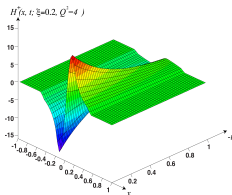
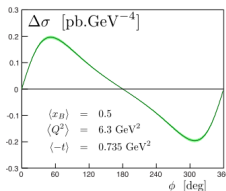
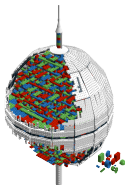
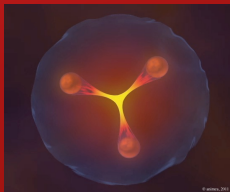


DE LA RECHERCHE À L'INDUSTRIE



NPQCD 2014 | Hervé MOUTARDE

Oct 3rd, 2014

Vector Meson Production at small x_B

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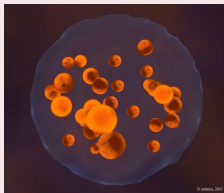
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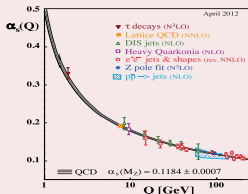
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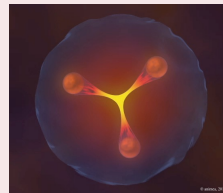
Perturbative QCD



Asymptotic freedom



Nonperturbative QCD



Perturbative AND nonperturbative QCD at work

- Define **universal** objects describing 3D nucleon structure:
Generalized Parton Distributions (GPD).
- Relate GPDs to measurements using **factorization**:
**Virtual Compton Scattering (DVCS, TCS),
Deeply Virtual Meson production (DVMP).**
- Get **experimental knowledge** of nucleon structure.

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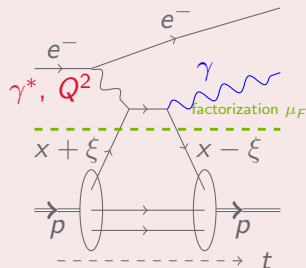
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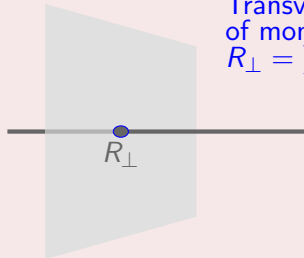
Conclusions

- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in the nucleon.
- Deeply Virtual Compton Scattering (DVCS) recognized as the theoretically cleanest channel to access GPDs.

DVCS and GPDs



Transverse center
of momentum $R_\perp$
 $R_\perp = \sum_i x_i r_{\perp i}$



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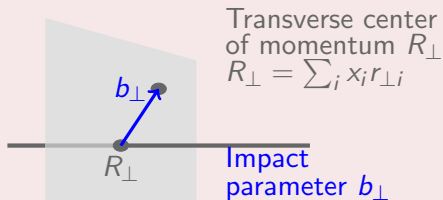
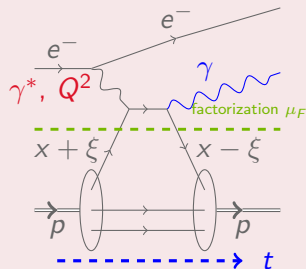
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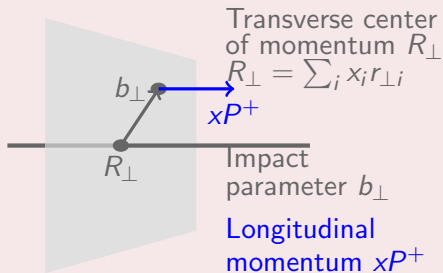
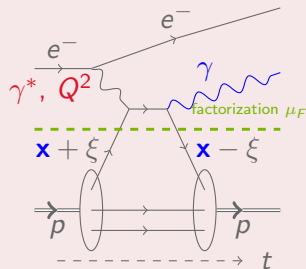
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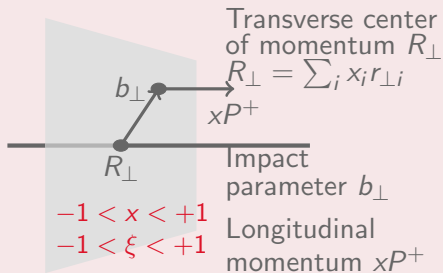
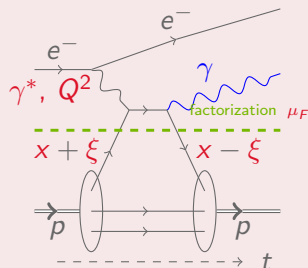
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- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in the nucleon.
- Deeply Virtual Compton Scattering (DVCS) recognized as the theoretically cleanest channel to access GPDs.

DVCS and GPDs



- GPD $F^i(x, \xi, t, \mu_F)$ for each parton type $i = g, u, d, \dots$

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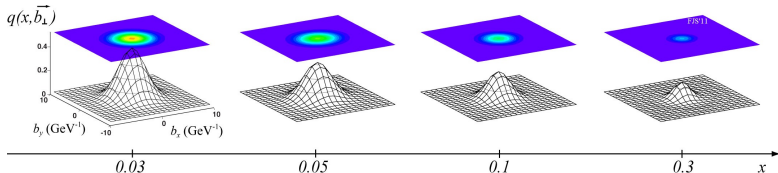
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- **Probabilistic interpretation** of Fourier transform of $\text{GPD}(x, \xi = 0, t)$ in **transverse plane**.

$$\rho(x, b_{\perp}, \lambda, \lambda_N) = \frac{1}{2} \left[H(x, b_{\perp}^2) + \frac{b_{\perp}^j \epsilon_{ji} S_{\perp}^i}{M} \frac{\partial E}{\partial b_{\perp}^2}(x, b_{\perp}^2) + \lambda \lambda_N \tilde{H}(x, b_{\perp}^2) \right]$$

- Notations : quark helicity λ , nucleon longitudinal polarization λ_N and nucleon transverse spin $S_{\perp}$.

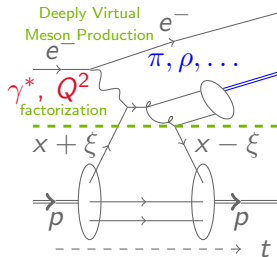
Burkardt, Phys. Rev. **D62**, 071503 (2000)



From Theory to Data

Exclusive processes of present interest (1/2). Factorization and universality.

Vector Meson Production at small x_B



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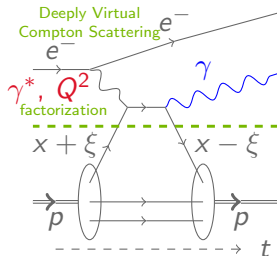
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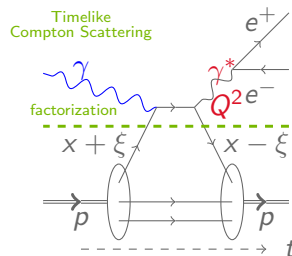
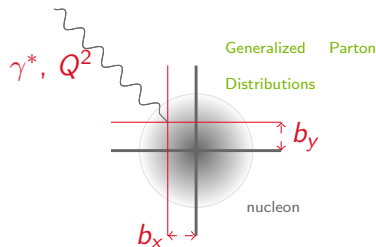
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Bjorken regime : large Q^2 and fixed $x_B \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on **factorization theorems**.
- All-order proofs for DVCS, TCS and some DVMP.
- GPDs depend on a (arbitrary) factorization scale μ_F .
- **Consistency** requires the study of **different channels**.

- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^1 dx \, C\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F)$$

for a given GPD F .

- CFF $\mathcal{F}$ is a **complex function**.

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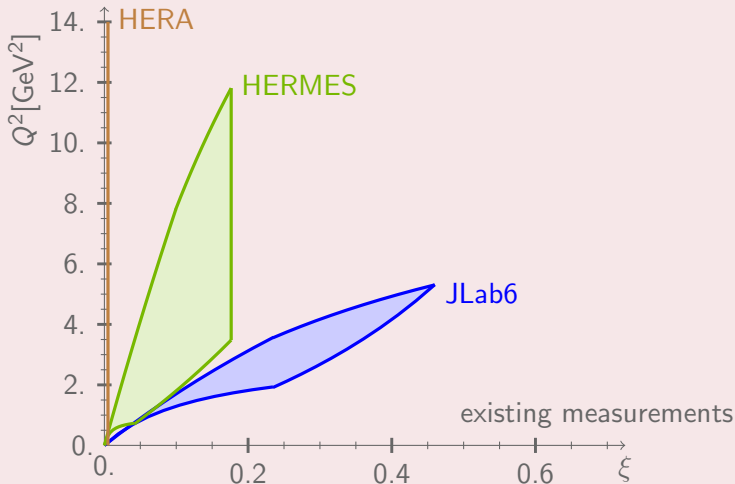
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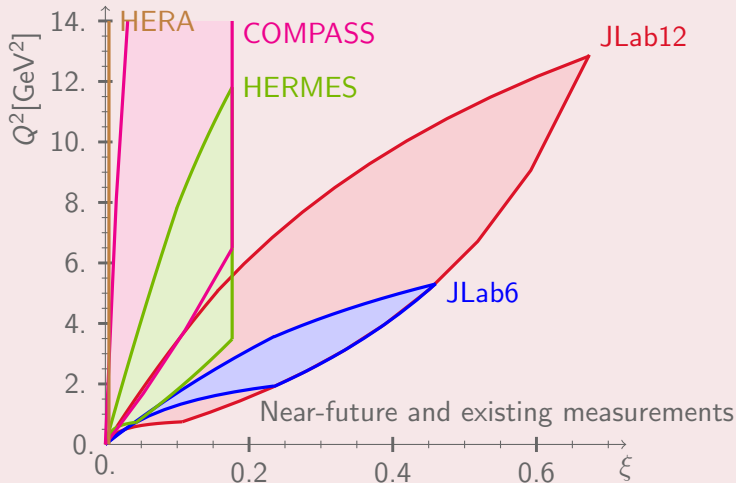
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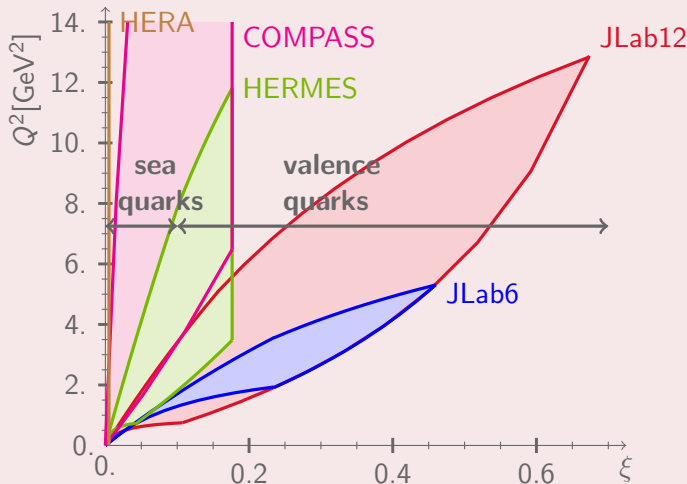
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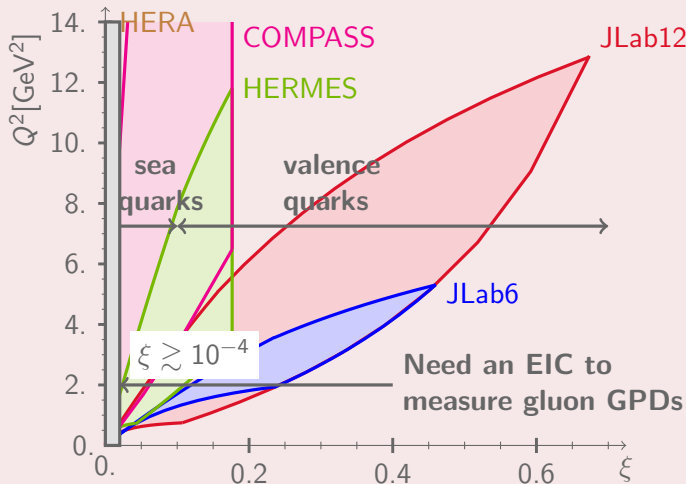
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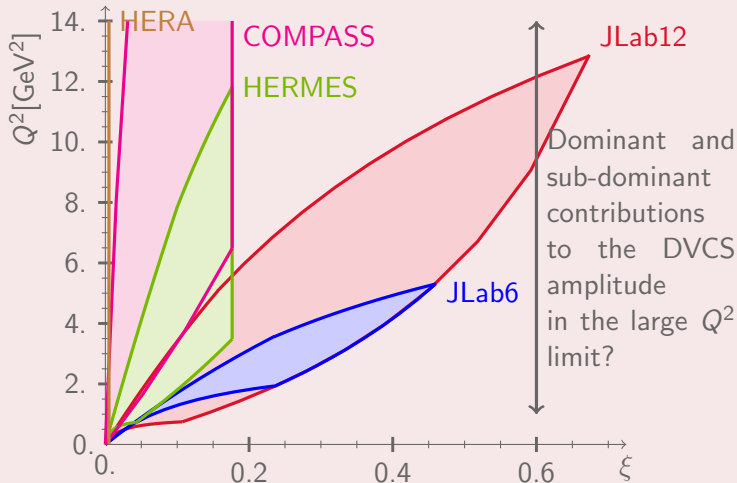
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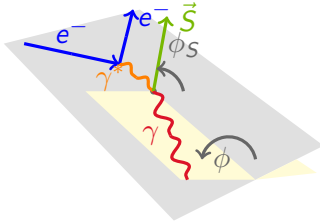
Kinematic reach of existing or near-future DVCS measurements



Definition of observables (1/3).

Harmonic structure of $ep \rightarrow ep\gamma$ amplitude.

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- Study the **harmonic structure** of $ep \rightarrow ep\gamma$ amplitude.

Diehl *et al.*,
Phys. Lett. **B411**, 193 (1997)

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$$|\mathcal{M}_{\text{BH}}|^2 \propto \frac{1}{|t|} \frac{1}{P(\cos \phi)} \sum_{n=0}^3 [c_n^{\text{BH}} \cos(n\phi) + s_n^{\text{BH}} \sin(n\phi)]$$

$$|\mathcal{M}_{\text{DVCS}}|^2 \propto \sum_{n=0}^3 [c_n^{\text{DVCS}} \cos(n\phi) + s_n^{\text{DVCS}} \sin(n\phi)]$$

$$\mathcal{M}_{\text{I}} \propto \frac{1}{|t|} \frac{1}{P(\cos \phi)} \sum_{n=0}^3 [c_n^{\text{I}} \cos(n\phi) + s_n^{\text{I}} \sin(n\phi)]$$

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- Combined beam-spin and charge asymmetries :

$$d\sigma^{h_e, Q_e}(\phi) = d\sigma_{UU}(\phi) [1 + h_e A_{LU, DVCS}(\phi) + Q_e h_e A_{LU, I}(\phi) + Q_e A_C(\phi)]$$

- Single beam-spin asymmetry :

$$A_{LU}^{Q_e}(\phi) = \frac{d\sigma^{\overrightarrow{Q_e}} - d\sigma^{\overleftarrow{Q_e}}}{d\sigma^{\overrightarrow{Q_e}} + d\sigma^{\overleftarrow{Q_e}}}$$

- Relation between observables :

$$A_{LU}^{Q_e}(\phi) = \frac{Q_e A_{LU, I}(\phi) + A_{LU, DVCS}(\phi)}{1 + Q_e A_C(\phi)}$$

- Compute Fourier coefficients of asymmetries.

Definition of observables (3/3).

What are the probed combinations of CFFs ?

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Typical kinematics

Experiment	Kinematics		
	x_B	Q^2 [GeV ²]	t [GeV ²]
HERA	0.001	8.00	-0.30
COMPASS	0.05	2.00	-0.20
HERMES	0.09	2.50	-0.12
CLAS	0.19	1.25	-0.19
HALL A	0.36	2.30	-0.23

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Selection of observables

Experiment	Observable	Normalized CFF dependence
HERMES	$A_C^{\cos 0\phi}$	$\text{Re}\mathcal{H} + 0.06\text{Re}\mathcal{E} + 0.24\text{Re}\tilde{\mathcal{H}}$
	$A_C^{\cos \phi}$	$\text{Re}\mathcal{H} + 0.05\text{Re}\mathcal{E} + 0.15\text{Re}\tilde{\mathcal{H}}$
	$A_{\text{LU},\text{I}}^{\sin \phi}$	$\text{Im}\mathcal{H} + 0.05\text{Im}\mathcal{E} + 0.12\text{Im}\tilde{\mathcal{H}}$
	$A_{\text{UL}}^{+,\sin \phi}$	$\text{Im}\tilde{\mathcal{H}} + 0.10\text{Im}\mathcal{H} + 0.01\text{Im}\mathcal{E}$
CLAS	$A_{\text{LU}}^{-,\sin \phi}$	$\text{Im}\mathcal{H} + 0.06\text{Im}\mathcal{E} + 0.21\text{Im}\tilde{\mathcal{H}}$
	$A_{\text{UL}}^{-,\sin \phi}$	$\text{Im}\tilde{\mathcal{H}} + 0.12\text{Im}\mathcal{H} + 0.04\text{Im}\mathcal{E}$
HALL A	$\sigma^{\cos 0\phi}$	$1 + 0.05\text{Re}\mathcal{H} + 0.007\mathcal{H}\mathcal{H}^*$
	$\sigma^{\cos \phi}$	$1 + 0.12\text{Re}\mathcal{H} + 0.05\text{Re}\tilde{\mathcal{H}}$

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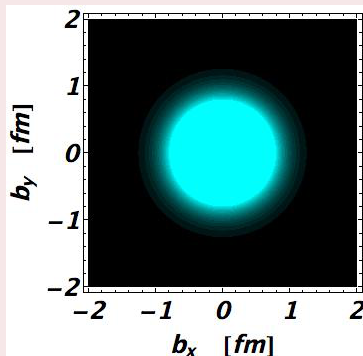
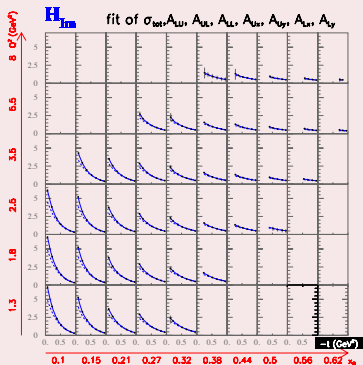
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Contour plot of the spatial charge density ($x_B = 0.25$)

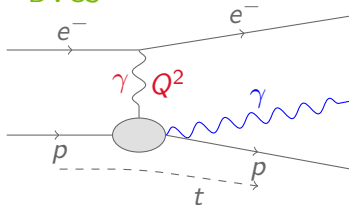


Guidal *et al.*, Rept. Prog. Phys. **76**, 066202 (2013)

Timelike and spacelike Compton Scattering.

Scattering amplitudes and their partonic interpretation.

DVCS



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Compton Form Factors (CFF)

- Parametrize amplitudes.

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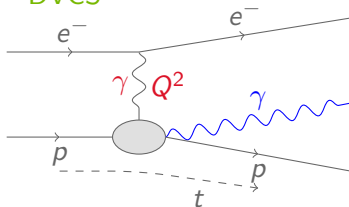
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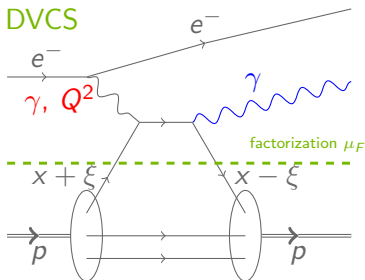
DVCS



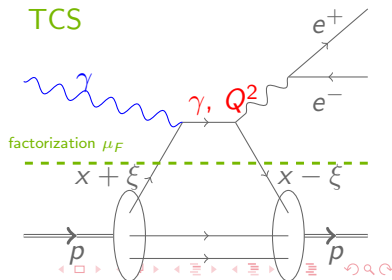
Compton Form Factors (CFF)

- Parametrize amplitudes.
- Evaluation at LO.

DVCS



TCS



Timelike and spacelike Compton Scattering.

Scattering amplitudes and their partonic interpretation.

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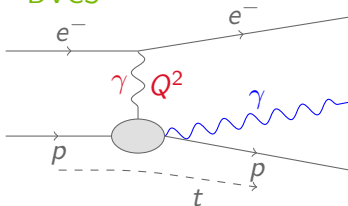
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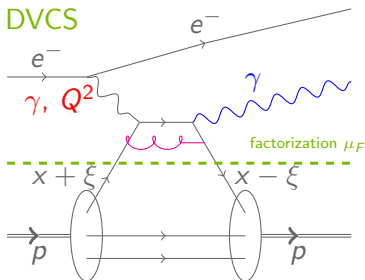
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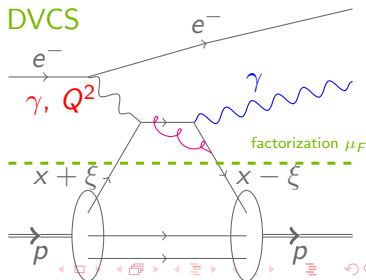
Compton Form Factors (CFF)

- Parametrize amplitudes.
- Evaluation at LO.
- Evaluation at **NLO**.

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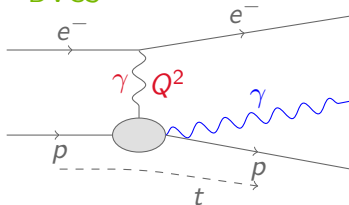
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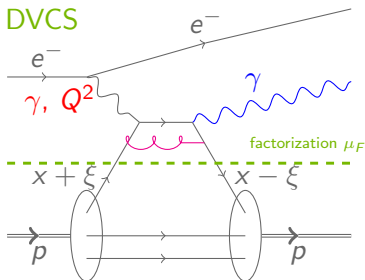
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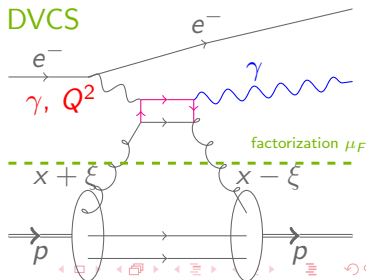
Compton Form Factors (CFF)

- Parametrize amplitudes.
- Evaluation at LO.
- Evaluation at **NLO**.
- Other diagrams at NLO, including **gluon GPDs**.

DVCS



DVCS



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- Convolution of singlet GPD $H_q^+(x) \equiv H_q(x) - H_q(-x)$:

$$\begin{aligned} \mathcal{H}_q(\xi, Q^2) &= \int_{-1}^{+1} dx H_q^+(x, \xi, \mu_F) T_q \left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F} \right) \\ &+ \int_{-1}^{+1} dx H_g(x, \xi, \mu_F) T_g \left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F} \right) \end{aligned}$$

Belitsky and Müller, Phys. Lett. **B417**, 129 (1998)
Pire *et al*, Phys. Rev. **D83**, 034009 (2011)

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- Convolution of singlet GPD $H_q^+(x) \equiv H_q(x) - H_q(-x)$:

$$\mathcal{H}_q(\xi, Q^2) \stackrel{\text{LO}}{=} \int_{-1}^{+1} dx H_q^+(x, \xi, \mu_F) C_0^q(x, \xi) + \int_{-1}^{+1} dx H_g(x, \xi, \mu_F) 0$$

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Belitsky and Müller, Phys. Lett. **B417**, 129 (1998)
Pire *et al*, Phys. Rev. **D83**, 034009 (2011)

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- Integration yields **imaginary** parts to $\mathcal{H}$:

$$\text{Im}\mathcal{H}_q(\xi, Q^2) \stackrel{\text{LO}}{=} \pi H_q^+(\xi, \xi, \mu_F)$$

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- Convolution of singlet GPD $H_q^+(x) \equiv H_q(x) - H_q(-x)$:

$$\mathcal{H}_q(\xi, Q^2) \stackrel{\text{NLO}}{=} \int_{-1}^{+1} dx H_q^+(x, \xi, \mu_F) \left[C_0^q + C_1^q + \frac{1}{2} \ln \frac{|Q^2|}{\mu_F^2} C_{\text{Coll}}^q \right] \\ + \int_{-1}^{+1} dx H_g(x, \xi, \mu_F) \left(0 + C_1^g + \frac{1}{2} \ln \frac{|Q^2|}{\mu_F^2} C_{\text{Coll}}^g \right)$$

Belitsky and Müller, Phys. Lett. **B417**, 129 (1998)

Pire *et al*, Phys. Rev. **D83**, 034009 (2011)

- Integration yields **imaginary** parts to $\mathcal{H}$:

$$\text{Im} \mathcal{H}_q(\xi, Q^2) \stackrel{\text{NLO}}{=} \mathcal{I}(\xi) H_q^+(\xi, \xi, \mu_F) \\ + \int_{-1}^{+1} dx \mathcal{T}^q(x) \left(H_q^+(x, \xi, \mu_F) - H_q^+(\xi, \xi, \mu_F) \right) \\ + \text{gluon contributions.}$$

Spacelike CFF $\mathcal{H}$: Large NLO corrections.

Mostly due to gluons, maximum in HERMES / COMPASS kinematic region.

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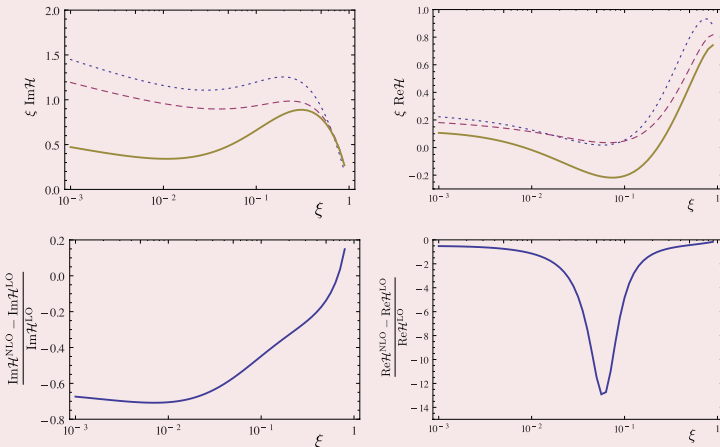
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$Re\mathcal{H}$ at LO and NLO ($t = -0.1 \text{ GeV}^2$, $Q^2 = \mu_F^2 = 4. \text{ GeV}^2$)



Moutarde *et al.*, Phys. Rev. **D87**, 054029 (2013)

dotted: LO dashed: NLO quark corrections solid: full NLO

Finite- t and target mass corrections.

Much more complex structure of DVCS to the twist-4 accuracy.

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BMP amplitudes

DVCS observables

Summary

BMP Compton form factors (CFFs)

- Photon helicity amplitudes can be expanded in a given set of spinor bilinears

$$\mathcal{A}_q^{a\pm} = \mathbb{H}_{a\pm}^q h + \mathbb{E}_{a\pm}^q e \mp \tilde{\mathbb{H}}_{a\pm}^q \tilde{h} \mp \tilde{\mathbb{E}}_{a\pm}^q \tilde{e}$$

with, e.g.

Belitsky, Müller, Ji: NPB 878 (2014) 214

$$h = \frac{\bar{u}(p') (\not{q} + \not{q}') u(p)}{P \cdot (\not{q} + \not{q}')} \quad \dots$$

- The results read

Braun, Manashov, Pirnay: PRL109 (2012) 242001

$$\begin{aligned} \mathbb{H}_{++} &= T_0 \otimes H + \frac{t}{Q^2} \left[-\frac{1}{2} T_0 + T_1 + 2\xi D_\xi T_2 \right] \otimes H + \frac{2t}{Q^2} \xi^2 \partial_\xi \xi T_2 \otimes (H+E) \\ \mathbb{H}_{0+} &= -\frac{4|\xi P_\perp|}{\sqrt{2}Q} \left[\xi \partial_\xi T_1 \otimes H + \frac{t}{Q^2} \partial_\xi \xi T_1 \otimes (H+E) \right] - \frac{t}{\sqrt{2}Q|\xi P_\perp|} \xi T_1 \otimes \left[\xi (H+E) - \tilde{H} \right] \\ \mathbb{H}_{-+} &= \frac{4|\xi P_\perp|^2}{Q^2} \left[\xi \partial_\xi^2 \xi T_1^{(+)} \otimes H + \frac{t}{Q^2} \partial_\xi^2 \xi^2 T_1^{(+)} \otimes (H+E) \right] \\ &\quad + \frac{2t}{Q^2} \xi \left[\xi \partial_\xi \xi T_1^{(+)} \otimes (H+E) + \partial_\xi \xi T_1 \otimes \tilde{H} \right] \end{aligned}$$

Finite- t and target mass corrections.

Much more complex structure of DVCS to the twist-4 accuracy.

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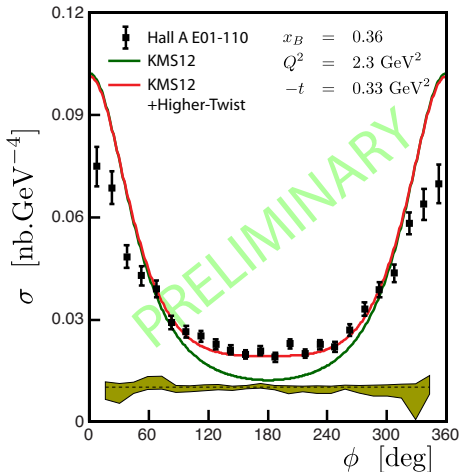
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- Large effect near $\phi = 180^\circ$.
- Only twist-2 GPDs.
- Similar effect for DVMP observables?

Sabatié, DIS2014

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- CFF extraction techniques offer **viable experimental information**, but the interpretation in terms of GPDs is obscured by **higher-order** and **higher-twist** contributions.
- CFF extraction **ready for DVCS and TCS**. DVMP and DIS should be taken into account to test the **universality** of GPDs.
- Sophisticated GPD modeling is highly desirable. Non trivial task because of numerous **theoretical constraints** and **curse of dimensionality**.
- These issues may become crucial with the **high precision measurements** expected after JLab's upgrade at 12 GeV.

Impact Parameter Description of Collinear Factorization and Modified Perturbative Approach

Vector Meson Production at small x_B

- Consider **longitudinal** virtual photon γ^* and vector meson V : $\lambda_\gamma = \lambda_V = 0$.

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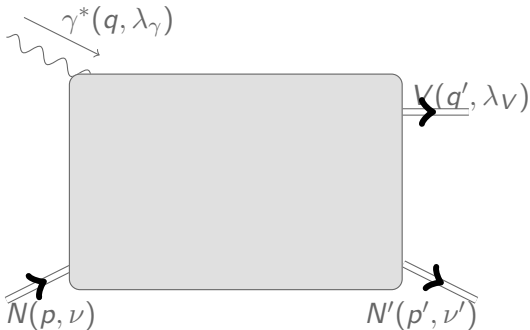
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Kinematics of hard meson production.

Hadronic process and parton picture.

Vector Meson
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- Consider **longitudinal** virtual photon γ^* and vector meson V : $\lambda_\gamma = \lambda_V = 0$.
- Assume **large virtuality** $Q^2 = -q^2$ of the incoming photon and invoke **collinear factorization**.

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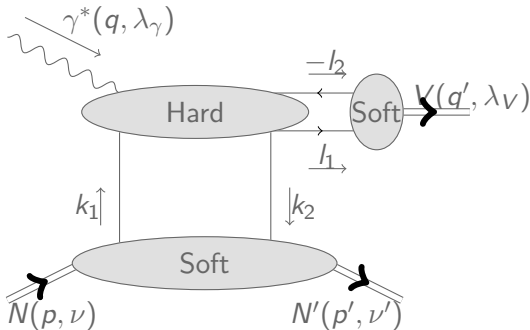
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Vector Meson Production at small x_B

- Consider **longitudinal** virtual photon γ^* and vector meson V : $\lambda_\gamma = \lambda_V = 0$.
- Assume **large virtuality** $Q^2 = -q^2$ of the incoming photon and invoke **collinear factorization**.

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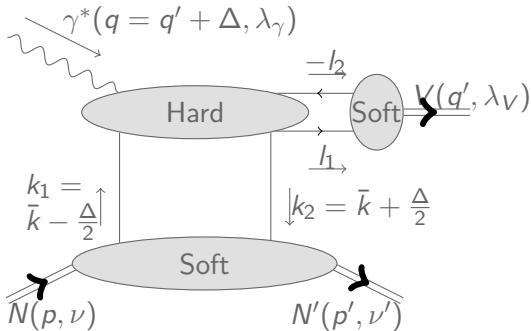
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Lightlike p_1 and p_2
s.t. $p_1 \cdot p_2 = 1$.

$$p \simeq (1 + \xi)p_2$$

$$p' \simeq (1 - \xi)p_2$$

$$\Delta \simeq -2\xi p_2$$

$$q \simeq \Delta + \frac{Q^2}{4\xi} p_1$$

$$q' \simeq \frac{Q^2}{4\xi} p_1$$

with $\Delta = p' - p$ and $\xi = -2\Delta^+ / (p + p')^+$. Also $n = 4\xi / Q^2 p_2$.

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- The transition between two states $(\lambda_\gamma, \lambda_V)$ with **different polarizations** requires a non-vanishing momentum in t -channel.
- Only available Lorentz structure $\underline{\epsilon}_V \cdot \underline{\Delta}$ which induces a $\sqrt{-t}$ **suppression for small t** .
- Focus on **small- t** limit and **longitudinal** photon and vector meson.
- From **parity invariance** of strong and electromagnetic interactions:

$$\mathcal{M}_{-\lambda_V - \nu', -\lambda_\gamma - \nu} = (-1)^{(\lambda_V - \nu') - (\lambda_\gamma - \nu)} \mathcal{M}_{\lambda_V \nu', \lambda_\gamma \nu}$$

- In the small- t limit:

$$\left. \frac{d\sigma_L}{dt} \right|_{t=0} = \frac{1}{\text{flux}} |\mathcal{M}_{0+,0+}|^2$$

Contribution of gluon GPDs.

Six diagrams associated to t -channel gluon exchange.

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- Separate contributions from different **quark flavors** to S -matrix element:

$$V_{V,\{0+,0+\}}^g = \sum_q C_V^q S_{q,\{0+,0+\}}^g$$

where, for $V \in \{\rho, \omega, \phi\}$:

$$C_\rho^u = -C_\rho^d = C_\omega^u = C_\omega^d = \frac{1}{\sqrt{2}} \text{ and } C_\phi^s = 1$$

- Compute only 2 out of 6 diagrams with t -channel gluon exchange due to **symmetry considerations**:
 - Exchange of quark and antiquark: $l_1 \leftrightarrow l_2$.
 - Exchange of gluon momenta: $k_1 \leftrightarrow k_2$.
- Work in the **light-cone gauge** $p_1 \cdot A = 0$.

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Contribution of gluon GPDs.

Leading twist result (1/2).

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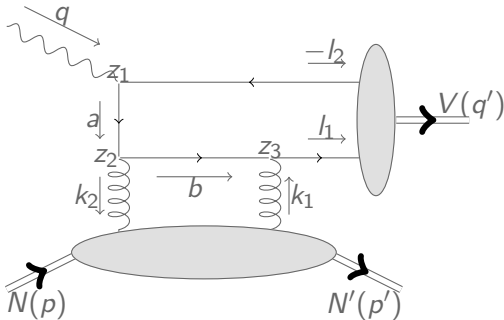
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- Longitudinal momentum fractions τ and $\bar{\tau} = 1 - \tau$ of the vector meson carried by the quarks.

Contribution of gluon GPDs.

Leading twist result (1/2).

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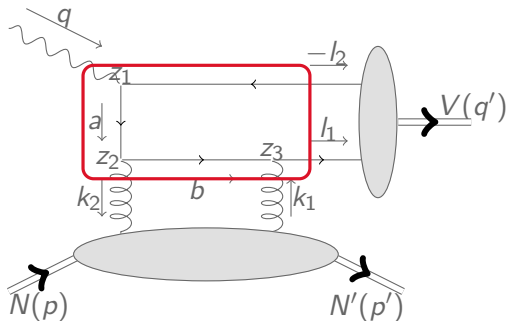
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- Longitudinal momentum fractions τ and $\bar{\tau} = 1 - \tau$ of the vector meson carried by the quarks.

- Collinear approximation in hard scattering part H :

$$H(l_1, l_2) = H(\tau q', \bar{\tau} q')$$

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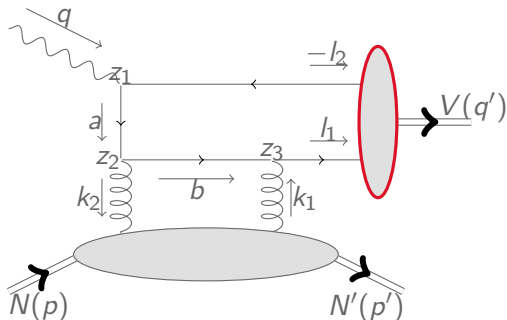
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- Longitudinal momentum fractions τ and $\bar{\tau} = 1 - \tau$ of the vector meson carried by the quarks.

- Collinear approximation in hard scattering part H :

$$H(l_1, l_2) = H(\tau q', \bar{\tau} q')$$

- Twist-2 meson DA $\phi_{\parallel}(\tau, \mu^2)$.

$$\left\langle \frac{1}{\tau} \right\rangle = \int_0^1 d\tau \frac{1}{\tau} \phi_{\parallel}(\tau, \mu^2)$$

Ball *et al.*,
Nucl. Phys. **B543**, 201 (1999)

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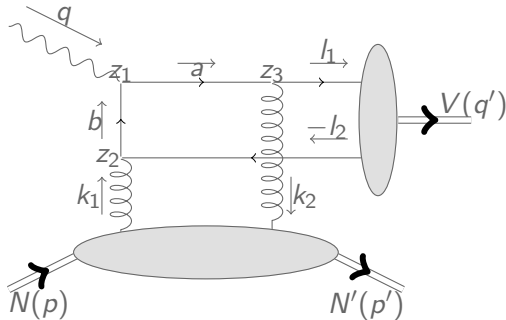
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- Trace of left diagram vanishes.
- Obtain result for all 6 diagrams by changing $\tau \rightarrow \bar{\tau}$ and $x \rightarrow -x$.

- Full **leading twist** gluon contribution $\mathcal{M}_{V,\{0+,0+\}}^g$:

$$\mathcal{M}_{V,\{0+,0+\}}^g = -e \frac{8\pi\alpha_s}{N_c Q} f_V \int_0^1 dx \frac{\sum_q e_q C_V^q H^g(x, \xi, t)}{(x + \xi - i\epsilon)(x - \xi + i\epsilon)} \langle \frac{1}{\tau} \rangle$$

$$\text{and } f_V \sum_q e_q C_V^q = \sqrt{3m_V \Gamma_{V \rightarrow e^+e^-} / (4\pi\alpha)}.$$

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- Introduced to overcome **end-point divergency** problem by keeping **transverse** degrees of freedom in the perturbative hard subprocess.

Botts and Stermann, Nucl. Phys. **B325**, 62 (1989)

Nan Li and Stermann, Nucl. Phys. **B381**, 129 (1989)

Goloskokov and Kroll, Eur. Phys. J. **C42**, 281 (2005)

Eur. Phys. J. **C50**, 829 (2007)

Eur. Phys. J. **C53**, 367 (2008)

- Difference between leading twist and MPA result in treatment of hard scattering part. Here keep outgoing quark and antiquark on the mass-shell:

$$l_1 = \tau q' + \bar{\tau} \frac{m_V^2}{2} n + l_\perp$$

$$l_2 = \bar{\tau} q' + \tau \frac{m_V^2}{2} n - l_\perp$$

Besse *et al.*, in preparation

Vector Meson Production at small x_B

- Introduce **vector meson wavefunction** $\Psi_{V,\bar{h}h}^{*(\lambda_V)}$ parameterizing the valence $q\bar{q}$ Fock state of the vector meson V :

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$$\langle V(p_V) | = \sqrt{4\pi N_c} \int dk^+ \int \frac{d^2 \underline{k}}{16\pi^3} \frac{\Psi_{V, \bar{h}h}^{*(\lambda_V)}(k^+/q'^+, \underline{k})}{\sqrt{k^+(q'^+ - k^+)}} \langle 0 | b_h d_{\bar{h}}$$

Forshaw and Sandapen, JHEP **1110**, 093 (2011)

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- Perturbative calculation of the virtual photon wavefunction $\Psi^{*(\lambda_\gamma)}_{\gamma^*, \hbar h}$:

$$\Psi_{\gamma^*, h\bar{h}}^{(\lambda_\gamma)}(\tau, \underline{l}) = \delta_{h, -\bar{h}} e e_q \sqrt{\frac{N_c}{4\pi}} \frac{2\tau\bar{\tau}Q}{l^2 + \tau\bar{\tau}Q^2}$$

Dosch *et al.*, Phys. Rev. **D55**, 2602 (1997)

Lepage and Brodsky, Phys. Rev. **D22**, 2157 (1980)

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- Express final result in terms of Fourier transforms $\hat{\Psi}$ of wavefunctions in **transverse coordinate space**:

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$$\mathcal{M}_{V,\{0+,0+\}}^g = \sum_q C_V^q \int_0^1 d\tau \int d^2 \underline{r} \sum_{h,\bar{h}} \hat{\Psi}_{V,\bar{h}h}^q(\tau, -\underline{r}) \hat{\Psi}_{\gamma^*, h\bar{h}}^q(\tau, \underline{r}) \\ \times \frac{4\pi\alpha_s}{N_c} \frac{1}{\tau\bar{\tau}} \int_0^1 dx \frac{H^g(x, \xi, t)}{(x + \xi - i\epsilon)(x - \xi + i\epsilon)}$$

where $r = |\underline{r}|$ is the size of the **color dipole** formed by the $q\bar{q}$ pair.

- Recover leading twist result by taking meson wavefunction at $\underline{r} = 0$.

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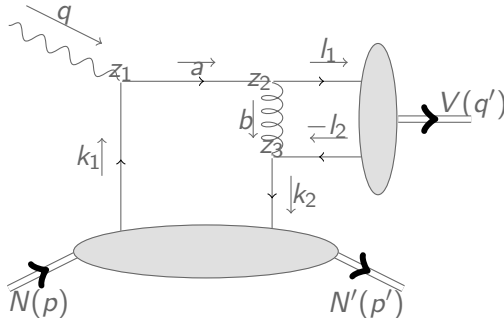
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- From symmetry consideration, restrict to 2 diagrams.



- Polarization of longitudinal virtual photon:

$$\epsilon^\mu(q) = \frac{q^\mu + 4\xi p_2}{Q}$$

- From Ward identity:

$$\not{\epsilon} \rightarrow \frac{4\xi}{Q} \not{p}_2$$

- Above diagram vanishes at leading twist due to traces with structure $\not{p}_2 \not{p}_2$.

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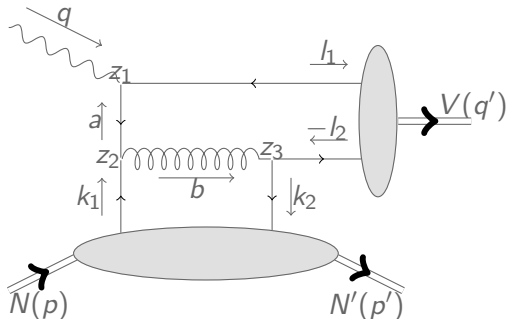
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■ Quark contribution:

- Compute only 1 diagram.
- Get other diagrams by exchanging roles of quark and antiquark:

$$x \leftrightarrow -x$$

$$\tau \leftrightarrow \bar{\tau}$$

$$\mathcal{M}_{V,\{0+,0+\}}^q = \sum_q C_V^q \int_0^1 d\tau \int d^2\bar{r} \sum_{h,\bar{h}} \hat{\Psi}_{V,\bar{h}h}^q(\tau, -\bar{r}) \hat{\Psi}_{\gamma^*, h\bar{h}}^q(\tau, \bar{r})$$

$$\times \frac{4\pi\alpha_s}{N_c} \frac{1}{\tau\bar{\tau}} \int_{-1}^1 dx \frac{C_F x H^q(x, \xi, t)}{(x + \xi - i\epsilon)(x - \xi + i\epsilon)}$$

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■ Modified Perturbative Approach result ($Q^2 \simeq 2\xi s$):

$$\text{Im } \mathcal{M}_{V,\{0+,0+\}} = \int_0^1 d\tau \int d^2 \underline{r} \sum_{h,\bar{h}} \sum_q C_V^q \left(\hat{\Psi}_{V,\bar{h}h}^q(\tau, -\underline{r}) \hat{\Psi}_{\gamma^*,h\bar{h}}^q(\tau, \underline{r}) \right)$$

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$$\times \left[s \frac{\alpha_s \pi^2}{N_c} \frac{4}{\tau \bar{\tau} Q^2} \left(H^g(\xi, \xi, 0) + C_F \xi H_S^g(\xi, \xi, 0) \right) \right]$$

with $H_S^g(x, \xi, t) = H^g(x, \xi, t) - H^g(-x, \xi, t)$.

Besse *et al.*, in preparation.

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■ k_T -factorization and dipole models result:

$$\text{Im } \mathcal{M}_{V,\{0+,0+\}}^g = \int_0^1 d\tau \int d^2 \underline{r} \sum_{h,\bar{h}} \sum_q C_V^q \left(\hat{\Psi}_{V,\bar{h}h}^q(\tau, -\underline{r}) \hat{\Psi}_{\gamma^*,h\bar{h}}^q(\tau, \underline{r}) \right)$$

$$\times \left[\text{Im } \mathcal{N}^{q\bar{q}} \left(x, \xi = \frac{x}{2}, \underline{r} \right) \right]$$

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Besse *et al.*, Nucl. Phys. **B867**, 19 (2013)

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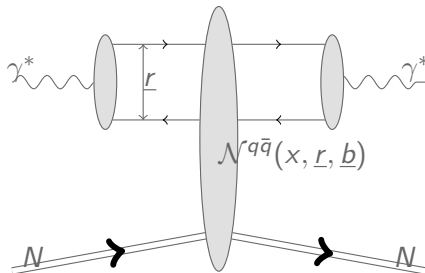
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- Identify gluon contributions:

$$\text{Im } \mathcal{N}^{q\bar{q}} \left(x, \xi = \frac{x}{2}, \underline{r} \right) = s \frac{\alpha_s \pi^2}{N_c} \frac{4}{\tau \bar{\tau} Q^2} H^g(\xi, \xi, 0)$$

Besse et al., in preparation.

- Connection** between two **different objects** coming from two **different formalisms**!



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- **Factorized Ansatz.** For $i = g$, sea or val :

$$H_i(x, \xi, t) = \int_{|\alpha|+|\beta|\leq 1} d\beta d\alpha \delta(\beta + \xi\alpha - x) f_i(\beta, \alpha, t)$$

$$f_i(\beta, \alpha, t) = e^{b_i t} \frac{1}{|\beta|^{\alpha' t}} h_i(\beta) \pi_{n_i}(\beta, \alpha)$$

$$\pi_{n_i}(\beta, \alpha) = \frac{\Gamma(2n_i + 2)}{2^{2n_i+1} \Gamma^2(n_i + 1)} \frac{(1 - |\beta|)^2 - \alpha^2)^{n_i}}{(1 - |\beta|)^{2n_i+1}}$$

- Expressions for h_i and n_i :

$$h_g(\beta) = |\beta| g(|\beta|) \quad n_g = 2$$

$$h_{\text{sea}}^q(\beta) = q_{\text{sea}}(|\beta|) \text{sign}(\beta) \quad n_{\text{sea}} = 2$$

$$h_{\text{val}}^q(\beta) = q_{\text{val}}(\beta) \Theta(\beta) \quad n_{\text{val}} = 1$$

Goloskokov and Kroll, Eur. Phys. J. **C42**, 281 (2005)

- Comparison to **existing DVCS measurements** at LO.

Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

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- Asymptotic leading twist DA:

$$\phi_{||}(\tau) = 6\tau\bar{\tau}$$

- Fit $B(Q^2)$ -slope from experimental data (H1 Coll.):

$$\frac{d\sigma_L}{dt}(t) \simeq e^{-B(Q^2)|t|} \left. \frac{d\sigma_L}{dt} \right|_{t=0}$$

Aaron *et al.*, JHEP **05**, 032 (2010)

- Factorization scale:

$$\mu = \sqrt{Q^2 + m_V^2}$$

with systematic uncertainty evaluated by covering the range between $\sqrt{Q^2 + m_V^2}/4$ and $4\sqrt{Q^2 + m_V^2}$.

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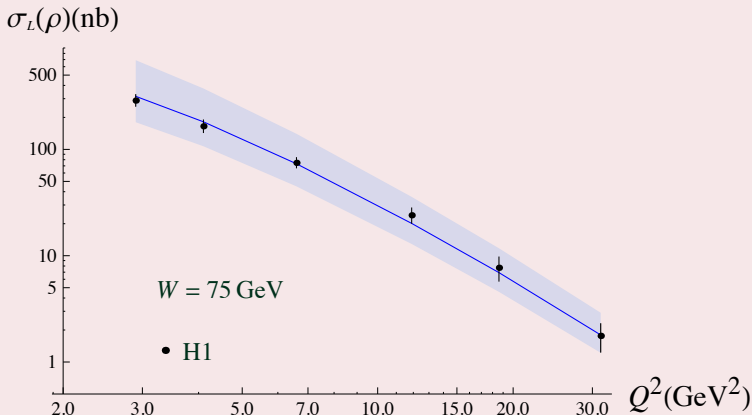
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Besse et al., in preparation

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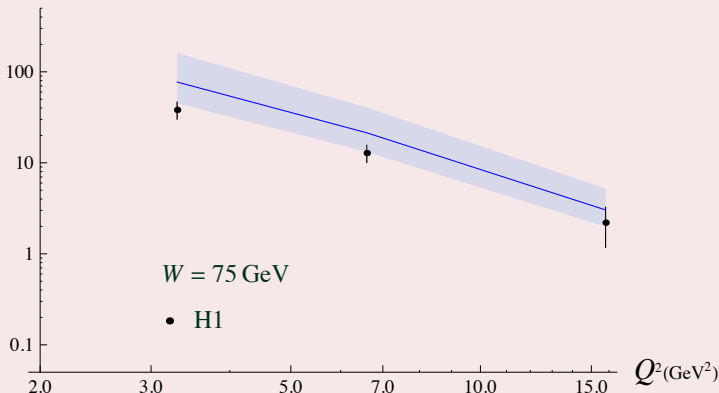
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$\sigma_L(\phi)(\text{nb})$



Besse et al., in preparation

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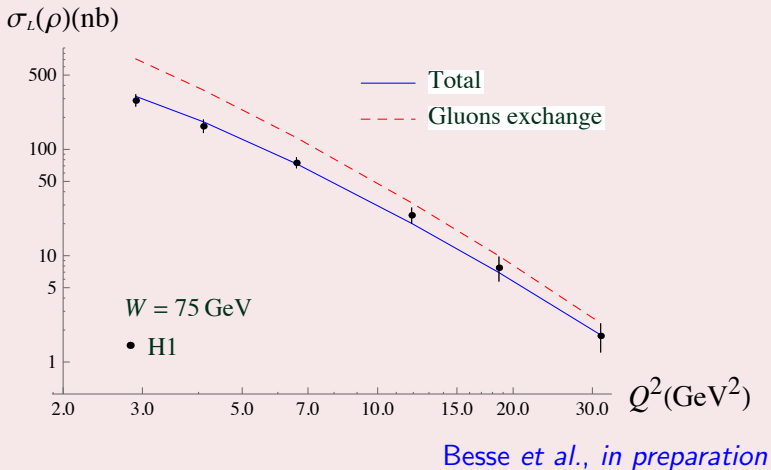
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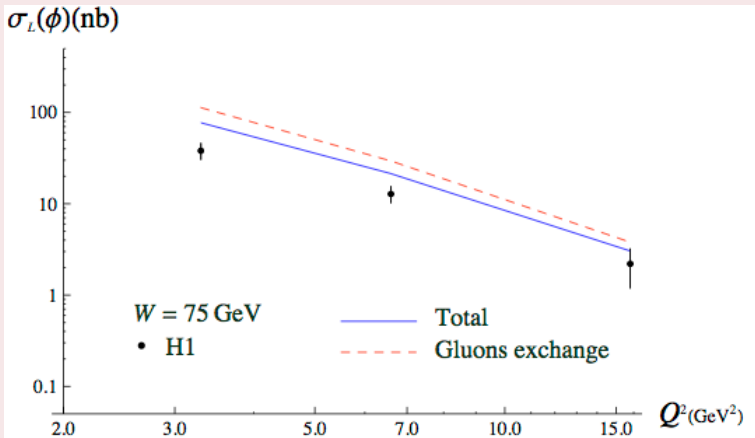
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- The GPD physics program is a mature field dealing with **well-defined** theoretical objects, **cleanly** related to experimental data, with present **experimental relevance**.
- To make one further step, one requires:
 - Better GPD and meson DA **models**.
 - Better control of **subdominant effects** on theoretical description of experimental channels.
 - Combined analysis of hard exclusive channels.
- Implementation of a DVMP computational code in an **software platform** for multi-channel GPD analysis.
- Intriguing relation between **GPD / collinear factorization** and **color dipole / k_T factorization** pictures. Open the way to alternative discussions of **saturation** physics?

