

Effects of Rotation on the Stability of Stalled Shocks in Accretion Flows of Core- Collapse Supernovae

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SASI in symmetric accretion flow

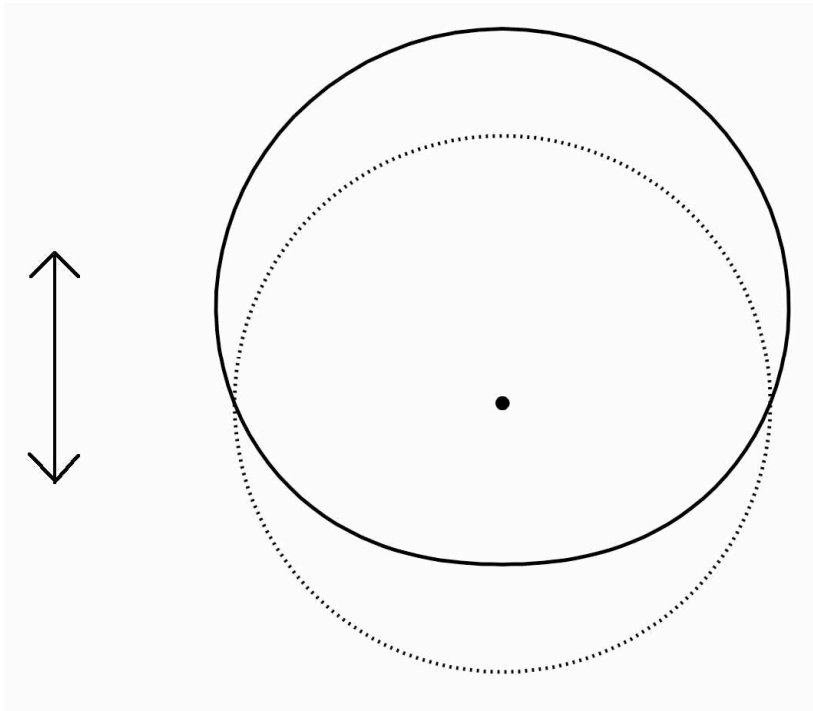
SASI is studied both by numerical simulation and by linear analysis. SASI for the spherically symmetric accretion flow is extensively studied.

According to them,

- Because of symmetry, sloshing mode and rotating modes are degenerated. And, 2-D simulations can reproduce only sloshing modes (But the consequences of them must be different between them).

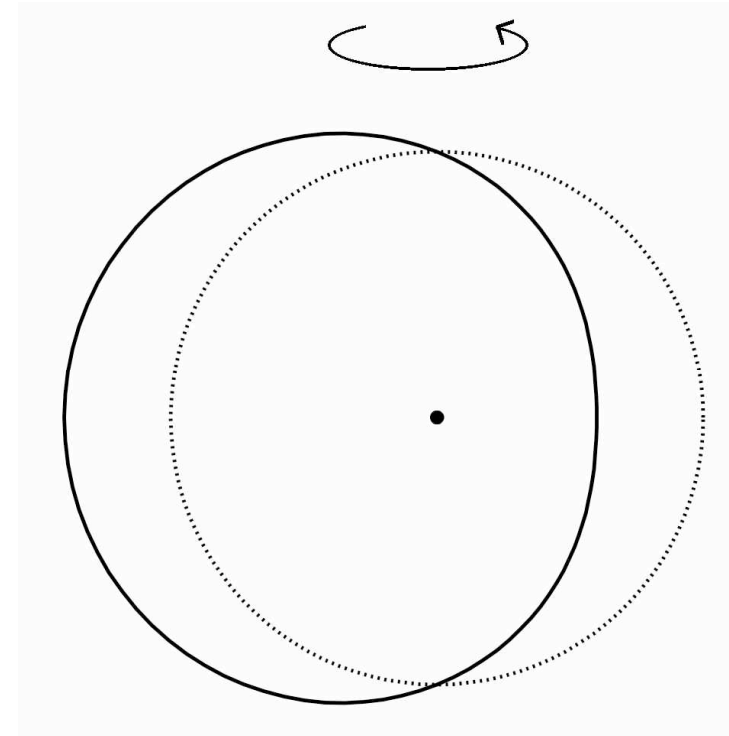
But, cf. Blondin et al. 2007, Iwakami et al. 2008....

Eigen modes of spherical flow



sloshing mode

$$(l = 1, m = 0)$$



rotating mode

$$(l = 1, m = \pm 1)$$

Consequences of SASI on SNe

- Enhancement of the neutrino heating which helps successful explosion (Marek and Janka 2008).
- Acoustic induced explosion (Burrows et al. 2006).
- Pulsar kick (Scheck et al. 2004).
- Pulsar spin up (Blondin and Mezzacappa 2007).

Rotation in SNe (this study)

- Simulations by Heger et al. 2000,2005 indicated the flow rotates rather rapidly.
- The angular momentum of young pulsars are rather large.

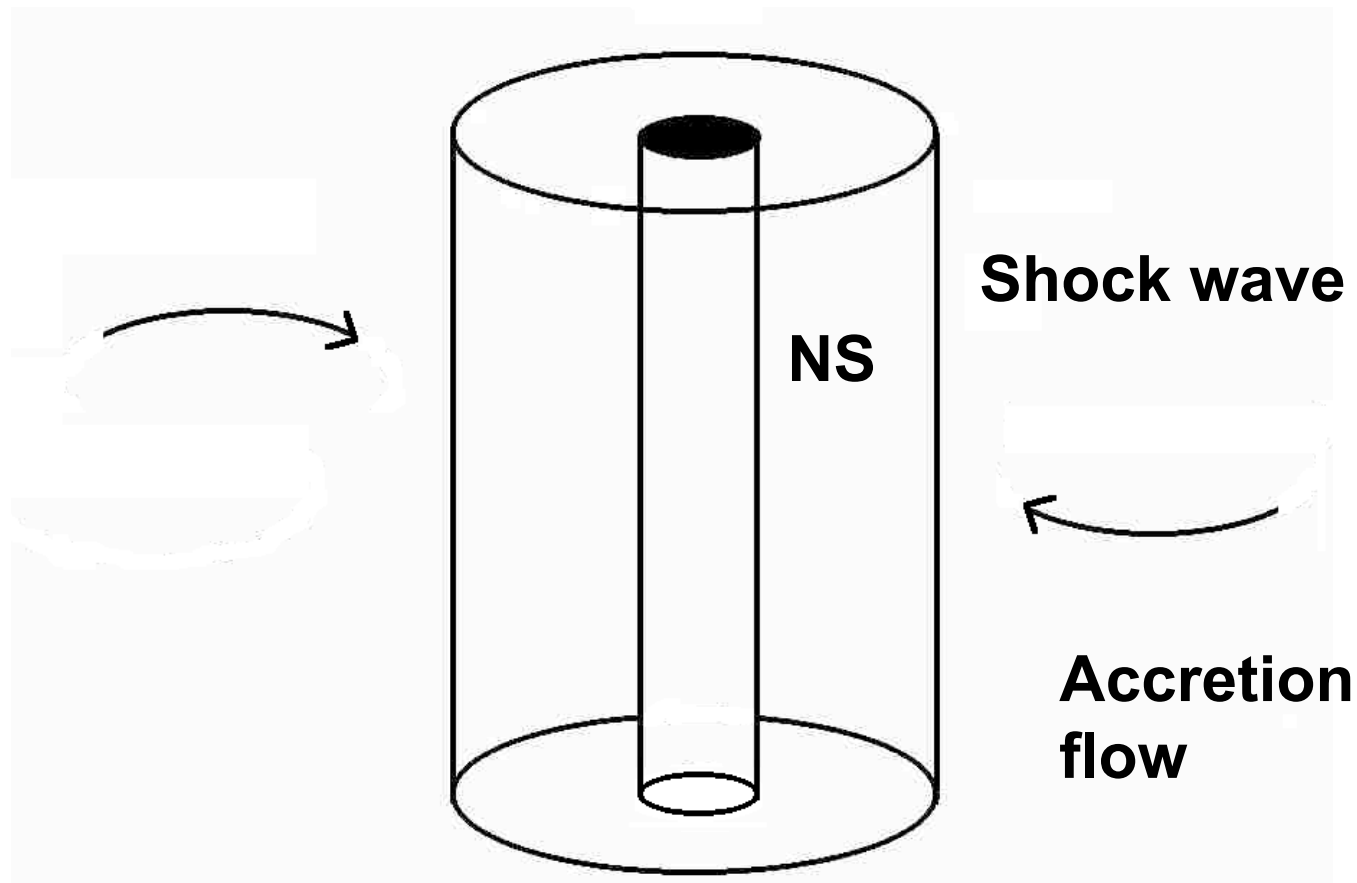
→ Rotation may be non-negligible and can affect SASI.

We investigated the effect of rotation on SASI, using linear analysis.

Calculations (assumptions)

- The accretion flow with steady shock is **cylindrical** (with infinite length).
- The upstream is a cold free-fall flow **with rotation** (not perturbed).
- Specific **angular momentum** in the steady flow **is given as a parameter**.
- Simple cooling law. $\propto \rho^{\beta-\alpha} P^\alpha$ ($\alpha = 3/2, \beta = 5/2$)
- Simple EOS. $P \propto \rho^{4/3}$
- Newtonian gravity (self-gravity is neglected).

Steady flow



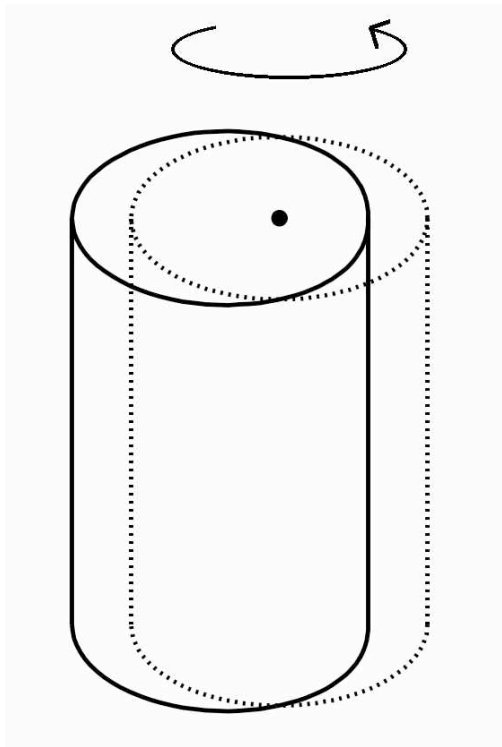
(Upon this flow, perturbations are superposed.)

Perturbations

- Perturbations $\propto \exp\{-i(\omega t - m\phi - k_z z)\}$ are superposed on the steady flow.
- Rankine-Hugoniot relations are imposed also for perturbed flow.
- At the inner boundary, $\delta v_r = 0$.

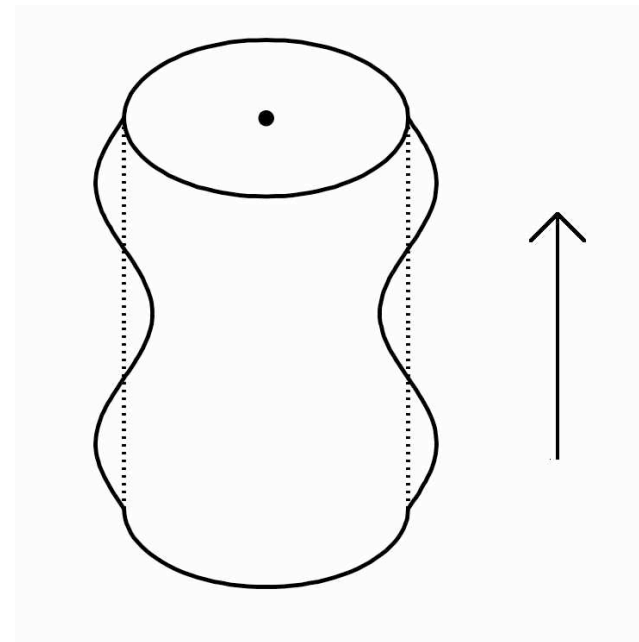
Eigen modes for cylindrical flow

perturbations $\propto \exp\{-i(\omega t - m\phi - k_z z)\}$



rotating mode

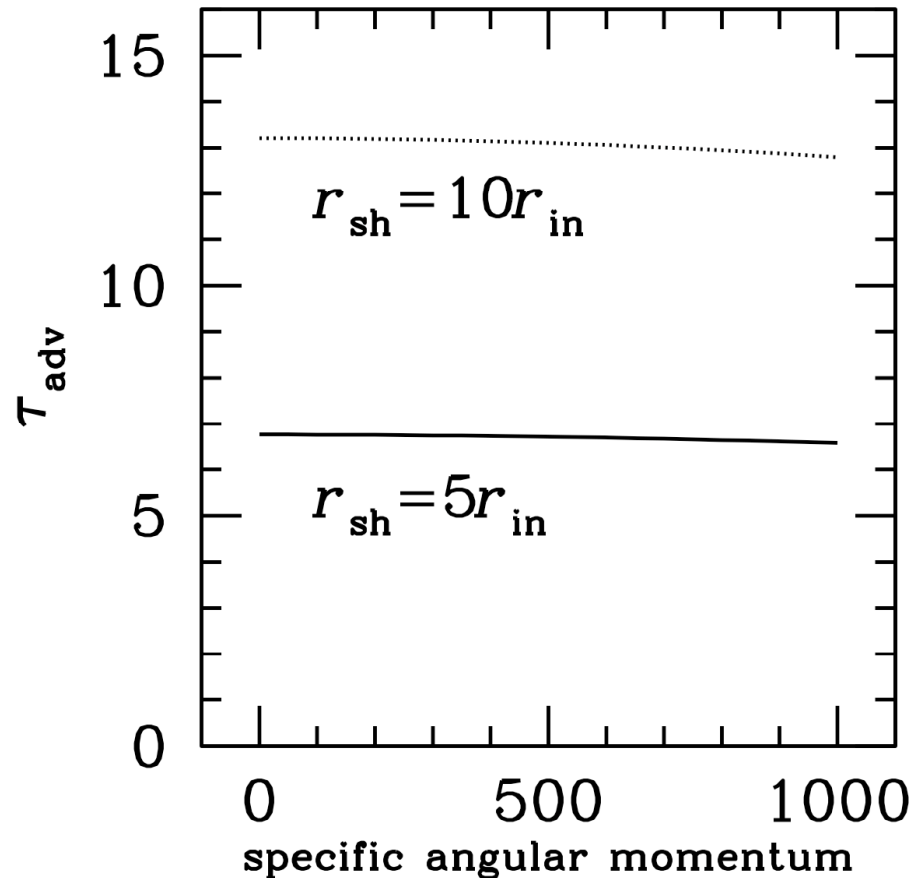
$$(m = \pm 1, k_z = 0)$$



sausage mode

$$(m = 0, k_z \neq 0)$$

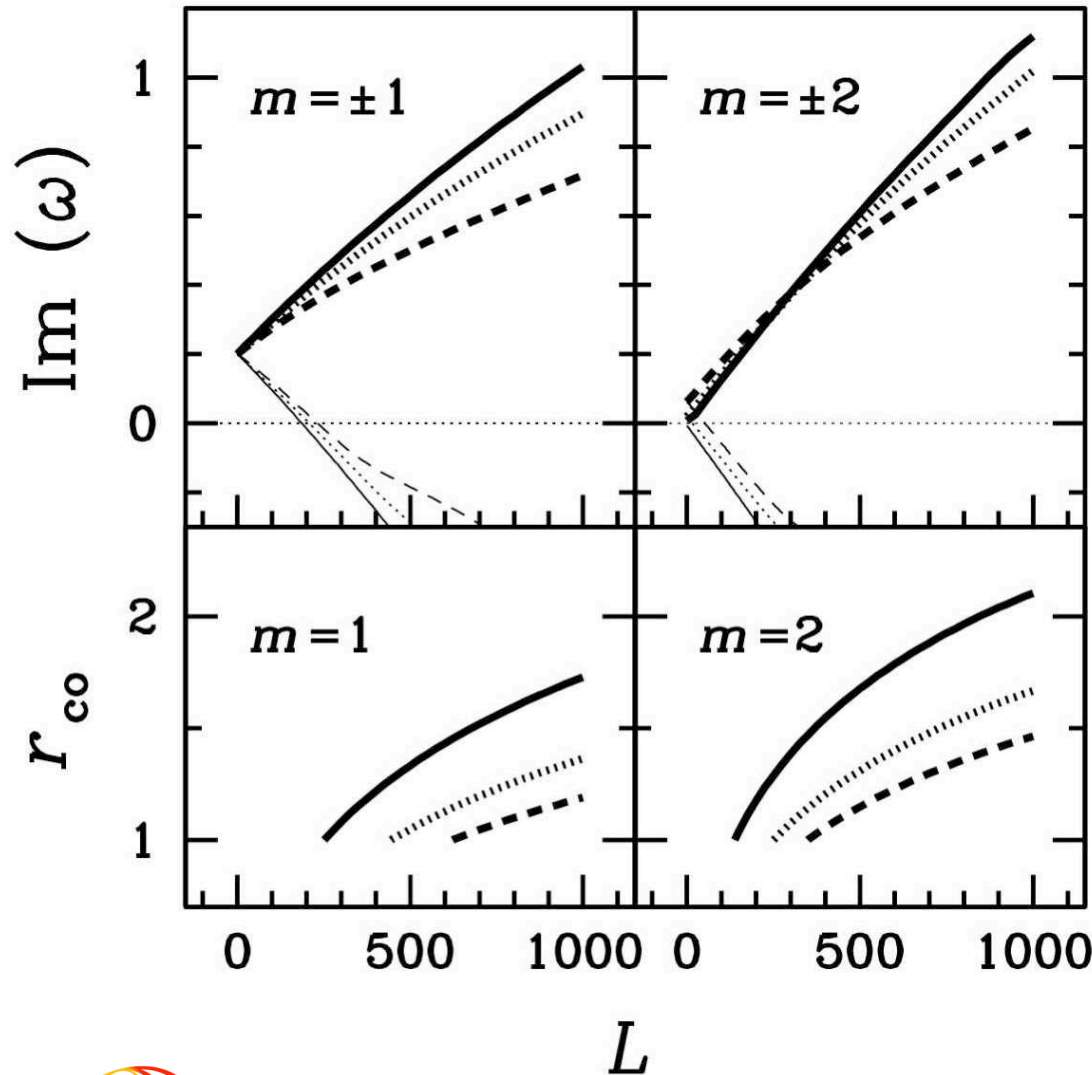
Results 1. Steady solutions (advection timescale)



$$(l_0 = 2\pi \cdot 10^{12} [\text{cm}^2/\text{s}])$$

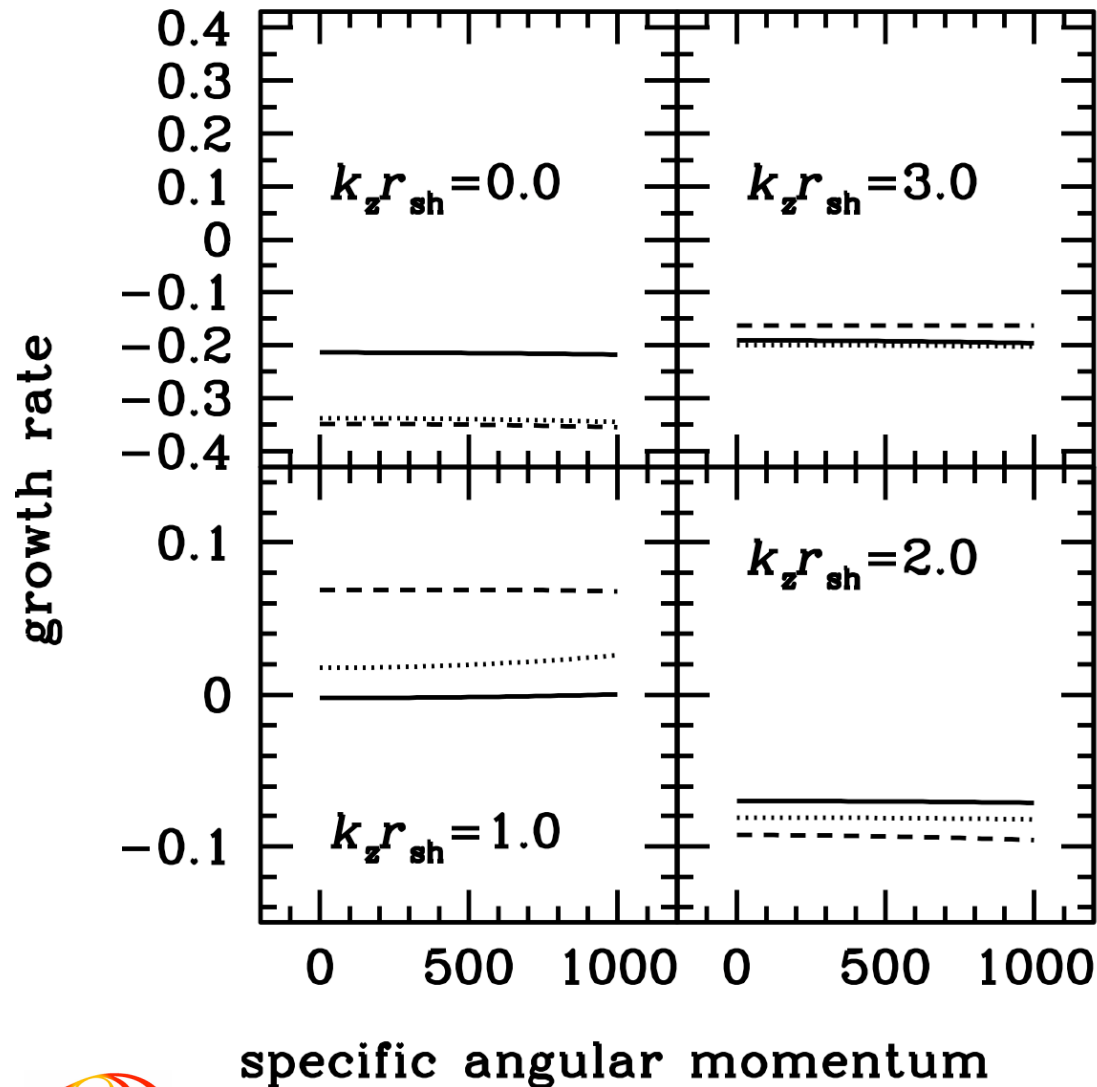
- Rotation hardly affects the structure of the flow.

Results 2. *Effects on stability ($m=1,2$)*



- Rotation destabilize the co-rotating modes, whereas stabilize the counter-rotating modes [cf. Laming (2007)].
- The dependence is almost linear.
- The effect is independent of existence of the co-rotation radius.

Results 3. *Effects on stability ($m=0$)*



- Stability of the axisymmetric (sausage) modes are hardly affected by rotation.

Implication on the realistic accretion flows

- The effects are attributed to the Doppler effect (because the steady flow is hardly affected by rotation).

Therefore, in the realistic (spherical) flow

- Co-rotating modes seem to be destabilized, whereas counter-rotating modes to be stabilized.
- Axi-symmetric modes (sloshing modes) seem to be hardly affected by rotation.

Conclusions

- Rotation works so as to destabilize the co-rotating modes. → co-rotating modes selectively grow.
- Rotation significantly increases the growth rate. → SASI grows in the earlier stage.
- Many unstable modes appears when the flow is rotating. → affects the nonlinear evolution.
- Different mode become dominant when the flow is rotating. → affects pulsar kick.

Magneto-rotational instability during core-collapse

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Introduction (MRI)

- Magneto-Rotational Instability (MRI) : an instability induced by angular momentum exchange during perturbations.
- As a result of the instability, magnetic field is amplified, turbulence can be developed and viscosity becomes effective (angular momentum transfer, dissipation).

MRI in accretion disks and SNe

- The MRI in accretion disks, is studied extensively since Bulbus & Hawley('91). MRI is local in nature and thought to induce the turbulence.
- Growth rate is of the order of angular velocity. Roughly, it is unstable when $d\Omega^2/dR < 0$.
- In SNe, some models in which MRI plays an important role are proposed (ex, Akiyama et al.2003), but only few simulation (Shibata et al. 2005) reproduced it because of short wavelength compared with the mesh size.

MRI in ADs and SNe

- ADs $v_\phi \gg c_s > c_A$
- SNe $c_s \gg v_\phi > c_A$

→ We should be careful to apply the results for accretion disks to SNe.

We perform a linear analysis, without adopting the Boussinesq approximation.

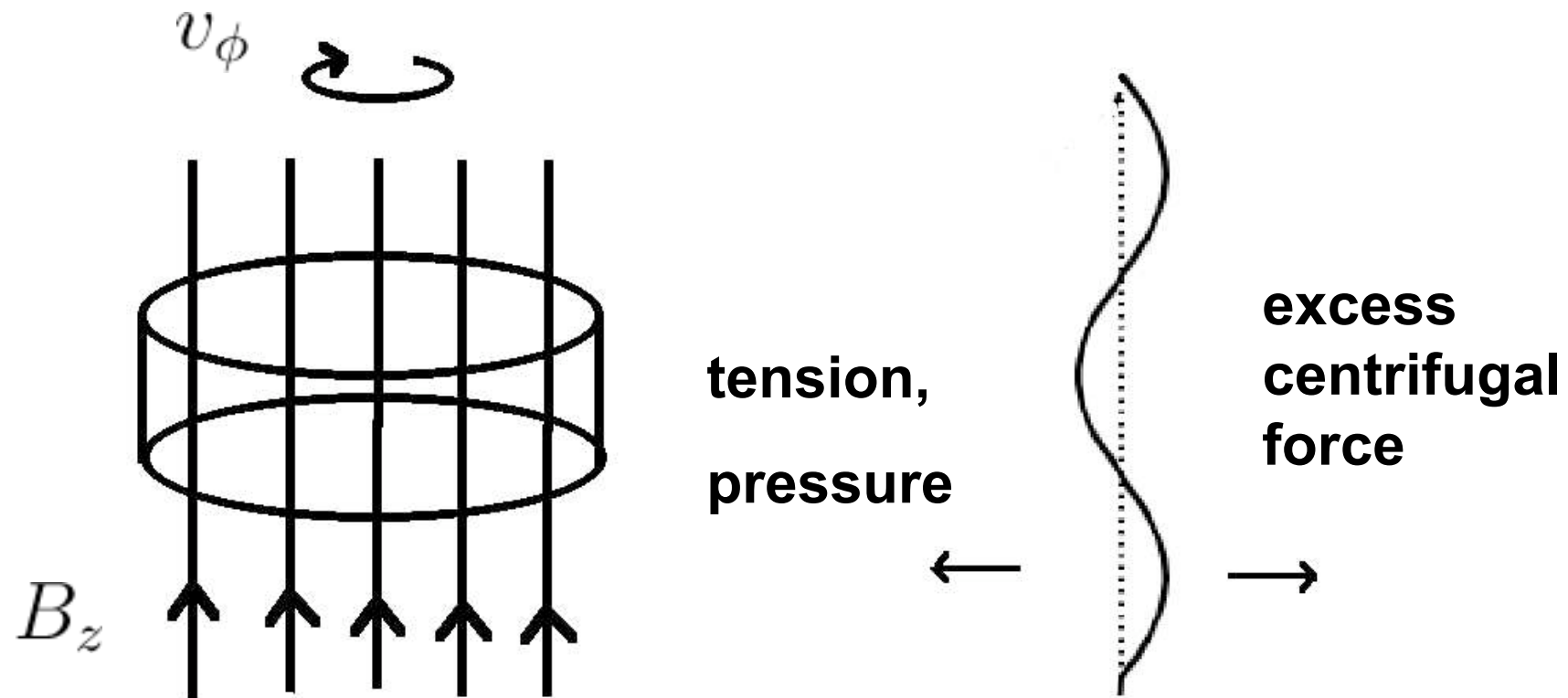
✂ We do not take into account the effect of advection.

MHD waves

- 3 waves exist (Alfven wave, fast mode and slow mode).
- When $c_s > c_A$, main restoring force for slow mode is the magnetic tension (to avoid the strong restoring force of pressure). So the slow mode is a nearly incompressible wave.
- MRI is the destabilizing of the slow mode.

MRI in the flow with vertical magnetic field

- Vertical magnetic field.



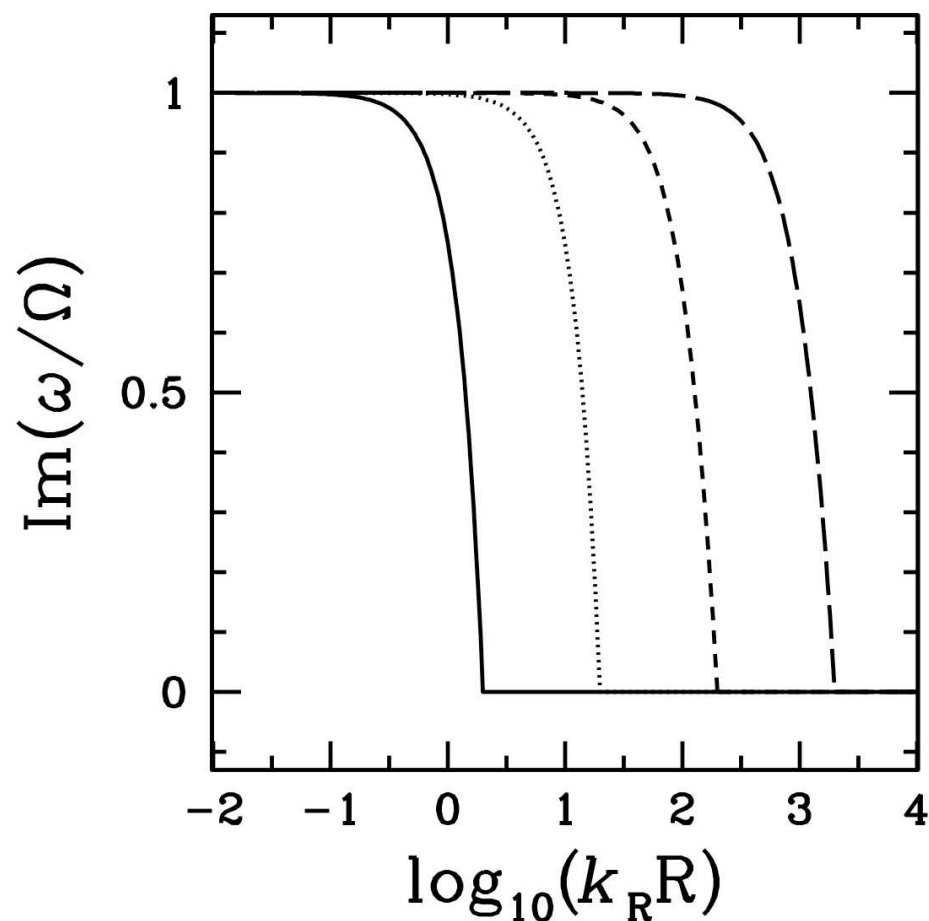
Axisymmetric MRI

- Magnetic tension works so as to transfer the angular momentum as well as to restore the flow.
- Centrifugal force is perturbed due to the effect and can work so as to grow the perturbations.
- If the destabilizing effect of perturbation of centrifugal force overcomes the magnetic tension and pressure (both thermal and magnetic), MRI happens.

Axisymmetric MRI, Calculations

- Steady flow --- homogeneous in the vertical direction. Flow is isentropic. Only B_z and v_ϕ are taken into account.
- Axisymmetric perturbations are considered. Linear stability is discussed by the normal mode local analysis.

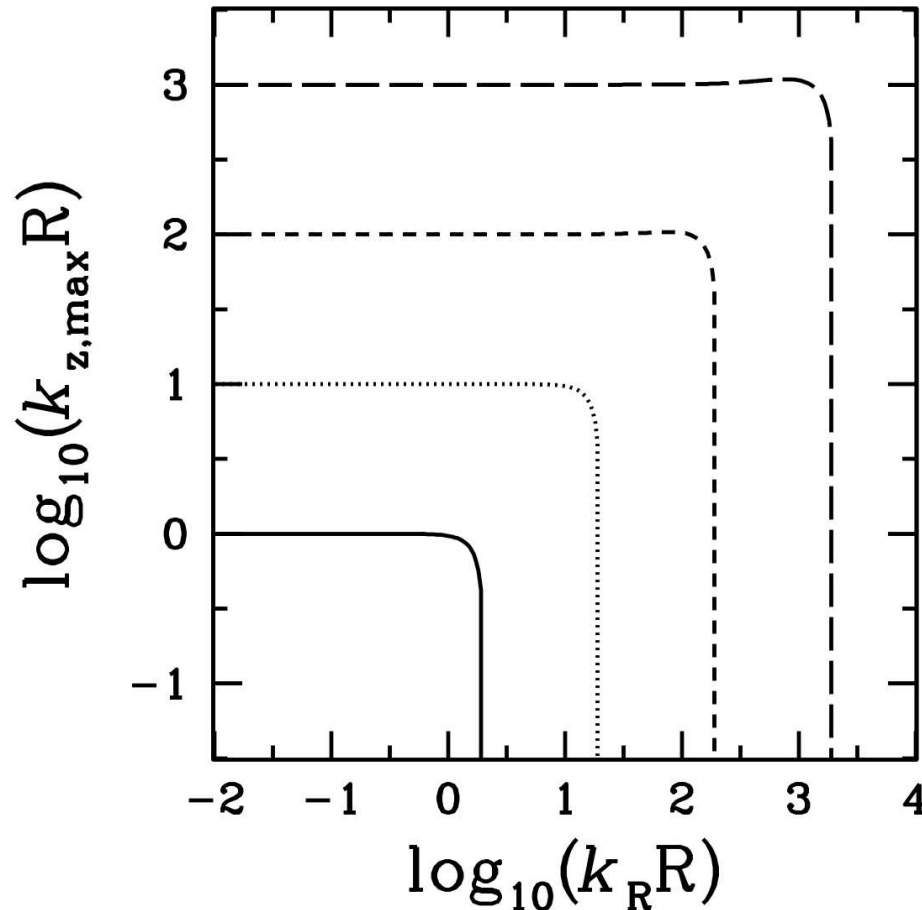
Axisymmetric MRI, Results 1



- Growth and the stability criterion of MRI is independent of the thermal pressure.
- This is because the slow mode is incompressible in nature.

$$v_\phi = 10^{-2} c_s, \quad \log_{10}(c_A/v_\phi) = 0, -1, -2, -3$$

Axisymmetric MRI, Results 2



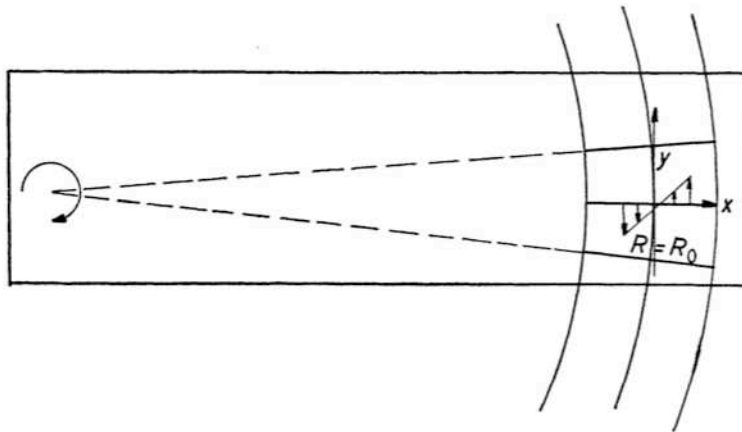
- Also vertical wavenumber for the mode with the maximum growth rate is independent of the thermal pressure ($\sim \Omega/c_A$).

$$v_\phi = 10^{-2} c_s, \quad \log_{10}(c_A/v_\phi) = 0, -1, -2, -3$$

Non-axisymmetric MRI

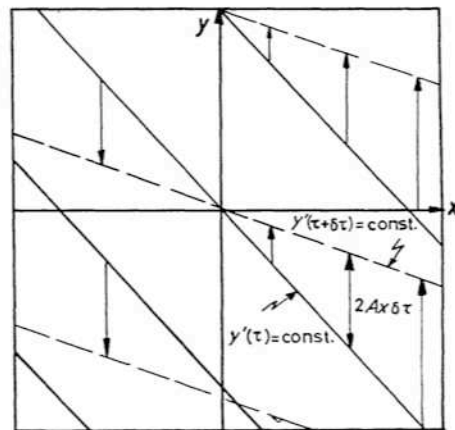
- Azimuthal magnetic field.
- Since pattern of the perturbation is sheared, normal mode analysis cannot be employed.
→ initial value problem
- For the local analysis of differentially rotating system, the shearing sheet approximation is often employed.
- In the approximation, the effect of the shearing of the perturbation is simply represented by the time dependence of radial wavenumber.
- We solved linear equations.

Shearing sheet approximation



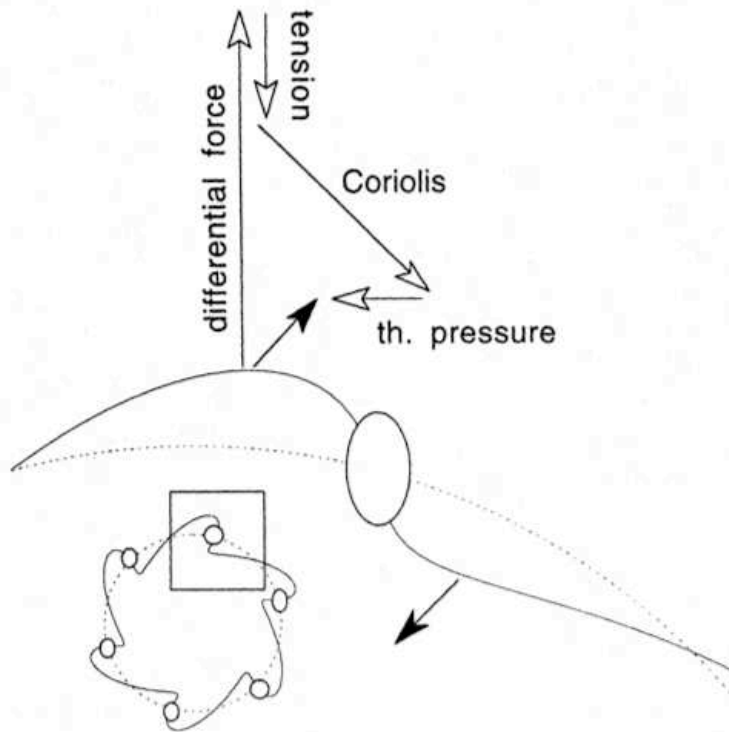
- Centrifugal force and Coriolis force are taken into account.

$$k_R(t) = k_R(0) - k_\phi R \frac{d\Omega}{dR} t.$$



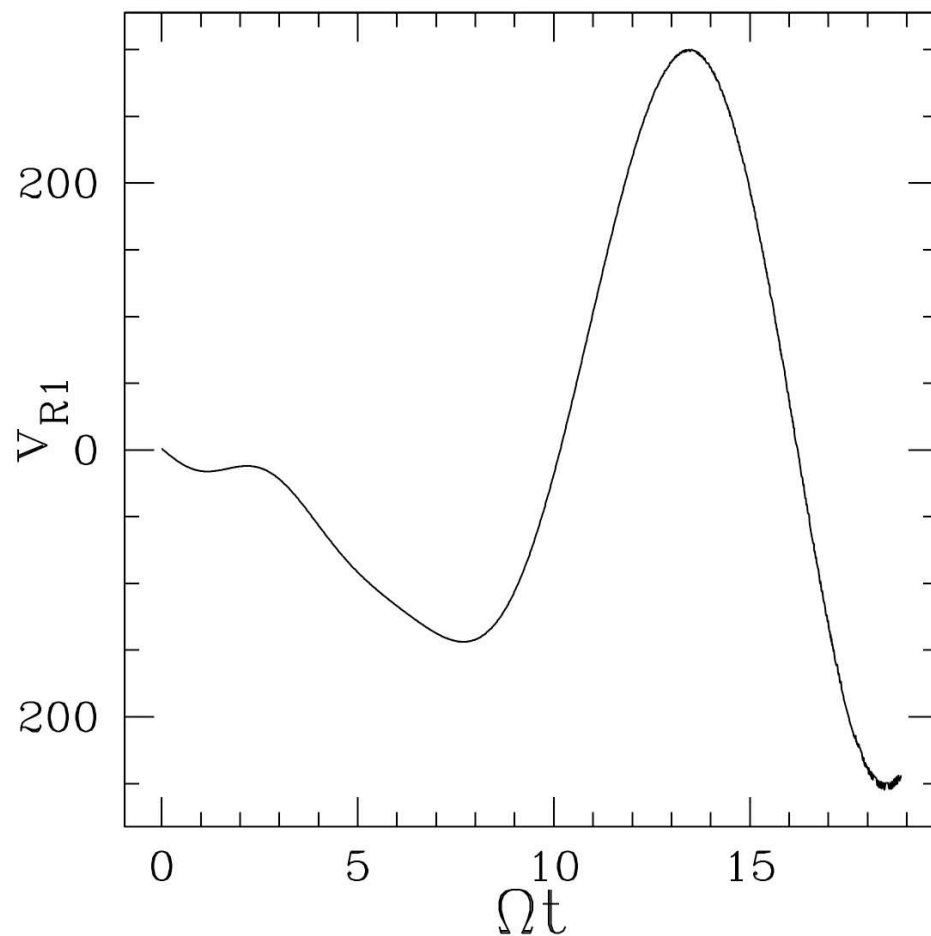
(Goldreich & Lynden-Bell, 1965)

Non-axisymmetric MRI with azimuthal field



Foglizzo & Tagger (1995)

Non axisymmetric MRI, Results

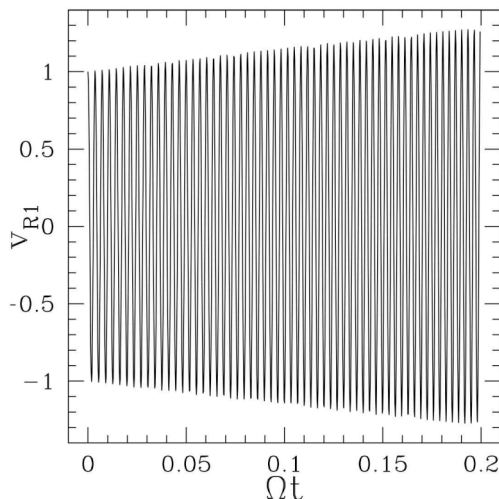
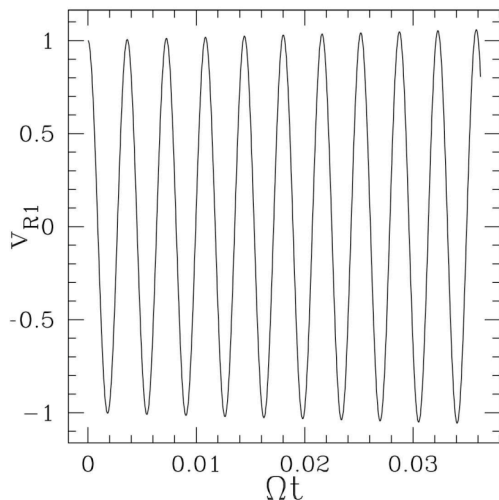


- Flow is unstable even when the thermal pressure is dominant in the steady flow.

$$v_{\phi} = 10^{-2} c_s, \quad c_A = 10^{-1} v_{\phi},$$

$$k(0)R = 10, k_{\phi}R = 10, k_zR = 100.$$

Amplification of the fast mode



- Not only the slow mode, but also the fast mode can grow.
- Growth rate $\sim \Omega$.
- The growth are caused by shear (increase of the wavenumber), rather than the centrifugal force.

$$v_\phi = 10^{-2} c_s, \quad c_A = 10^{-1} v_\phi,$$

$$k(0)R = 10, k_\phi R = 10, k_z R = 10.$$

Summary

- The stabilities of the rotating flow both with the vertical magnetic field against axisymmetric and with azimuthal magnetic field against non-axisymmetric perturbations are investigated.
- MRI is the growth of the slow mode. So the growths of them are nearly independent of the thermal pressure in the steady flow.
- We found the growth of the fast mode.