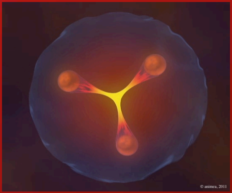
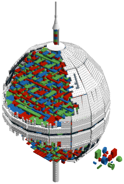
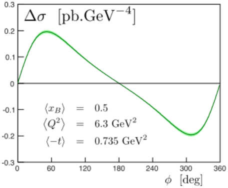
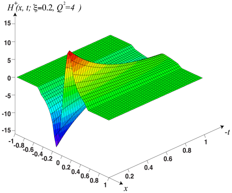
DE LA RECHERCHE À L'INDUSTRIE

Nucleon Reverse Engineering: Structuring hadrons with colored degrees of freedom









Habilitation thesis | Hervé MOUTARDE

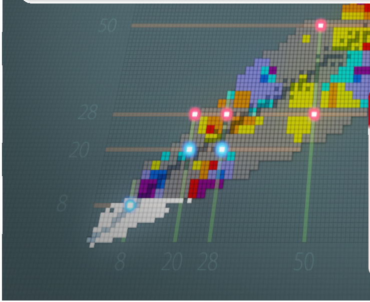
Dec. 11$^{\text{th}}$, 2015

Quantum Chromodynamics as a paradigm.

The **theory** (and not an *effective theory*) of the strong interaction.

Facts

- **Restricted number** of parameters.
- Mathematically **consistent**.
- **Large** scope.
- Validated up to **large energy** $\lesssim 8$ TeV.
- Accurate **algorithmic** answer.



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- **Confinement**.
- **Chiral symmetry** breaking.
- Existence of a **mass gap**.

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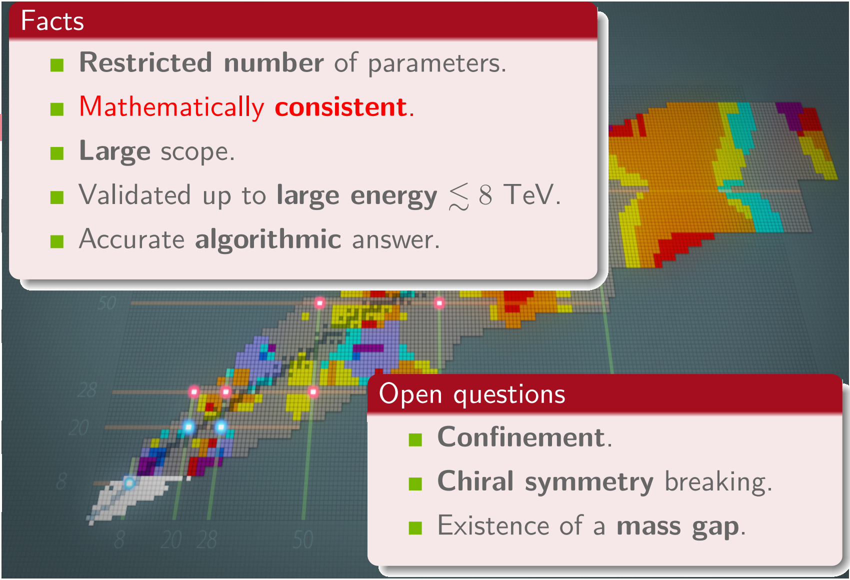

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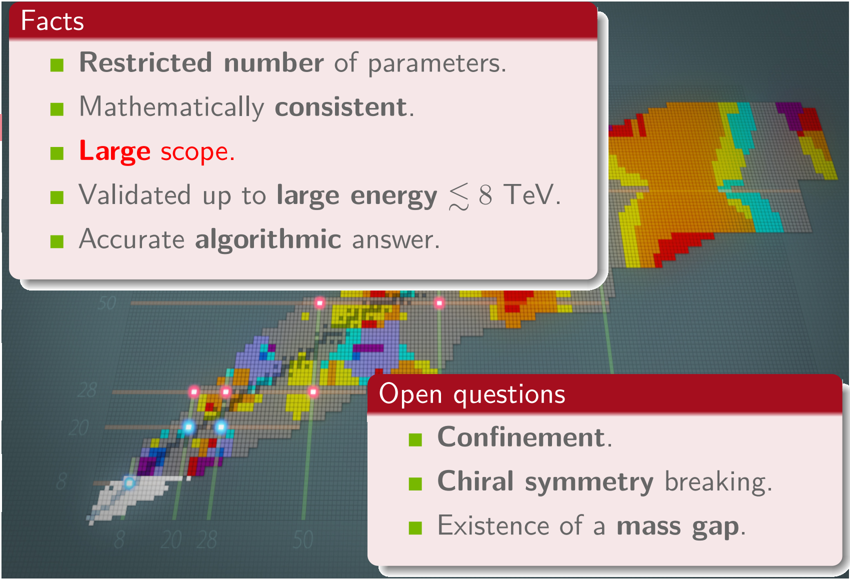

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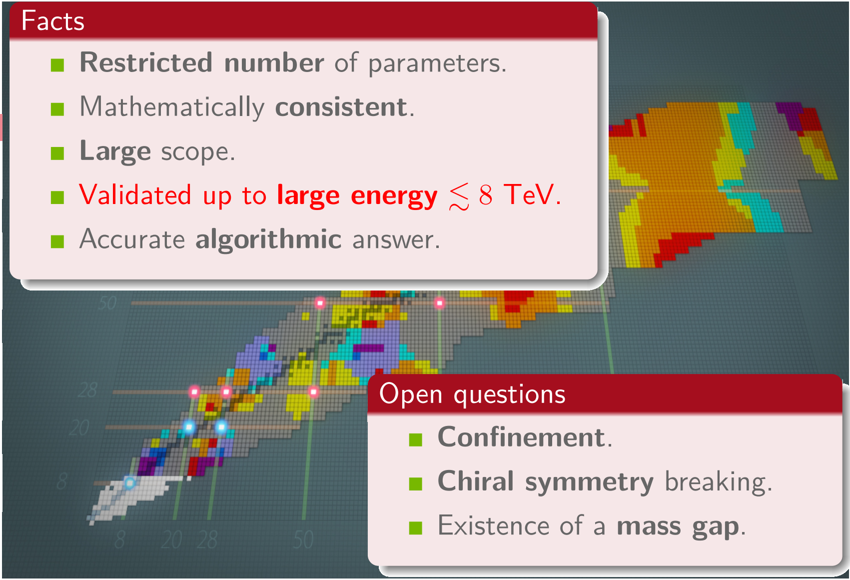

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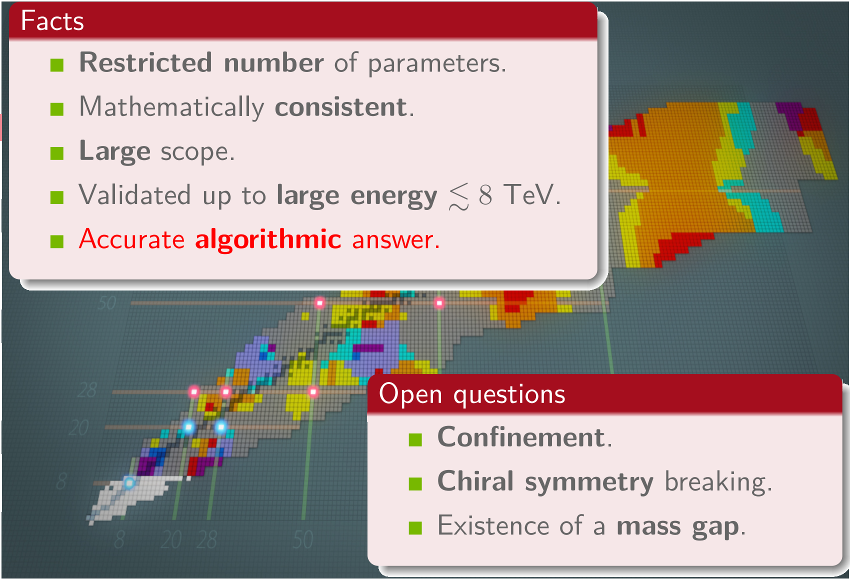

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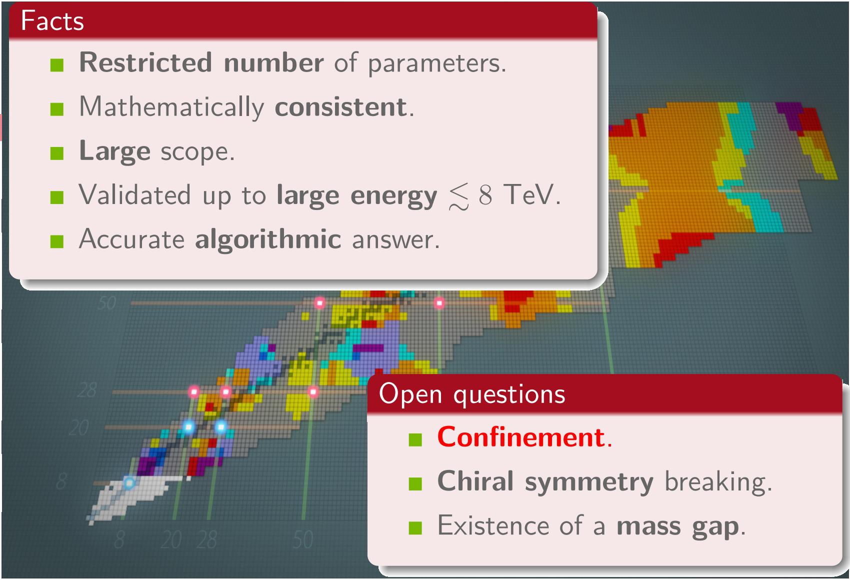

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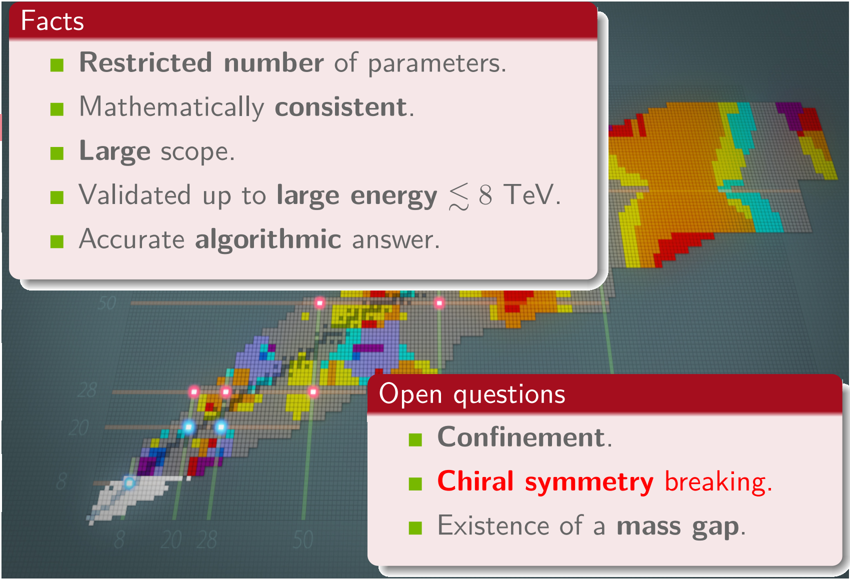

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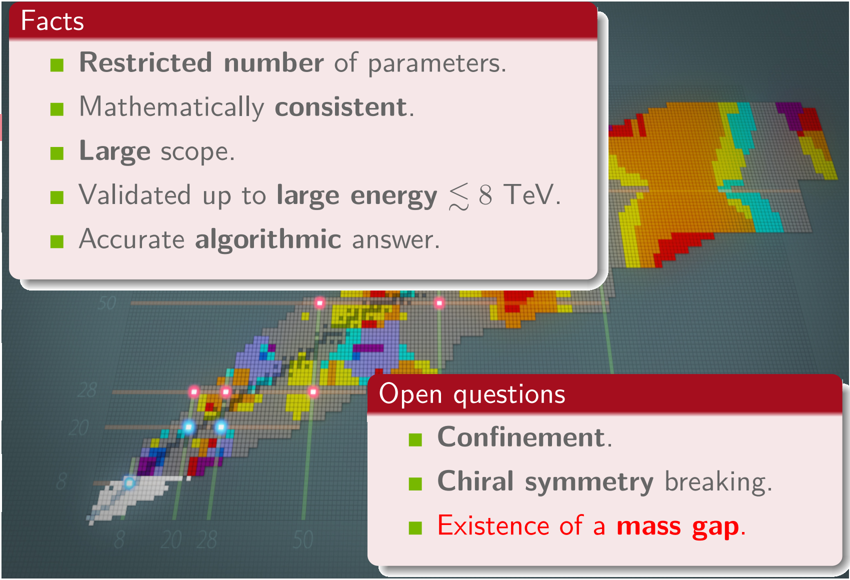

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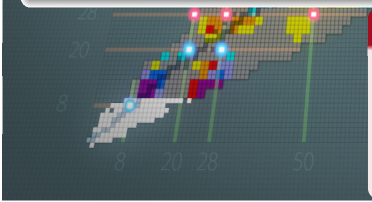
No observed free color charges (PDG 2009)

FREE QUARK SEARCHES

The basis for much of the theory of particle scattering and hadron spectroscopy is the construction of the hadrons from a set of fractionally charged constituents (quarks). A central but unproven hypothesis of this theory, Quantum Chromodynamics, is that quarks cannot be observed as free particles but are confined to mesons and baryons.

Experiments show that it is at best difficult to "unglue" quarks. Accelerator searches at increasing energies have produced no evidence for free quarks, while only a few cosmic-ray and matter searches have produced uncorroborated events.



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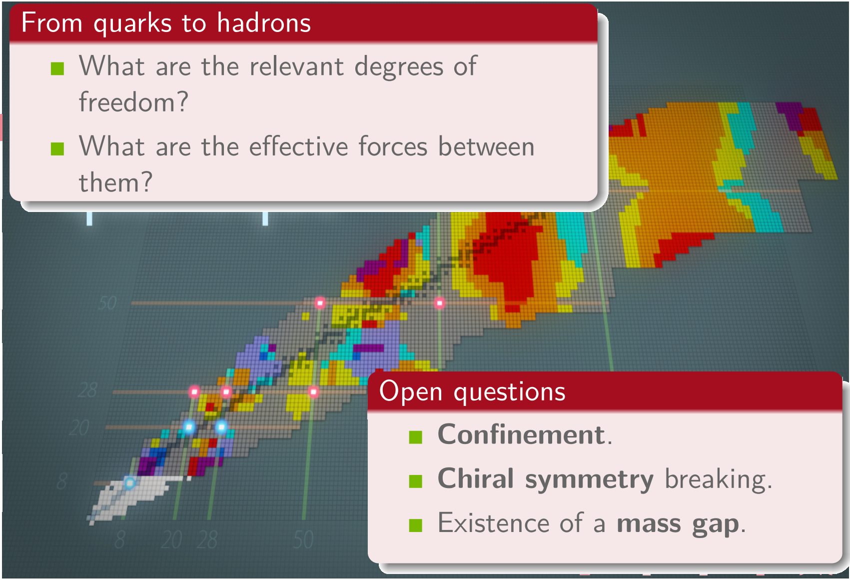

From quarks to hadrons

- What are the relevant degrees of freedom?
- What are the effective forces between them?

Open questions

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Clay Millenimum Prize (Jaffe et Witten)

QUANTUM YANG–MILLS THEORY 5

Finally, QFT is the jumping-off point for a quest that may prove central in 21st century physics—the effort to unify gravity and quantum mechanics, perhaps in string theory. For mathematicians to participate in this quest, or even to understand the possible results, QFT must be developed further as a branch of mathematics. It is important not only to understand the solution of specific problems arising from physics, but also to set such results within a new mathematical framework. One hopes that this framework will provide a unified development of several fields of mathematics and physics, and that it will also provide an arena for the development of new mathematics and physics.

For these reasons the Scientific Advisory Board of CMI has chosen a Millennium problem about quantum gauge theories. Solution of the problem requires both understanding one of the deep unsolved physics mysteries, the existence of a mass gap, and also producing a mathematically complete example of quantum gauge field theory in four-dimensional space-time.

Open questions

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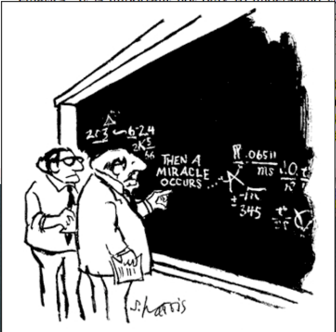
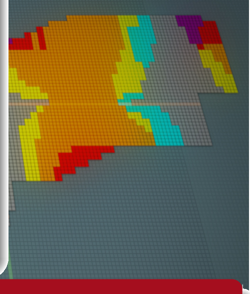


"I think you should be more explicit here in step two."



Open questions

- **Confinement**.
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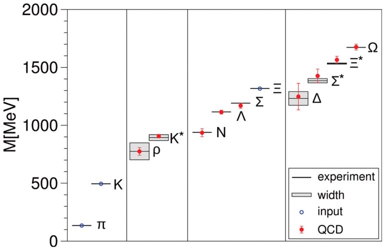
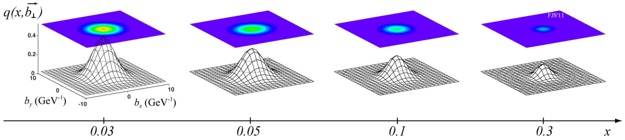
- Lattice QCD clearly shows that the mass of hadrons is generated by the **interaction**, not by the quark masses.

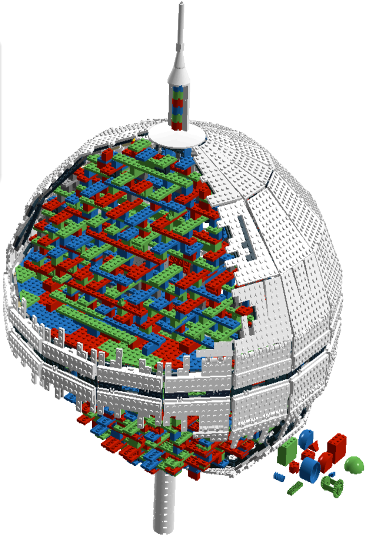


Durr *et al.*, Science **322**, 1224 (2008)

- Can we **map** the *location of mass* inside a hadron?



How can we recover the well-known characteristics of the nucleon from the properties of its **colored building blocks**?



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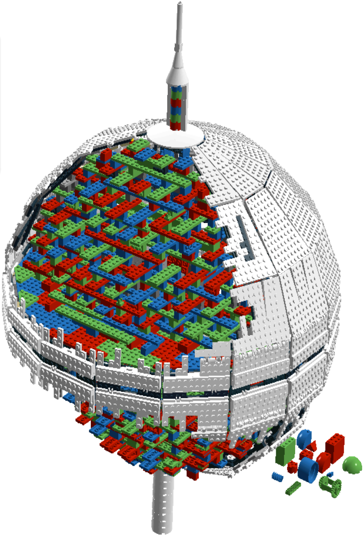
Mass?



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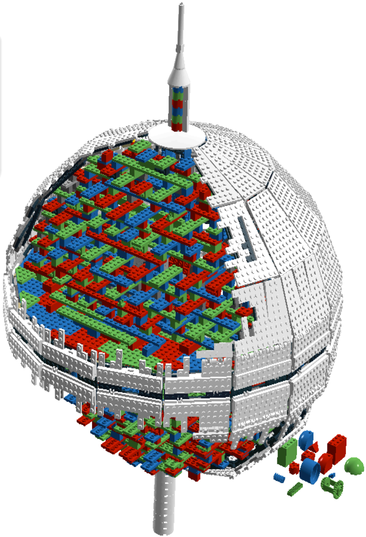
Mass?
Spin?



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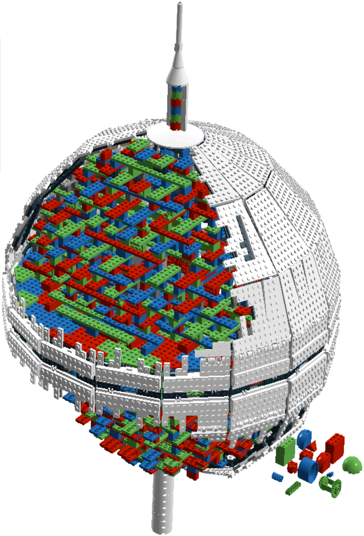
Mass?
Spin?
Charge?



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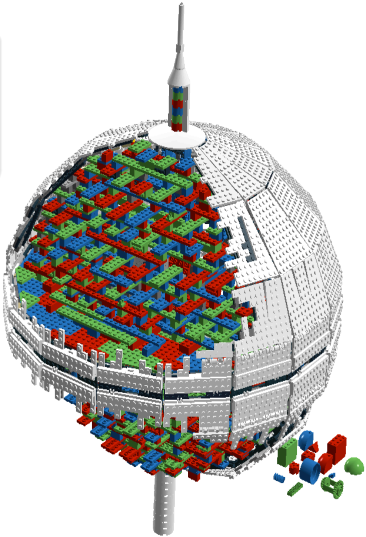

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Mass?
Spin?
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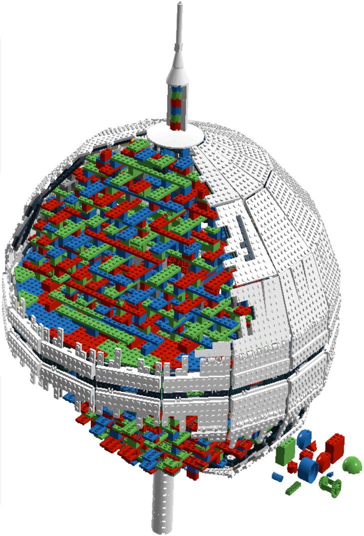
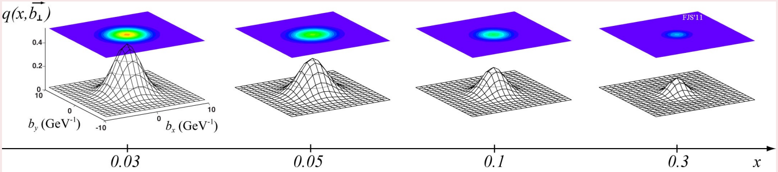
...

What are the relevant **effective degrees of freedom** and **effective interaction** at large distance?



Structuring questions for the hadron physics community

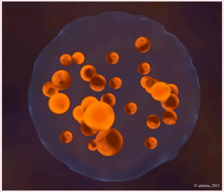
- QCD **mechanisms** behind the origin of **mas** in the **visible universe**?
- **Cartography** of interactions giving its mass to the nucleon?
- **Pressure** and **density** profiles of the nucleon as a continuous medium?
- **Localization** of quarks and gluons inside the nucleon?



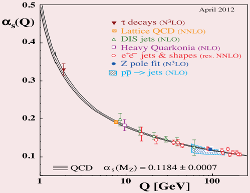
Motivation.

Study nucleon structure to shed new light on nonperturbative QCD.

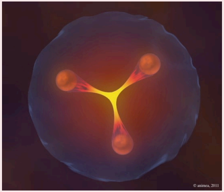
Perturbative QCD



Asymptotic freedom



Nonperturbative QCD

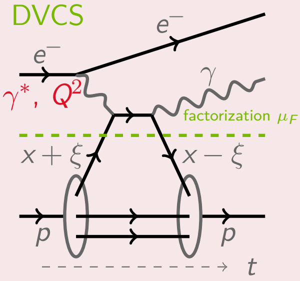


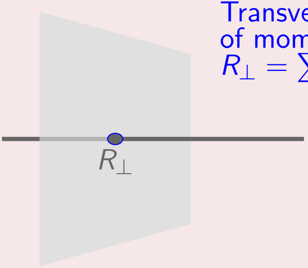
Perturbative AND nonperturbative QCD at work

- Define **universal** objects describing 3D nucleon structure: **Generalized Parton Distributions** (GPD).
- Relate GPDs to measurements using **factorization**: **Virtual Compton Scattering** (DVCS, TCS), **Deeply Virtual Meson production** (DVMP).
- Get **experimental knowledge** of nucleon structure.

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in a hadron.
- DVCS recognized as the cleanest channel to access GPDs.

Deeply Virtual Compton Scattering (DVCS)





Transverse center of momentum $R_\perp$

$R_\perp = \sum_i x_i r_{\perp i}$

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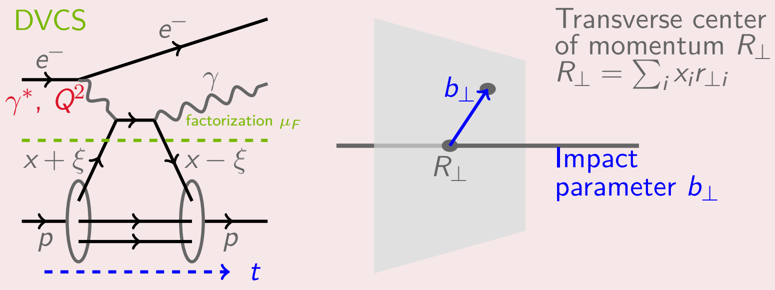

Anatomy of hadrons.

GPDs, 3D hadron imaging, and beyond (1/4).

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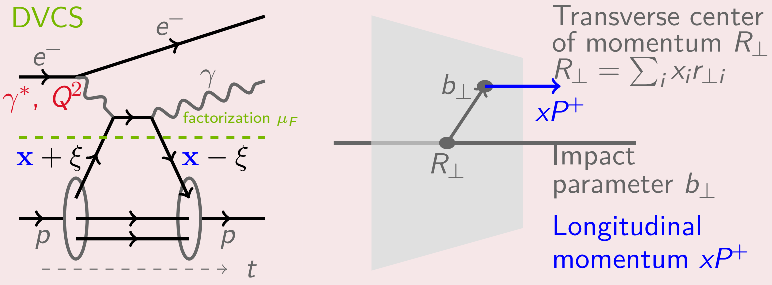

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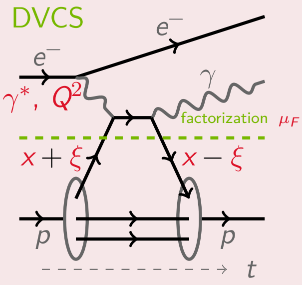
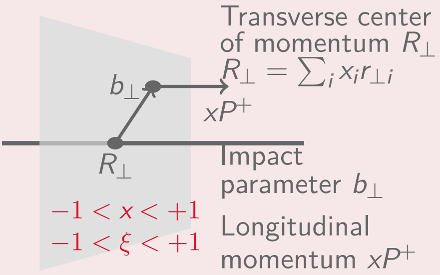




- **24 GPDs** $F^i(x, \xi, t, \mu_F)$ for each parton type $i = g, u, d, \ldots$ for leading and sub-leading twists.

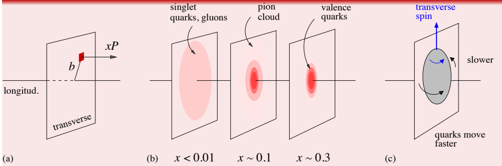
- **Probabilistic interpretation** of Fourier transform of GPD$(x, \xi = 0, t)$ in **transverse plane**.

$$\rho(x, b_\perp, \lambda, \lambda_N) = \frac{1}{2}\Bigg[H(x, 0, b_\perp^2) + \frac{b_\perp^j \epsilon_{ji} S_\perp^i}{M} \frac{\partial E}{\partial b_\perp^2}(x, 0, b_\perp^2) + \lambda\lambda_N \tilde{H}(x, 0, b_\perp^2)\Bigg]$$

- Notations : quark helicity λ, nucleon longitudinal polarization λ_N and nucleon transverse spin $S_\perp$.

Burkardt, Phys. Rev. **D62**, 071503 (2000)

Can we obtain this picture from exclusive measurements?



Weiss, AIP Conf. Proc. **1149**, 150 (2009)

- Most general structure of matrix element of energy momentum tensor between nucleon states:

$$\left\langle N, P+\frac{\Delta}{2}\right| T^{\mu\nu} \left|N, P-\frac{\Delta}{2}\right\rangle = \bar{u}\left(P+\frac{\Delta}{2}\right)\left[A(t)\gamma^{(\mu}P^{\nu)} + B(t)P^{(\mu}i\sigma^{\nu)\lambda}\frac{\Delta_\lambda}{2M} + \frac{C(t)}{M}(\Delta^\mu\Delta^\nu - \Delta^2\eta^{\mu\nu})\right] u\left(P-\frac{\Delta}{2}\right)$$

with $t = \Delta^2$.

- Key observation: **link between GPDs and gravitational form factors**

$$\int \mathrm{d}x\, x H^q(x,\xi,t) = A^q(t) + 4\xi^2 C^q(t)$$

$$\int \mathrm{d}x\, x E^q(x,\xi,t) = B^q(t) - 4\xi^2 C^q(t)$$

Ji, Phys. Rev. Lett. **78**, 610 (1997)

- Spin sum rule:

$$\int \mathrm{d}x\, x\big(H^q(x,\xi,0) + E^q(x,\xi,0)\big) = A^q(0) + B^q(0) = 2J^q$$

Ji, Phys. Rev. Lett. **78**, 610 (1997)

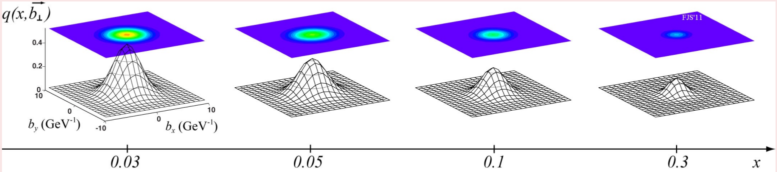
- **Shear** and **pressure** of a hadron considered as a continuous medium:

$$\langle N \,|\, T^{ij}(\vec{r}) \,|\, N \rangle\, N = s(r)\left(\frac{r^i r^j}{\vec{r}^2} - \frac{1}{3}\delta^{ij}\right) + p(r)\delta^{ij}$$

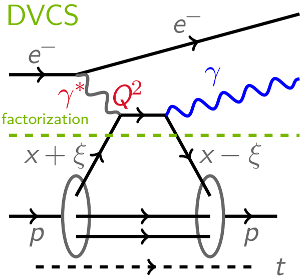
Polyakov and Shuvaev, hep-ph/0207153

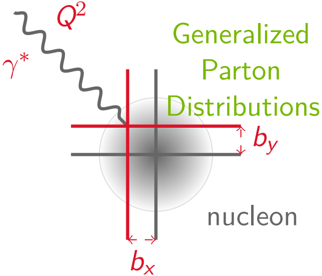
1. **Status of 3D imaging:** phenomenological relevance of the field.
2. **Building the tools:** preparing for the high precision era.
3. **Learning from GPDs:** steps towards new GPD models.

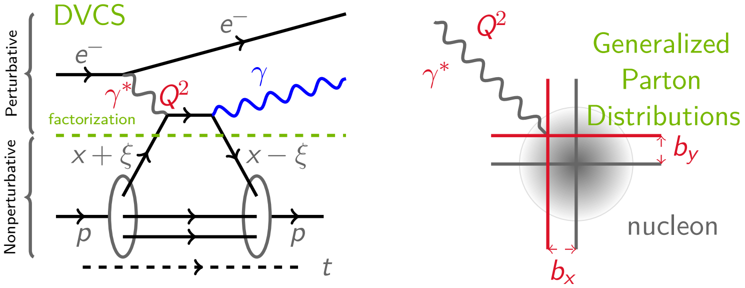
How can we make this picture? What do we learn from it?



Phenomenological status of nucleon 3D imaging


DVCS
e^-
e^-
γ^*
Q^2
γ
factorization
$x+\xi$
$x-\xi$
p
p
t


Q^2
γ^*
Generalized
Parton
Distributions
b_y
nucleon
b_x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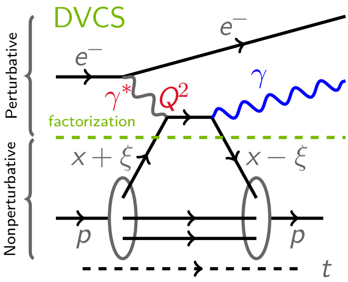
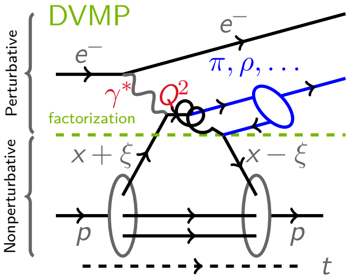
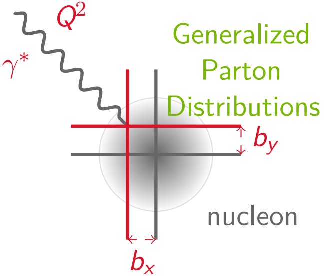
Generalized Parton Distributions
b_y
b_x
nucleon

Exclusive processes of current interest (1/2).

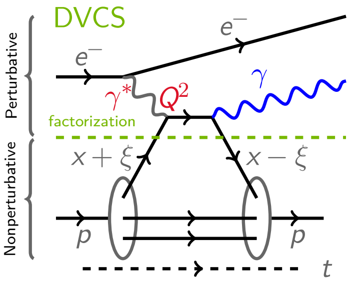
Factorization and universality.

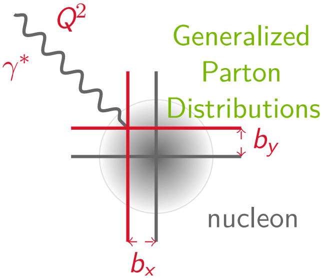


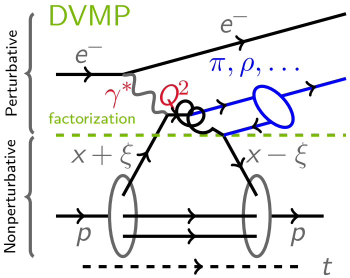


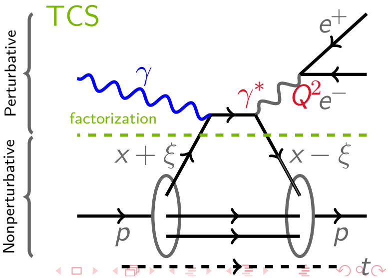


Exclusive processes of current interest (1/2).
Factorization and universality.









Exclusive processes of current interest (1/2).

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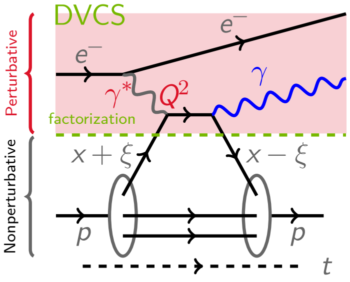
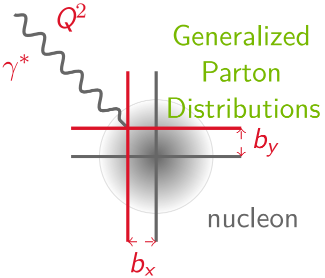
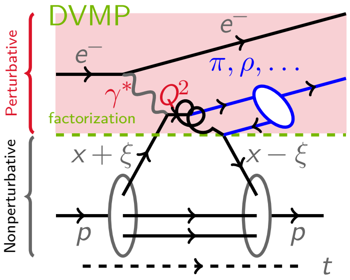
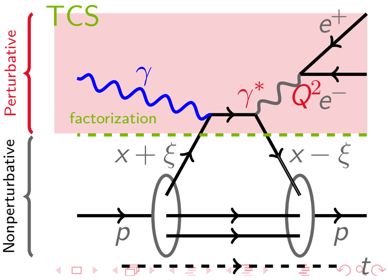


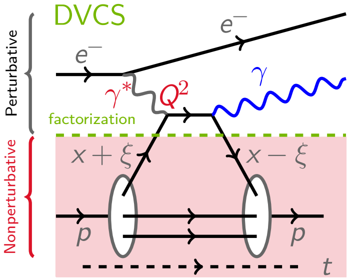


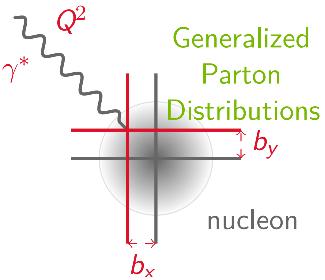


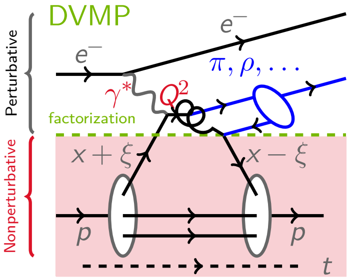


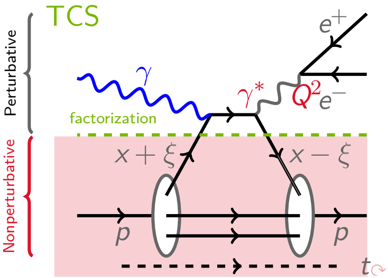
Exclusive processes of current interest (1/2).

Factorization and universality.









Bjorken regime : large Q^2 and fixed $xB \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on **factorization theorems**.
- All-order proofs for DVCS, TCS and some DVMP.
- GPDs depend on a (arbitrary) factorization scale μ_F.
- **Consistency** requires the study of **different channels**.

- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^{1} dx\, C\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F)$$

for a given GPD F.

- CFF $\mathcal{F}$ is a **complex function**.

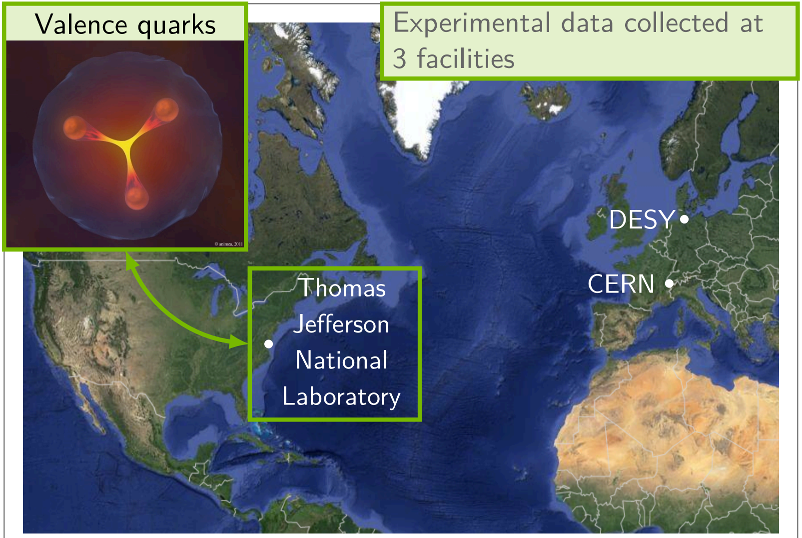
Need for global fits of world data.

Different facilities will probe different kinematic domains.

Experimental data collected at 3 facilities

Need for global fits of world data.

Different facilities will probe different kinematic domains.



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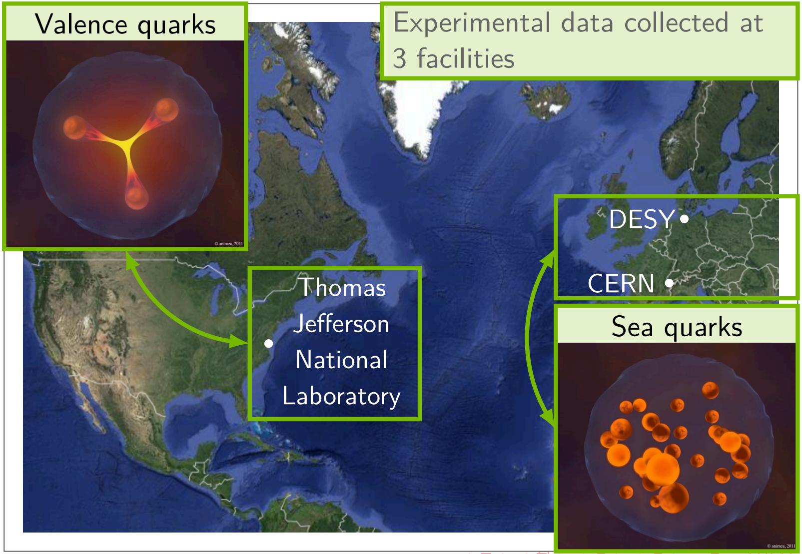
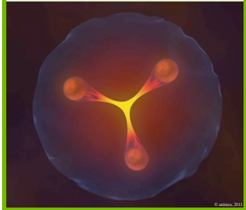


Need for global fits of world data.

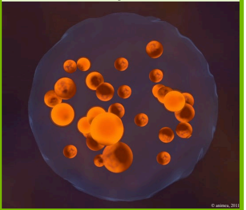
Different facilities will probe different kinematic domains.

Experimental data collected at 3 facilities, soon 4:EIC !

Valence quarks



Sea quarks

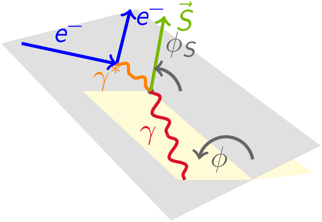


Gluons

NSAC, Long Range Plan 2015: "We recommend [...] EIC as the highest priority for new facility construction"

Typical DVCS kinematics.

Probing gluons, sea and valence quarks through DVCS.



- Study the **harmonic structure** of $ep \to ep\gamma$ amplitude.

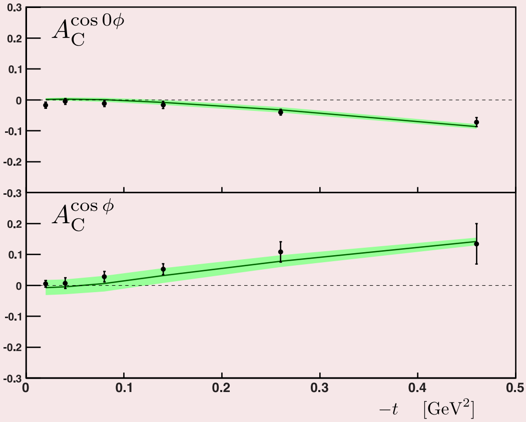
Diehl *et al.*, Phys. Lett. **B411**, 193 (1997)

Experiment	Kinematics		
	x_B	Q^2 [GeV^2]	t [GeV^2]
HERA	0.001	8.00	-0.30
COMPASS	0.05	2.00	-0.20
HERMES	0.09	2.50	-0.12
CLAS	0.19	1.25	-0.19
HALL A	0.36	2.30	-0.23

Goloskokov-Kroll (GK) model on DVCS.

No parameter of the GK model **was tuned** to analyse DVCS.

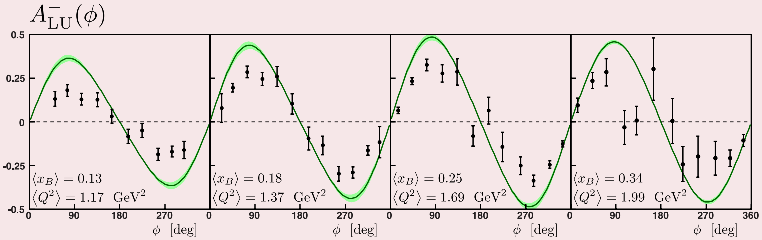
Beam Charge Asymmetry, HERMES



Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

No parameter of the GK model **was tuned** to analyse DVCS.

Beam Spin Asymmetry, CLAS



Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

Summary of first extractions.

Feasibility of twist-2 analysis of existing data.

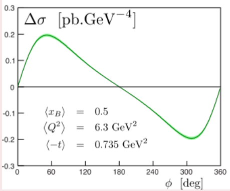
- **Dominance** of twist-2 and **validity** of a GPD analysis of DVCS data.
- $Im\mathcal{H}$ **best determined**. Large uncertainties on $Re\mathcal{H}$.
- However sizable **higher twist contamination** for DVCS measurements.
- Already some indications about the **invalidity** of the H-dominance hypothesis with **unpolarized data**.

See more on fits.

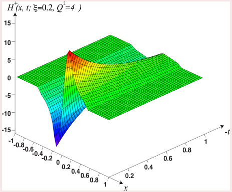
Imaging the nucleon. How?

Extracting GPDs is not enough…Need to extrapolate!

1. Experimental data fits



2. GPD extraction



3. Nucleon imaging

Images from Guidal et al., Rept. Prog. Phys. 76 (2013) 066202

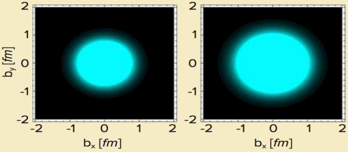
Reaching for the Horizon

The 2015 Long Range Plan for Nuclear Science

Sidebar 2.2: The First 3D Pictures of the Nucleon

A computed tomography (CT) scan can help physicians pinpoint minute cancer tumors, diagnose tiny broken bones, and spot the early signs of osteoporosis. Now physicists are using the principles behind the procedure to peer at the inner workings of the proton. This breakthrough is made possible by a relatively new concept in nuclear physics called generalized parton distributions.

An intense beam of high-energy electrons can be used



Imaging the nucleon. How?

Extracting GPDs is not enough...Need to extrapolate!

1. **Extract** $H(x, \xi, t, \mu_F^{\text{ref}})$ from experimental data.
2. **Extrapolate** to vanishing skewness $H(x, 0, t, \mu_F^{\text{ref}})$.
3. **Extrapolate** $H(x, 0, t, \mu_F^{\text{ref}})$ up to infinite t.
4. **Compute** 2D Fourier transform in transverse plane:

$$H(x, b_\perp) = \int_0^{+\infty} \frac{\mathrm{d}|\Delta_\perp|}{2\pi} |\Delta_\perp| J_0(|b_\perp||\Delta_\perp|) H(x, 0, -\Delta_\perp^2)$$

5. **Propagate** uncertainties.
6. **Control** extrapolations with an accuracy matching that of experimental data with **sound** GPD models.

The challenge of the high precision era.

Higher order and higher twist contributions, and GPD modeling.

- Evaluation of the impact of higher order effects.

See more on NLO evaluations.

- Evaluation of the impact of target mass and finite-t corrections.

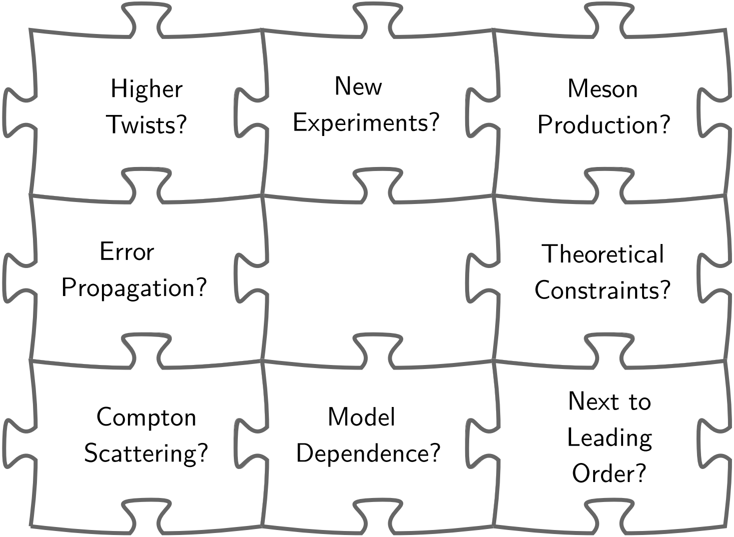
See more on DVCS kinematics.

- Evaluation of the contribution of higher twist GPDs.
- Extrapolations with GPD models.

See more on DVCS at LO.

Software for the phenomenology of GPDs.

Different questions to be answered with the same tools.



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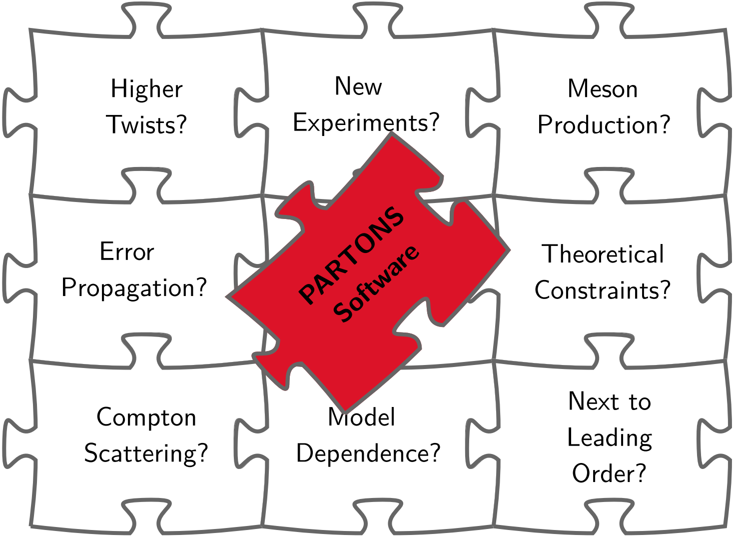


Building the tools for high precision: the PARTONS project

PARtonic
Tomography
Of
Nucleon
Software

Experimental data and phenomenology	Full processes
Computation of amplitudes	Small distance contributions
First principles and fundamental parameters	Large distance contributions

Computing chain design.

Differential studies: physical models and numerical methods.

Experimental data and phenomenology — **Full processes**

Computation of amplitudes — **Small distance contributions**

First principles and fundamental parameters — **Large distance contributions**

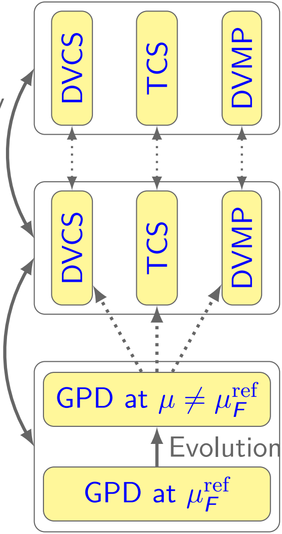
Computing chain design.

Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Computation of amplitudes

First principles and fundamental parameters



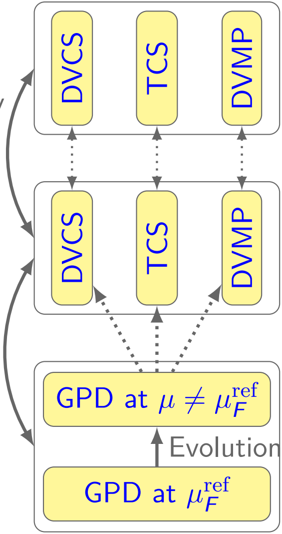
Computing chain design.

Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

Computing chain design.

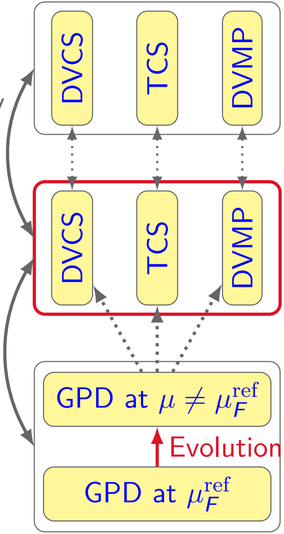
Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

Computing chain design.

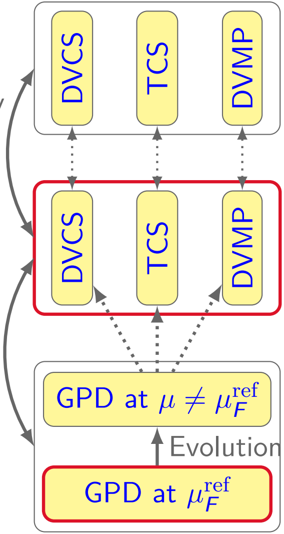
Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



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Computing chain design.

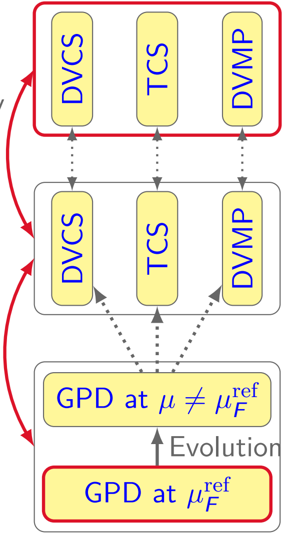
Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



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Computing chain design.

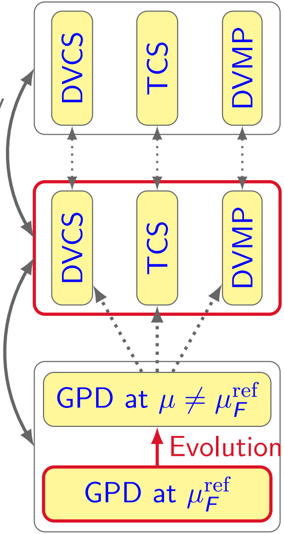
Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

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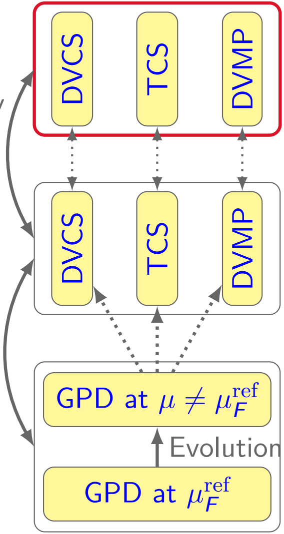
Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

- 3 stages:
  1. Design.
  2. Integration and validation.
  3. Production.
- Flexible software architecture.

  B. Berthou *et al.*, *PARTONS: a computing platform for the phenomenology of Generalized Parton Distributions*
- 1 new physical development = 1 new module.
- *Aggregate* **knowledge** and **know-how**. *Do not* reinvent the wheel!
- *Benefit* from the **experience** of code developers.
- What *can* be automated *will be* automated.

GPD computing made simple.

Each line of code corresponds to a physical hypothesis.

gpdExample()

```
// Lots of includes
#include <src/Partons.h>
...

// Retrieve GPD service
GPDService* pGPDService = ServiceObjectRegistry::getGPDService();
// Load GPD module with the BaseModuleFactory
GPDModule* pGK11Model = ModuleObjectFactory::newGPDModule(
GK11Model::classId);
// Create a GPDKinematic(x, xi, t, MuF, MuR)
GPDKinematic gpdKinematic(0.1, xBToXi(0.001), -0.3, 8., 8.);
// Compute data and store results
GPDResult gpdResult = pGPDService->
computeGPDModelRestrictedByGPDType(gpdKinematic, pGK11Model,
GPDType::ALL);
// Print results
std :: cout << gpdResult.toString() << std::endl;

delete pGK11Model;
pGK11Model = 0;
```

GPD computing automated.

Each line of code corresponds to a physical hypothesis.

computeOneGPD.xml

```xml
<?xml version="1.0" encoding="UTF-8" standalone="yes" ?>
<scenario id="01" date="" description="Example␣:␣computation␣of␣one␣GPD
␣model␣(GK11)␣without␣evolution">
    <!-- Select type of computation -->
    <task service="GPDService" method="computeGPDModel" >
        <!-- Specify kinematics -->
        <GPDKinematic>
            <param name="x" value="0.1" />
            <param name="xi" value="0.00050025" />
            <param name="t" value="-0.3" />
            <param name="MuF2" value="8" />
            <param name="MuR2" value="8" />
        </GPDKinematic>
        <!-- Choose GPD model and set parameters -->
        <GPDModule>
            <param name="id" value="GK11Model" />
        </GPDModule>
    </task>
</scenario>
```

GPD computing automated.

Each line of code corresponds to a physical hypothesis.

computeOneGP

```
<?xml version="1.0" encoding="UTF-8" stand
<scenario id="01" date="" description="Exam                    GPD
␣model␣(GK11)␣without␣evolution">
      <!-- Select type of computation -->
      <task service="GPDService" method=
            <!-- Specify kinematics -->
            <GPDKinematic>
                  <param name="x" valu
                  <param name="xi" va
                  <param name="t" valu
                  <param name="MuF2"
                  <param name="MuR2"
            </GPDKinematic>
            <!-- Choose GPD model and
            <GPDModule>
                  <param name="id" va
            </GPDModule>
      </task>
</scenario>
```

$H^u = 3.24235$

$H^{u(+)} = 3.65898$

$H^{u(-)} = 2.82571$

$H^d = 2.00354$

$H^{d(+)} = 2.42018$

$H^{d(-)} = 1.58691$

$H^s = 0.277787$

$H^{s(+)} = 0.555574$

$H^{s(-)} = 0$

$H^g = 0.660866$

and E, $\tilde{H}$, $\tilde{E}$, ...

CFF computing automated.

Each line of code corresponds to a physical hypothesis.

computeOneCFF.xml

```xml
<?xml version="1.0" encoding="UTF-8" standalone="yes" ?>
<scenario id="03" date="" description="Example␣:␣computation␣of␣one␣convol␣coeff␣function␣model␣(DVCSCFF)␣with␣GPD␣model␣(GK11)">
        <task service="DVCSConvolCoeffFunctionService" method="computeWithGPDModel">
                <DVCSConvolCoeffFunctionKinematic>
                        <param name="xi" value="0.5" />
                        <param name="t" value="-0.1346" />
                        <param name="Q2" value="1.5557" />
                        <param name="MuF2" value="4" />
                        <param name="MuR2" value="4" />
                </DVCSConvolCoeffFunctionKinematic>
                <GPDModule>
                        <param name="id" value="GK11Model" />
                </GPDModule>
                <DVCSConvolCoeffFunctionModule>
                        <param name="id" value="DVCSCFFModel" />
                        <param name="qcd_order_type" value="LO" />
                </DVCSConvolCoeffFunctionModule>
        </task>
</scenario>
```

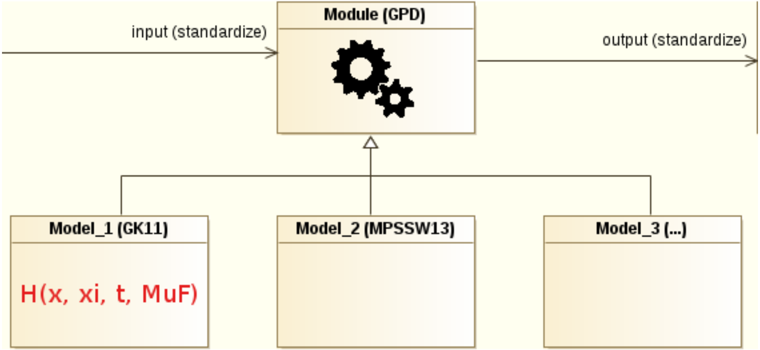
CFF computing automated.

Each line of code corresponds to a physical hypothesis.

computeOneCFF.xml

```
<?xml version="1.0" encoding="UTF-8" standalone="yes" ?>
<scenario id="03" date="" description="Example␣:␣computation␣of␣one␣convol␣coeff␣function␣model␣(DVCSCFF)␣with␣GPD␣model␣(GK11)">
    <task service="DVCSConvolCoeffFunctionService" method="computeWithGPDModel"
        <DVCSConvolCoeffFunctionKinematic>
            <param name="xi" value="0.5" />
            <param name="t" value="-0.1346" />
            <param name="Q2" value="1.5557" />
            <param name="MuF2" value="4" />
            <param name="MuR2" value="4" />
        </DVCSConvolCoeffFunctionKinematic>
        <GPDModule>
            <param name="id" value="GK11Model" />
        </GPDModule>
        <DVCSConvolC
            <para                                   >
            <para                                   />
        </DVCSConvol
    </task>
</scenario>
```

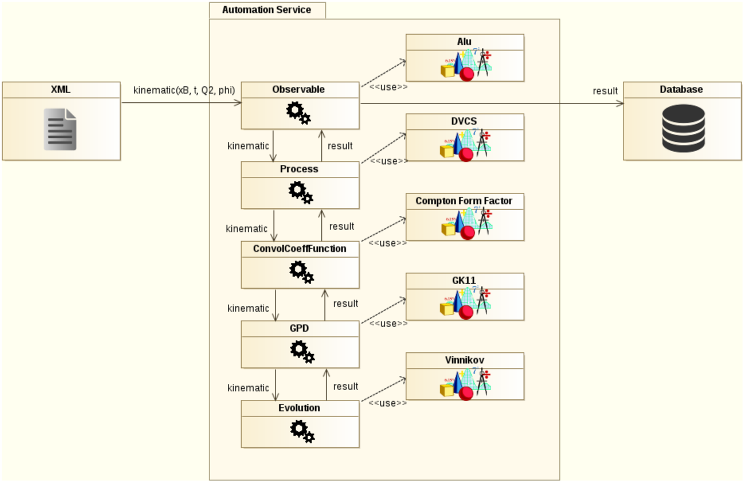
$$\mathcal{H} = 1.47722 + 1.76698\ i$$
$$\mathcal{E} = 0.12279 + 0.512312\ i$$
$$\widetilde{\mathcal{H}} = 1.54911 + 0.953728\ i$$
$$\widetilde{\mathcal{E}} = 18.8776 + 3.75275\ i$$



- Steps of logic sequence in parent class.
- Model description and related mathematical methods in daughter class.

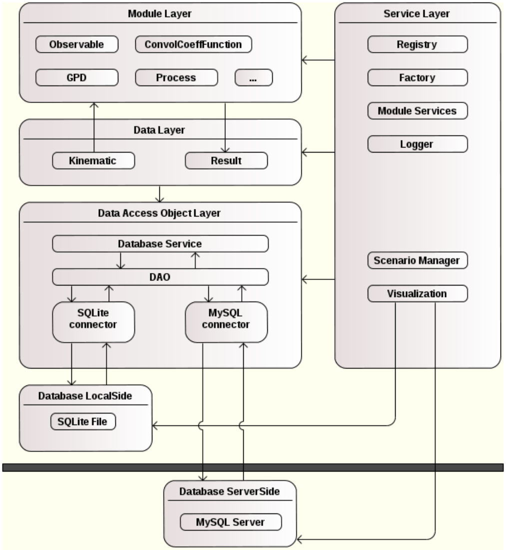
Modularity and automation.

Parse XML file, compute and store result in database.



Modularity and layer structure.

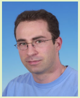
Modifying one layer does not affect the other layers.



Members and areas of expertise.

Collaborations at the national and international levels.

Development team

						
B. Berthou	N. Chouika	C. Mezrag	H. Moutarde	F. Sabatié	P. Sznajder	J. Wagner
(Irfu)	(Irfu)	(ANL)	(Irfu)	(Irfu)	(IPNO)	(NCBJ)

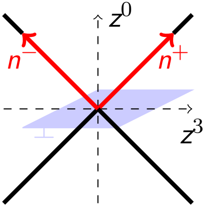
IPN and LPT (Orsay), Irfu (Saclay) and CPhT (Polytechnique)

Experimental data analysis	Perturbative QCD
World data fits	GPD modeling

Learning on the strong interaction from GPD models

$$H^q_\pi(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Ji, Phys. Rev. Lett. **78**, 610 (1997)
Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF **forward limit**

$$H^q(x, 0, 0) = q(x)$$

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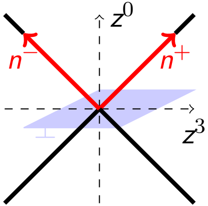

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- PDF **forward limit**
- Form factor **sum rule**

$$\int_{-1}^{+1} dx\, H^q(x, \xi, t) = F_1^q(t)$$

Spin-0 Generalized Parton Distribution.

Definition and simple properties.

$$H^q_\pi(x, \xi, t) = \frac{1}{2} \int \frac{\mathrm{d}z^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$

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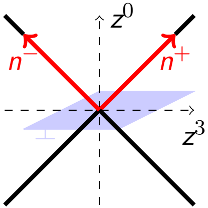

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- PDF **forward limit**
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- H^q is an **even function** of ξ from time-reversal invariance.

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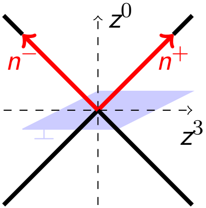

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Ji, Phys. Rev. Lett. **78**, 610 (1997)
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- PDF **forward limit**
- Form factor **sum rule**
- H^q is an **even function** of ξ from time-reversal invariance.
- H^q is **real** from hermiticity and time-reversal invariance.

Polynomiality

$$\int_{-1}^{+1} dx\, x^n H^q(x, \xi, t) = \text{polynomial in } \xi$$

Polynomiality

Lorentz covariance

See more on polynomiality.

Polynomiality

Lorentz covariance

See more on polynomiality.

Positivity

$$H^q(x, \xi, t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

- **Polynomiality**

  Lorentz covariance

  ▸ See more on polynomiality.

- **Positivity**

  Positivity of Hilbert space norm

  ▸ See more on positivity.

- **Polynomiality**

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Relativistic quantum mechanics

- **Polynomiality**

Lorentz covariance

See more on polynomiality.

- **Positivity**

Positivity of Hilbert space norm

See more on positivity.

- H^q has support $x \in [-1, +1]$.

Relativistic quantum mechanics

- **Soft pion theorem** (pion target)

$$H^q(x, \xi = 1, t = 0) = \frac{1}{2} \phi_\pi^q \left(\frac{1+x}{2} \right)$$

- **Polynomiality**

  Lorentz covariance

  See more on polynomiality.

- **Positivity**

  Positivity of Hilbert space norm

  See more on positivity.

- H^q has support $x \in [-1, +1]$.

  Relativistic quantum mechanics

- **Soft pion theorem** (pion target)

  Dynamical chiral symmetry breaking

- **Polynomiality**

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- H^q has support $x \in [-1, +1]$.

  Relativistic quantum mechanics

- **Soft pion theorem** (pion target)

  Dynamical chiral symmetry breaking

How can we implement *a priori* these theoretical constraints?

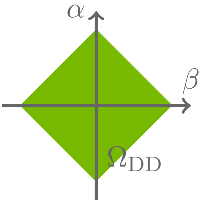
- There is no known GPD parameterization **relying only on first principles**.
- In the following, focus on **polynomiality** and **positivity**.

Double Distributions.

A convenient tool to encode GPD properties.

- Define Double Distributions F^q and G^q as matrix elements of **twist-2 quark operators**:

$$\left\langle P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^{\{\mu} i\overleftrightarrow{D}^{\mu_1} \ldots i\overleftrightarrow{D}^{\mu_m\}} q(0) \middle| P - \frac{\Delta}{2} \right\rangle = \sum_{k=0}^{m} \binom{m}{k} \left[F^q_{mk}(t) 2P^{\{\mu} - G^q_{mk}(t)\Delta^{\{\mu}\right] P^{\mu_1} \ldots P^{\mu_{m-k}} \left(-\frac{\Delta}{2}\right)^{\mu_{m-k+1}} \ldots \left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$



with

$$F^q_{mk} = \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, \alpha^k \beta^{m-k} F^q(\beta, \alpha)$$

$$G^q_{mk} = \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, \alpha^k \beta^{m-k} G^q(\beta, \alpha)$$

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Radyushkin, Phys. Rev. **D59**, 014030 (1999)
Radyushkin, Phys. Lett. **B449**, 81 (1999)

- Representation of GPD:

$$H^q(x,\xi,t) = \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, \delta(x-\beta-\alpha\xi)\big(F^q(\beta,\alpha,t)+\xi G^q(\beta,\alpha,t)\big)$$

- Support property: $x \in [-1,+1]$.
- Discrete symmetries: F^q is α-even and G^q is α-odd.
- **Pobylitsa gauge**: any representation (F^q, G^q) can be recast in one representation with a single DD f^q:

$$H^q(x,\xi,t) = (1-x)\int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha\, f^q(\beta,\alpha,t)\delta(x-\beta-\alpha\xi)$$

Pobylitsa, Phys. Rev. **D67**, 034009 (2003)
Müller, Few Body Syst. **55**, 317 (2014)

- Formalism: **Radon transform**.

See more on Radon transform.

- Decompose an hadronic state $|H; P, \lambda\rangle$ in a Fock basis:

$$|H; P, \lambda\rangle = \sum_{N,\beta} \int [\mathrm{d}x \mathrm{d}\mathbf{k}_\perp]_N \psi_N^{(\beta,\lambda)}(x_1, \mathbf{k}_{\perp 1}, \ldots, x_N, \mathbf{k}_{\perp N}) \,|\beta, k_1, \ldots, k_N\rangle$$

- Derive an expression for the pion GPD in the DGLAP region $\xi \leq x \leq 1$:

$$H^q(x, \xi, t) \propto \sum_{\beta,j} \int [\mathrm{d}\bar{x} \mathrm{d}\bar{\mathbf{k}}_\perp]_N \delta_{j,q} \delta(x - \bar{x}_j) \big(\psi_N^{(\beta,\lambda)}\big)^* (\hat{x}', \hat{\mathbf{k}}'_\perp) \psi_N^{(\beta,\lambda)}(\tilde{x}, \tilde{\mathbf{k}}_\perp)$$

with $\tilde{x}, \tilde{\mathbf{k}}_\perp$ (resp. $\hat{x}', \hat{\mathbf{k}}'_\perp$) generically denoting incoming (resp. outgoing) parton kinematics.

Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)

- Similar expression in the ERBL region $-\xi \leq x \leq \xi$, but with overlap of N- and $(N+2)$-body LFWFs.

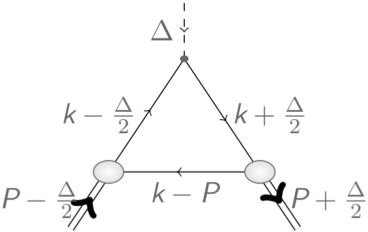
- Physical picture.
- Positivity relations are fulfilled **by construction**.
- Implementation of **symmetries of** N**-body problems**.

What is not obvious anymore

What is *not* obvious to see from the wave function representation is however the **continuity of GPDs at** $x = \pm\xi$ and the **polynomiality** condition. In these cases both the DGLAP and the ERBL regions must cooperate to lead to the required properties, and this implies **nontrivial relations between the wave functions** for the different Fock states relevant in the two regions. An *ad hoc* Ansatz for the wave functions would **almost certainly lead** to GPDs that **violate the above requirements**.

Diehl, Phys. Rept. **388**, 41 (2003)

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$

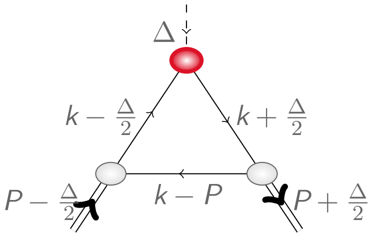


- Compute **Mellin moments** of the pion GPD H.

GPDs in the rainbow ladder approximation.

Evaluation of triangle diagrams.

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$



- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.

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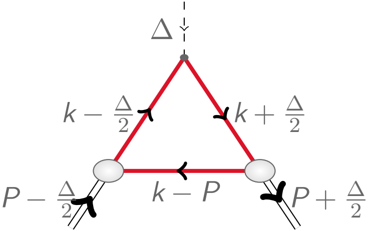
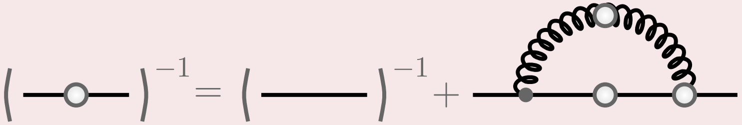


- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.

Dyson - Schwinger equation



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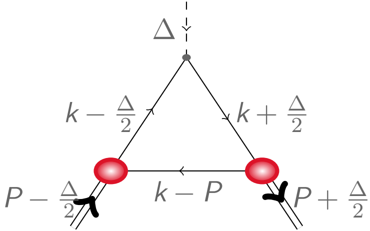

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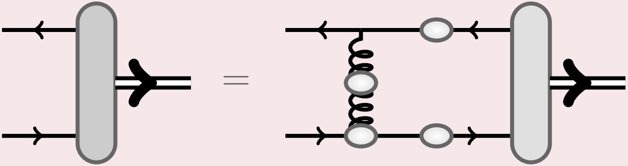
Bethe - Salpeter equation



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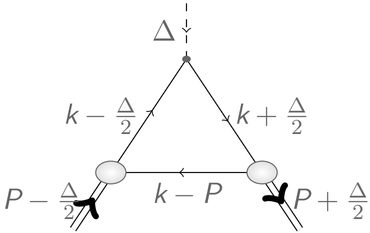

- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.
- **Nonperturbative** modeling.

- Most GPD properties **satisfied by construction**.

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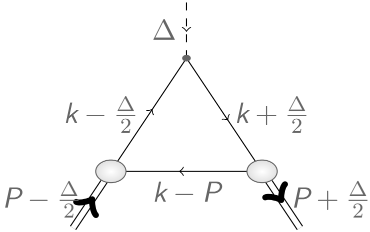


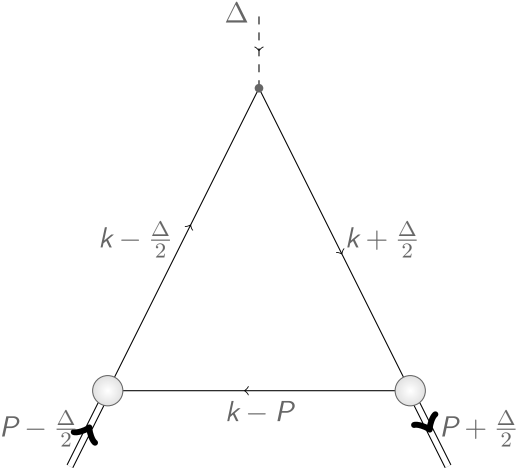
- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.
- **Nonperturbative** modeling.

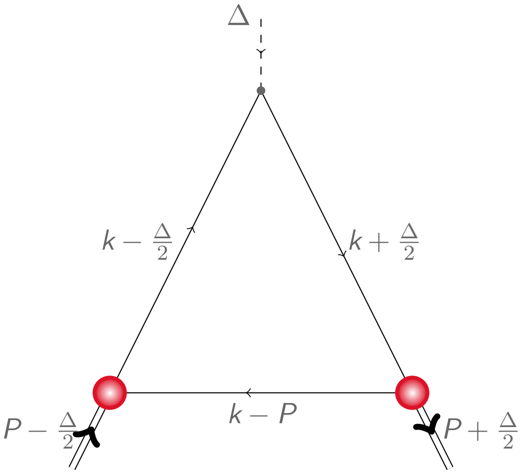
- Most GPD properties **satisfied by construction**.
- Also compute crossed triangle diagram.

Mezrag *et al.*, arXiv:1406.7425 [hep-ph] and Phys. Lett. **B741**, 190 (2015)

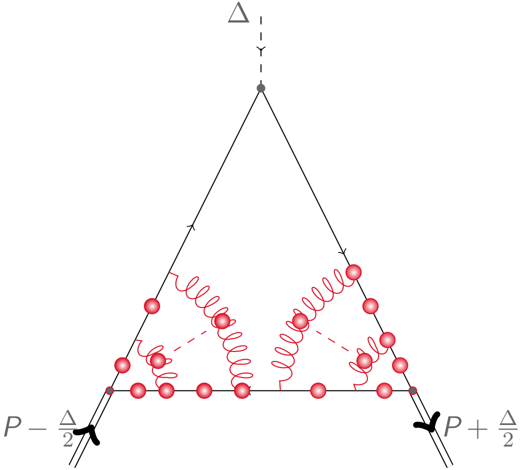
GPDs in the rainbow ladder approximation.

Physical content.





- Bethe-Salpeter vertex.



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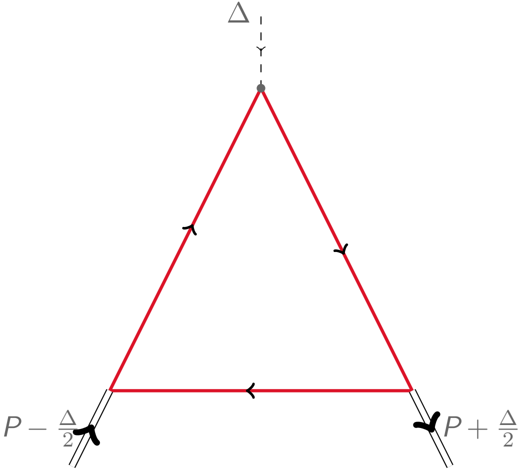

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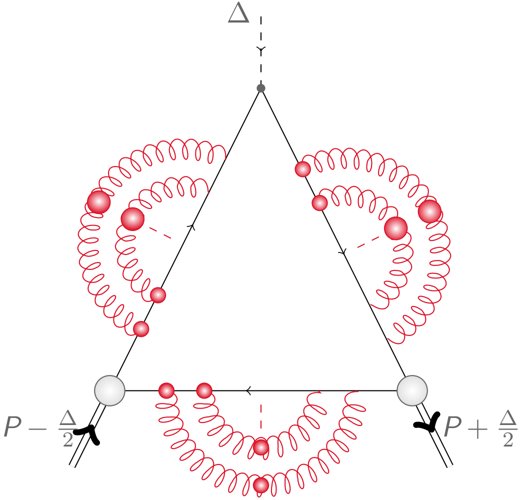

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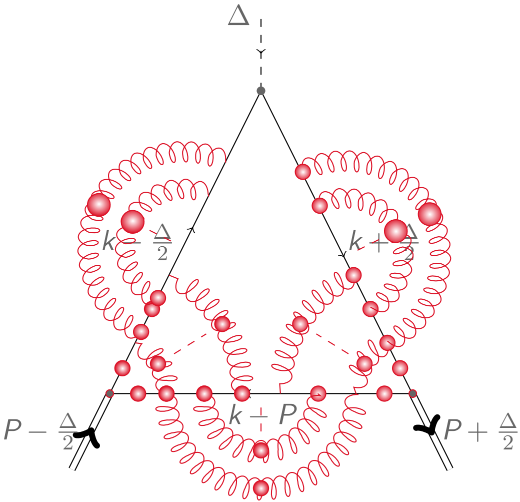

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- Dressed quark propagator.
- Much more than tree level perturbative diagram!

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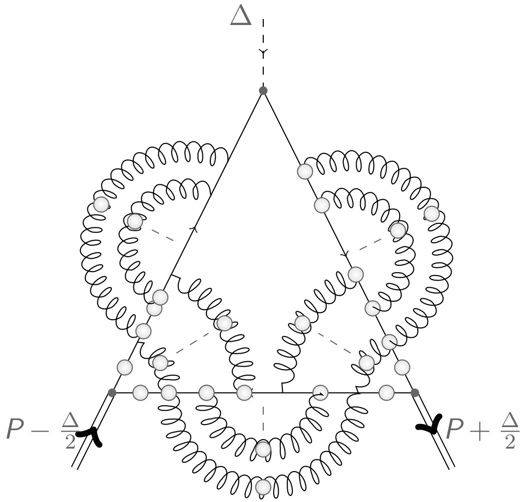

- Bethe-Salpeter vertex.
- Dressed quark propagator.
- Much more than tree level perturbative diagram!
- Enable description of **non perturbative phenomena**.

- **Polynomiality** from Poincaré covariance.

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- **Soft pion theorem** from **symmetry-preserving** truncation of Bethe-Salpeter and gap equations.

Mezrag *et al.*, Phys. Lett. **B741**, 190 (2015)

Symmetry-preserving truncation.

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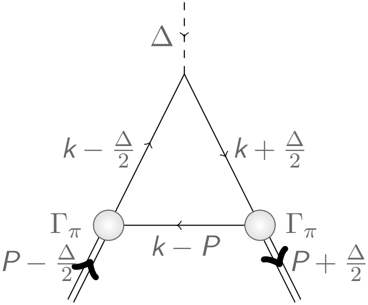
Mezrag *et al.*, Phys. Lett. **B741**, 190 (2015)

- Mellin moments.



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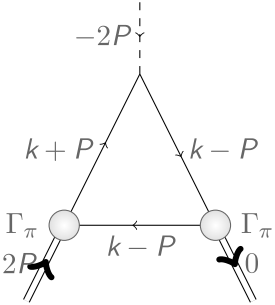

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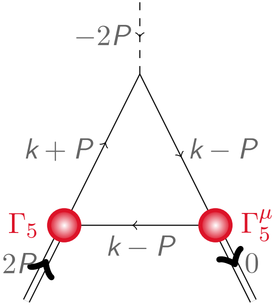

- Mellin moments.
- Soft pion kinematics.
- Axial and axial vector vertices Γ_5, Γ_5^μ in chiral limit.

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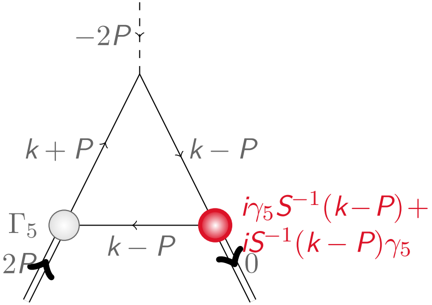
Mezrag *et al.*, Phys. Lett. **B741**, 190 (2015)



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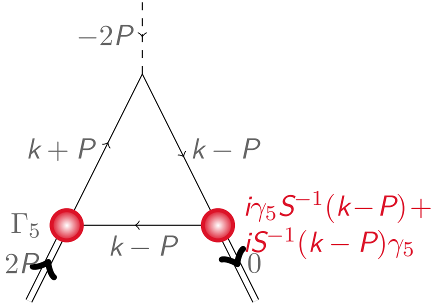
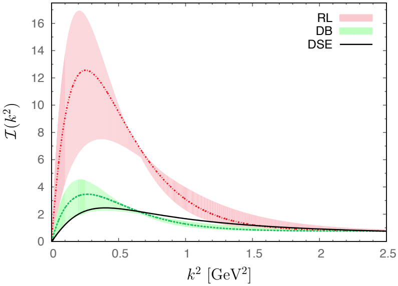
Mezrag *et al.*, Phys. Lett. **B741**, 190 (2015)



- Mellin moments.
- Soft pion kinematics.
- Axial and axial vector vertices Γ_5, Γ_5^μ in chiral limit.
- Axial-vector Ward identity.
- Recover pion DA Mellin moments.

- Gap equation kernel depends on **interaction strength** function $\mathcal{I}(k^2)$.
- Current model of $\mathcal{I}(k^2)$ yields ground and excited-state hadron masses with a **10-15 % accuracy** compared to experimental data.

Roberts *et al.*, Few Body Syst. **51**, 1 (2011)



- Good agreement with **independent evaluation** from lattice data + Dyson-Schwinger equations.

Binosi *et al.*, Phys. Lett. **B742**, 183 (2015)

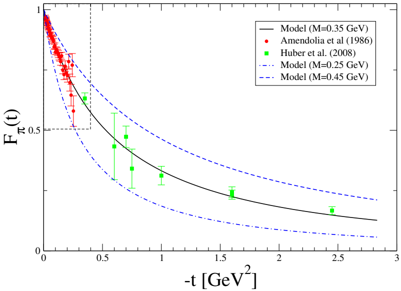
Pion form factor.

Good agreement with existing data with simple 1-parameter model.

- Pion form factor obtained from isovector GPD:

$$\int_{-1}^{+1} \mathrm{d}x \, H^{I=1}(x, \xi, t) = 2F_\pi(t)$$

- Single dimensionful parameter $M \simeq 350$ MeV.



Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

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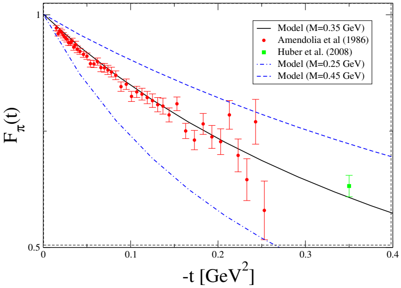

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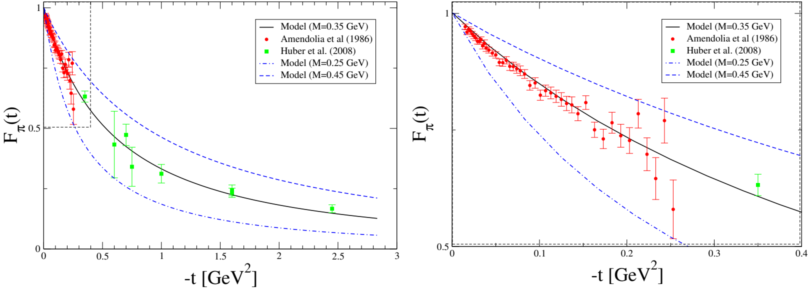
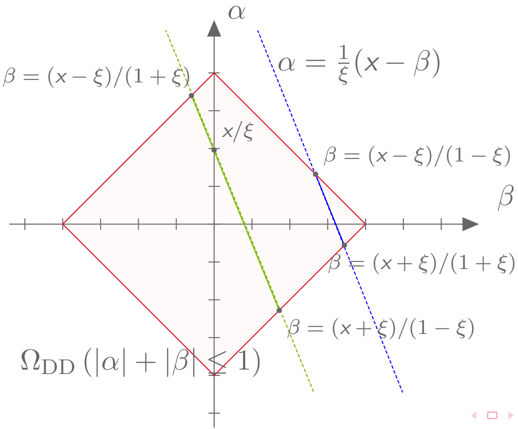


Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

DGLAP and ERBL regions

$$(x, \xi) \in \text{ DGLAP} \quad \Leftrightarrow \quad |s| \geq |\sin \phi| \,,$$
$$(x, \xi) \in \text{ ERBL} \quad \Leftrightarrow \quad |s| \leq |\sin \phi| \,.$$



Each point (β, α) with $\beta \neq 0$ contributes to **both** DGLAP and ERBL regions.

For **any model of LFWF**, one has to address the following three questions:

1. Does the extension exist?
2. If it exists, is it unique?
3. How can we compute this extension?

Work in progress!

See more on inverse Radon transform.

Conclusion

- Last decade demonstrated **maturity of GPD phenomenology**.
- **Good theoretical control** on the path between GPD models and experimental data.
- **Challenging constraints** expected from Jefferson Lab in the valence region.
- Building of **QCD-inspired models** to make progress.
- **Systematic** procedure to construct GPD models from any "reasonable" Ansatz of LFWFs.
- **Construction** of computing tools to make the best from future experimental data!

Appendix

Local fits

Take each kinematic bin independantly of the others. Extraction of $Re\mathcal{H}$, $Im\mathcal{H}$, ...as independent parameters.

Global fit

Take all kinematic bins at the same time. Use a parametrization of GPDs or CFFs.

Hybrid : Local / global fit

Start from local fits and add smoothness assumption.

Neural networks

Exploratory stage for GPDs.

Local fits

Take each kinematic bin independantly of the others.
Extraction of $Re\mathcal{H}$, $Im\mathcal{H}$, ...as independent parameters.

M. Guidal, Eur. Phys. J. **A39**, 5 (2009)

- **Almost model-independent**: relies on twist-2 dominance assumption and assume bounds for the fitting domain.
- Interpretation of **uncertainties** on extracted quantities? Contributions from measurements uncertainties, correlations between CFFs and fitting domain boundaries.
- Interpretation of **extracted quantities**? *e.g.*mixing of quark and gluon GPDs due to NLO effects.
- **Oscillations** between different (x_B, t, Q^2) bins may happen.
- **Extrapolation** problem left open.

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

where $a = \mathcal{O}(Q^{-1})$, $b = \mathcal{O}(Q^{-4})$, $c = \mathcal{O}(Q^{-1})$, $d = \mathcal{O}(Q^{-2})$, $e = \mathcal{O}(Q^{-5})$.

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c \cos\phi + d \cos 2\phi + e \cos 3\phi}$$

- **Underconstrained** problem (8 fit parameters : real and imaginary parts of 4 CFFs $\mathcal{H}$, $\mathcal{E}$, $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{E}}$).

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

- **Underconstrained** problem.
- Need other asymmetries on **same** kinematic bin to allow extraction of **all CFFs** (or **add** $\simeq$ 5-10 % **systematic uncertainty**).

Local fits: What can be achieved in principle?

- Structure of BSA at twist 2 :

$$\mathrm{BSA}(\phi) = \frac{a \sin\phi + b \sin 2\phi}{1 + c\cos\phi + d\cos 2\phi + e\cos 3\phi}$$

- **Underconstrained** problem.
- Need other asymmetries on **same** kinematic bin to allow extraction of **all CFFs**.
- Add physical input? **Dispersion relations**, etc.

Kumericki *et al.*, arXiv:1301.1230

Guidal *et al.*, Rept. Prog. Phys. **76**, 066202 (2013)

Global fit

Take all kinematic bins at the same time. Use a parametrization of GPDs or CFFs.

Kumericki, Nucl. Phys. **B841**, 1 (2010)

- **Model-dependent** approach.
- Allows the **implementation of theoretical constraints** on GPDs or CFFs.
- Guideline for **extrapolation** outside the physical domain.
- Compromise between number of parameters and number of described GPDs (flavor dependence, higher-twists, ...)?
- Impact on the **choice of a fitting strategy**?

Hybrid : Local / global fit

Start from local fits and add smoothness assumption.

Moutarde, Phys. Rev. **D79**, 094021 (2009)

- Avoid unphysical oscillations between different (x_B, t, Q^2) bins by comparing to a **global fit by a smooth function**:

$$H^+ = 2\sum_{n=0}^{N}\sum_{l=0}^{n+1} B_{nl}(t)\theta(|x| < \xi)\left(1-\frac{x^2}{\xi^2}\right) C_{2n+1}^{(3/2)}\left(\frac{x}{\xi}\right) P_{2l}\left(\frac{x}{\xi}\right)$$

- Number of fit parameters describing the B_{nl} coefficients **increases with** N^2...Extension to other GPDs seems difficult.
- **Extrapolation** problem left open.

Neural networks

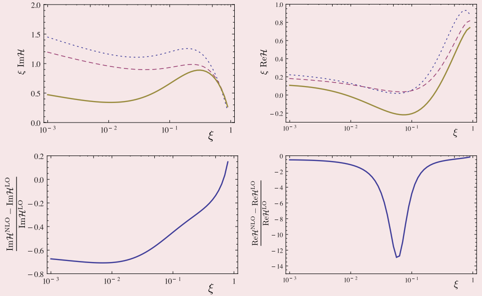
Exploratory stage for GPDs.

Kumericki *et al.*, JHEP **1107**, 073 (2011)

- Already used for PDF fits.
- **Almost model-independent**: neural network description, twist-2, H-dominance?
- Good agreement between model fit and neural network fit in the fitting domain.
- **More reliable uncertainties** in extrapolations?
- **Overtraining** as a generic feature of (too) flexible models.

Back to summary of first extractions.

$\mathcal{H}$ at LO and NLO ($t = -0.1$ GeV^2, $Q^2 = \mu_F^2 = 4.$ GeV^2)

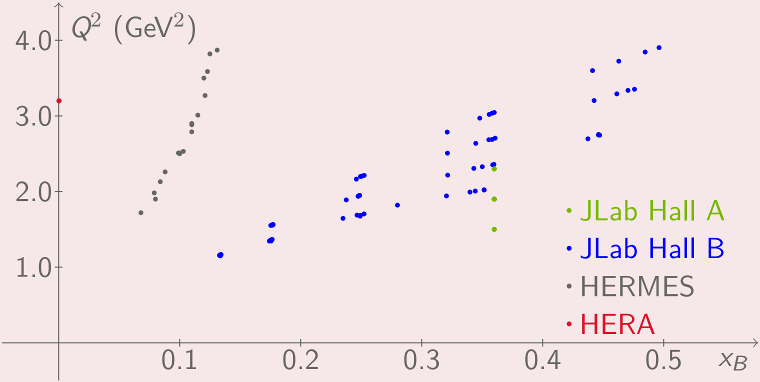


Moutarde *et al.*, Phys. Rev. **D87**, 054029 (2013)

- **Systematic** tests of perturbative QCD assumptions.
- **Wide kinematic range** (from JLab to EIC).
- **Accuracy** set by JLab 12 GeV expected statistical accuracy.
- **Model dependent** evaluations.

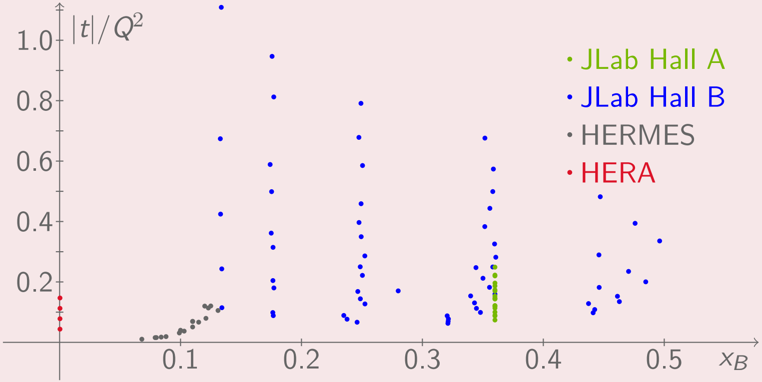
Back to challenges.

What is large Q^2 ?



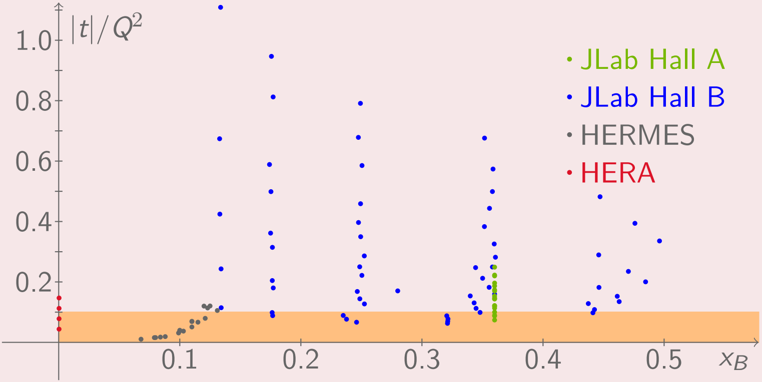
- World data cover **complementary kinematic regions.**

What is large Q^2 ?



- World data cover **complementary kinematic regions**.
- Q^2 is **not so large** for most of the data.

What is large Q^2 ?



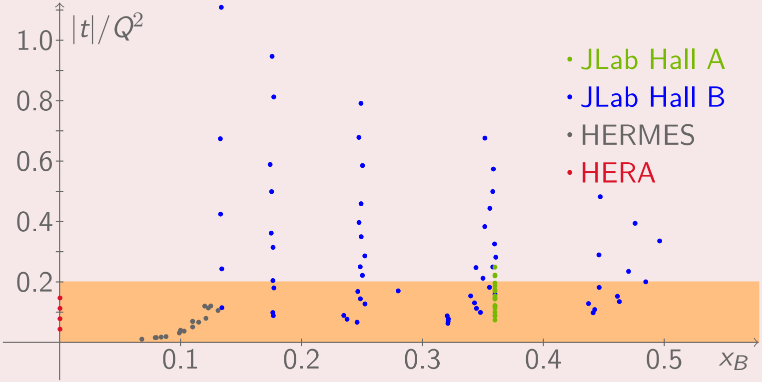
- World data cover **complementary kinematic regions**.
- Q^2 is **not so large** for most of the data.
- **Higher twists?**

Back to challenges.

Kinematics of existing DVCS measurements.

Looking for the Bjorken regime.

What is large Q^2 ?



- World data cover **complementary kinematic regions**.
- Q^2 is **not so large** for most of the data.
- **Higher twists?**

Back to challenges.

Dispersion relations and the cross-over line.

Existence of a relation between $Re\mathcal{H}(\xi)$ and $H(x, \xi = x)$.

- Write dispersion relation **at fixed** t **and** Q^2:

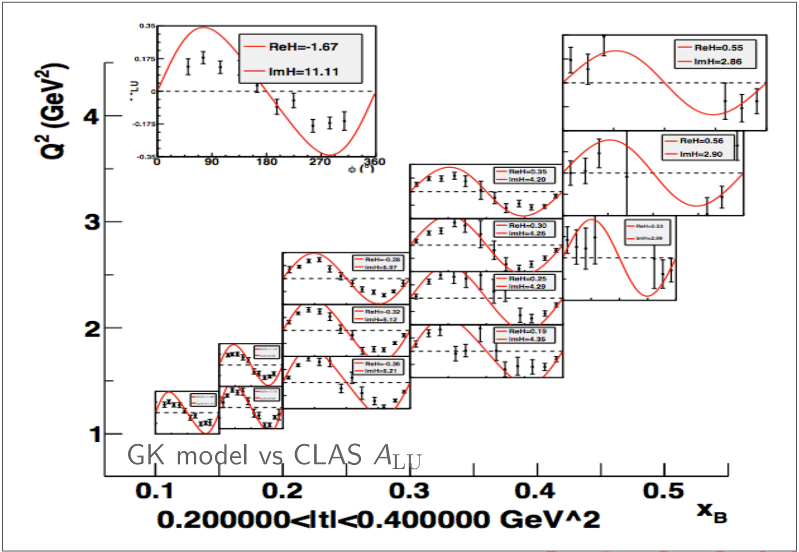
$$Re\mathcal{H}(\xi, t) = \Delta(t) + \frac{2}{\pi}\mathcal{P}\int_0^1 \frac{\mathrm{d}x}{x} \frac{Im\mathcal{H}(x, t)}{\left(\frac{\xi^2}{x^2} - 1\right)}$$

- Use LO relation $Im\mathcal{H}(x, t) = \pi\big(H(x, x, t) - H(-x, x, t)\big)$.
- Up to the D-term form factor $\Delta(t)$, all the information accessible **at LO and fixed** Q^2 is contained on the cross-over line.

Teryaev, hep-ph/0510031
Anikin and Teryaev, Phys. Rev. **D76**, 056007 (2007)
Diehl and Ivanov, Eur. Phys. J. **C52**, 919 (2007)

Dispersion relations and actual data.

Too few kinematic bins to provide model-independent constraints?



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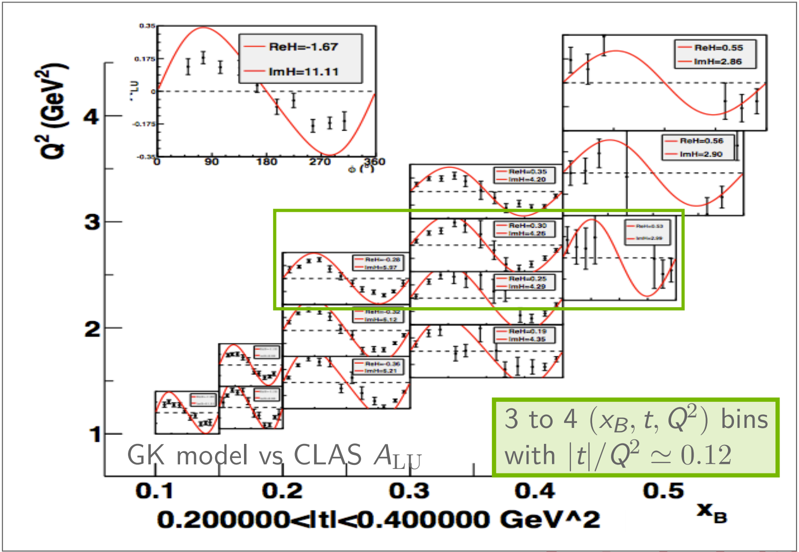

Dispersion relations and actual data.

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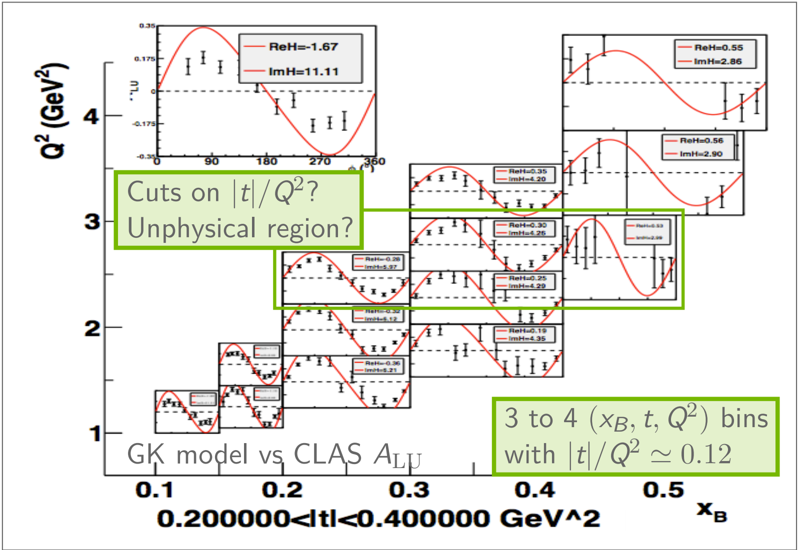
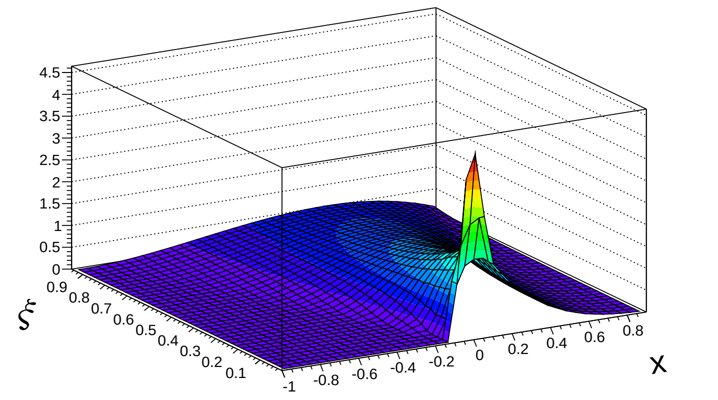
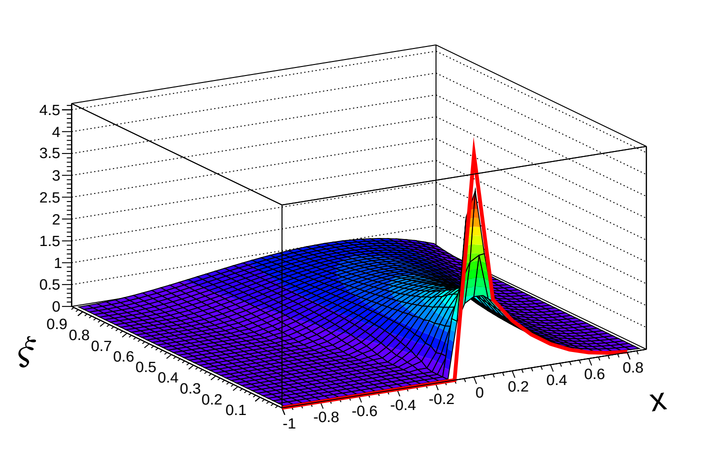


GPD H at $t = -0.23$ GeV^2 and $Q^2 = 2.3$ GeV^2.



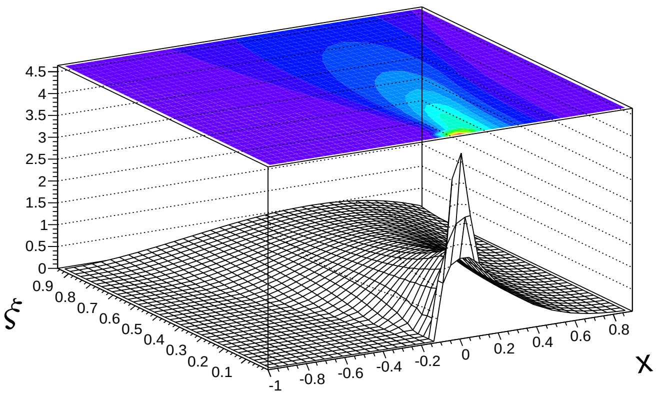
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

Need to know $H(x, \xi = 0, t)$ to do transverse plane imaging.



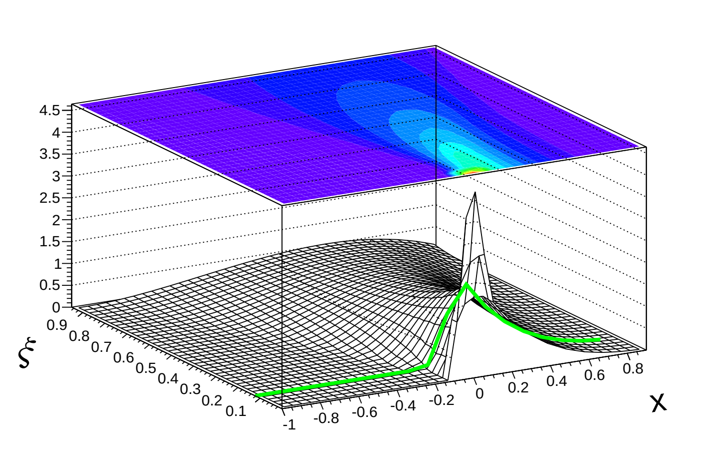
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

What is the physical region?



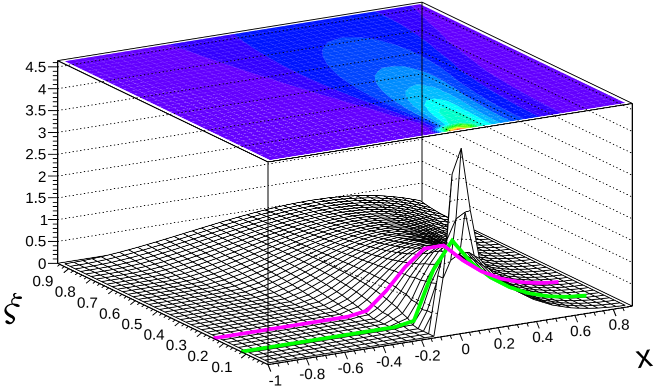
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

$\xi_{\min}$ from finite beam energy.



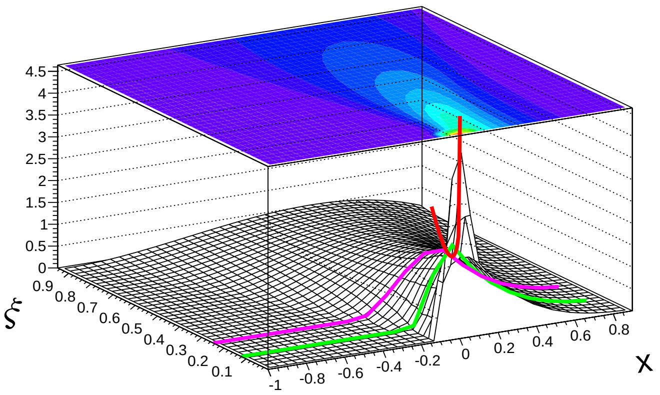
GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

ξ_{max} from kinematic constraint on 4-momentum transfer.



GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

The cross-over line $x = \xi$.

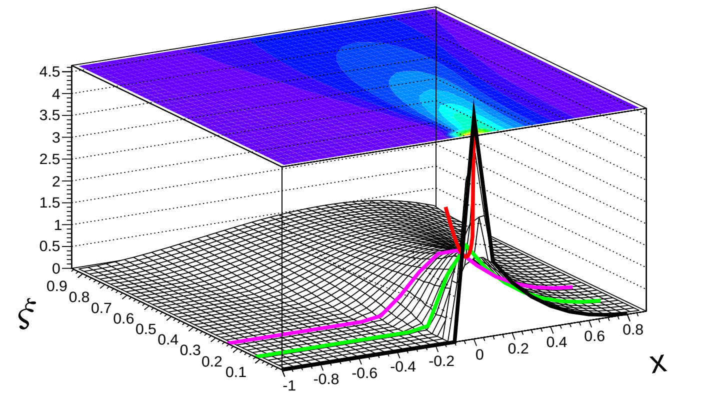


GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

Direct experimental access to a restricted kinematic domain.

The black curve is what is needed for transverse plane imaging!

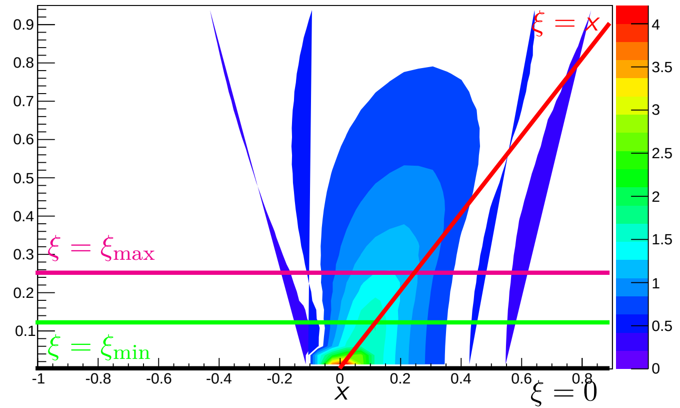


GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

From principles to actual data.

Direct experimental access to a restricted kinematic domain.

Density plot of H at $t = -0.23$ GeV^2 and $Q^2 = 2.3$ GeV^2



GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

Back to challenges.

Polynomiality.

Mixed constraint from Lorentz invariance and discrete symmetries.

- Express Mellin moments of GPDs as **matrix elements**:

$$\int_{-1}^{+1} \mathrm{d}x\, x^m H^q(x, \xi, t) = \frac{1}{2(P^+)^{m+1}} \left\langle P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \middle| P - \frac{\Delta}{2} \right\rangle$$

- Identify the **Lorentz structure** of the matrix element:

linear combination of $(P^+)^{m+1-k}(\Delta^+)^k$ for $0 \leq k \leq m+1$

- Remember definition of **skewness** $\Delta^+ = -2\xi P^+$.
- Select **even powers** to implement time reversal.
- Obtain **polynomiality condition**:

$$\int_{-1}^{1} \mathrm{d}x\, x^m H^q(x, \xi, t) = \sum_{\substack{i=0 \\ \text{even}}}^{m} (2\xi)^i C^q_{mi}(t) + (2\xi)^{m+1} C^q_{mm+1}(t) \,.$$

Double Distributions.

Lorentz covariance by example.

- Choose $F^q(\beta, \alpha) = 3\beta\theta(\beta)$ ad $G^q(\beta, \alpha) = 3\alpha\theta(\beta)$:

$$H^q(x, \xi) = 3x \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \, \delta(x - \beta - \alpha\xi)$$

- Simple analytic expressions for the GPD:

$$\begin{aligned} H(x, \xi) &= \frac{6x(1-x)}{1-\xi^2} \text{ if } 0 < |\xi| < x < 1, \\ H(x, \xi) &= \frac{3x(x+|\xi|)}{|\xi|(1+|\xi|)} \text{ if } -|\xi| < x < |\xi| < 1. \end{aligned}$$

Double Distributions.

Lorentz covariance by example.

- Compute first Mellin moments.

n	$\int_{-\xi}^{+\xi} \mathrm{d}x\, x^n H(x,\xi)$	$\int_{+\xi}^{+1} \mathrm{d}x\, x^n H(x,\xi)$	$\int_{-\xi}^{+1} \mathrm{d}x\, x^n H(x,\xi)$
0	$\frac{1+\xi-2\xi^2}{1+\xi}$	$\frac{2\xi^2}{1+\xi}$	1
1	$\frac{1+\xi+\xi^2-3\xi^3}{2(1+\xi)}$	$\frac{2\xi^3}{1+\xi}$	$\frac{1+\xi^2}{2}$
2	$\frac{3(1-\xi)(1+2\xi+3\xi^2+4\xi^3)}{10(1+\xi)}$	$\frac{6\xi^4}{5(1+\xi)}$	$\frac{3(1+\xi^2)}{10}$
3	$\frac{1+\xi+\xi^2+\xi^3+\xi^4-5\xi^5}{5(1+\xi)}$	$\frac{6\xi^5}{5(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{5}$
4	$\frac{1+\xi+\xi^2+\xi^3+\xi^4+\xi^5-6\xi^6}{7(1+\xi)}$	$\frac{6\xi^6}{7(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{7}$

- Expressions get more complicated as n increases... But they always yield polynomials!

Back to GPD properties.

Positivity.

A consequence of the positivity of the nom in a Hilbert space.

- Identify the matrix element defining a GPD as an **inner product** of two different states.
- Apply Cauchy-Schwartz inequality, and identify PDFs at specific kinematic points, *e.g.*:

$$|H^q(x,\xi,t)| \leq \sqrt{\frac{1}{1-\xi^2} q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

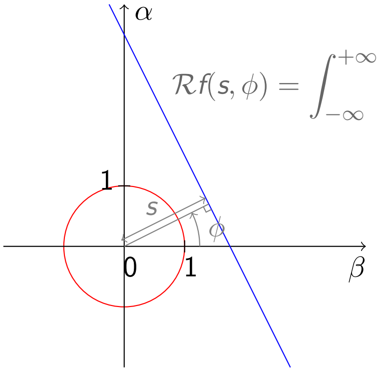
- This procedures yields **infinitely many inequalities** stable under LO evolution.

Pobylitsa, Phys. Rev. **D66**, 094002 (2002)

- The **overlap representation** guarantees *a priori* the fulfillment of positivity constraints.

Back to GPD properties.

The Radon transform.
Definition and properties.



For $s > 0$ and $\phi \in [0, 2\pi]$:

$$\mathcal{R}f(s, \phi) = \int_{-\infty}^{+\infty} \mathrm{d}\beta \mathrm{d}\alpha \, f(\beta, \alpha) \delta(s - \beta \cos\phi - \alpha \sin\phi)$$

and:

$$\mathcal{R}f(-s, \phi) = \mathcal{R}f(s, \phi \pm \pi)$$

Relation to GPDs:

$$x = \frac{s}{\cos\phi} \text{ and } \xi = \tan\phi$$

Relation between GPD and DD in Pobylitsa gauge

$$\frac{\sqrt{1+\xi^2}}{1-x} H(x, \xi) = \mathcal{R}f^{\mathrm{Pobylitsa}}(s, \phi) \; ,$$

The range of the Radon transform.

The polynomiality property a.k.a. the Ludwig-Helgason condition.

- The Mellin moments of a Radon transform are **homogeneous polynomials** in $\omega = (\sin\phi, \cos\phi)$.
- The converse is also true:

Theorem (Hertle, 1983)

Let $g(s, \omega)$ an even compactly-supported distribution. Then g is itself the Radon transform of a compactly-supported distribution if and only if the ***Ludwig-Helgason consistency condition*** *hold:*

(i) *g is C^∞ in ω,*

(ii) *$\int \mathrm{d}s\, s^m g(s, \omega)$ is a homogeneous polynomial of degree m for all integer $m \geq 0$.*

- Double Distributions and the Radon transform are the **natural solution** of the polynomiality condition.

Back to DDs.

Theorem

Let f be a compactly-supported locally summable function defined on $\mathbb{R}^2$ and $\mathcal{R}f$ its Radon transform.
Let $(s_0, \omega_0) \in \mathbb{R} \times S^1$ and U_0 an open neighborhood of ω_0 such that:

$$\text{for all } s > s_0 \text{ and } \omega \in U_0 \quad \mathcal{R}f(s, \omega) = 0 \,.$$

Then $f(\aleph) = 0$ on the half-plane $\langle \aleph \,|\, \omega_0 \rangle > s_0$ of $\mathbb{R}^2$.

Consider a GPD H being zero on the DGLAP region.

- Take ϕ_0 and s_0 *s.t.* $\cos\phi_0 \neq 0$ and $|s_0| > |\sin\phi_0|$.
- Neighborhood U_0 of ϕ_0 *s.t.* $\forall \phi \in U_0 \quad |\sin\phi| < |s_0|$.
- The underlying DD f has a zero Radon transform for all $\phi \in U_0$ and $s > s_0$ (DGLAP).
- Then $f(\beta, \alpha) = 0$ for all $(\beta, \alpha) \in \Omega_{\mathrm{DD}}$ with $\beta \neq 0$.
- Extension **unique** up to adding a **D-term**: $\delta(\beta)D(\alpha)$.

A discretized problem

Consider $N+1$ Hilbert spaces H, H_1, ..., H_N, and a family of continuous surjective operators $R_n : H \to H_n$ for $1 \leq n \leq N$. Being given $g_1 \in H_1$, ..., $g_n \in H_n$, we search f solving the following system of equations:

$$R_n f = g_n \quad \text{for } 1 \leq n \leq N$$

Fully discrete case

Assume f piecewise-constant with values f_m for $1 \leq m \leq M$. For a collection of lines $(L_n)_{1 \leq n \leq N}$ crossing Ω_{DD}, the Radon transform writes:

$$g_n = \mathcal{R}f = \int_{L_n} f = \sum_{m=1}^{M} f_m \times \mathrm{Measure}(L_n \cap C_m) \quad \text{for } 1 \leq n \leq N$$

Kaczmarz algorithm

Denote P_n the orthogonal projection on the *affine* subspace $R_n f = g_n$. Starting from $f^0 \in H$, the sequence defined iteratively by:

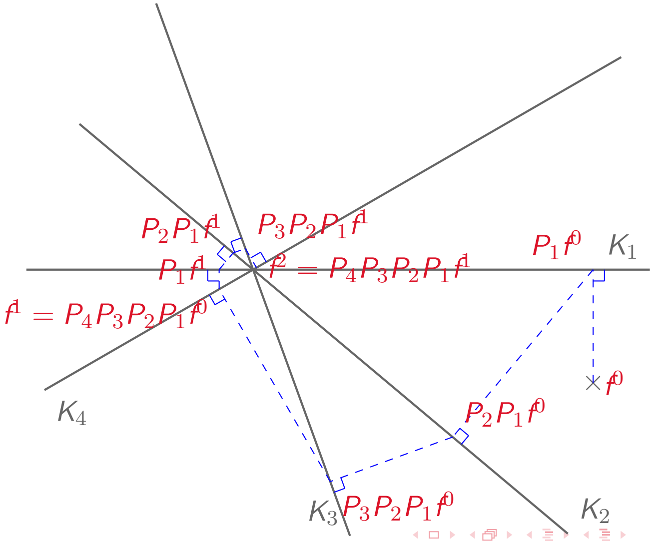
$$f^{k+1} = P_N P_{N-1} \dots P_1 f^k$$

converges to the solution of the system.
The convergence is exponential if the projections are randomly ordered.

Strohmer and Vershynin, Jour. Four. Analysis and Appl. **15**, 437 (2009)

Computation of the extension.

Numerical evaluation of the inverse Radon transform (2/3).



And if the input data are inconsistent?

- Instead of solving $g = \mathcal{R}f$, find f such that $\|g - \mathcal{R}f\|_2$ is **minimum**.
- The solution **always exists**.
- The input data are **inconsistent** if $\|g - \mathcal{R}f\|_2 > 0$.

Relaxed Kaczmarz algorithm

Let $\omega \in]0, 2[$ and:

$$P_n^\omega = (1 - \omega)\,\mathrm{Id}_H + \omega P_n \quad \text{for } 1 \leq n \leq N$$

Write:

$$RR^\dagger = (R_i R_j^\dagger)_{1 \leq i,j \leq N} = D + L + L^\dagger$$

where D is diagonal, and L is lower-triangular with zeros on the diagonal.

Theorem

Let $0 < \omega < 2$. *For* $f^0 \in \mathrm{Ran}\ R^\dagger$ *(e.g.* $f^0 = 0$*), the Kaczmarz method with relaxation converges to the unique solution* $f^\omega \in \mathrm{Ran}\ R^\dagger$ *of:*

$$R^\dagger (D + \omega L)^{-1} (g - Rf^\omega) = 0 \ ,$$

where the matrix D and L appear in the decomposition of $RR^\dagger$.
If $g = \mathcal{R}f$ *has a solution, then* f^ω *is its solution of minimal norm. Otherwise:*

$$f^\omega = f_{MP} + \mathcal{O}(\omega) \ ,$$

where f_{MP} *is the minimizer in H of:*

$$\langle g - \mathcal{R}f | g - \mathcal{R}f \rangle_D \ ,$$

the inner product being defined by:

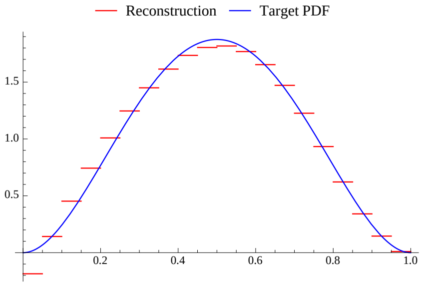
$$\langle h | k \rangle_D = \langle D^{-1} h | k \rangle \ .$$

Test on a 1D example.

Recovering a PDF from the knowledge of its Mellin moments.

A pion valence PDF-like example

Aim: reconstruct the PDF $q(x) = 30x^2(1-x)^2$ from the knowledge of its first 30 Mellin moments.



- Piecewise-constant PDF: 20 values.
- Input: 30 Mellin moments.
- Unrelaxed method $\omega = 1$.
- 10000 iterations.

- Extensive testing *in progress*
  - Various inputs: PDFs and LFWFs.
  - Numerical noise.

Back to covariant extensions.

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