Analytic Approach to the Stability of Standing Accretion Shocks

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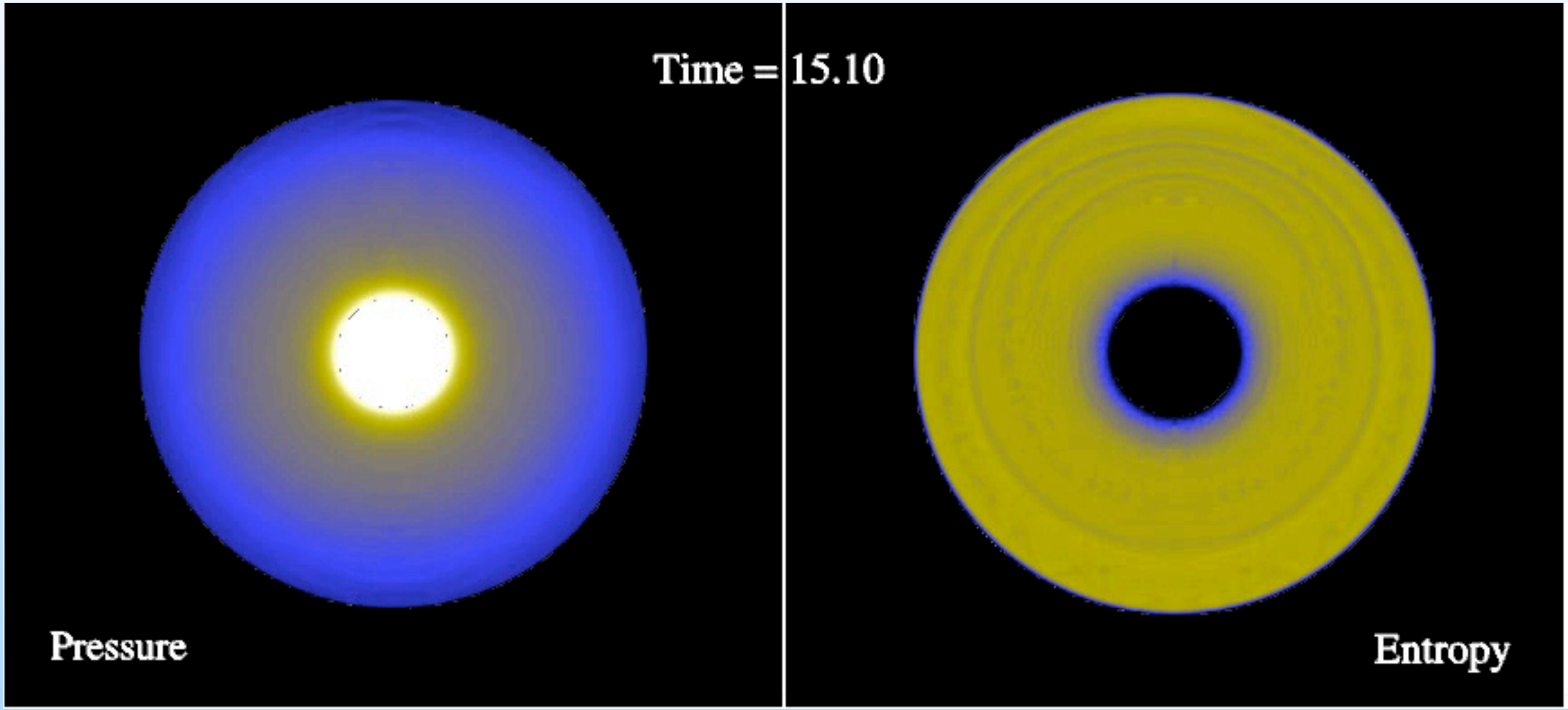
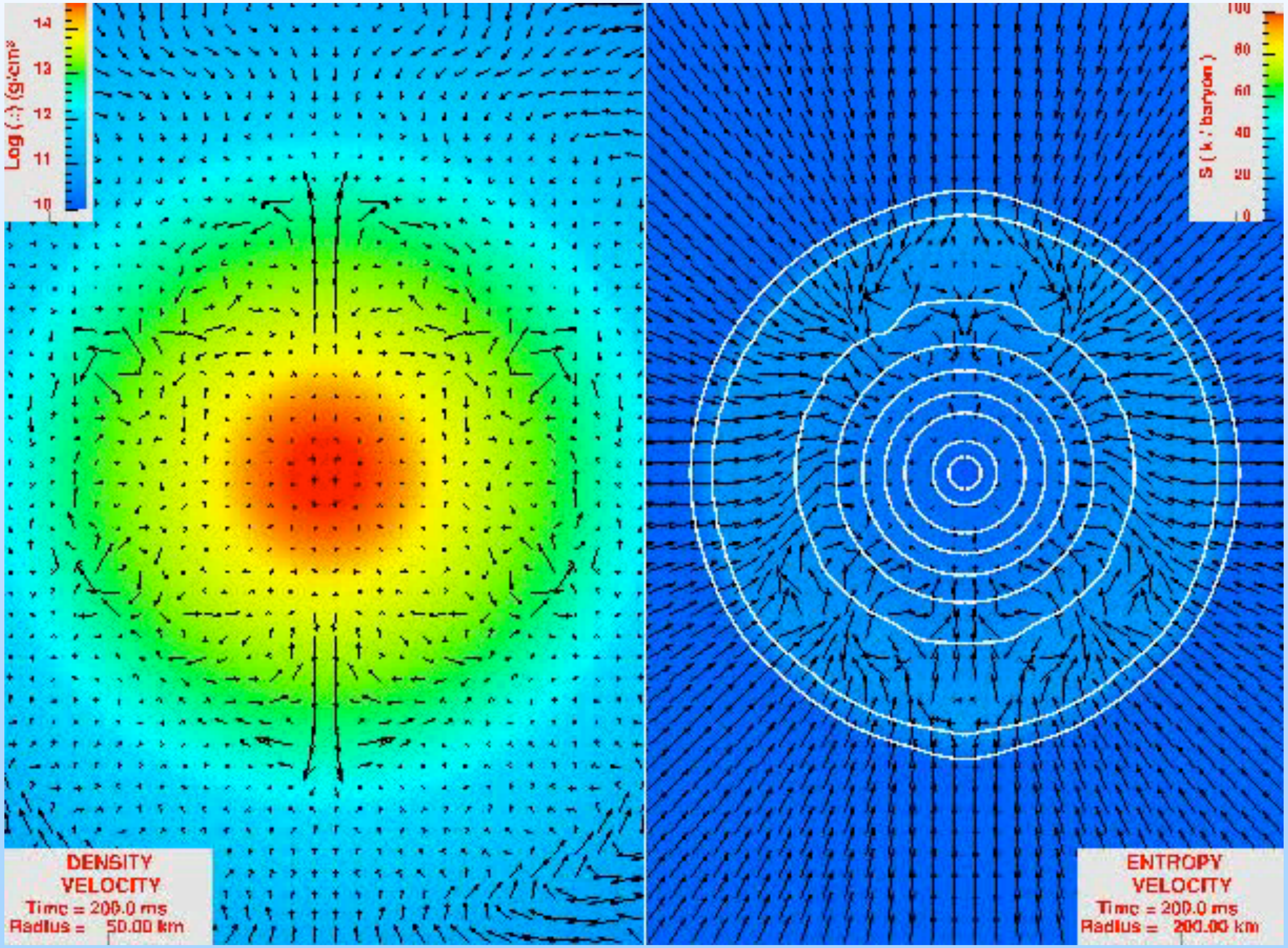
Standing Accretion Shock Instability I.

(courtesy John Blondin, North Carolina State University)



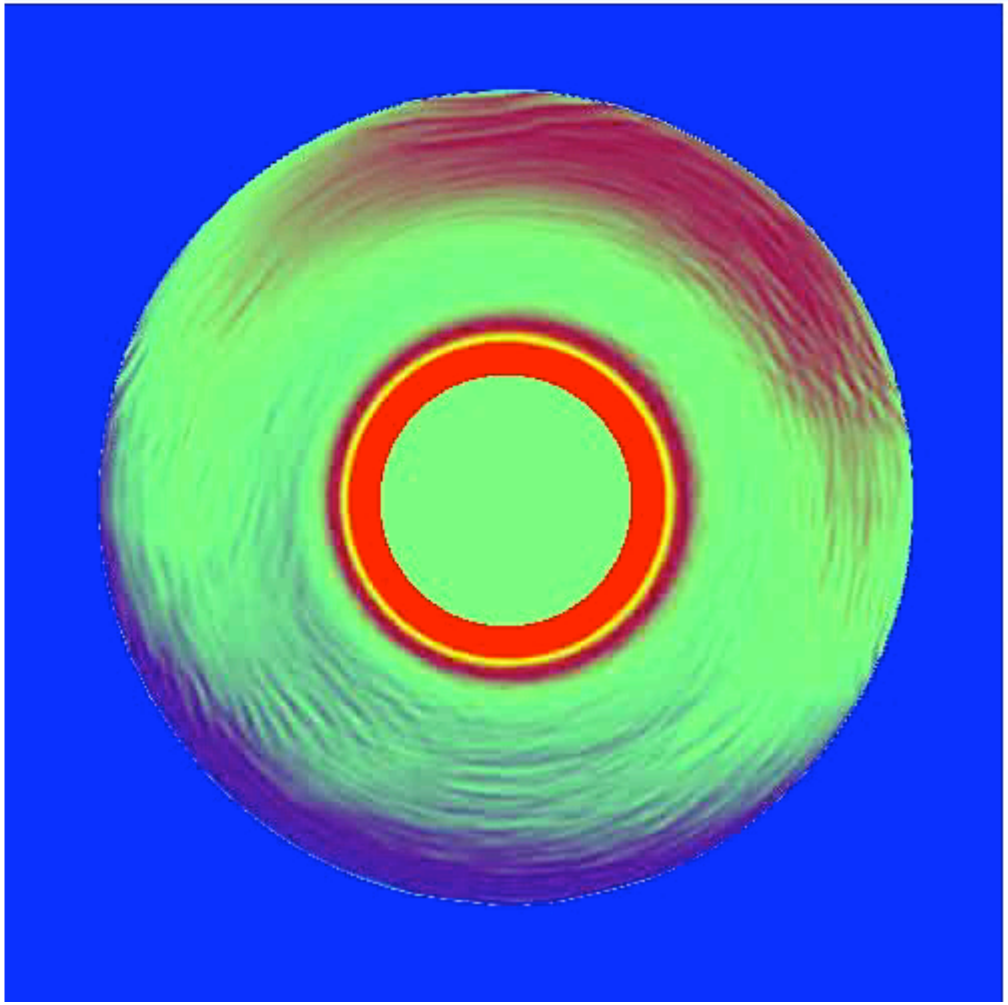
Standing Accretion Shock Instability II.

(courtesy Adam Burrows, University of Arizona;
25 solar mass progenitor)



Standing Accretion Shock Instability III.

Blondin & Shaw 2007, ApJ, 656, 366



To me this looks like a pressure wave propagating around the cavity - I see no sign of any radial propagation of anything. However, I have no doubt that Thierry can come up with some alternative explanation.

Have fun in Paris.

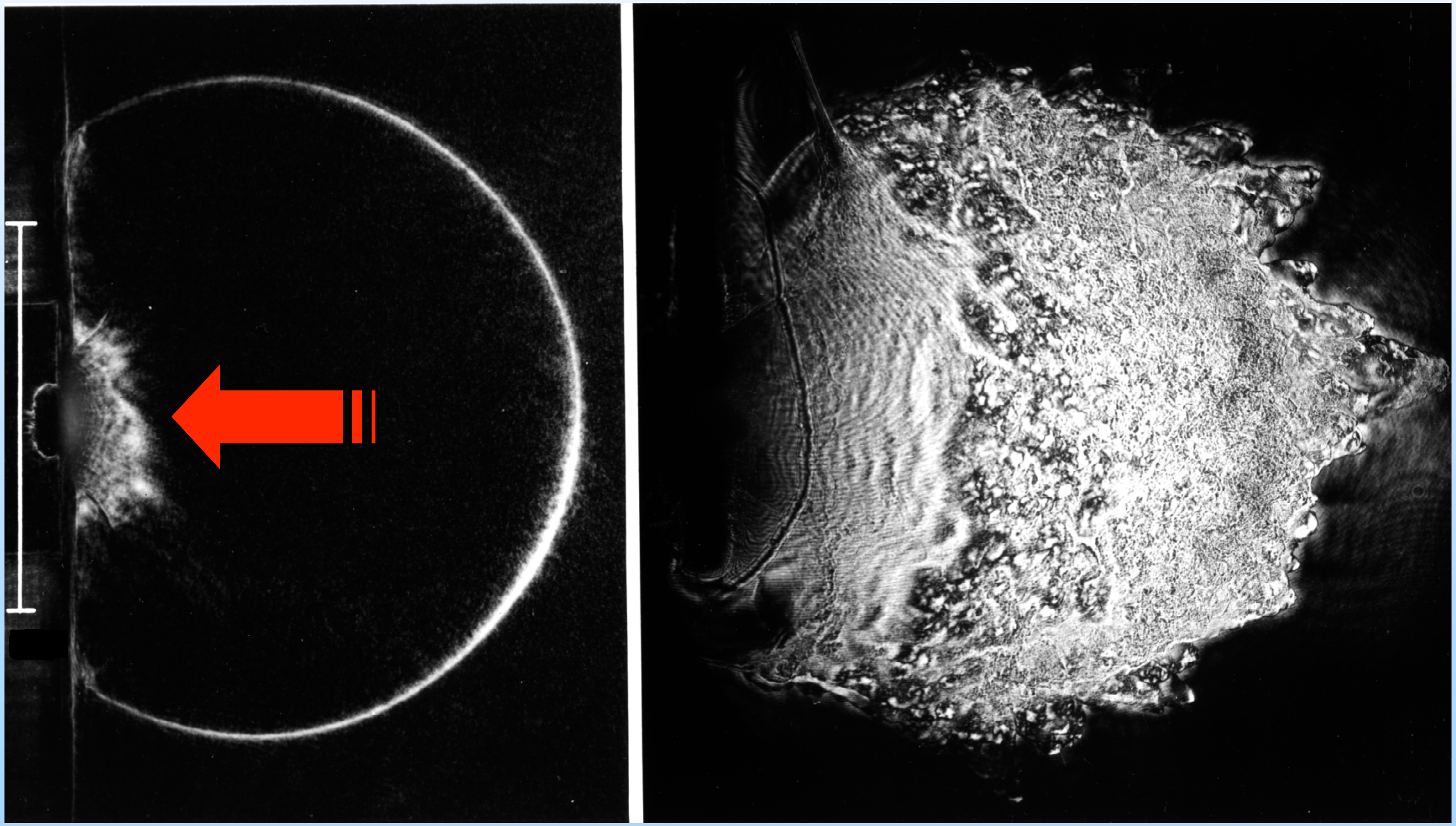
-John

… so pursue an analytic approximation following Vishniac & Ryu (1989) for more insight

Overstable Radiative Blast Waves first observed by Grun et al. 1991

Stable shock in N gas | Unstable shock in Xe gas

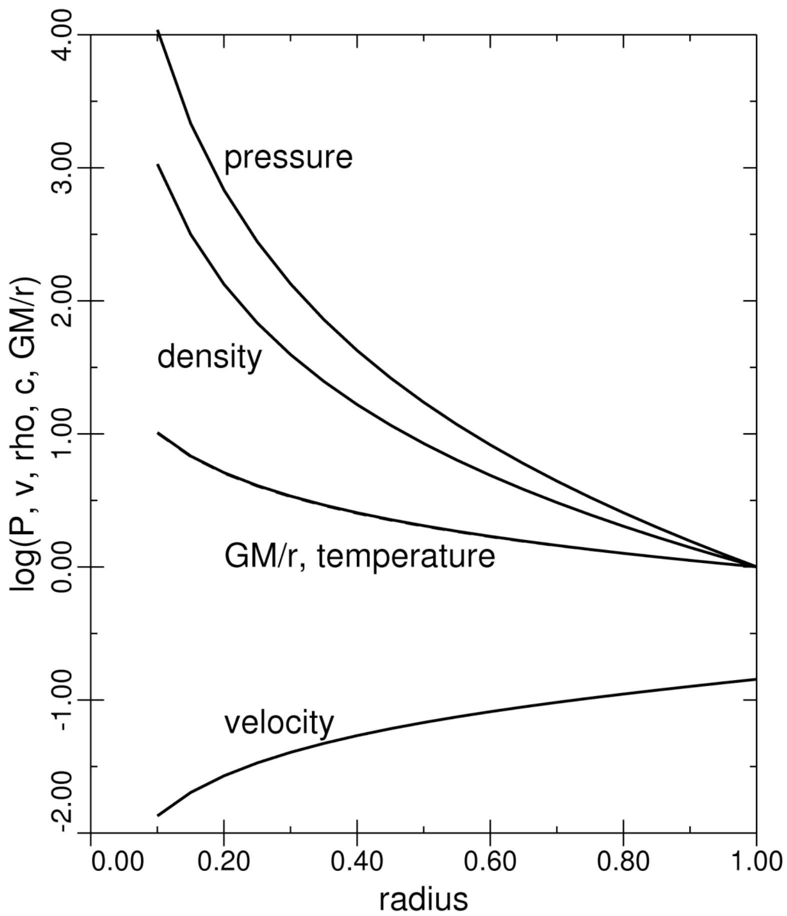
1 cm

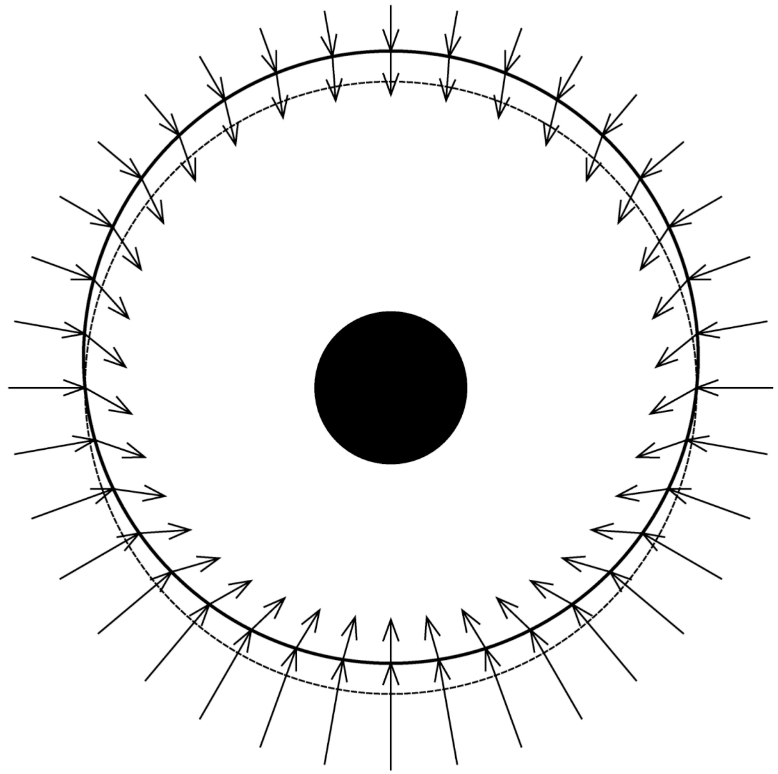


Basic Model, Instability Mechanism

Vishniac & Ryu 1989, ApJ, 337, 917

Velikovich et al. 2005, Phys. Rev. E, 72, 6306





Simple Dispersion Relation: assume postshock advection → 0

$\omega^4 - \omega^2[l(l+1)c_s^2/r_s^2 + \text{other terms going as } 1/a]$
$+l(l+1)c_s^4/ar_s^4(r_s/r_i)^Q/((r_s/r_i)^Q-1)$
$-\text{other term going as } 1/a = 0$

where "a" parameterizes density at r_i, $Q > 0$, and depends on model parameters. For moderate "a":

$\rightarrow \omega^2 \sim l(l+1)c_s^2/2r_s^2$ +/-
$[l^2(l+1)^2c_s^4/4r_s^4 - l(l+1)c_s^4/ar_s^4 (r_s/r_i)^Q/((r_s/r_i)^Q-1)]^{1/2}$

Complex roots for small l (but not l=0), which disappear as $r_i/r_s \rightarrow 0$
No advection involved, so presumably these are acoustic waves.

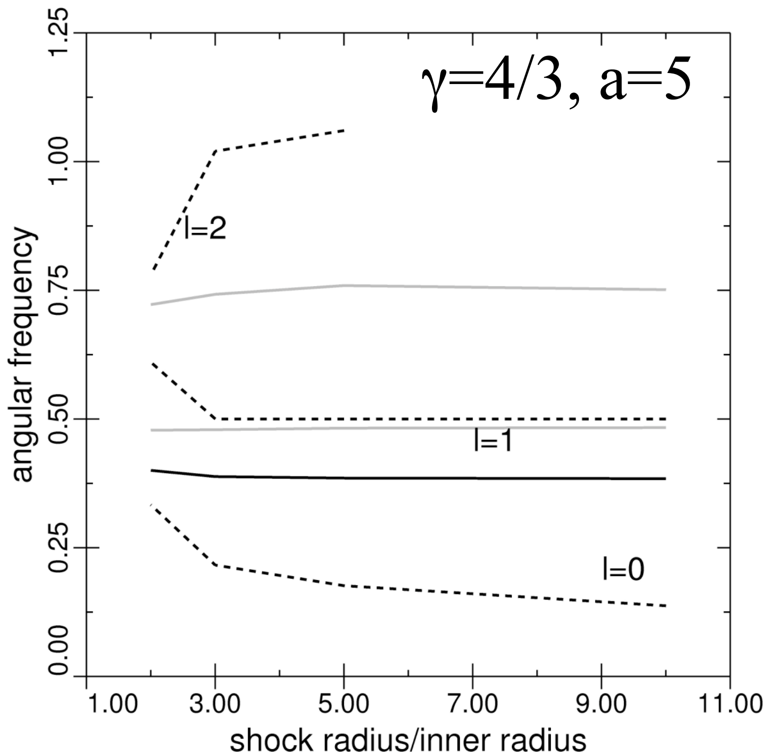
Not so Simple Dispersion Relation: assume postshock advection=u (const)

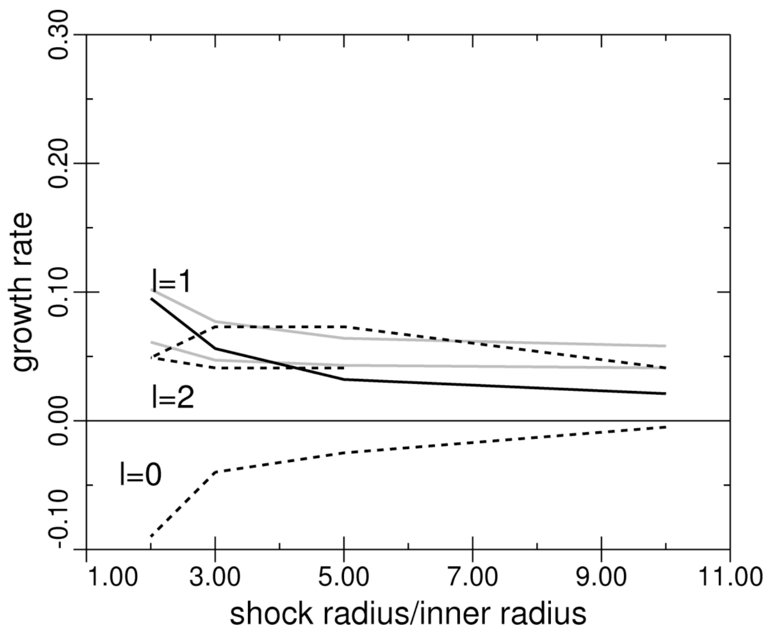
$\omega^4 + i\omega^3\, u/r_s$
$-\omega^2[l(l+1)c_s^{\,2}/r_s^{\,2}$ + other terms going as $1/a$]
$\quad +i\omega u[c_s^{\,2}/r_s^{\,3}$ + other terms going as $1/a]$
$\quad +l(l+1)c_s^{\,4}/ar_s^{\,4}(r_s/r_i)^Q/((r_s/r_i)^Q-1) + O(1/a)\ \ldots$
$-iu/\omega[r_s\ l(l+1)c_s^{\,4}/ar_s^{\,4}\ 2/((r_s/r_i)^Q-1) + \ldots] = 0$

Complex roots for wider range of r_i/r_s, in some cases $u < 0$ essential to instability. Advective-acoustic instability as in Foglizzo et al.?

Quantitative comparison with Blondin et al.

- Correct two known errors in Laming (2007, erratum forthcoming)
- Residual problems with l=0 stability for cases with advection included, probably due to expansion in $u_r/\omega r$ (Yamazaki & Foglizzo 2008)
- The γ=4/3 case can be done more exactly, with (so far) insignificant changes, but remains to be properly checked/implemented.

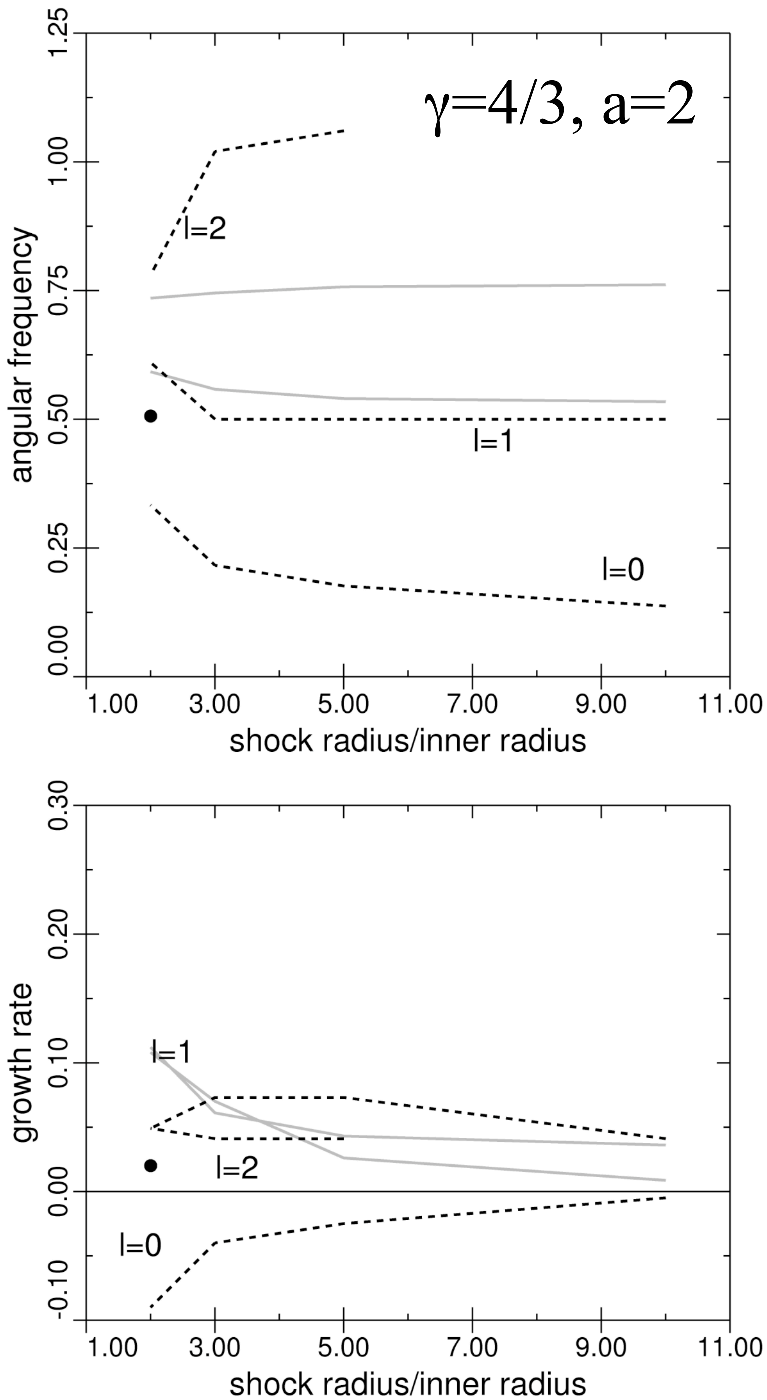




Best agreement with Blondin & Mezzacappa:

Dashed lines – Blondin/Mezza
Solid black – acoustic case (u_r=0)
Solid grey – $u_r < 0$

l=1 acoustic case is unstable for all radii, though the $u_r < 0$ case has significantly higher growth as radius increases



Lower interior cooling, modes move to slightly higher frequencies

l=1 acoustic case is only unstable at smallest radius. Elsewhere postshock advection is required.

Not so Simple Dispersion Relation: assume postshock advection=u (const)

$\omega^4 + i\omega^3\, u/r_s$

$-\omega^2[l(l+1)c_s^{\,2}/r_s^{\,2}$ + other terms going as $1/a\,]$

$\quad +i\omega u[c_s^{\,2}/r_s^{\,3}$ + other terms going as $1/a]$

$\quad +l(l+1)c_s^{\,4}/ar_s^{\,4}(r_s/r_i)^Q/((r_s/r_i)^Q-1) + O(1/a)\ \ldots$

$-iu/\omega[r_s\ l(l+1)c_s^{\,4}/ar_s^{\,4}\ 2/((r_s/r_i)^Q-1) + \ldots] = 0$

Complex roots for wider range of r_i/r_s, in some cases $u < 0$ essential to instability. Advective-acoustic instability as in Foglizzo et al.?

Add Rotation, ang. freq = Ω

$\omega^4 + i\omega^3\, u/r_s$

$-\omega^2[l(l+1)c_s^{\,2}/r_s^{\,2}$ + other terms going as 1/a]

$+i\omega u[c_s^{\,2}/r_s^{\,3}$ + other terms going as 1/a]

$-\omega m\Omega c_s^{\,2}/r_s^{\,2}$

$+l(l+1)c_s^{\,4}/ar_s^{\,4}(r_s/r_i)^Q/((r_s/r_i)^Q-1) + O(1/a)\ \ldots$

$-iu/\omega[r_s\ l(l+1)c_s^{\,4}/ar_s^{\,4}\ 2/((r_s/r_i)^Q-1) + \ldots]$

$+l(l+1)c_s^{\,4}/r_s^{\,4}(r_i/r_s)m\Omega/\omega$

$+2c_s^{\,4}/r_s^{\,4}(r_i/r_s)m\Omega/\omega = 0$

→ Stronger growth for m=+/-1 than for m=0. Compare with Yamazaki & Foglizzo (2008, ApJ, 679, 607)

Cassiopeia A in 1 Ms VLP Chandra Observation

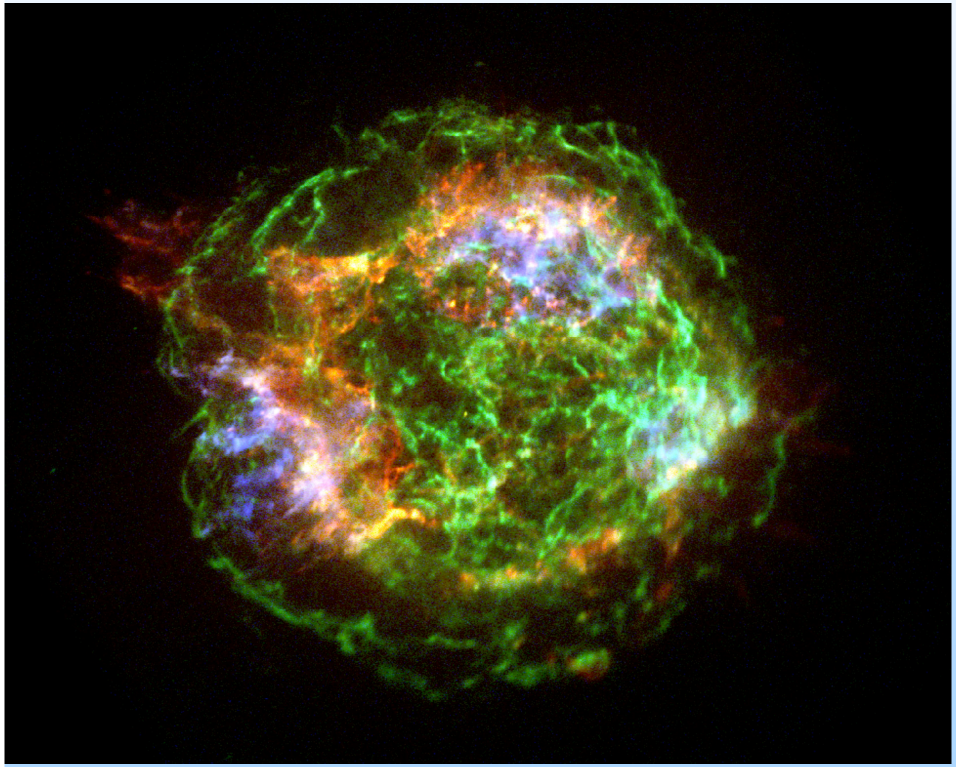


Image made by Una Hwang

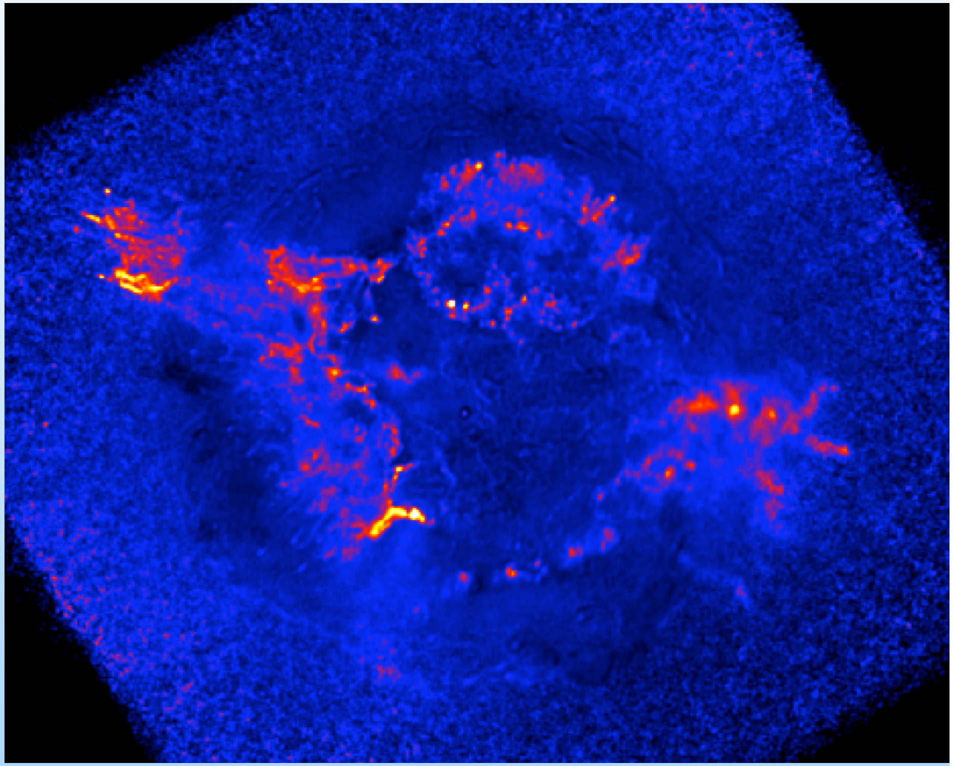
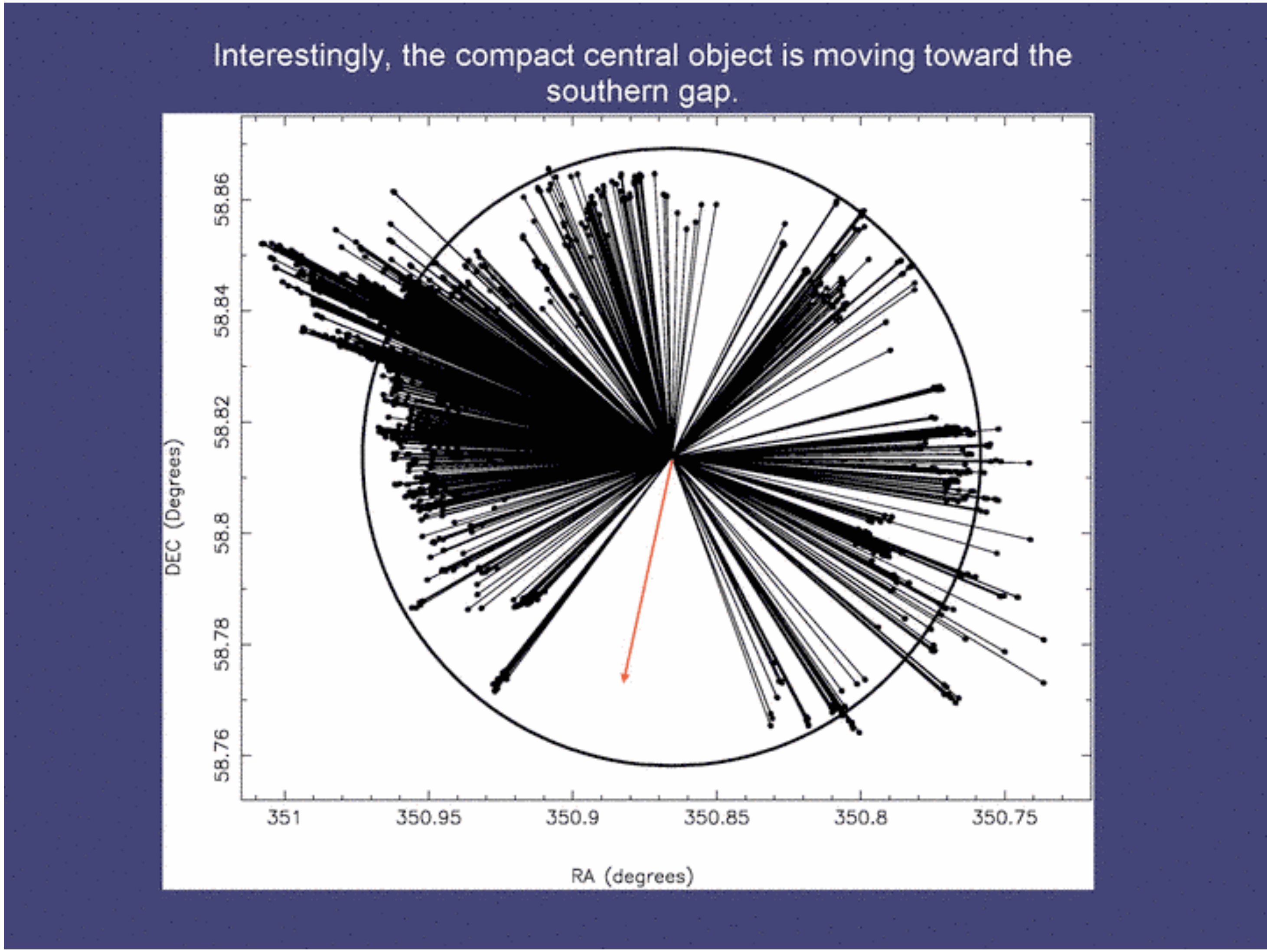


Image made by Jacco Vink

Optical Knot Motions and CCO kick

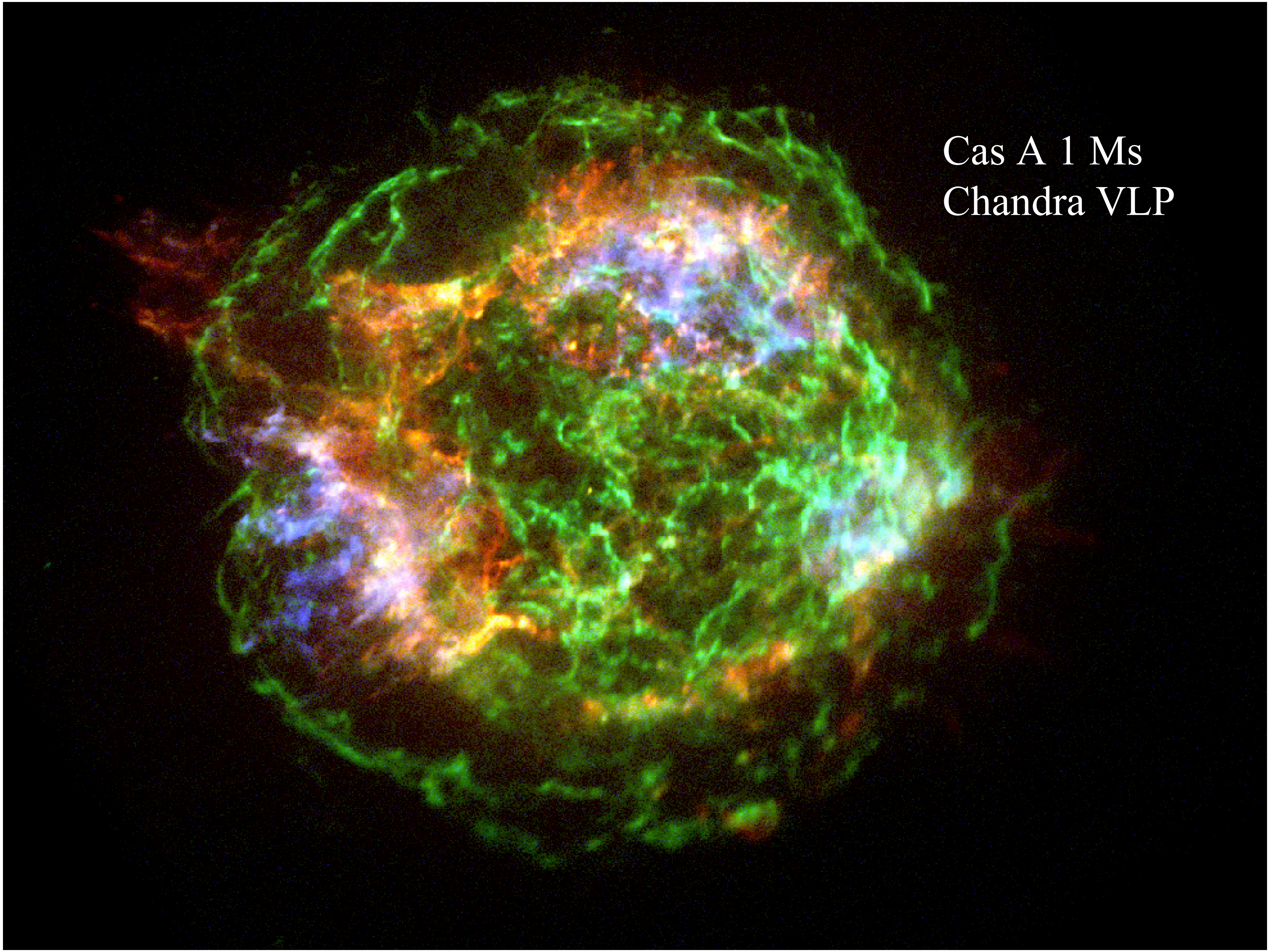
(presented by R. Fesen at KITP conference, 2/6/06)



See also Wex, Kalogera & Kramer (2000, ApJ, 528, 401) on PSR B1913+16

Conclusions

- Advective-acoustic cycle operates for large shock radii
- Not so clear at smaller shock radii, this work offers some support to Blondin's interpretation of an acoustic cycle, though the model here is not exactly the same as Blondin's. Seems to need special conditions at the inner boundary. Advective-acoustic cycle has more general applicability.
- In rotating cases, SASI grows strongest in equatorial plane, possibly relevant to pulsar natal kicks?
- How to relate "a" to real physics?


Cas A 1 Ms
Chandra VLP