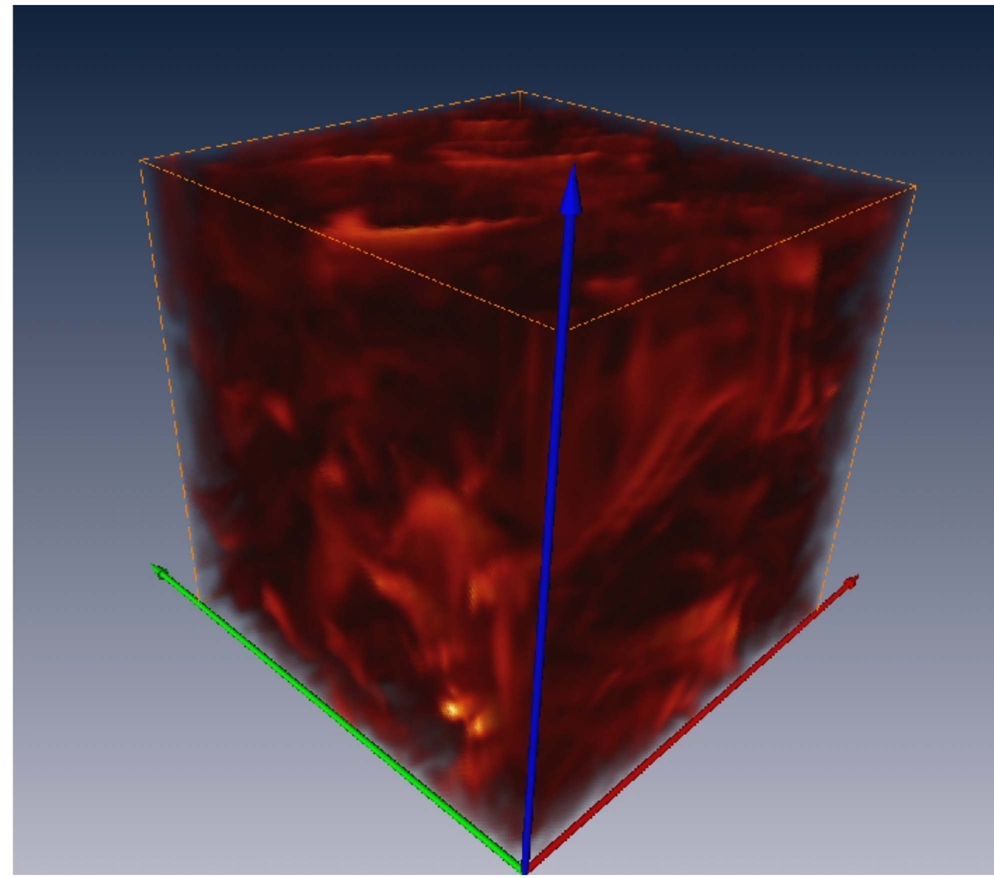
Field amplification mechanisms



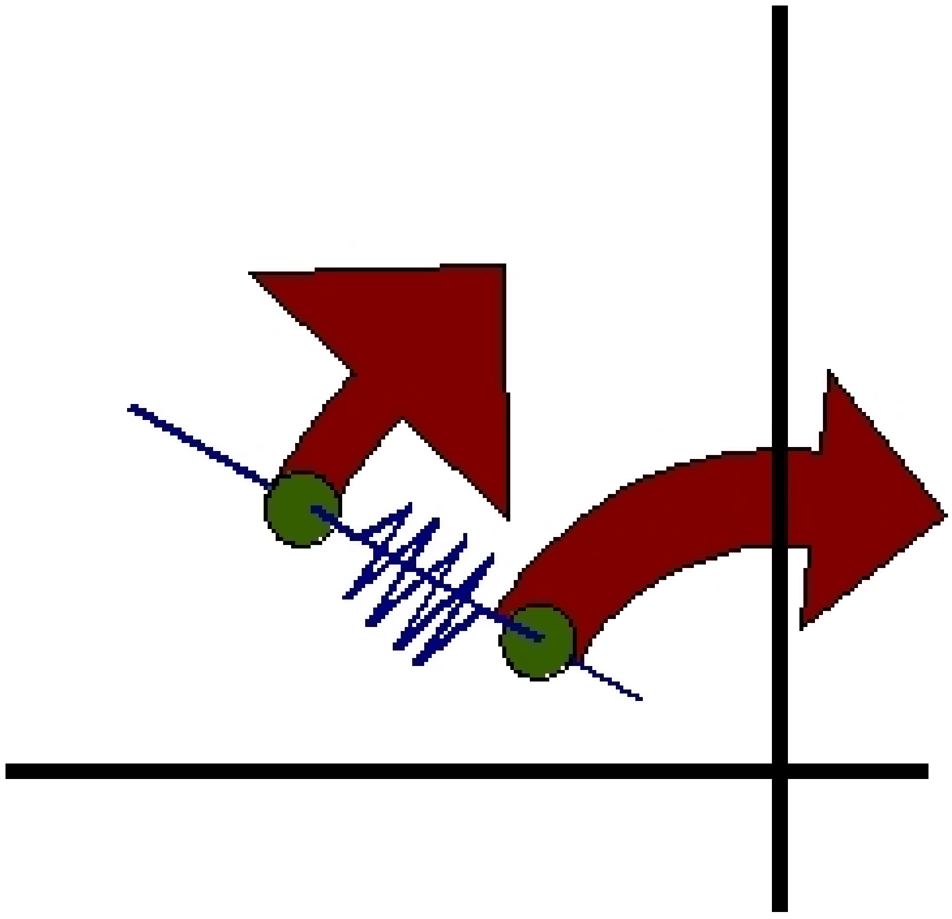
- winding by differential rotation amplifies toroidal component from a seed poloidal field
- energy source: differential rotation
- requires poloidal field
- linear in time; time scale set by rotational period

Field amplification mechanisms

- MHD version of HD instabilities, e.g.,
  - convection
  - SASI
- turbulent dynamo
  - energy source: turbulent kinetic energy; turbulence excited by different effects
  - genuinely 3d effect



Field amplification mechanisms



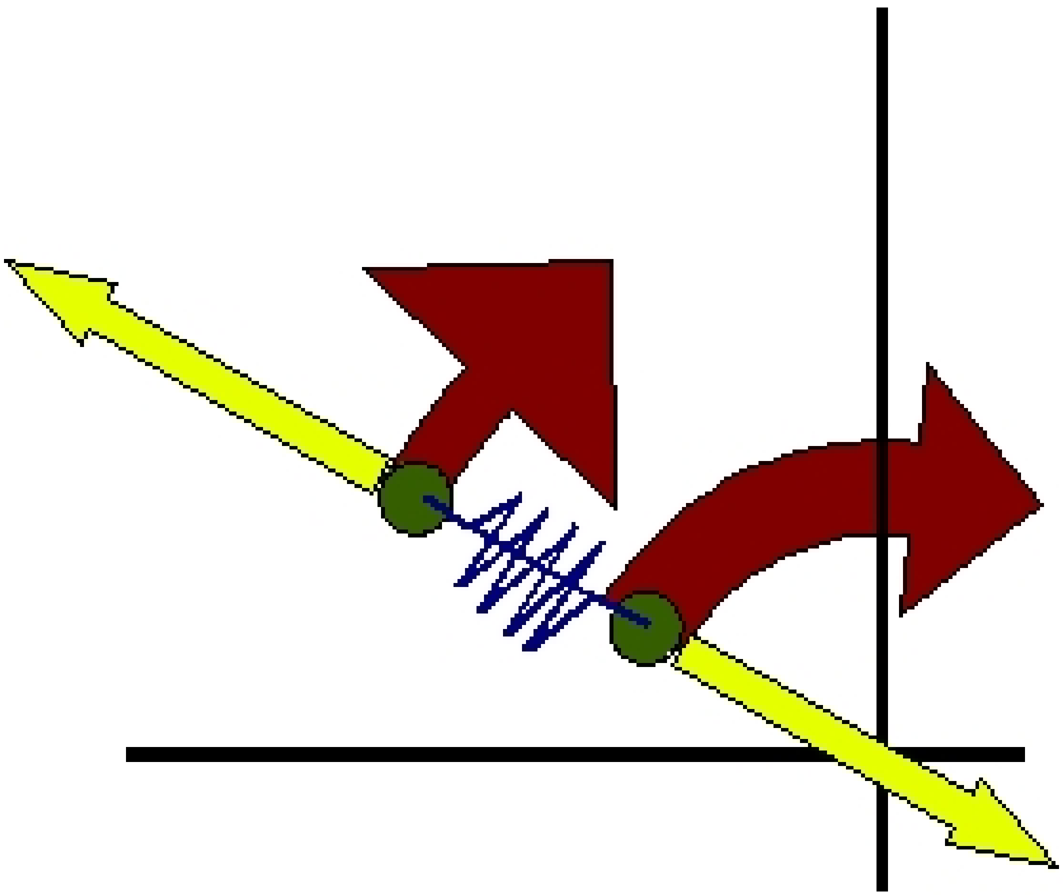
- magneto-rotational instabil-ity
- energy source: differential rotation
- instability criteria likely ful-filled
- local linear instability
- leading to exponential growth and turbulence

The magneto-rotational instability

- Balbus & Hawley et multi alii (1991*ff*) analysed the MRI in discs
- local, linear instability of differentially rotating fluids
- instability criterion: negative gradient of angular velocity
- criterion does not depend on the field strength

The magneto-rotational instability

- instability of the Alfvén and slow modes
- runaway of angular-momentum transport along field lines
- exponential growth, time scale set by the rotational period



The magneto-rotational instability

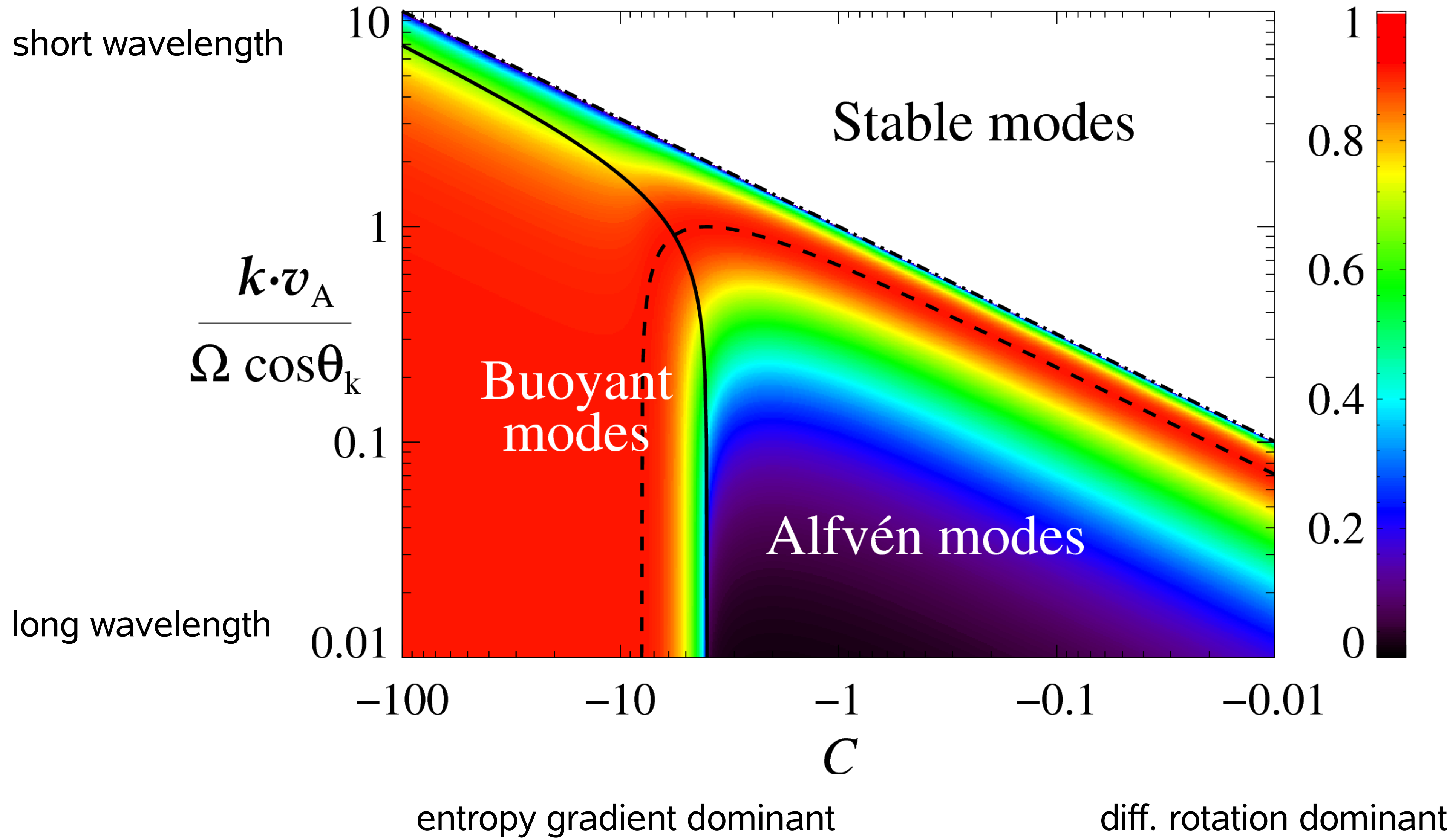
- main application: accretion discs
- Keplerian rotation unstable against MRI (but Rayleigh-stable)
- MRI-driven turbulence can explain (Shakura-Sunyaev-type) disc viscosity
- large mean Maxwell stresses transport angular-momentum outward (HD stresses lead to inward transport)
- well established, but still open problems, mostly centred around the mean stress and its dependence on the hydro-dynamic state of the disc

The MRI in supernovae

- post-collapse cores are rotating differentially (if at all) and fulfil the MRI instability criterion (Akiyama et al., 2001)
- possible saturation level estimated: > 10^{15} G
- MRI growth observed in a few simulations
- However: still only limited results
  - too strong initial fields
  - few detailed investigations
- questions:
  - how is the growth of the MRI modified?
  - what is the saturation level?

Physical and technical problems

- large difference of SN cores from accretion discs limit ap-plicability of results from accretion discs to cores
  - sub-Keplerian rotation
  - importance of pressure forces
  - thermal stratification and different microphysics
  - different global dynamics; competing with rapid global evolution
- modifications due to thermal stratification: stabilisation in radiative, destabilisation in convective regions



MRI dispersion relation based on Balbus (1995)

Physical and technical problems

- dispersion relation imposes severe resolution problem impossible to resolve properly the MRI
- relevant physical length scales
  - global scales: 10^1 km ... 10^3 km
  - scale heights of physical quantities: 1 km

Hydrodynamics

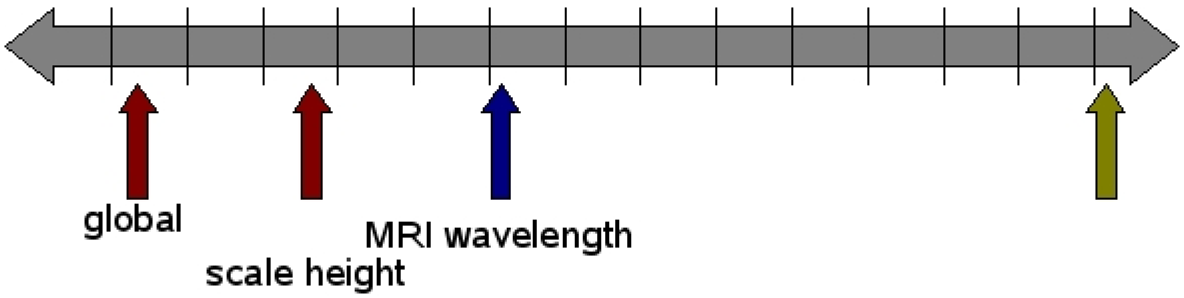
Physical and technical problems

- dispersion relation imposes severe resolution problem impossible to resolve properly the MRI
- relevant physical length scales
  - global scales: 10^1 km ... 10^3 km
  - scale heights of physical quantities: 1 km
  - MRI wavelength 1 cm ... 10 m

magnetic field

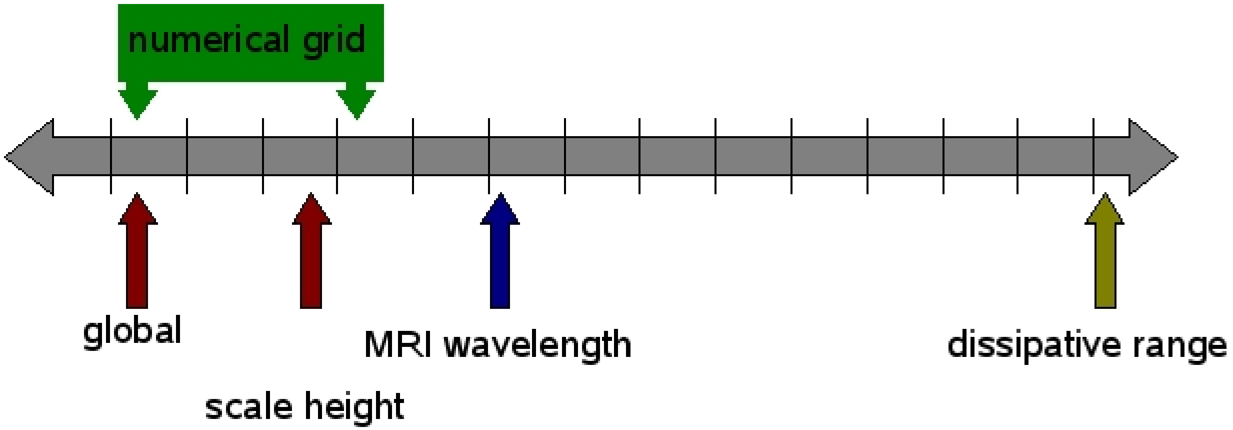
Physical and technical problems

- dispersion relation imposes severe resolution problem impossible to resolve properly the MRI
- relevant physical length scales
  - global scales: 10^1 km ... 10^3 km
  - scale heights of physical quantities: 1 km
  - MRI wavelength 1 cm ... 10 m
  - viscous and resistive scales $\ll$ 1 cm



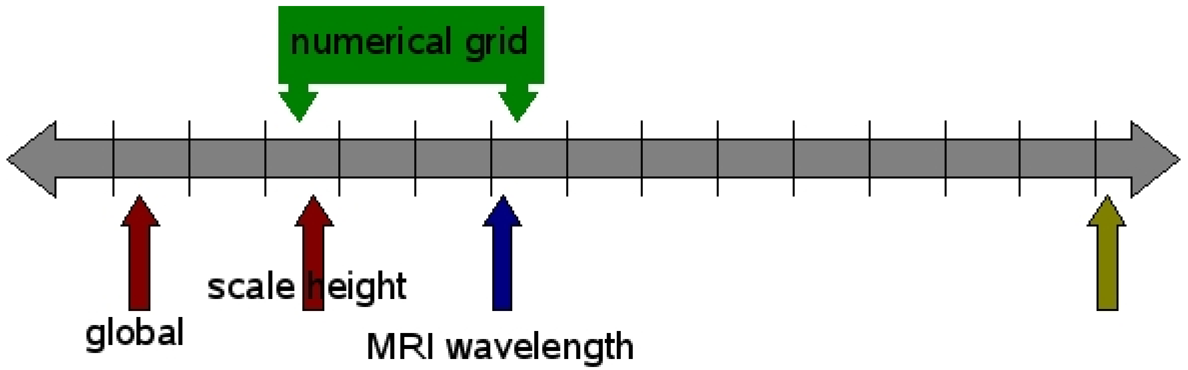
Physical and technical problems

- MRI differs strongly between 2d and 3d => 3d necessary
- numerical grid can cover a factor of ~ $10^{2..4}$ in length
- impossible to cover all scales appropriately
- where to place the numerical grid?
  - global simulations
    do not resolve the MRI properly, but follow global dynamics



Physical and technical problems

- MRI differs strongly between 2d and 3d => 3d necessary
- numerical grid can cover a factor of ~ $10^{2..4}$ in length
- impossible to cover all scales appropriately
- where to place the numerical grid?



  - local simulations
    (more or less) well resolved, but no global dynamics

Global simulations

- simulate a large region of the core at a fairly coarse resolution
  - 2d axisymmetric or 3d
  - include detailed physics
  - use detailed initial models
- MRI found; study its implications for dynamics
- BUT: MRI is underresolved
  => use stronger initial fields, relying on efficient amplification

Local simulations

- simulate a small box with a finer resolution
  - 2d, 3d Cartesian/spherical/cylindrical
  - simplified physics
  - representative initial conditions
  - choice of boundary conditions may be crucial periodic (*shearing box*), reflecting, outflow
- able to resolve the MRI for weak initial fields
- semi-global models: local with a physical scale

Local simulations

- problems: neglecting global dynamics may alter results
  - large parameter space
    - background models (rotation, stratification)
    - initial field strength and geometry
    - transport coefficients
    - grid size
    - resolution
    - boundary conditions
  - unclear dependence of the saturation on the numerical parameters, e.g., grid size, boundary treatment (Fromang et al., 2007; Umurhan & Regev, 2008)

Our setup

- small box of size 0.5, ..., 4 km and resolution 2.5, ..., 20 m in an idealised model of a post-collapse core
  - simplified EOS
  - no neutrino transport
- different initial magnetic field strength and geometry
  - initial fields correspond to $10^{10..11}$ G before collapse
  - uniform fields and fields with zero net flux through the box surface
  - shearing-disc boundary conditions in radius, otherwise periodic; radial damping may set a physical scale

The numerical method

- high-resolution finite-volume schemes
- high-order reconstruction:
  - monotonicity-preserving schemes (Suresh & Huynh 1997): MP5/7/9
  - the weighted essentially non-oscillatory scheme of Levy et al. (2002): WENO4
- approximate Riemann solvers based on the MUSTA method (Toro & Titarev 2006)
  - a predictor-corrector scheme minimising the numerical viscosity of approximate solvers

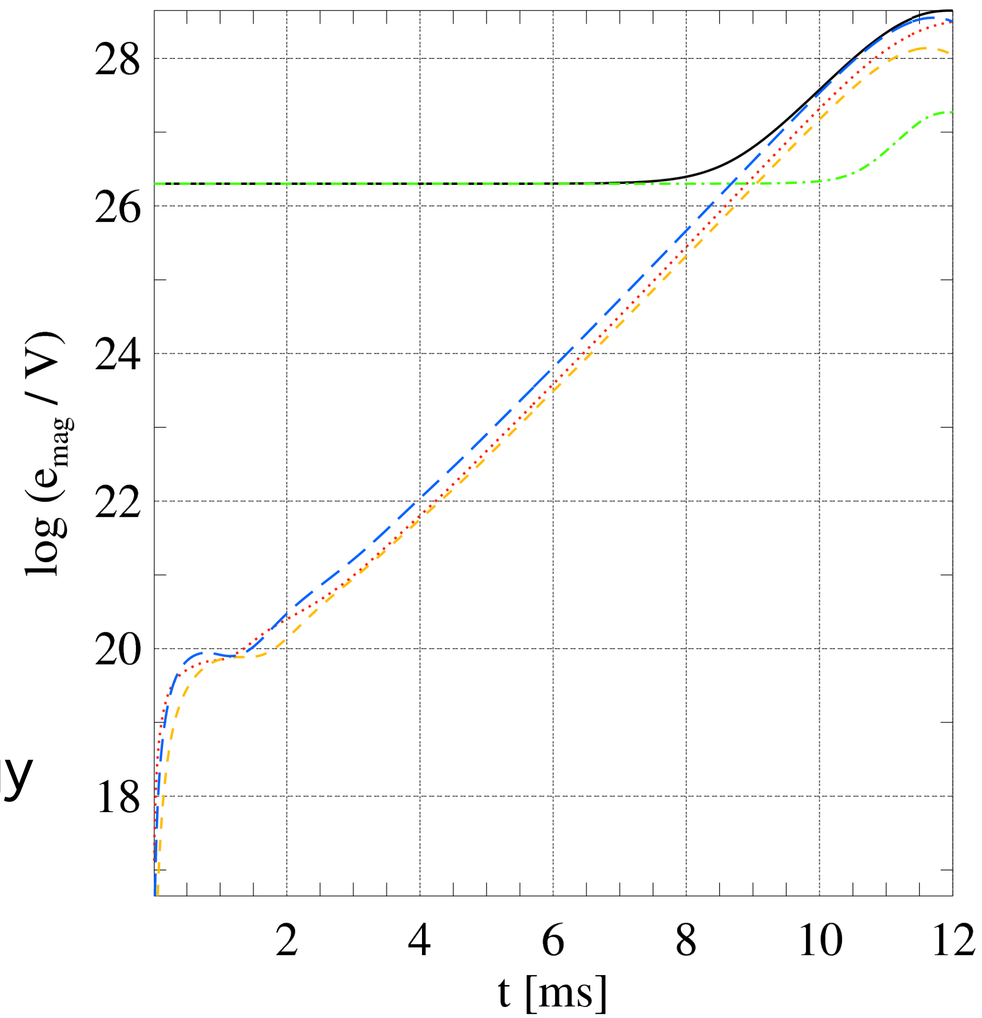
The numerical method

- divergence of the magnetic field:
  - constraint-transport scheme (Evans & Hawley 1988)
- coupling of magnetic field and hydro variables:
  - involves multiple mapping between staggered grids
  - upwind CT scheme (cf. Londrillo & Del Zanna, 2000)
- tested with different standard tests
- time stepping: Runge-Kutta of 2nd or 3rd order
- overall accuracy of the method: approx. 3rd -- 4th order
- Fortran 90, OpenMP and MPI parallel

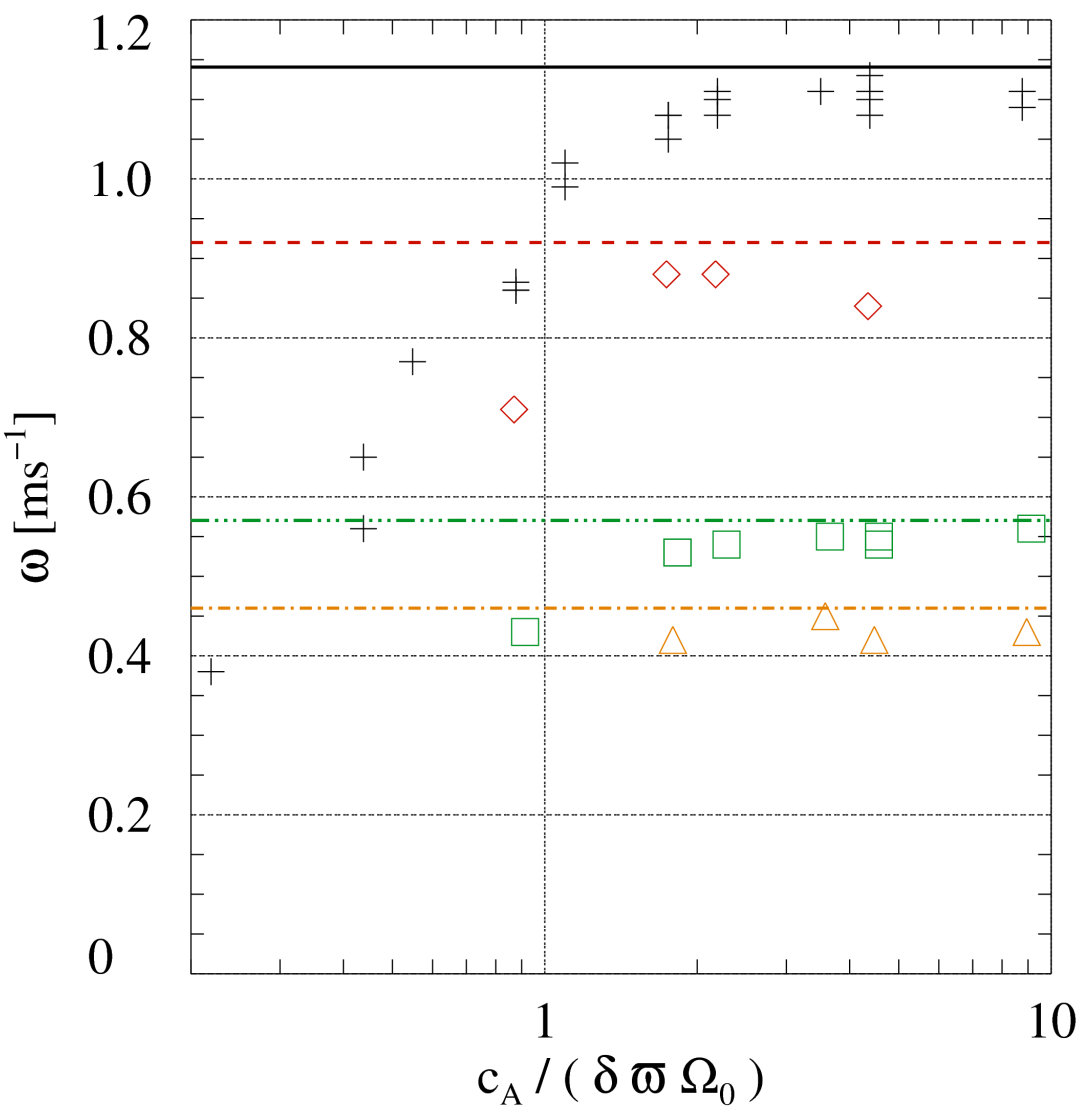
Results: growth rates

- We find growth rates in agreement with linear analysis, if sufficiently well resolved

early evolution of the magnetic energy in a local MRI simulation



Results: growth rates



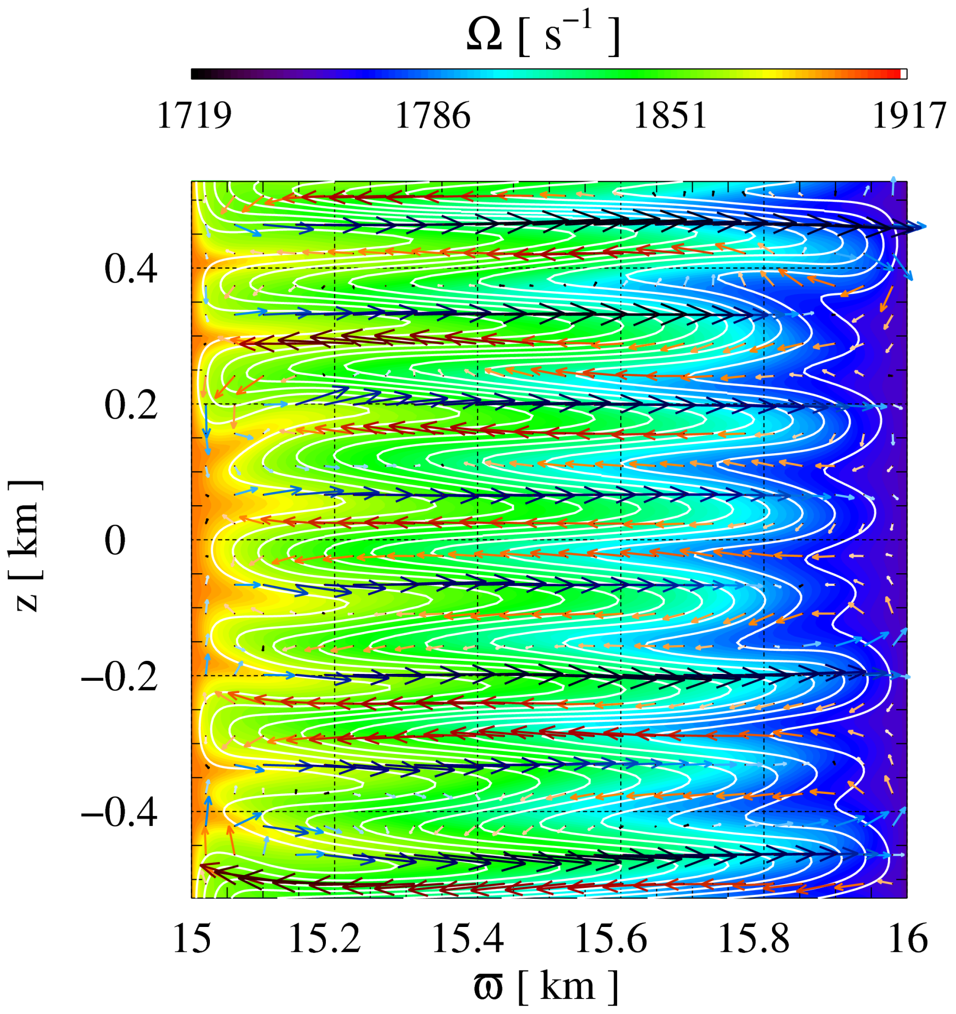
comparison of numerical (symbols) and theoretical (lines) growth rates
for different initial rotational profiles, different resolutions, and different initial fields

Once MRI wavelength is resolved, we find convergence of growth rates

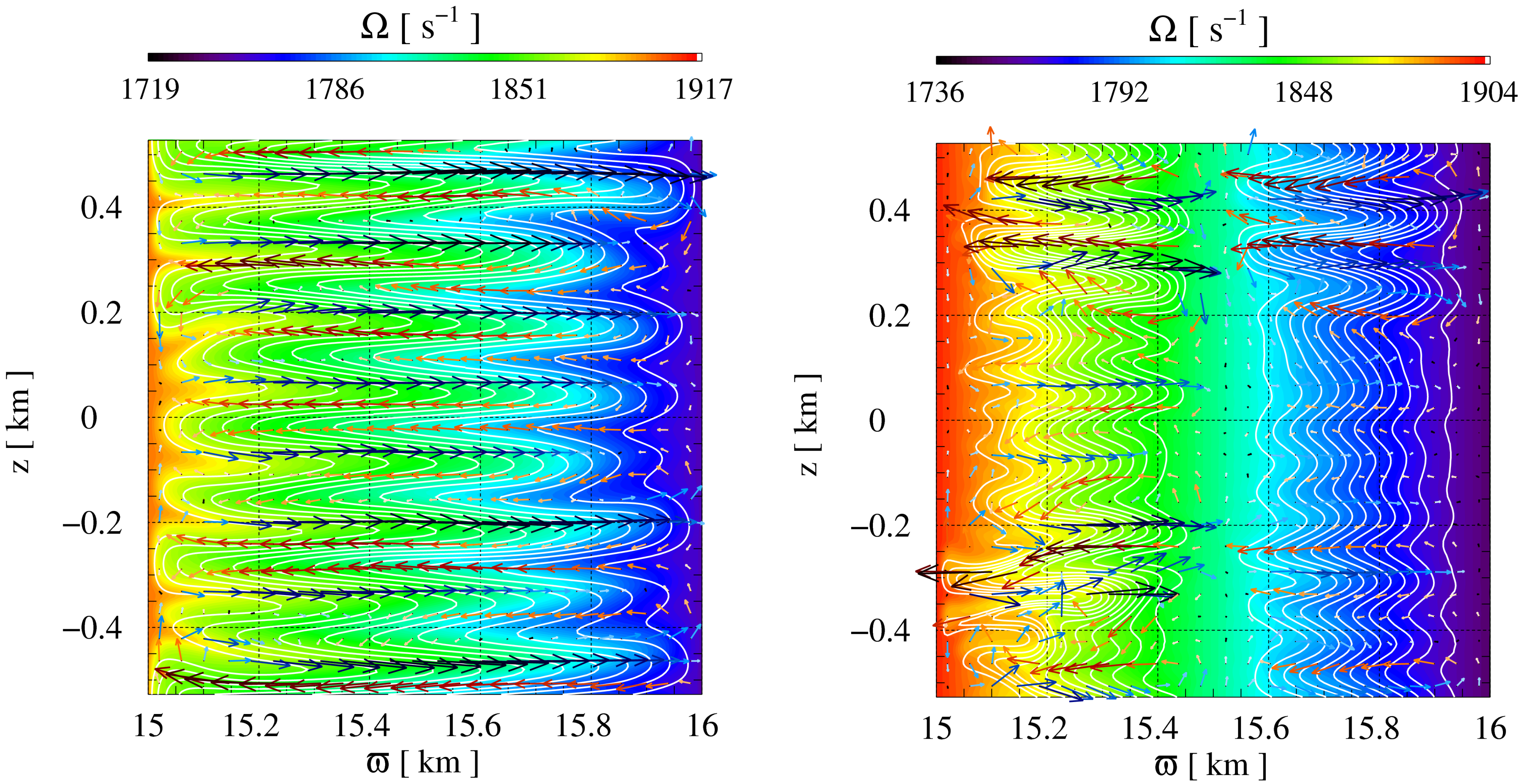
Results: channel modes

- characteristic flow pattern: channel modes
- pairs of upflows and downflows
- predominantely radial field created in the flows
- width set by the initial field
- exact solutions of full MHD equations, but may be unstable to *parasitic* instabilities (Goodman & Xu, 1994)

Results: channel modes



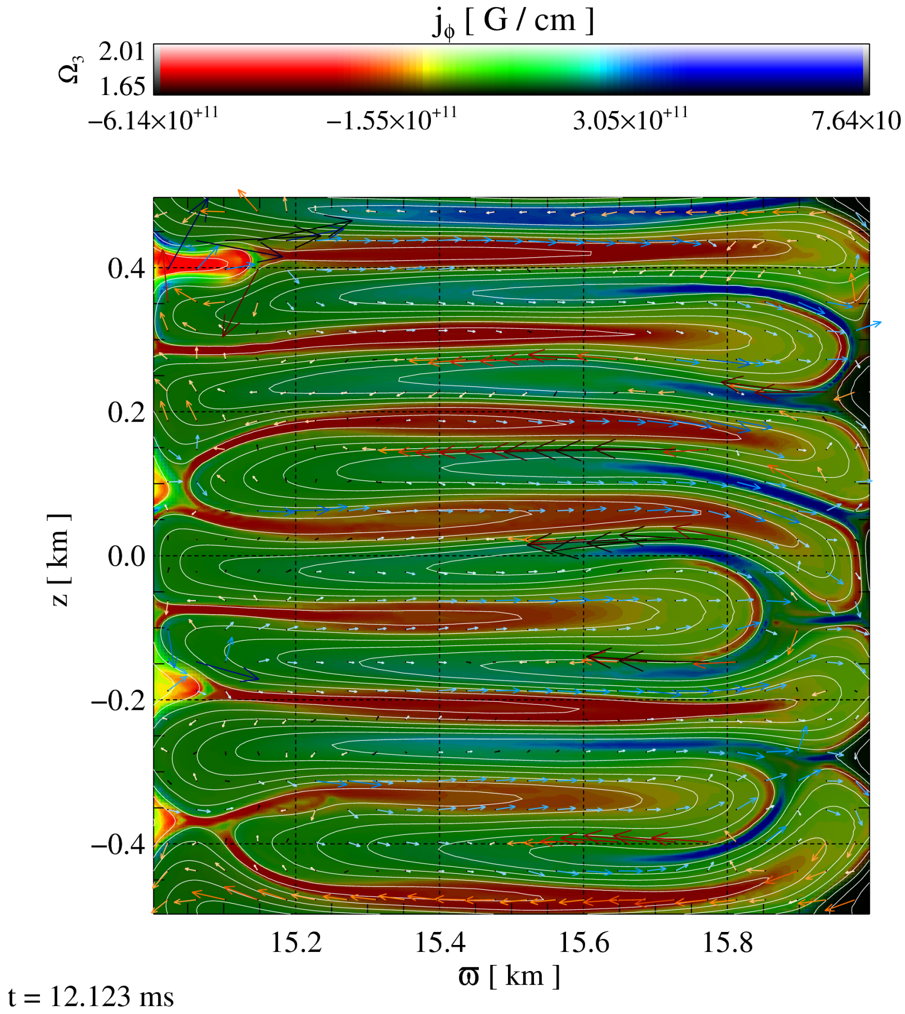
Results: channel modes

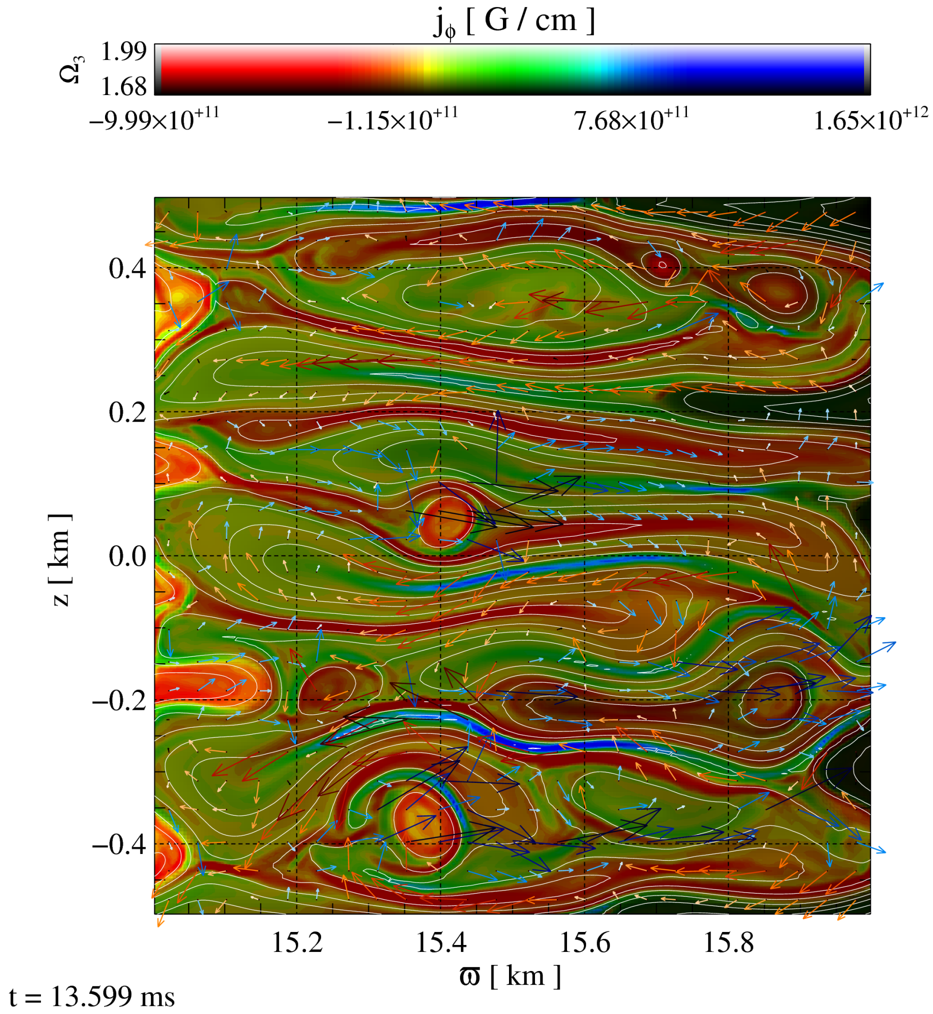


Results: channel modes

- channels may be disrupted
  - reconnection of field lines in, e.g., tearing modes occurring in current sheets
  - growth of these modes depends on thickness and length of the current sheet:
    - stable for short sheets
    - fast growth for long sheets
- => saturation and turbulence

Results: channel modes

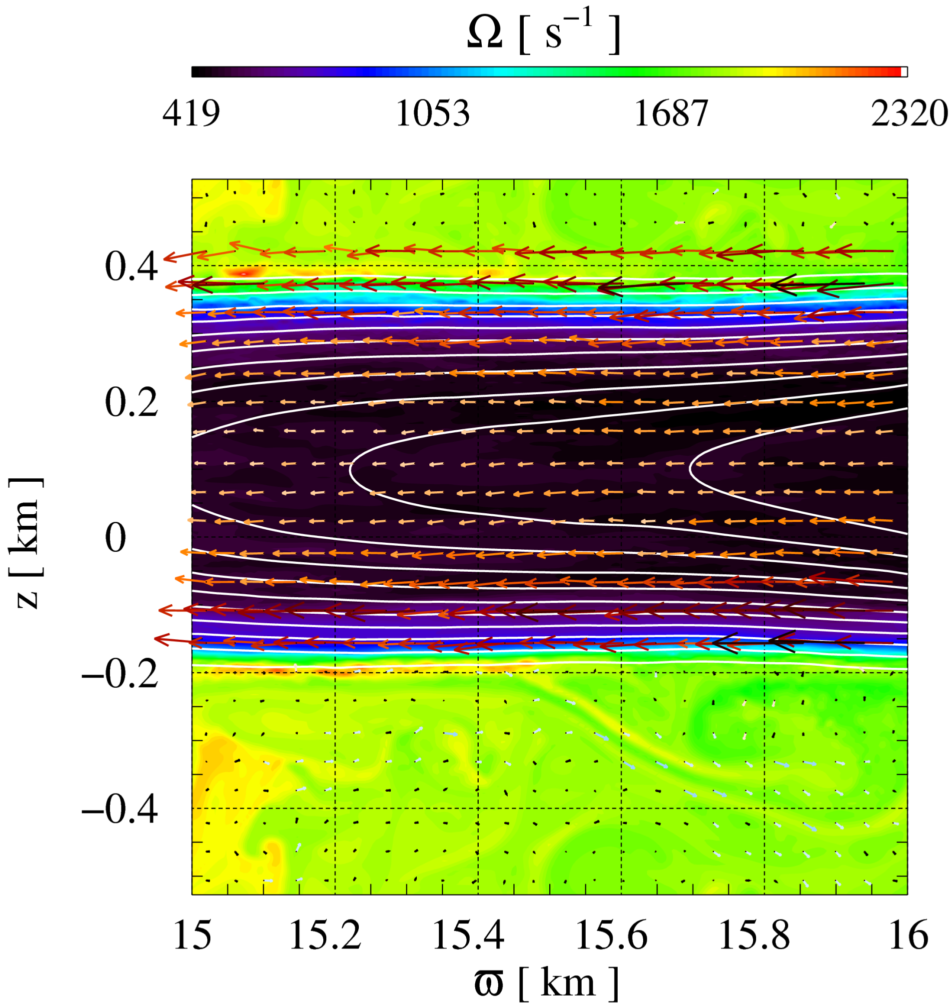


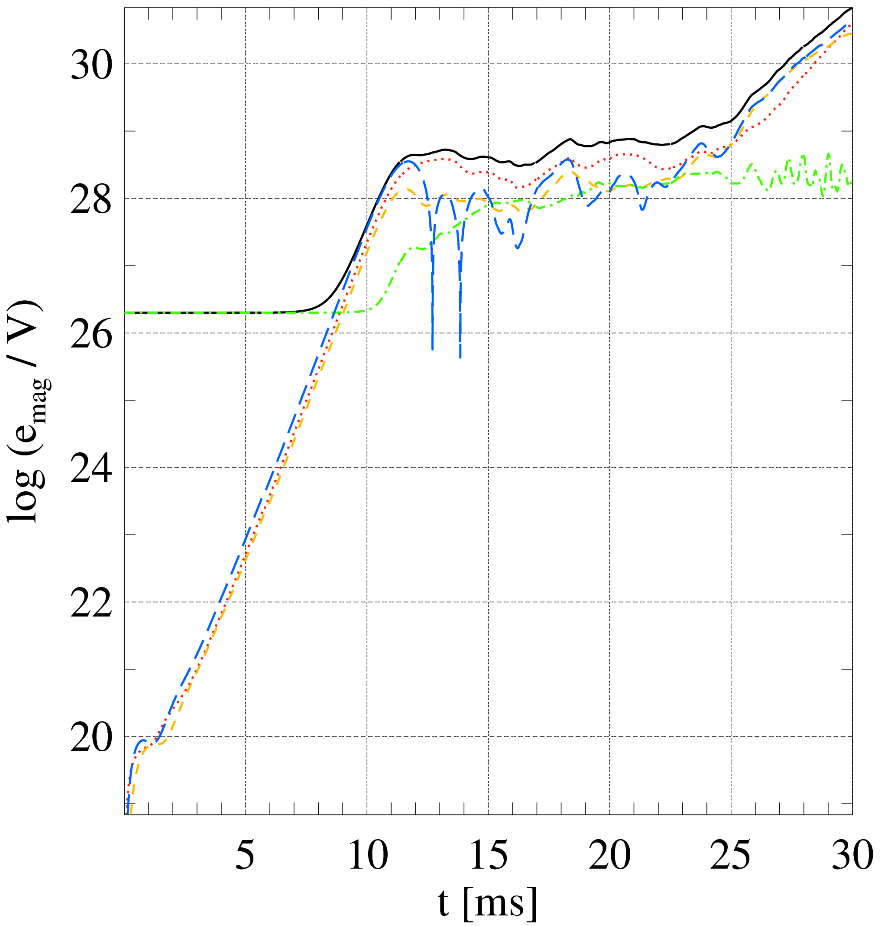


Results: channel modes

- channels may be disrupted
  - reconnection of field lines in, e.g., tearing modes occur-ing in current sheets
  - growth of these modes depends on thickness and length of the current sheet:
    - stable for short sheets
    - fast growth for long sheets
- => saturation and turbulence
- reorganisation of channels possible by merging of adjacent flows

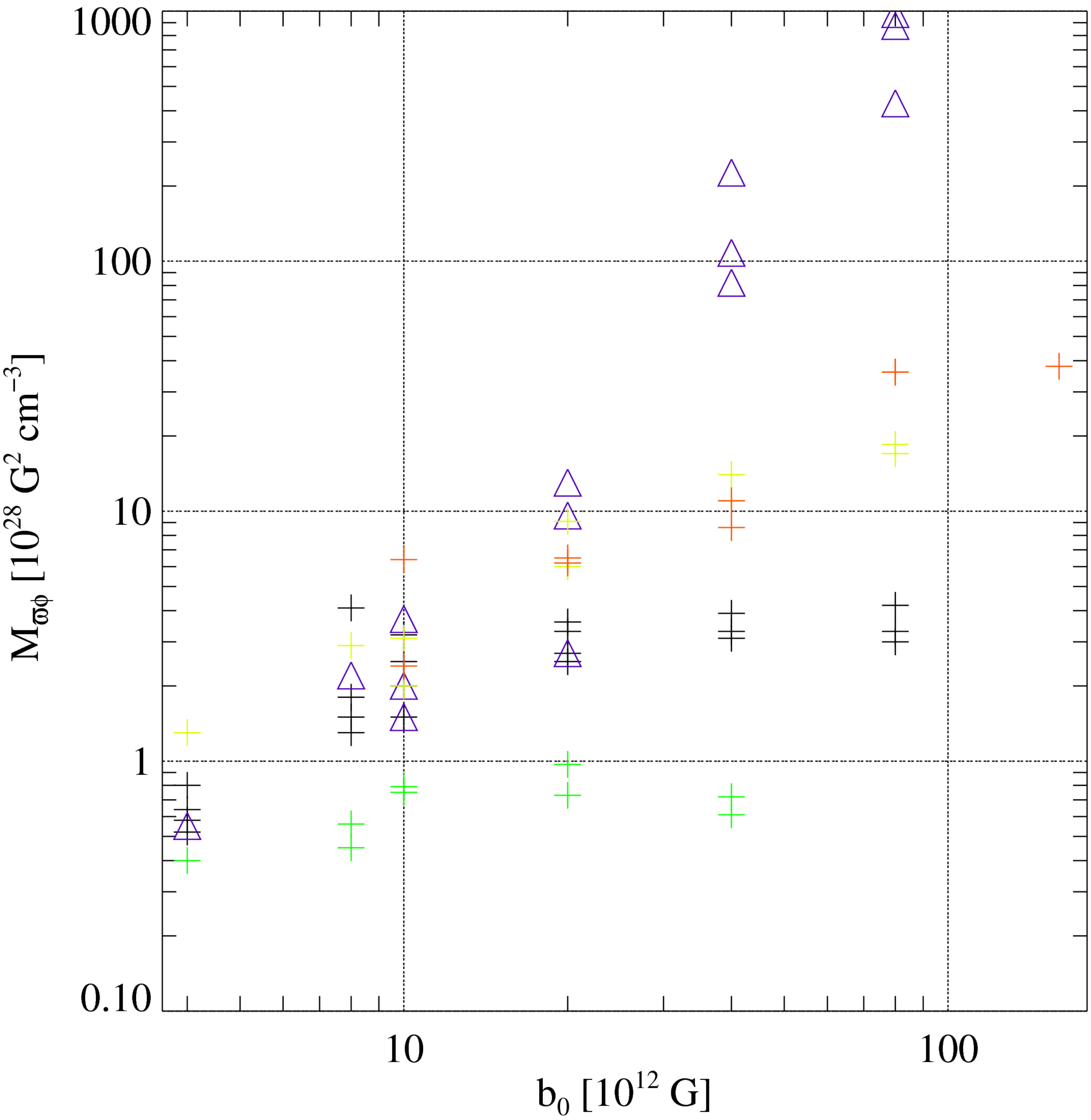
Results: channel modes





Results: channel modes

- saturation depends an various factors;
  as a first step, we concentrate on **numerical** ones:
  - box size and aspect ratio
  - boundary conditions
  - resolution
- for given initial field (channel thickness), the MRI has a lim-it set by the stability of associated current sheets
- earlier saturation may occur when the boundaries of the system interfer
- saturation can be prevented in periodic boxes

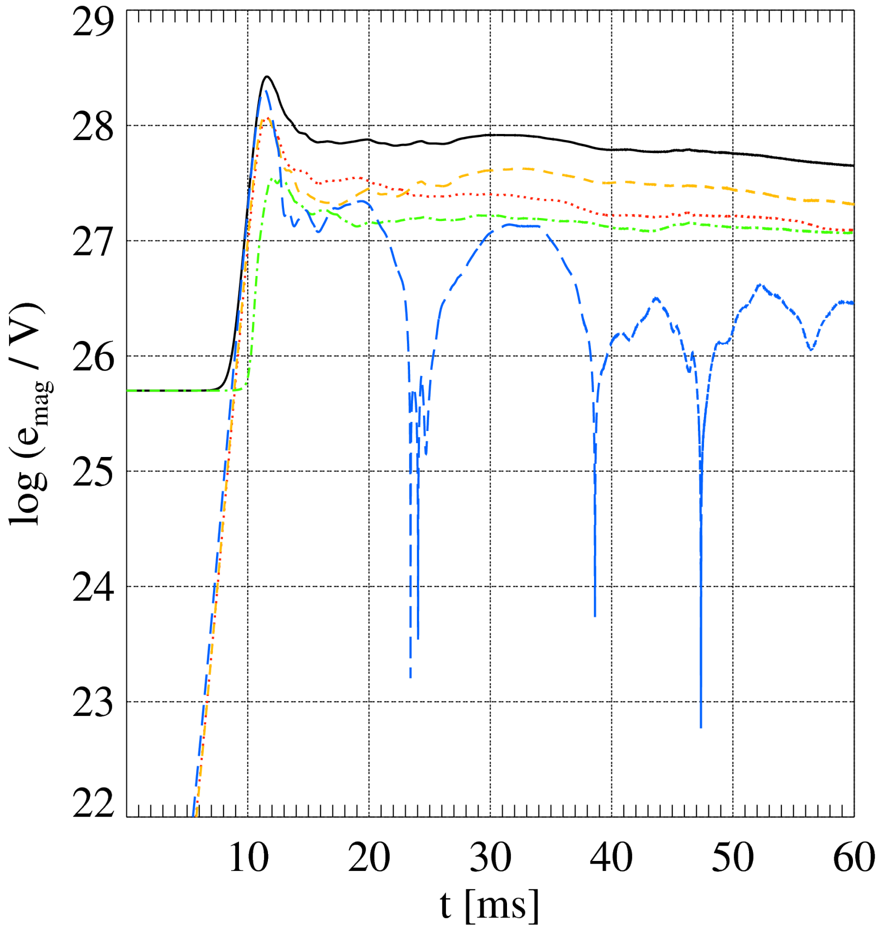


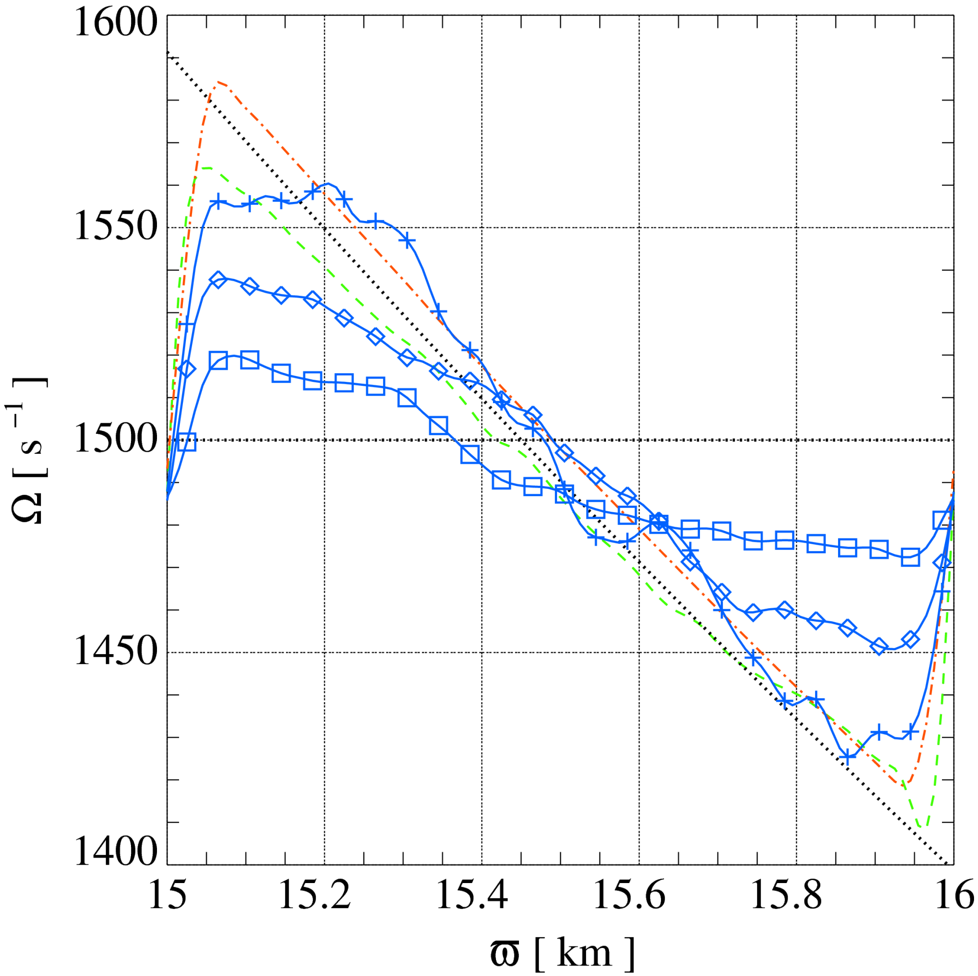
4. VII. 2008

Results: entropy gradients

- MRI is suppressed in convectively stable regions
- convectively unstable regions:
  - “standard” MRI modes for fast differential rotation
  - magnetoconvection for dominating entropy gradient
  - magneto-bouyant modes for fast rigid rotation
- distinguishing parameters are rotational and bouyancy fre-quencies

Results: entropy gradients

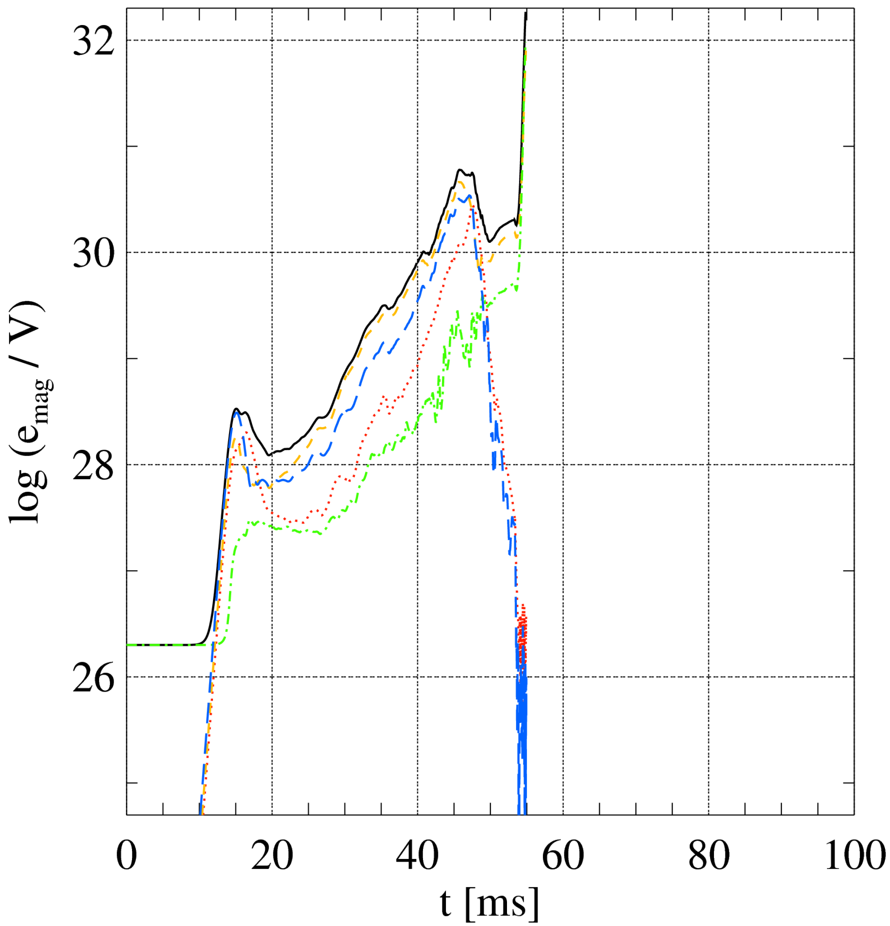


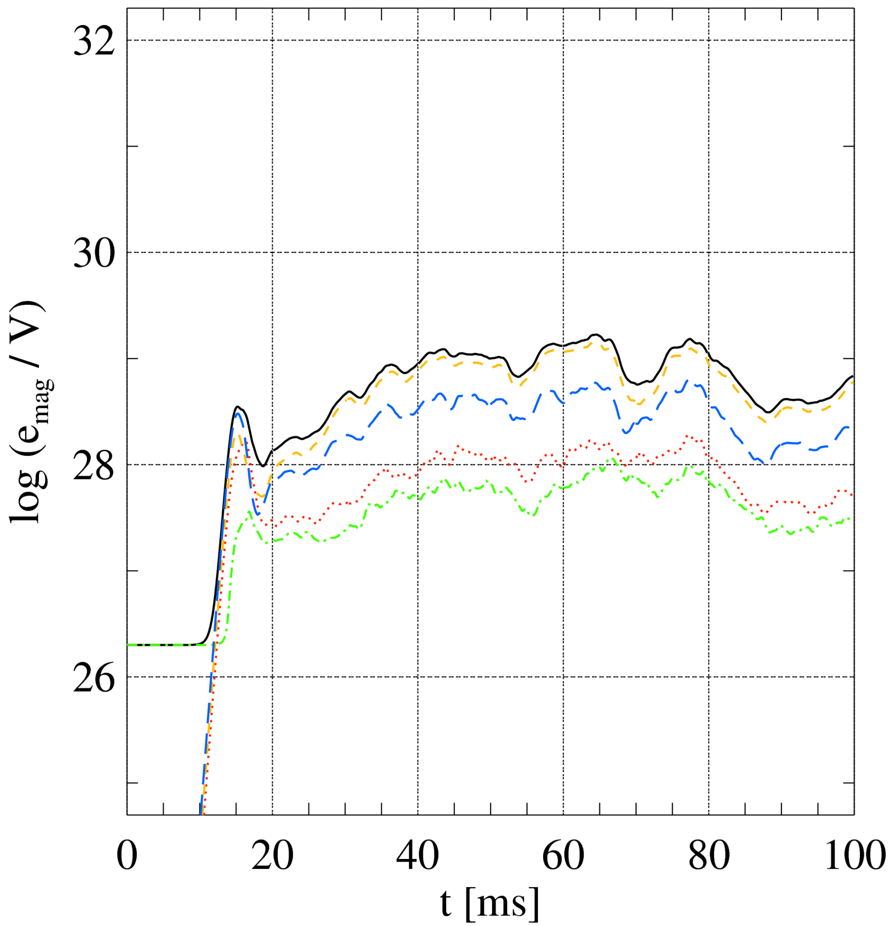


Results: axisymmetry vs. 3d

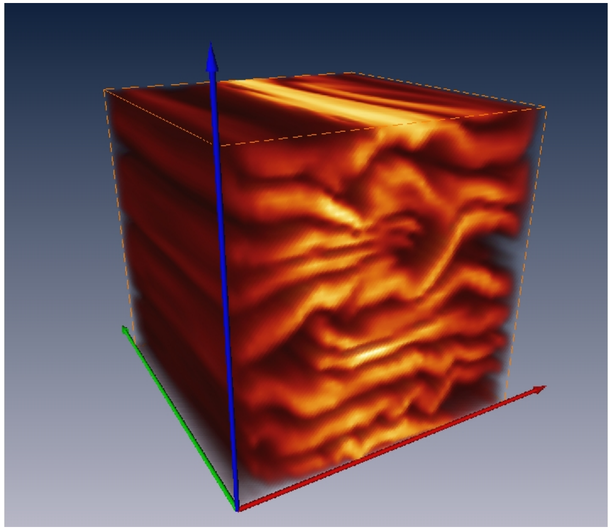
- several models
- 3d effects important for, e.g., dynamo
- further numerical parameter: aspect ratio of the box
  - “cubic” boxes: same as axisymmetry
  - “wide” boxes: turbulence develops

Results: axisymmetry vs. 3d

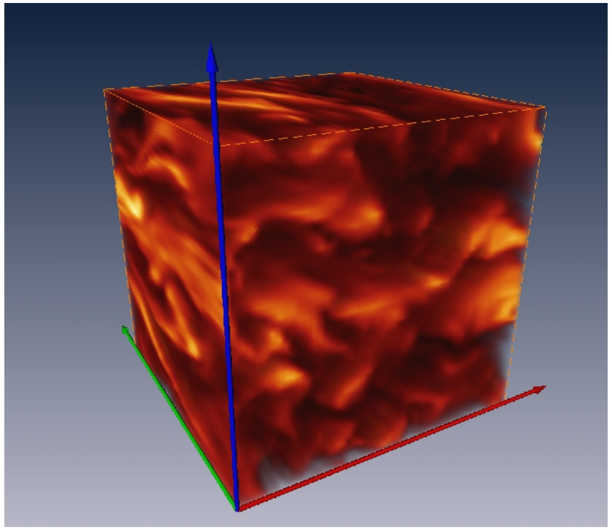




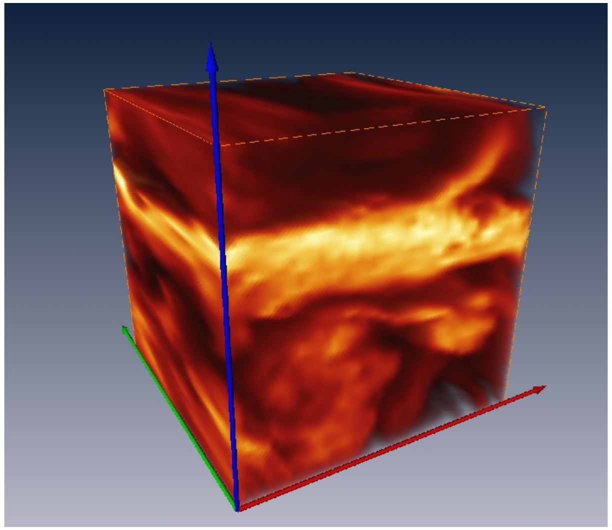
Magnetic field strength of a cubic 3d model



t = 11 ms: channel flows grow

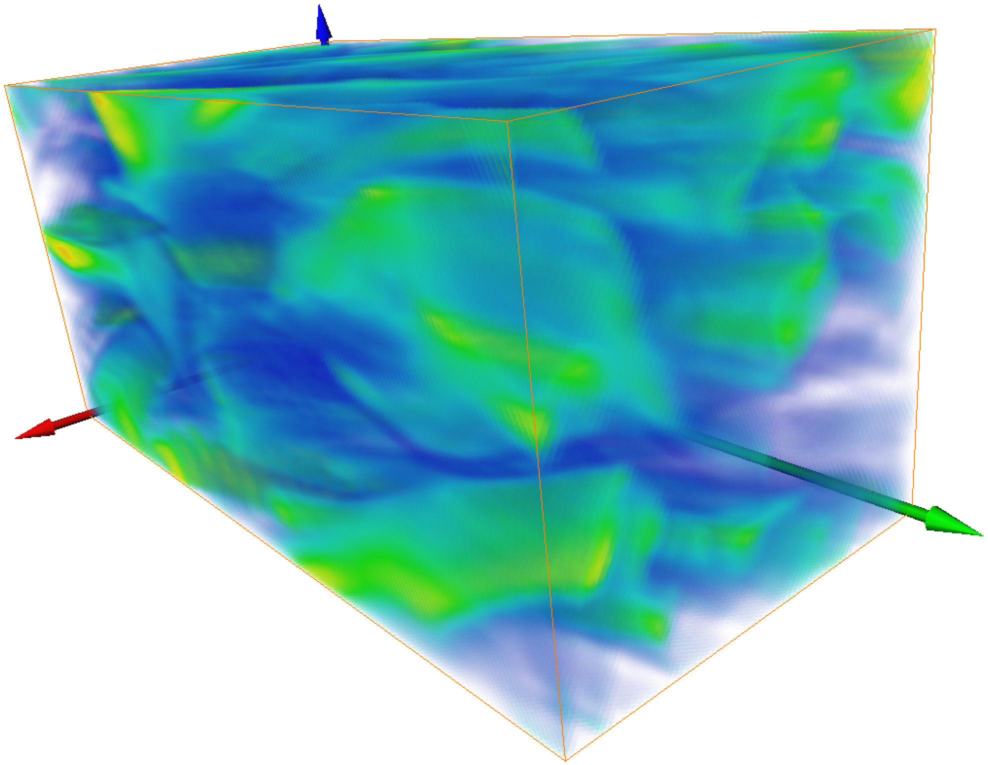


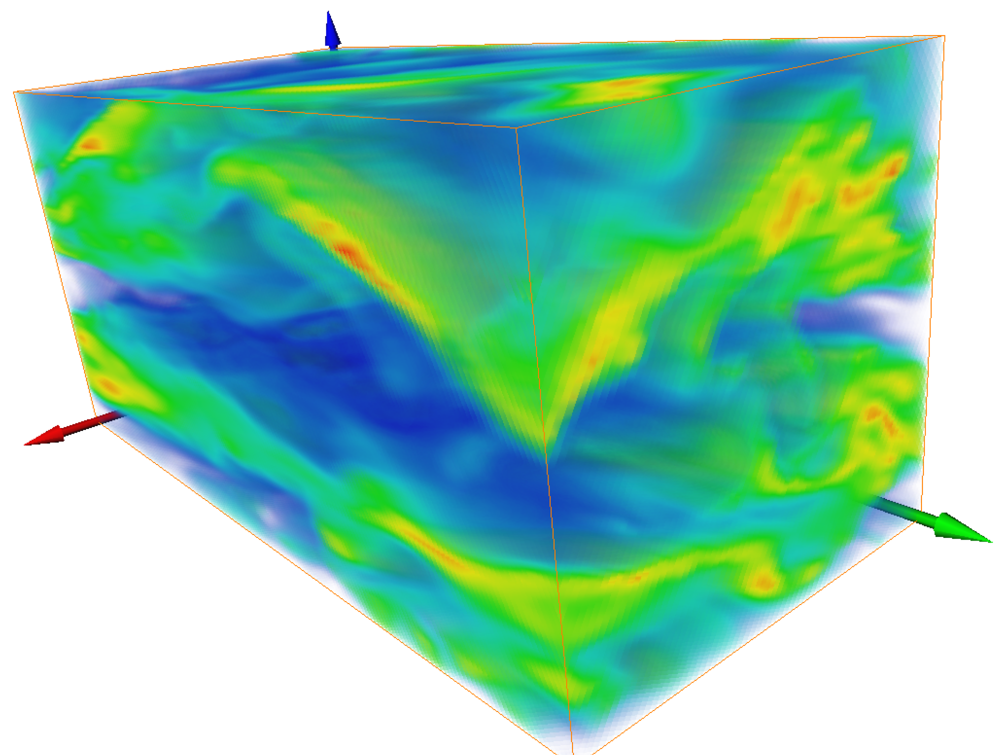
t = 35 ms: channels are disrupted



t = 52 ms: reappearance of coherent flows

Magnetic field strength of a wide 3d model





coherent flow patterns
may develop, but are
less prominent

Conclusions

- treatment of MRI hindered by resolution requirements
- combination of global and local simulations suggested
- a set of local simulations performed
  - confirm efficient amplification of magnetic stresses by the MRI, turbulence, and angular-momentum transport
  - still preliminary analysis, leaving open several issues
    - further numerical factors
    - dependence on physics, e.g., the initial conditions
    - effects of neglected physics