

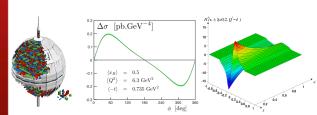


DE LA RECHERCHE À L'INDUSTRIE









www.cea.fr

QCD Workshop | Hervé MOUTARDE

May  $6^{\mathrm{th}}$ , 2016





# Motivation. QCD large distance dynamics from the hadron structure viewpoint.



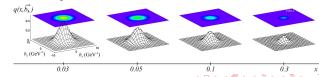
#### PARTONS Framework

■ Lattice QCD clearly shows that the mass of hadrons is generated by the interaction, not by the quark masses.

## 2000 Motivation 1500 Content of GPDs M[MeV] Imaging experiment 500 width input -- п QCD

Durr et al., Science 322, 1224 (2008)

Can we map the location of mass inside a hadron?



#### Mass without mass Nucleon structure

Experimental access **DVCS Kinematics** Towards 3D images

#### Building

Computing chain Examples

Architecture Team

#### Learning Definition

Overlan Radon transform Covariant extension

## Conclusion



## Imaging the origin of mass. Identification of underlying mechanisms from parton distributions.



## PARTONS Framework

Motivation Mass without mass Nucleon structure

Content of GPDs

#### Imaging Experimental access

**DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples

Architecture

Team

## Learning

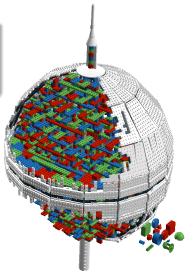
Definition Overlan

Radon transform Covariant extension

Conclusion

Appendix

How can we recover the wellknown characterics of the nucleon from the properties of its colored building blocks?





Identification of underlying mechanisms from parton distributions.



## PARTONS Framework

How can we recover the wellknown characterics of the nucleon from the properties of its colored building blocks?

Mass?

## Nucleon structure Content of GPDs Imaging

Motivation Mass without mass

Experimental access **DVCS Kinematics** Towards 3D images

#### Building

Computing chain

Examples

Architecture Team

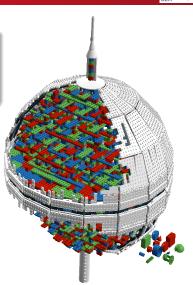
#### Learning

Definition Overlan

Radon transform

Covariant extension

### Conclusion





Identification of underlying mechanisms from parton distributions.



## PARTONS Framework

Mass without mass Nucleon structure

Content of GPDs

Motivation

Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

Computing chain

Examples Architecture

Team

#### Learning Definition

Overlan

Radon transform

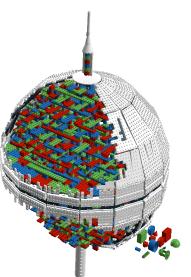
Covariant extension

#### Conclusion

Appendix

How can we recover the wellknown characterics of the nucleon from the properties of its colored building blocks?

> Mass? Spin?





Identification of underlying mechanisms from parton distributions.



### PARTONS Framework

Motivation Mass without mass

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

Nucleon structure Content of GPDs

#### Building

Computing chain Examples

Architecture

Team

## Learning

Definition Overlan

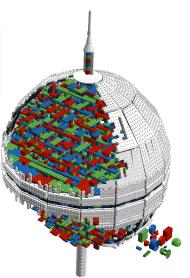
Radon transform Covariant extension

Conclusion

Appendix

How can we recover the wellknown characterics of the nucleon from the properties of its colored building blocks?

> Mass? Spin? Charge?





Identification of underlying mechanisms from parton distributions.



## PARTONS Framework

How can we recover the wellknown characterics of the

### Motivation Mass without mass Nucleon structure

Content of GPDs

#### Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building

Computing chain

Examples

Architecture Team

#### Learning

Definition Overlan

Radon transform Covariant extension

Conclusion

Appendix

nucleon from the properties of its colored building blocks?

> Mass? Spin? Charge?





Identification of underlying mechanisms from parton distributions.



## PARTONS Framework

How can we recover the wellknown characterics of the nucleon from the properties of its colored building blocks?

## Motivation Mass without mass Nucleon structure

Content of GPDs

#### Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building

Computing chain

Examples

Architecture Team

#### Learning

Definition

Overlan Radon transform

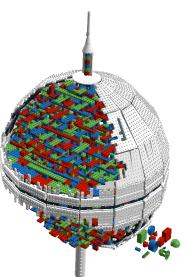
Covariant extension

#### Conclusion

Appendix

Mass? Spin? Charge?

What are the relevant **effec**tive degrees of freedom and effective interaction at large distance?





# Imaging the origin of mass. Identification of underlying mechanisms from parton distributions.



#### PARTONS Framework

# Motivation Mass without mass

Nucleon structure

Content of GPDs

#### Imaging

Experimental access
DVCS Kinematics
Towards 3D images

#### Building

Computing chain

Examples

Architecture Team

## Learning

Definition Overlan

Radon transform

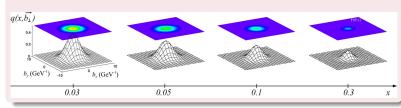
Covariant extension

### Conclusion

Appendix

# Structuring questions for the hadron physics community

- QCD mechanisms behind the origin of mas in the visible universe?
- Cartography of interactions giving its mass to the nucleon?
- Pressure and density profiles of the nucleon as a continuous medium?
- Localization of quarks and gluons inside the nucleon?





# Motivation.

Study nucleon structure to shed new light on nonperturbative QCD.



#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure

Content of GPDs

#### Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples

Architecture

Team

#### Learning Definition

Overlan Radon transform

Covariant extension

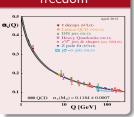
## Conclusion

Appendix

# Perturbative QCD



# Asymptotic freedom



## Nonperturbative QCD



## Perturbative AND nonperturbative QCD at work

- Define **universal** objects describing 3D nucleon structure: **Generalized Parton Distributions** (GPD).
- Relate GPDs to measurements using factorization: Virtual Compton Scattering (DVCS, TCS), Deeply Virtual Meson production (DVMP).
  - Get **experimental knowledge** of nucleon structure.





#### PARTONS Framework

- Correlation of the longitudinal momentum and the transverse position of a parton in a hadron.
- DVCS recognized as the cleanest channel to access GPDs.

#### Motivation Mass without mass

Nucleon structure

Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples

Architecture

Team Learning

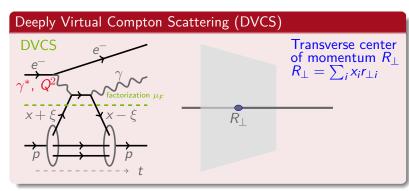
## Definition

Overlan

Radon transform

Covariant extension

## Conclusion







## PARTONS Framework

- Correlation of the longitudinal momentum and the transverse position of a parton in a hadron.
- DVCS recognized as the cleanest channel to access GPDs.

## Motivation Mass without mass

Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples

Architecture

Team

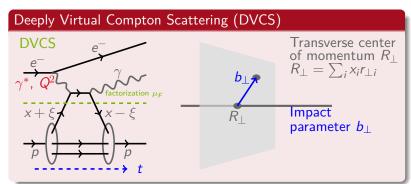
#### Learning Definition

Overlan

Radon transform

Covariant extension

## Conclusion







# PARTONS

- Correlation of the longitudinal momentum and the Framework transverse position of a parton in a hadron.
  - DVCS recognized as the cleanest channel to access GPDs.

## Motivation Mass without mass

Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building

Computing chain

Examples

Architecture Team

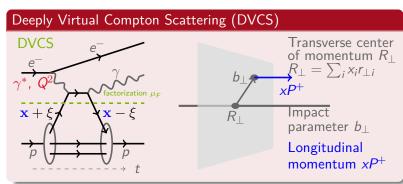
## Learning

Definition Overlan

Radon transform

Covariant extension

## Conclusion







## PARTONS Framework

Motivation

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in a hadron.
- DVCS recognized as the cleanest channel to access GPDs.

Mass without mass
Nucleon structure
Content of GPDs

Imaging

## Experimental access

DVCS Kinematics Towards 3D images

## Building

Computing chain Examples

Architecture Team

## Learning

Definition Overlan

Radon transform

Conclusion

Appendix

# 

■ 24 GPDs  $F^i(x, \xi, t, \mu_F)$  for each parton type i = g, u, d, ... for leading and sub-leading twists.





## PARTONS Framework

Probabilistic interpretation of Fourier transform of  $GPD(x, \xi = 0, t)$  in transverse plane.

$$\rho(\mathbf{x}, \mathbf{b}_{\perp}, \lambda, \lambda_{N}) = \frac{1}{2} \left[ \mathbf{H}(\mathbf{x}, 0, \mathbf{b}_{\perp}^{2}) + \frac{\mathbf{b}_{\perp}^{\mathbf{j}} \epsilon_{\mathbf{j} \mathbf{i}} S_{\perp}^{\mathbf{i}}}{M} \frac{\partial \mathbf{E}}{\partial \mathbf{b}_{\perp}^{2}} (\mathbf{x}, 0, \mathbf{b}_{\perp}^{2}) \right]$$

Content of GPDs Imaging

$$+\lambda\lambda_N ilde{ extbf{H}}( extbf{x},0, extbf{b}_{\perp}^2)\Big]$$

Experimental access **DVCS Kinematics** 

> ■ Notations : quark helicity  $\lambda$ , nucleon longitudinal polarization  $\lambda_N$  and nucleon transverse spin  $S_\perp$ .

Towards 3D images

Burkardt, Phys. Rev. **D62**, 071503 (2000)

## Building

Computing chain Examples

Architecture Team

#### Learning Definition

Overlan Radon transform

Covariant extension

## Conclusion

Appendix

## Can we obtain this picture from exclusive measurements? singlet quarks, gluons Weiss, AIP Conf. Proc. 1149. longitud. 150 (2009) quarks move $x \sim 0.3$ x < 0.01 $x \sim 0.1$



PARTONS

# Anatomy of hadrons. GPDs, 3D hadron imaging, and beyond (3/4).



# Framework

 Most general structure of matrix element of energy momentum tensor between nucleon states:

# Nucleon structure

 $\left\langle N, P + \frac{\Delta}{2} \middle| T^{\mu\nu} \middle| N, P - \frac{\Delta}{2} \right\rangle = \bar{u} \left( P + \frac{\Delta}{2} \right) \middle| A(t) \gamma^{(\mu} P^{\nu)}$ Motivation Mass without mass

## Content of GPDs Imaging

 $+B(t)P^{(\mu}i\sigma^{\nu)\lambda}\frac{\Delta_{\lambda}}{2M}+\frac{C(t)}{M}(\Delta^{\mu}\Delta^{\nu}-\Delta^{2}\eta^{\mu\nu})\right]u\left(P-\frac{\Delta}{2}\right)$ 

#### Experimental access **DVCS Kinematics**

Building Computing chain

Towards 3D images

with  $t = \Delta^2$ .

Examples Architecture Team

Key observation: link between GPDs and gravitational form factors

### Learning Definition

Overlan Radon transform

Covariant extension

Conclusion

$$\int dx x H^q(x, \xi, t) = A^q(t) + 4\xi^2 C^q(t)$$
$$\int dx x E^q(x, \xi, t) = B^q(t) - 4\xi^2 C^q(t)$$

Ji, Phys. Rev. Lett. **78**, 610 (1997) Appendix





## PARTONS Framework

Motivation Mass without mass Spin sum rule:

$$\int dx x (\mathbf{H}^{q}(x,\xi,0) + \mathbf{E}^{q}(x,\xi,0)) = \mathbf{A}^{q}(0) + \mathbf{B}^{q}(0) = 2J^{q}$$

Ji, Phys. Rev. Lett. **78**, 610 (1997)

## Nucleon structure Content of GPDs Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building Computing chain

Examples Architecture Team

#### Learning Definition

Overlan Radon transform Covariant extension

# Conclusion

Appendix

■ **Shear** and **pressure** of a hadron considered as a continuous medium:

$$\langle N | T^{ij}(\vec{r}) | N \rangle N = s(r) \left( \frac{r^i r^j}{\vec{r}^2} - \frac{1}{3} \delta^{ij} \right) + p(r) \delta^{ij}$$

Polyakov and Shuvaev, hep-ph/0207153



# Towards hadron tomography. GPDs as a scalpel-like probe of hadron structure.



## PARTONS Framework

1 Status of 3D imaging: phenomenological relevance of the field.

#### Motivation Mass without mass Nucleon structure Content of GPDs

**Building the tools:** preparing for the high precision era.

#### Imaging Experimental access

**3 Learning from GPDs:** steps towards new GPD models.

**DVCS Kinematics** Towards 3D images

## Building

Computing chain

Examples

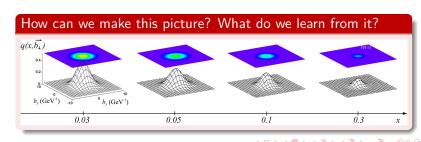
Architecture Team

## Learning

Definition Overlan

Radon transform Covariant extension

Conclusion



Phenomenological status of nucleon 3D imaging



# Exclusive processes of current interest. Factorization and universality.



### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## Imaging

#### Experimental access DVCS Kinematics

Towards 3D images

## Building

Computing chain Examples

Architecture Team

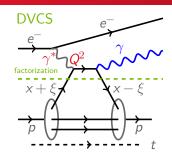
## Learning

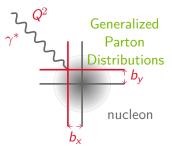
Definition

Overlan Radon transform

Covariant extension

#### Conclusion







# Exclusive processes of current interest. Factorization and universality.



#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## Imaging

# Experimental access DVCS Kinematics

Towards 3D images

#### Building

Computing chain Examples

Architecture

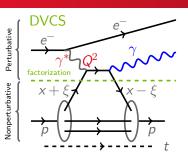
#### Learning Definition

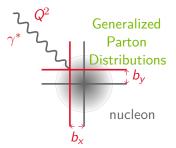
Overlap

Radon transform Covariant extension

Conclusion

Appendix







# Exclusive processes of current interest. Factorization and universality.



#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

#### Imaging Experimental access

## DVCS Kinematics

Towards 3D images

#### Building

Computing chain

Examples Architecture

## Learning

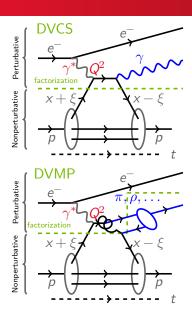
Definition Overlan

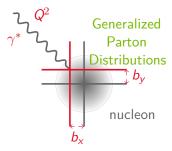
Radon transform

Covariant extension

## Conclusion

Appendix







# Exclusive processes of current interest. Factorization and universality.



## PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

#### Imaging Experimental access

#### Experimen

DVCS Kinematics Towards 3D images

#### Building

#### Computing chain Examples

Architecture

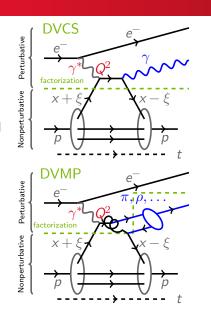
## Learning

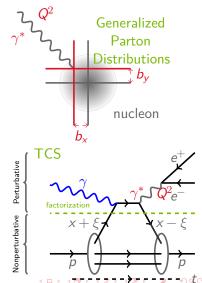
#### Definition Overlan

Radon transform Covariant extension

## Conclusion

Appendix





Ilhabela

11 / 49

H. Moutarde



# Exclusive processes of current interest. Factorization and universality.



### PARTONS Framework

# Motivation

Mass without mass Nucleon structure Content of GPDs

#### Imaging Experimental access

**DVCS** Kinematics Towards 3D images

## Building

#### Computing chain

Examples Architecture

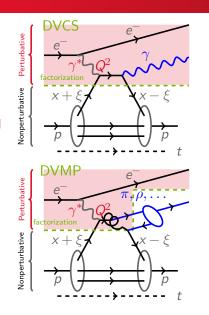
Team

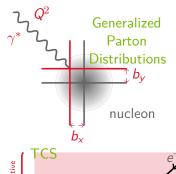
#### Learning Definition

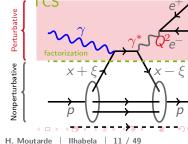
## Overlan

Radon transform Covariant extension

## Conclusion









# Exclusive processes of current interest. Factorization and universality.



## PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## Imaging

## Experimental access

**DVCS** Kinematics Towards 3D images

#### Building

#### Computing chain

Examples Architecture

Team

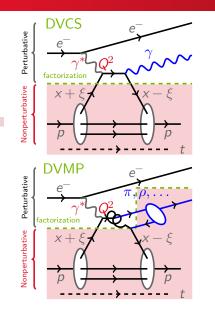
#### Learning Definition

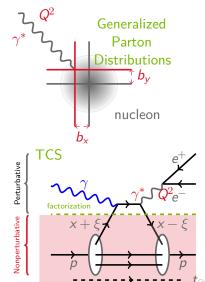
Overlan

Radon transform Covariant extension

## Conclusion

Appendix









### PARTONS Framework

## Motivation

Mass without mass
Nucleon structure
Content of GPDs

#### Imaging

DVCS Kinematics
Towards 3D images

#### Building

Computing chain Examples

Architecture

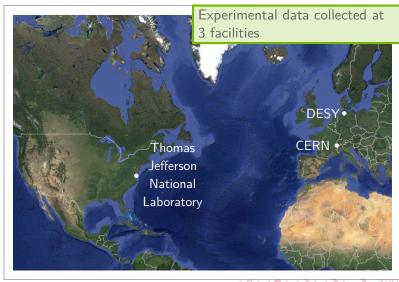
## Learning

Definition
Overlap
Radon transform

Covariant extension

## Conclusion

Appendix







### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

#### Imaging

Experimental access **DVCS Kinematics** 

Towards 3D images

#### Building Computing chain

Examples

Architecture Team

#### Learning

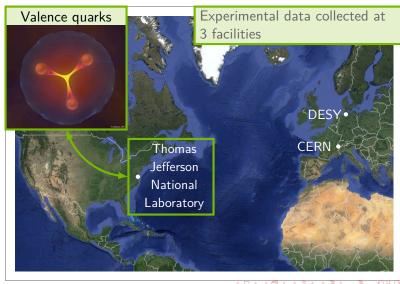
Definition Overlan

Radon transform

Covariant extension

## Conclusion

Appendix







#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples

Architecture Team

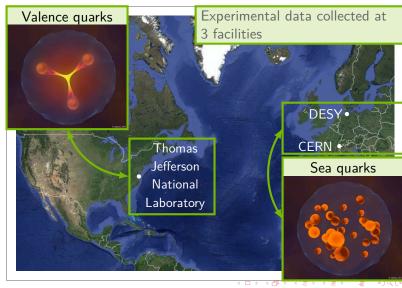
## Learning

Definition Overlan Radon transform

Covariant extension

## Conclusion

Appendix







#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

# Building

Computing chain Examples

Architecture

Team

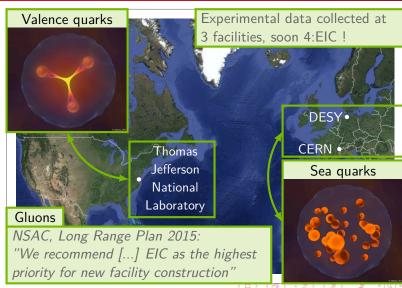
## Learning

Definition Overlan

Radon transform Covariant extension

Conclusion

Appendix





# Imaging the nucleon. How? Extracting GPDs is not enough...Need to extrapolate!

## PARTONS Framework

## Motivation Mass without mass

Nucleon structure Content of GPDs

## **Imaging**

Experimental access **DVCS Kinematics** 

## Towards 3D images

## Building

Computing chain

Examples

Architecture Team

## Learning

#### Definition

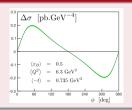
Overlan Radon transform

Covariant extension

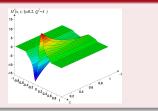
## Conclusion

Appendix

# 1. Experimental data fits



## 2. GPD extraction



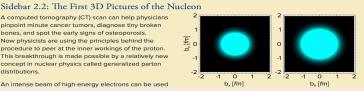
## 3. Nucleon imaging

Images from Guidal et al., Rept. Prog. Phys. 76 (2013) 066202

A computed tomography (CT) scan can help physicians pinpoint minute cancer tumors, diagnose tiny broken bones, and spot the early signs of osteoporosis. Now physicists are using the principles behind the procedure to peer at the inner workings of the proton. This breakthrough is made possible by a relatively new concept in nuclear physics called generalized parton distributions.

An intense beam of high-energy electrons can be used







# Imaging the nucleon. How? Extracting GPDs is not enough...Need to extrapolate!



## PARTONS Framework

- **1 Extract**  $H(x, \xi, t, \mu_F^{ref})$  from experimental data.
- **2** Extrapolate to vanishing skewness  $H(x, 0, t, \mu_{\mathcal{E}}^{ref})$ .
- **3 Extrapolate**  $H(x, 0, t, \mu_F^{ref})$  up to infinite t.
- **Compute** 2D Fourier transform in transverse plane:

$$H(x, b_{\perp}) = \int_0^{+\infty} \frac{\mathrm{d}|\Delta_{\perp}|}{2\pi} |\Delta_{\perp}| J_0(|b_{\perp}||\Delta_{\perp}|) H(x, 0, -\Delta_{\perp}^2)$$

- 5 Propagate uncertainties.
- 6 Control extrapolations with an accuracy matching that of experimental data with sound GPD models.

#### Motivation Mass without mass

Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples

Architecture Team

#### Learning Definition

Overlan Radon transform

Covariant extension

## Conclusion



# The challenge of the high precision era. Higher order and higher twist contributions, and GPD modeling.



#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building Computing chain

Examples Architecture

## Team Learning

Definition

Overlan Radon transform Covariant extension

## Conclusion

- Evaluation of the impact of **higher order** effects.
- Evaluation of the impact of target mass and finite-t corrections.
- Evaluation of the contribution of higher twist GPDs.
- DVMP: sensitivity to **DA models**.
- Extrapolations with **GPD models**.



# Software for the phenomenology of GPDs. Different questions to be answered with the same tools.

Infu CEA - Saciay

#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## Imaging

Experimental access

DVCS Kinematics

#### Towards 3D images

## Building

Computing chain

Examples

Architecture

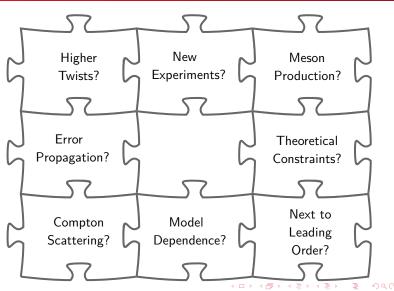
## Learning

#### Definition

Overlap Radon transform

Covariant extension

## Conclusion





# Software for the phenomenology of GPDs. Different questions to be answered with the same tools.

#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples

Architecture

Team

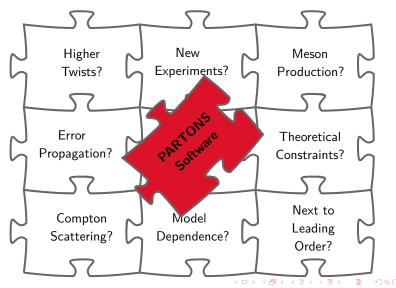
## Learning

#### Definition

Overlan Radon transform

Covariant extension

Conclusion



# Building the tools for high precision: the PARTONS project



PARtonic Tomography Of Nucleon Software



# Computing chain design. Differential studies: physical models and numerical methods.



## PARTONS Framework

Motivation

Mass without mass

Nucleon structure

## Imaging

Experimental access
DVCS Kinematics
Towards 3D images

## Building

#### Computing chain Examples

Architecture

## Learning

Definition
Overlap
Radon transform
Covariant extension

#### Conclusion

Appendix

Experimental data and phenomenology

Computation of amplitudes

First principles and fundamental parameters

Full processes

Small distance contributions

Large distance contributions





## PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## **Imaging**

Experimental access **DVCS Kinematics** Towards 3D images

## Building

#### Computing chain

Examples Architecture Team

#### Learning Definition

Overlan Radon transform Covariant extension

Conclusion

Appendix

# Experimental data and phenomenology

Computation of amplitudes

# First principles and fundamental parameters

Full processes

Small distance contributions

Large distance contributions





#### PARTONS Framework

# Motivation Mass without mass

Nucleon structure Content of GPDs

### Imaging

Experimental access
DVCS Kinematics
Towards 3D images

## Building

#### Computing chain

Examples Architecture Team

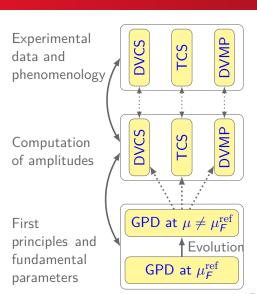
#### Learning Definition

Overlap Radon transform

Covariant extension

## Conclusion

 ${\sf Appendix}$ 







#### PARTONS Framework

# Motivation Mass without mass

Mass without mass
Nucleon structure
Content of GPDs

#### Imaging

Experimental access
DVCS Kinematics
Towards 3D images

### Building

### Computing chain

Examples Architecture Team

## Learning

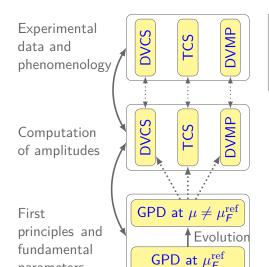
Definition Overlan

Radon transform Covariant extension

parameters

## Conclusion

Appendix



- Many observables.
- Kinematic reach.





#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

#### Computing chain

Examples Architecture Team

#### Learning

Definition Overlan

Radon transform Covariant extension

## Conclusion

Appendix

Experimental data and phenomenology Need for

modularity

Computation of amplitudes

First principles and fundamental parameters

DVMP DVMP GPD at  $\mu \neq \mu_F^{\text{ref}}$ **Evolution** 

GPD at  $\mu_{F}^{\text{ref}}$ 

- Many observables.
- Kinematic reach.
  - Perturbative approximations.
- Physical models.
  - Fits.
- Numerical methods.
- Accuracy and speed.





#### PARTONS Framework

## Motivation

Mass without mass
Nucleon structure
Content of GPDs

#### Imaging

Experimental access
DVCS Kinematics
Towards 3D images

## Building

#### Computing chain Examples

Architecture

# Learning

Overlap Radon transform

Covariant extension

## Conclusion

Appendix

Experimental data and phenomenology Need for

modularity

Computation of amplitudes

First principles and fundamental parameters

DVMP DVMP GPD at  $\mu \neq \mu_F^{\text{ref}}$ 

GPD at  $\mu_F^{\rm ref}$ 

- Many observables.
- Kinematic reach.
  - Perturbative approximations.
- Physical models.
  - Fits.
- Numerical methods.
- Accuracy and speed.

Evolution



#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

#### Computing chain Examples

Architecture Team

#### Learning Definition

Overlan Radon transform

Covariant extension

## Conclusion

Appendix

Experimental data and phenomenology Need for

modularity

Computation of amplitudes

First principles and fundamental parameters

DVMP DVCS DVCS GPD at  $\mu \neq \mu_F^{\text{ref}}$ 

GPD at  $\mu_F^{\rm ref}$ 

- Many observables.
- Kinematic reach.
- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

Evolution





#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

#### Computing chain Examples

Architecture Team

#### Learning

Definition Overlan

Radon transform Covariant extension

## Conclusion

Appendix

Experimental data and phenomenology Need for modularity

Computation of amplitudes

First principles and fundamental parameters

DVMP DVMP GPD at  $\mu \neq \mu_F^{\text{ref}}$ 

GPD at  $\mu_F^{\rm ref}$ 

- Many observables.
- Kinematic reach.
- Perturbative approximations.
- Physical models.
  - Fits.
- Numerical methods.
- Accuracy and speed.

Evolution



#### PARTONS Framework

#### Motivation Mass without mass

Nucleon structure Content of GPDs

#### Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

#### Computing chain Examples

Architecture Team

#### Learning

Definition Overlan

Radon transform Covariant extension

## Conclusion

Appendix

Experimental data and phenomenology Need for

modularity

Computation of amplitudes

First principles and fundamental parameters

DVMP DVCS

> GPD at  $\mu \neq \mu_F^{\text{ref}}$ Evolution GPD at  $\mu_{F}^{\text{ref}}$

- Many observables.
- Kinematic reach.
- Perturbative approximations.
- Physical models.
  - Fits.
- Numerical methods.
- Accuracy and speed.



# Towards the first release. Currently: tests, benchmarking, documentation, tutorials.



#### PARTONS Framework

# Motivation Mass without mass Nucleon structure

Content of GPDs

#### Imaging

Experimental access
DVCS Kinematics
Towards 3D images

## Building

#### Computing chain

Examples Architecture Team

## Learning

#### Definition

Overlap Radon transform

Covariant extension

#### Conclusion

Appendix

# 3 stages:

- Design.
- 2 Integration and validation.
- 3 Benchmarking and production.
- Flexible software architecture.

B. Berthou et al., PARTONS: a computing platform for the phenomenology of Generalized Parton Distributions arXiv:1512.06174, to appear in Eur. Phys. J. C.

- 1 new physical development = 1 new module.
- *Aggregate* **knowledge** and **know-how**:
  - Models
  - Measurements
    - Numerical techniques
  - Validation
- What can be automated will be automated.



# Towards the first release. Debugging and flexibility: the path to controlled results.

Infu CEA - Saciay

#### PARTONS Framework

## Motivation

Mass without mass
Nucleon structure

## Imaging

Experimental access
DVCS Kinematics
Towards 3D images

## Building

#### Computing chain

Examples Architecture

## Learning

#### Definition Overlan

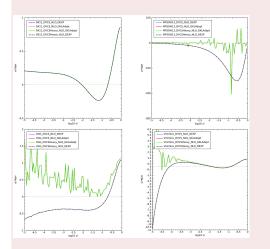
Radon transform

Covariant extension

## Conclusion

Appendix

# Coefficient functions, from EIC to Jefferson Lab kinematics



- Work in progress!
- Validation and preparation of nonregression tools.
- Flexibility at work: physical models and numerical techniques.
- $\simeq 2 \times 10^4 \; \mathrm{GPD}$  computed in  $\lesssim 1'$ .



# GPD computing made simple.

Each line of code corresponds to a physical hypothesis.



```
gpdExample()
 PARTONS
 Framework
             1 // Lots of includes
             2 #include <src/Partons.h>
Motivation
Mass without mass
             4 // Retrieve GPD service
Nucleon structure
             5 GPDService* pGPDService = Partons::getInstance()->getServiceObjectRegistry
Content of GPDs
                ()—>getGPDService();
Imaging
             6 // Load GPD module with the BaseModuleFactory
Experimental access
             7 GPDModule* pGK11Model = Partons::getInstance()->getModuleObjectFactory
DVCS Kinematics
Towards 3D images
                ()—>newGPDModule(GK11Model::classId);
             8 // Create a GPDKinematic(x, xi, t, MuF, MuR) to compute
Building
Computing chain
             9 GPDKinematic gpdKinematic(0.1, 0.00050025, -0.3, 8., 8.);
Examples
             10 // Compute data and store results
Architecture
             11 GPDResult gpdResult = pGPDService->
Team
                computeGPDModelRestrictedByGPDType(gpdKinematic, pGK11Model,
Learning
                GPDType::ALL);
Definition
             12 // Print results
Overlan
Radon transform
             13 std::cout << gpdResult.toString() << std::endl;
Covariant extension
             14
Conclusion
                delete pGK11Model:
                pGK11Model = 0;
Appendix
```



PARTONS

Framework

# GPD computing automated.



Each line of code corresponds to a physical hypothesis.

```
<?xml version="1.0" encoding="UTF-8" standalone="yes" ?>
             2 <scenario id="01" date="" description="Example_::ucomputation_of_one_GPD
Motivation
               _model_(GK11)_without_evolution">
Mass without mass
                        <!-- Select type of computation -->
             3
Nucleon structure
Content of GPDs
                        <task service="GPDService" method="computeGPDModel" >
             4
                                <!-- Specify kinematics -->
Imaging
                                <GPDKinematic>
Experimental access
DVCS Kinematics
                                        <param name="x" value="0.1" />
Towards 3D images
                                        <param name="xi" value="1.00050025" />
Building
                                        <param name="t" value="-0.3" />
             g
Computing chain
                                        <param name="MuF2" value="8" />
Examples
                                        <param name="MuR2" value="8" />
Architecture
Team
                                </GPDKinematic>
            12
                                <!-- Choose GPD model and set parameters -->
Learning
            13
Definition
                                <GPDModule>
            14
Overlan
                                        <param name="id" value="GK11Model" />
            15
Radon transform
                                </GPDModule>
            16
Covariant extension
                        </task>
            17
Conclusion
               </scenario>
Appendix
```

computeOneGPD.xml

H. Moutarde | Ilhabela

21 / 49



# GPD computing automated.



Each line of code corresponds to a physical hypothesis.

PARTONS Framework		computeOneGPI	$H^u = 0.822557$	-
	1	<pre><?xml version="1.0" encoding="UTF-8" stand</pre></pre>	$H^{u(+)} = 0.165636$	
Motivation  Mass without mass	2	<pre><scenario date="" description="Exam&lt;/td&gt;&lt;td&gt;&lt;math&gt;H^{u(-)} = 1.47948&lt;/math&gt;&lt;/td&gt;&lt;td&gt;&lt;/td&gt;&lt;/tr&gt;&lt;tr&gt;&lt;td&gt;Nucleon structure&lt;/td&gt;&lt;td&gt;3&lt;/td&gt;&lt;td&gt;&lt;&lt;/math&gt;!—— Select type of computation ——&lt;math&gt;&gt;&lt;/td&gt;&lt;td&gt;&lt;/td&gt;&lt;td&gt;&lt;/td&gt;&lt;/tr&gt;&lt;tr&gt;&lt;td&gt;Content of GPDs&lt;/td&gt;&lt;td&gt;4&lt;/td&gt;&lt;td&gt;&lt;pre&gt;&lt;task service=" gpdservice"="" id="01" method="&lt;/pre"></scenario></pre>	$H^d = 0.421431$	
Imaging	5	<!—— Specify kinematics —— $>$		
Experimental access	6	<gpdkinematic></gpdkinematic>	$H^{d(+)} = 0.0805182$	
DVCS Kinematics Towards 3D images	7	<param name="x" td="" valu<=""/> <td><math>H^{d(-)} = 0.762344</math></td> <td></td>	$H^{d(-)} = 0.762344$	
_	8	<pre><param name="xi" pre="" va<=""/></pre>	11 \ \ \ = 0.702344	
Building	9	<pre><param name="t" pre="" valu<=""/></pre>		
Computing chain  Examples	10	<pre><param <="" name="MuF2" pre=""/></pre>	$H^s = 0.00883408$	
Architecture	11	<pre><param <="" name="MuR2" pre=""/></pre>		
Team	12		$H^{s(+)} = 0.0176682$	
Learning	13	Choose GPD model and</td <td><math>H^{s(-)} = 0</math></td> <td></td>	$H^{s(-)} = 0$	
Definition	14	<gpdmodule></gpdmodule>	- 0	
Overlap Radon transform	15	<param name="id" td="" va<=""/> <td></td> <td></td>		
Covariant extension	16		$H^g = 0.385611$	
Conclusion	17 18		and $E$ , $\tilde{H}$ , $\tilde{E}$ ,	
Appendix	-	7		-



PARTONS

Framework

# CFF computing automated.



Each line of code corresponds to a physical hypothesis.

1 <2vml version="1 0" encoding="IIITE-8" standalone="weg" 2

Framework	1	<pre><!--xml version="1.0" encoding="UTF-8" standalone="yes" !--></pre>
	2	$<$ scenario id="03" date="" description="Example $_{\sqcup}$ : $_{\sqcup}$ computation $_{\sqcup}$ of $_{\sqcup}$ one $_{\sqcup}$
Motivation		${\tt convol}_{\sqcup}{\tt coeff}_{\sqcup}{\tt function}_{\sqcup}{\tt model}_{\sqcup}({\tt DVCSCFF})_{\sqcup}{\tt with}_{\sqcup}{\tt GPD}_{\sqcup}{\tt model}_{\sqcup}({\tt GK11})$ " $>$
Mass without mass	3	<pre><task <="" method="&lt;/pre&gt;&lt;/td&gt;&lt;/tr&gt;&lt;tr&gt;&lt;td&gt;Nucleon structure&lt;/td&gt;&lt;td&gt;&lt;/td&gt;&lt;td&gt;computeWithGPDModel" service="DVCSConvolCoeffFunctionService" td=""></task></pre>
Content of GPDs	4	<pre>- <dvcsconvolcoefffunctionkinematic></dvcsconvolcoefffunctionkinematic></pre>
Imaging	5	<pre><param name="xi" value="0.5"/></pre>
Experimental access	6	<pre><param name="t" value="-0.1346"/></pre>
DVCS Kinematics Towards 3D images	7	<pre>. <pre><param name="Q2" value="1.5557"/></pre></pre>
_	8	<pre><param name="MuF2" value="4"/></pre>
Building Computing chain	9	<pre><param name="MuR2" value="4"/></pre>
Examples	10	$$
Architecture	11	<gpdmodule></gpdmodule>
Team	12	<pre><param name="id" value="GK11Model"/></pre>
Learning	13	
Definition	14	<dvcsconvolcoefffunctionmodule></dvcsconvolcoefffunctionmodule>
Overlap Radon transform	15	<pre><param name="id" value="DVCSCFFModel"/></pre>
Covariant extension	16	<pre><param name="qcd_order_type" value="L0"/></pre>
Conclusion	17	
Conclusion	18	
Appendix	19	
	19	√3certailo /
		H. Moutarde   IIhabela   22 / 49

computeOneCFF.xml



# CFF computing automated.



Each line of code corresponds to a physical hypothesis.

```
computeOneCFF.xml
 PARTONS
 Framework
             1 <?xml version="1.0" encoding="UTF-8" standalone="yes" ?>
             2 <scenario id="03" date="" description="Example_:..computation_of_lone_..
                convolucoeffufunction_modelu(DVCSCFF)uwithuGPDumodelu(GK11)">
Motivation
                        <task service="DVCSConvolCoeffFunctionService" method="</pre>
Mass without mass
             3
Nucleon structure
                computeWithGPDModel"
Content of GPDs
                                <DVCSConvolCoeffFunctionKinematic>
Imaging
                                          <param name="xi" value="0.5" />
Experimental access
                                          <param name="t" value="-0.1346" />
DVCS Kinematics
                                          <param name="Q2" value="1.5557" />
Towards 3D images
                                          <param name="MuF2" value="4" />
Building
                                          <param name="MuR2" value="4" />
Computing chain
                                </DVCSConvolCoeffFunctionKinematic>
Examples
Architecture
                                <GPDModule>
Team
                                          <param name="id" value="GK11Model" />
            12
Learning
                                </GPDModule
            13
Definition
                                                    \mathcal{H} = 1.47722 + 1.76698 i
                                < DVCSConvolC
            14
Overlan
            15
                                          < para
                                                    \mathcal{E} = 0.12279 + 0.512312 i
Radon transform
Covariant extension
            16
                                          < para
                                                    \mathcal{H} = 1.54911 + 0.953728 i
                                </DVCSConvol
Conclusion
            17
                        </task>
            18
                                                    \widetilde{\mathcal{E}} = 18.8776 + 3.75275 i
Appendix
                </scenario>
```



Learning

D-6-141---

14

# Observable computing automated.



	_	lafa	
		CEA - Saciay	
PARTONS computeManyKinematicsOneModel.xml			
Framework	1	<pre><scenario date="" description="Compute_Alu_E1-DVCS_kinematics" id="5"></scenario></pre>	
	2	<pre><task method="&lt;/pre&gt;&lt;/td&gt;&lt;/tr&gt;&lt;tr&gt;&lt;td&gt;Motivation&lt;/td&gt;&lt;td&gt;&lt;/td&gt;&lt;td&gt;computeManyKinematicOneModel" service="ObservableService" storeindb="0"></task></pre>	
Mass without mass	3	<observablekinematic></observablekinematic>	
Nucleon structure	4	<pre><param <="" name="file" pre=""/></pre>	
Content of GPDs	5	. value="/home/debian/workspace/PARTONS/data/E1DVCS.dat" $/>$	
Imaging	6		

Content of GPDs		param name 1110
	5	value="/home/debian/workspace/PARTONS/data/E1DVCS.dat"/>
Imaging	6	
Experimental access	7	<observable></observable>
DVCS Kinematics Towards 3D images	8	<param name="id" value="Alu"/>
	9	

Towards 5D images		< / / / / / / / / / / / / / / / / / / /
Distriction in	9	
Building	10	<dvcsmodule></dvcsmodule>
Computing chain	10	
Examples	11	<param name="id" value="BMJ2012Model"/>
Architecture	12	$<$ param name $=$ "beam_energy" value $=$ "5.75" $/>$
Team	13	

<DVCSConvolCoeffFunctionModule>

Delilition	15	<pre><param name="id" value="DVCSCFFMode1"/></pre>
Overlap Radon transform	16	<pre><param name="qcd_order_type" value="LO"/></pre>
Covariant extension	17	</DVCSConvolCoeffFunctionModule>
Conclusion	18	<gpdmodule></gpdmodule>
		Consider the second of the sec

	21		H. Moutarde │ Ilhabela │ 23 / 49
penuix	20		
pendix	19		" value="GK11Model" />



# Modularity. Inheritance, standardized inputs and outputs.



#### PARTONS Framework

#### Motivation Mass without mass

Nucleon structure Content of GPDs

## **Imaging**

Experimental access **DVCS Kinematics** Towards 3D images

### Building

Computing chain Examples Architecture

Team

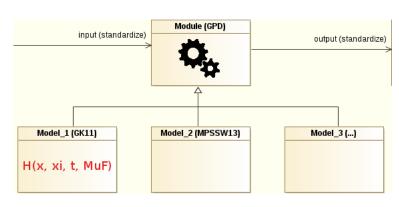
## Learning

Definition Overlan

Radon transform Covariant extension

Conclusion

Appendix



- Steps of logic sequence in parent class.
- Model description and related mathematical methods in daughter class.



# Modularity and automation.

Parse XML file, compute and store result in database.



#### PARTONS Framework

#### Motivation Mass without mass

Nucleon structure Content of GPDs

## **Imaging**

Experimental access **DVCS Kinematics** Towards 3D images

#### Building

Computing chain Examples

Architecture Team

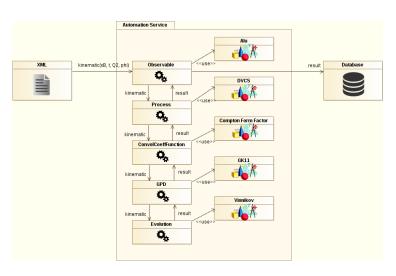
## Learning

Definition Overlan

Radon transform Covariant extension

Conclusion

## Appendix





# Modularity and layer structure. Modifying one layer does not affect the other layers.



#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

#### Imaging Experimental access

DVCS Kinematics
Towards 3D images

#### Building

Computing chain Examples

Architecture Team

## Learning

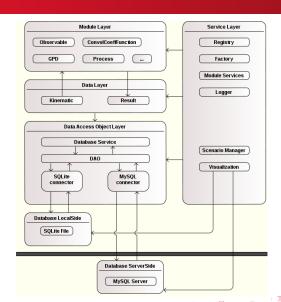
Definition Overlan

Radon transform

Covariant extension

## Conclusion

 ${\bf Appendix}$ 





# Pre-PARTONS times.

Irfu

æ

saclav

Pivotal year

for GPDs

2011 situation

GPDs and DVCS

Leading twist, leading order

Selected data

analysis

methods Universality

Key results

orientations

COMPASS-II

JLab's 12 GeV upgrade

Spin observables on an EIC The PROPHET

Conclusions

Status of GPD

First mention of the PARTONS project in a conference.



#### PARTONS Framework

# Motivation Mass without mass

Nucleon structure Content of GPDs

## Imaging

Experimental access
DVCS Kinematics
Towards 3D images

## Building

Computing chain Examples

Architecture Team

#### Learning Definition

Overlan

Radon transform Covariant extension

Conclusion

Appendix

## PROPHET.

Platform for Representing the Organization of Partons inside Hadrons and Experimental Tomographies.

- Omprehensive database of experimental results.
- Omprehensive database of theoretical predictions.
- Fitting engine.
- Propagation of statistic and systematic uncertainties.
- Visualizing software to compare experimental results and model expectations.
- Connection to experimental set-up descriptions to design new experiments.
- Interactive website providing free access to model and experimental values.

H. MOUTARDE (Irfu/SPhN, CEA-Saclay)



イロト イボト イミト イモト



## 1 multidisciplinary team over 5 countries Theorists, experimentalists, 1 mathematician + 1 software engineer



#### PARTONS Framework

## Motivation

Mass without mass Nucleon structure Content of GPDs

## **Imaging**

Experimental access DVCS Kinematics Towards 3D images

#### Building

Computing chain Examples

Architecture Team

#### Learning

## Definition Overlan

Radon transform

Covariant extension

## Conclusion

Appendix



Ilhabela

Learning on the strong interaction from GPD models





## PARTONS Framework

Mass without mass Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building Computing chain

Examples Architecture Team

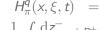
### Learning

#### Definition Overlan

Radon transform Covariant extension

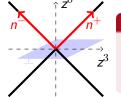
## Conclusion

## Appendix



$$\frac{1}{2} \int \frac{\mathrm{d}z^{-}}{2\pi} e^{ixP^{+}z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q} \left( -\frac{z}{2} \right) \gamma^{+} q \left( \frac{z}{2} \right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{z^{+}=0}$$

with 
$$t=\Delta^2$$
 and  $\xi=-\Delta^+/(2P^+).$ 



# References

Müller et al., Fortschr. Phys. 42, 101 (1994)

Ji, Phys. Rev. Lett. 78, 610 (1997) Radyushkin, Phys. Lett. **B380**, 417 (1996)

PDF forward limit

$$H^q(x,0,0) = q(x)$$





## PARTONS Framework

Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building Computing chain

Examples Architecture Team

## Learning

#### Definition Overlan Radon transform

Covariant extension

## Conclusion

Appendix

 $H_{\pi}^{q}(x,\xi,t) =$ 

 $\frac{1}{2} \int \frac{\mathrm{d}z^{-}}{2\pi} e^{ixP^{+}z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q} \left( -\frac{z}{2} \right) \gamma^{+} q \left( \frac{z}{2} \right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{z^{+}=0}$ 

with  $t = \Delta^2$  and  $\xi = -\Delta^+/(2P^+)$ .

References Müller et al., Fortschr. Phys. 42, 101 (1994)

Ji, Phys. Rev. Lett. 78, 610 (1997)

Radyushkin, Phys. Lett. **B380**, 417 (1996)

PDF forward limit

Form factor sum rule

 $dx H^{q}(x, \xi, t) = F_{1}^{q}(t)$ 





#### PARTONS Framework

$$H_{\pi}^{q}(x,\xi,t) = \frac{1}{2} \int \frac{\mathrm{d}z^{-}}{2\pi} e^{ixP^{+}z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q} \left( -\frac{z}{2} \right) \gamma^{+} q \left( \frac{z}{2} \right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{z^{+}=0}$$

#### Motivation Mass without mass

Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples Architecture

## Team Learning

#### Definition Overlan

Radon transform Covariant extension

## Conclusion

Appendix

with 
$$t = \Delta^2$$
 and  $\xi = -\Delta^+/(2P^+)$ .

References

Müller et al., F

## References

Müller et al., Fortschr. Phys. 42, 101 (1994) Ji, Phys. Rev. Lett. 78, 610 (1997) Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF forward limit
- Form factor sum rule
- $H^q$  is an **even function** of  $\xi$  from time-reversal invariance.





## PARTONS Framework

#### Mass without mass Nucleon structure

Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building Computing chain

Examples Architecture

## Team Learning

#### Definition Overlan

Radon transform Covariant extension

## Conclusion

Appendix

# $H_{\pi}^{q}(x,\xi,t) =$ $\frac{1}{2} \int \frac{\mathrm{d}z^{-}}{2\pi} e^{ixP^{+}z^{-}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q} \left( -\frac{z}{2} \right) \gamma^{+} q \left( \frac{z}{2} \right) \middle| \pi, P - \frac{\Delta}{2} \right\rangle_{z^{+}=0}$

with 
$$t=\Delta^2$$
 and  $\xi=-\Delta^+/(2P^+)$ .

Müller et al., Fortschr. Phys. 42, 101 (1994) Ji, Phys. Rev. Lett. 78, 610 (1997) Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF forward limit
- Form factor sum rule
- $H^q$  is an **even function** of  $\xi$  from time-reversal invariance.

H. Moutarde

 $H^q$  is **real** from hermiticity and time-reversal invariance.

Ilhabela





#### PARTONS Framework

# **Polynomiality**

## Motivation

Mass without mass Nucleon structure Content of GPDs

## **Imaging**

Experimental access **DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples

Architecture Team

## Learning

#### Definition Overlan

Radon transform Covariant extension

## Conclusion

## Appendix



 $\int_{-1}^{+1} dx x^n H^q(x, \xi, t) = \text{polynomial in } \xi$ 





#### PARTONS Framework

Motivation Mass without mass Nucleon structure

**Polynomiality** 

# Lorentz covariance

#### Content of GPDs

**Imaging** Experimental access **DVCS** Kinematics Towards 3D images

## Building

Computing chain

Examples Architecture

Team

## Learning

#### Definition Overlap

Radon transform Covariant extension

#### Conclusion

## Appendix





#### PARTONS Framework

Motivation

Mass without mass Nucleon structure

**Polynomiality** 

**Positivity** 

## Lorentz covariance

## Content of GPDs **Imaging**

Experimental access **DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples Architecture

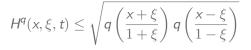
## Team Learning

#### Definition Overlan

Radon transform Covariant extension

#### Conclusion

## Appendix







## PARTONS Framework

**Polynomiality** 

I orentz covariance

## Motivation

Mass without mass Nucleon structure

# **Positivity**

#### **Imaging** Experimental access

**DVCS Kinematics** Towards 3D images

Content of GPDs

#### Building

Computing chain Examples Architecture

## Team Learning

#### Definition Overlan

Radon transform Covariant extension

#### Conclusion

## Appendix



Positivity of Hilbert space norm





#### PARTONS Framework

**Polynomiality** 

 $\blacksquare$   $H^q$  has support  $x \in [-1, +1]$ .

Lorentz covariance

## Motivation

**Positivity** Mass without mass Nucleon structure

Positivity of Hilbert space norm

## Content of GPDs Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building

Computing chain Examples Architecture Team

## Learning

#### Definition Overlan

Radon transform Covariant extension

#### Conclusion

## Appendix

Ilhabela





#### PARTONS Framework

**Polynomiality** 

Lorentz covariance

## Motivation

**Positivity** Mass without mass Nucleon structure

Positivity of Hilbert space norm

 $\blacksquare$   $H^q$  has support  $x \in [-1, +1]$ .

Relativistic quantum mechanics

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

Content of GPDs

#### Building

Computing chain Examples

Architecture Team

## Learning

#### Definition Overlan

Radon transform Covariant extension

#### Conclusion

## Appendix

Ilhabela





#### PARTONS Framework

**Polynomiality** 

Lorentz covariance

**Positivity** 

Positivity of Hilbert space norm

 $\blacksquare$   $H^q$  has support  $x \in [-1, +1]$ .

Relativistic quantum mechanics

**Soft pion theorem** (pion target)

$$H^{q}(x, \xi = 1, t = 0) = \frac{1}{2} \phi_{\pi}^{q} \left( \frac{1+x}{2} \right)$$

#### Motivation Mass without mass

Nucleon structure Content of GPDs

## Imaging Experimental access

**DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples Architecture Team

#### Learning Definition

Overlan Radon transform Covariant extension

#### Conclusion

Appendix





#### PARTONS Framework

**Polynomiality** 

Lorentz covariance

## Motivation

Mass without mass Nucleon structure Content of GPDs

## **Imaging**

Experimental access **DVCS Kinematics** Towards 3D images

### Building

Computing chain Examples Architecture Team

#### Learning Definition

Overlan Radon transform Covariant extension

#### Conclusion

Appendix

# **Positivity**

Positivity of Hilbert space norm

 $\blacksquare$   $H^q$  has support  $x \in [-1, +1]$ .

Relativistic quantum mechanics

■ **Soft pion theorem** (pion target)

Dynamical chiral symmetry breaking





#### PARTONS Framework

**Polynomiality** 

Lorentz covariance

#### Motivation Mass without mass

Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS Kinematics** Towards 3D images

## Building

Computing chain Examples Architecture Team

## Learning

#### Definition Overlan

Radon transform Covariant extension

#### Conclusion

Appendix

**Positivity** 

Positivity of Hilbert space norm

 $\blacksquare$   $H^q$  has support  $x \in [-1, +1]$ .

Relativistic quantum mechanics

**Soft pion theorem** (pion target)

Dynamical chiral symmetry breaking

## How can we implement a priori these theoretical constraints?

- There is no known GPD parameterization relying only on first principles.
- In the following, focus on **polynomiality** and **positivity**.



## Double Distributions. Relation to Generalized Parton Distributions.



### PARTONS Framework

#### Motivation Mass without mass

Nucleon structure Content of GPDs

#### Imaging Experimental access

**DVCS Kinematics** Towards 3D images

#### Building Computing chain

Examples Architecture Team

#### Learning Definition

## Overlan

Radon transform Covariant extension

## Conclusion

Appendix

# Representation of GPD:

$$H^{q}(x,\xi,t) = \int_{\Omega_{DD}} d\beta d\alpha \, \delta(x-\beta-\alpha\xi) \big( F^{q}(\beta,\alpha,t) + \xi G^{q}(\beta,\alpha,t) \big)$$

- Support property:  $x \in [-1, +1]$ .
- Discrete symmetries:  $F^q$  is  $\alpha$ -even and  $G^q$  is  $\alpha$ -odd.
- **Gauge**: any representation  $(F^q, G^q)$  can be recast in one representation with a single DD  $f^q$ :

$$H^{q}(x,\xi,t) = x \int_{\Omega_{\rm DD}} d\beta d\alpha \, f_{\rm BMKS}^{q}(\beta,\alpha,t) \delta(x-\beta-\alpha\xi)$$

Belitsky et al., Phys. Rev. **D64**, 116002 (2001)

$$H^{q}(x,\xi,t) = (1-x) \int_{\Omega_{\mathrm{DD}}} \mathrm{d}\beta \mathrm{d}\alpha \, f_{\mathrm{P}}^{q}(\beta,\alpha,t) \delta(x-\beta-\alpha\xi)$$

Pobylitsa, Phys. Rev. **D67**, 034009 (2003) Müller, Few Body Syst. **55**, 317 (2014)



## Double Distributions. Lorentz covariance by example.



#### PARTONS Framework

#### Motivation Mass without mass

Nucleon structure Content of GPDs

#### Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building

Computing chain Examples Architecture Team

#### Learning

#### Definition

Overlan Radon transform Covariant extension

#### Conclusion

### Appendix

■ Choose  $F^q(\beta, \alpha) = 3\beta\theta(\beta)$  ad  $G^q(\beta, \alpha) = 3\alpha\theta(\beta)$ :

$$H^{q}(x,\xi) = 3x \int_{\Omega} d\beta d\alpha \, \delta(x - \beta - \alpha \xi)$$

Simple analytic expressions for the GPD:

$$H(x,\xi) = \frac{6x(1-x)}{1-\xi^2} \text{ if } 0 < |\xi| < x < 1,$$

$$H(x,\xi) = \frac{3x(x+|\xi|)}{|\xi|(1+|\xi|)} \text{ if } -|\xi| < x < |\xi| < 1.$$



## Double Distributions. Lorentz covariance by example.



Р	Α	R.	T	IC	VS	,
F	ra	m	e١	ΝC	ork	

Compute first Mellin moments.

PARTUNS								
Framework	n	$\int_{-\xi}^{+\xi} \mathrm{d}x  x^n H(x,\xi)$	$\int_{+\xi}^{+1} \mathrm{d}x x^n H(x,\xi)$	$\int_{-\xi}^{+1} \mathrm{d}x  x^n H(x,\xi)$				
Motivation  Mass without mass Nucleon structure Content of GPDs	0	$\frac{1+\xi-2\xi^2}{1+\xi}$	$\frac{2\xi^2}{1+\xi}$	1				
Imaging Experimental access DVCS Kinematics Towards 3D images	1	$\frac{1+\xi+\xi^2-3\xi^3}{2(1+\xi)}$	$\frac{2\xi^3}{1+\xi}$	$\frac{1+\xi^2}{2}$				
Building Computing chain Examples Architecture	2	$\frac{3(1-\xi)(1+2\xi+3\xi^2+4\xi^3)}{10(1+\xi)}$	$\frac{6\xi^4}{5(1+\xi)}$	$\frac{3(1+\xi^2)}{10}$				
Team Learning Definition	3	$\frac{1+\xi+\xi^2+\xi^3+\xi^4-5\xi^5}{5(1+\xi)}$	$\frac{6\xi^5}{5(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{5}$				
Overlap Radon transform Covariant extension	4	$\frac{1+\xi+\xi^2+\xi^3+\xi^4+\xi^5-6\xi^6}{7(1+\xi)}$	$\frac{6\xi^6}{7(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{7}$				

## Radon transform Conclusion

Expressions get more complicated as *n* increases... But they always yield polynomials!

H. Moutarde

Ilhabela

33 / 49



## Overlap representation.



#### PARTONS Framework

■ Decompose an hadronic state  $|H; P, \lambda\rangle$  in a Fock basis:

## Motivation

 $H; P, \lambda \rangle = \sum \int [\mathrm{d}x \mathrm{d}\mathbf{k}_{\perp}]_{N} \psi_{N}^{(\beta,\lambda)}(x_{1}, \mathbf{k}_{\perp 1}, \dots, x_{N}, \mathbf{k}_{\perp N}) | \beta, k_{1}, \dots, k_{N} \rangle$ 

A first-principle connection with Light Front Wave Functions.

#### Nucleon structure Content of GPDs Imaging

Mass without mass

Derive an expression for the pion GPD in the DGLAP region  $\xi < x < 1$ :

#### Experimental access **DVCS Kinematics** Towards 3D images Building Computing chain

 $H^{q}(x,\xi,t) \propto \sum_{\lambda} \int [d\bar{x}d\bar{k}_{\perp}]_{N} \delta_{j,q} \delta(x-\bar{x}_{j}) (\psi_{N}^{(\beta,\lambda)})^{*} (\hat{x}',\hat{k}'_{\perp}) \psi_{N}^{(\beta,\lambda)} (\tilde{x},\tilde{k}_{\perp})$ 

#### Examples Architecture Team Learning Definition

with  $\tilde{x}, \tilde{k}_{\perp}$  (resp.  $\hat{x}', \tilde{k}'_{\perp}$ ) generically denoting incoming (resp. outgoing) parton kinematics.

#### Overlan Radon transform Covariant extension

Diehl et al., Nucl. Phys. **B596**, 33 (2001) ■ Similar expression in the ERBL region  $-\xi \le x \le \xi$ , but with overlap of N- and (N+2)-body LFWFs.

#### Conclusion Appendix



# Overlap representation. Advantages and drawbacks.



#### PARTONS Framework

Motivation

Mass without mass

Nucleon structure

- Physical picture.
- Positivity relations are fulfilled **by construction**.
- Implementation of **symmetries of** *N***-body problems**.

## What is not obvious anymore

What is *not* obvious to see from the wave function representation is however the **continuity of GPDs at**  $x=\pm\xi$  and the **polynomiality** condition. In these cases both the DGLAP and the ERBL regions must cooperate to lead to the required properties, and this implies **nontrivial relations between the wave functions** for the different Fock states relevant in the two regions. An *ad hoc* Ansatz for the wave functions would **almost certainly lead** to GPDs that **violate the above requirements**.

## Content of GPDs

Experimental access
DVCS Kinematics
Towards 3D images

#### Building

Computing chain Examples Architecture

#### Learning

Overlap

Radon transform Covariant extension

#### Conclusion

Appendix

Diehl, Phys. Rept. **388**, 41 (2003)



## The Radon transform. Definition and properties.



#### PARTONS Framework

### Motivation

Mass without mass Nucleon structure Content of GPDs

#### Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building

Computing chain Examples

Architecture Team

#### Learning

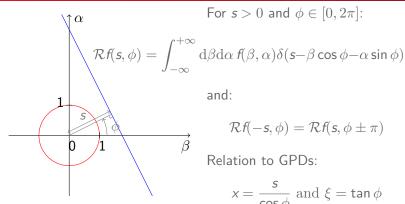
Definition Overlan

Radon transform

Covariant extension

#### Conclusion

Appendix



For s > 0 and  $\phi \in [0, 2\pi]$ :

and:

 $\mathcal{R}f(-s,\phi) = \mathcal{R}f(s,\phi \pm \pi)$ 

Relation to GPDs:

$$x = \frac{s}{\cos \phi}$$
 and  $\xi = \tan \phi$ 

## Relation between GPD and DD in Belistky et al. gauge

$$\frac{\sqrt{1+\xi^2}}{x}H(x,\xi)=\mathcal{R}f_{\mathrm{BMKS}}(s,\phi)\;,$$



## The Radon transform. Definition and properties.



#### PARTONS Framework

#### Motivation

Mass without mass Nucleon structure Content of GPDs

#### Imaging

Experimental access **DVCS Kinematics** Towards 3D images

Building Computing chain Examples Architecture Team

#### Learning

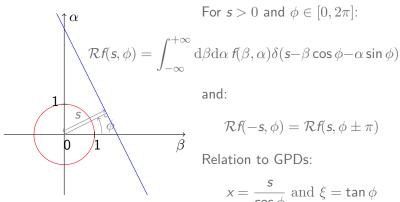
Definition

Overlan Radon transform

Covariant extension

#### Conclusion

Appendix



For s > 0 and  $\phi \in [0, 2\pi]$ :

and:

$$\mathcal{R}f(-s,\phi) = \mathcal{R}f(s,\phi \pm \pi)$$

Relation to GPDs:

$$x = \frac{s}{\cos \phi}$$
 and  $\xi = \tan \phi$ 

## Relation between GPD and DD in Pobylitsa gauge

$$\frac{\sqrt{1+\xi^2}}{1-x}H(x,\xi)=\mathcal{R}f_{\mathcal{P}}(s,\phi)\;,$$



## The range of the Radon transform.

The polynomiality property a.k.a. the Ludwig-Helgason condition.

### PARTONS Framework

## Motivation Mass without mass

Nucleon structure Content of GPDs

#### Imaging Experimental access

DVCS Kinematics Towards 3D images

#### Building

Computing chain Examples Architecture

#### Team Learning

Definition Overlan

Radon transform

Covariant extension

#### Conclusion

Appendix

- The Mellin moments of a Radon transform are homogeneous polynomials in  $\omega = (\sin \phi, \cos \phi)$ .
- The converse is also true:

## Theorem (Hertle, 1983)

Let  $g(s,\omega)$  an even compactly-supported distribution. Then g is itself the Radon transform of a compactly-supported distribution if and only if the **Ludwig-Helgason consistency** condition hold:

- (i) g is  $C^{\infty}$  in  $\omega$ ,
- (ii)  $\int \mathrm{d} s \, s^m g(s,\omega)$  is a homogeneous polynomial of degree m for all integer m>0.

H. Moutarde

Double Distributions and the Radon transform are the natural solution of the polynomiality condition.



## Implementing Lorentz covariance. Extend an overlap in the DGLAP region to the whole GPD domain.



#### PARTONS Framework

#### Motivation Mass without mass

### Imaging

Experimental access **DVCS Kinematics** 

#### Towards 3D images

Building Computing chain

### Examples

Architecture Team

#### Learning Definition

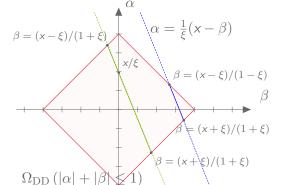
Overlan Radon transform Covariant extension

### Conclusion

Appendix

## DGLAP and ERBL regions

$$(x,\xi) \in \text{DGLAP} \Leftrightarrow |s| \ge |\sin \phi|,$$
  
 $(x,\xi) \in \text{ERBL} \Leftrightarrow |s| \le |\sin \phi|.$ 



Each point  $(\beta, \alpha)$  with

 $\beta \neq 0$ contributes to **both** DGLAP and

ERBL regions.

Expressed in support theorem.



## III-posedness in the sense of Hadamard. A first glimpse at the inverse Radon transform.



#### PARTONS Framework

Motivation

Imaging

Building Computing chain

Examples

Architecture

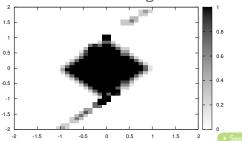
Mass without mass Nucleon structure Content of GPDs

Experimental access **DVCS Kinematics** Towards 3D images

## Numerical evaluation almost unavoidable (polar vs cartesian coordinates).

## Ill-posedness by lack of continuity.

- The **unlimited** Radon inverse problem is **mildly** ill-posed while the **limited** one is **severely** ill-posed.
- Careful selection of algorithms and numerical methods.



#### Team Learning Definition

Overlan Radon transform

Covariant extension

Conclusion

Appendix

PhD dissertation

Mezrag

## **Conclusion**



## Conclusions and prospects. Towards a unifying framework for GPD studies.



#### PARTONS Framework

#### Motivation Mass without mass

Nucleon structure Content of GPDs

#### Imaging Experimental access

**DVCS Kinematics** Towards 3D images

#### Building Computing chain

Examples Architecture Team

#### Learning Definition

Overlan Radon transform Covariant extension

Appendix

## ■ Challenging constraints expected from Jefferson Lab in valence region and later from EIC in gluon sector.

- **Good theoretical control** on the path between GPD models and experimental data: from theory to measurements, and conversely.
- Development of the PARTONS framework for phenomenology and theory purposes.
- **Systematic** procedure to construct GPD models from any "reasonable" Ansatz of I FWFs.
- First release of PARTONS in summer 2016!

## **Appendix**



# Implementing Lorentz covariance. Extend an overlap in the DGLAP region to the whole GPD domain.



#### PARTONS Framework

## Motivation Mass without mass

Nucleon structure Content of GPDs

## Imaging

Experimental access
DVCS Kinematics
Towards 3D images

#### Building

Computing chain Examples

Architecture

#### Learning

Definition Overlan

Radon transform

Covariant extension

#### Conclusion

Appendix

For **any model of LFWF**, one has to address the following three questions:

- Does the extension exist?
- 2 If it exists, is it unique?
- 3 How can we compute this extension?

Work in progress!



## Implementing Lorentz covariance. Uniqueness of the extension.



#### PARTONS Framework

### Motivation

Mass without mass Nucleon structure Content of GPDs

#### Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building

Computing chain Examples Architecture

#### Team Learning

#### Definition

Overlan Radon transform Covariant extension

#### Conclusion

**Appendix** 

### Theorem

Let f be a compactly-supported locally summable function defined on  $\mathbb{R}^2$  and  $\mathcal{R}f$  its Radon transform.

Let  $(s_0, \omega_0) \in \mathbb{R} \times S^1$  and  $U_0$  an open neighborhood of  $\omega_0$  such that:

for all 
$$s > s_0$$
 and  $\omega \in U_0$   $\mathcal{R}f(s,\omega) = 0$ .

Then  $f(\aleph) = 0$  on the half-plane  $\langle \aleph | \omega_0 \rangle > s_0$  of  $\mathbb{R}^2$ .

Consider a GPD H being zero on the DGLAP region.

- Take  $\phi_0$  and  $s_0$  s.t.  $\cos \phi_0 \neq 0$  and  $|s_0| > |\sin \phi_0|$ .
- Neighborhood  $U_0$  of  $\phi_0$  s.t.  $\forall \phi \in U_0$   $|\sin \phi| < |s_0|$ .
- The underlying DD f has a zero Radon transform for all  $\phi \in U_0$  and  $s > s_0$  (DGLAP).
- Then  $f(\beta, \alpha) = 0$  for all  $(\beta, \alpha) \in \Omega_{DD}$  with  $\beta \neq 0$ .
  - Extension **unique** up to adding a **D-term**:  $\delta(\beta)D(\alpha)$ .



## Computation of the extension.

Numerical evaluation of the inverse Radon transform (1/3).



#### PARTONS Framework

### Motivation

Mass without mass Nucleon structure Content of GPDs

#### Imaging

Experimental access
DVCS Kinematics
Towards 3D images

#### Building

Computing chain

Examples Architecture

Architecture Team

#### Learning Definition

Overlan

Radon transform

#### Conclusion

Appendix

## A discretized problem

Consider N+1 Hilbert spaces H,  $H_1$ , ...,  $H_N$ , and a family of continuous surjective operators  $R_n: H \to H_n$  for  $1 \le n \le N$ . Being given  $g_1 \in H_1$ , ...,  $g_n \in H_n$ , we search f solving the following system of equations:

$$R_n f = g_n \quad \text{for } 1 \le n \le N$$

## Fully discrete case

Assume f piecewise-constant with values  $f_m$  for  $1 \le m \le M$ .

For a collection of lines  $(L_n)_{1 \le n \le N}$  crossing  $\Omega_{\mathrm{DD}}$ , the Radon transform writes:

$$g_n = \mathcal{R}f = \int_{L_n} f = \sum_{m=1}^M f_m \times \text{Measure}(L_n \cap C_m)$$
 for  $1 \le n \le N$ 



## Computation of the extension.

Numerical evaluation of the inverse Radon transform (2/3).



#### PARTONS Framework

### Motivation

Mass without mass Nucleon structure Content of GPDs

#### Imaging

Experimental access
DVCS Kinematics
Towards 3D images

#### Building

Computing chain Examples

Architecture Team

#### Learning Definition

Overlap Radon transform

Covariant extension

#### Conclusion

Appendix

## Kaczmarz algorithm

iteratively by:

Denote  $P_n$  the orthogonal projection on the *affine* subspace  $R_n f = g_n$ . Starting from  $f^0 \in H$ , the sequence defined

$$f^{k+1} = P_N P_{N-1} \dots P_1 f^k$$

converges to the solution of the system.

The convergence is exponential if the projections are randomly ordered.

Strohmer and Vershynin, Jour. Four. Analysis and Appl. **15**, 437 (2009)



## Computation of the extension. Numerical evaluation of the inverse Radon transform (2/3).

#### PARTONS Framework

#### Motivation Mass without mass

Nucleon structure Content of GPDs

## Imaging

Experimental access **DVCS** Kinematics Towards 3D images

#### Building

Computing chain Examples

Architecture Team

## Learning

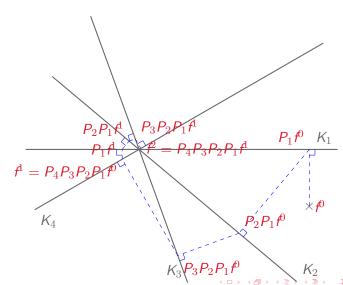
Definition

Overlap Radon transform

Covariant extension

#### Conclusion

Appendix



H. Moutarde



# Computation of the extension. Numerical evaluation of the inverse Radon transform (3/3).

Isfu CEA - Saciay

#### PARTONS Framework

## Motivation Mass without mass

Nucleon structure
Content of GPDs

#### Imaging

Experimental access

DVCS Kinematics

Towards 3D images

#### Building

Computing chain Examples

Architecture Team

#### Learning

Definition Overlap

Radon transform Covariant extension

#### Conclusion

Appendix

## And if the input data are inconsistent?

- Instead of solving  $g = \mathcal{R}f$ , find f such that  $||g \mathcal{R}f||_2$  is minimum.
- The solution always exists.
- The input data are **inconsistent** if  $||g \mathcal{R}f||_2 > 0$ .



## Computation of the extension. Numerical evaluation of the inverse Radon transform (3/3).

#### PARTONS Framework

### Motivation

Mass without mass Nucleon structure Content of GPDs

#### Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building

Computing chain

Examples Architecture Team

#### Learning

Definition Overlan

Radon transform Covariant extension

Conclusion

**Appendix** 

## Relaxed Kaczmarz algorithm

Let  $\omega \in ]0,2[$  and:

$$P_n^{\omega} = (1 - \omega) \operatorname{Id}_H + \omega P_n \quad \text{ for } 1 \le n \le N$$

Write:

$$RR^{\dagger} = (R_i R_i^{\dagger})_{1 \le i,j \le N} = D + L + L^{\dagger}$$

where D is diagonal, and L is lower-triangular with zeros on the diagonal.



## Computation of the extension. Numerical evaluation of the inverse Radon transform (3/3).



#### PARTONS Framework

Theorem

#### Motivation Mass without mass Nucleon structure Content of GPDs

#### Imaging

Experimental access **DVCS Kinematics** Towards 3D images

#### Building Computing chain

Examples Architecture Team

#### Learning

Definition Overlan Radon transform

Covariant extension

Conclusion

**Appendix** 

Let  $0 < \omega < 2$ . For  $f^0 \in \text{Ran } R^{\dagger}$  (e.g.  $f^0 = 0$ ), the Kaczmarz method with relaxation converges to the unique solution  $f^{\omega} \in \text{Ran } R^{\dagger} \text{ of } \cdot$ 

$$R^{\dagger}(D+\omega L)^{-1}(g-Rf^{\omega})=0,$$

where the matrix D and L appear in the decomposition of  $RR^{\dagger}$ . If  $g = \mathcal{R}f$  has a solution, then  $f^{\omega}$  is its solution of minimal

norm. Otherwise:  $f^{\omega} = f_{MP} + \mathcal{O}(\omega)$ .

where  $f_{MP}$  is the minimizer in H of:

$$\langle g - \mathcal{R}f | g - \mathcal{R}f \rangle_D$$
,

the inner product being defined by:

$$\langle h|k\rangle_D = \langle D^{-1}h|k\rangle$$
.



## Test on a 1D example. Recovering a PDF from the knowledge of its Mellin moments.



#### PARTONS Framework

#### Motivation

Mass without mass Nucleon structure Content of GPDs

### **Imaging**

Experimental access **DVCS Kinematics** Towards 3D images Building Computing chain Examples

#### Architecture Team

Learning Definition Overlan

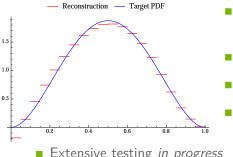
Radon transform Covariant extension

#### Conclusion

**Appendix** 

## A pion valence PDF-like example

Aim: reconstruct the PDF  $q(x) = 30x^2(1-x)^2$  from the knowledge of its first 30 Mellin moments.



- Piecewise-constant PDF: 20 values.
  - Input: 30 Mellin moments.
- Unrelaxed method  $\omega = 1$ 
  - 10000 iterations.
- Extensive testing in progress
  - Various inputs: PDFs and LFWFs.
  - Numerical noise.

Commissariat à l'énergie atomique et aux énergies alternatives Centre de Saclay 91191 Gif-sur-Yvette Cedex

Etablissement public à caractère industriel et commercial R.C.S. Paris B 775 68

**◀□▶ ◀圖▶ ◀필▶ ◀필▶**