Theory of tidal dissipation in stars and giant planets

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Review article

Tidal Dissipation in Stars and Giant Planets

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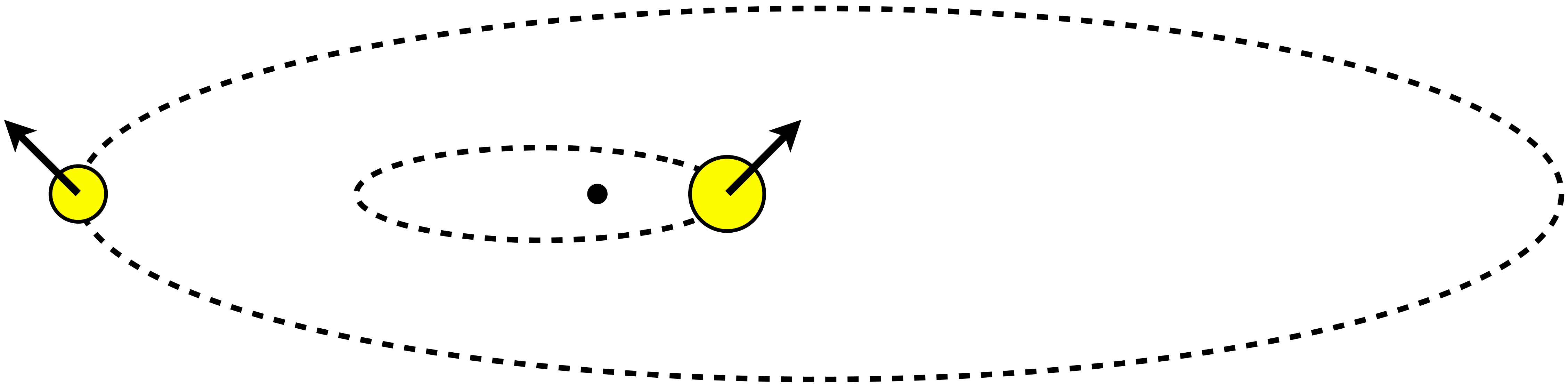
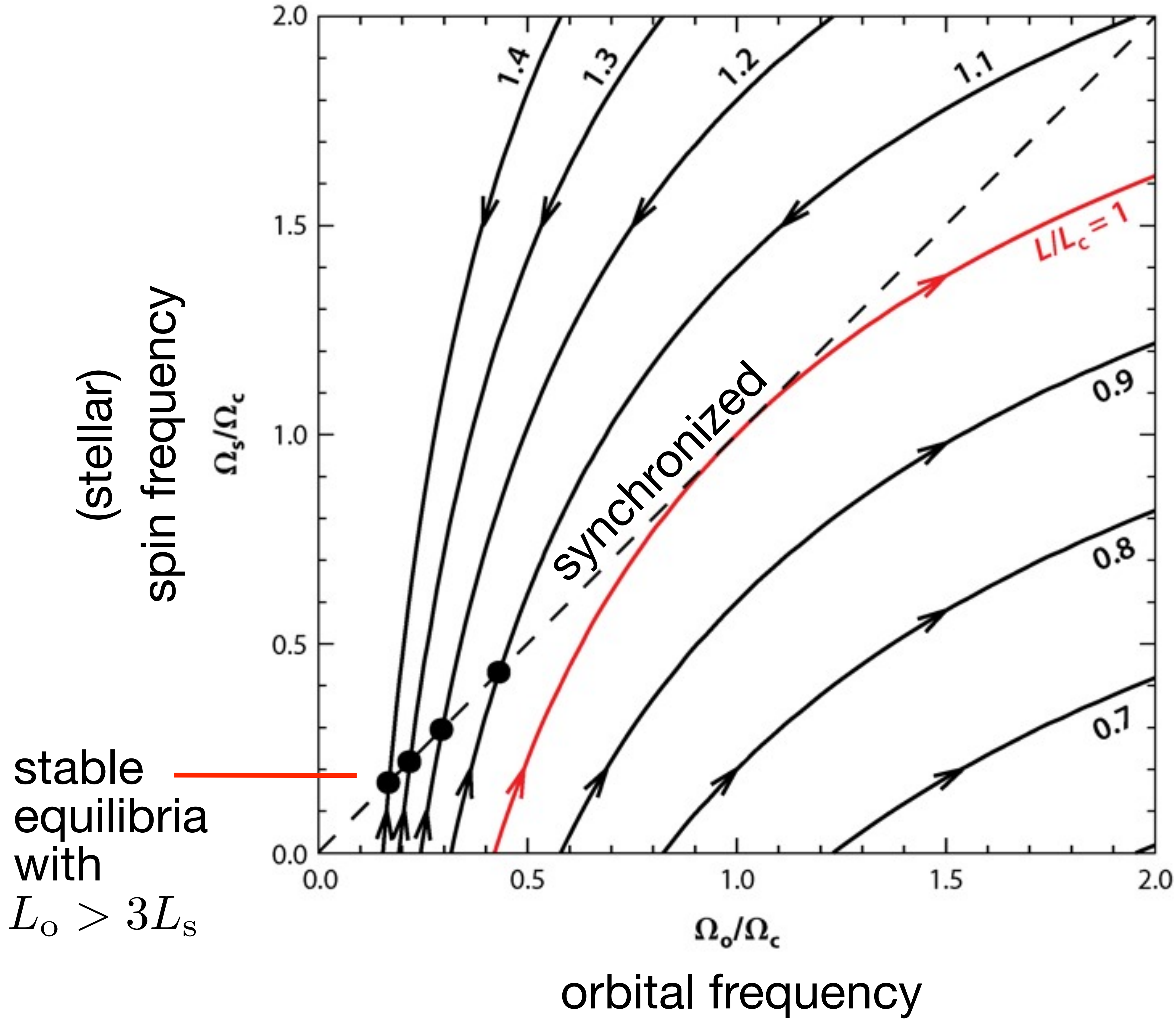
The tidal problem



- Time-dependent deformation: dissipation, power and torque
- How do the spins and orbit evolve on astronomical time-scales?
- Typical outcomes: synchronization, alignment, circularization
- No tidal equilibrium if total angular momentum too small
- Tidal equilibrium may be inaccessible in practice



(stellar)
spin frequency
Ω_s/Ω_c
Ω_o/Ω_c
orbital frequency
synchronized
$L/L_c = 1$
1.4
1.3
1.2
1.1
0.9
0.8
0.7
stable equilibria with $L_{\rm o} > 3L_{\rm s}$
0.0
0.5
1.0
1.5
2.0

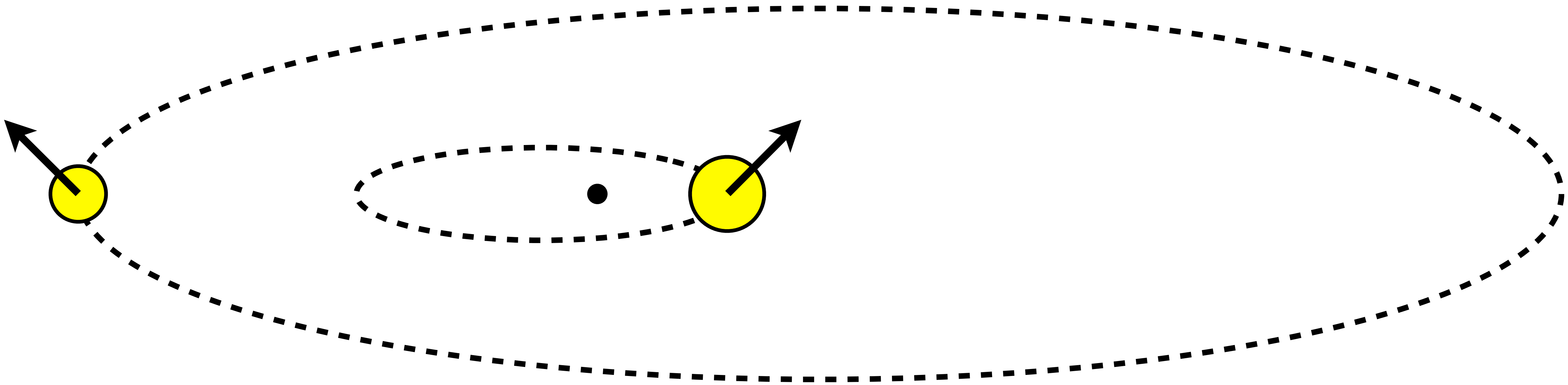
Critical angular momentum and tidal equilibria

$$L_\mathrm{c} = 4I\Omega_\mathrm{c}, \qquad \Omega_\mathrm{c} = (GM)^{1/2}\left(\frac{\mu}{3I}\right)^{3/4}$$

$$M = M_1 + M_2, \quad \mu = \frac{M_1 M_2}{M}, \quad I = I_1 + I_2$$

- For $M_1 = 1\,M_\odot$, stable tidal equilibria have $P \gtrsim 7\left(\frac{M_2}{M_\mathrm{J}}\right)^{-3/4}\ \mathrm{day}$ and so are inaccessible for $M_2 \lesssim M_\mathrm{J}$
- Stable tidal equilibria most relevant for planets of 10+ Jupiter masses and orbital periods of 3+ days

The tidal problem



- Linear versus nonlinear tides ➜
- Q, Q', Im(k), etc. (limitations)
- Tide in star versus tide in planet
- Nearly circular (harmonic) versus nearly parabolic (impulsive)

Tidal amplitudes

$$\epsilon = \frac{M_2}{M_1}\left(\frac{R_1}{d}\right)^3$$

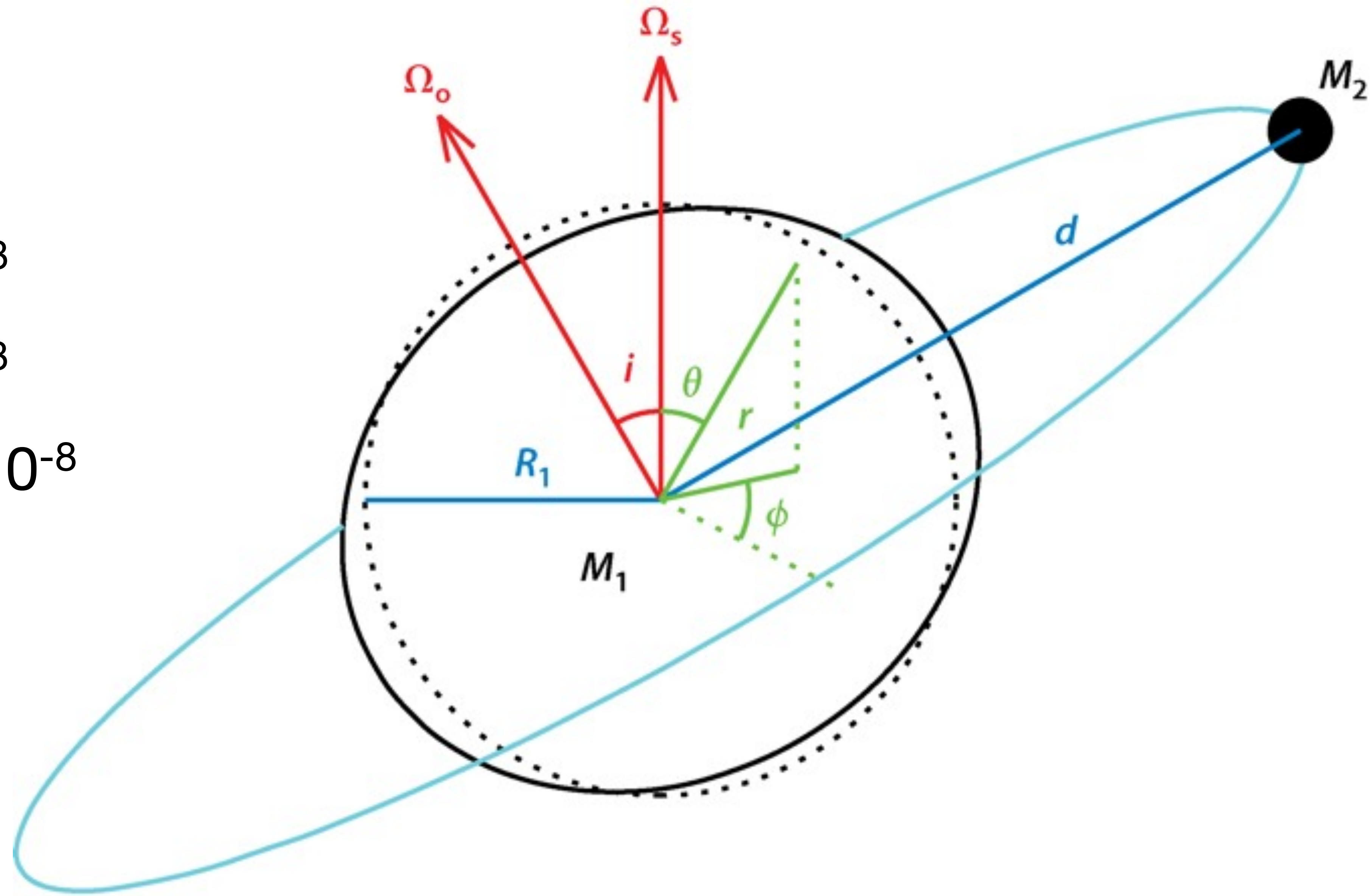
Jupiter–Io: 2 x 10^{-7}

Saturn–Titan: 3 x 10^{-8}

Uranus–Ariel: 4 x 10^{-8}

Neptune–Triton: 8 x 10^{-8}

WASP-19 b: 6 x 10^{-2}



- Internal nonlinearities can occur even when $\epsilon \ll 1$

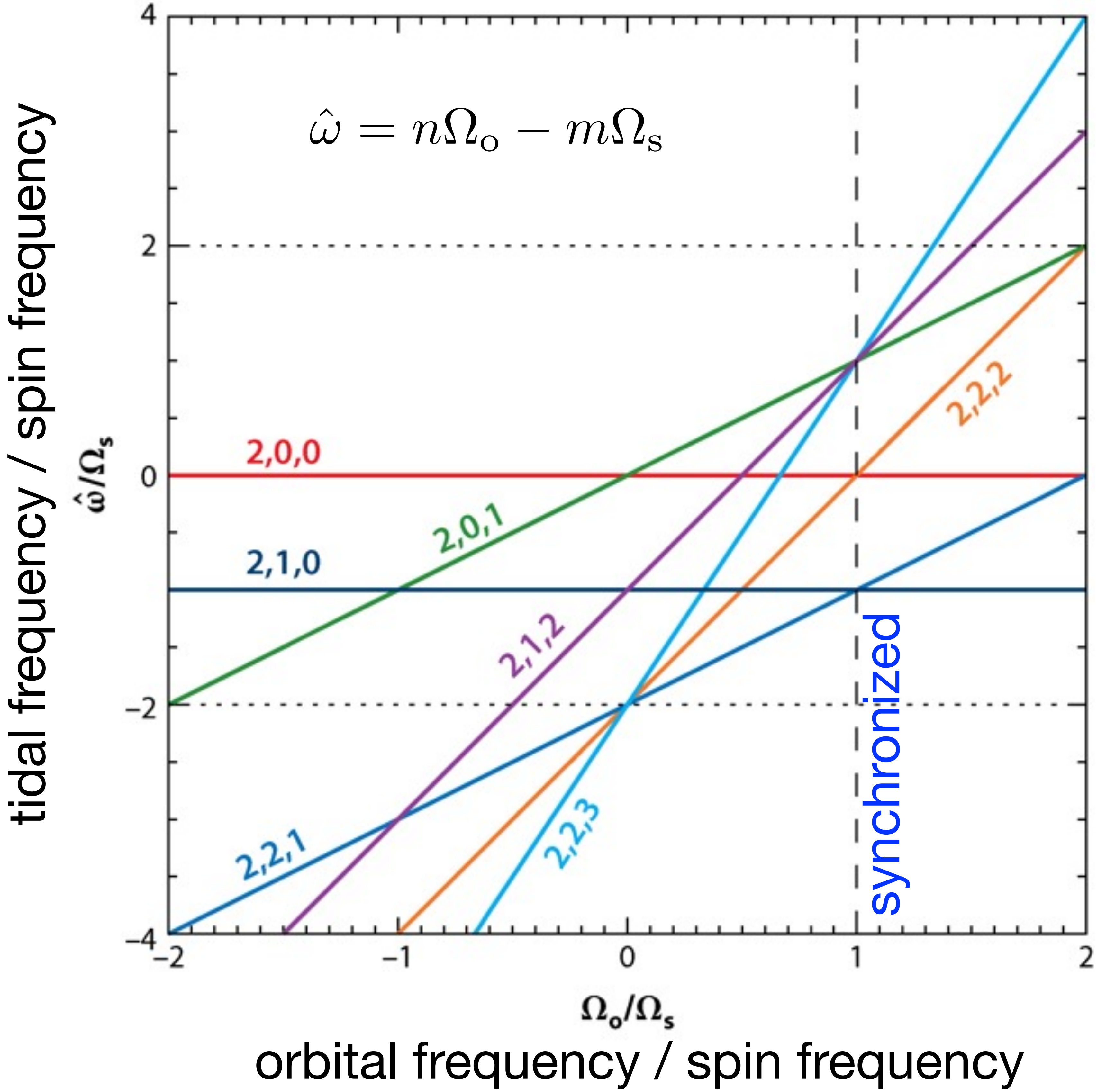
Tidal forcing

$$\Psi = \mathrm{Re} \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{n=-\infty}^{\infty} \frac{GM_2}{a} A_{l,m,n}(e,i) \left(\frac{r}{a}\right)^l Y_l^m(\theta,\phi)\, \mathrm{e}^{-in\Omega_o t}$$

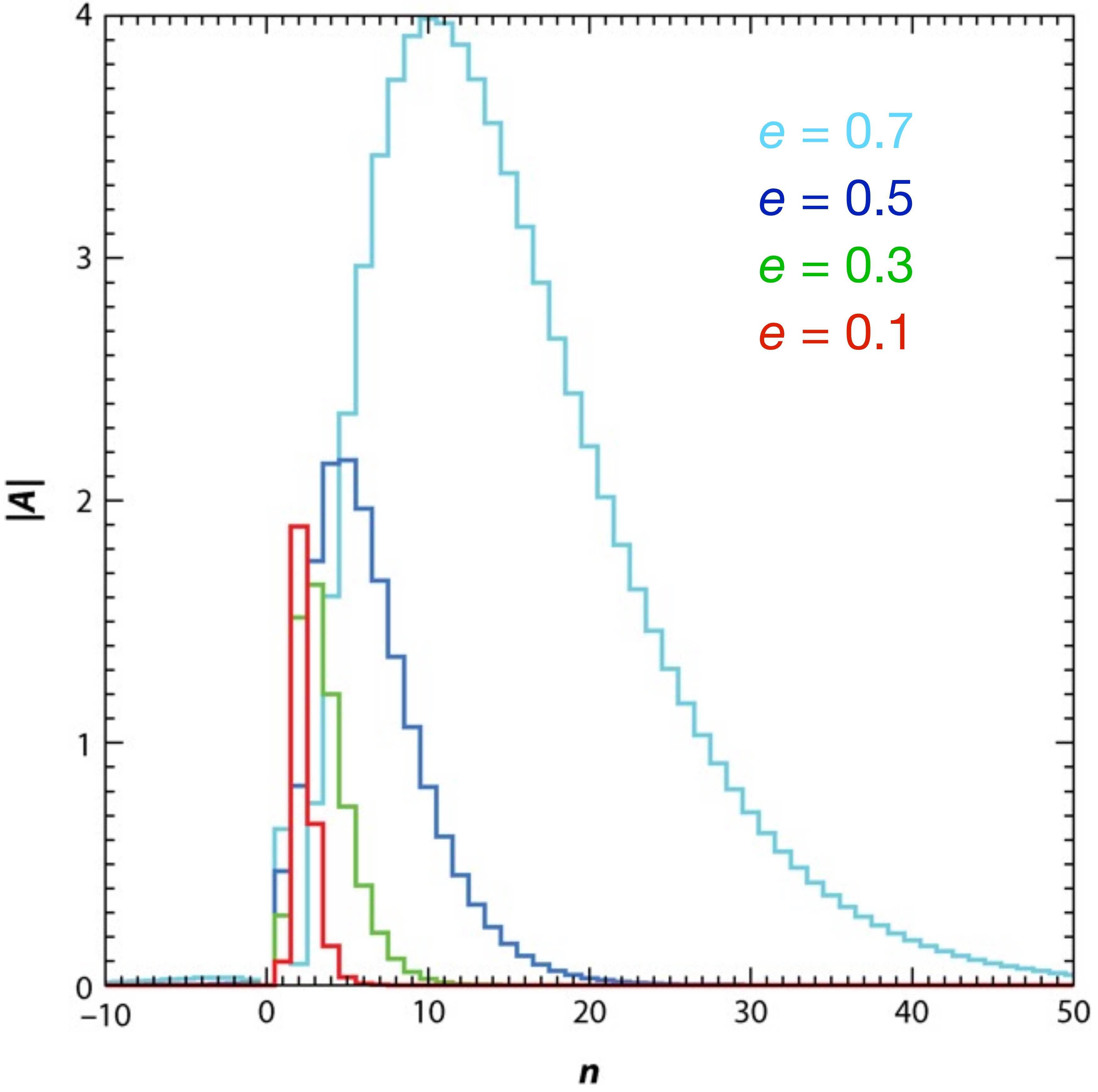
Quadrupolar components up to first order in *e* and *i* :

l	m	n	$\lvert A \rvert$	Description
2	0	0	$\sqrt{\frac{\pi}{5}}$	Static tide
2	2	2	$\sqrt{\frac{6\pi}{5}}$	Asynchronous tide
2	0	1	$3e\sqrt{\frac{\pi}{5}}$	Eccentricity tides
2	2	1	$\frac{1}{2}e\sqrt{\frac{6\pi}{5}}$	
2	2	3	$\frac{7}{2}e\sqrt{\frac{6\pi}{5}}$	
2	1	0	$i\sqrt{\frac{6\pi}{5}}$	Obliquity tides
2	1	2	$i\sqrt{\frac{6\pi}{5}}$	

Tidal forcing frequencies



Higher eccentricities



Tidal response

$$\Psi = \mathrm{Re} \sum_{l=2}^{\infty} \sum_{m=0}^{l} \sum_{n=-\infty}^{\infty} \frac{GM_2}{a} A_{l,m,n}(e,i) \left(\frac{r}{a}\right)^l Y_l^m(\theta,\phi)\, \mathrm{e}^{-\mathrm{i}n\Omega_\mathrm{o} t}$$

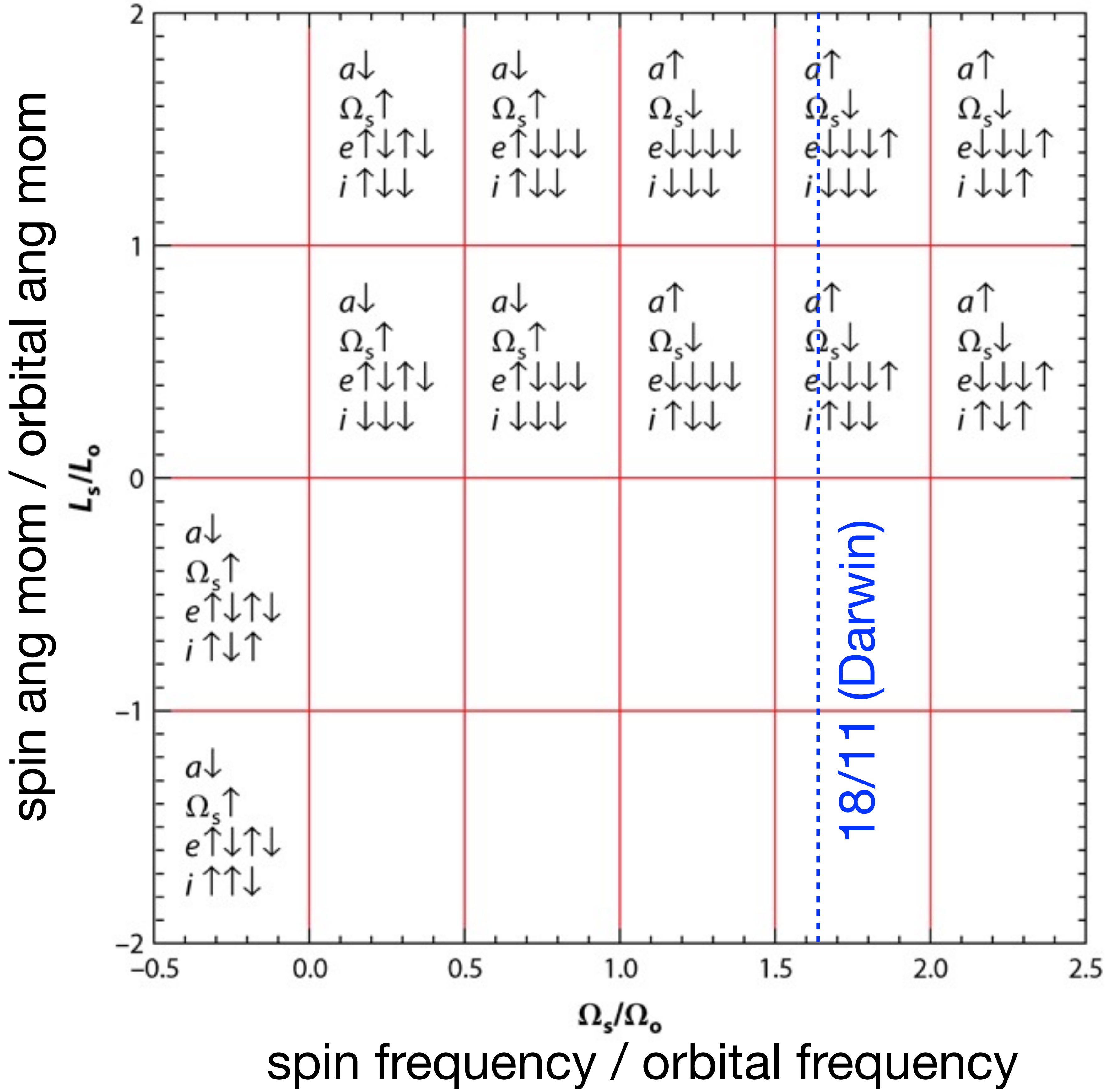
- Love number (response function) :

$$\Psi = \mathrm{Re}\left[\left(\frac{r}{R}\right)^l Y_l^m(\theta,\phi)\, \mathrm{e}^{-\mathrm{i}\omega t}\right]$$

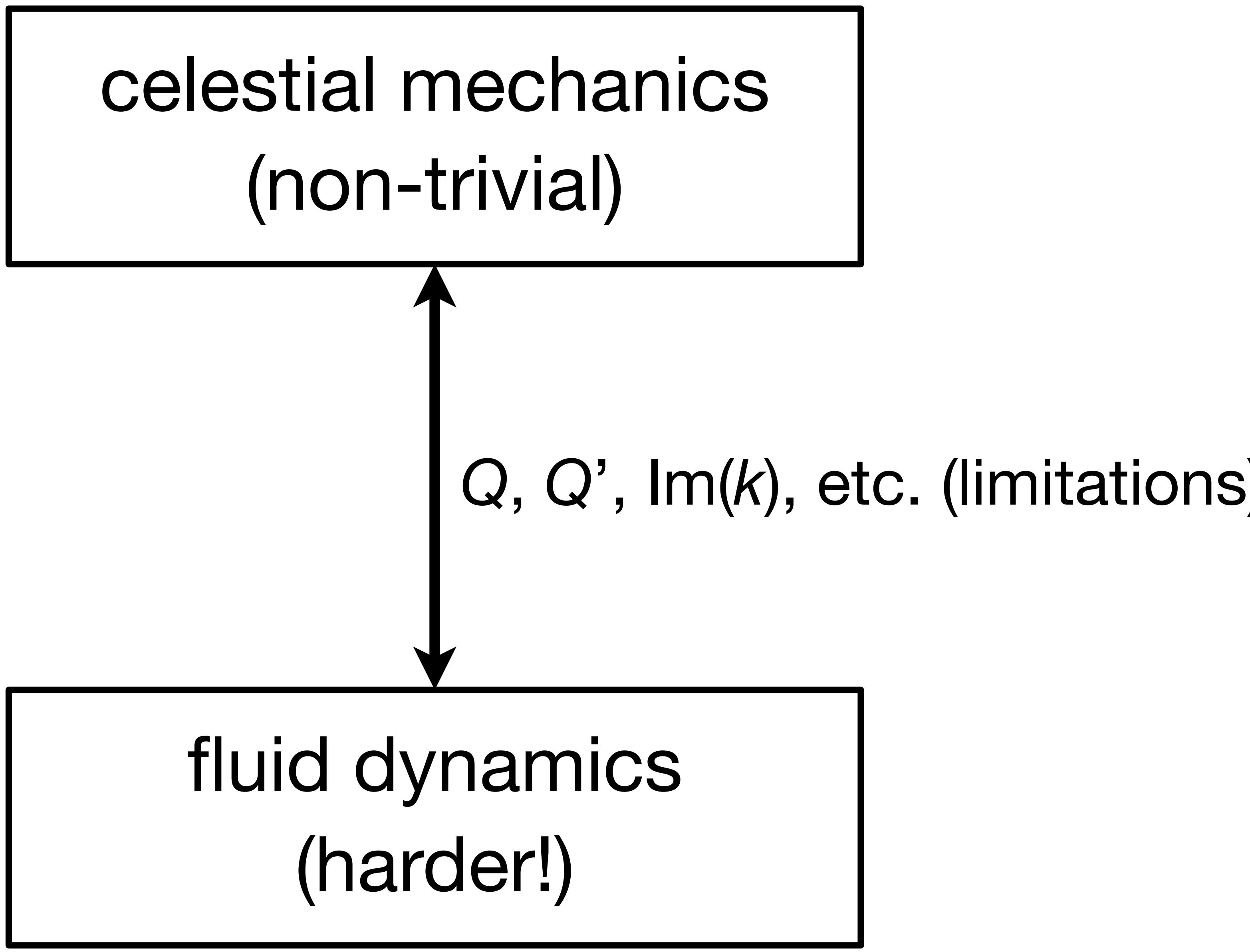
$$\Phi' = \mathrm{Re}\left[k_l^m(\omega)\left(\frac{r}{R}\right)^{-l-1} Y_l^m(\theta,\phi)\, \mathrm{e}^{-\mathrm{i}\omega t}\right] + \text{orthogonal terms}$$

- $\mathrm{Im}(k)$ determines power, torque and dissipation rate

Direction of tidal evolution



Tides in stars and giant planets



Observed quantities relevant to tides

- Orbital period and eccentricity
- Orbital period and planetary mass
- Stellar spin period
- Stellar obliquity (spin–orbit misalignment)
- Planetary radius
- Orbital period decay (?)

...etc.

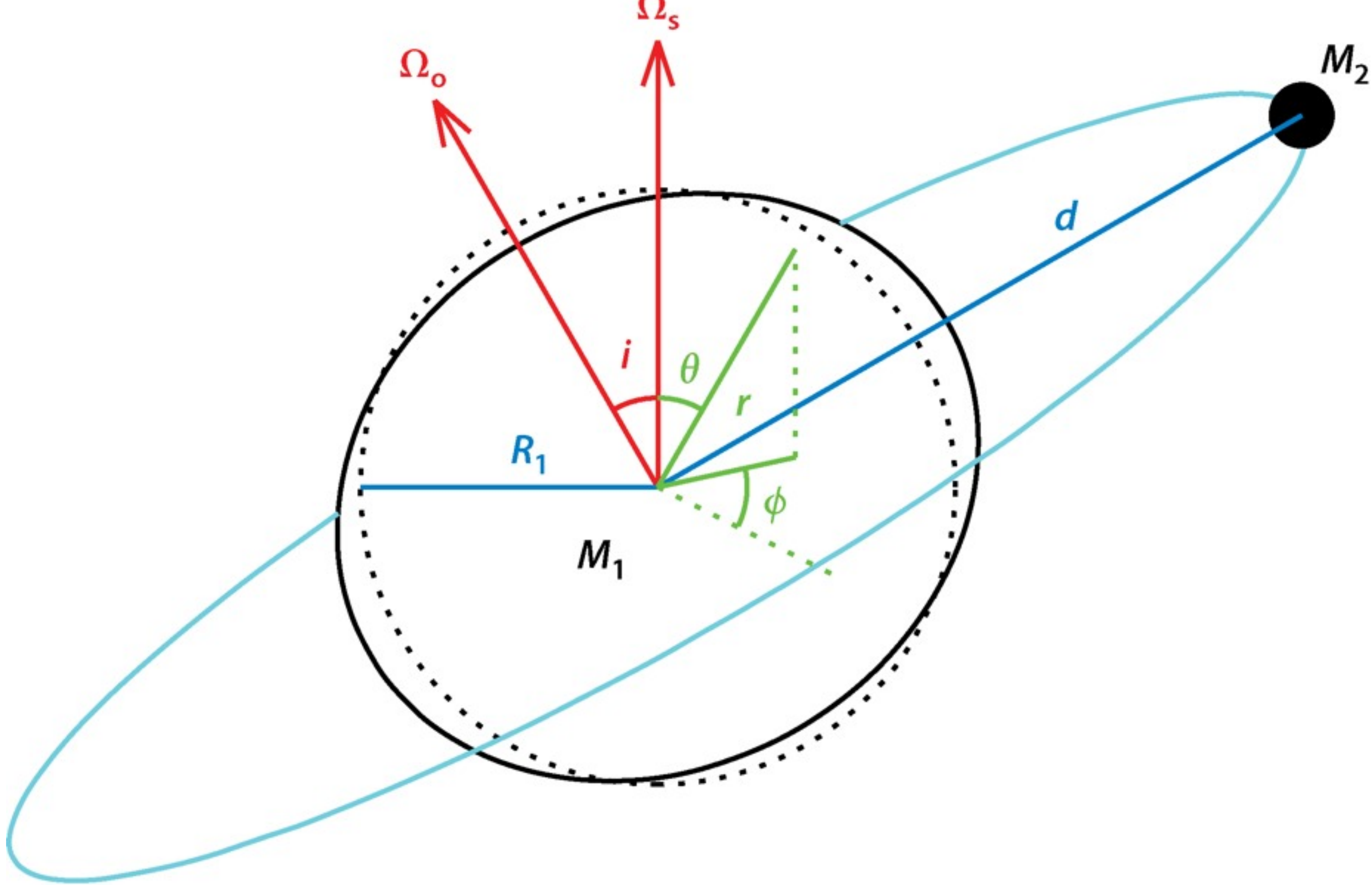
Tidal decomposition

Equilibrium / non-wavelike tide:

- Quasi-hydrostatic spheroidal bulge ➜
- Accompanied by large-scale flow
- Not a complete solution of equations
- Not uniquely defined in neutrally stratified (convective?) regions

Dynamical / wavelike tide:

- Completes solution of equations
- Involves internal (gravity / inertial) waves excited by periodic forcing
- May involve resonances and short length-scales

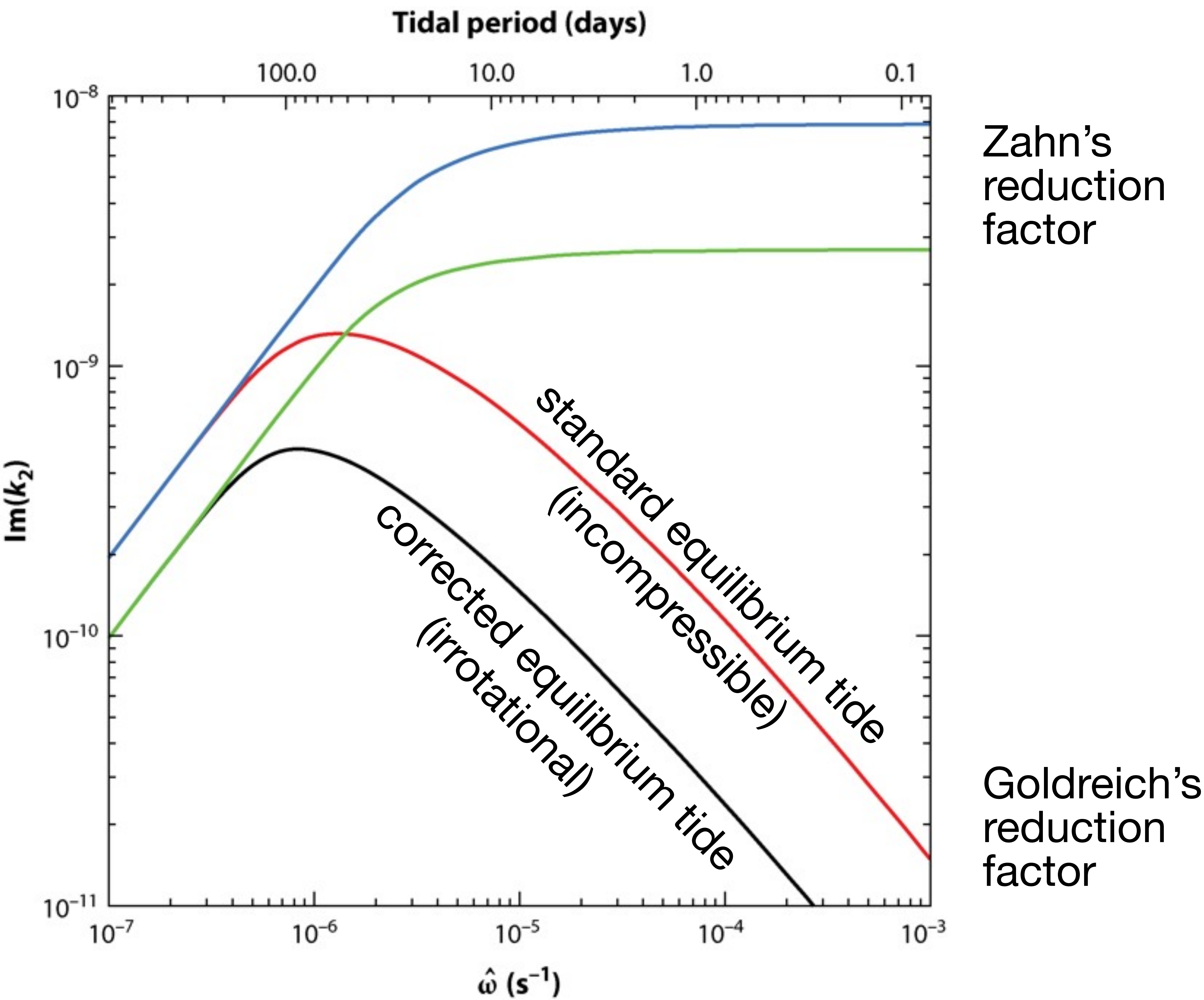


Ω_s
Ω_o
M_2
d
i
θ
r
R_1
ϕ
M_1

Mechanisms of tidal dissipation

Equilibrium / non-wavelike tide:

- Convective turbulent viscosity (MLT x reduction factors) ➜
- Hydrodynamic instability (elliptical / parametric, etc.)
- Multiphase fluids in giant planets (Stevenson)
- Viscoelastic dissipation in solid cores (if present)

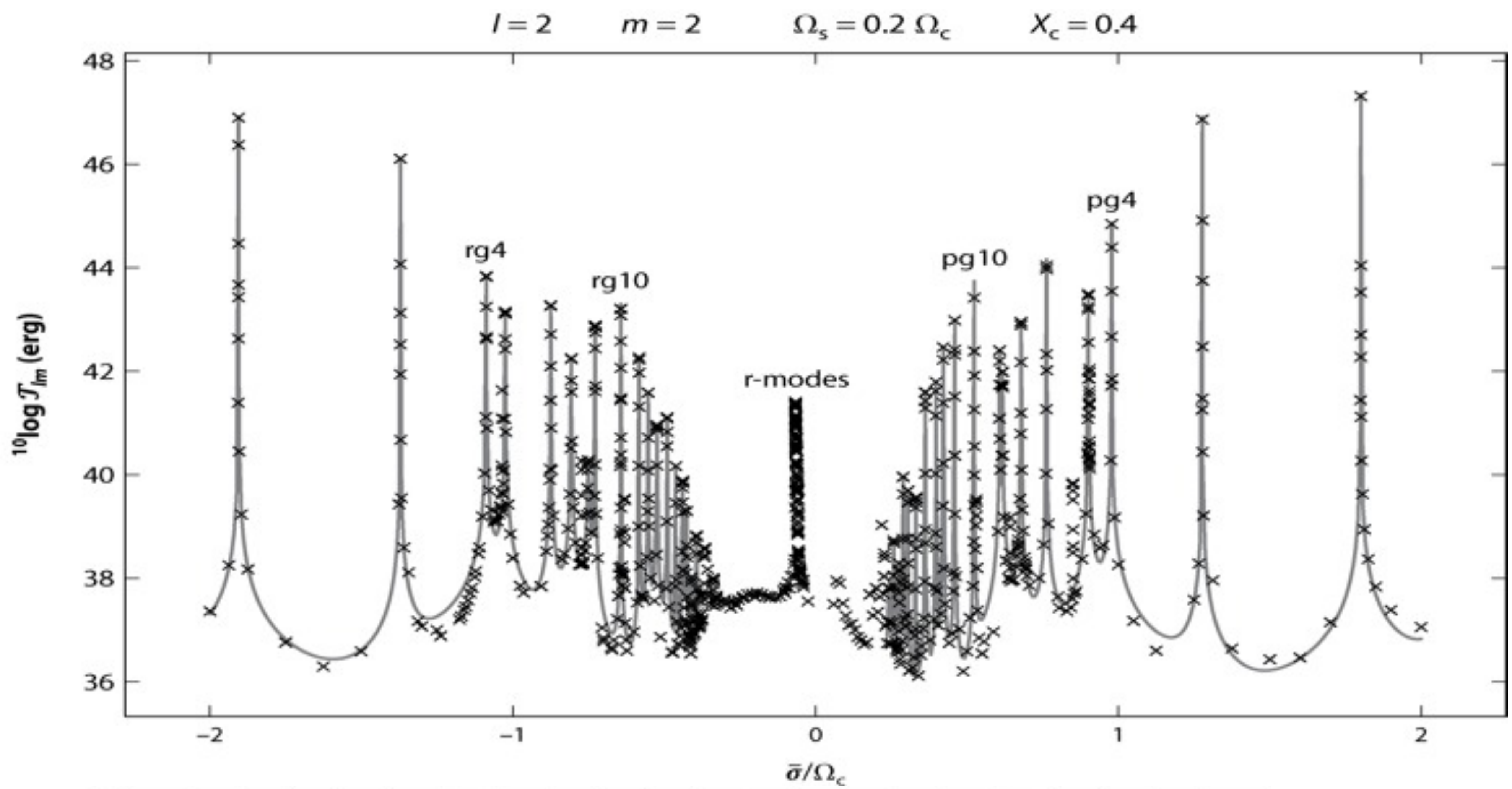


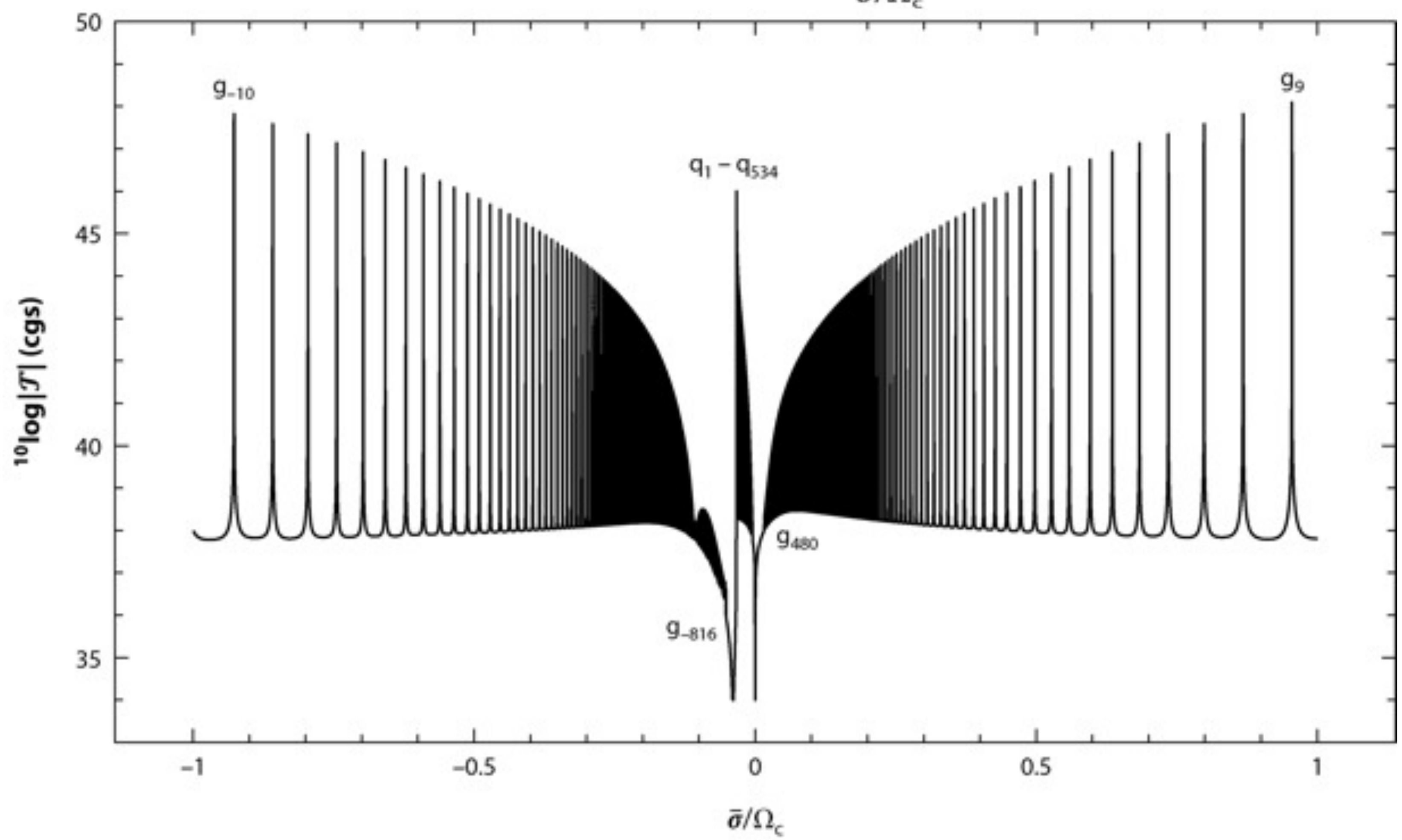
Tidal period (days)
100.0
10.0
1.0
0.1
10^-8
10^-9
10^-10
10^-11
Im(k_2)
10^-7
10^-6
10^-5
10^-4
10^-3
ω̂ (s^-1)
Zahn's reduction factor
Goldreich's reduction factor
standard equilibrium tide (incompressible)
corrected equilibrium tide (irrotational)

Mechanisms of tidal dissipation

Dynamical / wavelike tide:

- Radiative zones: internal gravity waves
  - Radiative damping (Zahn; Savonije) ➜
  - Wave breaking / critical-layer absorption (Goldreich; Barker)
  - More effective with deep radiative–convective transition
  - Hot Jupiters (Lubow+ 1997): detailed calculations needed!
- Convective zones: inertial waves
  - Complicated linear response (wave singularities)
  - More effective with larger core (solid or fluid)
- Importance of internal structure (stratification, core, interfaces, etc.)
- Zonal flows / differential rotation (Favier+ 2014)
- Complex response curves: resonance locking?





Savonije & Witte

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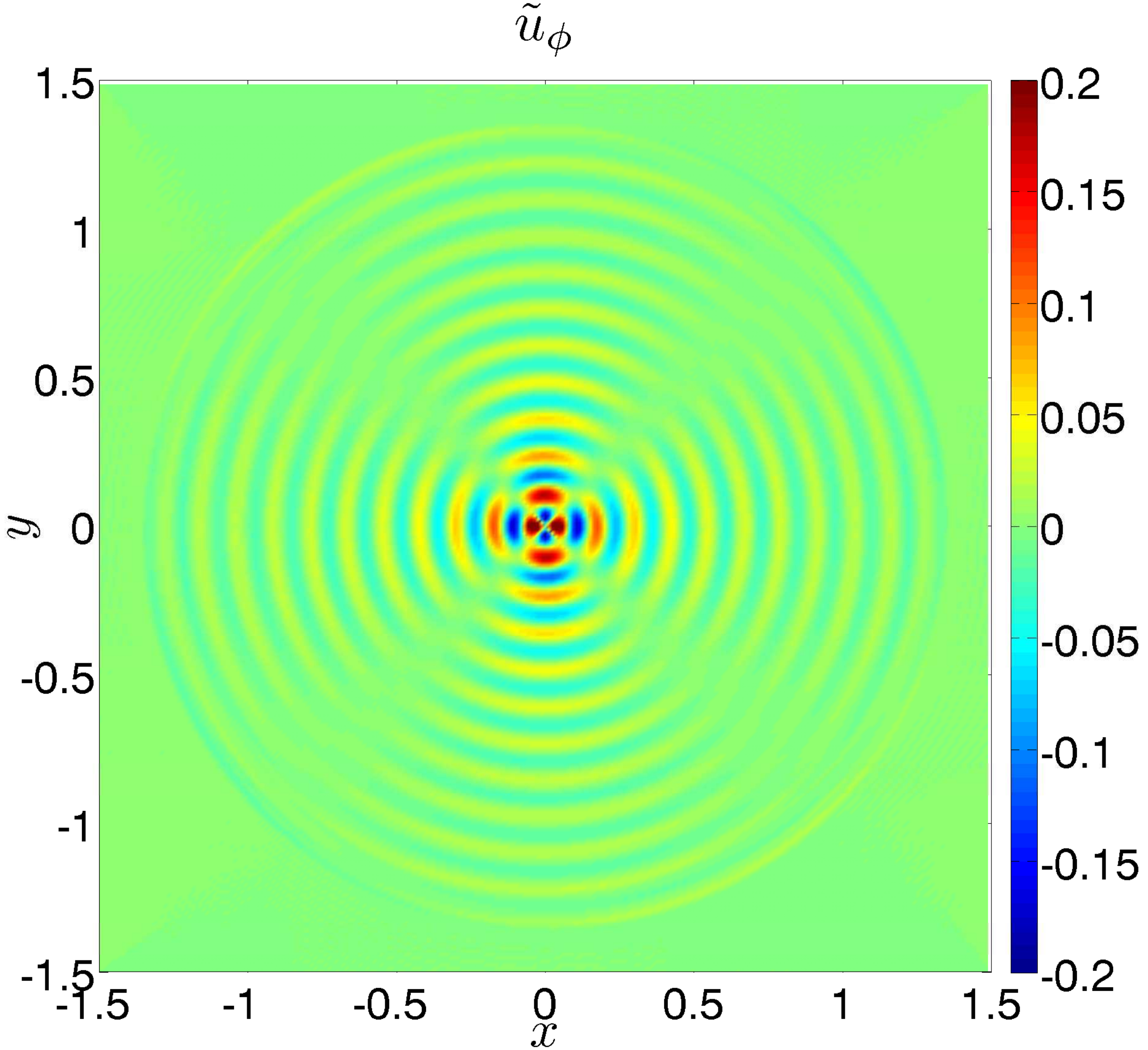
3D numerical simulations

Barker & Ogilvie 2011

Lower amplitude: standing wave

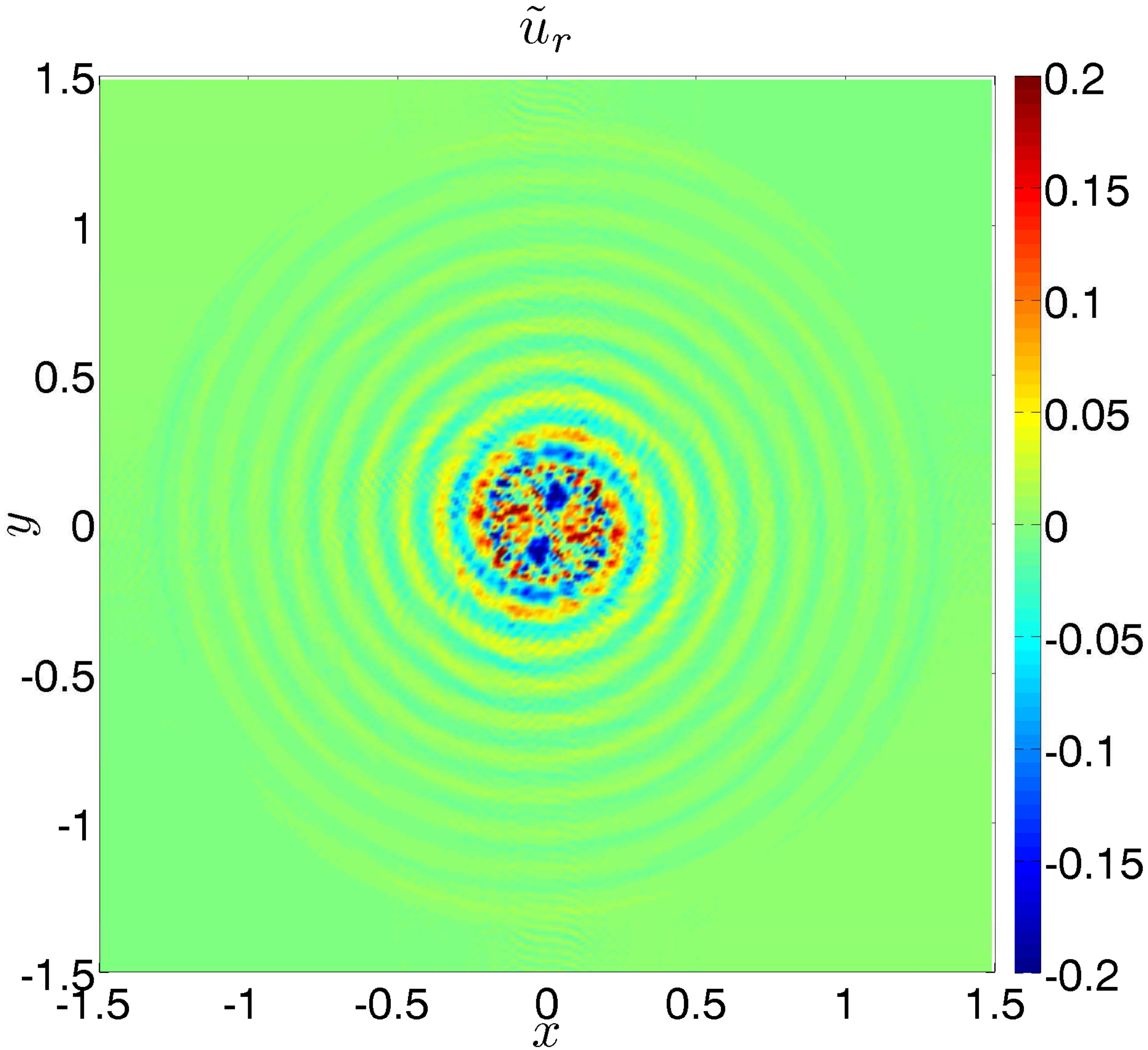


equatorial plane

3D numerical simulations

Barker & Ogilvie 2011

Higher amplitude: breaking wave

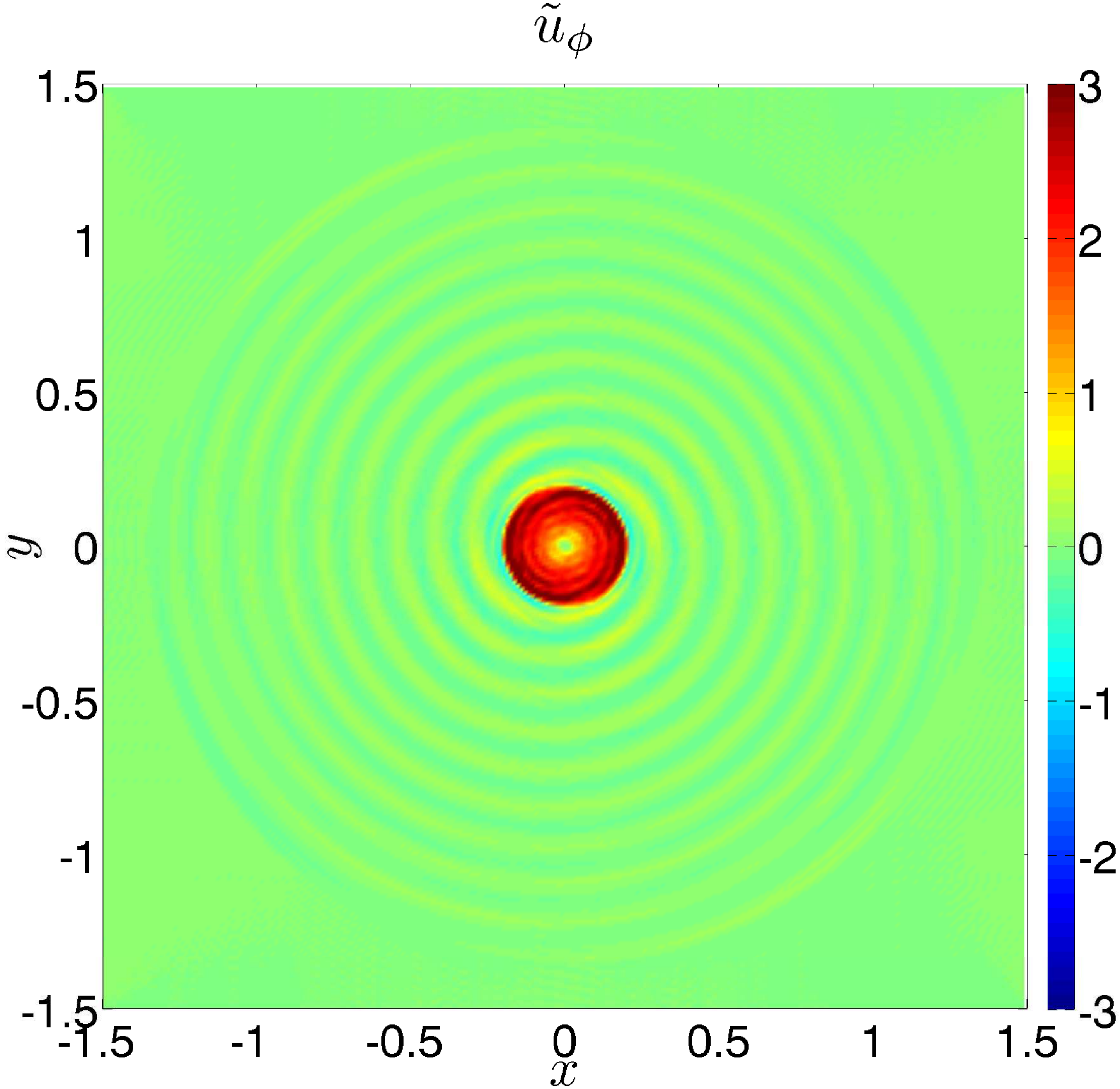


equatorial plane

3D numerical simulations

Barker & Ogilvie 2011

Higher amplitude: breaking wave



meridional plane

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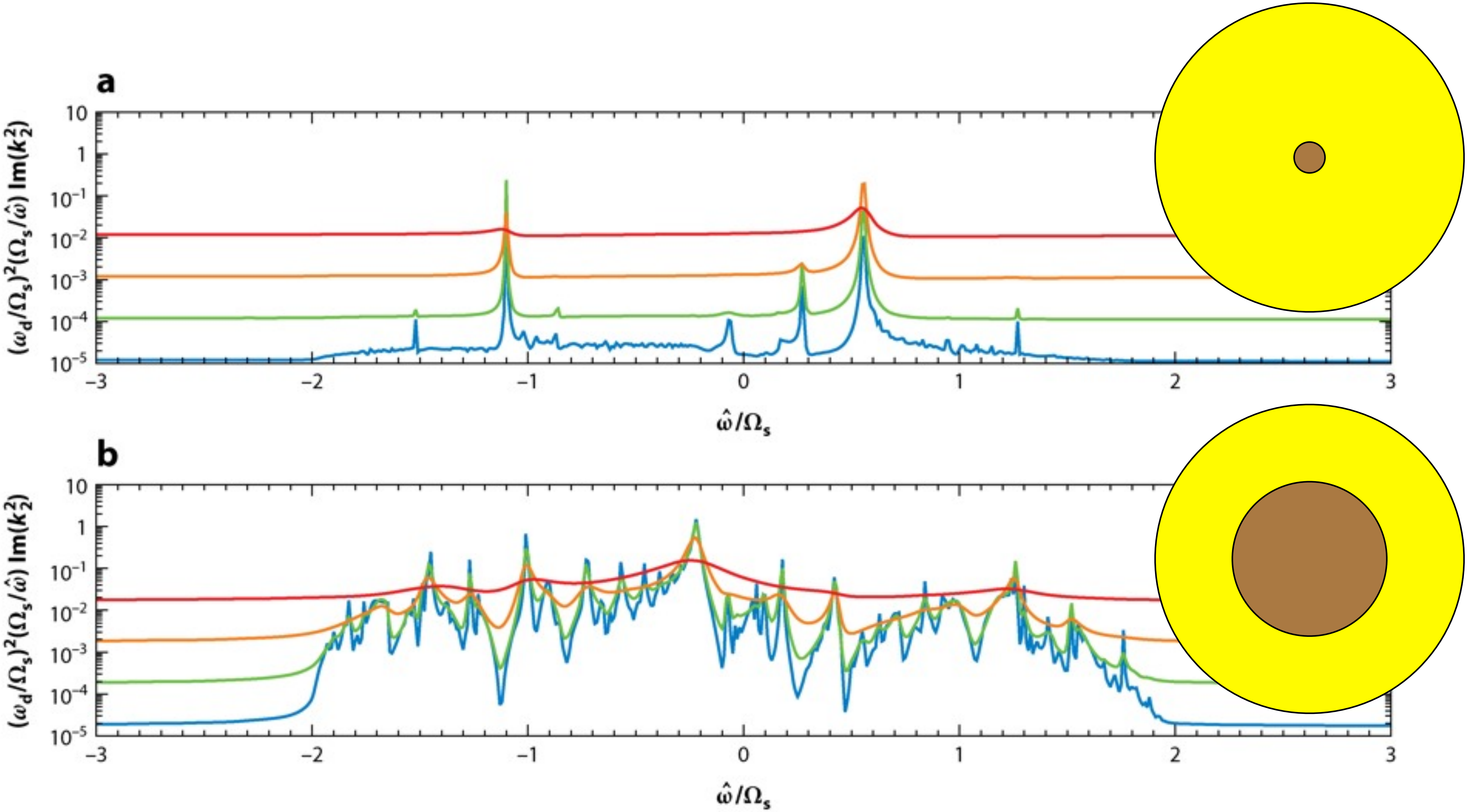
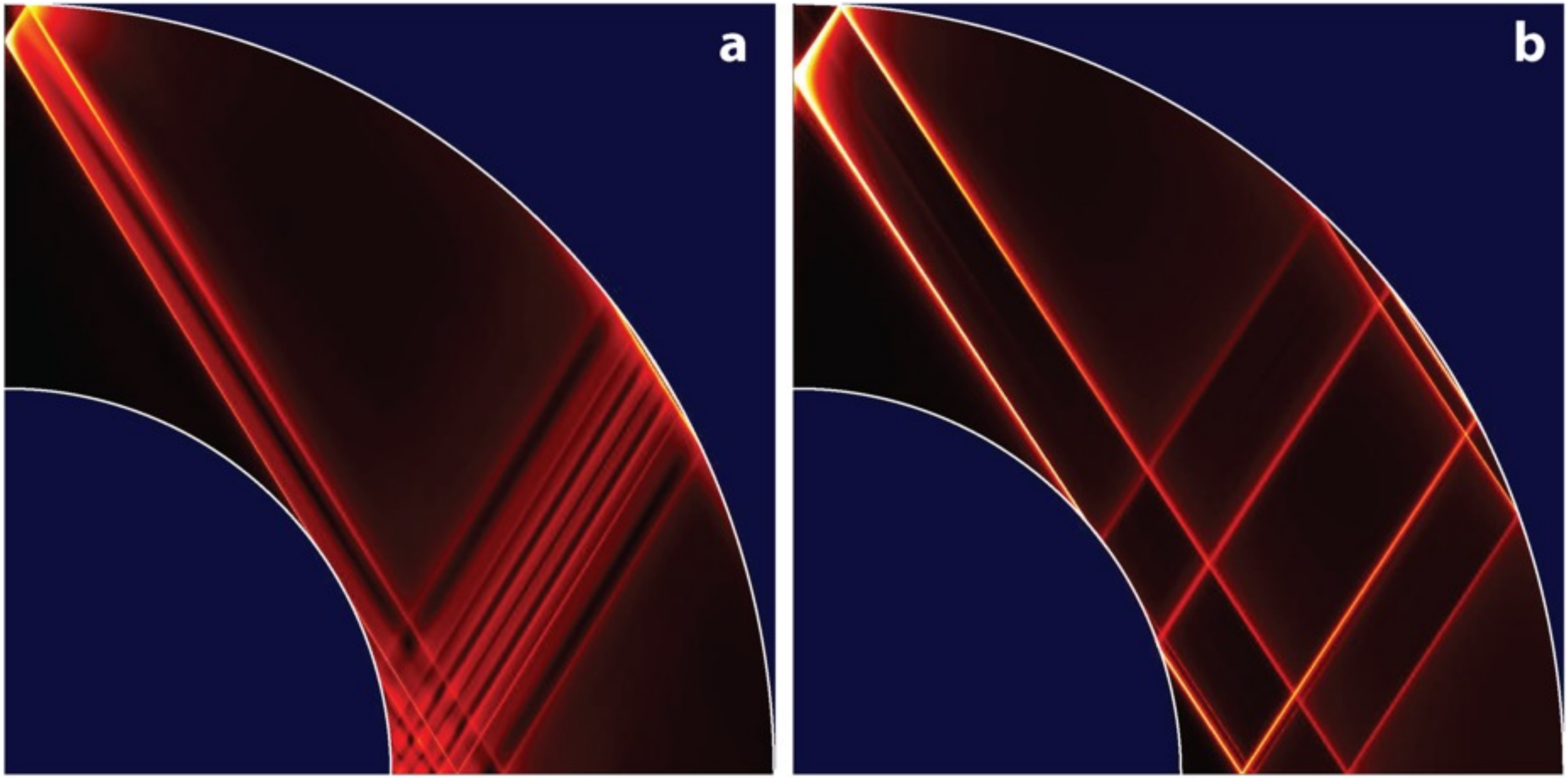


tidal frequency / spin frequency

Ogilvie


a
b

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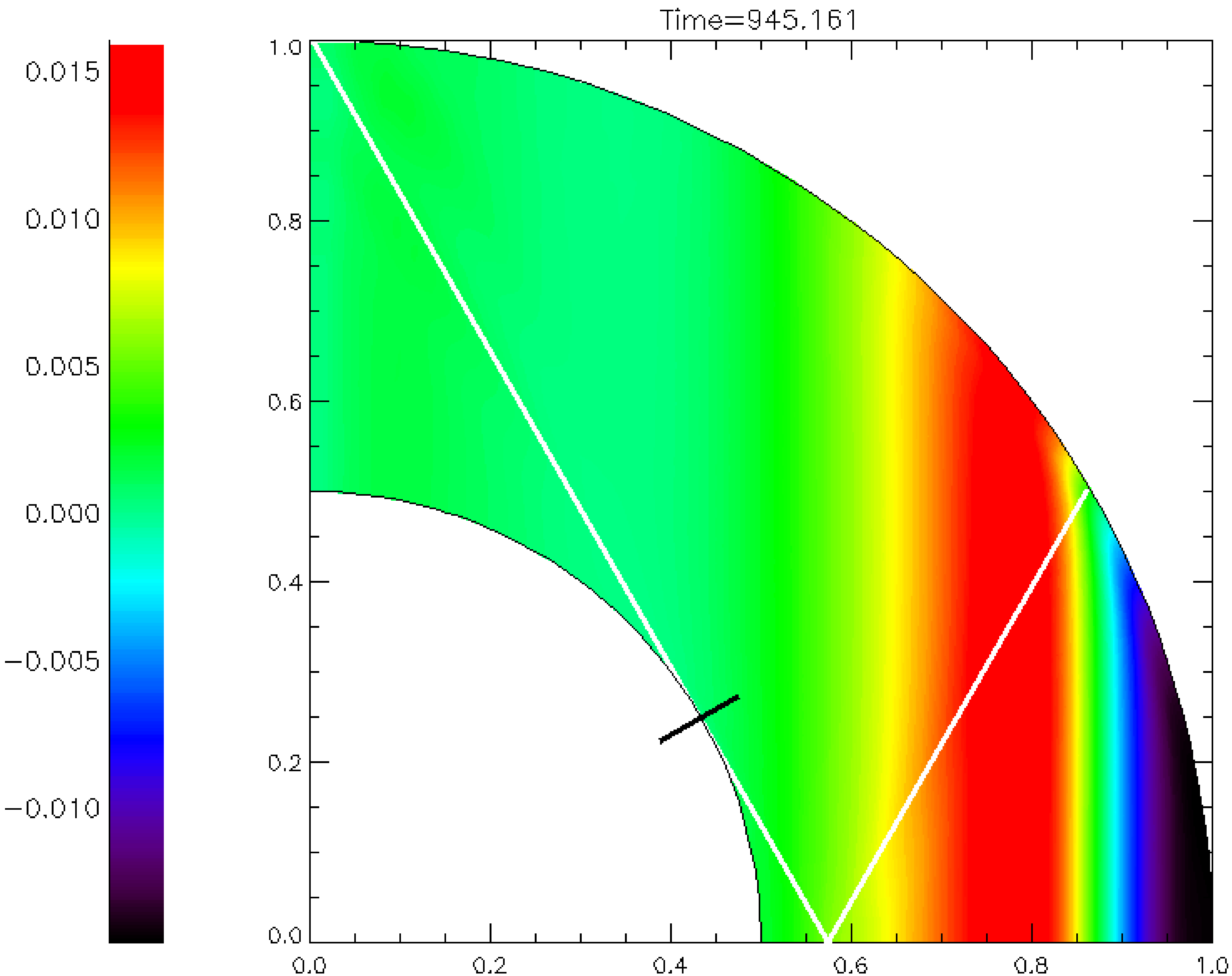
Tidally forced inertial waves and zonal flows



$\omega/\Omega = 1.0$

Favier+ 2014

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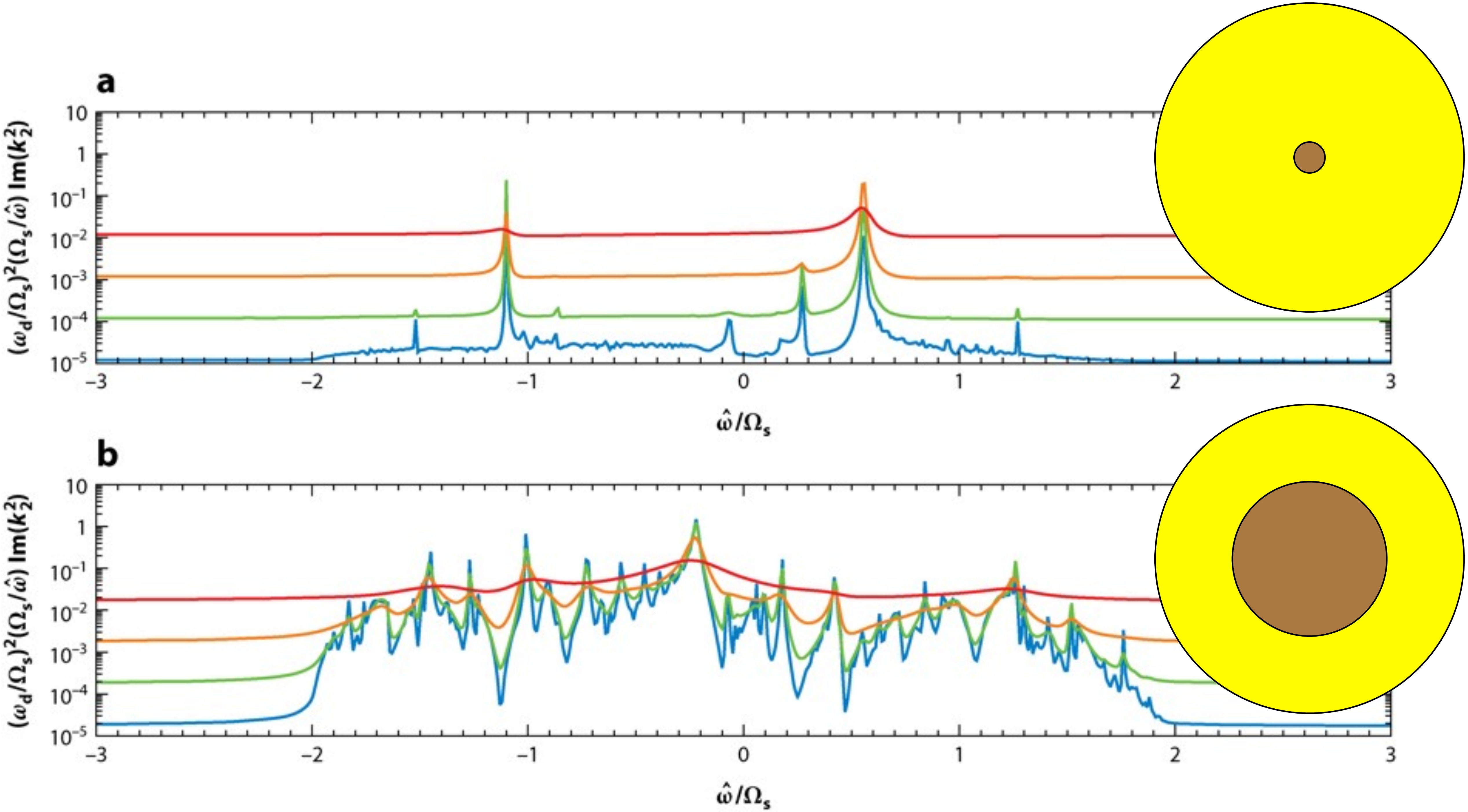

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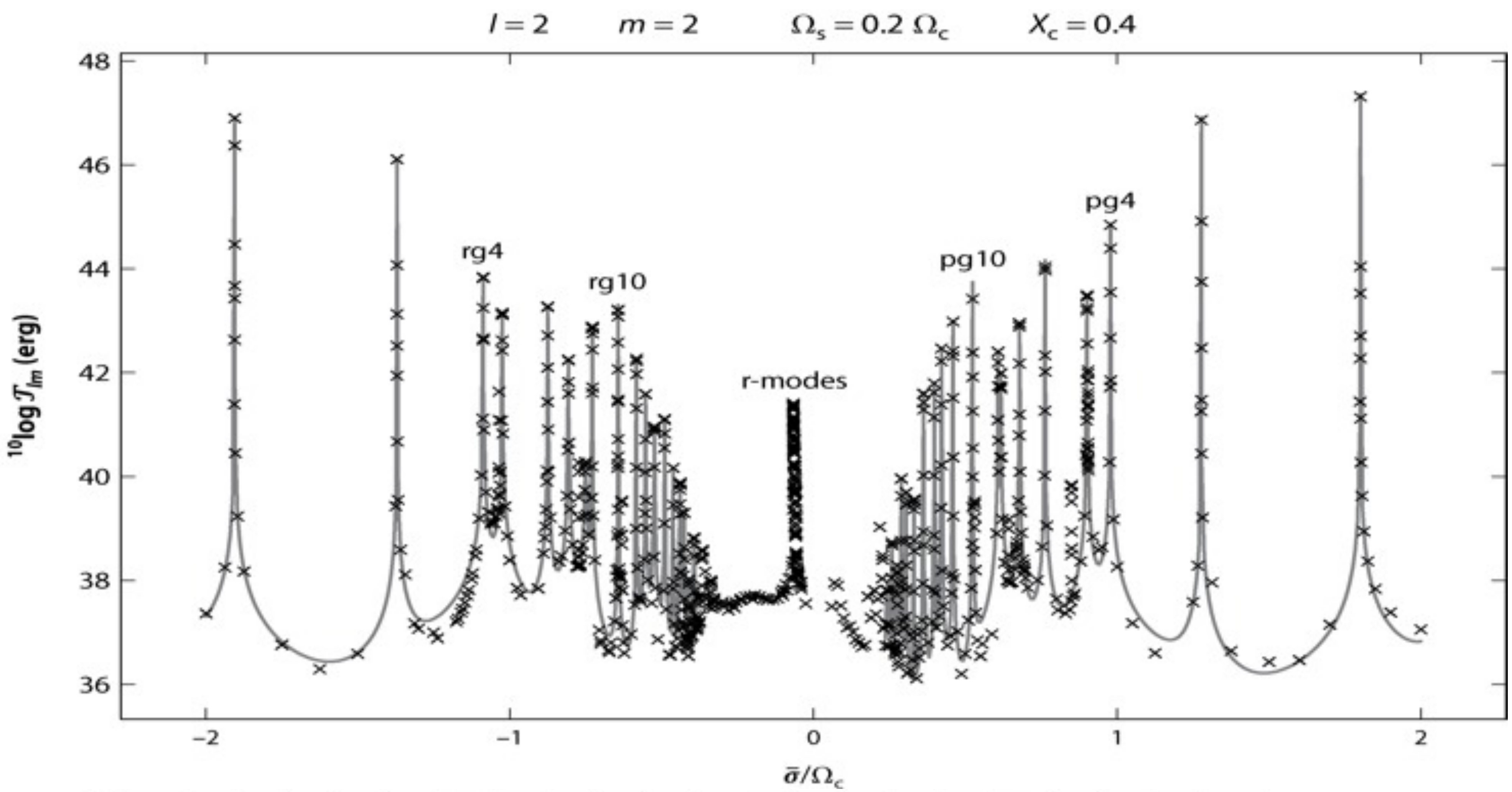
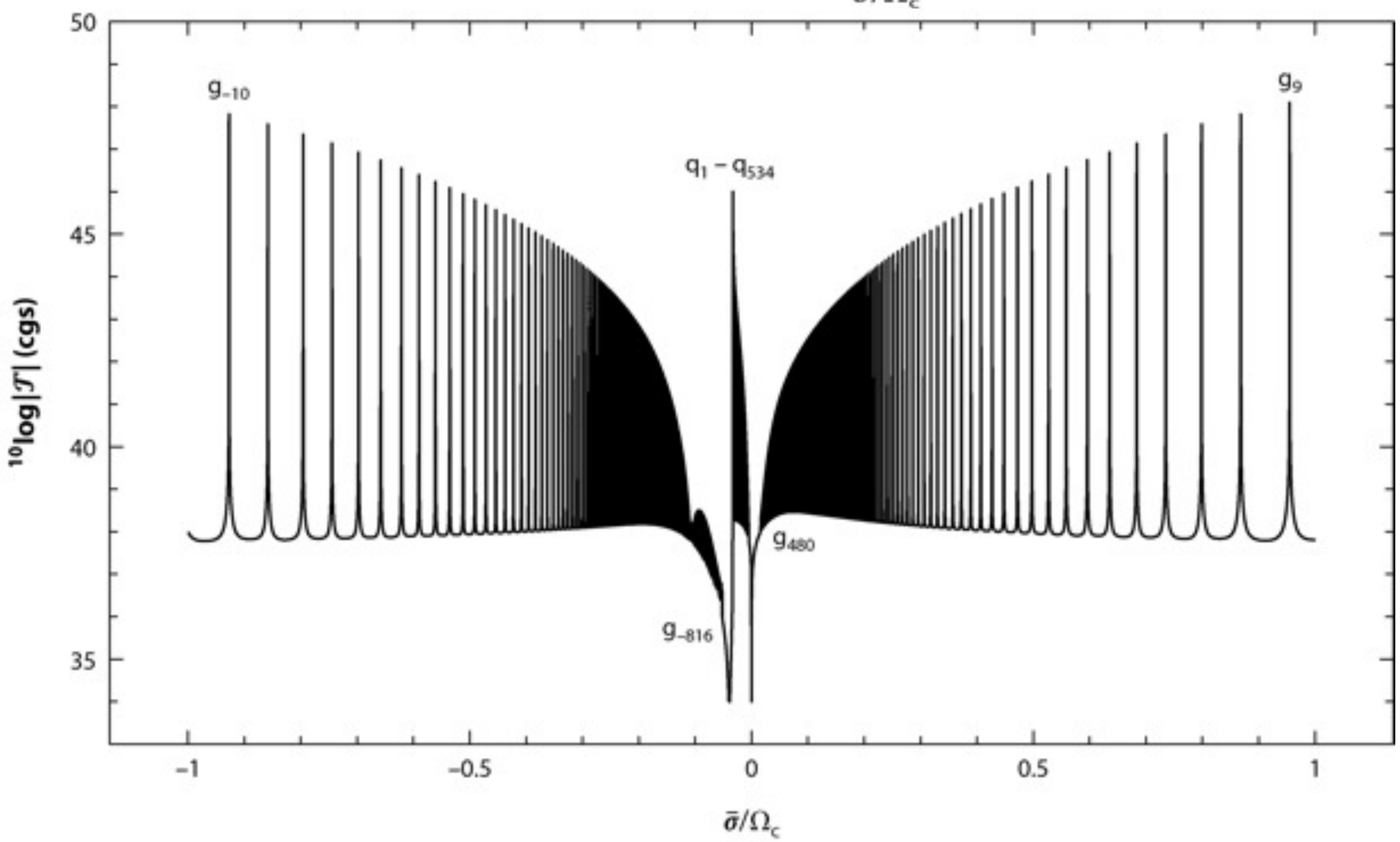
Ogilvie





Savonije & Witte

Recommendations for future theoretical work

- Interaction of waves / tides with convection
- Atmospheric gravitational and thermal tides in hot Jupiters
- Local and global simulations
- Various codes with different capabilities
- Nonlinear regimes
- Differential rotation
- Applications to a variety of more realistic interior models
- Applications to systems with large obliquity