Sparse representations and Filtering

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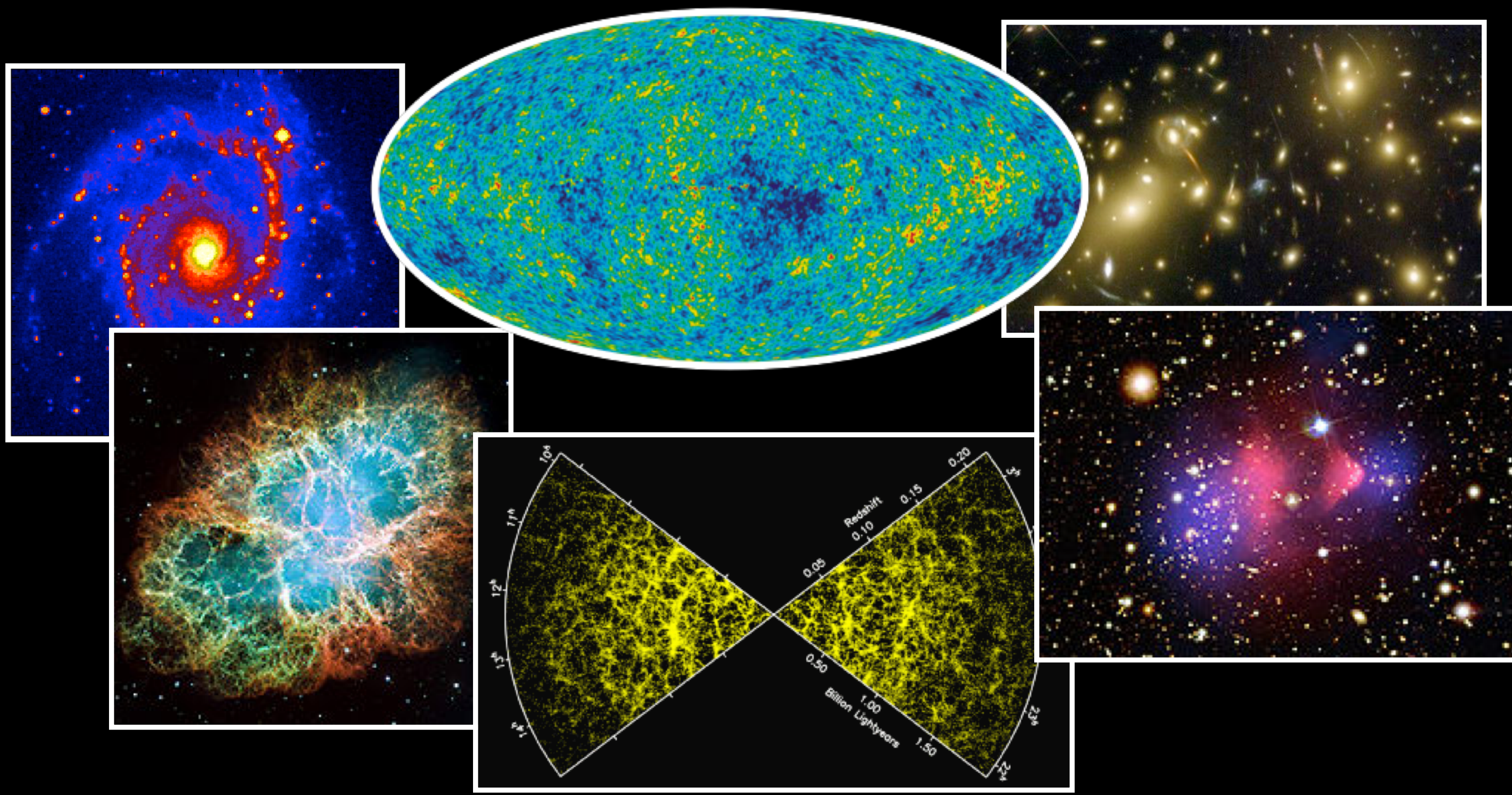


The Fourier Transform

$$f(k) = \int_{-\infty}^{+\infty} f(x) e^{-2\pi ikx} dx$$

- ✓ Where *f(k)* are the Fourier coefficient of the function *f(x)*
- ✓ The analysing function is $e^{-2\pi ikx}$
- ✓ *k* is the frequence parameter

➡ The Fourier Transform is the best for representing harmonic components of a signal

➡ The Fourier Transform provides a poor representation of non stationary signals and discontinuities.

The Wavelet Transform

$$W(a,b) = K \int_{-\infty}^{+\infty} \psi^{*}(\frac{x-b}{a}) f(x) dx$$

- ✓ Where *W(a,b)* are the Wavelet coefficients of the function *f(x)*
- ✓ The analysing function is $\psi(x)$
- ✓ *a (>0)* is the scale parameter and *b* is the position parameter

When the scale *a* varies the filter $\psi(x)$ is only reduced or dilated while keeping the same pattern.

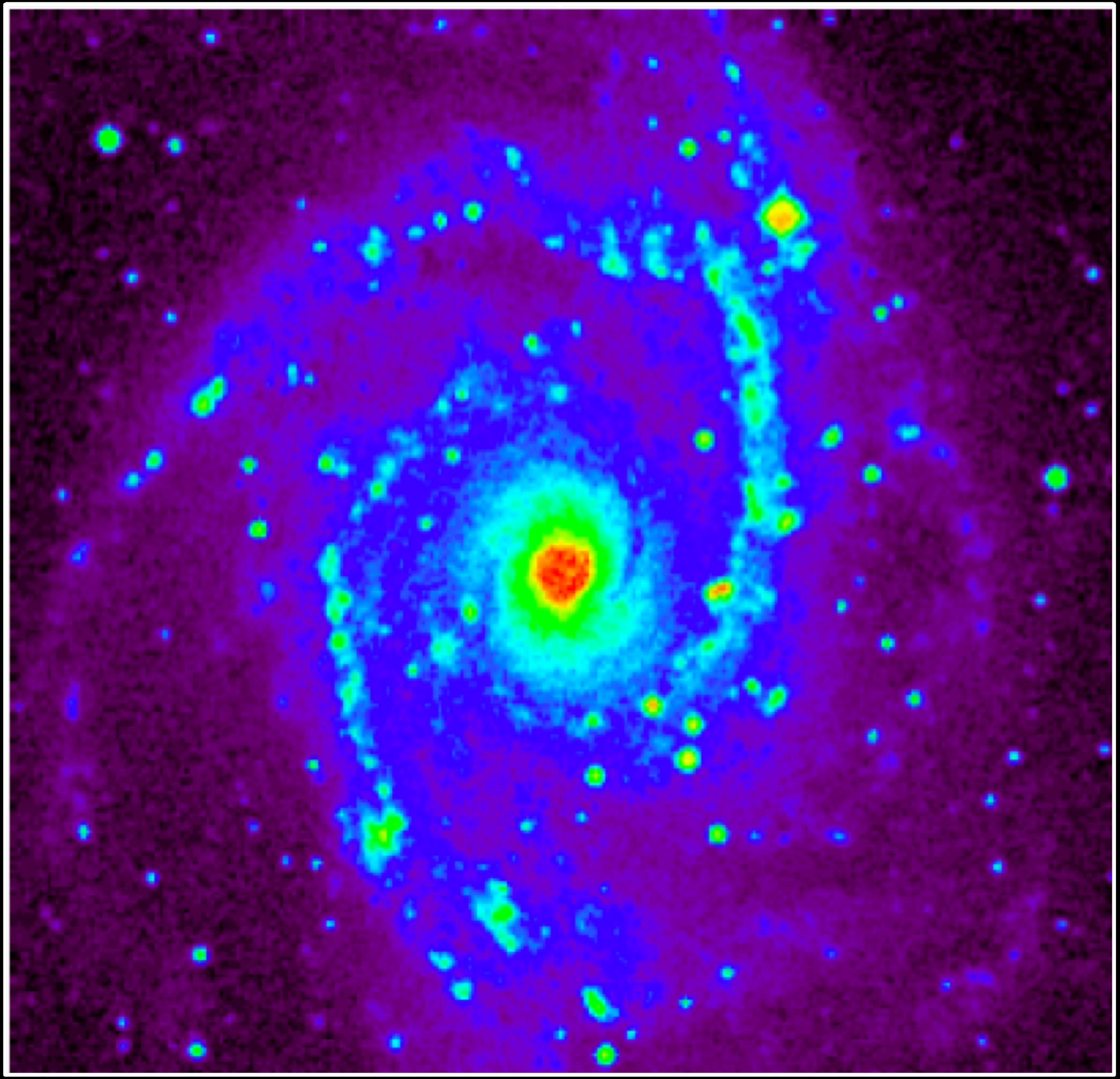
The inverse transform is:

$$f(x) = \frac{1}{C_\psi} \int_{-\infty}^{+\infty} \int_{0}^{+\infty} \frac{1}{\sqrt{a}} W(a,b) \psi(\frac{x-b}{a}) \frac{dadb}{a^2}$$

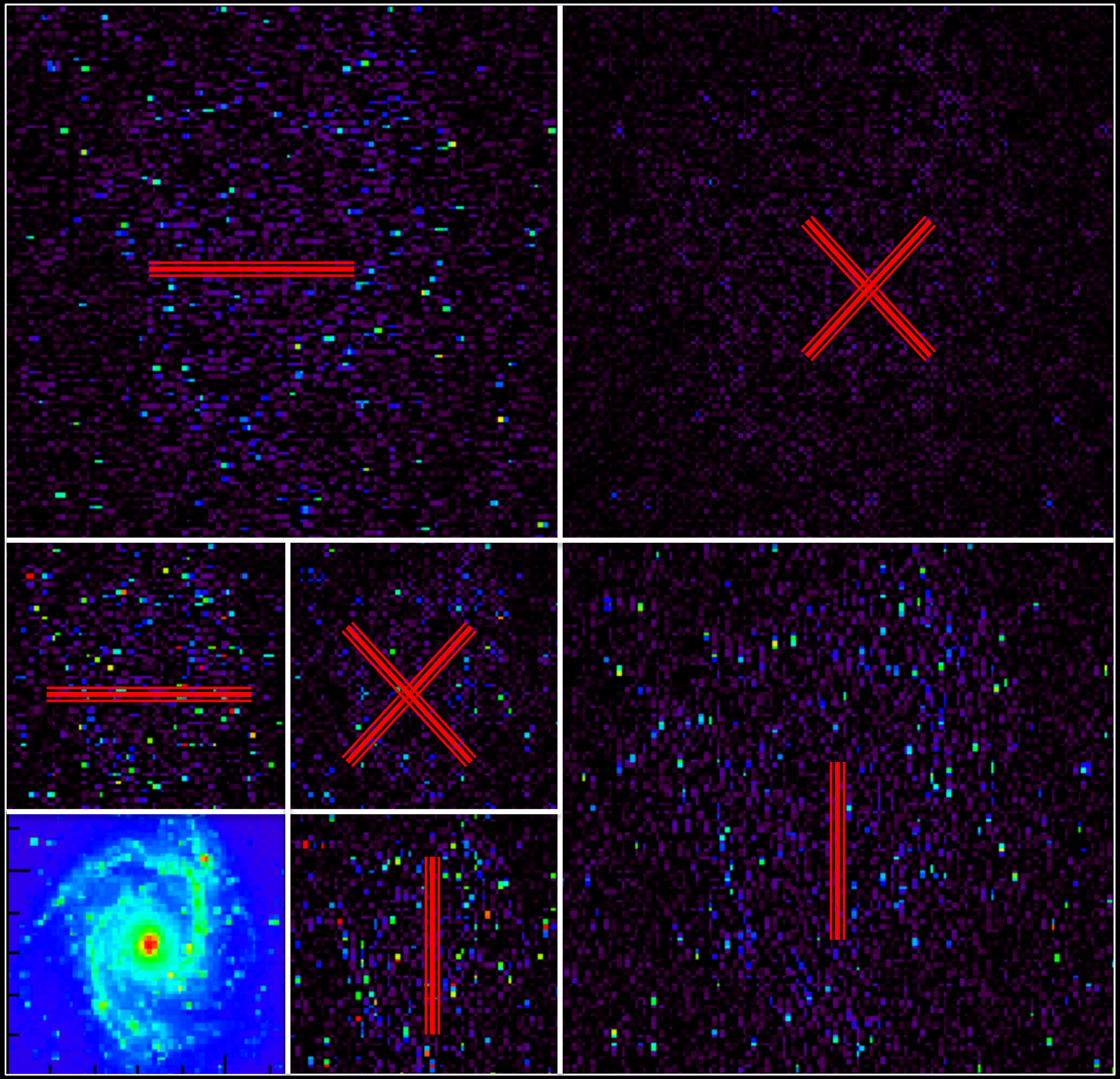
where: $C_\psi = \int_{-\infty}^{+\infty} |\hat{\psi}(t)|^2 \frac{dt}{t} < +\infty$

➡ The Wavelet Transform is the best for representing piecewise smooth images

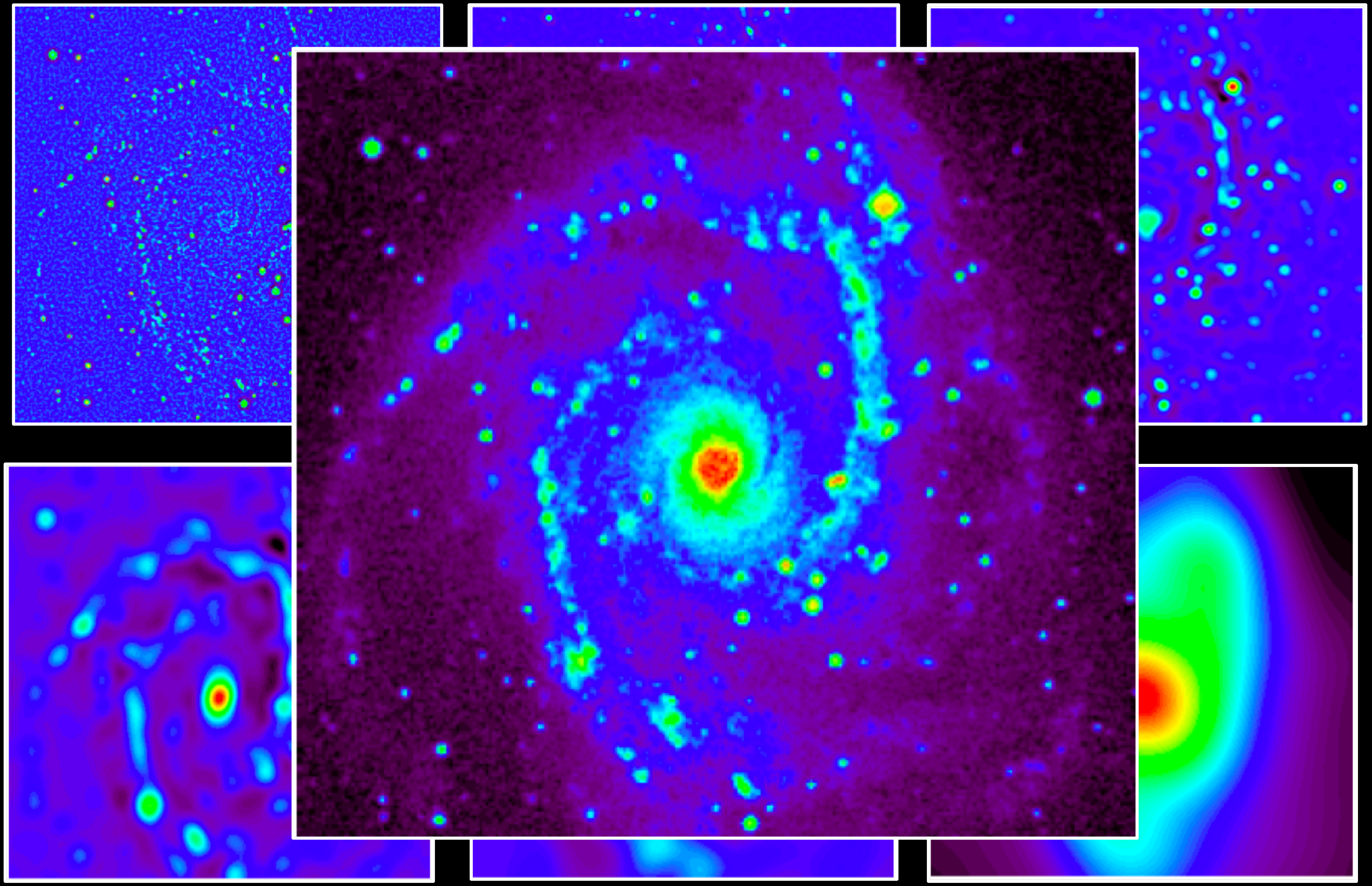
Orthogonal Wavelet Transform



Orthogonal Wavelet Transform

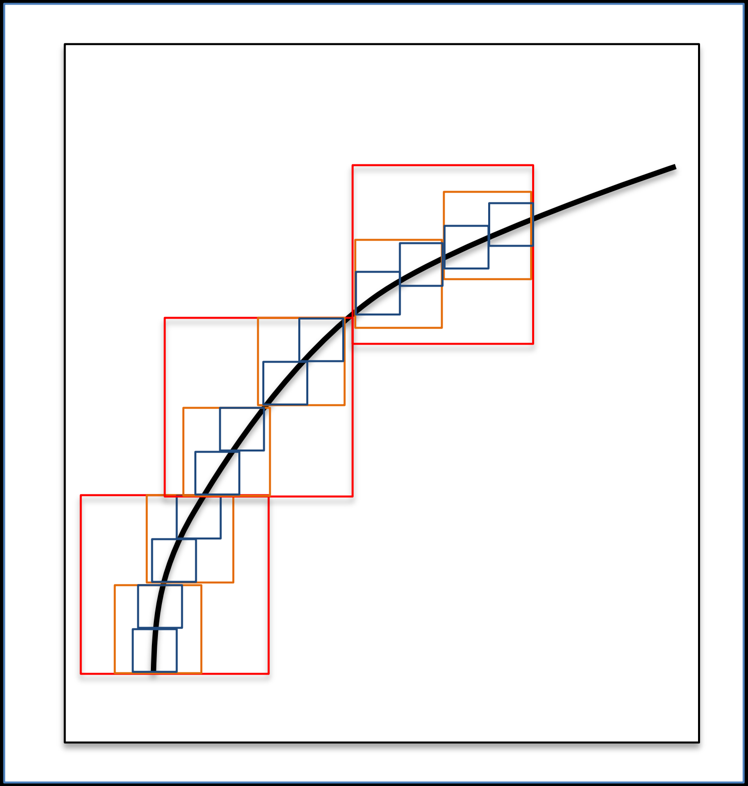


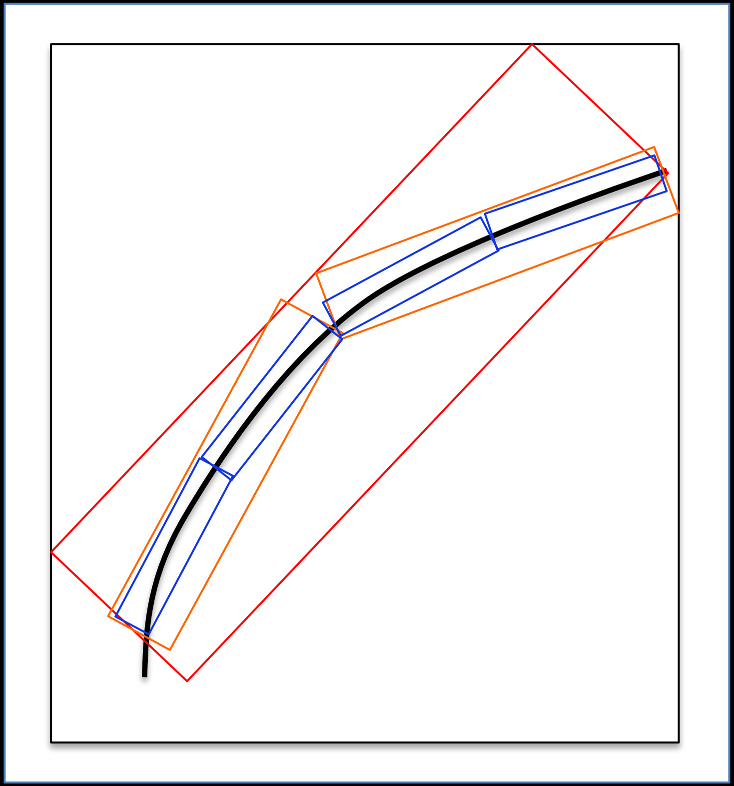
Undecimated Isotropic Wavelet Transform



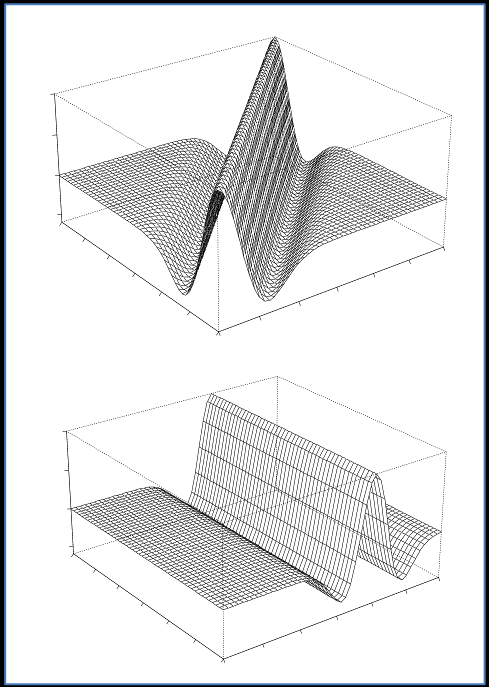
WAVELET TUTORIAL

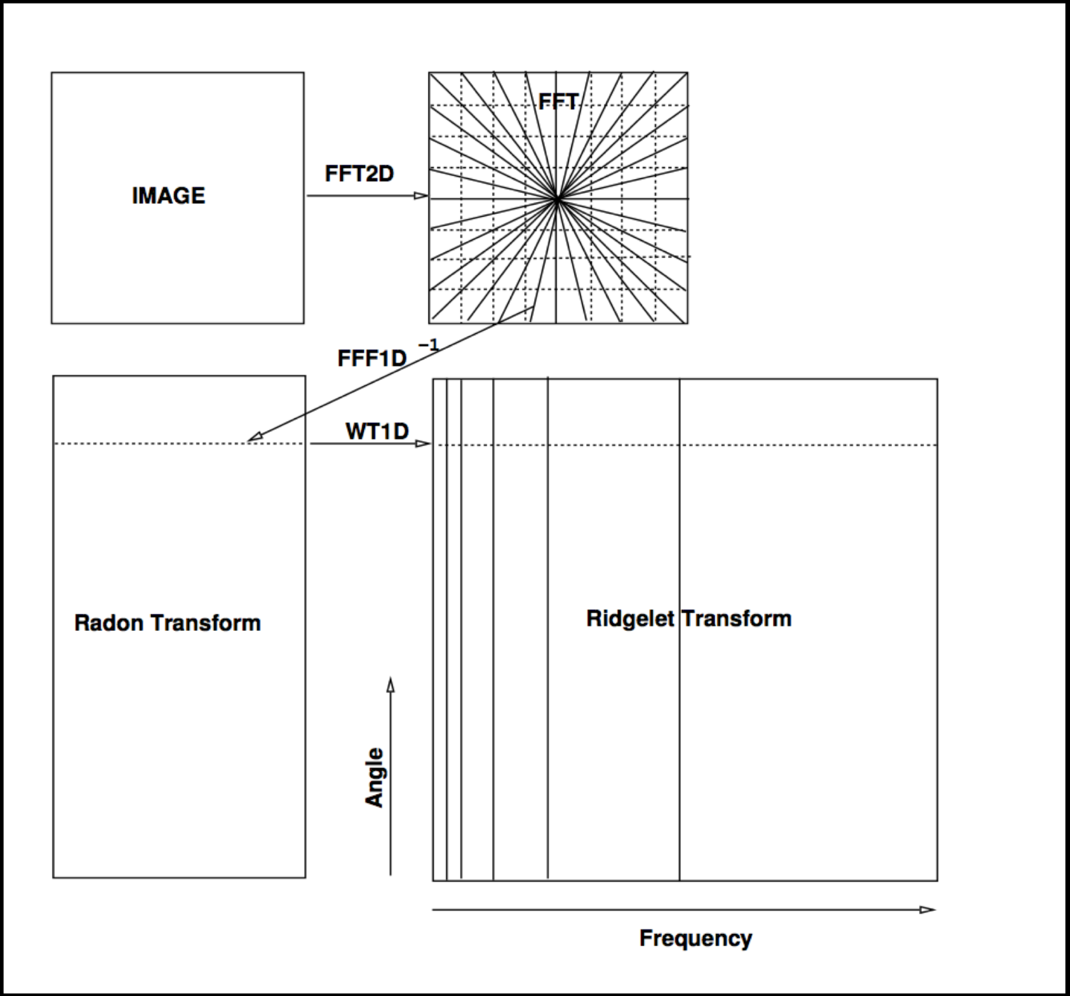
Other representations


Wavelet representation

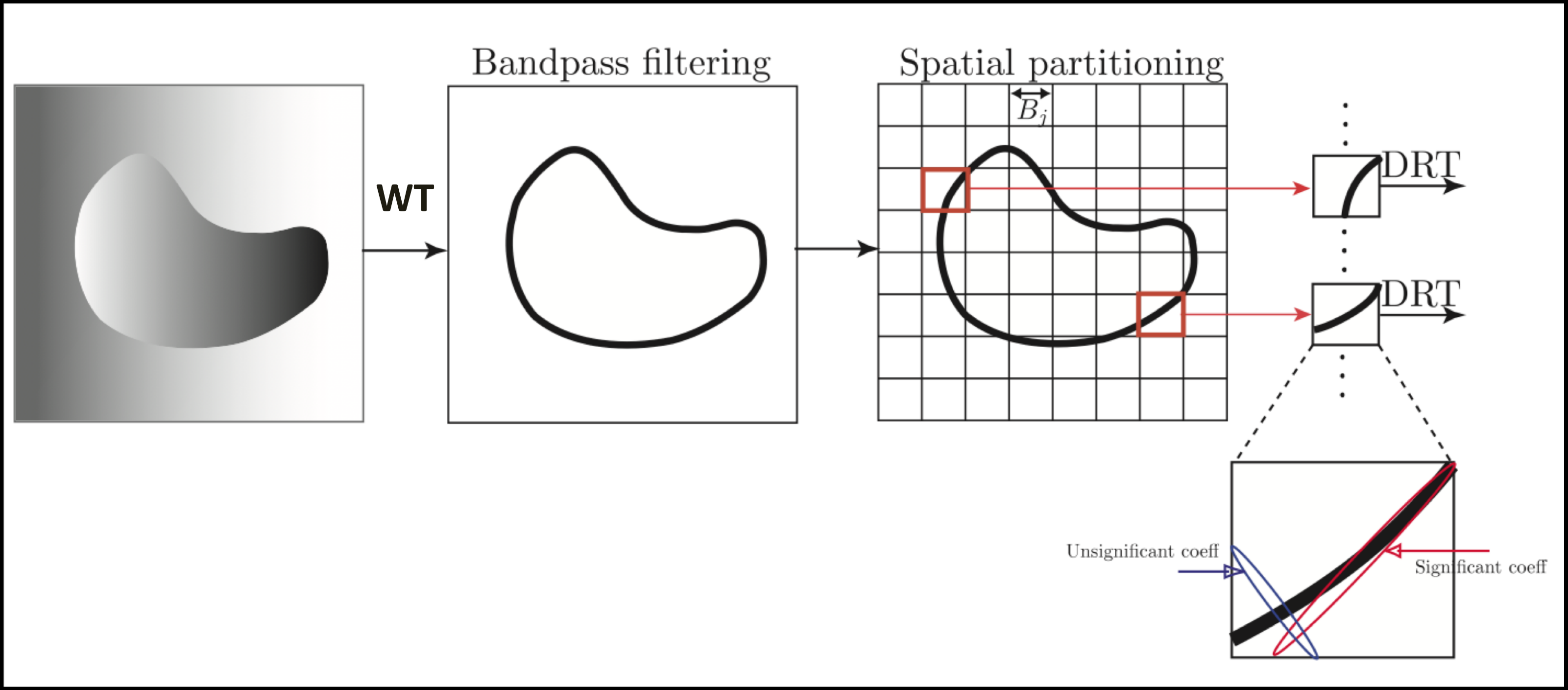

Anisotropic representation

Other transformations : Ridgelet transform





Other transformations : Curvelet transform



What is a good representation for data?

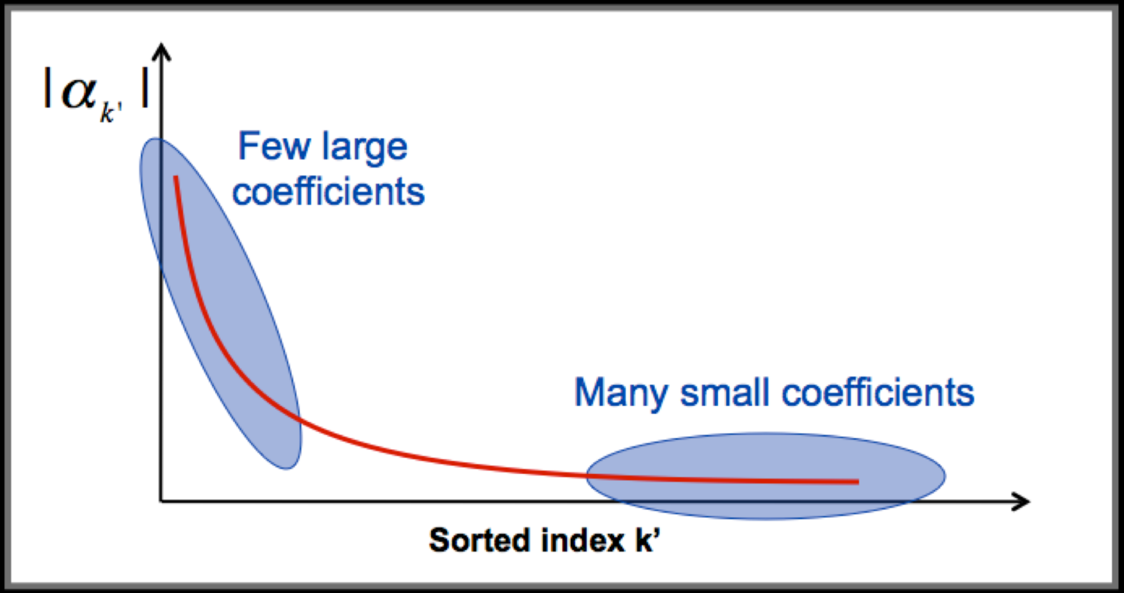
- ✓ We seek representations of the signal (*f*) as linear combination of:
  - ✓ basis elements
  - ✓ frames
  - ✓ dictionary elements

$$f = \sum_k \underbrace{a_k}_{\text{coefficients}} \underbrace{\mathbf{b}_k}_{\text{basis, frame}}$$

- ✓ The analyzing functions should extract the features of interest:
  - ✓ Harmonic features
  - ✓ Isotropic features
  - ✓ Anisotropic feature
- ✓ Recent methods exploit the sparsity of the coefficients

What is sparsity ?

Considering a transform : $\alpha = \Phi^T X$

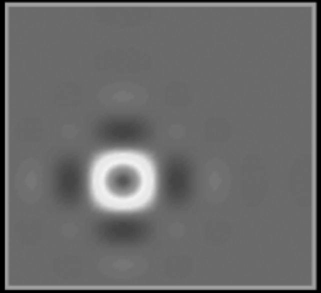


- ✓ Why do you need sparsity:
  - ✓ Data compression
  - ✓ Feature extraction, detection
  - ✓ Image restoration
  - ✓ ….

Signal and image representations

- ✓ Local DCT :
  - ✓ Stationary textures
  - ✓ Locally oscillatory

- ✓ Wavelet Transform
  - ✓ Piecewise smooth
  - ✓ Isotropic structures

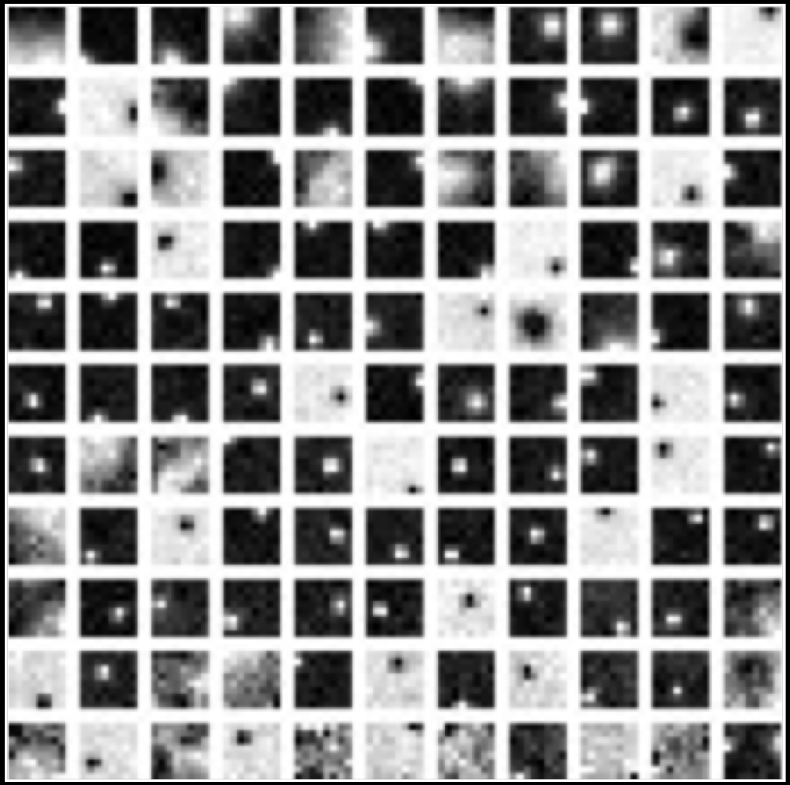


- ✓ Curvelet Transform
  - ✓ Piecewise smooth
  - ✓ Edge structures

Dictionary Learning

(by Simon Beckouche)

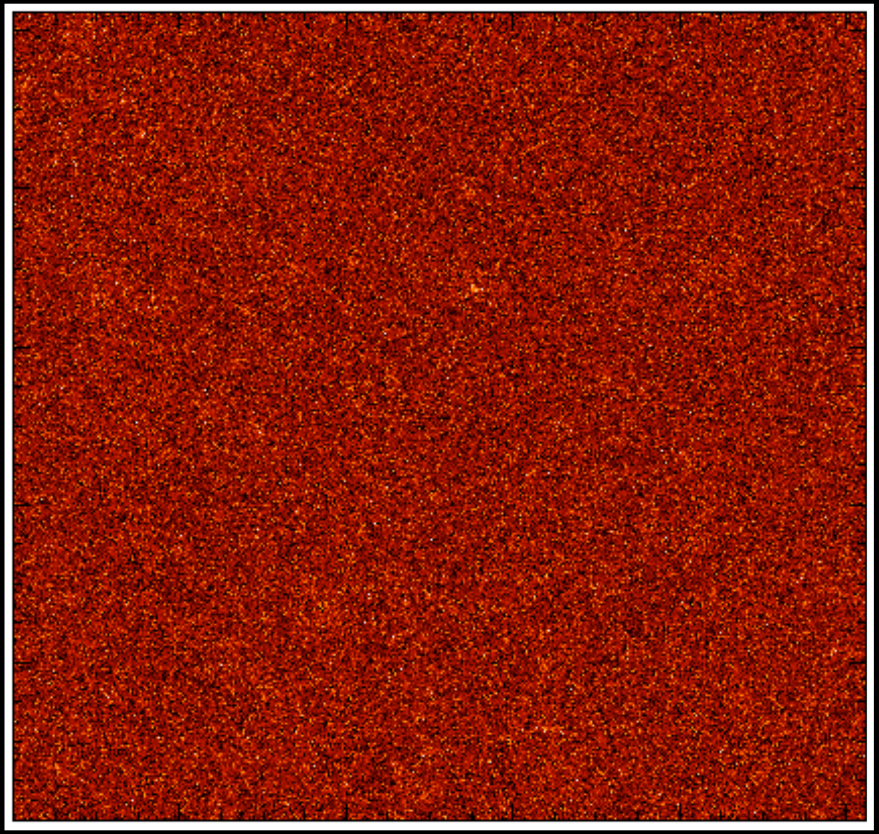
Data



Learned Dictionary

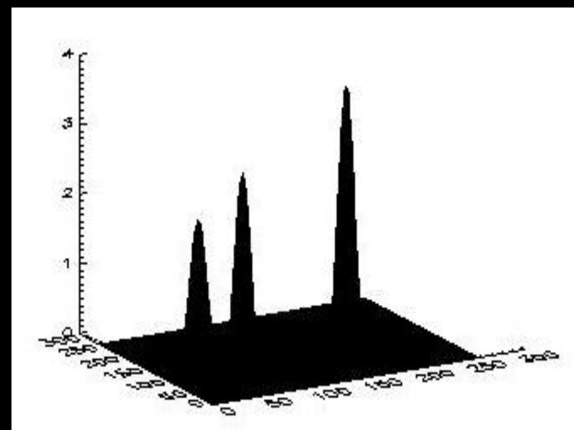
SPARSE TUTORIAL

How to reduce the observational noise ?

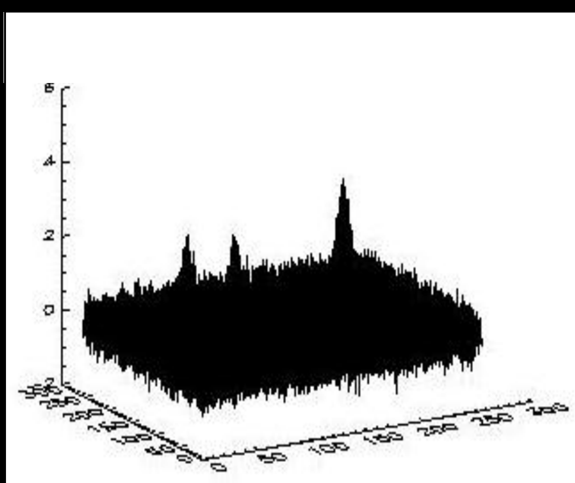


Standard methods based on a linear filter

(i.e. Gaussian filtering)

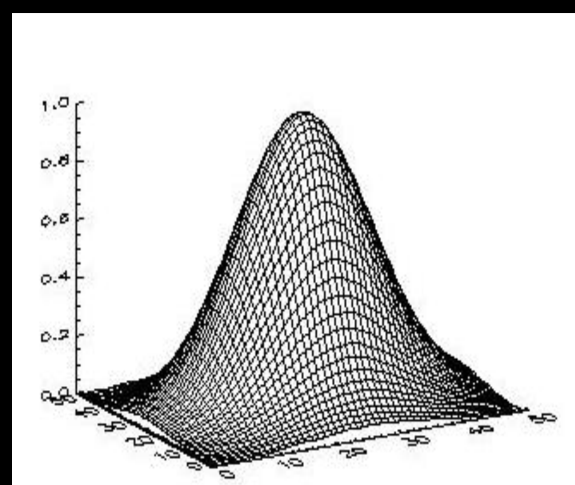


Signal



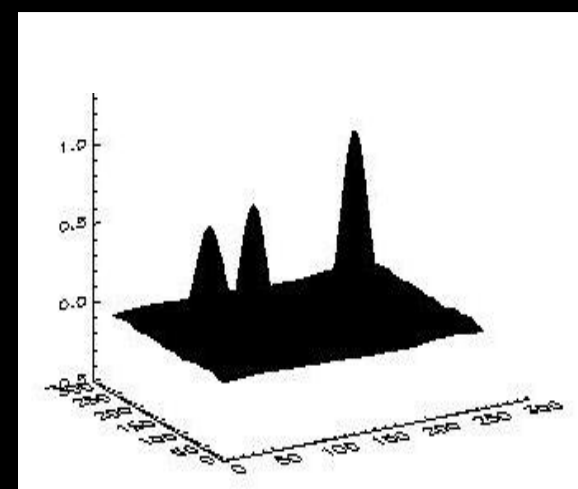
Signal + noise

⊗



Gaussian function (σ)

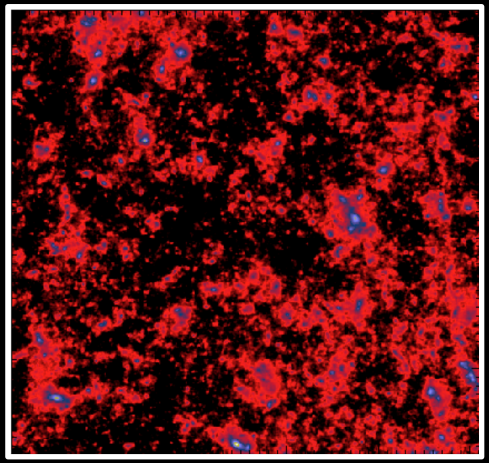
=



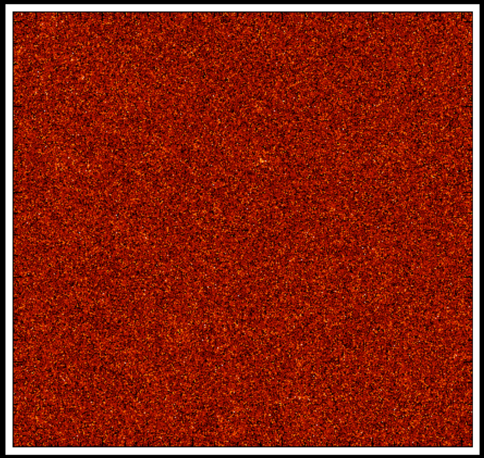
Filtered signal

Standard methods based on a linear filter

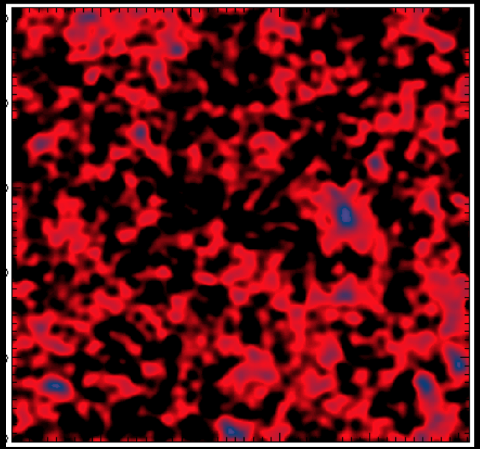
(i.e. Gaussian filtering)



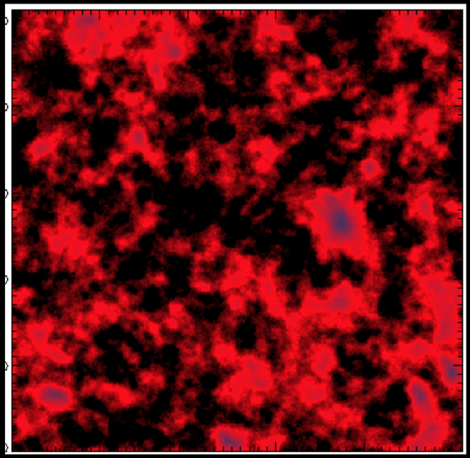
Signal



Signal + noise

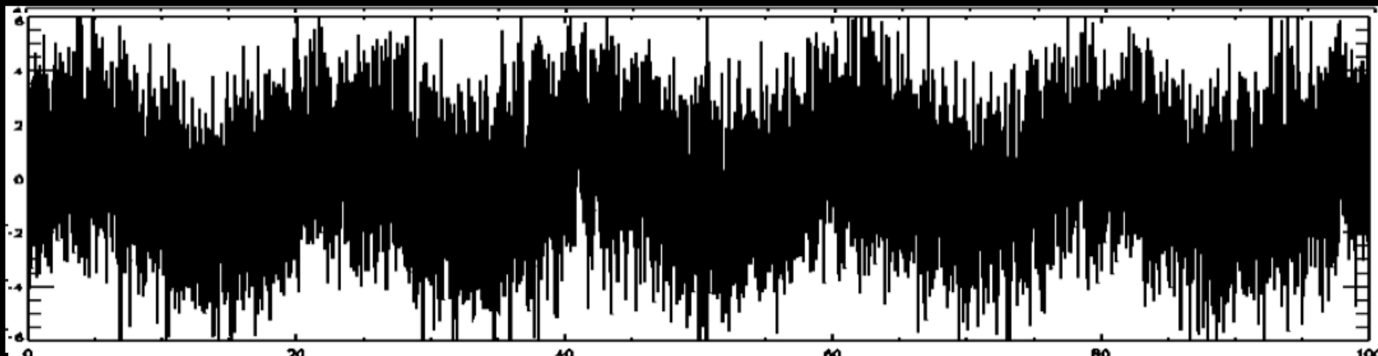


Gaussian filtered (σ =0.6 arcmin)



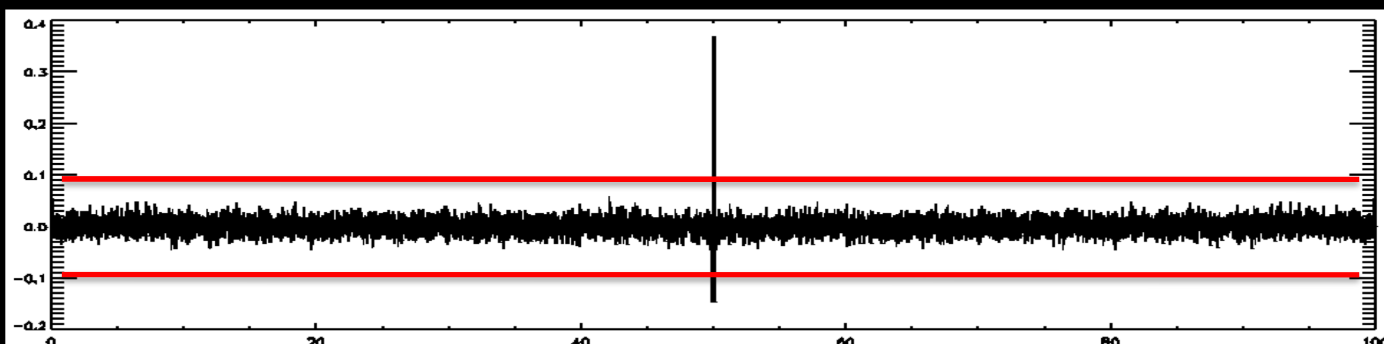
Wiener filter

Basic Example

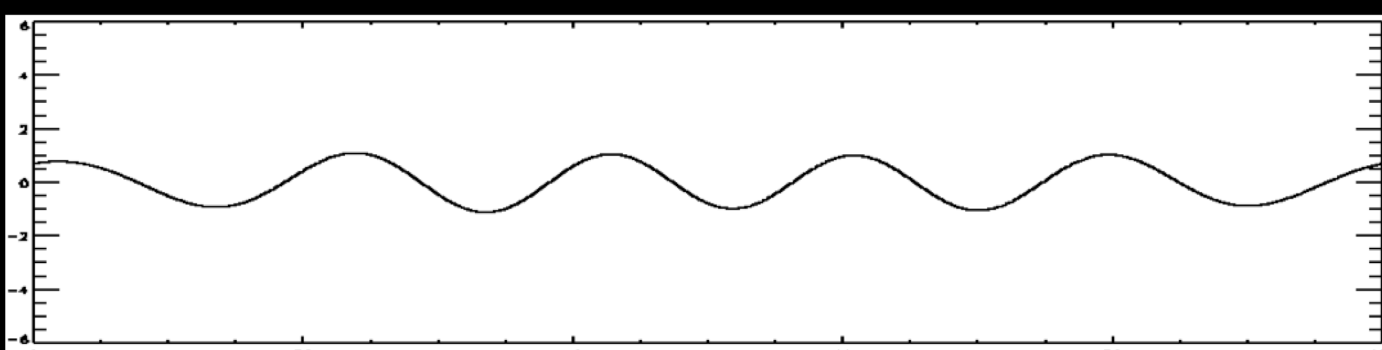


Noisy signal

Original signal

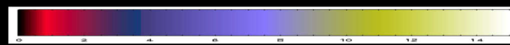


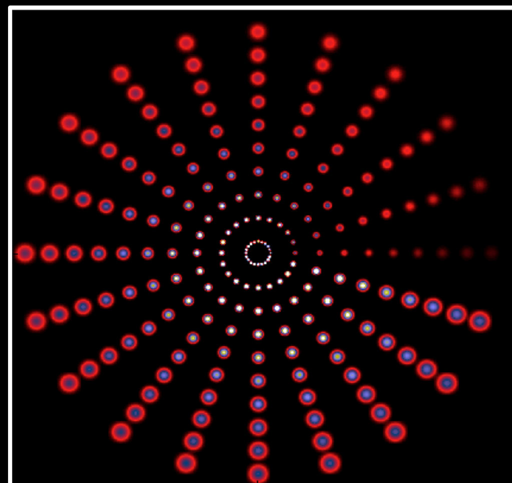
Fourier Transform



Filtered Signal

Adapted Representations





Test image 1

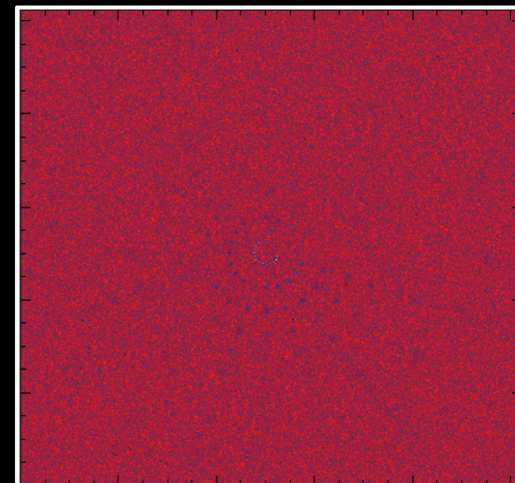
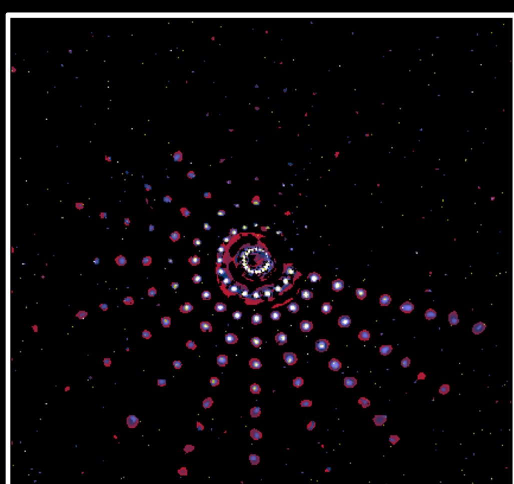
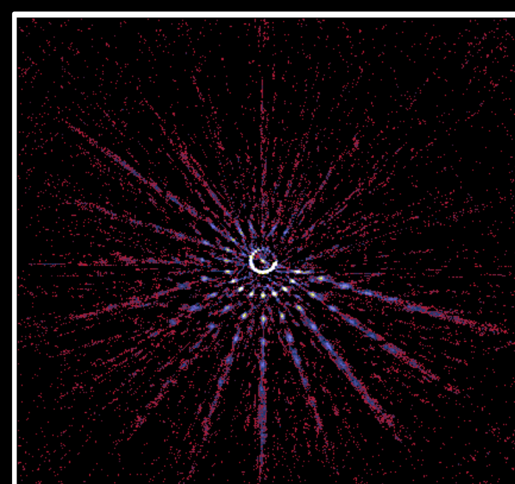


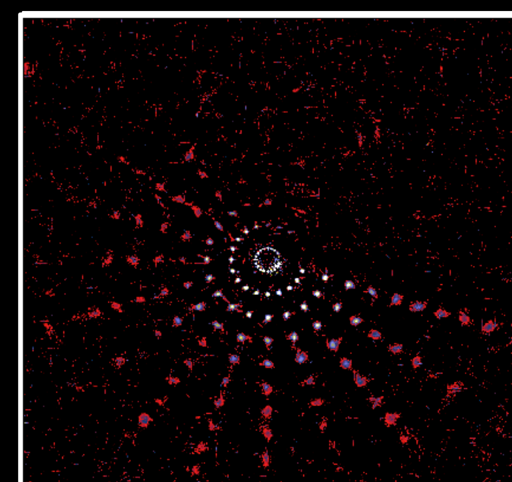
Image test 1 + noise



Wavelet filtering



Ridgelet filtering



Curvelet filtering

Adapted Representations

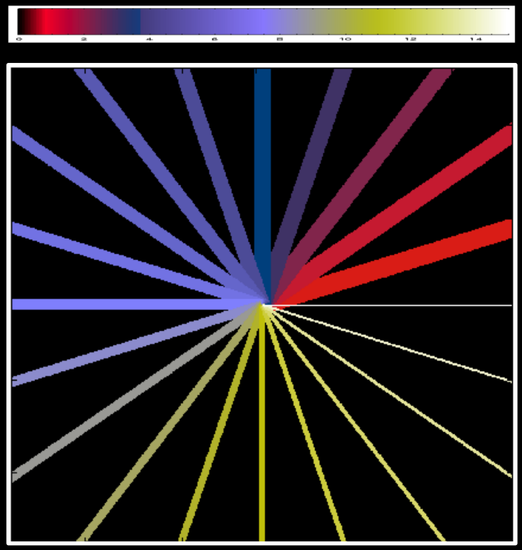


Image test 2

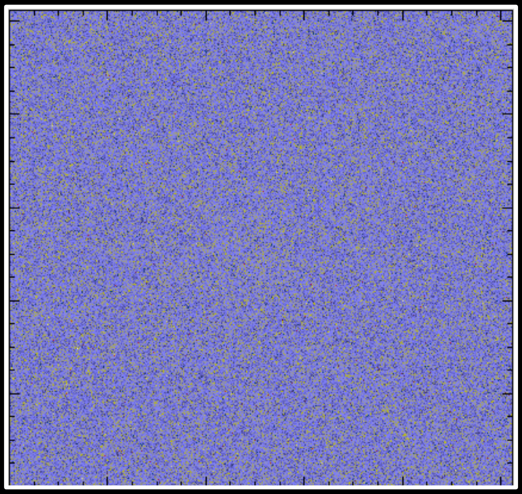
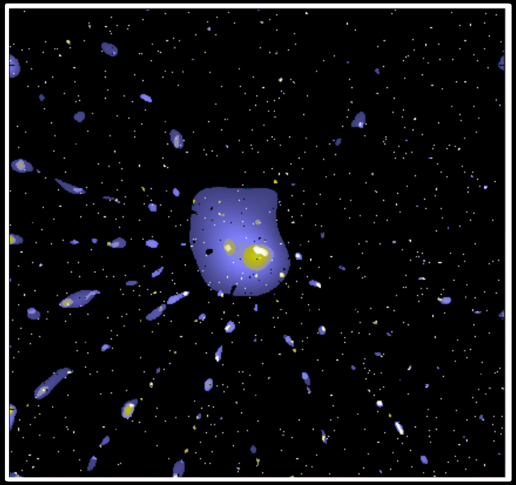
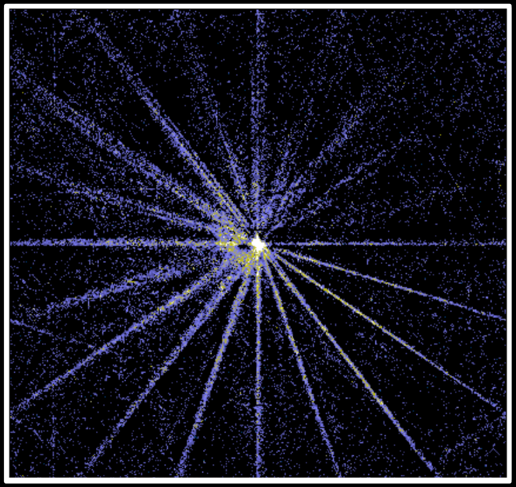


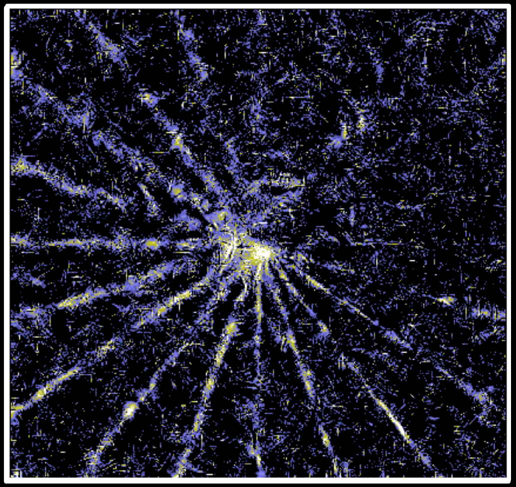
Image test 2 + noise



Wavelet filtering



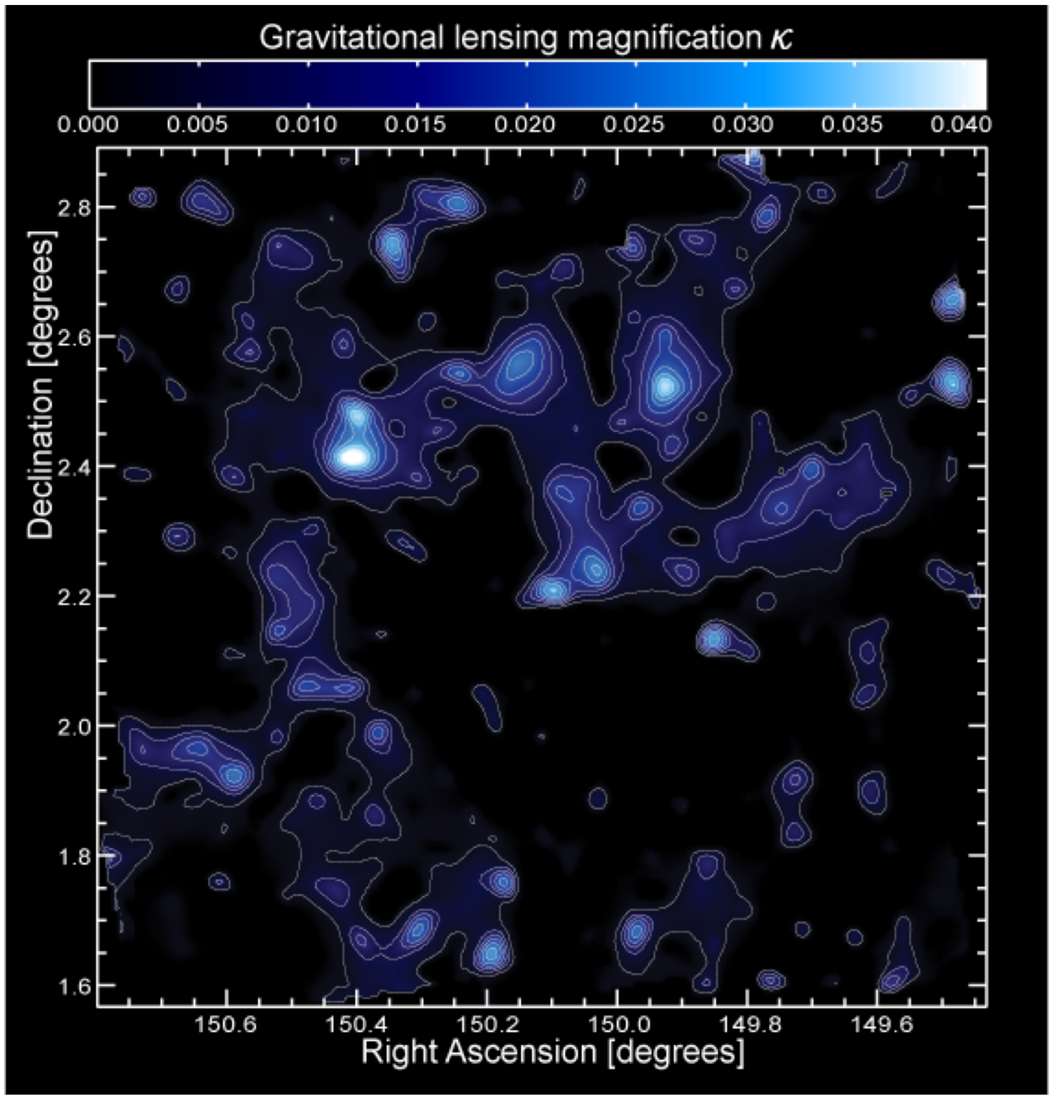
Ridgelet filtering



Curvelet filtering

Dark matter Map

- HST observations -



FILTERING TUTORIAL