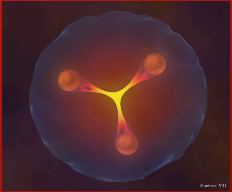
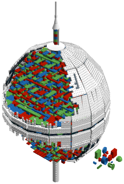
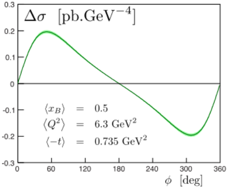
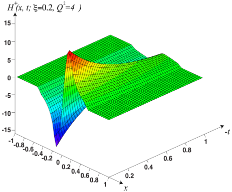
DE LA RECHERCHE À L'INDUSTRIE

Nucleon Reverse Engineering: Towards the mechanisms of mass generation









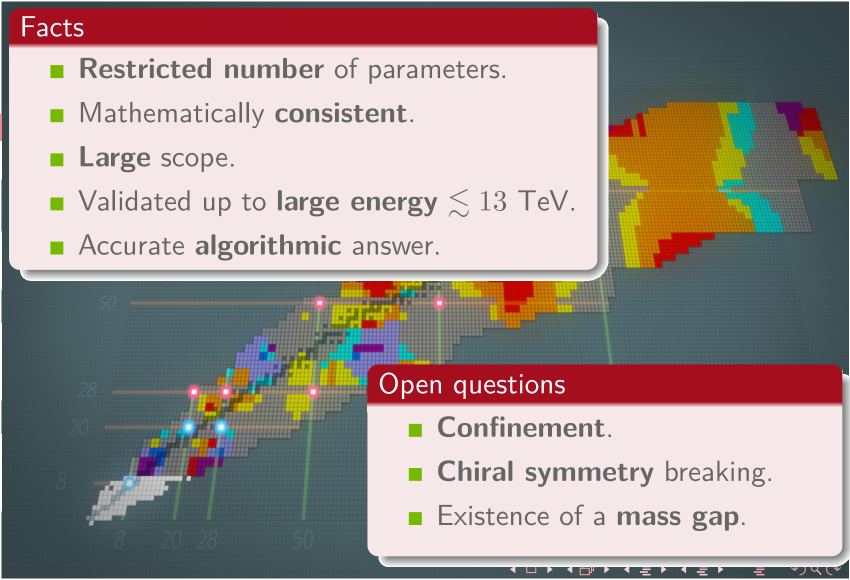
DPTA Seminar | Hervé MOUTARDE

May 22$^{\text{nd}}$, 2017

www.cea.fr

Quantum Chromodynamics as a paradigm.

The **theory** (and not an *effective theory*) of the strong interaction.

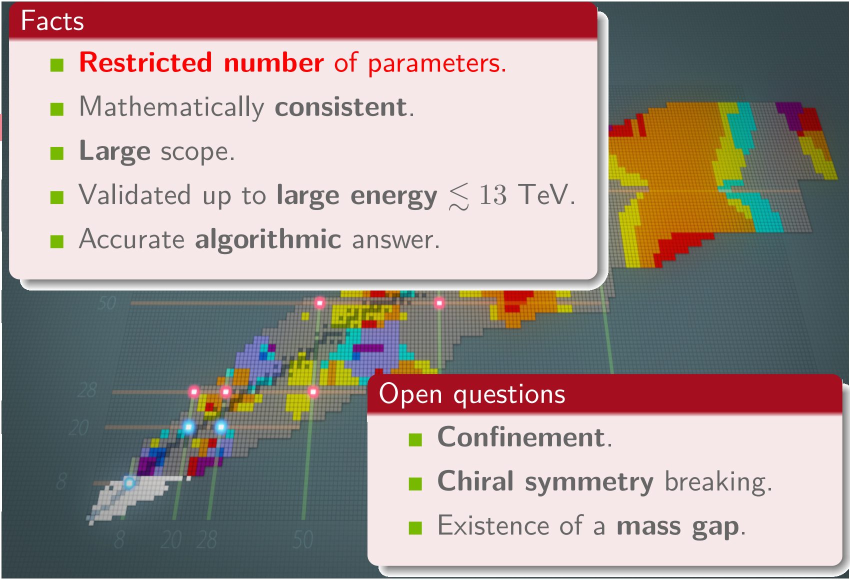


Facts

- **Restricted number** of parameters.
- Mathematically **consistent**.
- **Large** scope.
- Validated up to **large energy** $\lesssim 13$ TeV.
- Accurate **algorithmic** answer.

Open questions

- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

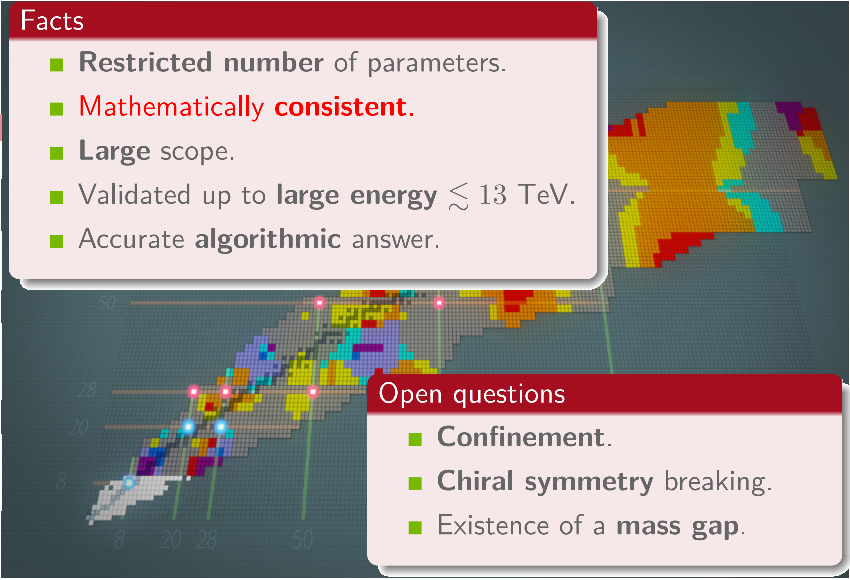


Facts

- **Restricted number** of parameters.
- Mathematically **consistent**.
- **Large** scope.
- Validated up to **large energy** $\lesssim 13$ TeV.
- Accurate **algorithmic** answer.

Open questions

- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.



Facts

- **Restricted number** of parameters.
- Mathematically **consistent**.
- **Large** scope.
- Validated up to **large energy** $\lesssim 13$ TeV.
- Accurate **algorithmic** answer.

Open questions

- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

Quantum Chromodynamics as a paradigm.

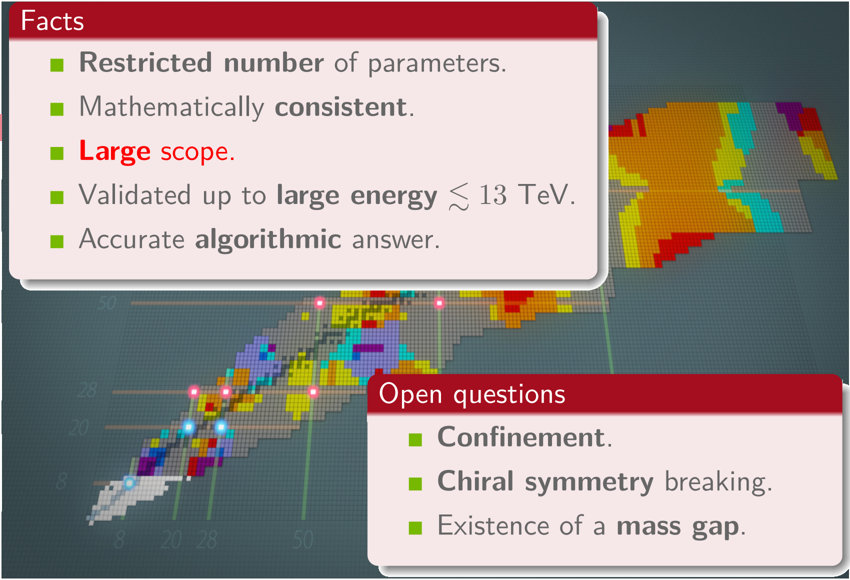
The **theory** (and not an *effective theory*) of the strong interaction.

Facts

- **Restricted number** of parameters.
- Mathematically **consistent**.
- **Large** scope.
- Validated up to **large energy** $\lesssim 13$ TeV.
- Accurate **algorithmic** answer.

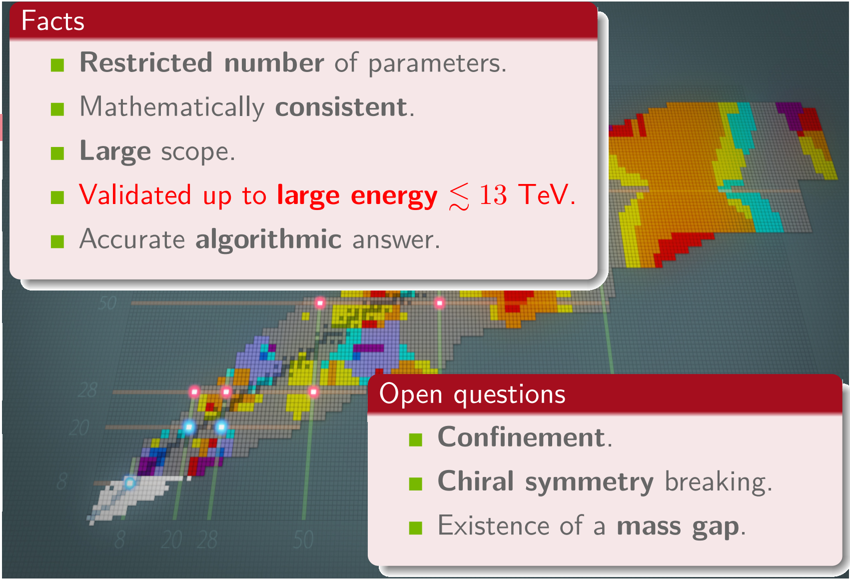
Open questions

- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.



Quantum Chromodynamics as a paradigm.

The **theory** (and not an *effective theory*) of the strong interaction.



Facts

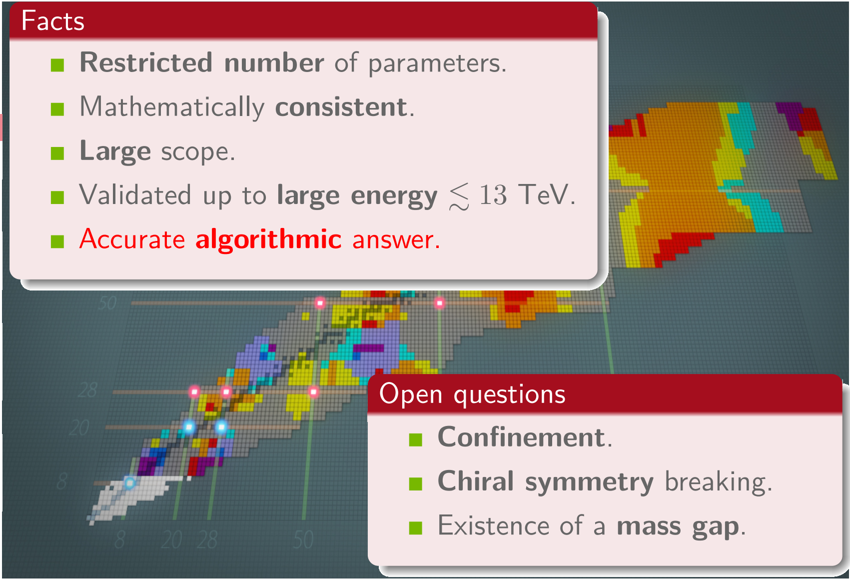
- **Restricted number** of parameters.
- Mathematically **consistent**.
- **Large** scope.
- Validated up to **large energy** $\lesssim 13$ TeV.
- Accurate **algorithmic** answer.

Open questions

- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

Quantum Chromodynamics as a paradigm.

The **theory** (and not an *effective theory*) of the strong interaction.



Facts

- **Restricted number** of parameters.
- Mathematically **consistent**.
- **Large** scope.
- Validated up to **large energy** $\lesssim 13$ TeV.
- Accurate **algorithmic** answer.

Open questions

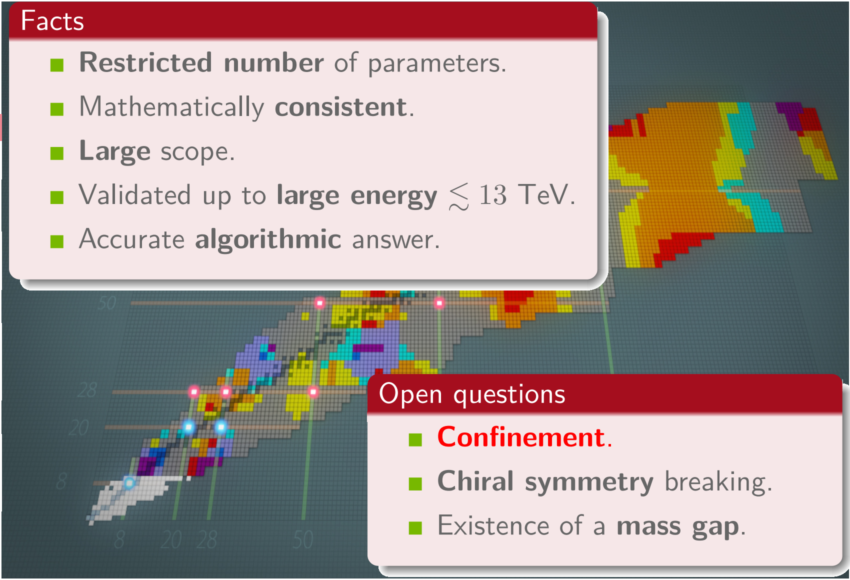
- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

Quantum Chromodynamics as a paradigm.

The **theory** (and not an *effective theory*) of the strong interaction.

Facts

- **Restricted number** of parameters.
- Mathematically **consistent**.
- **Large** scope.
- Validated up to **large energy** $\lesssim 13$ TeV.
- Accurate **algorithmic** answer.

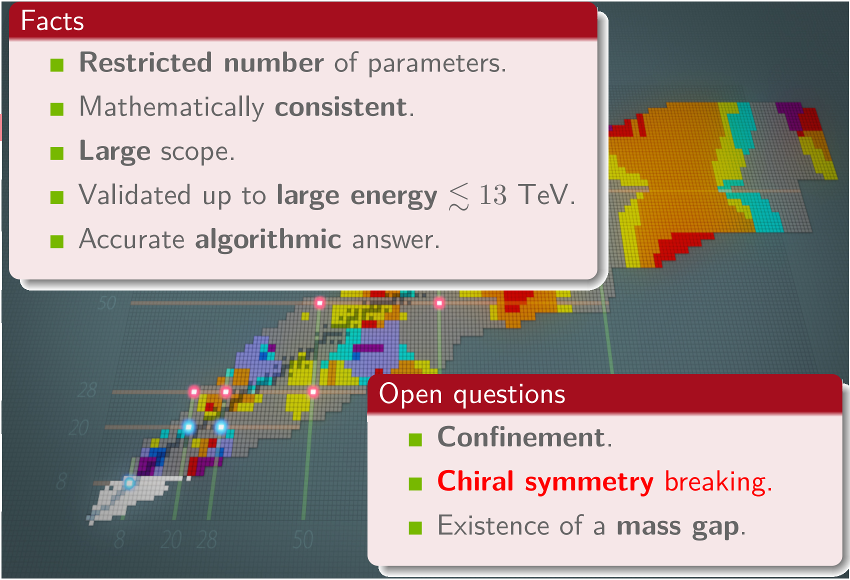


Open questions

- **Confinement.**
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

Quantum Chromodynamics as a paradigm.

The **theory** (and not an *effective theory*) of the strong interaction.



Facts

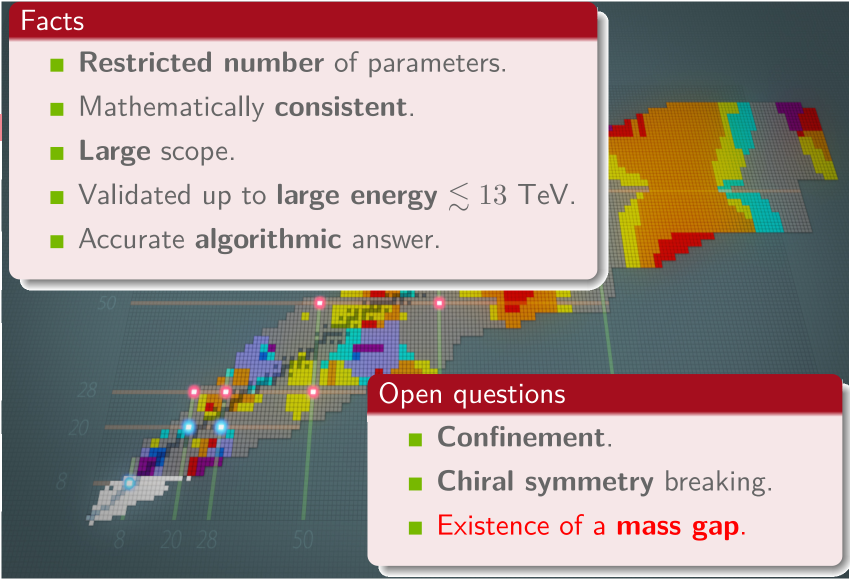
- **Restricted number** of parameters.
- Mathematically **consistent**.
- **Large** scope.
- Validated up to **large energy** $\lesssim 13$ TeV.
- Accurate **algorithmic** answer.

Open questions

- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

Quantum Chromodynamics as a paradigm.

The **theory** (and not an *effective theory*) of the strong interaction.



Facts

- **Restricted number** of parameters.
- Mathematically **consistent**.
- **Large** scope.
- Validated up to **large energy** $\lesssim 13$ TeV.
- Accurate **algorithmic** answer.

Open questions

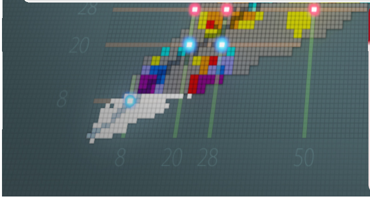
- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

No observed free color charges (PDG 2009)

FREE QUARK SEARCHES

The basis for much of the theory of particle scattering and hadron spectroscopy is the construction of the hadrons from a set of fractionally charged constituents (quarks). A central but unproven hypothesis of this theory, Quantum Chromodynamics, is that quarks cannot be observed as free particles but are confined to mesons and baryons.

Experiments show that it is at best difficult to "unglue" quarks. Accelerator searches at increasing energies have produced no evidence for free quarks, while only a few cosmic-ray and matter searches have produced uncorroborated events.



Open questions

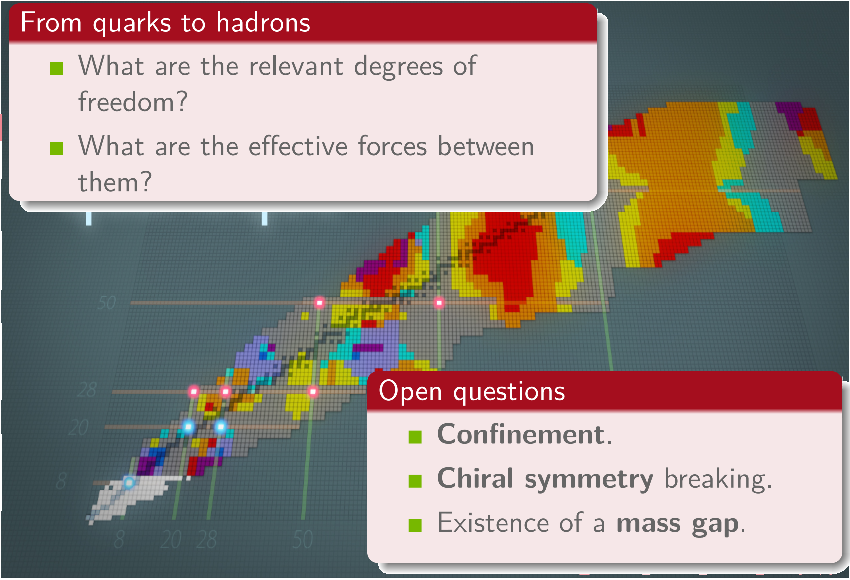
- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

Quantum Chromodynamics as a paradigm.

The **theory** (and not an *effective theory*) of the strong interaction.

From quarks to hadrons

- What are the relevant degrees of freedom?
- What are the effective forces between them?



Open questions

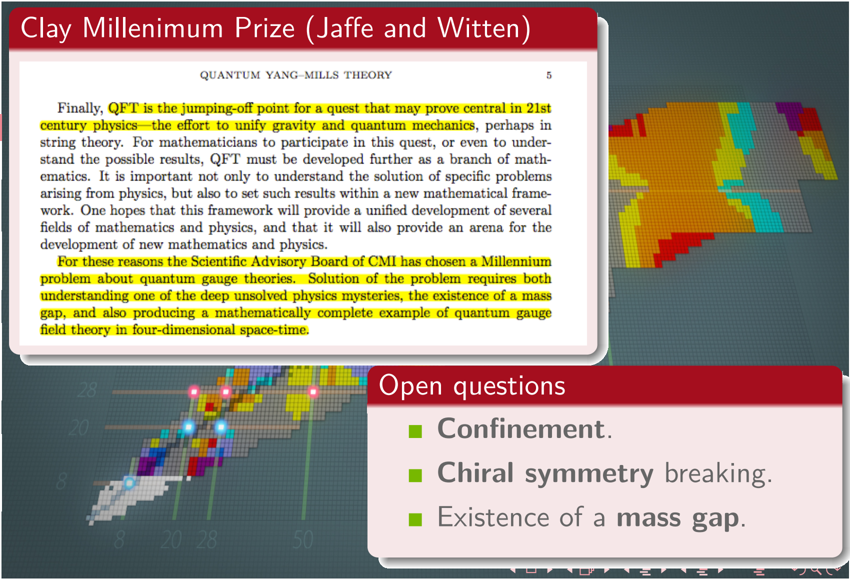
- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

Clay Millenimum Prize (Jaffe and Witten)

QUANTUM YANG–MILLS THEORY 5

Finally, QFT is the jumping-off point for a quest that may prove central in 21st century physics—the effort to unify gravity and quantum mechanics, perhaps in string theory. For mathematicians to participate in this quest, or even to understand the possible results, QFT must be developed further as a branch of mathematics. It is important not only to understand the solution of specific problems arising from physics, but also to set such results within a new mathematical framework. One hopes that this framework will provide a unified development of several fields of mathematics and physics, and that it will also provide an arena for the development of new mathematics and physics.

For these reasons the Scientific Advisory Board of CMI has chosen a Millennium problem about quantum gauge theories. Solution of the problem requires both understanding one of the deep unsolved physics mysteries, the existence of a mass gap, and also producing a mathematically complete example of quantum gauge field theory in four-dimensional space-time.



Open questions

- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

Quantum Chromodynamics as a paradigm.

The **theory** (and not an *effective theory*) of the strong interaction.

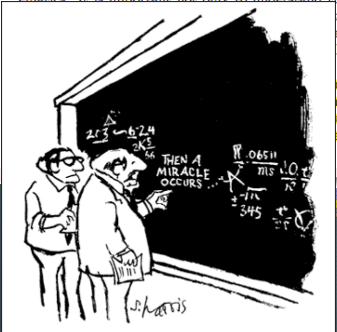
Clay Millenimum Prize (Jaffe and Witten)

QUANTUM YANG–MILLS THEORY 5

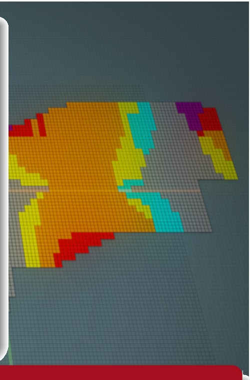
Finally, QFT is the jumping-off point for a quest that may prove central in 21st century physics—the effort to unify gravity and quantum mechanics, perhaps in string theory. For mathematicians to participate in this quest, or even to understand the possible results, QFT must be developed further as a branch of mathematics. It is important not only to understand the solution of specific problems

...thin a new mathematical frame-
...a unified development of several
...l also provide an arena for the

...f CMI has chosen a Millennium
...of the problem requires both
...ysteries, the existence of a mass
...ete example of quantum gauge



"I think you should be more explicit here in step two."

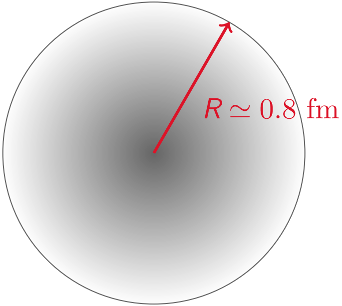


Open questions

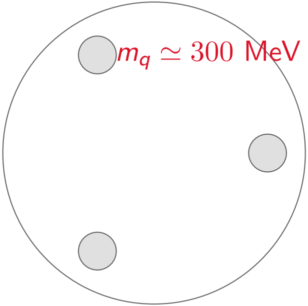
- **Confinement.**
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

Hadrons and partons.

The nucleon: a **quantum relativistic** system of **confined** particles.



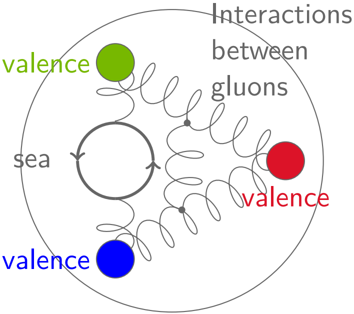
- **Composite** object with an **electric charge** spread over a spherical region.



- **Composite** object with an **electric charge** spread over a spherical region.
- Quark model description: **nonrelativistic** bound state of **3 massive quarks**.

Hadrons and partons.

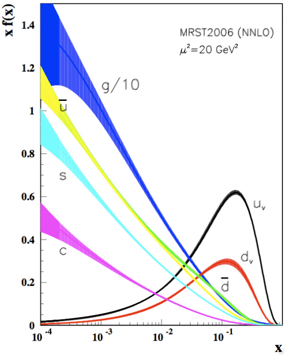
The nucleon: a **quantum relativistic** system of **confined** particles.



- **Composite** object with an **electric charge** spread over a spherical region.
- Quark model description: **nonrelativistic** bound state of **3 massive quarks**.
- Modern description (QCD): **relativistic** bound state of **colored light** quarks and **massless** gluons (**partons**).

Hadrons and partons.

The nucleon: a **quantum relativistic** system of **confined** particles.



Partons number densities vs longitudinal momentum

- **Composite** object with an **electric charge** spread over a spherical region.
- Quark model description: **nonrelativistic** bound state of **3 massive quarks**.
- Modern description (QCD): **relativistic** bound state of **colored light** quarks and **massless** gluons (**partons**).
- **Arbitrarily many** quarks, antiquarks and gluons in nucleons.

Hadrons and partons.

The nucleon: a **quantum relativistic** system of **confined** particles.

QCD generates $\gtrsim 90$ % of the visible universe mass

- **Composite** object with an **electric charge** spread over a spherical region.
- Quark model description: **nonrelativistic** bound state of **3 massive quarks**.
- Modern description (QCD): **relativistic** bound state of **colored light** quarks and **massless** gluons (**partons**).
- **Arbitrarily many** quarks, antiquarks and gluons in nucleons.
- QCD: few **principles**, wide **scope** and puzzling **properties**:
  - ✓ Asymptotic freedom,
  - ✗ Confinement.

What are the hadrons lighter than 10 GeV?

Origin of mass, or *mass without mass*.

			Status as seen in —						
Particle	$L_{2I \cdot 2J}$	Overall status	$N\pi$	$N\eta$	ΛK	ΣK	$\Delta\pi$	$N\rho$	$N\gamma$
$N(939)$	P_{11}	****							
$N(1440)$	P_{11}	****	****	*			***	*	***
$N(1520)$	D_{13}	****	****	***			****	****	****
$N(1535)$	S_{11}	****	****	****			*	**	***
$N(1650)$	S_{11}	****	****	*	***	**	***	**	***
$N(1675)$	D_{15}	****	****	*	*		****	*	****
$N(1680)$	F_{15}	****	****	*			****	****	****
$N(1700)$	D_{13}	***	***	*	**	*	**	*	**
$N(1710)$	P_{11}	***	***	**	**	*	**	*	***
$N(1720)$	P_{13}	****	****	*	**	*	*	**	**
$N(1900)$	P_{13}	**	**					*	
$N(1990)$	F_{17}	**	**	*	*	*			*
$N(2000)$	F_{15}	**	**	*	*	*	*	**	
$N(2080)$	D_{13}	**	**	*	*				*
$N(2090)$	S_{11}	*	*						
$N(2100)$	P_{11}	*	*	*					
$N(2190)$	G_{17}	****	****	*	*	*		*	*
$N(2200)$	D_{15}	**	**	*	*				
$N(2220)$	H_{19}	****	****	*					
$N(2250)$	G_{19}	****	****	*					
$N(2600)$	$I_{1\,11}$	***	***						
$N(2700)$	$K_{1\,13}$	**	**						
$\Delta(1232)$	P_{33}	****	****	F					****
$\Delta(1600)$	P_{33}	***	***	o			***	*	**
$\Delta(1620)$	S_{31}	****	****	r			****	****	***
$\Delta(1700)$	D_{33}	****	****	b		*	***	**	***
$\Delta(1750)$	P_{31}	*	*	i					
$\Delta(1900)$	S_{31}	**	**		d	*	*	**	*
$\Delta(1905)$	F_{35}	****	****		d	*	**	**	***
$\Delta(1910)$	P_{31}	****	****		e	*	*	*	*
$\Delta(1920)$	P_{33}	***	***		n	*	**		*
$\Delta(1930)$	D_{35}	***	***			*			**
$\Delta(1940)$	D_{33}	*	*	F					
$\Delta(1950)$	F_{37}	****	****	o		*	****	*	****
$\Delta(2000)$	F_{35}	**		r			**		
$\Delta(2150)$	S_{31}	*	*	b					
$\Delta(2200)$	G_{37}	*	*	i					
$\Delta(2300)$	H_{39}	**	**		d				
$\Delta(2350)$	D_{35}	*	*		d				
$\Delta(2390)$	F_{37}	*	*		e				
$\Delta(2400)$	G_{39}	**	**		n				
$\Delta(2420)$	$H_{3\,11}$	****	****						*
$\Delta(2750)$	$I_{3\,13}$	**	**						
$\Delta(2950)$	$K_{3\,15}$	**	**						

- Resonances described by QCD (**quark masses** + **coupling constant**).

Status as seen in —

Particle		Overall			
Δ(2390)	F_{37}	*	*	e	
Δ(2400)	G_{39}	**	**	n	
Δ(2420)	$H_{3\,11}$	****	****		*
Δ(2750)	$I_{3\,13}$	**	**		
Δ(2950)	$K_{3\,15}$	**	**		

Numerous bound states

"Quarks. Neutrinos. Mesons. All those damn particles you can't see. That's what drove me to drink. But now I can see them!"

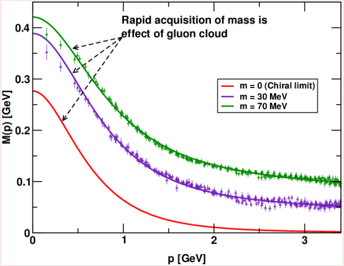
- Resonances described by QCD (**quark masses** + **coupling constant**).
- Success of the quark model but **missing resonances**.

Status as seen in —

Overall

Δ(2350)	D_{35}	*	*	d
Δ(2390)	F_{37}	*	*	e
Δ(2400)	G_{39}	**	**	n
Δ(2420)	H_{311}	****	****	*
Δ(2750)	I_{313}	**	**	
Δ(2950)	K_{315}	**	**	

Current mass or constituant mass?



Roberts, arXiv:1203.5341 [nucl-th]

- Resonances described by QCD (**quark masses** + **coupling constant**).
- Success of the quark model but **missing resonances**.
- International **experimental** and **theoretical** programs to answer this question.

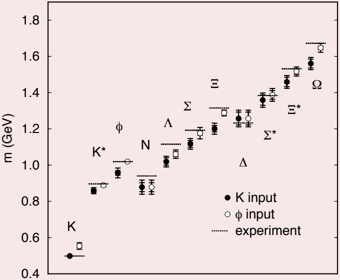
What are the hadrons lighter than 10 GeV?

Origin of mass, or *mass without mass*.

Status as seen in —

Δ(2390)	F_{37}	*	*	c	
Δ(2400)	G_{39}	**	**	n	
Δ(2420)	$H_{3\,11}$	****	****		*
Δ(2750)	$I_{3\,13}$	**	**		
Δ(2950)	$K_{3\,15}$	**	**		

Light hadrons (quenched)

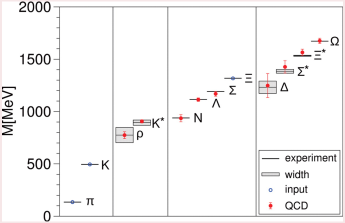


Aoki *et al.* Phys. Rev. **D67**, 034503 (2003)

- Resonances described by QCD (**quark masses** + **coupling constant**).
- Success of the quark model but **missing resonances**.
- International **experimental** and **theoretical** programs to answer this question.
- Essential role of **lattice QCD**.

Particle	$L_{2I\cdot 2J}$	Overall status	Status as seen in — $N\pi$	$N\eta$	ΛK	ΣK	$\Delta\pi$	$N\rho$	$N\gamma$
$\Delta(2200)$	G_{37}	*	*		i				
$\Delta(2300)$	H_{39}	**	**		d				
$\Delta(2350)$	D_{35}	*	*		d				
$\Delta(2390)$	F_{37}	*	*		e				
$\Delta(2400)$	G_{39}	**	**		n				
$\Delta(2420)$	$H_{3\,11}$	****	****						*
$\Delta(2750)$	$I_{3\,13}$	**	**						
$\Delta(2950)$	$K_{3\,15}$	**	**						

Light hadrons (unquenched)



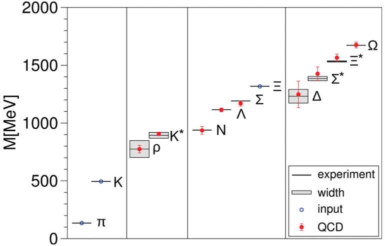
Durr *et al.* Science **322**, 1224 (2008)

- Resonances described by QCD (**quark masses** + **coupling constant**).
- Success of the quark model but **missing resonances**.
- International **experimental** and **theoretical** programs to answer this question.
- Essential role of **lattice QCD**.
- **Low impact** of the *u* and *d* quark masses.

Motivation.

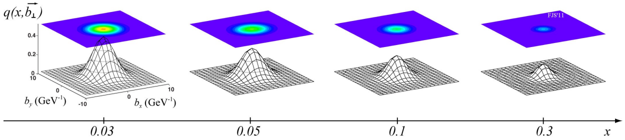
QCD large distance dynamics from the hadron structure viewpoint.

- Lattice QCD clearly shows that the mass of hadrons is generated by the **interaction**, not by the quark masses.



Durr *et al.*, Science **322**, 1224 (2008)

- Can we **map** the *location of mass* inside a hadron?



1 Basics of partonic structure of hadrons

We introduce the basics of partons.

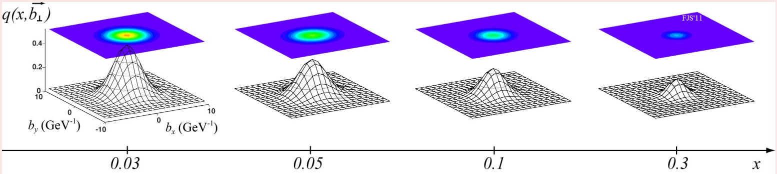
2 Experimental hadron tomography

We can apply the GPD formalism to existing data.

3 Toolbox for high precision: the PARTONS project

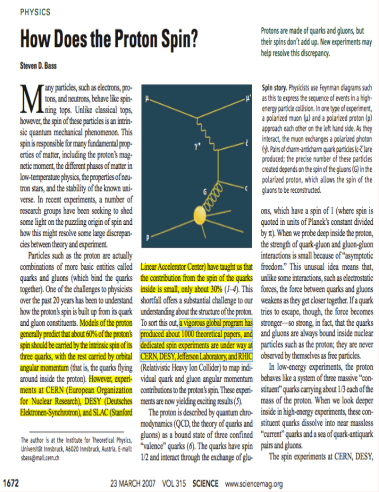
We develop the tools to analyze near-future data.

How can we make this picture? What do we learn from it?



Basics of partonic structure of hadrons

Quark spin contribution


PHYSICS

How Does the Proton Spin?

Protons are made of quarks and gluons, but their spins don't add up. New experiments may help resolve this discrepancy.

Steven D. Bass

Many particles, such as electrons, protons, and neutrons, behave like spinning tops. Unlike classical tops, however, the spin of these particles is an intrinsic quantum mechanical phenomenon. This spin is responsible for many fundamental properties of matter, including the proton's magnetic moment, the different phases of matter in low-temperature physics, the properties of neutron stars, and the stability of the known universe. In recent experiments, a number of research groups have been seeking to shed some light on the puzzling origin of spin and how this might resolve some large discrepancies between theory and experiment.

Particles such as the proton are actually combinations of more basic entities called quarks and gluons (which bind the quarks together). One of the challenges to physicists over the past 20 years has been to understand how the proton's spin is built up from its quark and gluon constituents. Models of the proton generally predict that about 60% of the proton's spin should be carried by the intrinsic spin of its three quarks, with the rest carried by orbital angular momentum (that is, the quarks flying around inside the proton). However, experiments at CERN (European Organization for Nuclear Research), DESY (Deutsches Elektronen-Synchrotron), and SLAC (Stanford Linear Accelerator Center) have taught us that the contribution from the spin of the quarks inside is small, only about 30% (*1–4*). This shortfall offers a substantial challenge to our understanding about the structure of the proton. To sort this out, a vigorous global program has produced about 1000 theoretical papers, and dedicated spin experiments are under way at CERN, DESY, Jefferson Laboratory, and RHIC (Relativistic Heavy Ion Collider) to map individual quark and gluon angular momentum contributions to the proton's spin. These experiments are now yielding exciting results (*5*).

The proton is described by quantum chromodynamics (QCD, the theory of quarks and gluons) as a bound state of three confined "valence" quarks (*6*). The quarks have spin 1/2 and interact through the exchange of glu-

Spin story. Physicists use Feynman diagrams such as this to express the sequence of events in a high-energy particle collision. In one type of experiment, a polarized muon (μ) and a polarized proton (p) approach each other on the left hand side. As they interact, the muon exchanges a polarized photon (γ). Pairs of charm-anticharm quark particles (c-$\bar{c}$) are produced; the precise number of these particles created depends on the spin of the gluons (G) in the polarized proton, which allows the spin of the gluons to be reconstructed.

ons, which have a spin of 1 (where spin is quoted in units of Planck's constant divided by π). When we probe deep inside the proton, the strength of quark-gluon and gluon-gluon interactions is small because of "asymptotic freedom." This unusual idea means that, unlike some interactions, such as electrostatic forces, the force between quarks and gluons weakens as they get closer together. If a quark tries to escape, though, the force becomes stronger—so strong, in fact, that the quarks and gluons are always bound inside nuclear particles such as the proton; they are never observed by themselves as free particles.

In low-energy experiments, the proton behaves like a system of three massive "constituent" quarks carrying about 1/3 each of the mass of the proton. When we look deeper inside in high-energy experiments, these constituent quarks dissolve into near massless "current" quarks and a sea of quark-antiquark pairs and gluons.

The spin experiments at CERN, DESY,

The author is at the Institute for Theoretical Physics, Universität Innsbruck, A6020 Innsbruck, Austria. E-mail: sbass@mail.cern.ch

1672 23 MARCH 2007 VOL 315 SCIENCE www.sciencemag.org

Gluon spin contribution


actualités PHYSIQUE

Les gluons ont leur part dans le spin des protons

Le spin des protons, addition de plusieurs contributions de ses constituants élémentaires, intrigue toujours. Une simulation numérique vient d'évaluer la fraction due aux gluons.

Le tout est parfois plus que la somme des parties. Cette maxime illustre bien l'énigme du spin du proton. Le spin est une caractéristique quantique propre à chaque particule. Il y a trente ans, des expériences ont montré que le spin des quarks, les composants élémentaires des protons, n'expliquait que 30 % du spin du proton. Récemment, des théoriciens ont calculé que d'autres particules élémentaires, les gluons, seraient responsables de la moitié de la valeur du spin du proton (1). *« On présente souvent le proton comme étant composé de trois quarks, une vision qui a émergé dans les années 1960, mais la réalité est un peu plus compliquée que cela »*, explique Hervé Moutarde, responsable du laboratoire sur la structure du nucléon au CEA. Au sein du proton, les quarks interagissent entre eux et il y a création constante de paires de quark-antiquark. Il y a aussi les gluons, qui ne sont pas des particules de matière, mais les médiateurs de l'interaction forte, celle entre les quarks, de la même manière que les photons transmettent l'interaction électromagnétique. Ces gluons peuvent eux-mêmes donner naissance à des paires de quark-antiquark. *« À chaque instant, il y a certes toujours un avantage de plus trois pour les quarks contre les antiquarks, mais le proton est un édifice complexe et dynamique »*, poursuit Hervé Moutarde. Cette complexité se reflète dans la façon dont les différentes parties se combinent pour former la totalité du spin du proton.

Après une remise à plat du formalisme de la théorie à la fin des années 2000, les physiciens ont identifié quatre sources au spin du proton : le spin des quarks, le spin des gluons et les contributions du mouvement de ces quarks et de ces gluons (ce qu'on appelle le moment angulaire orbital). La tâche consiste désormais à déterminer ces quantités à partir des expériences mais aussi à partir de simulations numériques.

Yi-Bo Yang, de l'université du Kentucky, et ses collègues ont donc calculé théoriquement la part des gluons au spin du proton. C'est le premier calcul numérique de ce terme. Les ordinateurs ne peuvent pas simuler l'espace comme un milieu continu, mais seulement comme une grille de points. Plus les points sont resserrés, plus les chiffres sont précis et plus la puissance requise est importante. Pendant longtemps, les physiciens devaient faire des approximations pas très réalistes pour réaliser ces simulations dans des temps raisonnables. Les améliorations des capacités de calculs et des algorithmes ont changé la donne. Il y a aussi eu ces dernières années des avancées théoriques originales (2) pour rendre ces calculs complexes, qui mêlent physique quantique et relativité restreinte, réalisables avec les ordinateurs.

CE SUCCÈS ILLUSTRE LA PRÉCISION DES OUTILS DONT SONT DÉSORMAIS DOTÉS LES PHYSICIENS

Ce nouveau résultat est compatible avec des mesures déjà effectuées au Cern et au Laboratoire national de Brookhaven, aux États-Unis, de la fraction du spin du proton due aux gluons. Cependant, les barres d'erreur sur ces expériences sont encore assez importantes et les espoirs des physiciens reposent notamment sur le futur collisionneur électrons-ions, projet prévu aux États-Unis mais qui n'existe pour l'instant que sur le papier. Outre l'éclairage porté sur le spin du proton, ce succès illustre la précision des outils dont sont désormais dotés les physiciens pour étudier l'interaction forte et comprendre le bouillonnement des particules à l'échelle subatomique. **Sylvain Guilbaud**

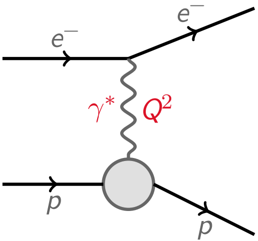
(1) Yi-Bo Yang *et al.*, *PRL*, 118, 102001, 2017.
(2) X. Ji *et al.*, *PRL*, 110, 262002, 2013.

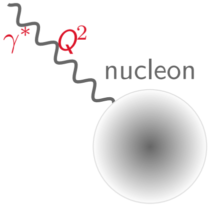
50 % DE LA VALEUR du spin du proton serait due au spin des gluons.

N°523 • Mai 2017 | La Recherche • 31

Electron Ion Collider

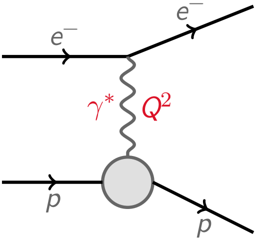
Quark transverse position

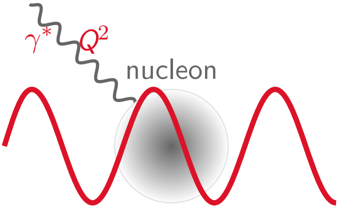

e^-
e^-
γ^*
Q^2
p
p


γ^*
Q^2
nucleon

Elastic scattering.

Nucleon form factors and transverse plane charge densities.

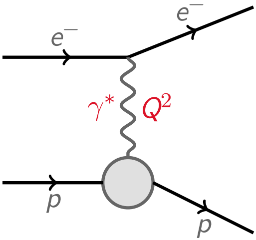


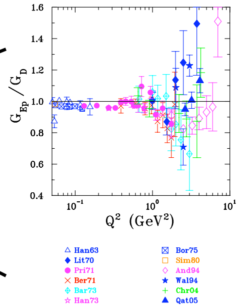


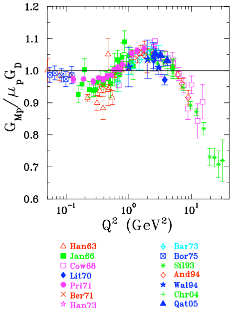
wavelength $\simeq$ nucleon size

Elastic scattering.

Nucleon form factors and transverse plane charge densities.



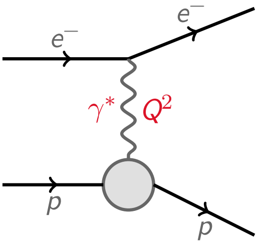


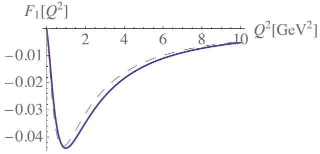


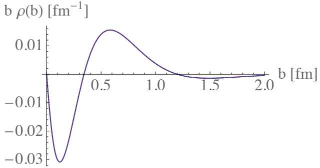
- **Elastic scattering** electron / nucleon described with Q^2-dependent **form factors**.

Elastic scattering.

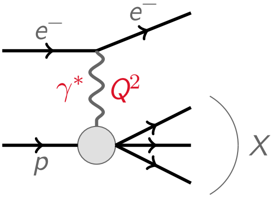
Nucleon form factors and transverse plane charge densities.

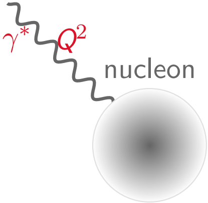


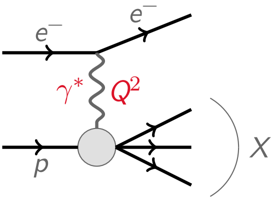


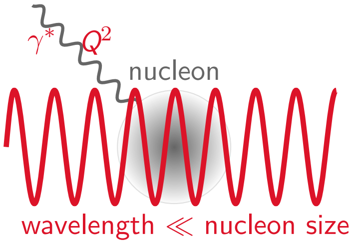


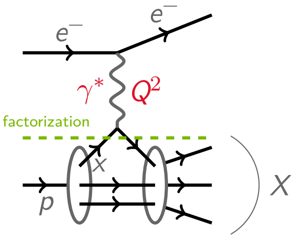
- **Elastic scattering** electron / nucleon described with Q^2-dependent **form factors**.
- Transverse plane **charge density**.

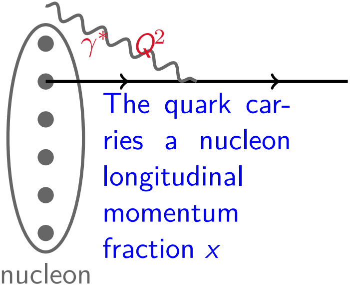

e−
e−
γ*
Q2
p
X


γ*
Q2
nucleon


e−
e−
γ*
Q2
p
X

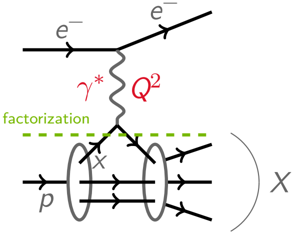

γ*
Q2
nucleon
wavelength ≪ nucleon size

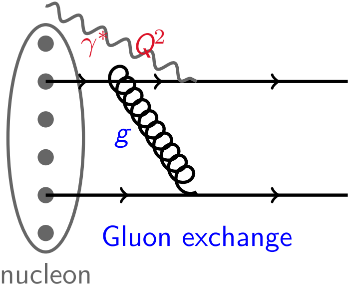




The quark carries a nucleon longitudinal momentum fraction x

- **Parton distribution** $q(x)$: quark number density.

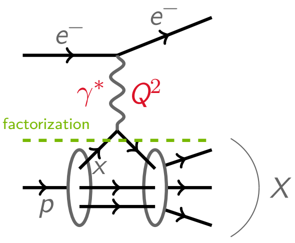


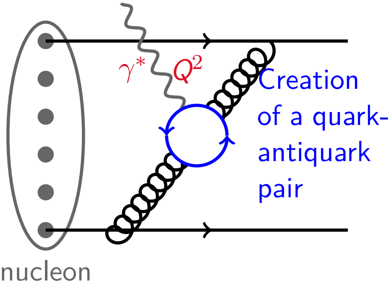


- **Parton distribution** $q(x)$: quark number density.
- Measurement of the gluon contribution $g(x)$.

Deep Inelastic Scattering (DIS).

Parton Distribution Function (PDF) and longitudinal momentum.

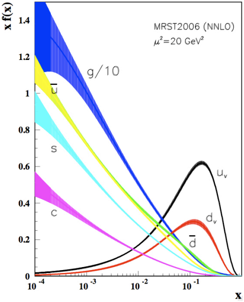


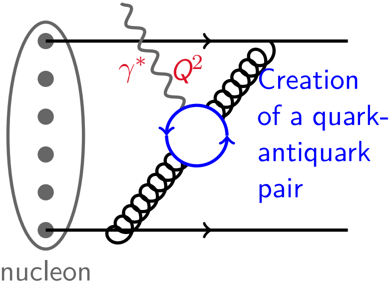


- **Parton distribution** $q(x)$: quark number density.
- Measurement of the gluon contribution $g(x)$.
- The nucleon has a strange content!

Deep Inelastic Scattering (DIS).

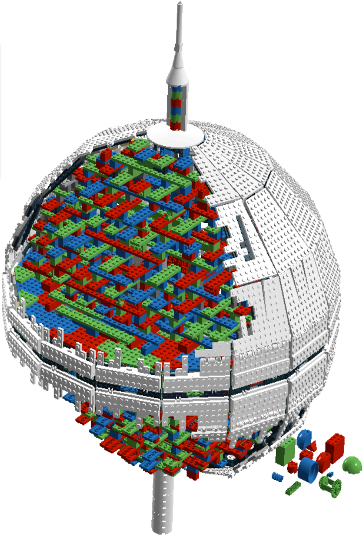
Parton Distribution Function (PDF) and longitudinal momentum.





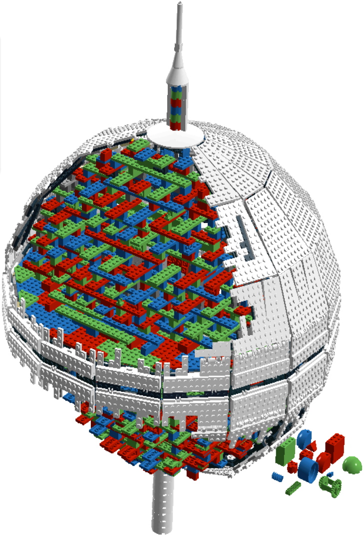
- **Parton distribution** $q(x)$: quark number density.
- Measurement of the gluon contribution $g(x)$.
- The nucleon has a strange content!
- The nucleon is a **dynamical** object.

How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?



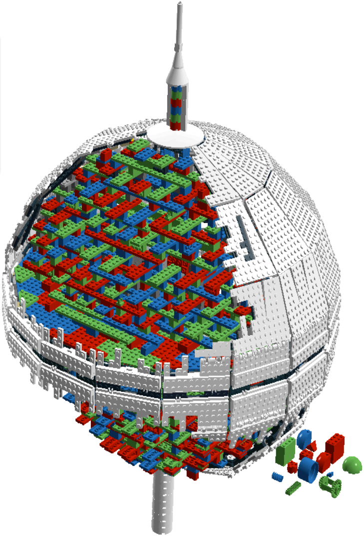
How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?

Mass?



How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?

Mass?
Spin?

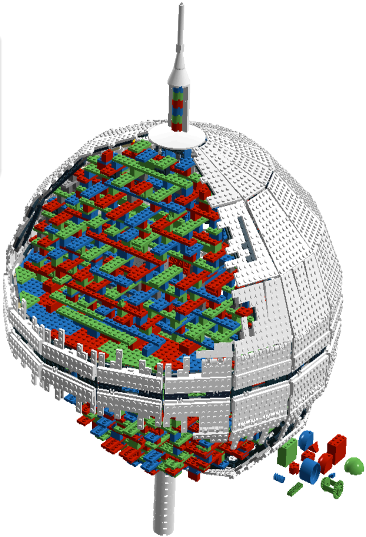


Imaging the origin of mass.

Identification of underlying mechanisms from parton distributions.

How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?

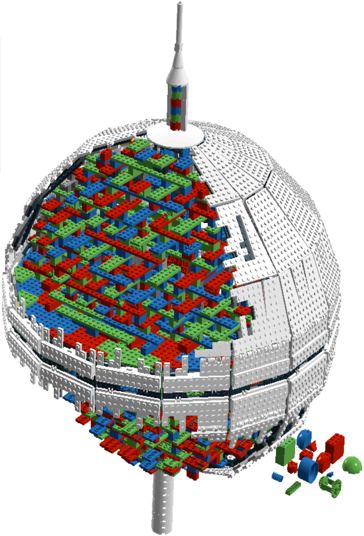
Mass?
Spin?
Charge?



How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?

Mass?
Spin?
Charge?

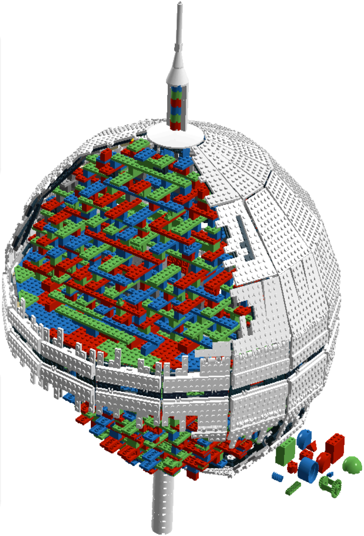
...



How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?

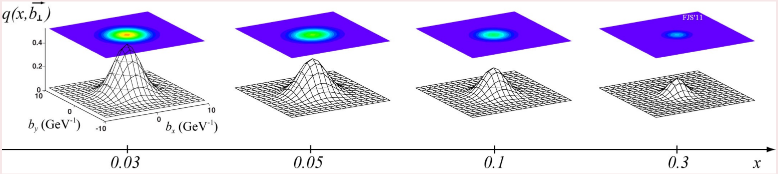
Mass?
Spin?
Charge?
...

What are the relevant **effective degrees of freedom** and **effective interaction** at large distance?



Structuring questions for the hadron physics community

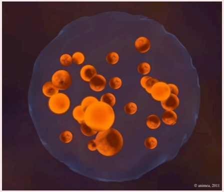
- QCD **mechanisms** behind the origin of **mass** in the **visible universe**?
- **Cartography** of interactions giving its mass to the nucleon?
- **Pressure** and **density** profiles of the nucleon as a continuous medium?
- **Localization** of quarks and gluons inside the nucleon?



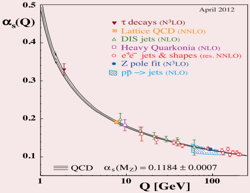
Motivation.

Study nucleon structure to shed new light on nonperturbative QCD.

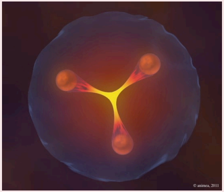
Perturbative QCD



Asymptotic freedom



Nonperturbative QCD

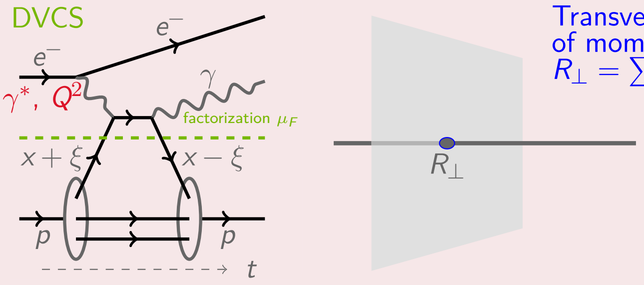


Perturbative AND nonperturbative QCD at work

- Define **universal** objects describing 3D nucleon structure:
  Generalized Parton Distributions (GPD).
- Relate GPDs to measurements using **factorization**:
  Virtual Compton Scattering (DVCS, TCS),
  Deeply Virtual Meson production (DVMP).
- Get **experimental knowledge** of nucleon structure.

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in a hadron.
- DVCS recognized as the cleanest channel to access GPDs.

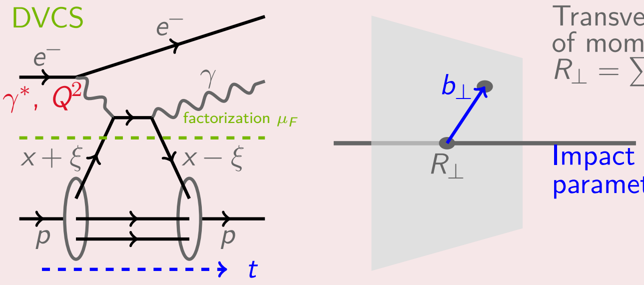
Deeply Virtual Compton Scattering (DVCS)



Transverse center of momentum $R_\perp$
$R_\perp = \sum_i x_i r_{\perp i}$

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in a hadron.
- DVCS recognized as the cleanest channel to access GPDs.

Deeply Virtual Compton Scattering (DVCS)

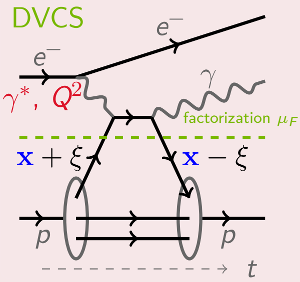


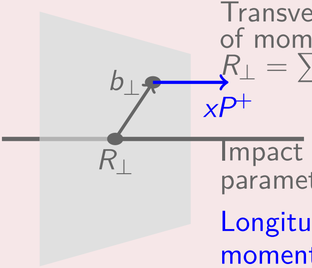
Transverse center of momentum $R_\perp$

$R_\perp = \sum_i x_i r_{\perp i}$

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in a hadron.
- DVCS recognized as the cleanest channel to access GPDs.

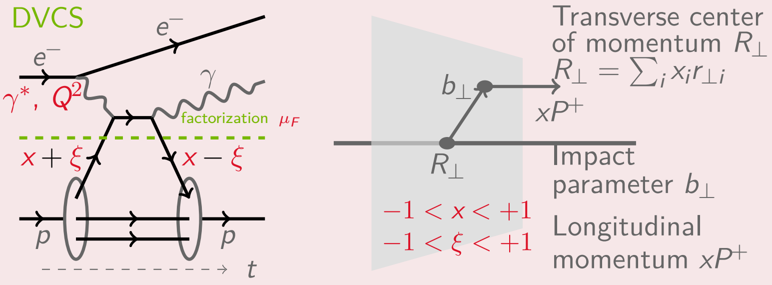
Deeply Virtual Compton Scattering (DVCS)





- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in a hadron.
- DVCS recognized as the cleanest channel to access GPDs.

Deeply Virtual Compton Scattering (DVCS)



- **24 GPDs** $F^i(x, \xi, t, \mu_F)$ for each parton type $i = g, u, d, \ldots$ for leading and sub-leading twists.

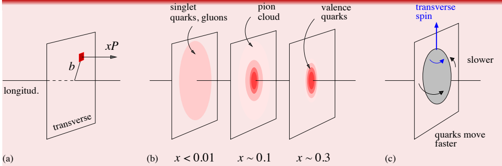
- **Probabilistic interpretation** of Fourier transform of GPD$(x, \xi = 0, t)$ in **transverse plane**.

$$\rho(x, b_\perp, \lambda, \lambda_N) = \frac{1}{2}\left[H(x, 0, b_\perp^2) + \frac{b_\perp^j \epsilon_{ji} S_\perp^i}{M} \frac{\partial E}{\partial b_\perp^2}(x, 0, b_\perp^2) + \lambda\lambda_N \tilde{H}(x, 0, b_\perp^2)\right]$$

- Notations : quark helicity λ, nucleon longitudinal polarization λ_N and nucleon transverse spin $S_\perp$.

Burkardt, Phys. Rev. **D62**, 071503 (2000)

Can we obtain this picture from exclusive measurements?



Weiss, AIP Conf. Proc. **1149**, 150 (2009)

- Most general structure of matrix element of energy momentum tensor between nucleon states:

$$\left\langle N, P+\frac{\Delta}{2}\right| T^{\mu\nu} \left| N, P-\frac{\Delta}{2}\right\rangle = \bar{u}\left(P+\frac{\Delta}{2}\right)\left[A(t)\gamma^{(\mu}P^{\nu)} + B(t)P^{(\mu}i\sigma^{\nu)\lambda}\frac{\Delta_\lambda}{2M} + \frac{C(t)}{M}(\Delta^\mu\Delta^\nu - \Delta^2\eta^{\mu\nu})\right]u\left(P-\frac{\Delta}{2}\right)$$

with $t = \Delta^2$.

- Key observation: **link between GPDs and gravitational form factors**

$$\int \mathrm{d}x\, x H^q(x,\xi,t) = A^q(t) + 4\xi^2 C^q(t)$$

$$\int \mathrm{d}x\, x E^q(x,\xi,t) = B^q(t) - 4\xi^2 C^q(t)$$

Ji, Phys. Rev. Lett. **78**, 610 (1997)

- Spin sum rule:

$$\int \mathrm{d}x\, x\big(H^q(x,\xi,0) + E^q(x,\xi,0)\big) = A^q(0) + B^q(0) = 2J^q$$

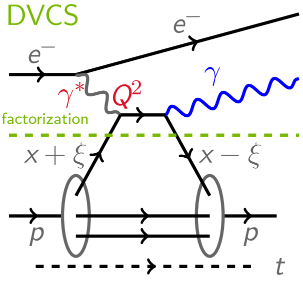
Ji, Phys. Rev. Lett. **78**, 610 (1997)

- **Shear** and **pressure** of a hadron considered as a continuous medium:

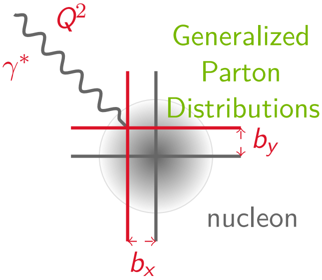
$$\langle N | T^{ij}(\vec{r}) | N \rangle N = s(r)\left(\frac{r^i r^j}{\vec{r}^2} - \frac{1}{3}\delta^{ij}\right) + p(r)\delta^{ij}$$

Polyakov and Shuvaev, hep-ph/0207153

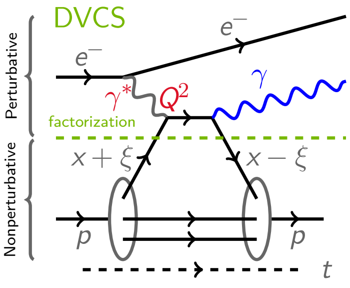
Experimental hadron tomography

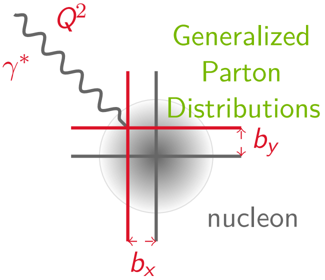


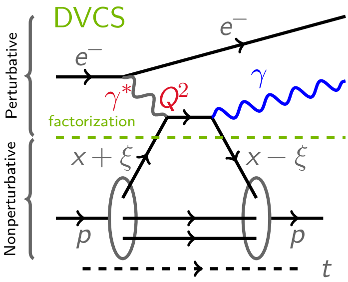
DVCS
e^-
e^-
γ^*
Q^2
γ
factorization
$x+\xi$
$x-\xi$
p
p
t

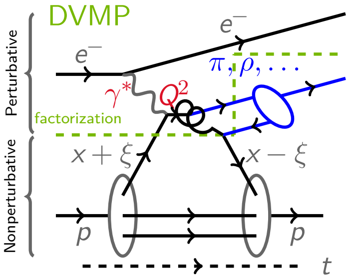


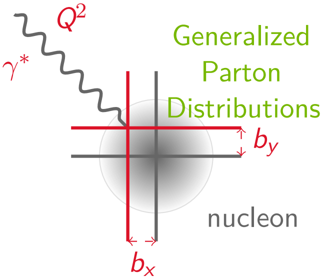
Q^2
γ^*
Generalized Parton Distributions
b_y
nucleon
b_x


DVCS
Perturbative
Nonperturbative
e^-
e^-
γ^*
Q^2
γ
factorization
$x+\xi$
$x-\xi$
p
p
t


Q^2
γ^*
Generalized
Parton
Distributions
b_y
nucleon
b_x

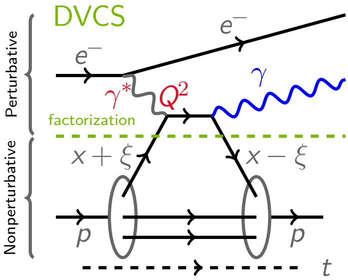

DVCS
Perturbative
Nonperturbative
e^-
e^-
γ^*
Q^2
γ
factorization
$x+\xi$
$x-\xi$
p
p
t

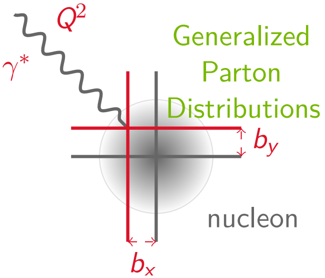

DVMP
Perturbative
Nonperturbative
e^-
e^-
$\pi, \rho, \ldots$
γ^*
Q^2
factorization
$x+\xi$
$x-\xi$
p
p
t

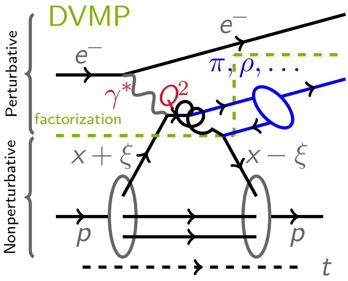

Q^2
γ^*
Generalized
Parton
Distributions
b_y
nucleon
b_x

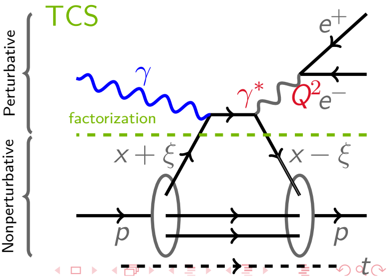
Exclusive processes of current interest (1/2).

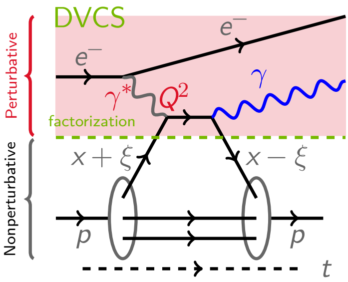
Factorization and universality.



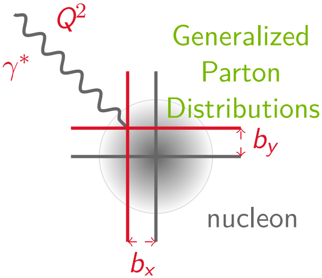




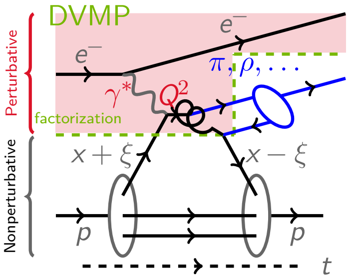




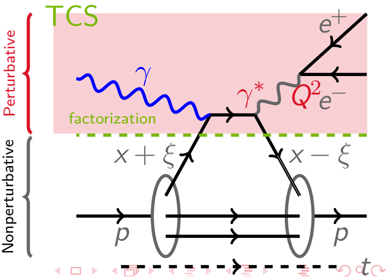
DVCS
Perturbative
Nonperturbative
e^-
e^-
γ^*
Q^2
γ
factorization
$x+\xi$
$x-\xi$
p
p
t



Q^2
γ^*
Generalized
Parton
Distributions
b_y
nucleon
b_x



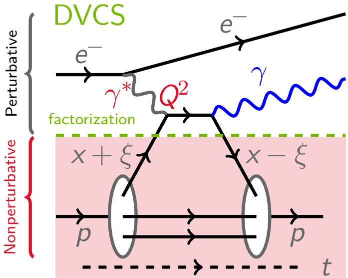
DVMP
Perturbative
Nonperturbative
e^-
e^-
$\pi, \rho, \ldots$
γ^*
Q^2
factorization
$x+\xi$
$x-\xi$
p
p
t

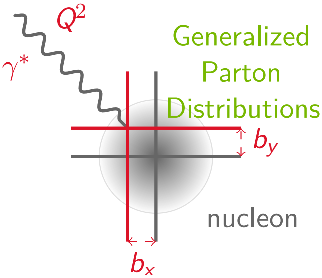


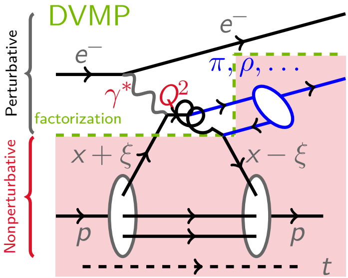
TCS
Perturbative
Nonperturbative
e^+
γ
γ^*
Q^2
e^-
factorization
$x+\xi$
$x-\xi$
p
p
t

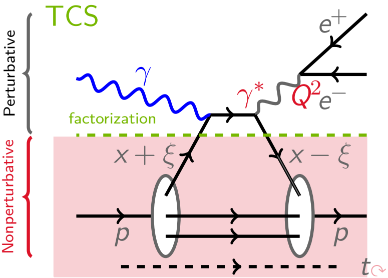
Exclusive processes of current interest (1/2).

Factorization and universality.









Bjorken regime : large Q^2 and fixed $xB \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on **factorization theorems**.
- All-order proofs for DVCS, TCS and some DVMP.
- GPDs depend on a (arbitrary) factorization scale μ_F.
- **Consistency** requires the study of **different channels**.

- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^{1} dx\, C\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F)$$

for a given GPD F.

- CFF $\mathcal{F}$ is a **complex function**.

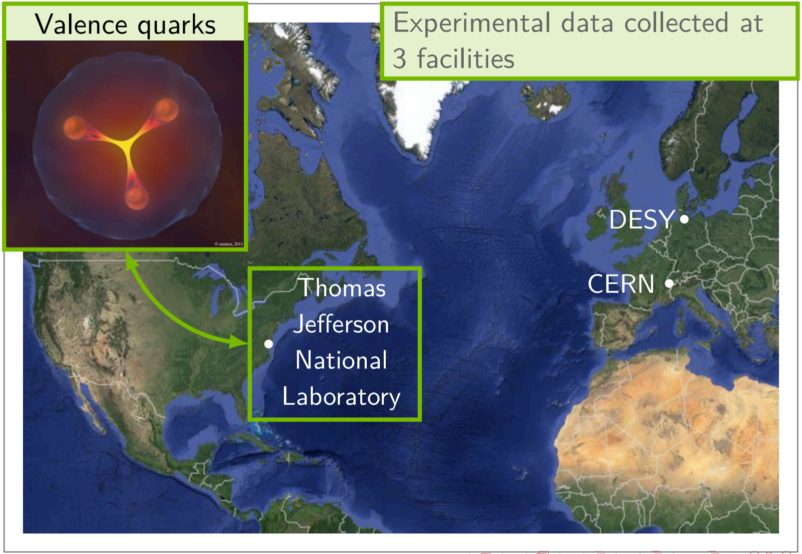
Need for global fits of world data (1/2).

Different facilities will probe different kinematic domains.

Experimental data collected at 3 facilities

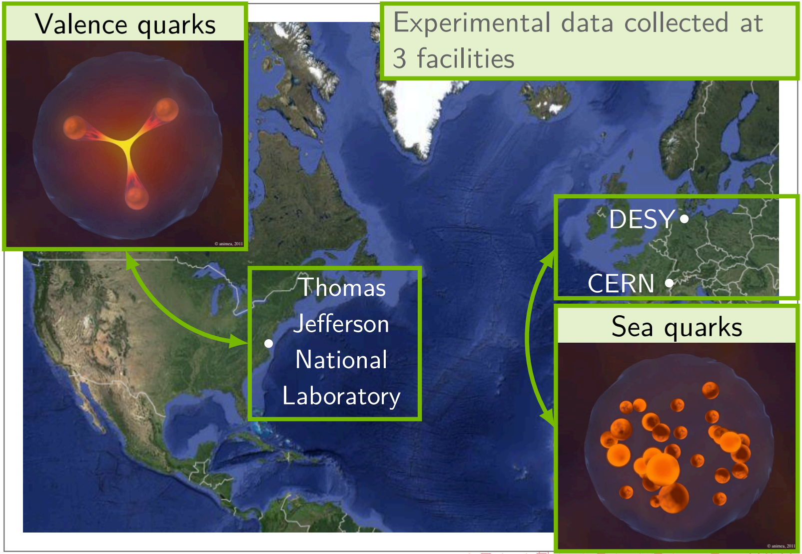
Need for global fits of world data (1/2).

Different facilities will probe different kinematic domains.



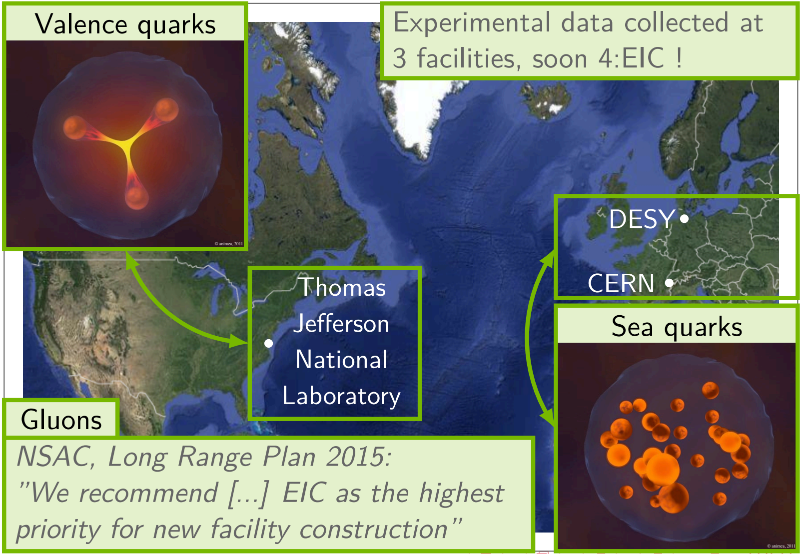
Need for global fits of world data (1/2).

Different facilities will probe different kinematic domains.

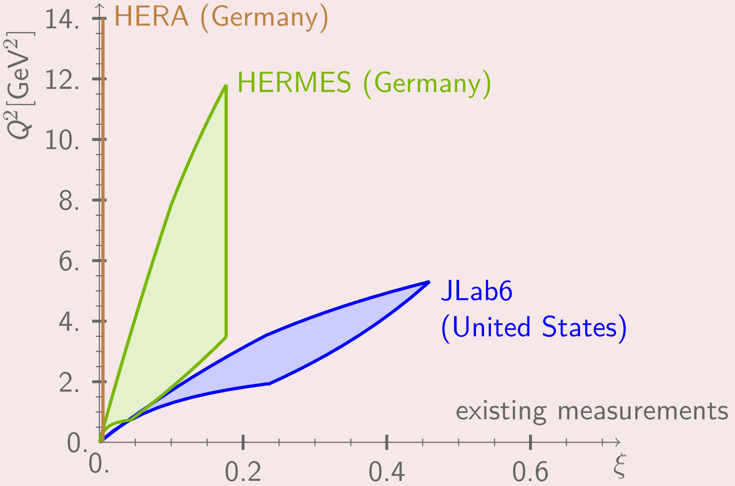


Need for global fits of world data (1/2).

Different facilities will probe different kinematic domains.



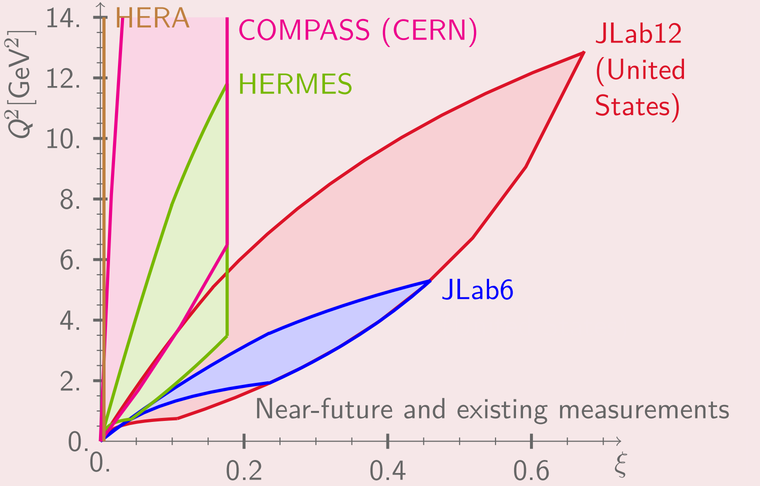
Kinematic reach of existing or near-future DVCS measurements



Need for global fits of world data (2/2).

Sparse data in a large functional space.

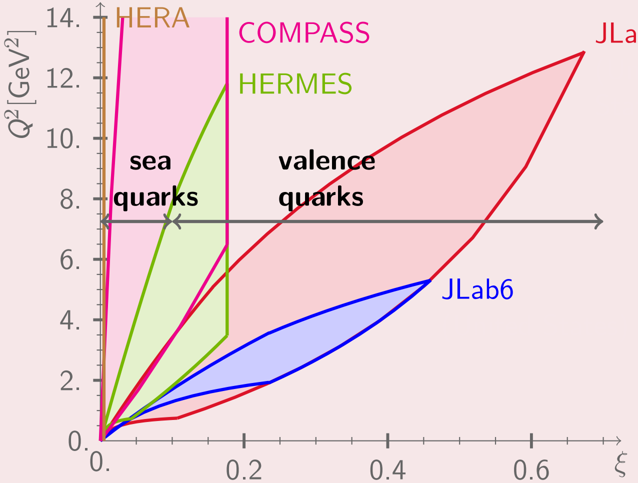
Kinematic reach of existing or near-future DVCS measurements



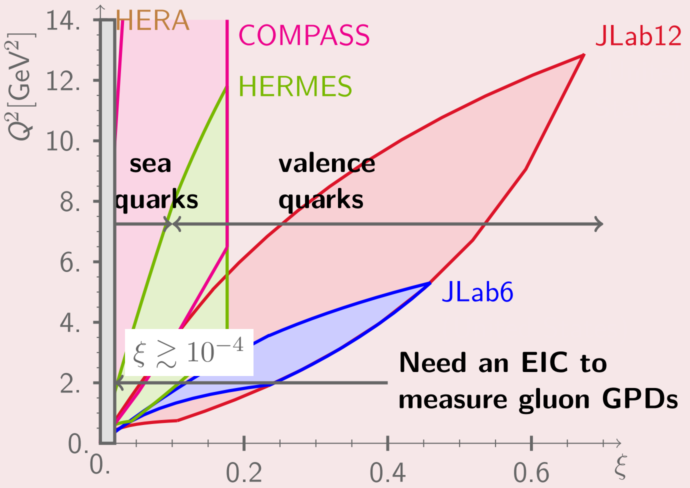
Need for global fits of world data (2/2).

Sparse data in a large functional space.

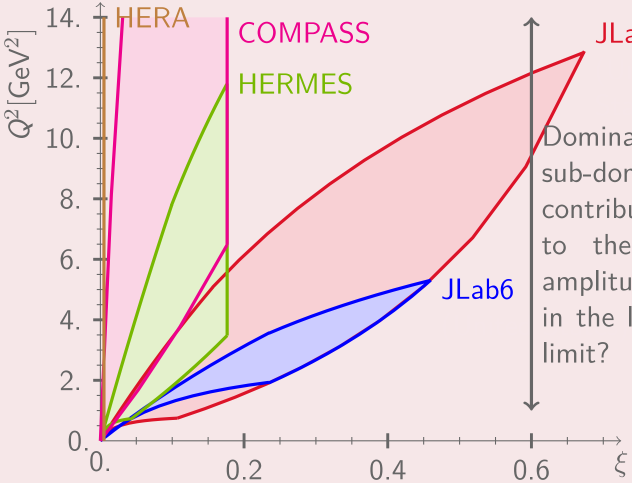
Kinematic reach of existing or near-future DVCS measurements



Kinematic reach of existing or near-future DVCS measurements



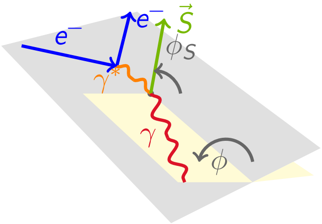
Kinematic reach of existing or near-future DVCS measurements



Dominant and sub-dominant contributions to the DVCS amplitude in the large Q^2 limit?

Definition of observables (1/3).

Harmonic structure of $ep \to ep\gamma$ amplitude.



- Study the **harmonic structure** of $ep \to ep\gamma$ amplitude.

Diehl *et al.*, Phys. Lett. **B411**, 193 (1997)

$$|\mathcal{M}_{\mathrm{BH}}|^2 \propto \frac{1}{|t|}\frac{1}{P(\cos\phi)}\sum_{n=0}^{3}\left[c_n^{\mathrm{BH}}\cos(n\phi) + s_n^{\mathrm{BH}}\sin(n\phi)\right]$$

$$|\mathcal{M}_{\mathrm{DVCS}}|^2 \propto \sum_{n=0}^{3}\left[c_n^{\mathrm{DVCS}}\cos(n\phi) + s_n^{\mathrm{DVCS}}\sin(n\phi)\right]$$

$$\mathcal{M}_{\mathrm{I}} \propto \frac{1}{|t|}\frac{1}{P(\cos\phi)}\sum_{n=0}^{3}\left[c_n^{\mathrm{I}}\cos(n\phi) + s_n^{\mathrm{I}}\sin(n\phi)\right]$$

- Combined beam-spin and charge asymmetries :

$$d\sigma^{h_e, Q_e}(\phi) = d\sigma_{\mathrm{UU}}(\phi) \left[1 + h_e A_{\mathrm{LU,DVCS}}(\phi) + Q_e h_e A_{\mathrm{LU,I}}(\phi) + Q_e A_{\mathrm{C}}(\phi)\right]$$

- Single beam-spin asymmetry :

$$A_{\mathrm{LU}}^{Q_e}(\phi) = \frac{d\sigma^{Q_e}_{\rightarrow} - d\sigma^{Q_e}_{\leftarrow}}{d\sigma^{Q_e}_{\rightarrow} + d\sigma^{Q_e}_{\leftarrow}}$$

- Relation between observables :

$$A_{\mathrm{LU}}^{Q_e}(\phi) = \frac{Q_e A_{\mathrm{LU,I}}(\phi) + A_{\mathrm{LU,DVCS}}(\phi)}{1 + Q_e A_{\mathrm{C}}(\phi)}$$

- Compute Fourier coefficients of asymmetries.

Typical kinematics

Experiment	Kinematics		
	x_B	Q^2 [GeV^2]	t [GeV^2]
HERA	0.001	8.00	-0.30
COMPASS	0.05	2.00	-0.20
HERMES	0.09	2.50	-0.12
CLAS	0.19	1.25	-0.19
HALL A	0.36	2.30	-0.23

Definition of observables (3/3).

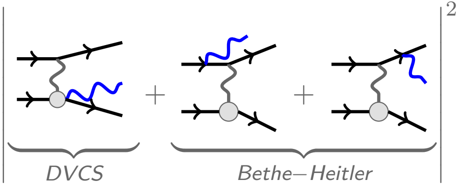
What are the probed combinations of CFFs?

Selection of observables

Experiment	Observable	Normalized CFF dependence
HERMES	$A_{\rm C}^{\cos 0\phi}$	$\mathrm{Re}\mathcal{H} + 0.06\mathrm{Re}\mathcal{E} + 0.24\mathrm{Re}\widetilde{\mathcal{H}}$
	$A_{\rm C}^{\cos \phi}$	$\mathrm{Re}\mathcal{H} + 0.05\mathrm{Re}\mathcal{E} + 0.15\mathrm{Re}\widetilde{\mathcal{H}}$
	$A_{\rm LU,I}^{\sin \phi}$	$\mathrm{Im}\mathcal{H} + 0.05\mathrm{Im}\mathcal{E} + 0.12\mathrm{Im}\widetilde{\mathcal{H}}$
	$A_{\rm UL}^{+,\sin \phi}$	$\mathrm{Im}\widetilde{\mathcal{H}} + 0.10\mathrm{Im}\mathcal{H} + 0.01\mathrm{Im}\mathcal{E}$
CLAS	$A_{\rm LU}^{-,\sin \phi}$	$\mathrm{Im}\mathcal{H} + 0.06\mathrm{Im}\mathcal{E} + 0.21\mathrm{Im}\widetilde{\mathcal{H}}$
	$A_{\rm UL}^{-,\sin \phi}$	$\mathrm{Im}\widetilde{\mathcal{H}} + 0.12\mathrm{Im}\mathcal{H} + 0.04\mathrm{Im}\mathcal{E}$
HALL A	$\sigma^{\cos 0\phi}$	$1 + 0.05\mathrm{Re}\mathcal{H} + 0.007\mathcal{H}\mathcal{H}^*$
	$\sigma^{\cos \phi}$	$1 + 0.12\mathrm{Re}\mathcal{H} + 0.05\mathrm{Re}\widetilde{\mathcal{H}}$

- The DVCS and BH processes have the **same incoming and outgoing states**:

$$\sigma(ep \to ep\gamma) = \left| \underbrace{\mathcal{A}}_{DVCS} + \underbrace{\mathcal{A}_1 + \mathcal{A}_2}_{Bethe-Heitler} \right|^2$$

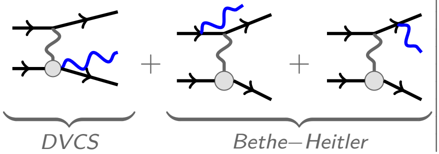


A quantum mechanical measurement .

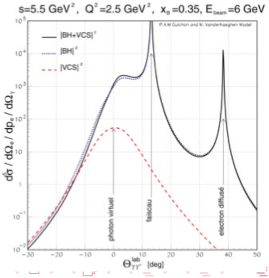
Quantum interference and amplification.

- The DVCS and BH processes have the **same incoming and outgoing states**:

$$\sigma(ep \to ep\gamma) = \left| \underbrace{\cdots}_{DVCS} + \underbrace{\cdots + \cdots}_{Bethe-Heitler} \right|^2$$



- Measurement of the **interference**.
- BH under control thanks to **form factors**.

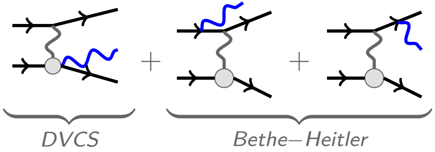


A quantum mechanical measurement .

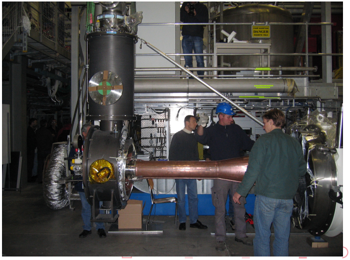
Quantum interference and amplification.

- The DVCS and BH processes have the **same incoming and outgoing states**:

$$\sigma(ep \to ep\gamma) = \Big| \quad + \quad + \quad \Big|^2$$



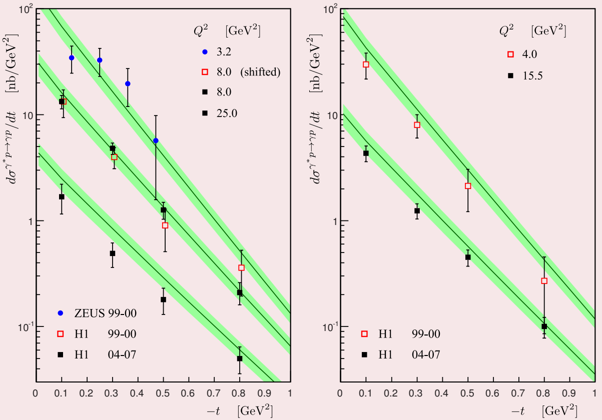
- Measurement of the **interference**.
- BH under control thanks to **form factors**.
- **Polarized** beam or target.



Goloskokov-Kroll (GK) model on DVCS.

No parameter of the GK model **was tuned** to analyse DVCS.

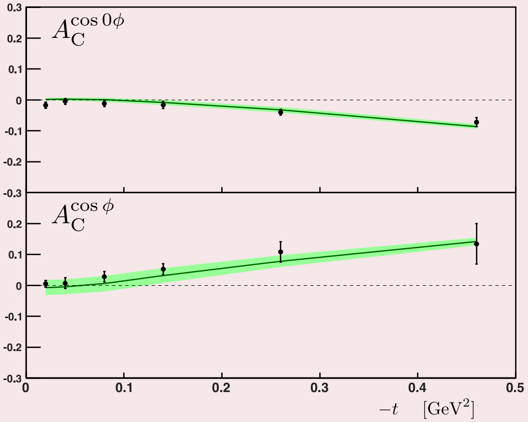
Differential cross section, HERA



Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

No parameter of the GK model **was tuned** to analyse DVCS.

Beam Charge Asymmetry, HERMES

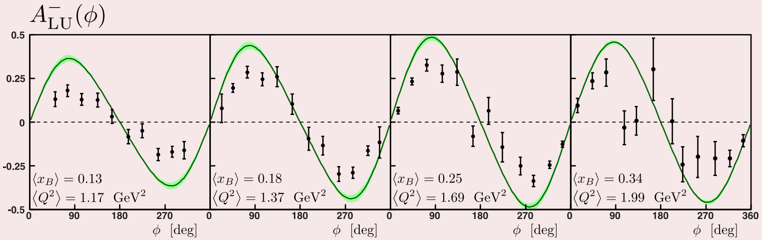


Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

Goloskokov-Kroll (GK) model on DVCS.

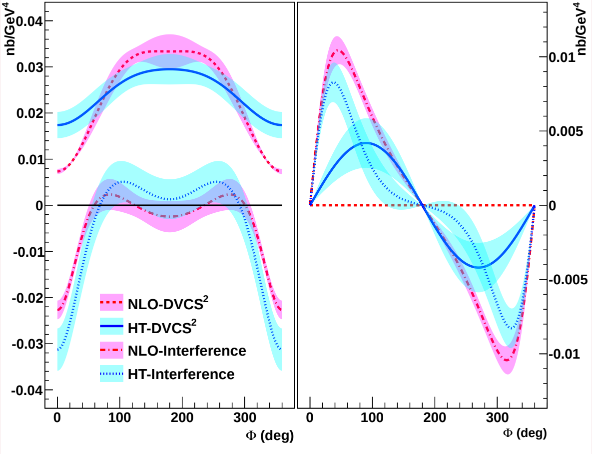
No parameter of the GK model **was tuned** to analyse DVCS.

Beam Spin Asymmetry, CLAS



Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

t/Q^2 and M^2/Q^2 are not small!



Defurne *et al.*, arXiv:1703.09442 [hep-ex]

Summary of first extractions.

Feasibility of twist-2 analysis of existing data.

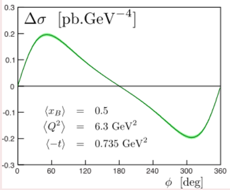
- **Dominance** of twist-2 and **validity** of a GPD analysis of DVCS data.
- $Im\mathcal{H}$ **best determined**. Large uncertainties on $Re\mathcal{H}$.
- However sizable **higher twist contamination** for DVCS measurements.

See more on fits.

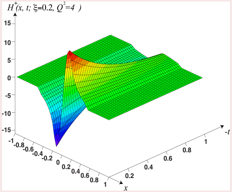
Imaging the nucleon. How?

Extracting GPDs is not enough...Need to extrapolate!

1. Experimental data fits



2. GPD extraction



3. Nucleon imaging

Images from Guidal et al.,
Rept. Prog. Phys. 76 (2013) 066202

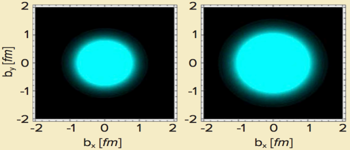
Reaching for the Horizon

The 2015 Long Range Plan for Nuclear Science

Sidebar 2.2: The First 3D Pictures of the Nucleon

A computed tomography (CT) scan can help physicians pinpoint minute cancer tumors, diagnose tiny broken bones, and spot the early signs of osteoporosis. Now physicists are using the principles behind the procedure to peer at the inner workings of the proton. This breakthrough is made possible by a relatively new concept in nuclear physics called generalized parton distributions.

An intense beam of high-energy electrons can be used



1. **Extract** $H(x, \xi, t, \mu_F^{\mathrm{ref}})$ from experimental data.
2. **Extrapolate** to vanishing skewness $H(x, 0, t, \mu_F^{\mathrm{ref}})$.
3. **Extrapolate** $H(x, 0, t, \mu_F^{\mathrm{ref}})$ up to infinite t.
4. **Compute** 2D Fourier transform in transverse plane:

$$H(x, b_\perp) = \int_0^{+\infty} \frac{\mathrm{d}|\Delta_\perp|}{2\pi} |\Delta_\perp| J_0(|b_\perp||\Delta_\perp|) H(x, 0, -\Delta_\perp^2)$$

5. **Propagate** uncertainties.
6. **Control** extrapolations with an accuracy matching that of experimental data with **sound** GPD models.

The challenge of the high precision era.

Higher order and higher twist contributions, and GPD modeling.

- Evaluation of the impact of higher order effects.

▸ See more on NLO evaluations.

- Evaluation of the impact of target mass and finite-t corrections.

▸ See more on DVCS kinematics.

- Extrapolations with **GPD models**.

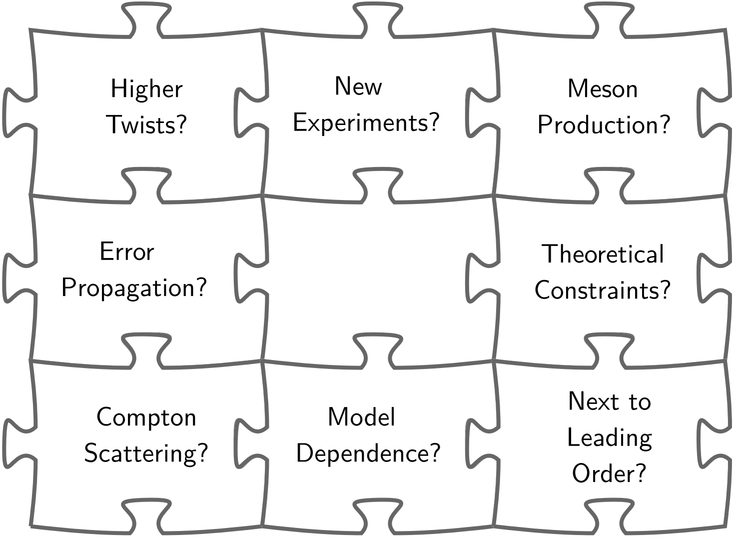
▸ See more on DVCS at LO.

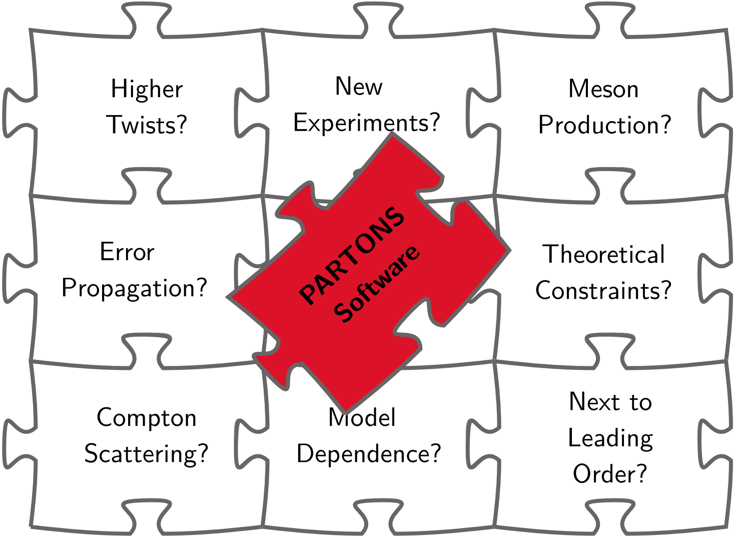
- Evaluation of the contribution of **higher twist** GPDs.

- DVMP: sensitivity to **DA models**.

Software for the phenomenology of GPDs.

Different questions to be answered with the same tools.




Higher Twists?
New Experiments?
Meson Production?
Error Propagation?
PARTONS Software
Theoretical Constraints?
Compton Scattering?
Model Dependence?
Next to Leading Order?

Toolbox for high precision: the PARTONS project

PARtonic
Tomography
Of
Nucleon
Software

Experimental data and phenomenology	Full processes
Computation of amplitudes	Small distance contributions
First principles and fundamental parameters	Large distance contributions

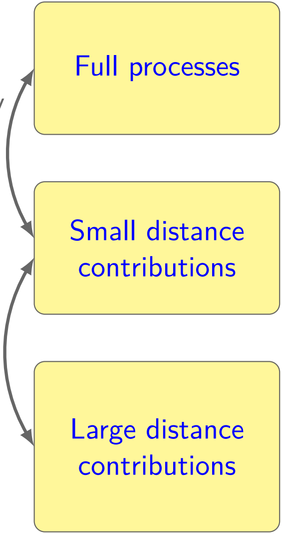
Computing chain design.

Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Computation of amplitudes

First principles and fundamental parameters

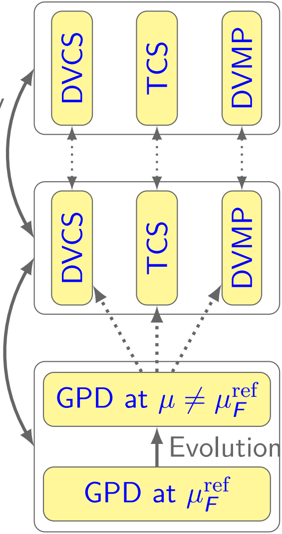


Differential studies: physical models and numerical methods.

Experimental data and phenomenology

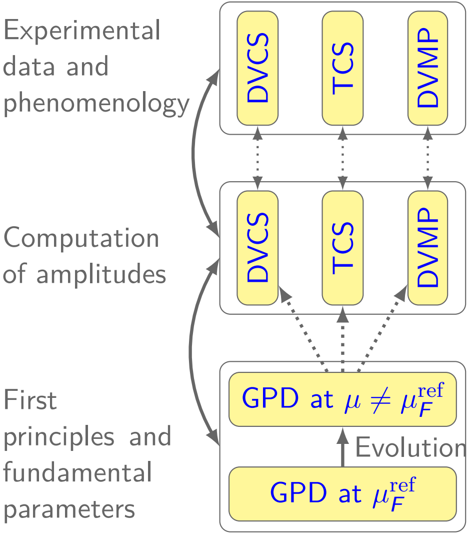
Computation of amplitudes

First principles and fundamental parameters



Computing chain design.

Differential studies: physical models and numerical methods.



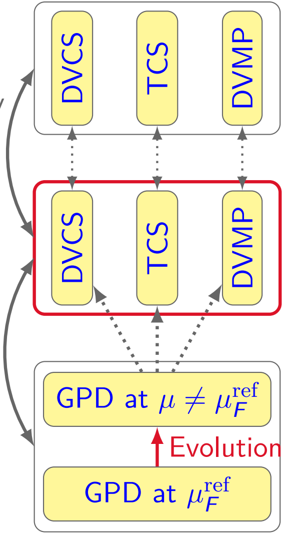
- Many observables.
- Kinematic reach.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

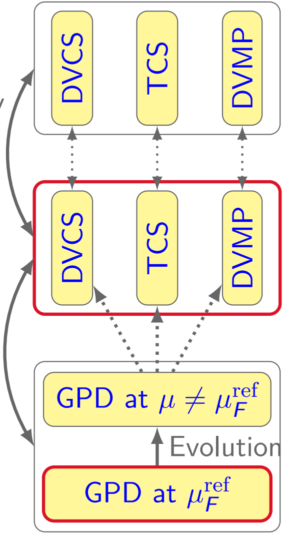
- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

Computing chain design.

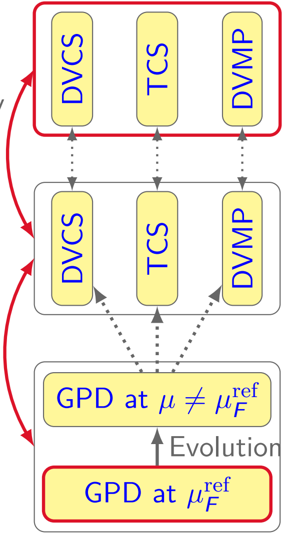
Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

Computing chain design.

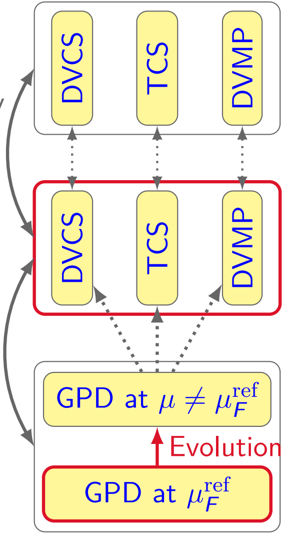
Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

Computing chain design.

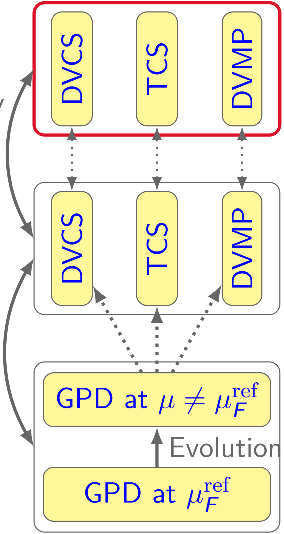
Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

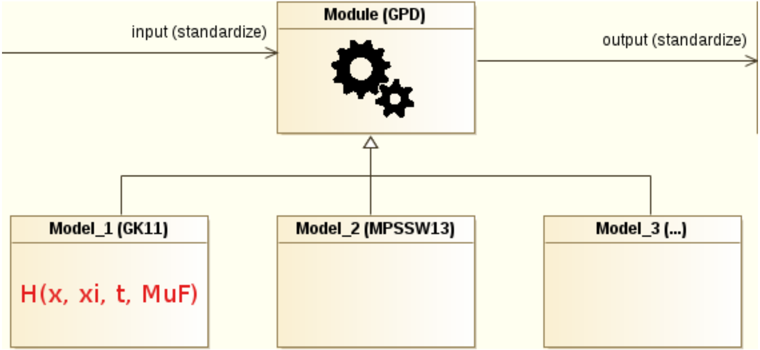
- 3 stages:
  1. Design.
  2. Integration and validation.
  3. Benchmarking and production.
- Flexible software architecture.

  B. Berthou *et al.*, *PARTONS: a computing platform for the phenomenology of Generalized Parton Distributions* arXiv:1512.06174, *to appear in Eur. Phys. J. C.*

  See more on software architecture.
- 1 new physical development = 1 new module.
- *Aggregate* **knowledge** and **know-how**:
  - Models
  - Measurements
  - Numerical techniques
  - Validation
- What *can* be automated *will be* automated.

Modularity.

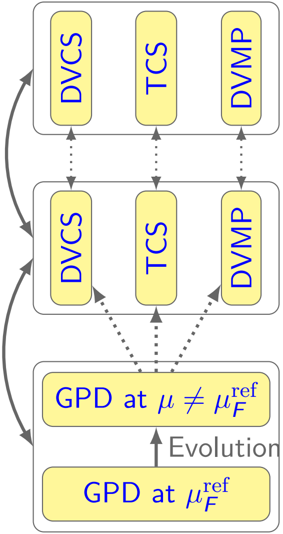
Inheritance, standardized inputs and outputs.



- Steps of logic sequence in parent class.
- Model description and related mathematical methods in daughter class.

Flexibility.

Example: implementation of new coefficient functions.



- A DVCS coefficient function module generically outputs a complex number when provided $(\xi, t, Q^2, \mu_F^2, \mu_R^2)$.

ConvolCoeffFunctionModule.h

```
virtual std::complex<double> compute(
double xi, double t, double Q2, double MuF2,
 double MuR2, GPDType::Type gpdType) =
0;
```

- This module can be anything:
  - Constant CFFs for local fits.
  - CFFs for massless quarks.
  - CFFs for heavy quarks.
  - CFFs with TMC.
  - ...

Handling experimental data.

Using the power of SQL for data selections.

- **All fixed-target DVCS data collected at Jefferson Lab** are stored in the database used for fits.
- No data about TCS or DVMP so far.
- Including **all existing DVCS data sets** in a database is **a matter of hours**.
- Data selection from SQL requests for fits.

insert_CLAS_asymmetries.sql

```sql
-- Kinematics --
INSERT INTO observable_kinematic (bin_id, xB, t, Q2, E, phi) VALUES(0, 0.19400, -0.11000, 1.68000, 5.93200, 25.00000);
SET @last_observable_kinematic_id = LAST_INSERT_ID();
-- Value and uncertainties --
INSERT INTO observable_result (observable_name, observable_value, stat_error_lb, stat_error_ub, syst_error_lb, syst_error_ub, total_error, observable_kinematic_id) VALUES('Alu', 0.37000, 0.23000, 0.23000, 0.01000, 0.01000, 0.00000, @last_observable_kinematic_id);
```

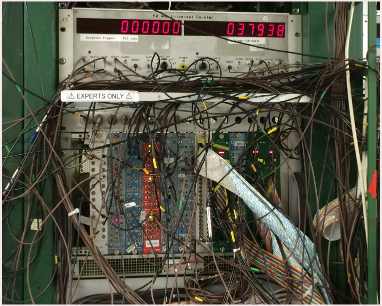
Systematic studies made easy.

A faster and safer way to GPD phenomenology.

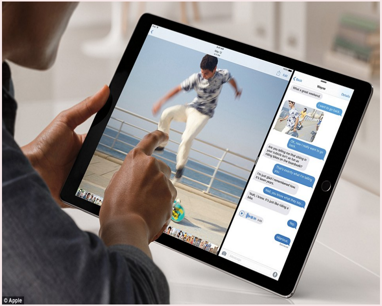
Automation allows...:

- to run **numerous computations** with various physical assumptions,
- to run **nonregression** tests.
- to perform **fits** with various models.
- physicists to **focus on physics**!

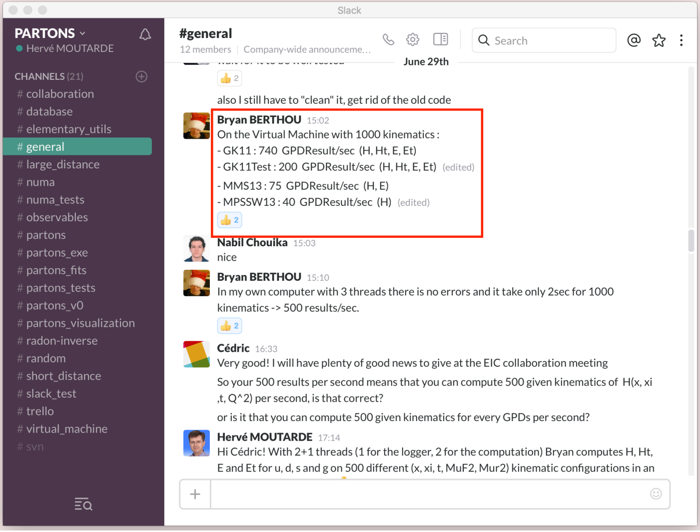
Without PARTONS



With PARTONS



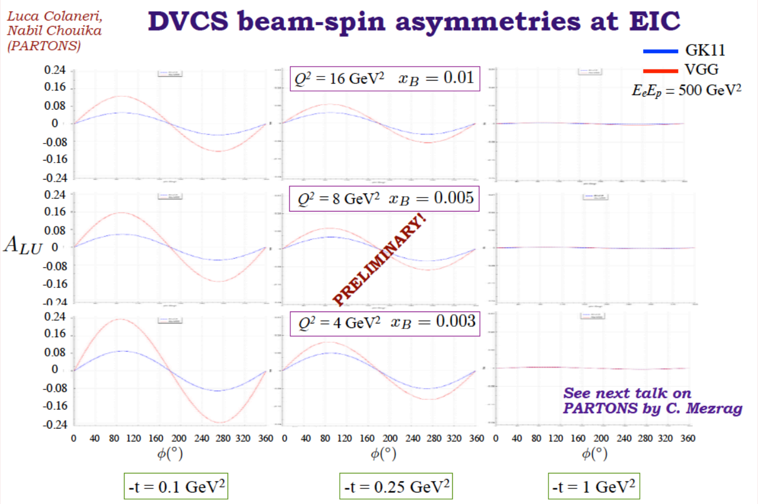
GPD computations with or without threads



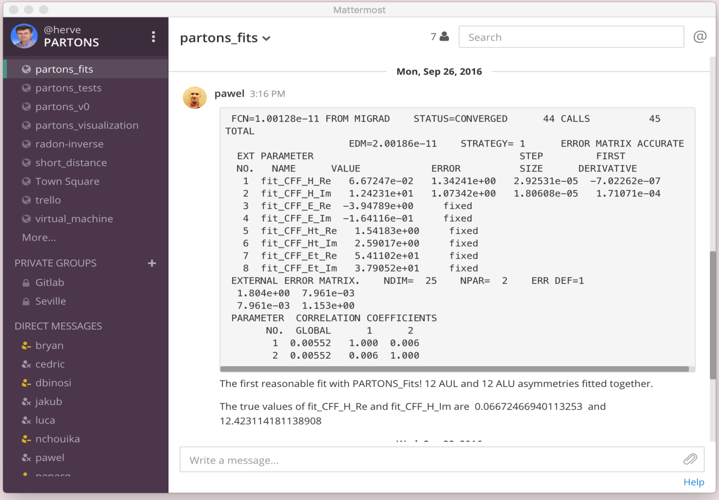
Systematic studies made fast.

What can be done from scratch in about 1 hour.

From D. Sokhan's talk, EIC User Group Meeting, ANL, 2016



First local fit of pseudo DVCS data, Sep. 26th, 2016



Succesful global fit of Jefferson Lab DVCS data, Apr. 5th, 2017

RESULTS

- Kinematic cuts
  - $Q^2 > 1.5$ GeV^2 (*where we can rely on LO approximation*)
  - $-t / Q^2 < 0.25$ (*where we can rely on GPD factorization*)
- χ^2 / ndf: $3272.6 / (3433 - 7) \approx 0.96$
- Free parameters: a_{Hsea}, $a_{\tilde{H}val}$, $a_{\tilde{H}sea}$, C_{sub}, a_{sub}, N_E, $N_{\tilde{E}}$
- χ^2 / ndf per data set

[1] Phys. Rev. C 92, 052202 (2015)
[2] Phys. Rev. Lett. 115, 212003 (2015)
[3] Phys. Rev. D 91, 052014 (2015)

Experiment	Reference	Observables	N points all	N points selected	chi2	chi2 / ndf
Hall A	[1] KINX2	σUU	120	120	135.0	1.19
Hall A	[1] KINX2	ΔσLU	120	120	98.9	0.88
Hall A	[1] KINX3	σUU	108	108	274.8	2.72
Hall A	[1] KINX3	ΔσLU	108	108	107.3	1.06
CLAS	[2]	σUU	1933	1333	1089.2	0.82
CLAS	[2]	ΔσLU	1933	1333	1171.9	0.88
CLAS	[3]	AUL, ALU, ALL	498	305	338.1	1.13

GPD or CFF fits (3/5).

The second global fit of CFFs in the valence region.

Careful statistical analysis

RESULTS

- Values of parameters and correlation matrix

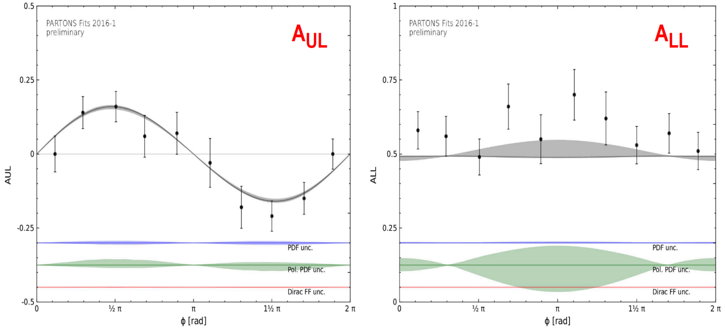
GPD	Parameter	Value	Error
H	Cu val	1.21	-
H	Cu sea	1.27	-
H	Cd val	1.2	-
H	Cd sea	1.27	-
Htilde	Cu val	1.07	-
Htilde	Cu sea	1.06	-
Htilde	Cd val	1.11	-
Htilde	Cd sea	1.07	-
H	a val	0.74	-
H	a sea	52.7	62.2
Htilde	a val	2.51	0.35
Htilde	a sea	0	1.35
H	C sub	-0.81	0.16
H	a sub	-0.39	0.6
E	N	-8.08	0.57
Etilde	N	-0.45	0.07

Comparison to CLAS data

RESULTS

CLAS: A_{UL} and A_{LL}
@ $x_B = 0.26$, $t = -0.23$ GeV², $Q^2 = 2.0$ GeV², $E = 5.9$ GeV

0.68 c.l.



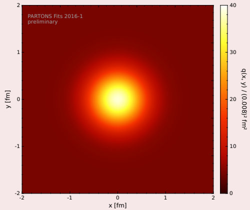
Good description of experimental data, large systematics coming from Δq

Paweł Sznajder DIS 2017 16

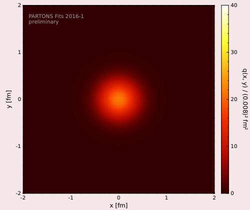
GPD or CFF fits (5/5).

Proton tomography from experimental data.

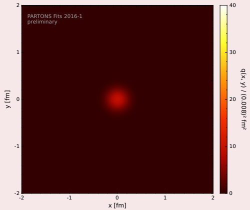
$x = 0.01$

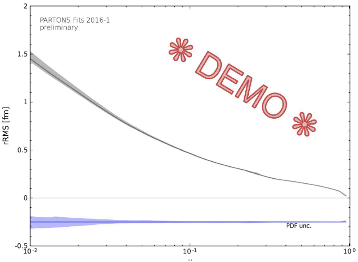


$x = 0.1$



$x = 0.5$





GPD computing made simple.

Each line of code corresponds to a physical hypothesis.

gpdExample()

```
// Lots of includes
#include <src/Partons.h>
...
// Retrieve GPD service
GPDService* pGPDService = Partons::getInstance()−>getServiceObjectRegistry
()−>getGPDService();
// Load GPD module with the BaseModuleFactory
GPDModule* pGK11Model = Partons::getInstance()−>getModuleObjectFactory
()−>newGPDModule(GK11Model::classId);
// Create a GPDKinematic(x, xi, t, MuF, MuR) to compute
GPDKinematic gpdKinematic(0.1, 0.00050025, −0.3, 8., 8.);
// Compute data and store results
GPDResult gpdResult = pGPDService−>
computeGPDModelRestrictedByGPDType(gpdKinematic, pGK11Model,
GPDType::ALL);
// Print results
std::cout << gpdResult.toString() << std::endl;

delete pGK11Model;
pGK11Model = 0;
```

GPD computing automated.

Each line of code corresponds to a physical hypothesis.

computeOneGPD.xml

```xml
<?xml version="1.0" encoding="UTF-8" standalone="yes" ?>
<scenario id="01" date="" description="Example␣:␣computation␣of␣one␣GPD␣model␣(GK11)␣without␣evolution">
    <!-- Select type of computation -->
    <task service="GPDService" method="computeGPDModel">
        <!-- Specify kinematic -->
        <kinematics type="GPDKinematic">
            <param name="x" value="0.1" />
            <param name="xi" value="0.00050025" />
            <param name="t" value="-0.3" />
            <param name="MuF2" value="8" />
            <param name="MuR2" value="8" />
        </kinematics>
    <!-- Select GPD model and set parameters -->
    <computation_configuration>
            <module type="GPDModule">
                <param name="className" value="GK11Model" />
            </module>
    </computation_configuration>
</task>
</scenario>
```

GPD computing automated.

Each line of code corresponds to a physical hypothesis.

computeOneGPD.xml

```
<?xml version="1.0" encoding="UTF-8" stand
<scenario id="01" date="" description="Exam                     D
␣model␣(GK11)␣without␣evolution">
    <!-- Select type of computation -->
    <task service="GPDService" method="com
        <!-- Specify kinematic -->
        <kinematics type="GPDKinematic">
            <param name="x" value="0.1" /
            <param name="xi" value="0.000
            <param name="t" value="-0.3" /
            <param name="MuF2" value="8" /
            <param name="MuR2" value="8" /
        </kinematics>
    <!-- Select GPD model and set paramete
    <computation_configuration>
            <module type="GPDModule">
                <param name="className" va
            </module>
    </computation_configuration>
</task>
</scenario>
```

$H^u = 0.822557$
$H^{u(+)} = 0.165636$
$H^{u(-)} = 1.47948$

$H^d = 0.421431$
$H^{d(+)} = 0.0805182$
$H^{d(-)} = 0.762344$

$H^s = 0.00883408$
$H^{s(+)} = 0.0176682$
$H^{s(-)} = 0$

$H^g = 0.385611$
and E, $\tilde{H}$, $\tilde{E}$, ...

Observable computing automated.

Each line of code corresponds to a physical hypothesis.

computeManyKinematicsOneModel.xml

```
<scenario date="2016-10-18" description="Use␣kinematics␣list">
    <task service="ObservableService" method="computeManyKinematicOneModel" storeInDB="1">
        <kinematics type="ObservableKinematic">
            <param name="file" value="observable_kinematics.dat" />
        </kinematics>
        <computation_configuration>
            <module type="Observable">
                <param name="className" value="Alu" />
            </module>
            <module type="DVCSModule">
                <param name="className" value="BMJ2012Model" />
                <param name="beam_energy" value="1066" />
            </module>
            <module type="DVCSConvolCoeffFunctionModule">
                <param name="className" value="DVCSCFFModel" />
                <param name="qcd_order_type" value="LO" />
            </module>
            <module type="GPDModule">
                <param name="className" value="GK11Model" />
            </module>
        </computation_configuration>
```

Pre-PARTONS times...

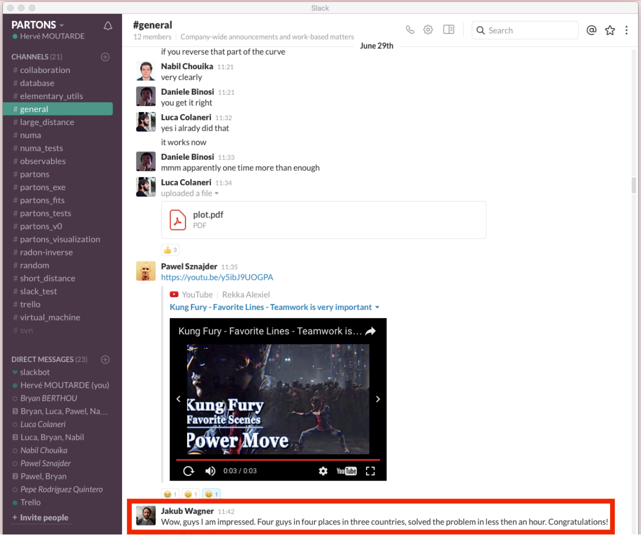
First mention of the PARTONS project in a conference.

PROPHET.

Platform for **R**epresenting the **O**rganization of **P**artons inside **H**adrons and **E**xperimental **T**omographies.

1. Comprehensive **database of experimental results**.
2. Comprehensive **database of theoretical predictions**.
3. **Fitting engine**.
4. **Propagation** of statistic and systematic **uncertainties**.
5. **Visualizing software** to compare experimental results and model expectations.
6. Connection to **experimental set-up descriptions** to design new experiments.
7. **Interactive website** providing free access to model and experimental values.

Many (unfortunately not all!) problems can be solved fast



1 multidisciplinary team over 5 countries.

Future experiments in the US, most project members in Europe...

Active developpers N. Chouika (Irfu, France), L. Colaneri (IPNO, France), P. Sznajder (NCBJ, Poland).

Not-so-active developpers D. Binosi (ECT*/FBK, Italy), C. Mezrag (ANL, USA), H. Moutarde (Irfu, France), J. Wagner (NCBJ, Poland).

Advising P. Lafitte (ECP, France), J. Rodríguez-Quintero (U. Huelva, Spain).

Funding K. Joo (U. Conn., USA), M. Pennington, then J. Qiu (Jefferson Lab, USA)?

Friends M. Guidal (IPNO, France), B. Pire (CPhT, France), L. Szymanowski (NCBJ, Poland), S. Wallon (LPT, France).

Future projects C. Lorcé (CPhT, France), G. Duplancic (IRB, Croatia), K. Passek-Kumerički (IRB, Croatia).

Requests Event generators (COMPASS, Jefferson Lab, EIC).

Nonpermanent members

Conclusion

- What makes hadron structure studies so interesting:
  - Deep **physical questions** waiting for answers!
  - Well-defined **theoretical framework** and **observables**.
  - Active **experimental programs** worldwide.
- **Challenging constraints** expected from:
  - Jefferson Lab in the valence sector,
  - CERN in the sea sector,
  - EIC (later) in the gluon sector.
- Development of the PARTONS framework for **phenomenology** and **theory** purposes.
- Initiated as an experimentalist companion, grown as a **multidisciplinary project** attracting theorists to the field.
- **Fitting engine** ready for global and local fits. **Original global CFF fits** recently achieved, meeting initial aim!
- **First release** of PARTONS... *as soon as possible!*

Appendix

Local fits

Take each kinematic bin independantly of the others.
Extraction of $Re\mathcal{H}$, $Im\mathcal{H}$, ...as independent parameters.

Global fit

Take all kinematic bins at the same time. Use a parametrization of GPDs or CFFs.

Hybrid : Local / global fit

Option 1 Local fits and then smoothness assumption.

Option 2 Local fits and then 1-parameter fit.

Neural networks

Exploratory stage for GPDs.

Local fits

Take each kinematic bin independantly of the others.
Extraction of $Re\mathcal{H}$, $Im\mathcal{H}$, ...as independent parameters.

M. Guidal, Eur. Phys. J. **A39**, 5 (2009)

- **Almost model-independent**: relies on twist-2 dominance assumption and assume bounds for the fitting domain.
- Interpretation of **uncertainties** on extracted quantities? Contributions from measurements uncertainties, correlations between CFFs and fitting domain boundaries.
- Interpretation of **extracted quantities**? *e.g.*mixing of quark and gluon GPDs due to NLO effects.
- **Oscillations** between different (x_B, t, Q^2) bins may happen.
- **Extrapolation** problem left open.

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

where $a = \mathcal{O}(Q^{-1}), \quad b = \mathcal{O}(Q^{-4}), \quad c = \mathcal{O}(Q^{-1}),$
$d = \mathcal{O}(Q^{-2}), \quad e = \mathcal{O}(Q^{-5}).$

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

- **Underconstrained** problem (8 fit parameters : real and imaginary parts of 4 CFFs $\mathcal{H}$, $\mathcal{E}$, $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$).

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

- **Underconstrained** problem.
- Need other asymmetries on **same** kinematic bin to allow extraction of **all CFFs** (or **add** $\simeq$ 5-10 % **systematic uncertainty**).

Overview of current extraction methods.

Problems: Model dependence? Uncertainties?

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

- **Underconstrained** problem.
- Need other asymmetries on **same** kinematic bin to allow extraction of **all CFFs**.
- Add physical input? **Dispersion relations**, etc.

  Kumericki *et al.*, Phys. Part. Nucl. **45**, 723 (2014)

  Guidal *et al.*, Rept. Prog. Phys. **76**, 066202 (2013)

Global fit

Take all kinematic bins at the same time. Use a parametrization of GPDs or CFFs.

Kumericki, Nucl. Phys. **B841**, 1 (2010)

- **Model-dependent** approach.
- Allows the **implementation of theoretical constraints** on GPDs or CFFs.
- Guideline for **extrapolation** outside the physical domain.
- Compromise between number of parameters and number of described GPDs (flavor dependence, higher-twists, ...)?
- Impact on the **choice of a fitting strategy**?

Hybrid : Local / global fit

Option 1 Local fits and then smoothness assumption.

Moutarde, Phys. Rev. **D79**, 094021 (2009)

- Avoid unphysical oscillations between different (x_B, t, Q^2) bins by comparing to a **global fit by a smooth function**:

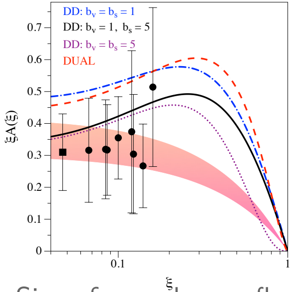
$$H^+ = 2 \sum_{n=0}^{N} \sum_{l=0}^{n+1} B_{nl}(t) \theta(|x| < \xi) \left(1 - \frac{x^2}{\xi^2}\right) C_{2n+1}^{(3/2)}\left(\frac{x}{\xi}\right) P_{2l}\left(\frac{x}{\xi}\right)$$

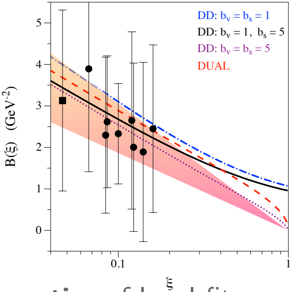
- Number of fit parameters describing the B_{nl} coefficients **increases with** N^2...Extension to other GPDs seems difficult.
- **Extrapolation** problem left open.

Hybrid : Local / global fit

Option 2 Local fits and then 1-parameter fit.

Dupré *et al.*, Phys. Rev. **D95**, 011501 (2017)





- Size of error bars reflects **systematics** of local fits.
- **Extrapolation** problem taken care of by model-dependent 1-parameter parameterization.

Neural networks

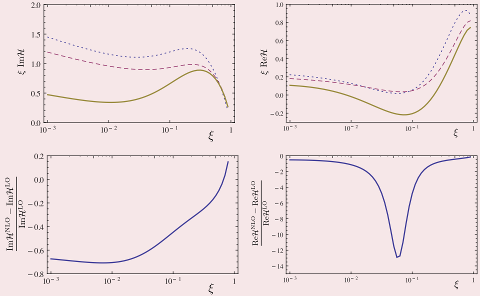
Exploratory stage for GPDs.

Kumericki *et al.*, JHEP **1107**, 073 (2011)

- Already used for PDF fits.
- **Almost model-independent**: neural network description, twist-2, *H*-dominance?
- Good agreement between model fit and neural network fit in the fitting domain.
- **More reliable uncertainties** in extrapolations?
- **Overtraining** as a generic feature of (too) flexible models.

Back to summary of first extractions.

$\mathcal{H}$ at LO and NLO ($t = -0.1$ GeV^2, $Q^2 = \mu_F^2 = 4.$ GeV^2)

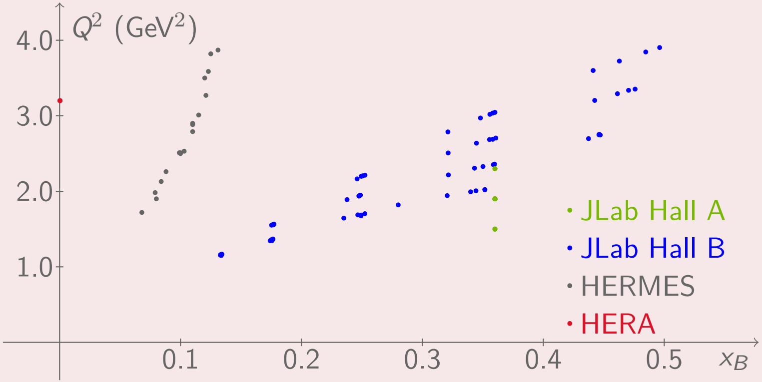


Moutarde *et al.*, Phys. Rev. **D87**, 054029 (2013)

- **Systematic** tests of perturbative QCD assumptions.
- **Wide kinematic range** (from JLab to EIC).
- **Accuracy** set by JLab 12 GeV expected statistical accuracy.
- **Model dependent** evaluations.

Back to challenges.

What is large Q^2 ? *Measurements before 2015...*

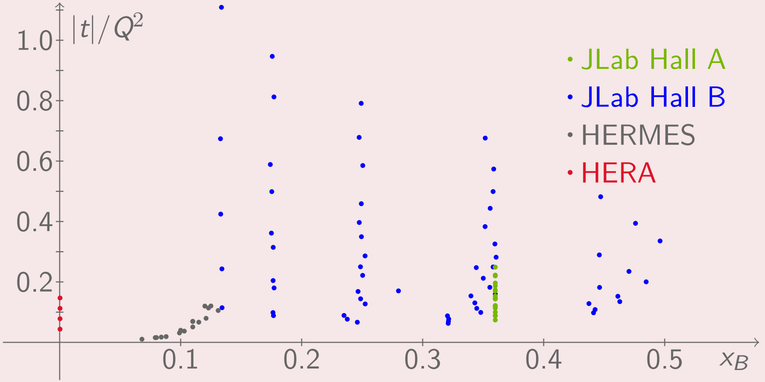


- World data cover **complementary kinematic regions**.

Kinematics of existing DVCS measurements.

Looking for the Bjorken regime.

What is large Q^2 ? *Measurements before 2015...*



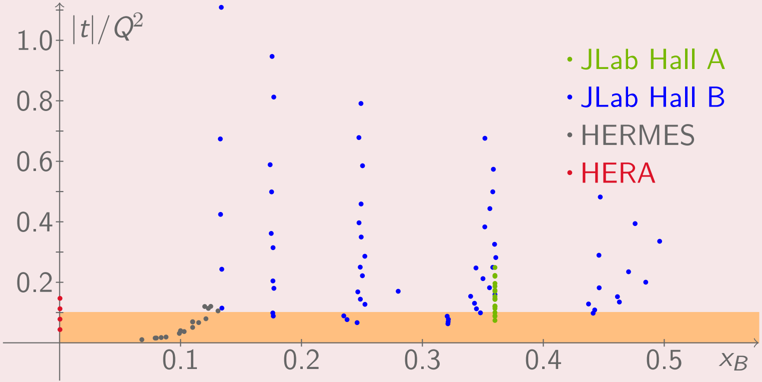
- World data cover **complementary kinematic regions**.
- Q^2 is **not so large** for most of the data.

Back to challenges.

Kinematics of existing DVCS measurements.

Looking for the Bjorken regime.

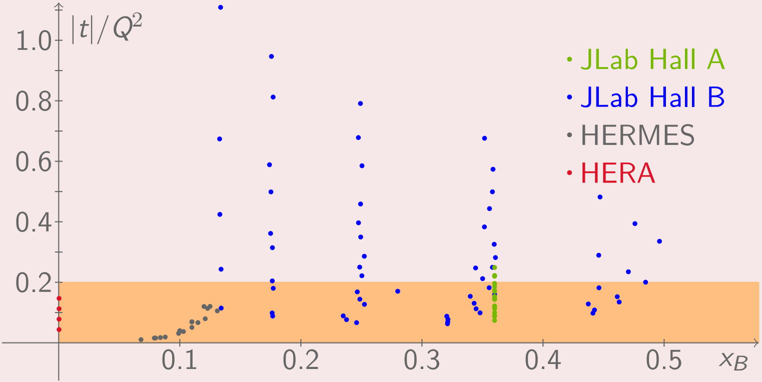
What is large Q^2 ? *Measurements before 2015...*



- World data cover **complementary kinematic regions**.
- Q^2 is **not so large** for most of the data.
- **Higher twists? Finite-t and target mass corrections?**

Back to challenges.

What is large Q^2 ? *Measurements before 2015...*



- World data cover **complementary kinematic regions.**
- Q^2 is **not so large** for most of the data.
- **Higher twists? Finite-t and target mass corrections?**

Back to challenges.

- Write dispersion relation **at fixed** t **and** Q^2:

$$Re\mathcal{H}(\xi, t) = \Delta(t) + \frac{2}{\pi}\mathcal{P}\int_0^1 \frac{\mathrm{d}x}{x} \frac{Im\mathcal{H}(x, t)}{\left(\frac{\xi^2}{x^2} - 1\right)}$$

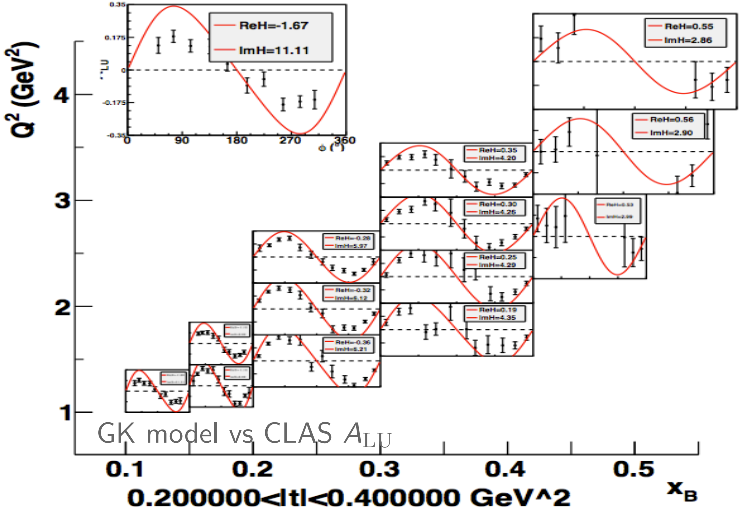
- Use LO relation $Im\mathcal{H}(x, t) = \pi\big(H(x, x, t) - H(-x, x, t)\big)$.

- Up to the D-term form factor $\Delta(t)$, all the information accessible **at LO and fixed** Q^2 is contained on the cross-over line.

Teryaev, hep-ph/0510031
Anikin and Teryaev, Phys. Rev. **D76**, 056007 (2007)
Diehl and Ivanov, Eur. Phys. J. **C52**, 919 (2007)

Dispersion relations and actual data.

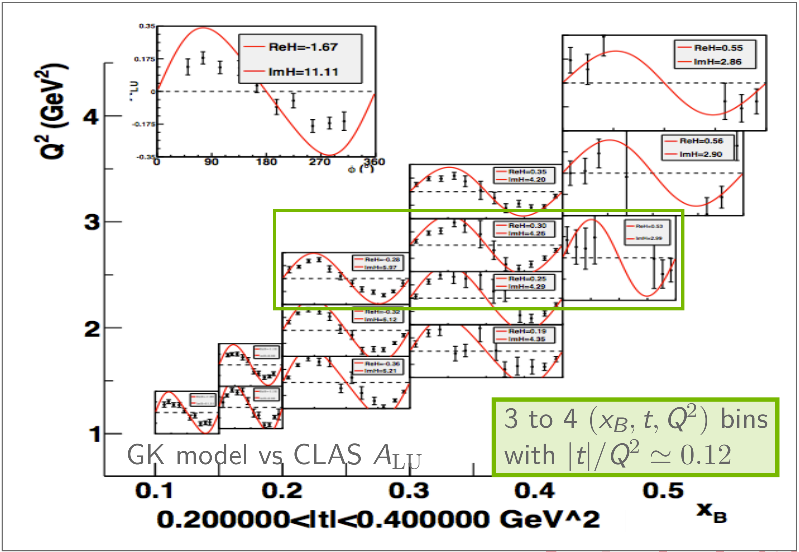
Too few kinematic bins to provide model-independent constraints?



GK model vs CLAS A_{LU}

Dispersion relations and actual data.

Too few kinematic bins to provide model-independent constraints?

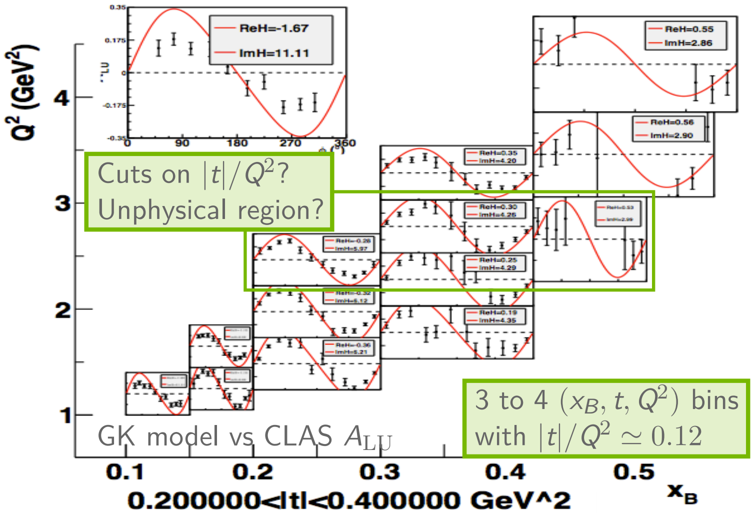


GK model vs CLAS A_{LU}

3 to 4 (x_B, t, Q^2) bins with $|t|/Q^2 \simeq 0.12$

Dispersion relations and actual data.

Too few kinematic bins to provide model-independent constraints?

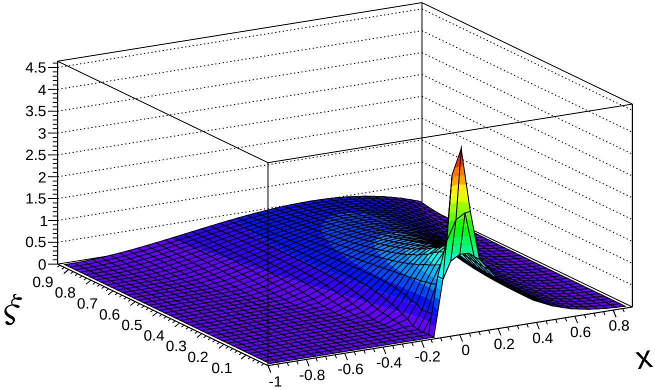


Cuts on $|t|/Q^2$?
Unphysical region?

GK model vs CLAS A_{LU}

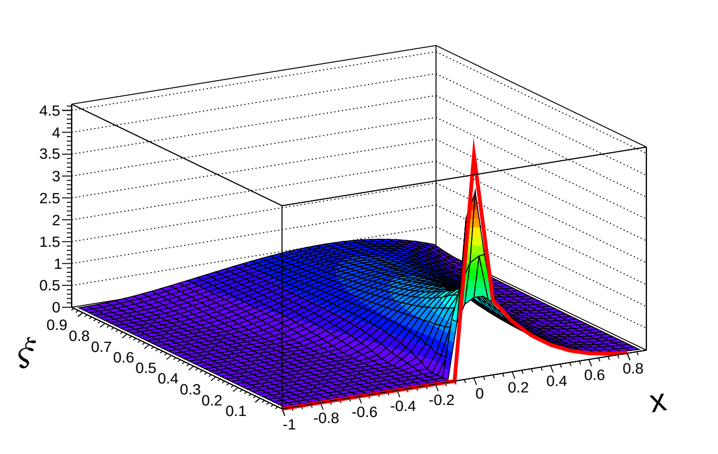
3 to 4 (x_B, t, Q^2) bins with $|t|/Q^2 \simeq 0.12$

GPD H at $t = -0.23$ GeV^2 and $Q^2 = 2.3$ GeV^2.



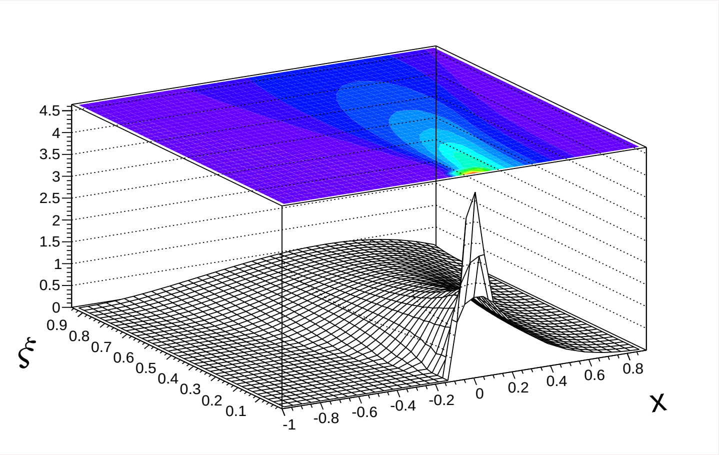
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

Need to know $H(x, \xi = 0, t)$ to do transverse plane imaging.



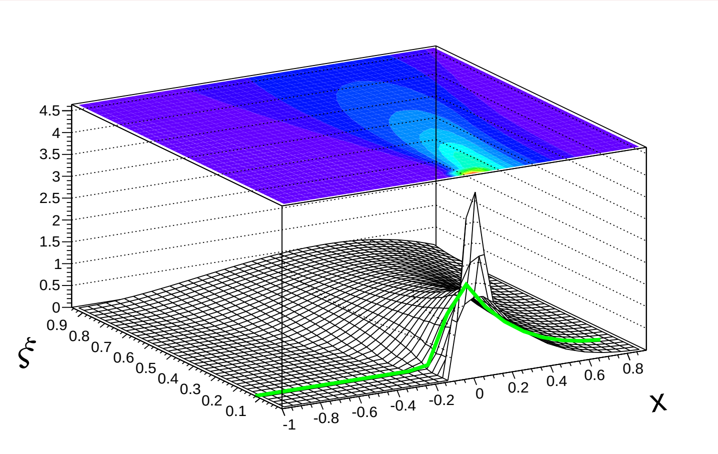
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

What is the physical region?



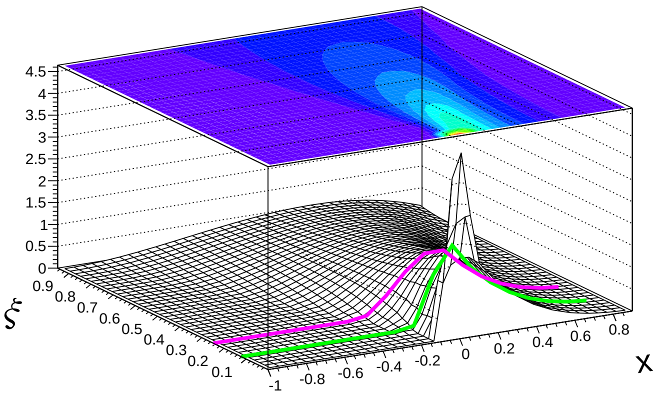
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

ξ_{min} from finite beam energy.



GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

$\xi_{\max}$ from kinematic constraint on 4-momentum transfer.

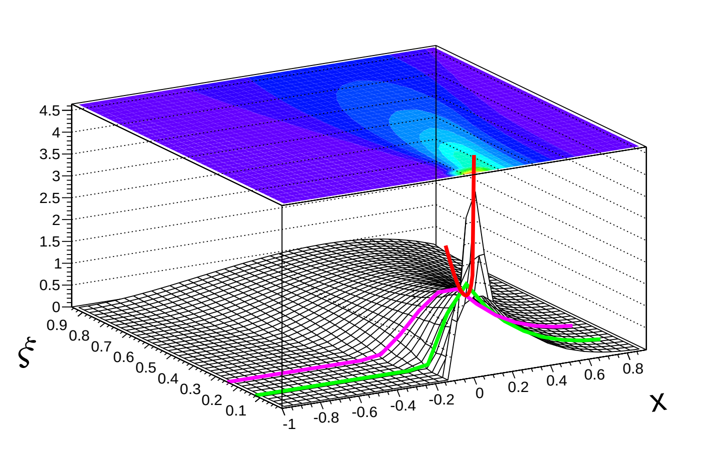


GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

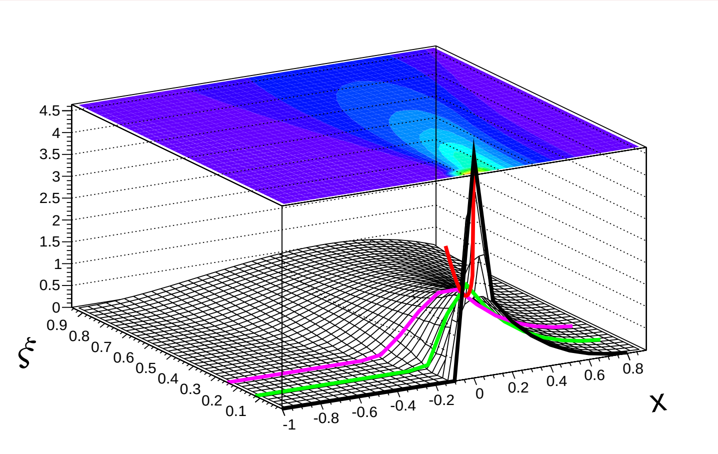
Direct experimental access to a restricted kinematic domain.

The cross-over line $x = \xi$.



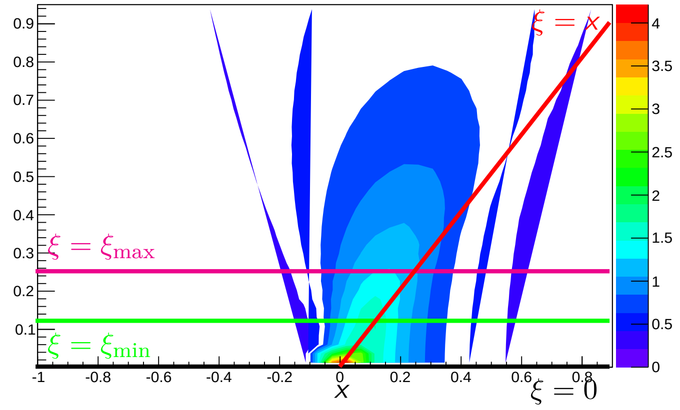
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

The black curve is what is needed for transverse plane imaging!



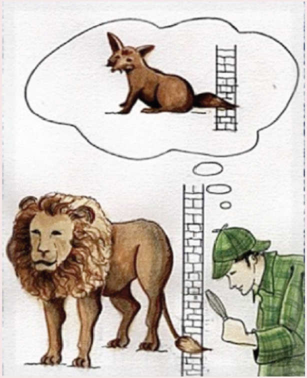
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

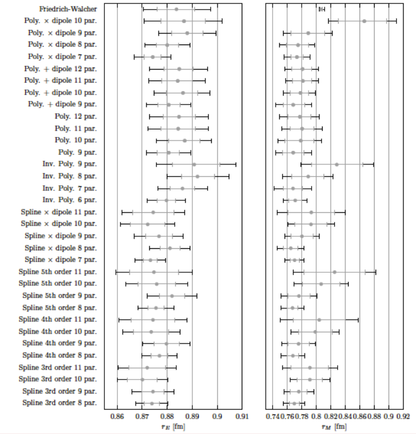
Density plot of H at $t = -0.23$ GeV^2 and $Q^2 = 2.3$ GeV^2



GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

Extrapolations...





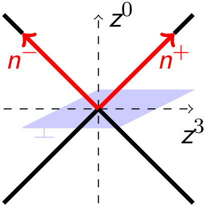
Bernauer *et al.*(A1 Coll.), Phys. Rev. **C90**, 015206 (2014)

General idea

- Assume $H(x, \xi, t)$ is known for all x and $\xi \in [\xi_{\mathrm{min}}, \xi_{\mathrm{max}}]$.
- Then all Mellin moments are known for $\xi \in [\xi_{\mathrm{min}}, \xi_{\mathrm{max}}]$.
- Mellin moments are **polynomials** in ξ and in particular can be evaluated at $\xi = 0$.
- The knowledge of the Mellin moments at $\xi = 0$ **uniquely determines** the transverse plane density $H(x, 0, b_\perp)$.
- *Caveat*: **ill-posed problem** in the sense of Hadamard.

$$H^q_\pi(x,\xi,t) = \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P-\frac{\Delta}{2} \right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

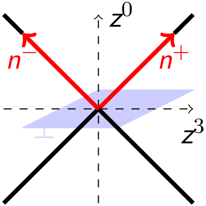
Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF **forward limit**

$$H^q(x,0,0) = q(x)$$

$$H^q_\pi(x, \xi, t) \quad = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2} \right| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \left| \pi, P-\frac{\Delta}{2} \right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF **forward limit**
- Form factor **sum rule**

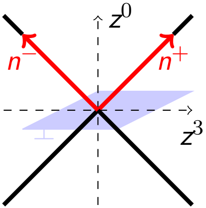
$$\int_{-1}^{+1} dx\, H^q(x, \xi, t) = F^q_1(t)$$

Spin-0 Generalized Parton Distribution.

Definition and simple properties.

$$H^q_\pi(x, \xi, t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



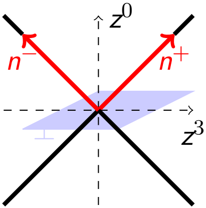
References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF **forward limit**
- Form factor **sum rule**
- H^q is an **even function** of ξ from time-reversal invariance.

$$H^q_\pi(x, \xi, t) = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2}\right| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\left|\pi, P-\frac{\Delta}{2}\right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF **forward limit**
- Form factor **sum rule**
- H^q is an **even function** of ξ from time-reversal invariance.
- H^q is **real** from hermiticity and time-reversal invariance.

Polynomiality.

Mixed constraint from Lorentz invariance and discrete symmetries.

- Express Mellin moments of GPDs as **matrix elements**:

$$\int_{-1}^{+1} \mathrm{d}x\, x^m H^q(x, \xi, t) = \frac{1}{2(P^+)^{m+1}} \left\langle P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \middle| P - \frac{\Delta}{2} \right\rangle$$

- Identify the **Lorentz structure** of the matrix element:

linear combination of $(P^+)^{m+1-k}(\Delta^+)^k$ for $0 \leq k \leq m+1$

- Remember definition of **skewness** $\Delta^+ = -2\xi P^+$.
- Select **even powers** to implement time reversal.
- Obtain **polynomiality condition**:

$$\int_{-1}^{1} \mathrm{d}x\, x^m H^q(x, \xi, t) = \sum_{\substack{i=0 \\ \text{even}}}^{m} (2\xi)^i C^q_{mi}(t) + (2\xi)^{m+1} C^q_{mm+1}(t) \ .$$

- Choose $F^q(\beta, \alpha) = 3\beta\theta(\beta)$ ad $G^q(\beta, \alpha) = 3\alpha\theta(\beta)$:

$$H^q(x, \xi) = 3x \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \, \delta(x - \beta - \alpha\xi)$$

- Simple analytic expressions for the GPD:

$$H(x, \xi) = \frac{6x(1-x)}{1-\xi^2} \text{ if } 0 < |\xi| < x < 1,$$
$$H(x, \xi) = \frac{3x(x+|\xi|)}{|\xi|(1+|\xi|)} \text{ if } -|\xi| < x < |\xi| < 1.$$

Double Distributions.

Lorentz covariance by example.

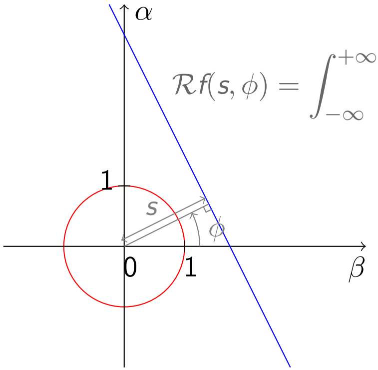
- Compute first Mellin moments.

n	$\int_{-\xi}^{+\xi} \mathrm{d}x\, x^n H(x,\xi)$	$\int_{+\xi}^{+1} \mathrm{d}x\, x^n H(x,\xi)$	$\int_{-\xi}^{+1} \mathrm{d}x\, x^n H(x,\xi)$
0	$\frac{1+\xi-2\xi^2}{1+\xi}$	$\frac{2\xi^2}{1+\xi}$	1
1	$\frac{1+\xi+\xi^2-3\xi^3}{2(1+\xi)}$	$\frac{2\xi^3}{1+\xi}$	$\frac{1+\xi^2}{2}$
2	$\frac{3(1-\xi)(1+2\xi+3\xi^2+4\xi^3)}{10(1+\xi)}$	$\frac{6\xi^4}{5(1+\xi)}$	$\frac{3(1+\xi^2)}{10}$
3	$\frac{1+\xi+\xi^2+\xi^3+\xi^4-5\xi^5}{5(1+\xi)}$	$\frac{6\xi^5}{5(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{5}$
4	$\frac{1+\xi+\xi^2+\xi^3+\xi^4+\xi^5-6\xi^6}{7(1+\xi)}$	$\frac{6\xi^6}{7(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{7}$

- Expressions get more complicated as n increases… But they always yield polynomials!

The Radon transform.

Definition and properties.



For $s > 0$ and $\phi \in [0, 2\pi]$:

$$\mathcal{R}f(s,\phi) = \int_{-\infty}^{+\infty} \mathrm{d}\beta \mathrm{d}\alpha\, f(\beta,\alpha)\delta(s - \beta\cos\phi - \alpha\sin\phi)$$

and:

$$\mathcal{R}f(-s,\phi) = \mathcal{R}f(s,\phi\pm\pi)$$

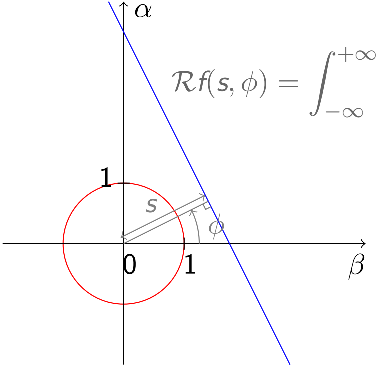
Relation to GPDs:

$$x = \frac{s}{\cos\phi} \text{ and } \xi = \tan\phi$$

Relation between GPD and DD in Belistky *et al.* gauge

$$\frac{\sqrt{1+\xi^2}}{x} H(x,\xi) = \mathcal{R}f_{\mathrm{BMKS}}(s,\phi) \,,$$

The Radon transform.
Definition and properties.



For $s > 0$ and $\phi \in [0, 2\pi]$:

$$\mathcal{R}f(s, \phi) = \int_{-\infty}^{+\infty} \mathrm{d}\beta \mathrm{d}\alpha \, f(\beta, \alpha) \delta(s - \beta \cos\phi - \alpha \sin\phi)$$

and:

$$\mathcal{R}f(-s, \phi) = \mathcal{R}f(s, \phi \pm \pi)$$

Relation to GPDs:

$$x = \frac{s}{\cos\phi} \text{ and } \xi = \tan\phi$$

Relation between GPD and DD in Pobylitsa gauge

$$\frac{\sqrt{1+\xi^2}}{1-x} H(x, \xi) = \mathcal{R}f_{\mathrm{P}}(s, \phi) \,,$$

The range of the Radon transform.

The polynomiality property a.k.a. the Ludwig-Helgason condition.

- The Mellin moments of a Radon transform are **homogeneous polynomials** in $\omega = (\sin\phi, \cos\phi)$.
- The converse is also true:

Theorem (Hertle, 1983)

Let $g(s, \omega)$ an even compactly-supported distribution. Then g is itself the Radon transform of a compactly-supported distribution if and only if the ***Ludwig-Helgason consistency condition*** *hold:*

(i) *g is C^∞ in ω,*

(ii) *$\int \mathrm{d}s\, s^m g(s, \omega)$ is a homogeneous polynomial of degree m for all integer $m \geq 0$.*

- Double Distributions and the Radon transform are the **natural solution** of the polynomiality condition.

Support theorem.

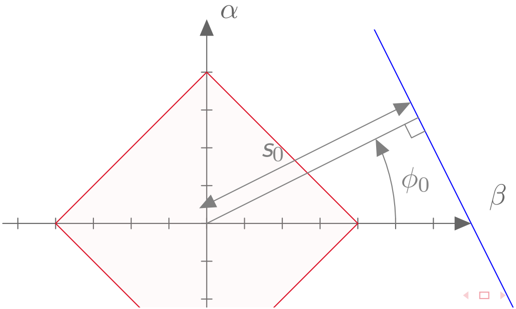
We don't need to know the GPD everywhere to image the proton!

Theorem

Let f be a compactly-supported summable function defined on $\mathbb{R}^2$ and $\mathcal{R}f$ its Radon transform.
Let $(s_0, \omega_0) \in \mathbb{R} \times S^1$ and U_0 an open neighborhood of ω_0 s.t.:

$$\textit{for all } s > s_0 \textit{ and } \omega \in U_0 \quad \mathcal{R}f(s, \omega) = 0 \, .$$

Then $f(\aleph) = 0$ on the half-plane $\langle \aleph | \omega_0 \rangle > s_0$ of $\mathbb{R}^2$.

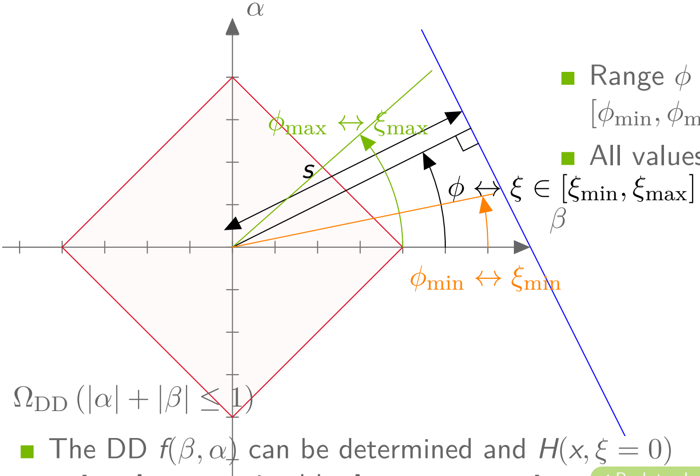


- $\omega_0 = (\cos \phi_0, \sin \phi_0)$.
- $\aleph = (\beta, \alpha)$.

Support theorem.

We don't need to know the GPD everywhere to image the proton!

- Assume deconvolution of CFF achieved.
- Data: $H(x, \xi)$ for all $x \in [-1, +1]$ and $\xi \in [\xi_{\min}, \xi_{\max}]$.



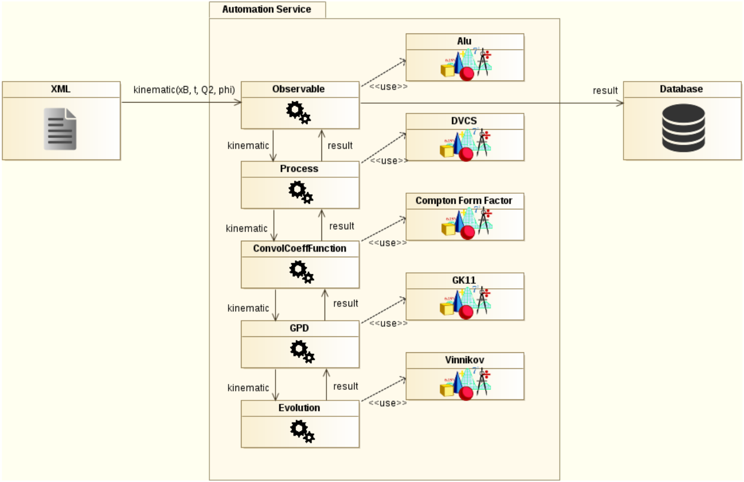
- Range $\phi \in [\phi_{\min}, \phi_{\max}]$.
- All values of s.

- The DD $f(\beta, \alpha)$ can be determined and $H(x, \xi = 0)$ **uniquely** constrained by **Lorentz covariance**.

Back to challenges.

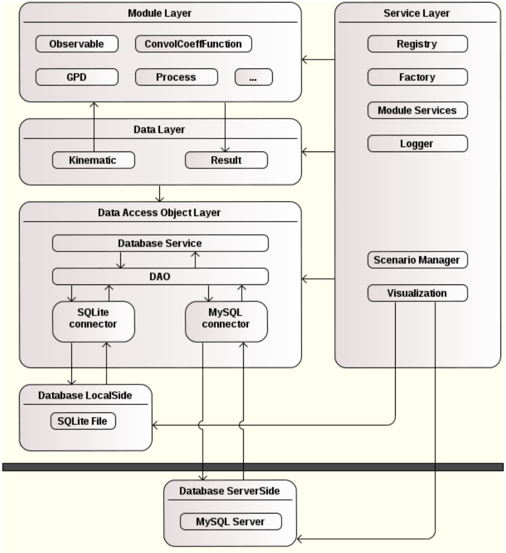
Modularity and automation.

Parse XML file, compute and store result in database.



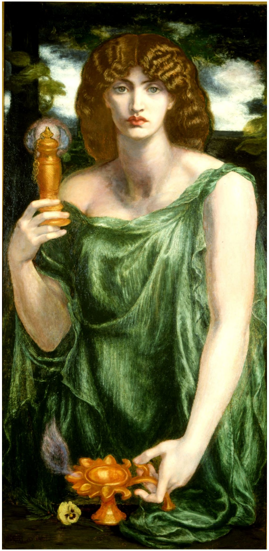
Modularity and layer structure.

Modifying one layer does not affect the other layers.



- Keep track of validated results.
- Systematic nonregression tests.
- Help preparing new releases.
- Store experimental data.
- Store grids of new models.
- Post processing?
- Time consuming fits?

Back to computing chain.



Commissariat à l'énergie atomique et aux énergies alternatives
Centre de Saclay | 91191 Gif-sur-Yvette Cedex
T. +33(0)1 69 08 73 88 | F. +33(0)1 69 08 75 84

DRF
Irfu
SPhN