Magnetic field amplification by the stationary accretion shock instability

Christian Y. Cardall

Oak Ridge National Laboratory

Physics Division

University of Tennessee, Knoxville

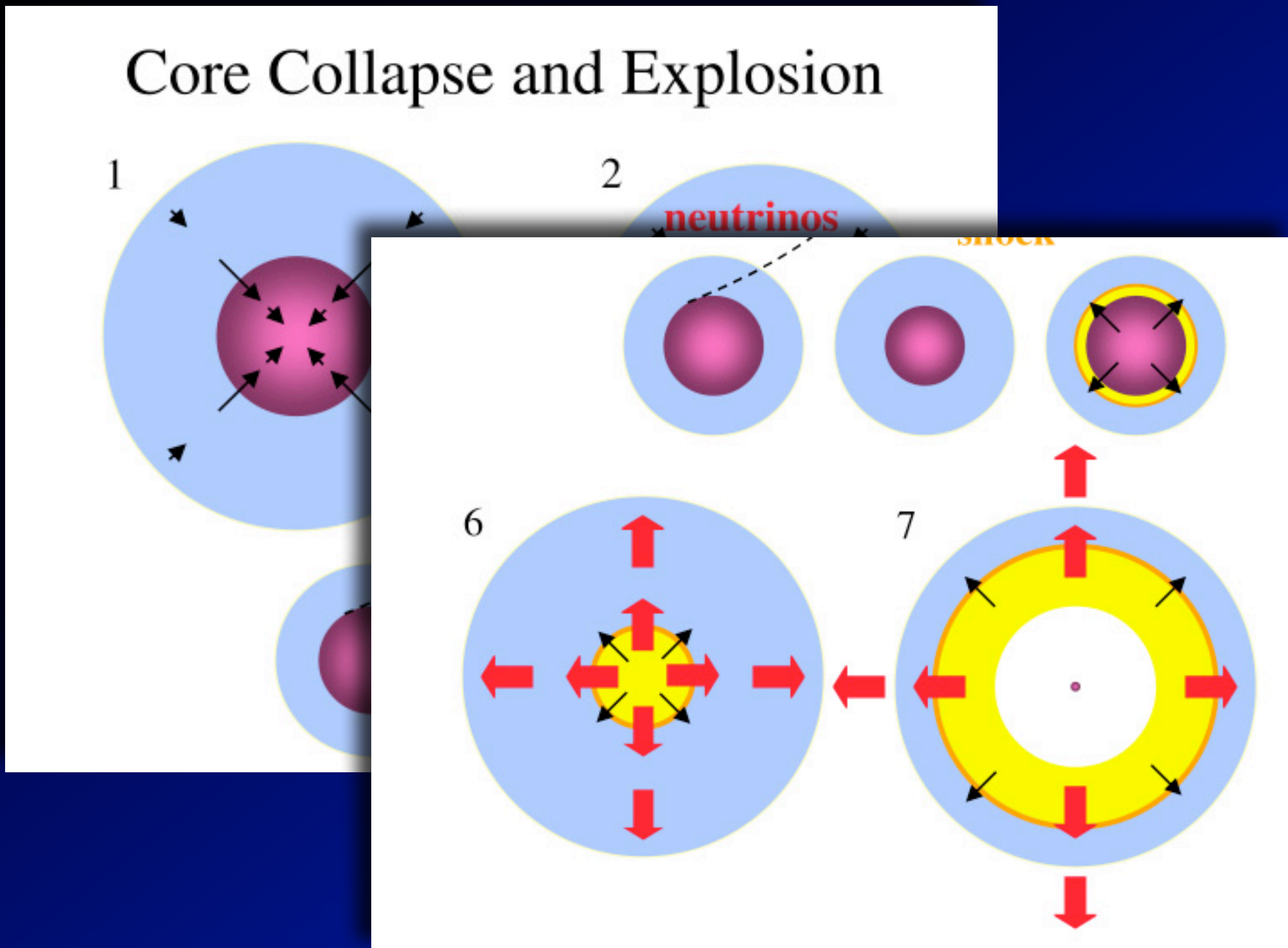
Department of Physics and Astronomy

Eirik Endeve

Reuben Budiardja

Anthony Mezzacappa

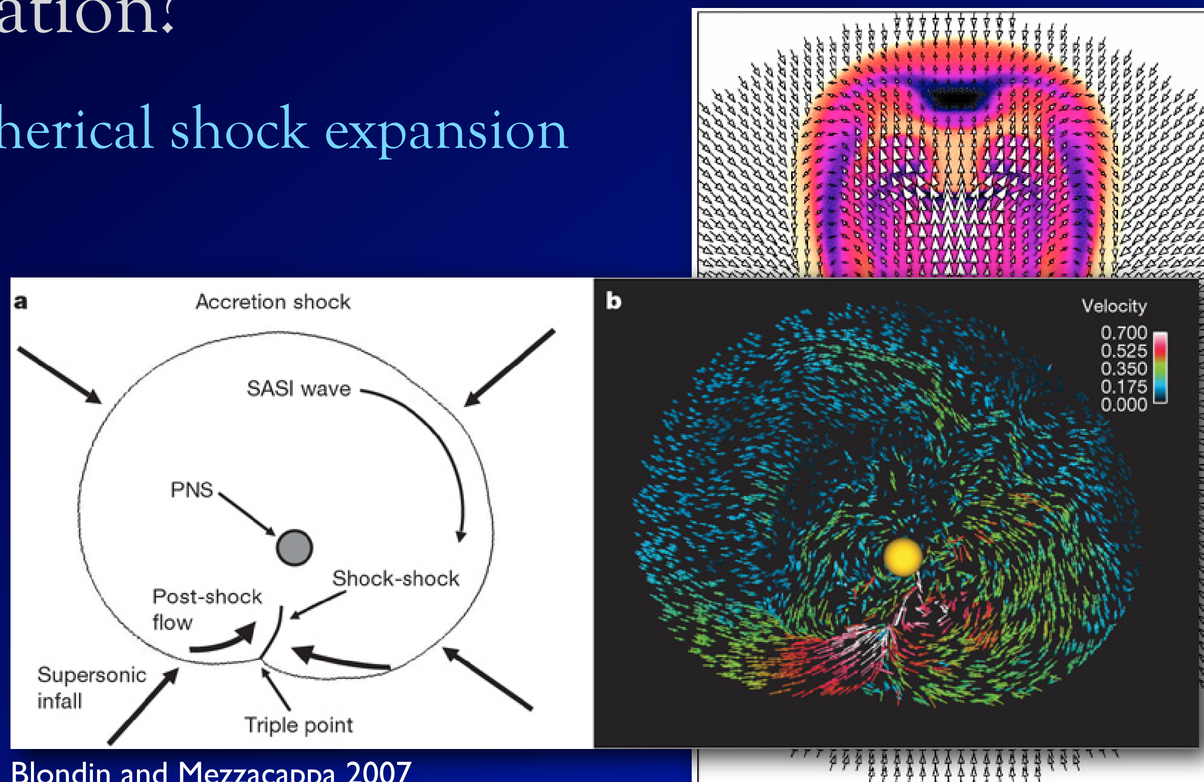
Introduction


Core Collapse and Explosion
1
2
neutrinos
6
7

What can the stationary accretion shock instability (SASI) accomplish, even in the absence of initial rotation?

Generate aspherical shock expansion

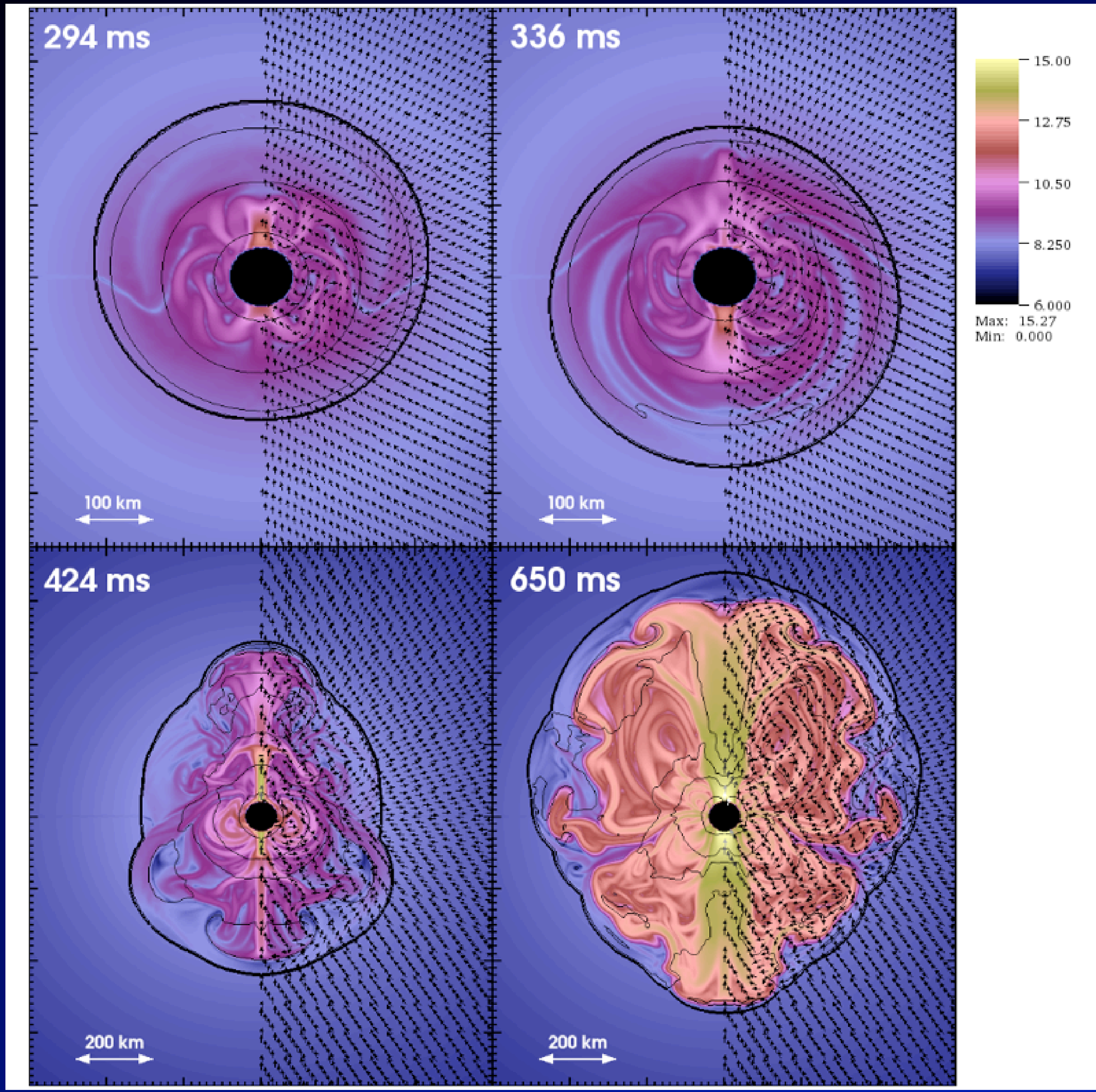
Account for pulsar spin



Blondin and Mezzacappa 2007

Blondin, Mezzacappa, and DeMarino (2003)

Amplify the magnetic field to dynamically significant strength, at least in axisymmetry


294 ms
336 ms
424 ms
650 ms
100 km
100 km
200 km
200 km
15.00
12.75
10.50
8.250
6.000
Max: 15.27
Min: 0.000

The model

Initial steady state toy model

Shock placed at $R_{\rm sh} = 200$ km

Accretion rate of $0.36\ M_\odot\ {\rm s}^{-1}$

Outside the shock

Supersonic: Mach number 300

Free fall: $u = \sqrt{\dfrac{2GM}{r}}$, with $M = 1.2\ M_\odot$

Inside the shock

Conditions immediately inside the shock given by Rankine-Hugoniot jump conditions

Structure given by the Bernoulli equation

Initial steady state toy model

- Polytropic equation of state with $\gamma = 4/3$
- Adiabatic evolution
- Inner boundary conditions at $R_{\rm PNS} = 40$ km
  - Density: $\rho \propto r^{-3}$ as found in the adiabatic "settling solution," but amplitude allowed to float
  - Pressure: $p \propto r^{-4}$ as found in the adiabatic "settling solution," but amplitude allowed to float
  - Velocity: fixed to analytic solution, including zero tangential velocity

Initial steady state toy model

Magnetic field

- "Split monopole": purely radial with opposite directions in northern and southern hemispheres, and magnitude $\propto r^{-2}$
- Magnitude of 10^{10} G at $R_{\rm PNS}$

Inner boundary conditions

- Parallel components just inside a "cutout" face set equal to parallel components just outside
- Component perpendicular to a cutout face lives on that face and is allowed to evolve

Numerical scheme

Ideal magnetohydrodynamics (MHD) (zero viscosity and resistivity, except for numerical dissipation)

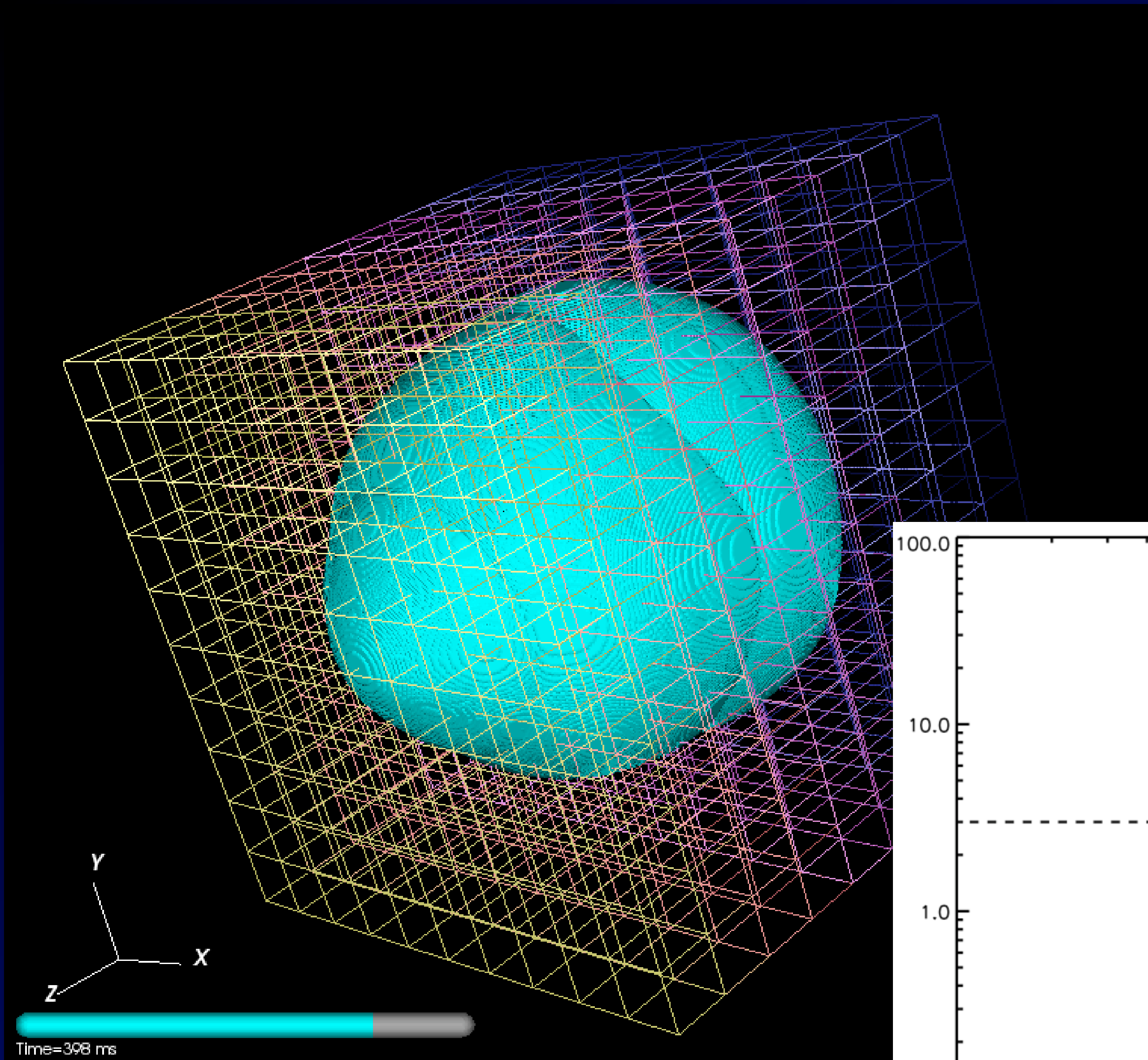
Time: semi-discrete formulation evolved with second order Runge-Kutta scheme

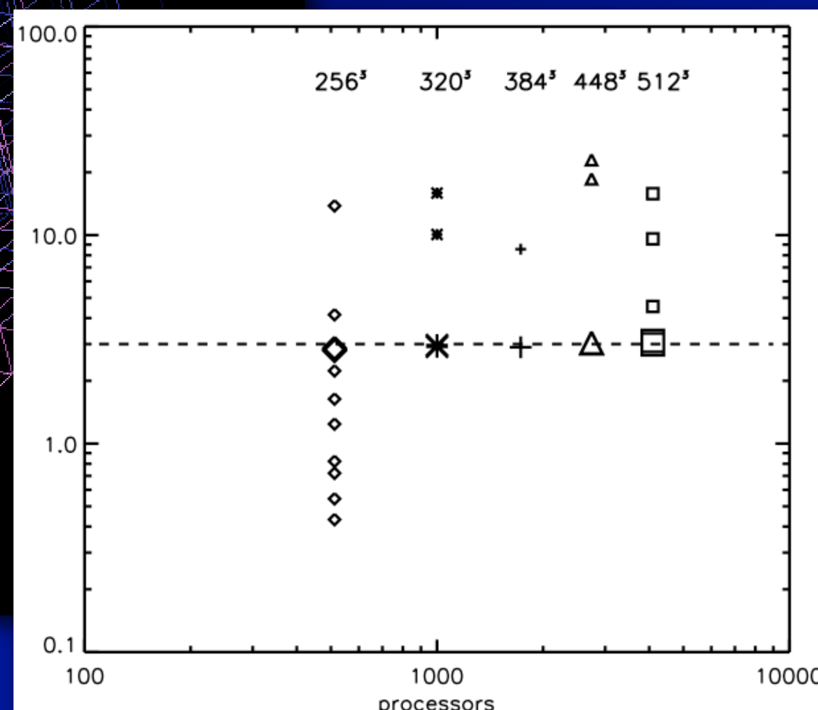
Space: central-upwind (finite volume) scheme, second order with generalized minmod slope limiter

Divergence-free evolution of Faraday's law via constrained transport scheme

HLL solvers for fluxes on zone faces and electric field on zone edges

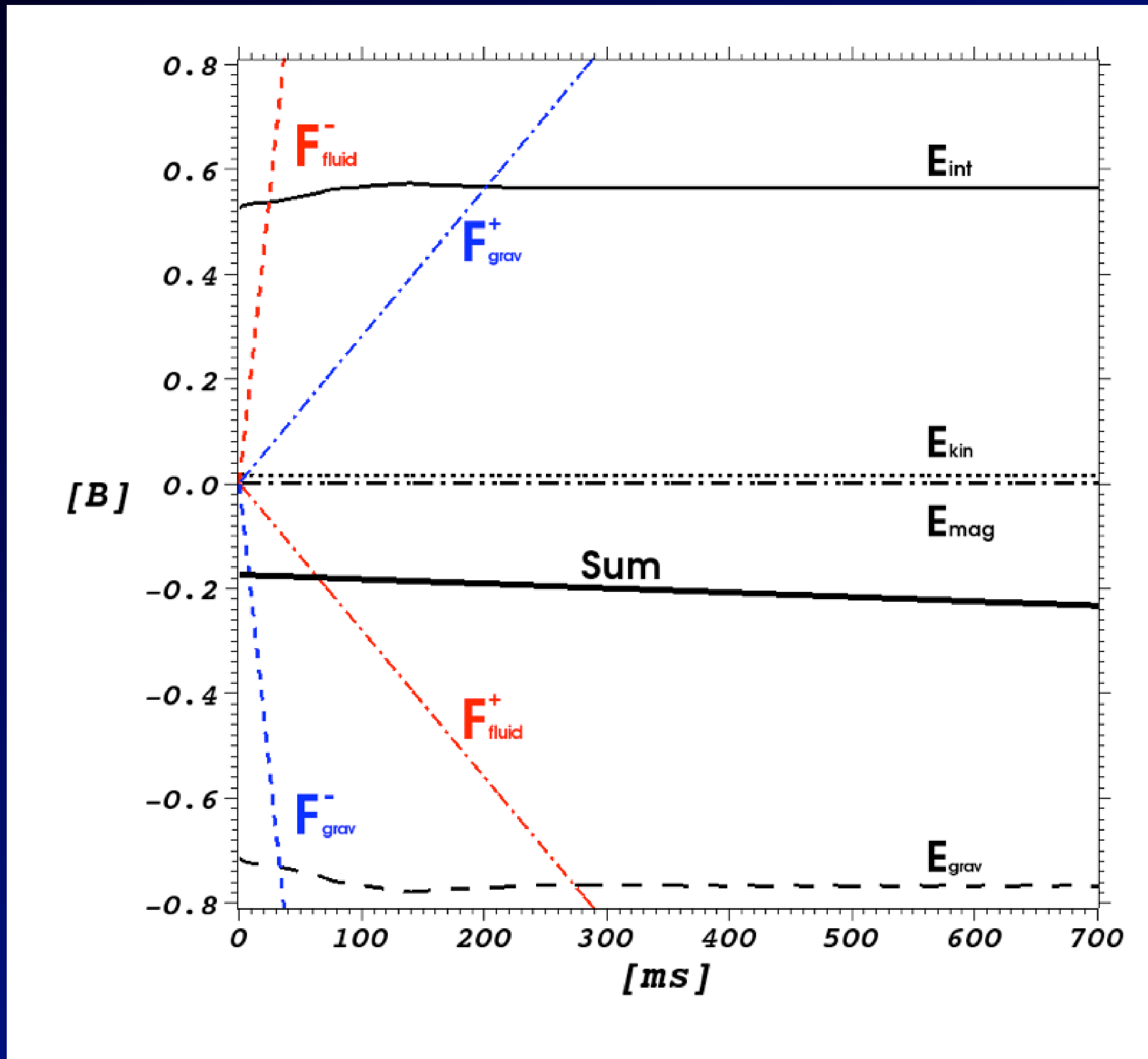
Cartoon method (axisymmetry with Cartesian coordinates)


Y
X
Z
Time=398 ms

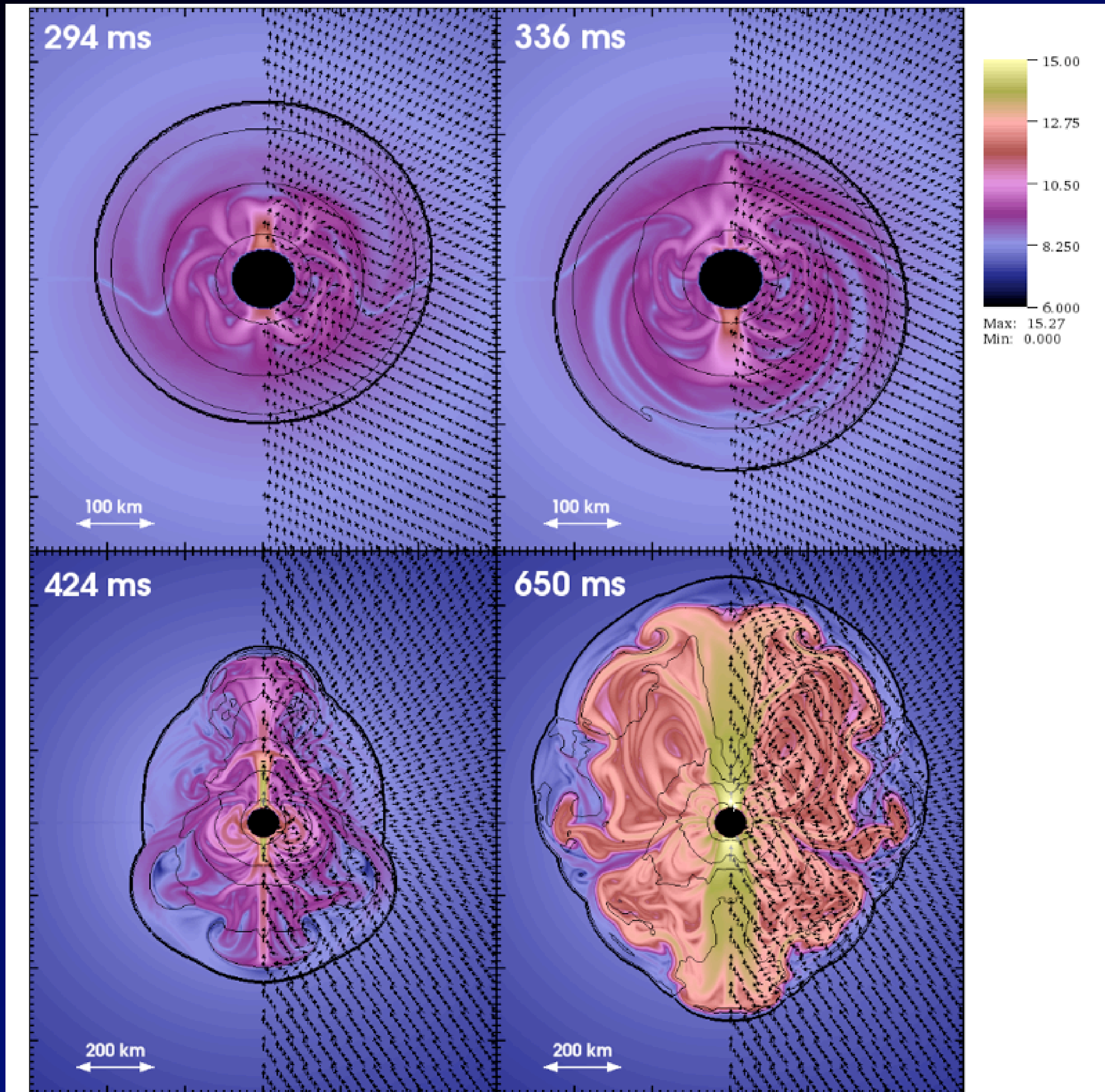

100.0
10.0
1.0
0.1
256³
320³
384³
448³
512³
100
1000
10000
processors

Energy of an unperturbed 2D magnetic model

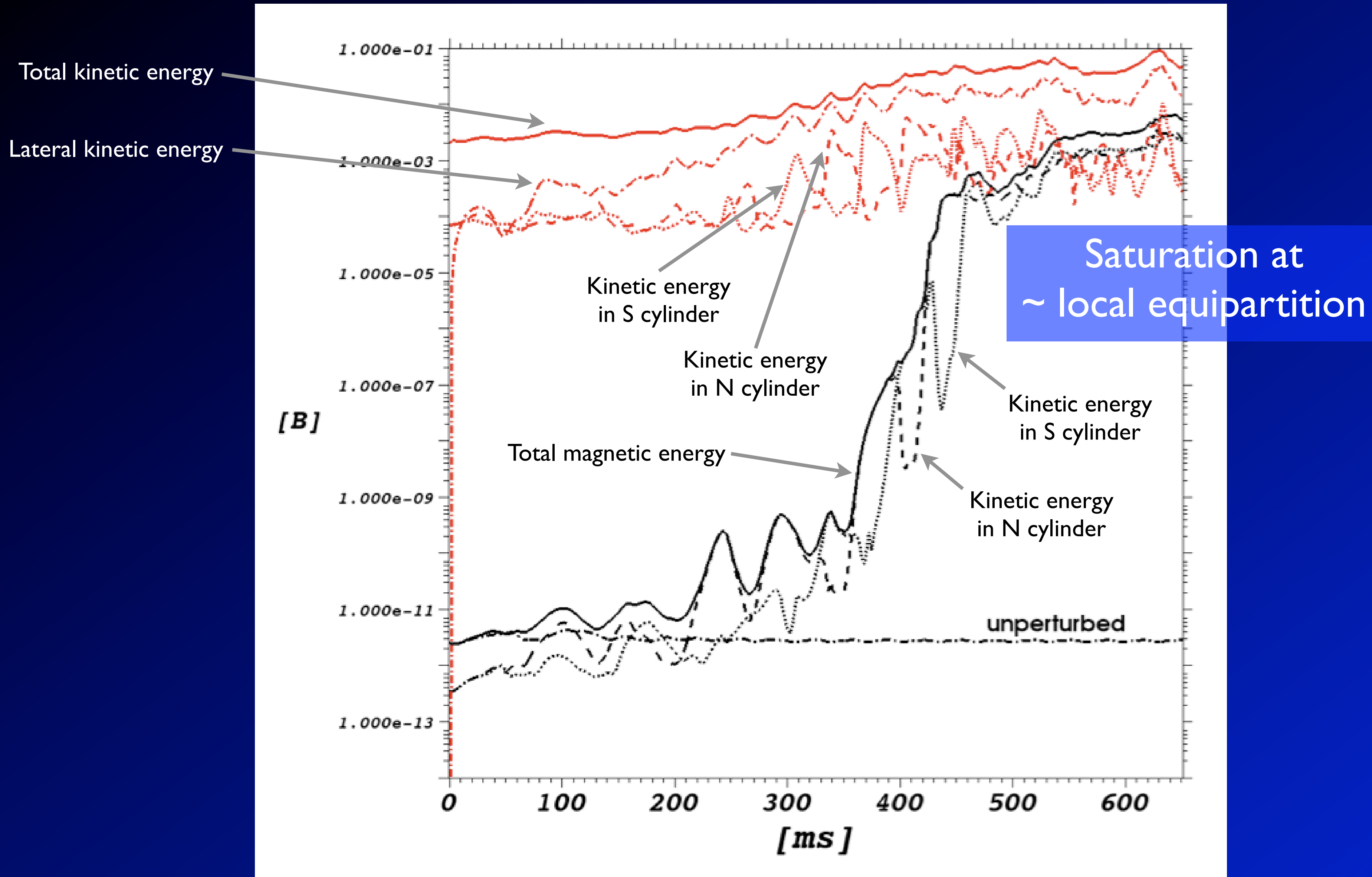
$(1 \text{ B} = 10^{51} \text{ erg})$



Growth of the magnetic field


294 ms
336 ms
424 ms
650 ms
100 km
100 km
200 km
200 km
15.00
12.75
10.50
8.250
6.000
Max: 15.27
Min: 0.000

Energies over time inside the shock



A brief physical explanation

A SASI-induced lateral flow advects radial magnetic field lines towards the symmetry axis, resulting in amplification by compression

Constrained by axisymmetry, and without an (initially) symmetry-breaking initiation of a toroidal flow, the fluid has no choice but to turn parallel to the symmetry axis

But a fluid flow parallel to the magnetic field cannot advect the field, so it remains “deposited” at the site of impact

The flow will eventually turn back away from the axis; but to the extent the field is weaker at this position, less magnetic field is advected away than was originally delivered

A brief mathematical explanation

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

where for vanishing resistivity

$$\mathbf{E} = -\mathbf{u} \times \mathbf{B}$$

Initially $\mathbf{u}$ and $\mathbf{B}$ are both radial, in accordance with stationarity

In axisymmetry and without rotation, any SASI-induced lateral flows give rise to a toroidal $\mathbf{E}$, which manifestly has a curl parallel to the symmetry axis

A brief mathematical explanation

$$\frac{\partial B_z}{\partial t} = -\frac{E_\phi}{r_\perp} - \cancel{\frac{\partial E_\phi}{d r_\perp}}$$

where

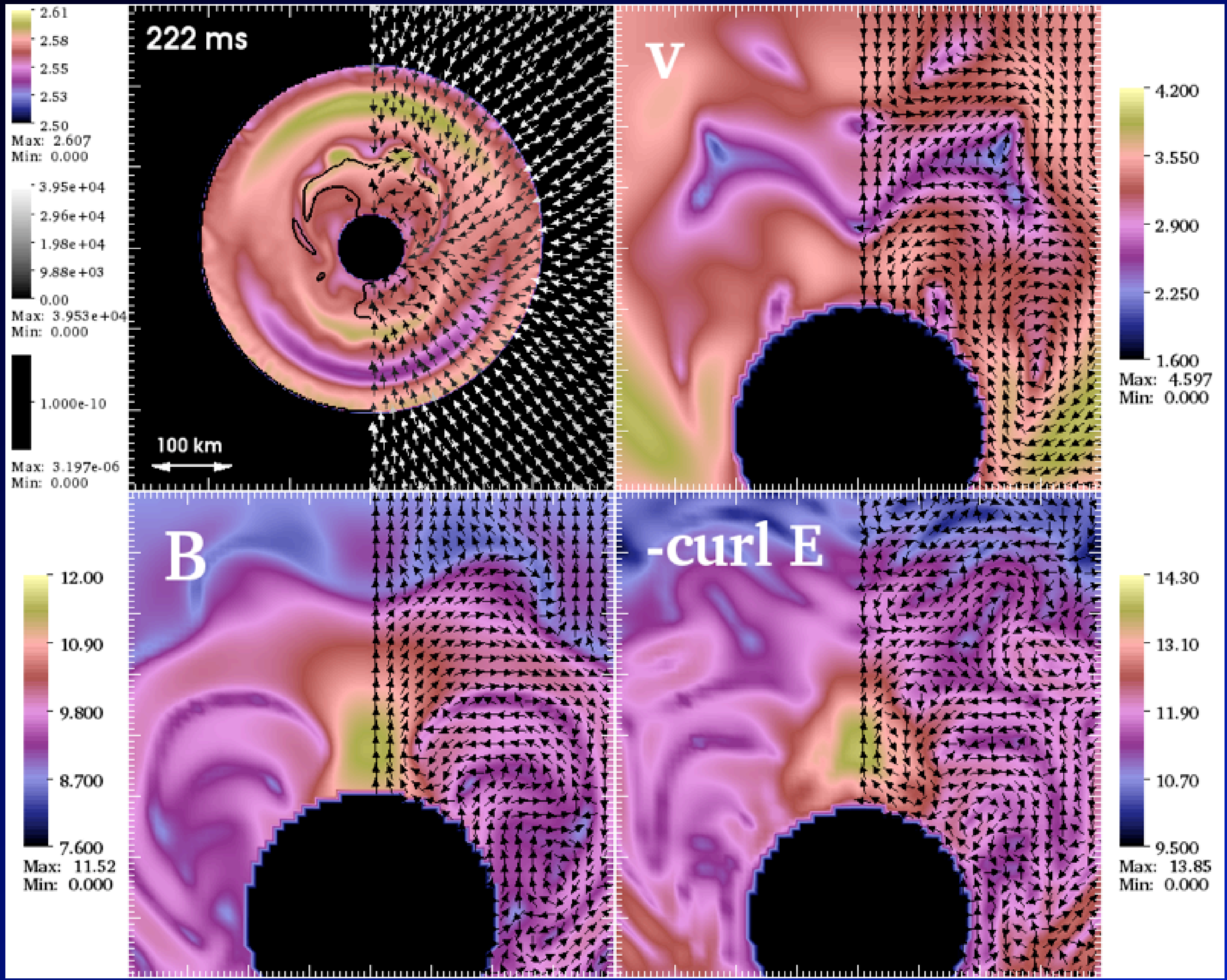
$$E_\phi = u_{r_\perp} B_z - \cancel{u_z B_{r_\perp}}$$

Near the axis $\partial E_\phi / \partial r_\perp \rightarrow 0$ while $u_{r_\perp}/r_\perp$ remains finite, and $B_z \gg B_{r_\perp}$

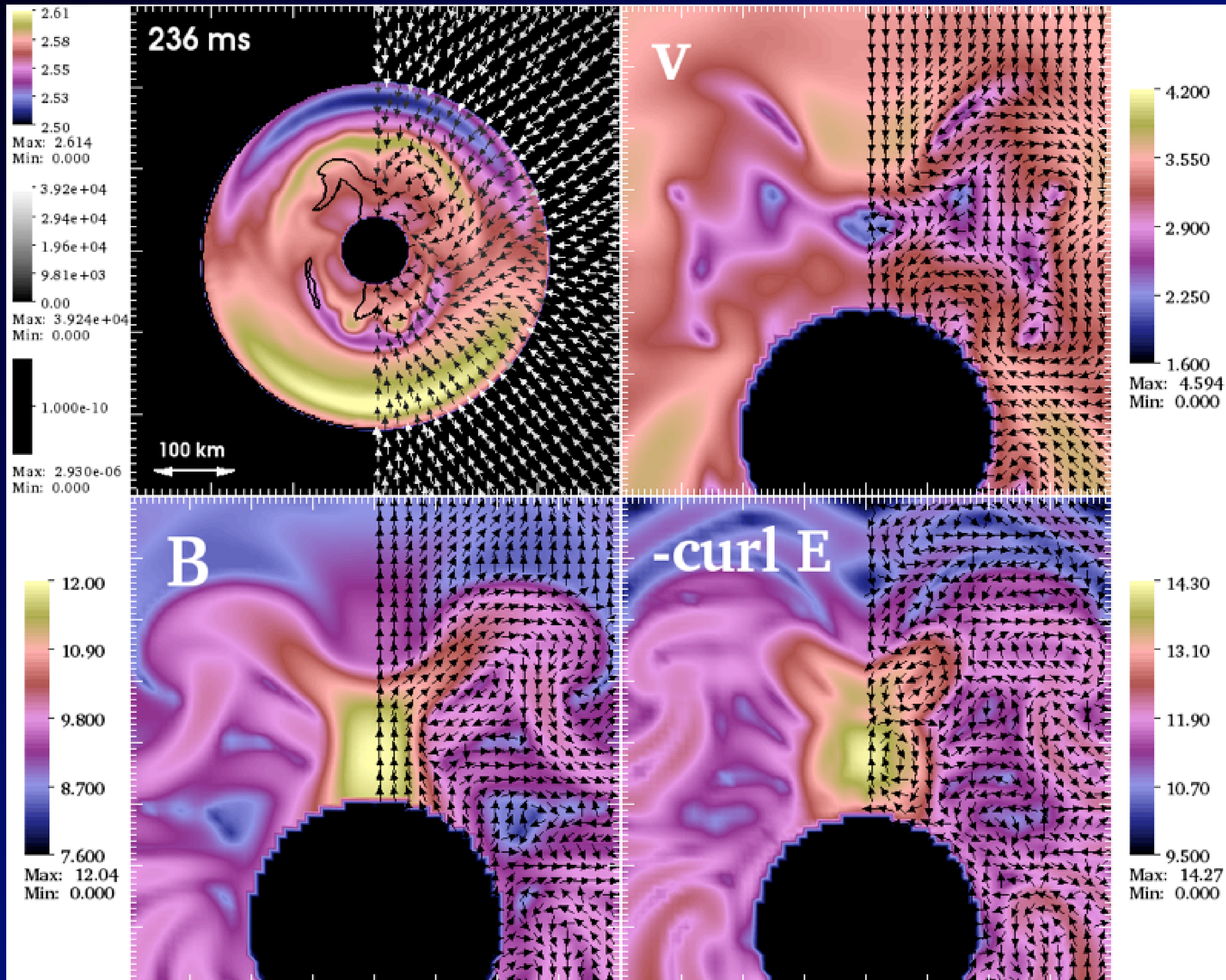
$$\frac{\partial B_z}{\partial t} \rightarrow -\frac{u_{r_\perp}}{r_\perp} B_z$$

B_z *is subject to episodes of exponential growth (or decline) near the symmetry axis*

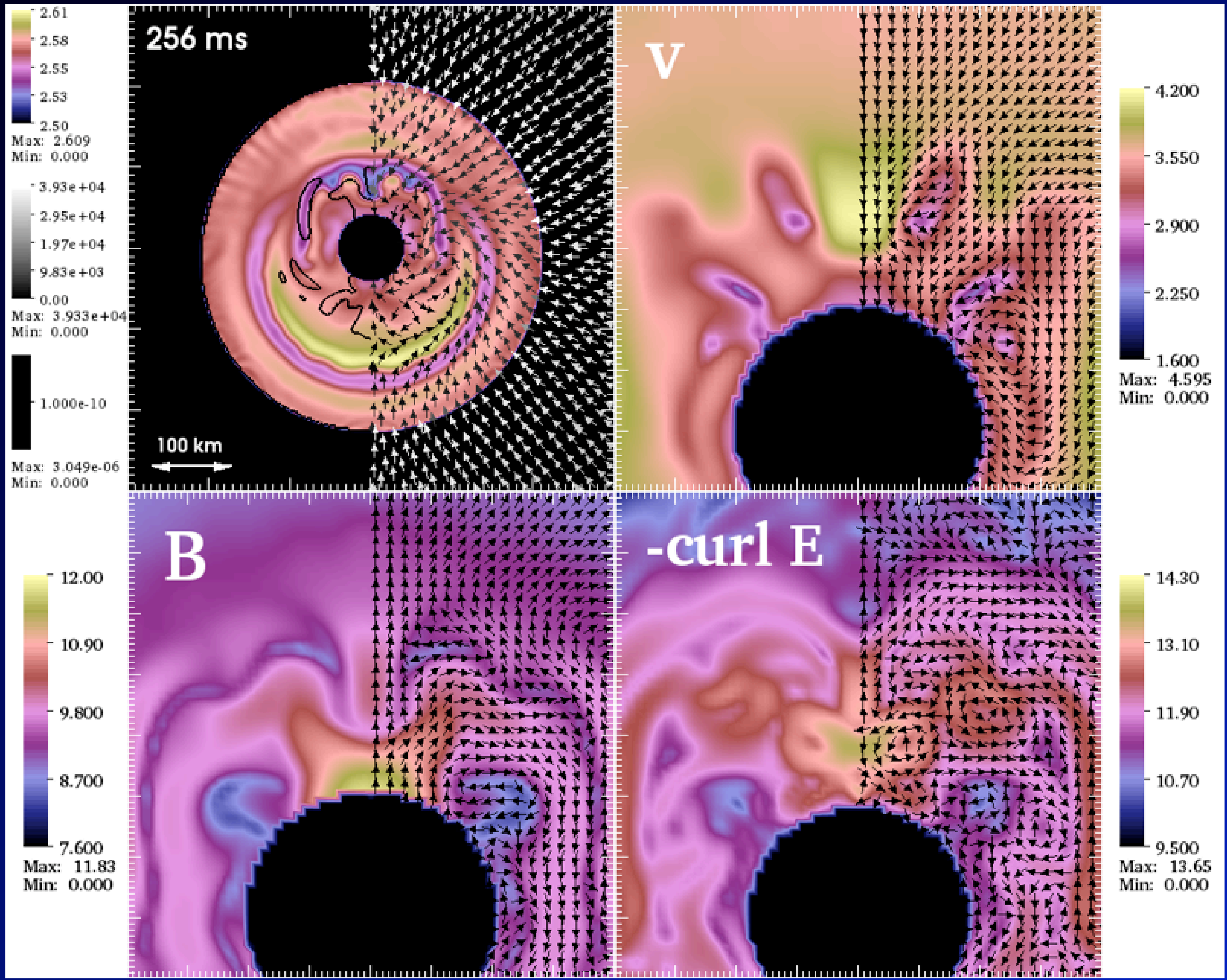
A growth episode begins (zenith of an upward slosh)



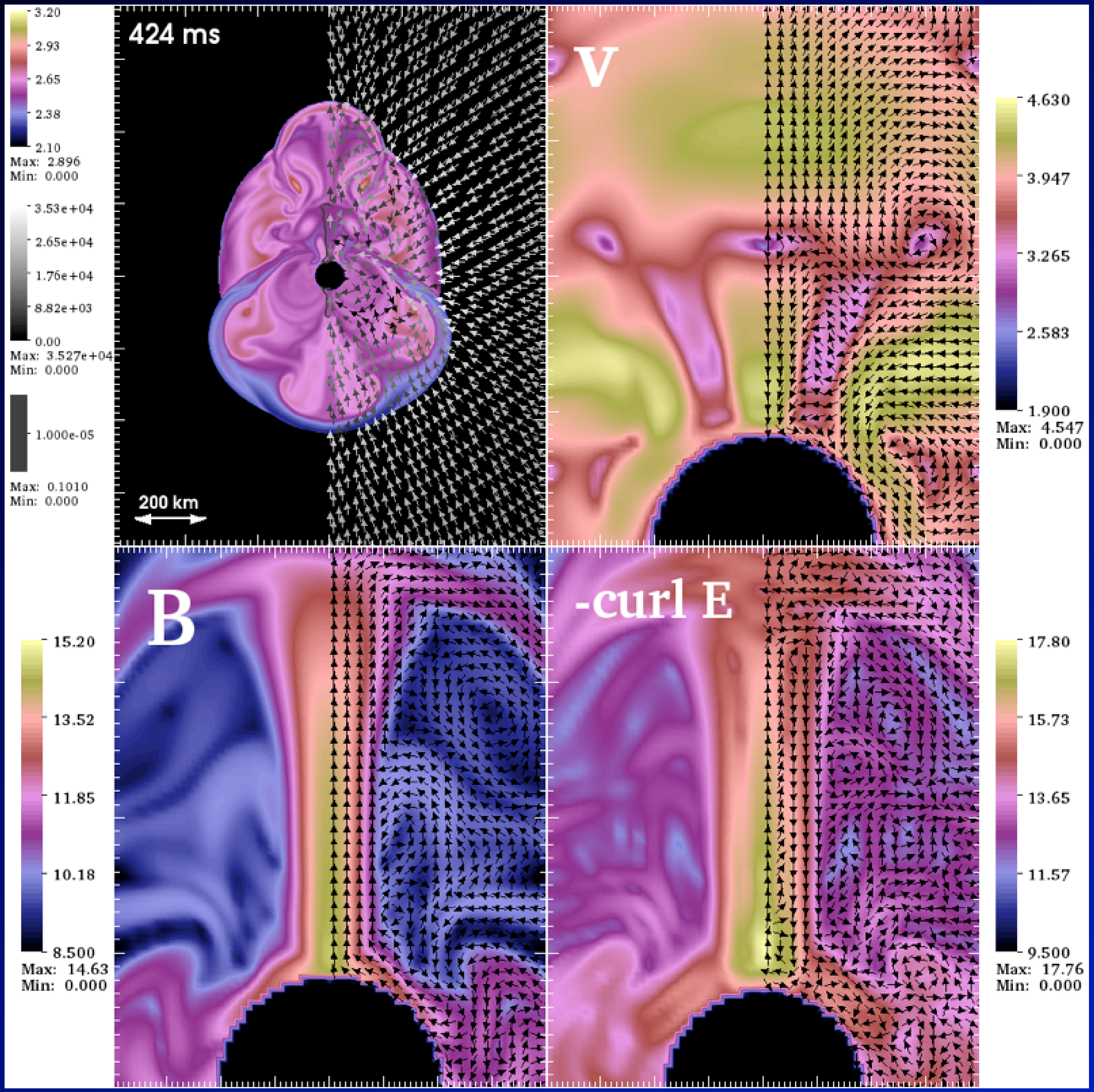
Near peak, decline begins (downward slosh underway)



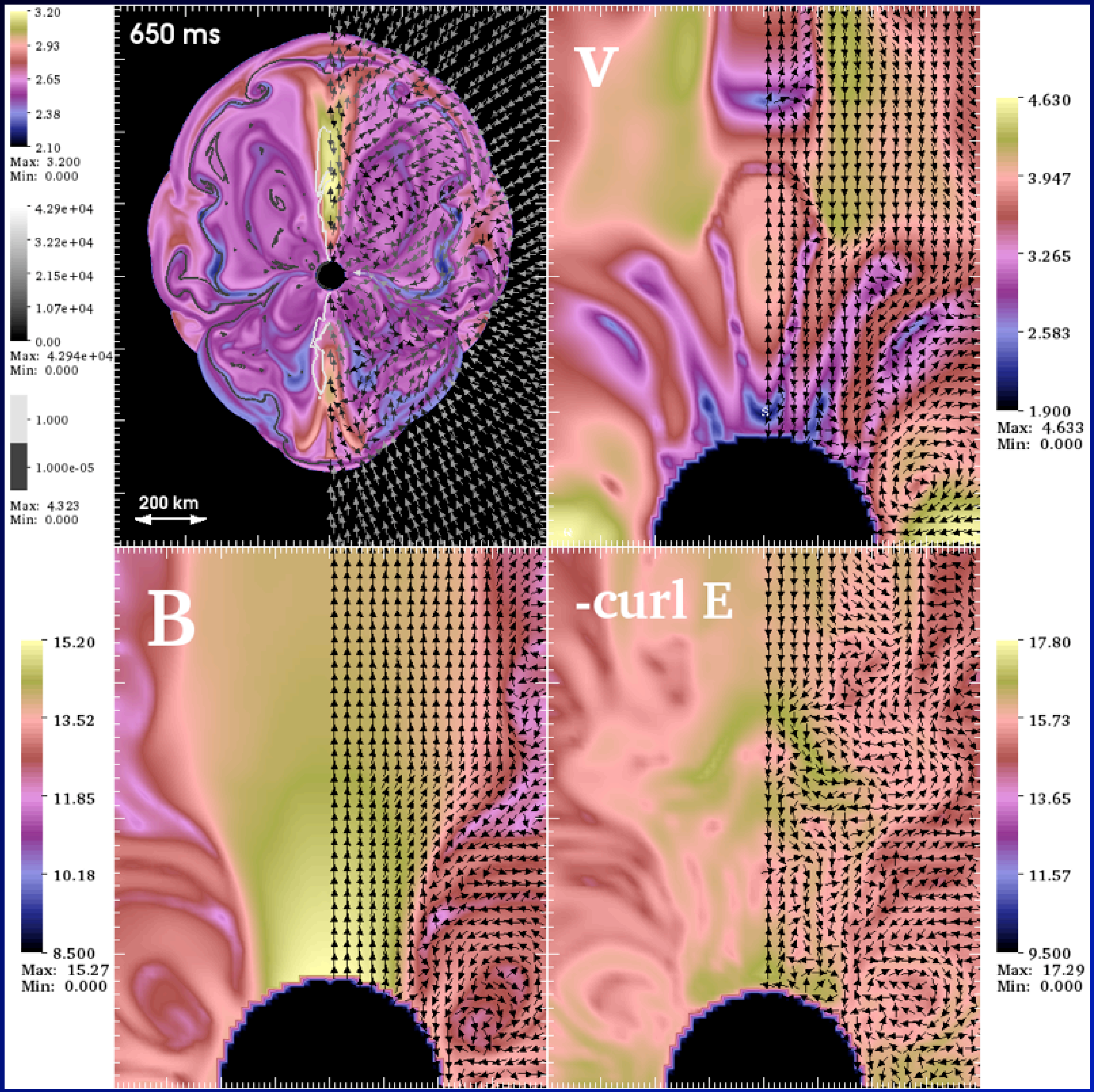
Field significantly erased (nadir of downward slosh)



Continuing growth enabled (persistent plunging streams)

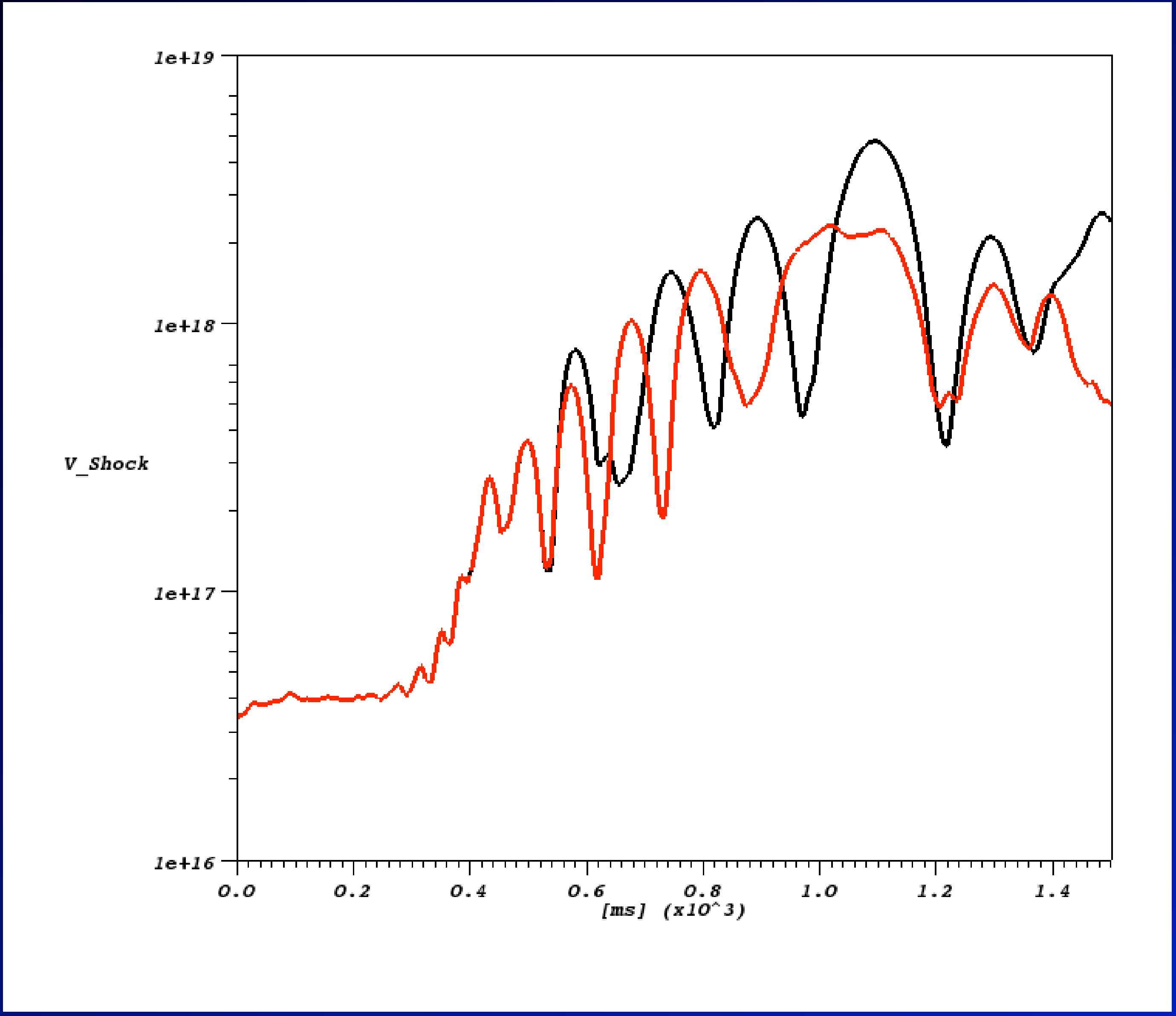


Growth saturates (field resists further flows towards axis)

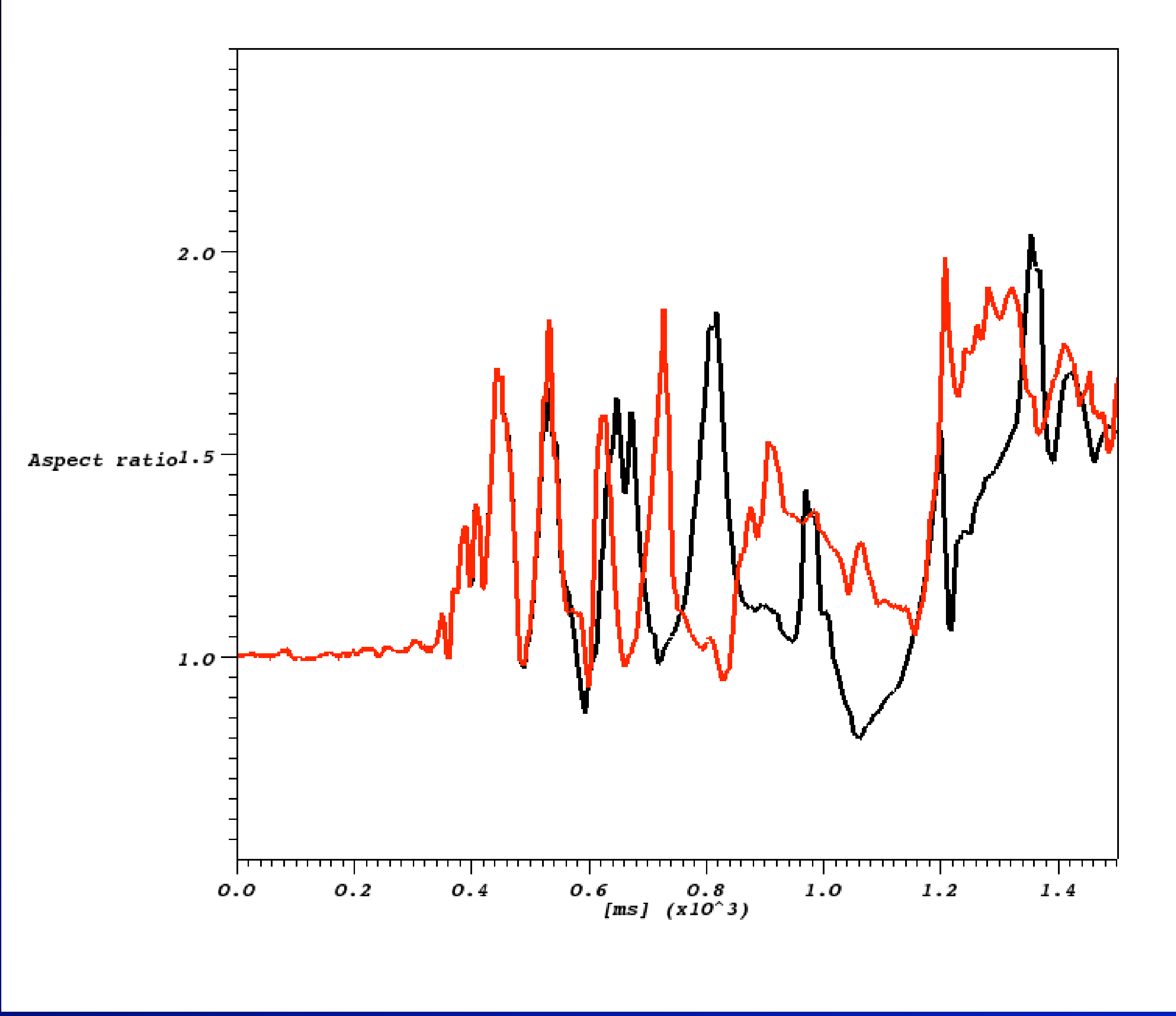


Dynamical consequences

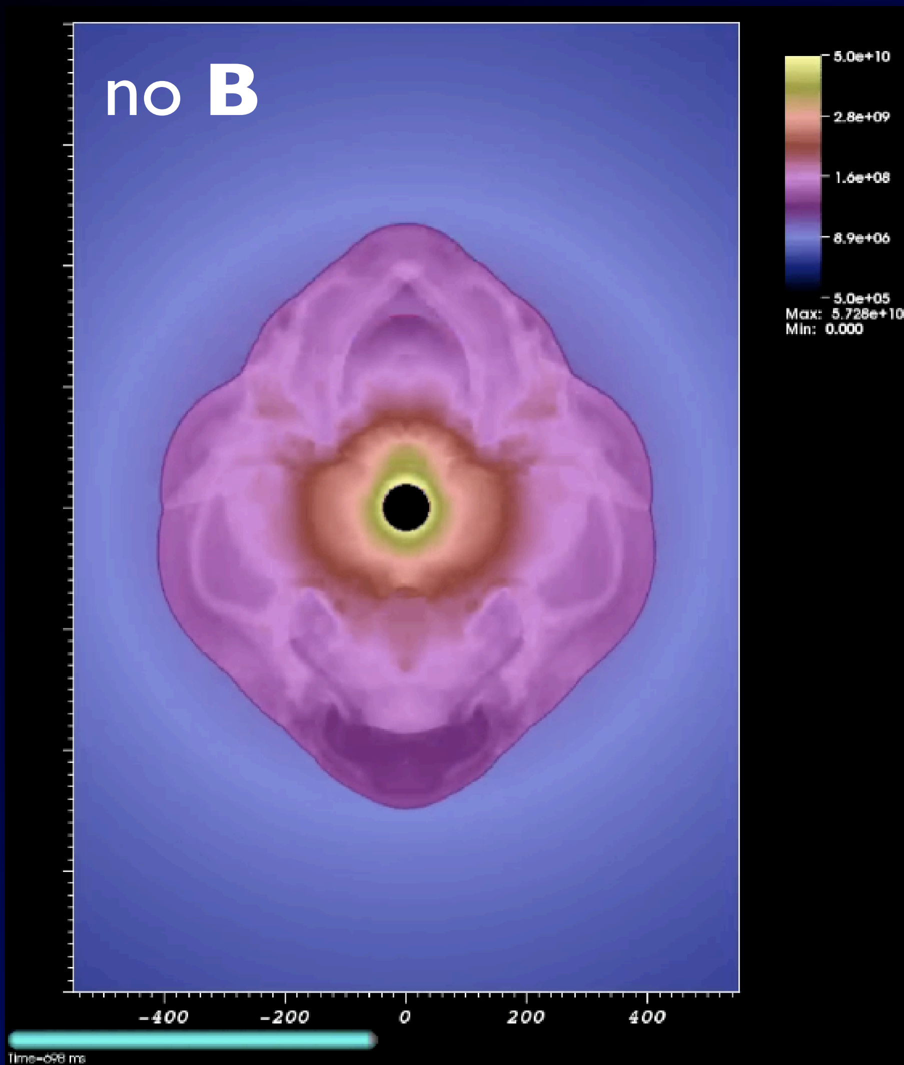
Shock expansion

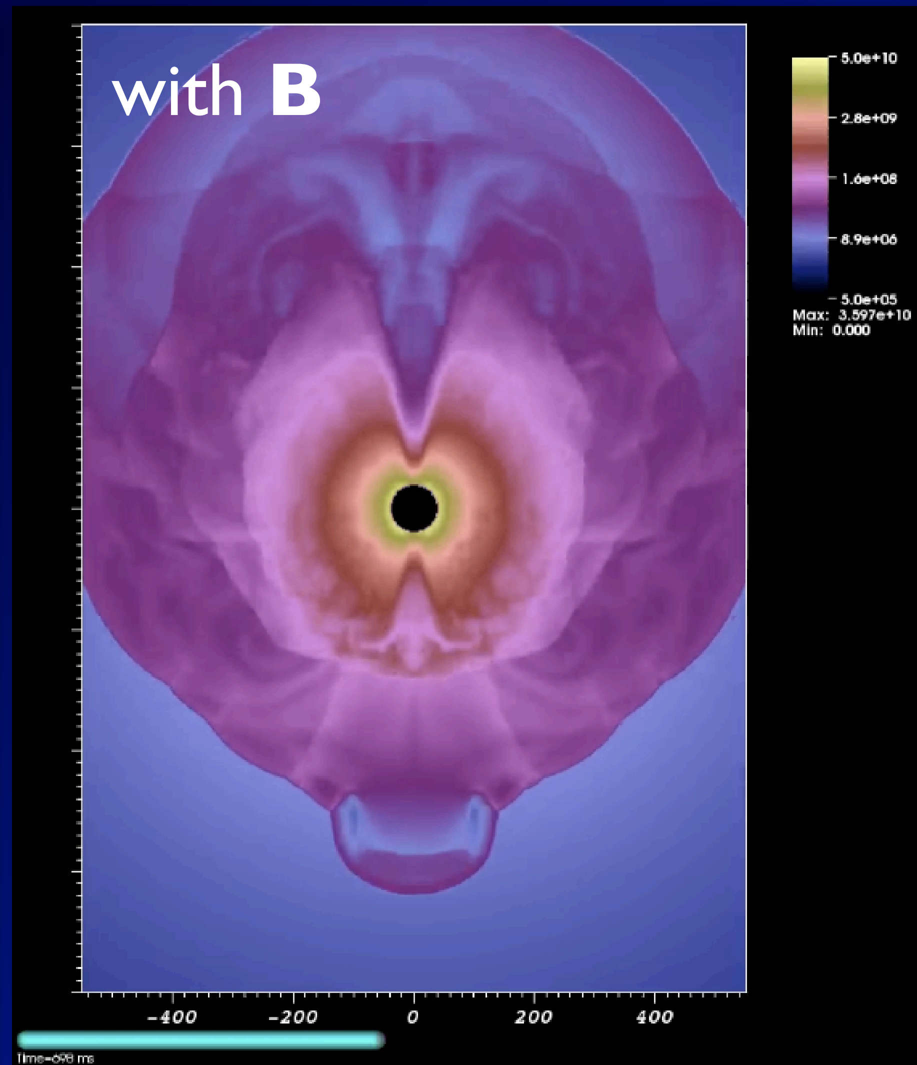


Shock aspect ratio

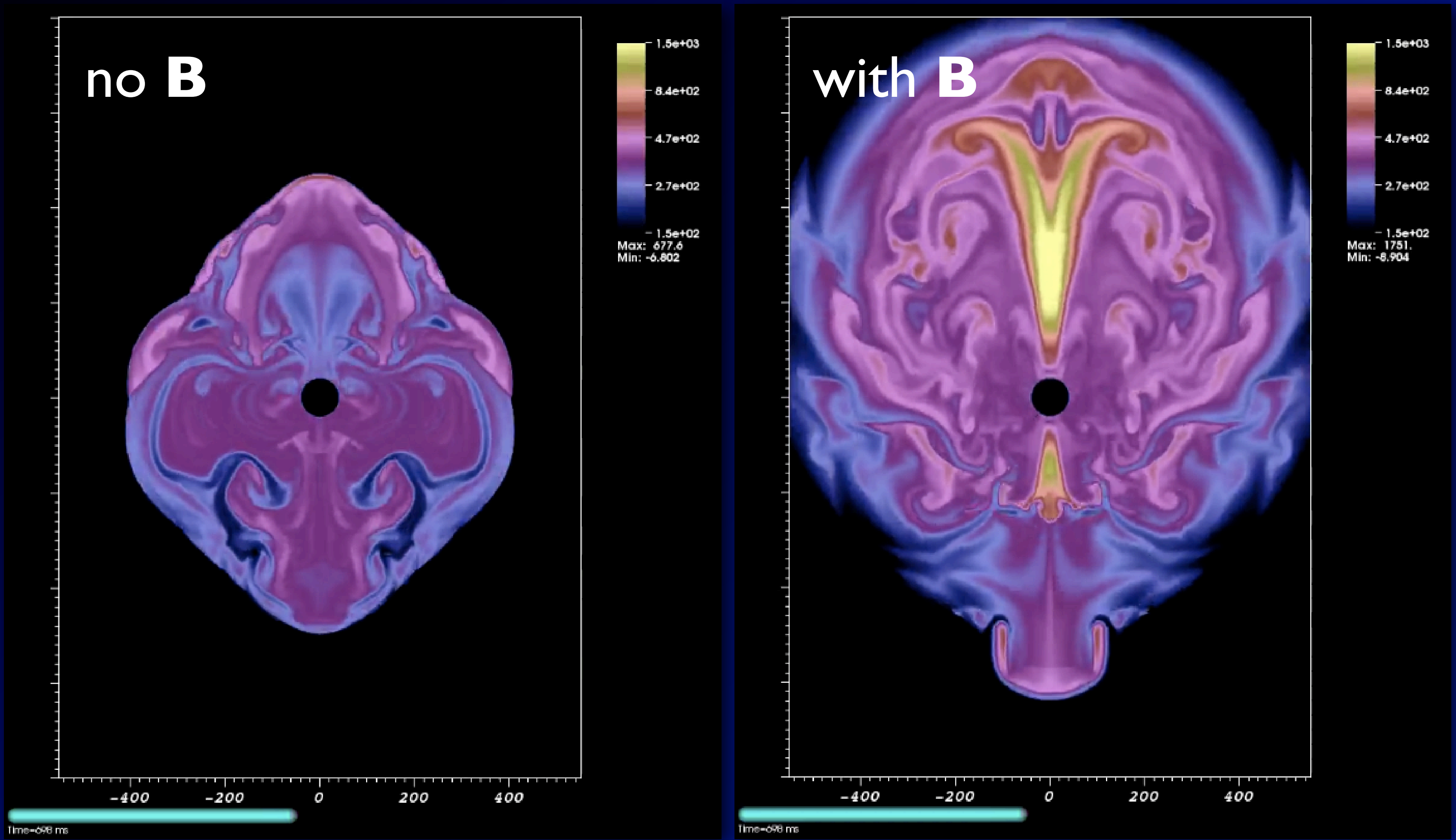


Low-density funnel





“Entropy” generation



Conclusions

The SASI can accomplish what previously had been attributed to strong rotation

- Asphericity
- Pulsar spin
- Magnetic field amplification

Magnetic field amplification in axisymmetry

- Exponential amplification of B_z to dynamical significance $(\sim 10^{15}\ \mathrm{G})$ in polar regions by compression and deposition
- Some field strength advected throughout the shock volume
- This amplification mechanism seems to depend upon axisymmetry in an essential way

Impact of amplified magnetic field in axisymmetry

Modest but noticeable increase in overall shock expansion

No obvious impact on overall trend in shock aspect ratio

No direct driving of jets, but low-density funnel could facilitate collimation of (for example) neutrino-heated ejecta

Significant entropy generation, probably by waves in the "magnetic trunk" steepening into shocks

Some questions we hope to answer soon

What happens in 2D with rotation—and in 3D?

Even if this amplification mechanism does not survive in 3D, can the shear associated with plunging streams trigger the MRI?

Angular momentum of a perturbed 3D magnetic model