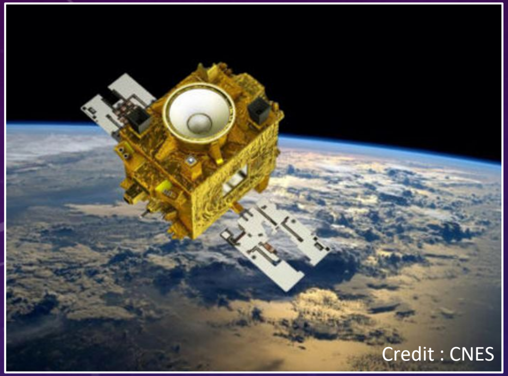


THE MICROSCOPE MISSION

DEALING WITH MISSING DATA

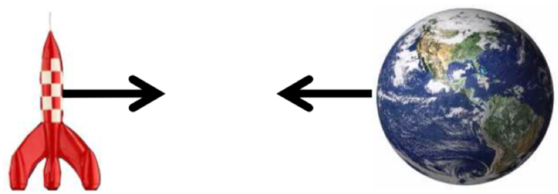
SANDRINE PIRES

http://www.cosmostat.org/people/sandrine-pires

EQUIVALENCE PRINCIPLE

The **Equivalence Principle (EP)** is a fundamental law of physics that states that gravitational mass is equal or proportional to inertial mass.

Gravitation Law



$$F = G\frac{M_g m_g}{r^2}$$

Newton law

$$\Gamma = \frac{F}{m_i}$$

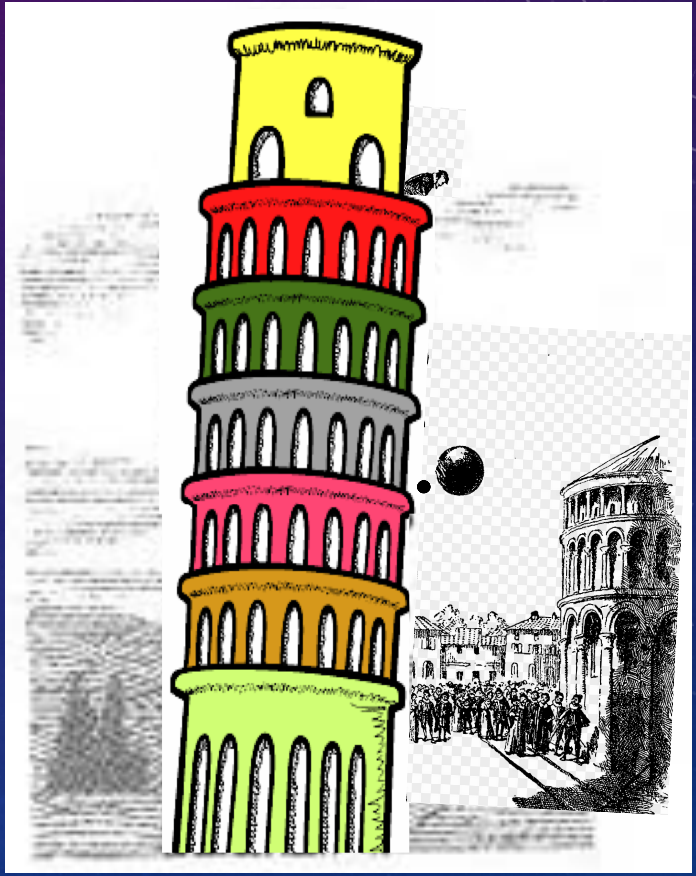
$$\Gamma = G\frac{M_g}{r^2}\frac{m_g}{m_i}$$

$\Rightarrow$ All the bodies subject to a same gravitational field, fall with the same acceleration.

WEAK EQUIVALENCE PRINCIPLE

Galileo (XVII Century):

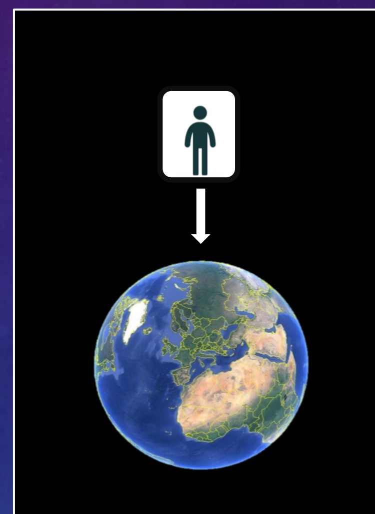
"In a vacuum with an uniform gravitational field, all the bodies fall with the same acceleration whatever their mass or composition"



STRONG EQUIVALENCE PRINCIPLE

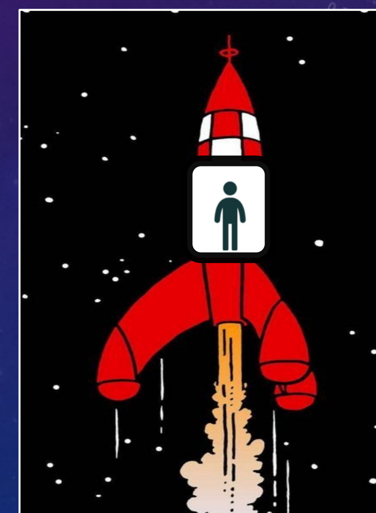
Einstein (XX Century) :

"There is no experiment that can distinguish a uniform acceleration from a uniform gravitational field.



On Earth

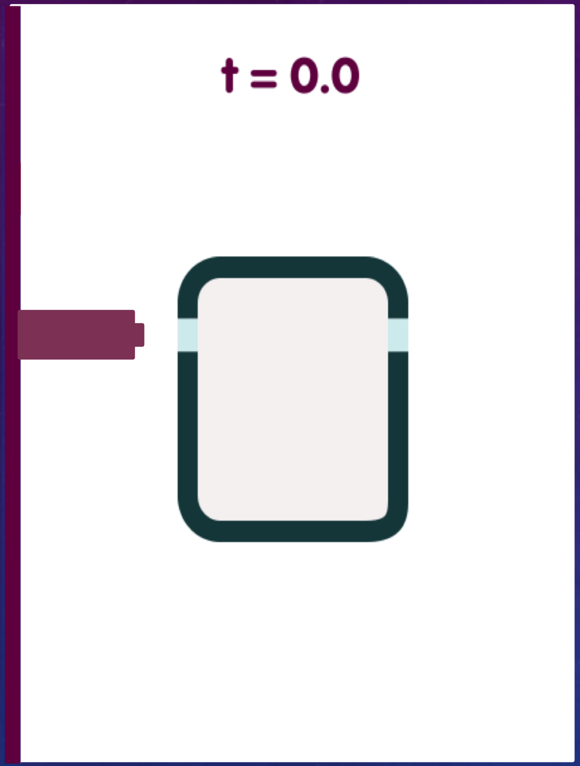
⇔



In Space

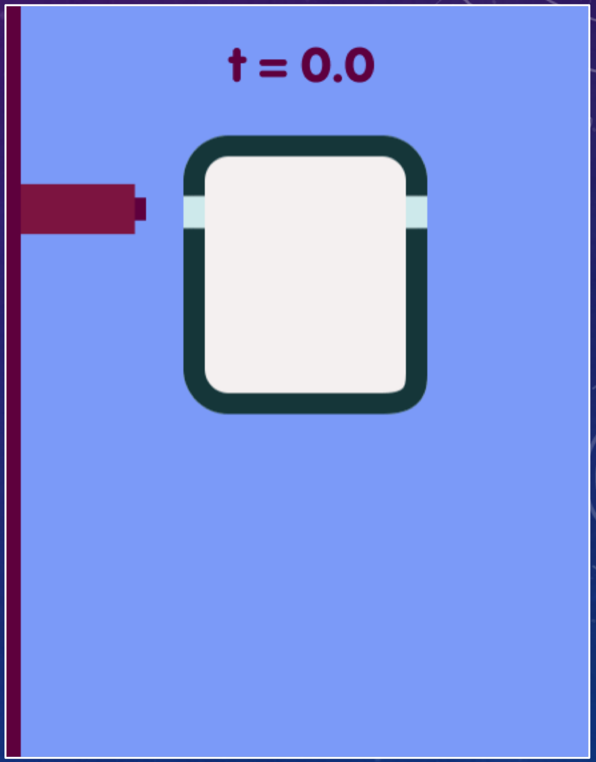
EQUIVALENCE PRINCIPLE AND DEFLECTION OF LIGHT

Both observers have to agree that the light finally leave the elevator



Inside view

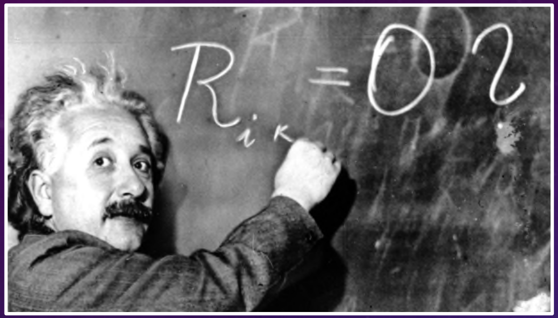
Light travels along straight light.
Laser beam exits the elevator at the same height as it enters the elevator.

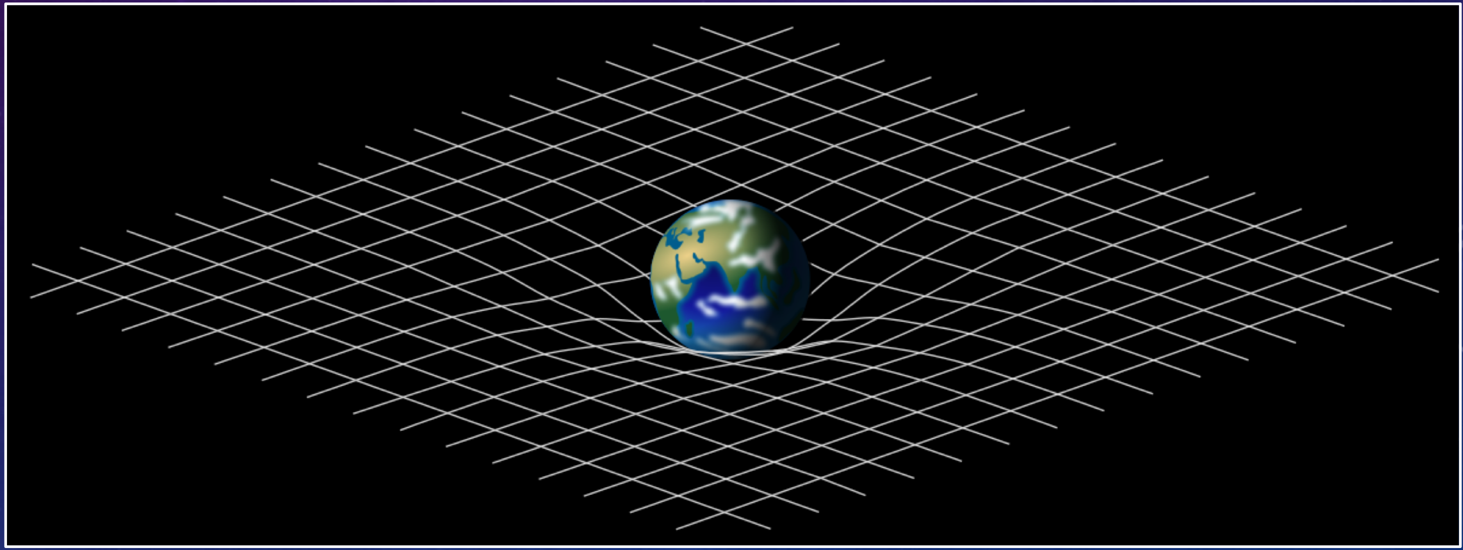


Outside view

The light ray cannot remain horizontally. It has to bent

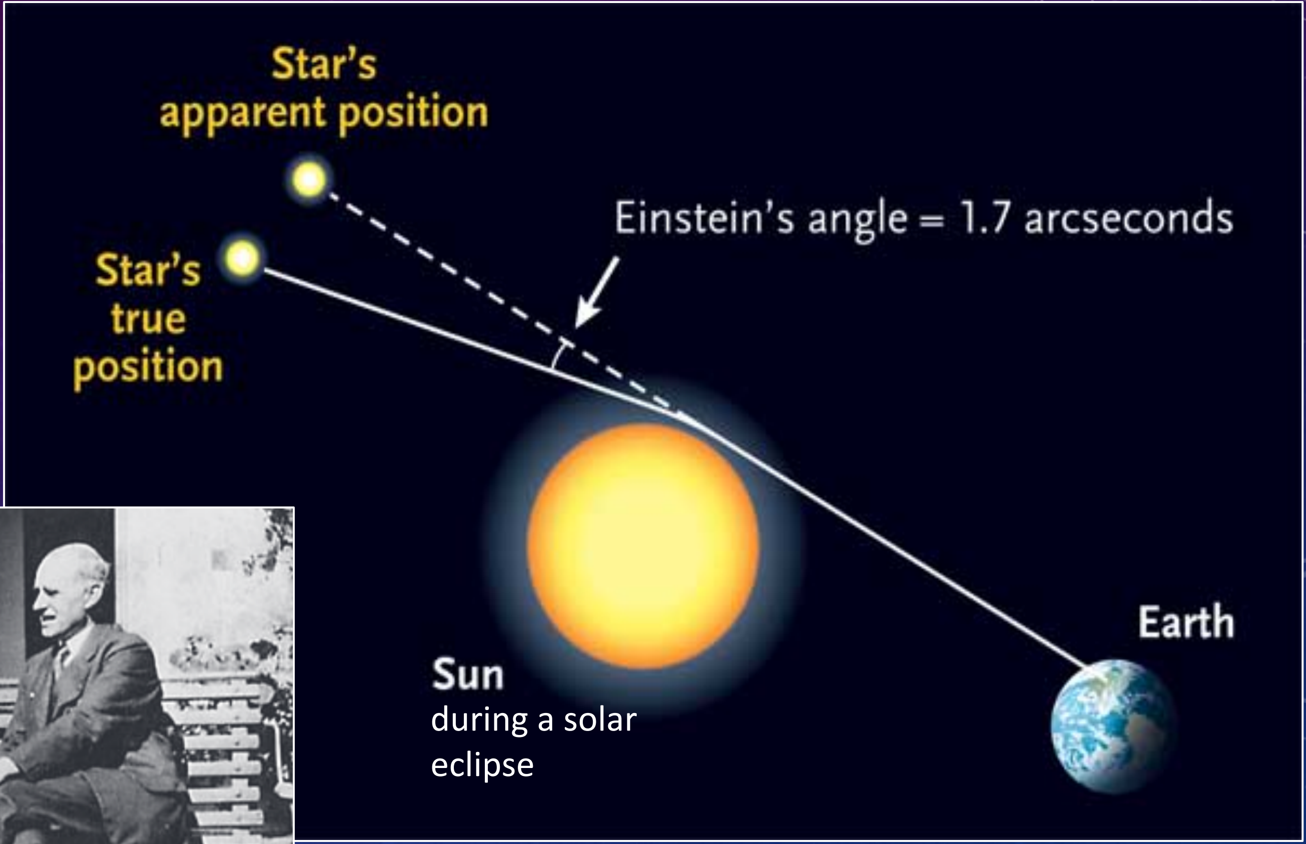
FUNDAMENTAL PILAR OF THE GENERAL RELATIVITY

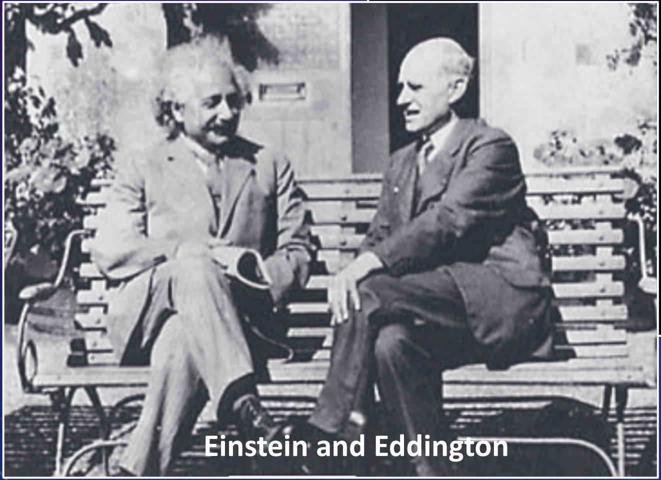




The **equivalence principle** implies (at least, suggests) that the action of gravity should be attributed to the curvature of **space-time**.

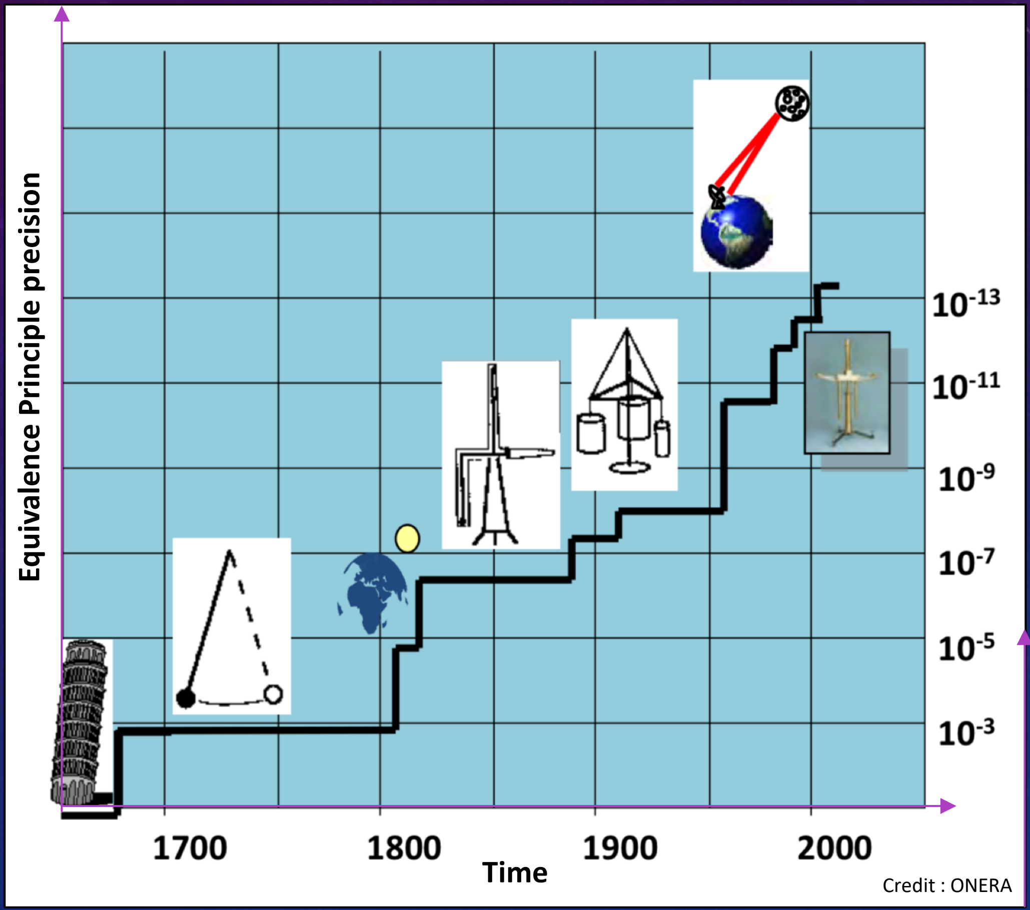
VERIFICATION OF THE SPACE-TIME CURVATURE



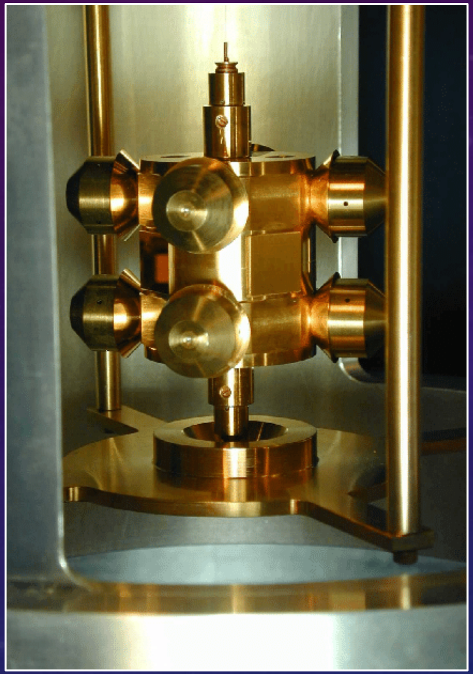


Einstein and Eddington

TESTS OF EQUIVALENCE PRINCIPLE



MOST RECENT EP TESTS

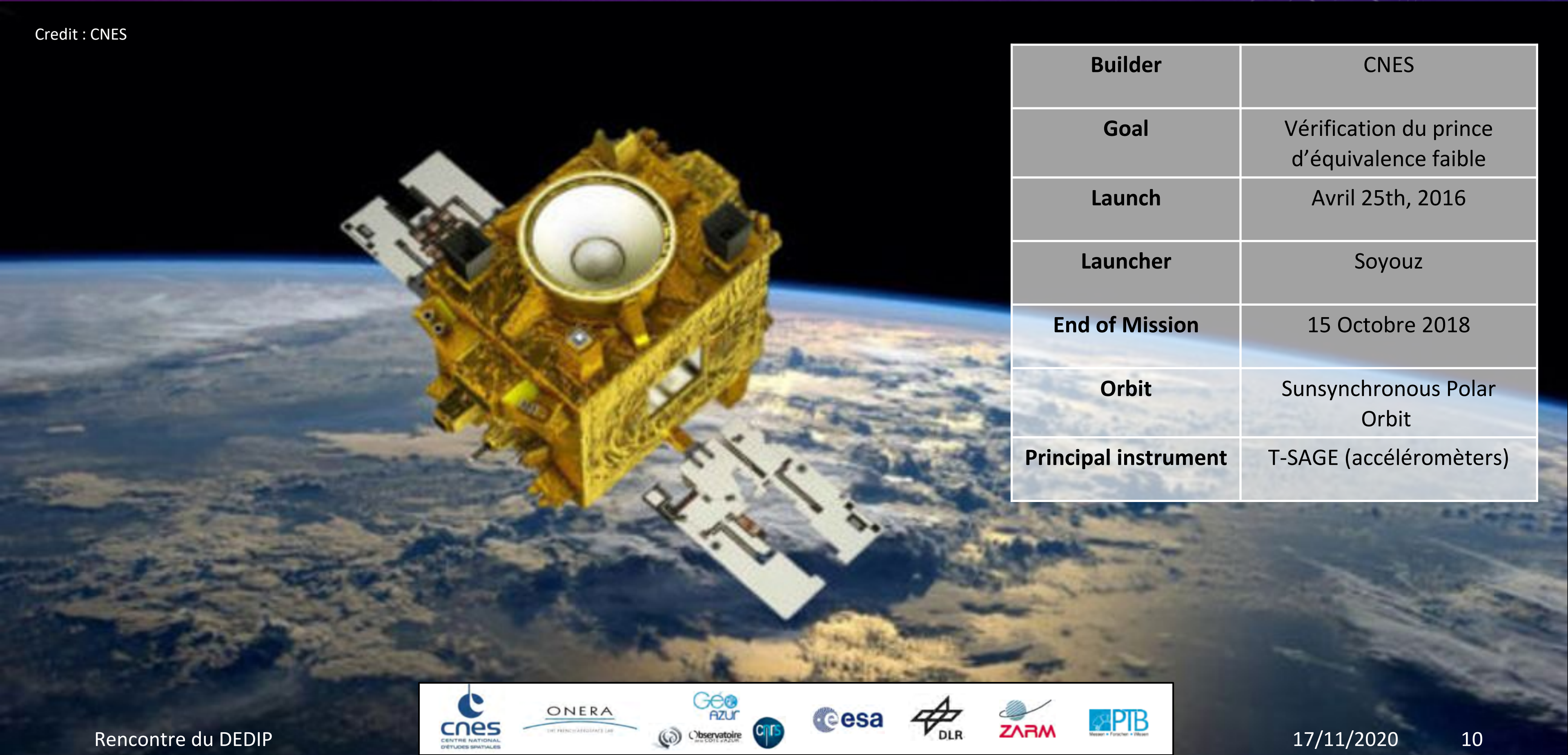


Experience Eöt-Wash: 10^{-13}
In 2008, the Eöt-Wash group reached a precision of 10^{-13}, using a torsion balance

The Lunar Laser Ranging experiment: 10^{-13}
Measurements of the distance between the Earth and the Moon allows to reach a precision of 10^{-13}, using laser ranging. This uses reflectors planted in the Moon during the Apollo program and Russian missions.

MICROSCOPE MISSION

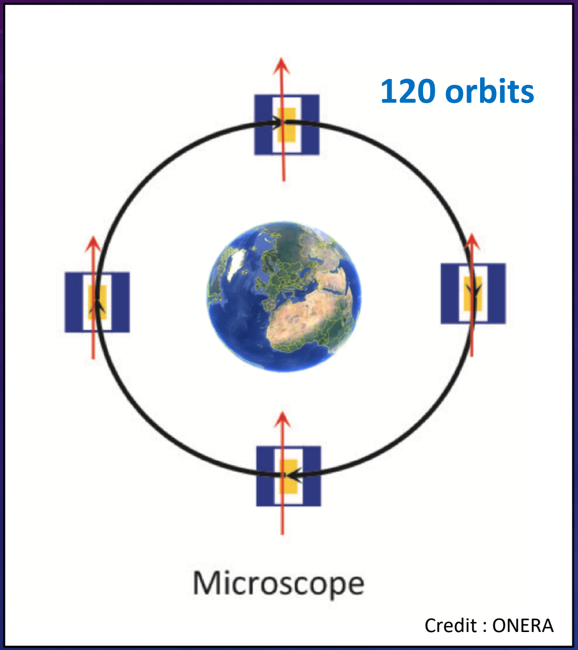
MICROSATELLITE À TRAINÉE COMPENSÉE POUR L'OBSERVATION DU PRINCIPE D 'EQUIVALENCE

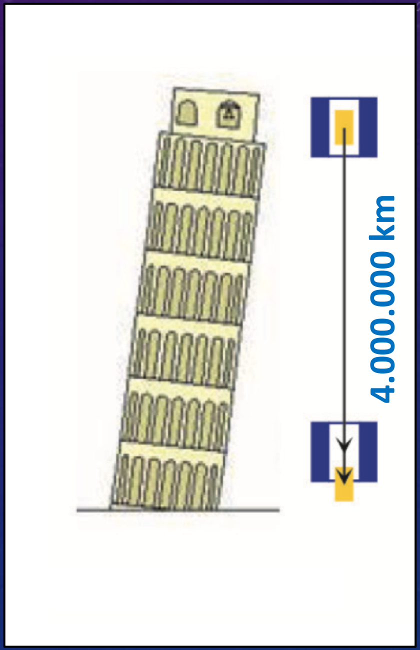


Credit : CNES

Builder	CNES
Goal	Vérification du prince d'équivalence faible
Launch	Avril 25th, 2016
Launcher	Soyouz
End of Mission	15 Octobre 2018
Orbit	Sunsynchronous Polar Orbit
Principal instrument	T-SAGE (accéléromèters)

MICROSCOPE PRINCIPLE

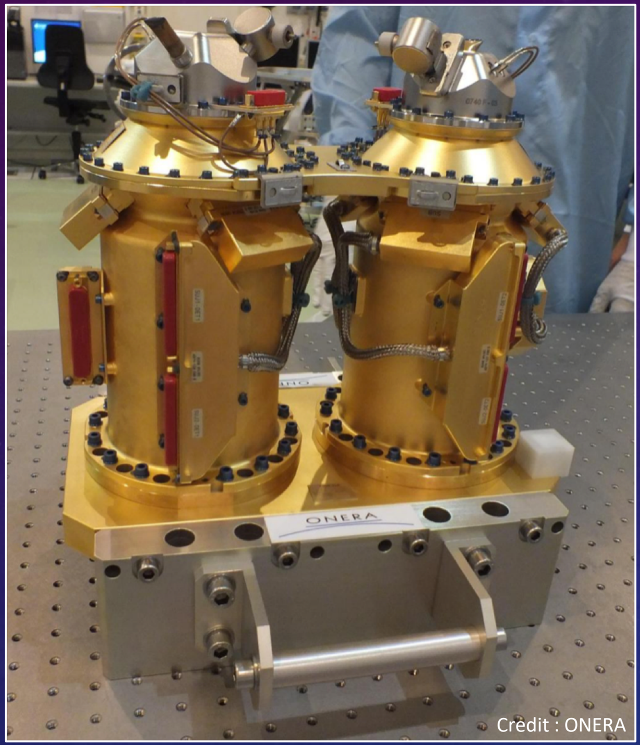




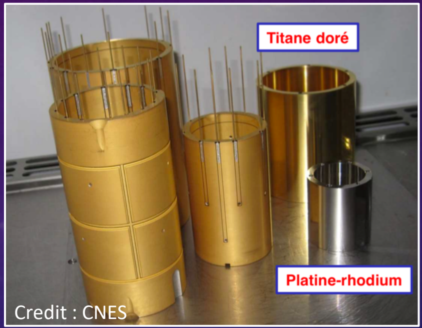
T-SAGE

TWIN SPACE ACCELEROMETER FOR SPACE GRAVITY EXPERIMENT

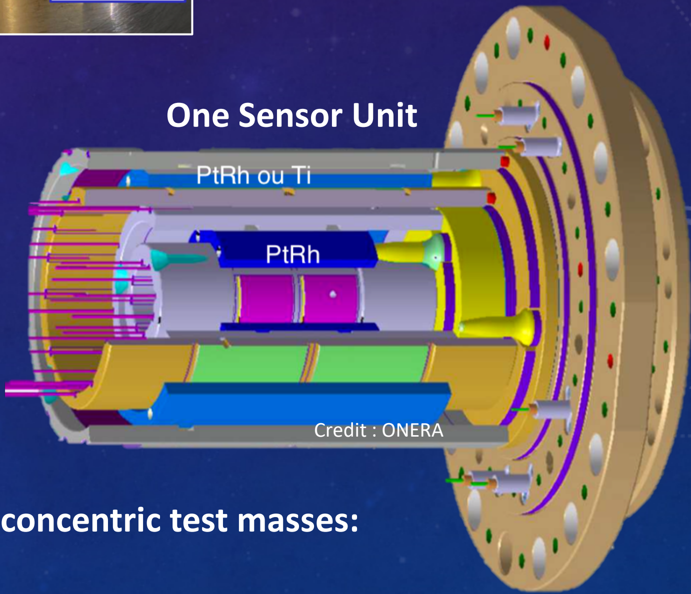
T-SAGE Flight model



Concentric test masses



One Sensor Unit



The two Accelerometers (or Sensor Unit) each composed of two concentric test masses:

- **SU-EP : Equivalence Principle in Titanim/Platinium (on left)**
- **SU-REF : REFerence in Platinim/Platinium (on right)**

EOTVOS PARAMETER

Acceleration signal :

$$\Gamma = G\frac{M_g}{r^2}\frac{m_g}{m_i}$$

The Eötvös parameter:

$$\delta(A,B) = \left(\frac{m_g}{m_i}\right)_A - \left(\frac{m_g}{m_i}\right)_B$$

THE MICROSCOPE MEASUREMENT

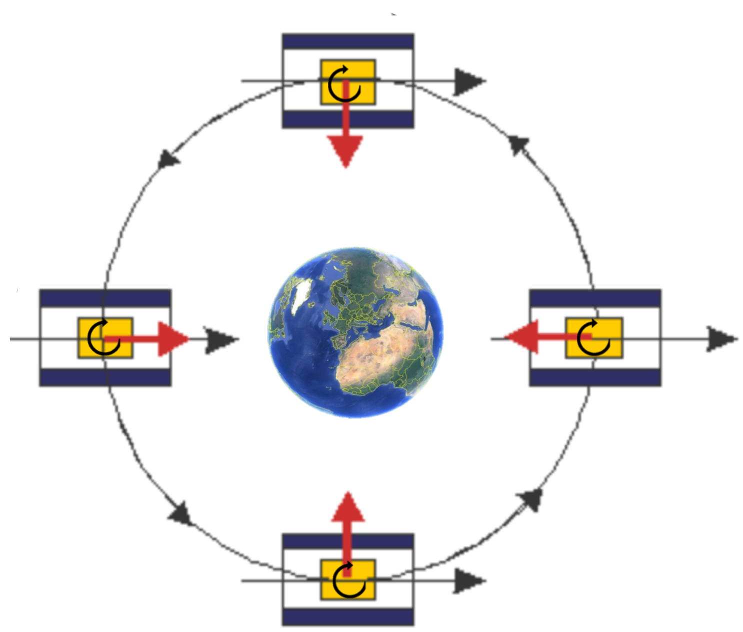
$$\vec{\Gamma}^{\rm meas}_{\rm diff} \simeq \vec{\Gamma}^{\rm meas}_{A} - \vec{\Gamma}^{\rm meas}_{B}$$

$$\begin{aligned}\vec{\Gamma}^{\rm meas}_{\rm diff} &\simeq \vec{K}_{0,\rm diff} \\ &+ [M_{\rm c}]\left(([T]-[In])\vec{\Delta} - 2[\omega]\dot{\vec{\Delta}} - \ddot{\vec{\Delta}} + \delta({\rm A},{\rm B})\vec{g}(O_{\rm sat})\right) \\ &+ 2[M_{\rm d}]\vec{\Gamma}^{\rm app}_{c} + \vec{\Gamma}^{\rm quad}_{d} + [{\rm Coupl}_{\rm d}]\dot{\vec{\Omega}} + \Gamma^{\rm n}_{\rm d}\end{aligned}$$

DESCRIPTION OF THE TERMS

Terms	Description of the terms
$\overrightarrow{K}_{0,d}$	Vector of the difference of the inertial sensor measurement bias.
$\overrightarrow{\Delta} = (\Delta_x, \Delta_y, \Delta_z)^T$	Vector (in the satellite frame) connecting the center of the inner mass to that of the outer mass.
$\dot{\overrightarrow{\Delta}}$ and $\ddot{\overrightarrow{\Delta}}$	First and second time derivatives of $\overrightarrow{\Delta}$. They are nullified in the instrument's bandwidth when the instruments servo-controls maintain the masses motionless versus the satellite frame.
$[\Omega]$	Satellite's angular velocity matrix, $\vec{\Omega} \times \vec{r} = [\Omega]\overrightarrow{r}$
$[T]$	Gravity gradient tensor in the satellite frame.
$[In]$	Matrix gradient of inertia defined in the satellite frame by $[In] = \left[\dot{\Omega}\right] + [\Omega]\,[\Omega]$.
$\overrightarrow{g} = (g_x, g_y, g_z)^T$	Gravity acceleration vector in the satellite frame of $7.9\,\mathrm{m\,s^{-2}}$ in magnitude at the 710 km altitude.
$\delta\,(2,1)$	Eötvös parameter of the outer mass (2) with respect to the inner mass (1).
$2\,[\Omega]\,\dot{\overrightarrow{\Delta}}$	Coriolis effect in the satellite frame. Very weak because the relative velocity of the test-masses at the test frequency is limited by the integral term of the accelerometer' servo-loops and because the angular velocity is well controlled by the satellite DFACS loops.
$\overrightarrow{\Gamma}_c^{\mathrm{app}}$	Mean acceleration applied on both masses in the satellite frame. Limited by the satellite DFACS.
$\overrightarrow{\Gamma}_d^{\mathrm{quad}}$	Difference of the non-linear terms in the measurement, mainly the difference of the quadratic responses of the inertial sensors.
$[\mathrm{Coupl}_d]$	Matrix of the difference, between the two sensors, of the coupling from the angular acceleration $\dot{\overrightarrow{\Omega}}$ to the linear acceleration.
$\overrightarrow{\Gamma}_d^{n}$	Difference of the acceleration measurement noises of the two sensors (coming from thermal noise, electronics noise, parasitic forces,...), comprising stochastic and systematic error sources.

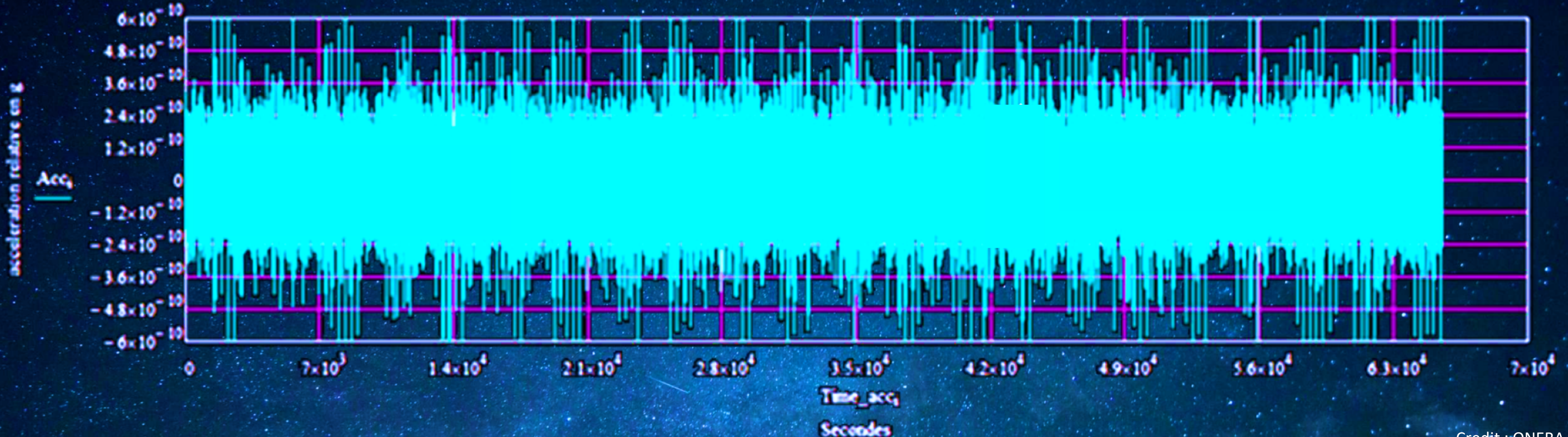
POSSIBLE EQUIVALENCE PRINCIPLE VIOLATION



Credit : ONERA

- Quasi-circular orbit
- Two different modes:
  - **Inertial mode** (no satellite rotation)
  - **Spin mode** (satellite spinned about its axis normal to the orbital plane)

MICROSCOPE DATA

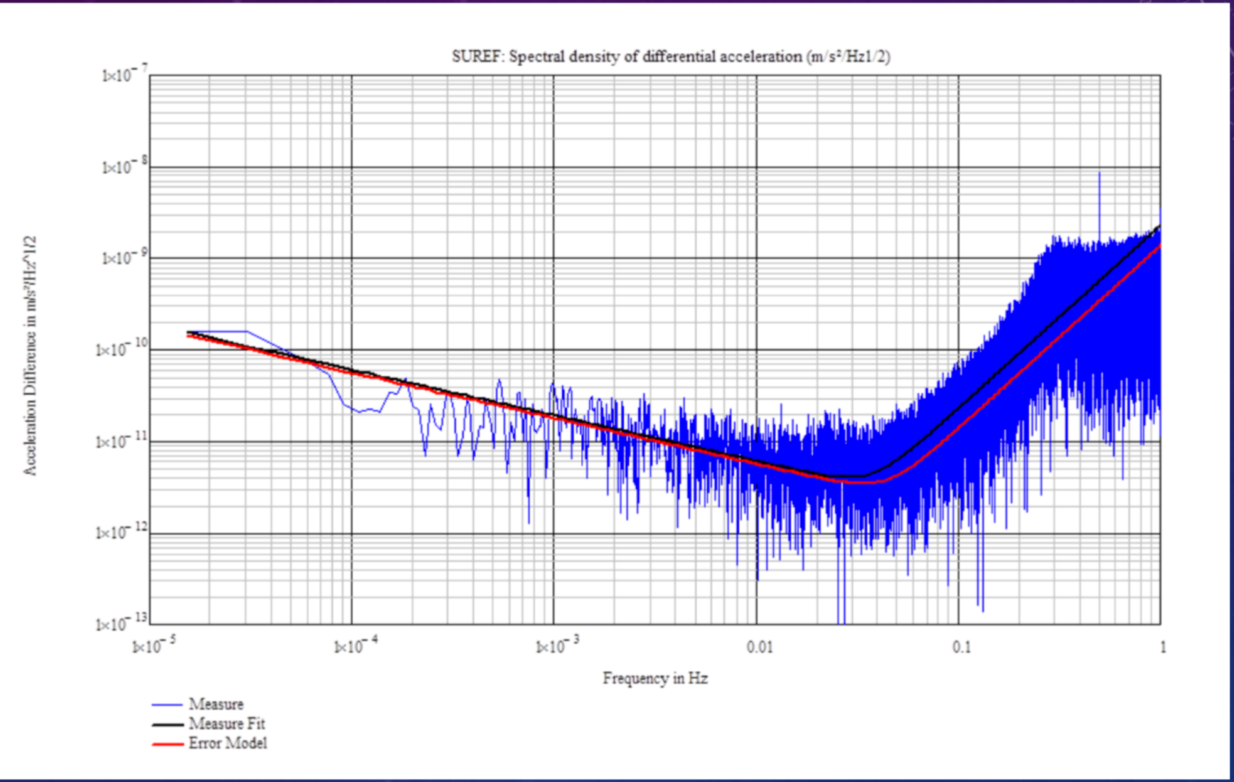


Credit : ONERA

120 orbits = 8.25 days

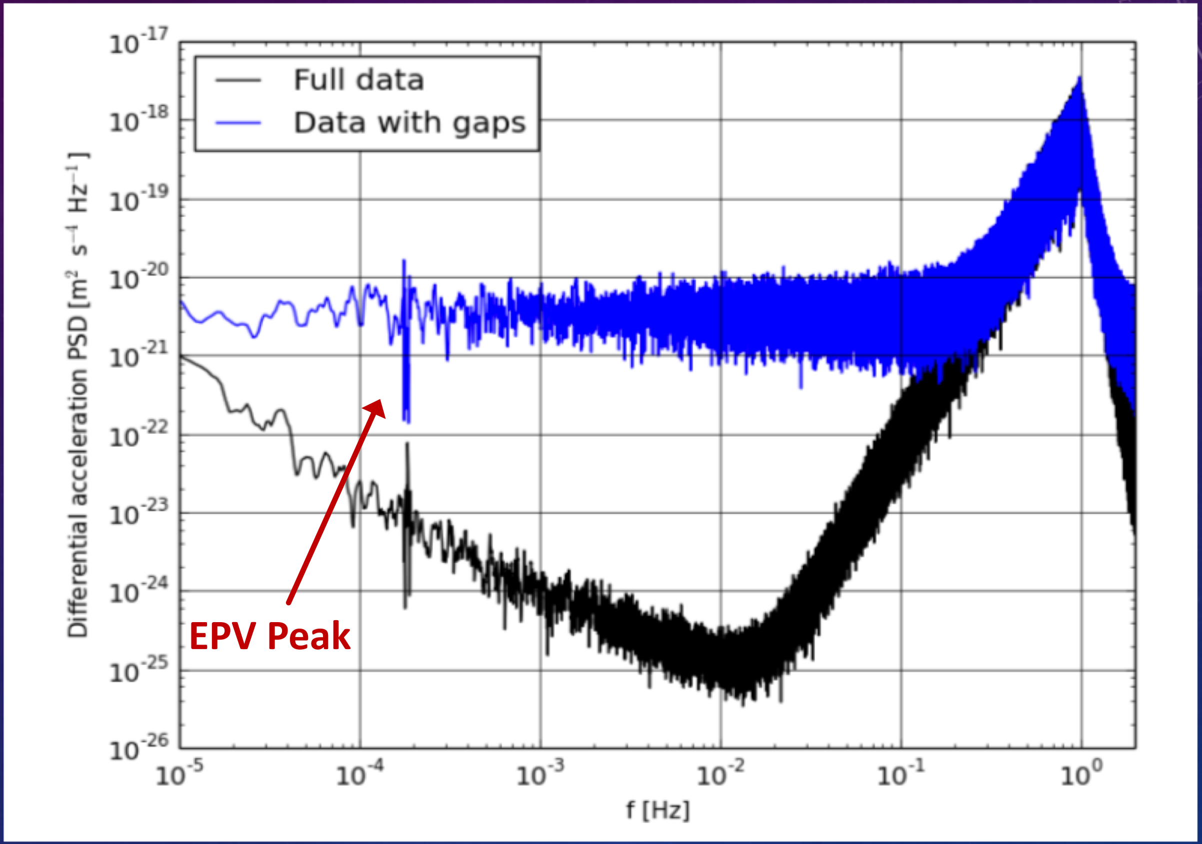
DIFFERENTIAL ACCELERATION POWER SPECTRUM

Touboul et al, 2019



Square root of the measured Power Spectral Density of the differential acceleration along X of SUEP

MISSING DATA IN MICROSCOPE



Square root of the measured Power Spectral Density of the differential acceleration along X of SUEP (Inertial mode)

THE MISSING DATA PROBLEM

$$Y(t) = M(t)X(t)$$

- The effect of missing data depends on :
  - The percentage of missing data
  - The power spectrum of the noise
- Methods
  - Autoregressive fit of the noise to build a Generalized Least Square estimator (KARMA)
  - Interpolation of the missing data (Sparse Inpainting method)

SPARSE INPAINTING

Elad et al., JACHA, 2005

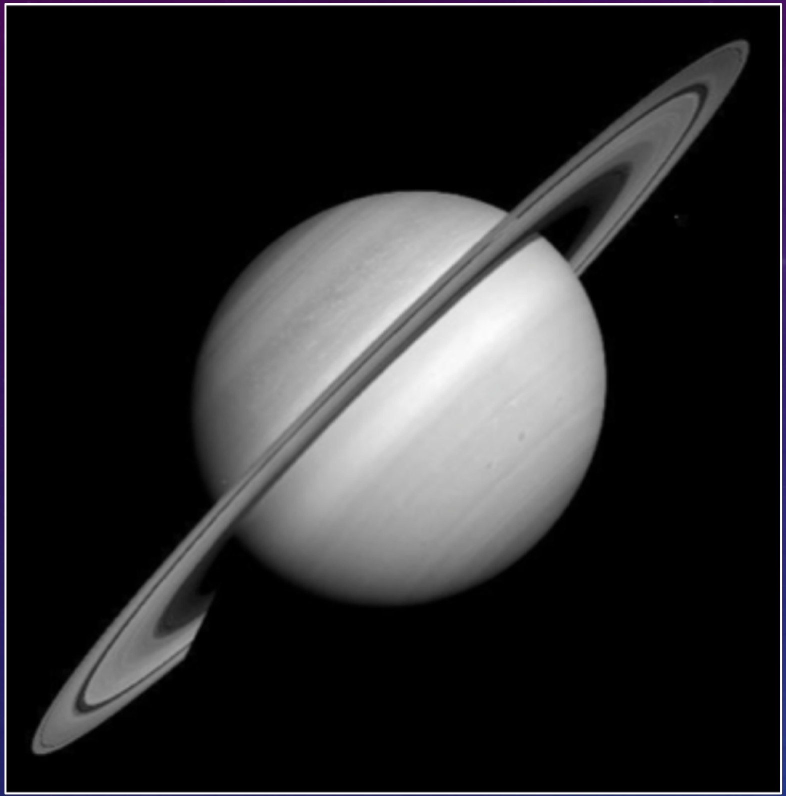
$$\min ||\alpha||_0 \text{ s.t. } ||\Gamma_{\text{diff}}^{\text{meas}} - M\Phi\alpha|| \leq \sigma^2$$

Sparse constraint

Fidelity term

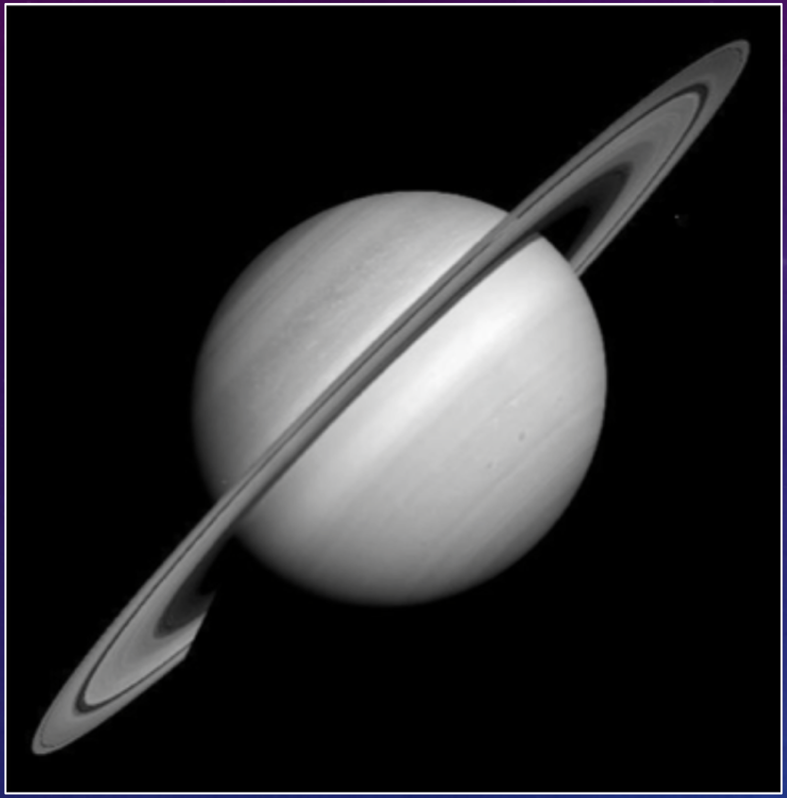
$$\Gamma_{\text{diff}} = \Phi\alpha$$

SPARSE INPAINTING AND COMPRESSIBLE SIGNALS



The top 1% of the coefficients concentrate only 8.66% of the energy: not sparse in the direct space

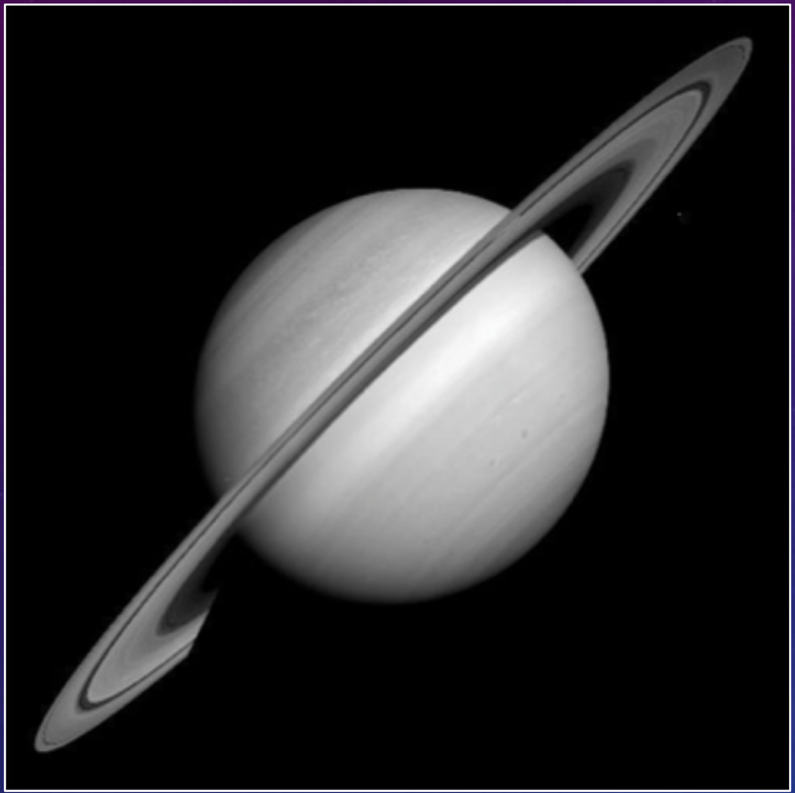
COMPRESSIBLE SIGNALS

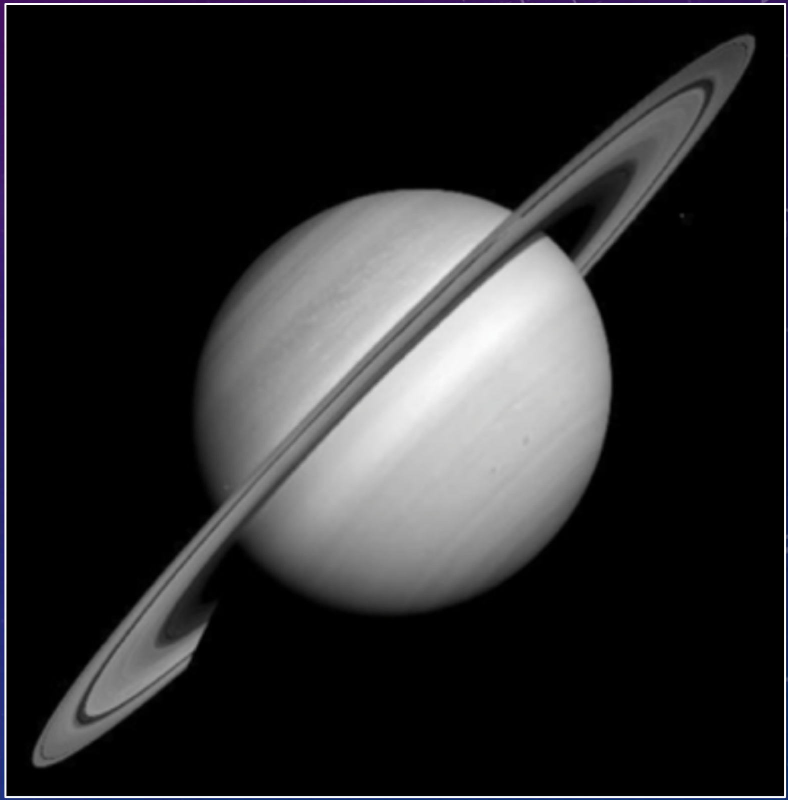


Wavelet transform

1% of the largest coefficients in the Wavelet space (the others are set to zero)

COMPRESSIBLE SIGNALS

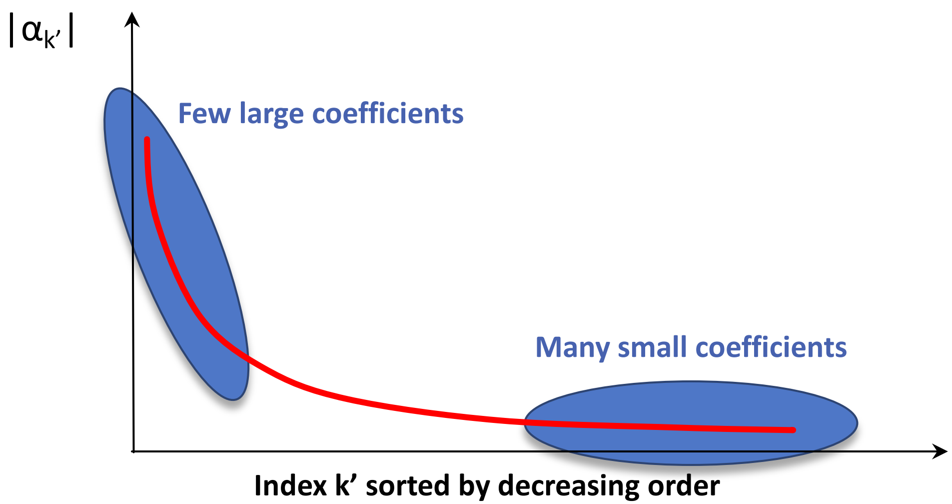




The top 1% of the coefficients concentrate 99.96% of the energy: Sparse in the wavelet space

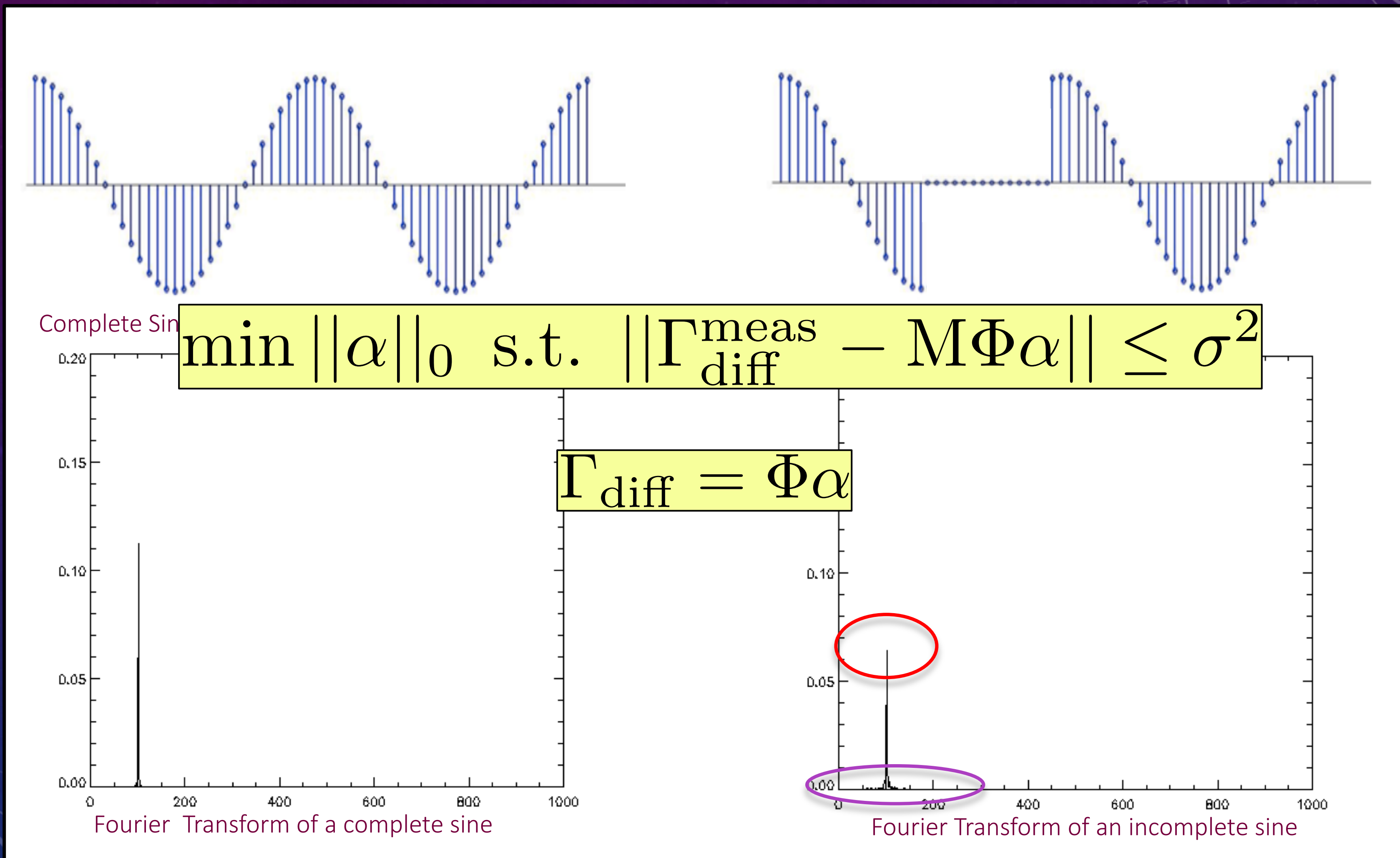
WHAT IS A GOOD REPRESENTATION?

$$\underbrace{W(a,b)}_{\alpha} = K \int_{-\infty}^{+\infty} \underbrace{\psi^*\left(\frac{x-b}{a}\right)}_{\Phi^{T}} \underbrace{f(x)}_{\Gamma_{\mathrm{diff}}} dx$$



SPARSE INPAINTING

Elad et al., JACHA, 2005



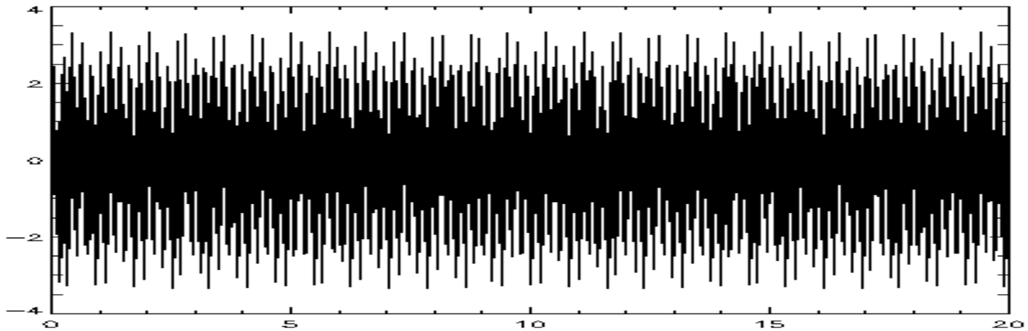
$$\min ||\alpha||_0 \text{ s.t. } ||\Gamma_{\text{diff}}^{\text{meas}} - \text{M}\Phi\alpha|| \leq \sigma^2$$

$$\Gamma_{\text{diff}} = \Phi\alpha$$

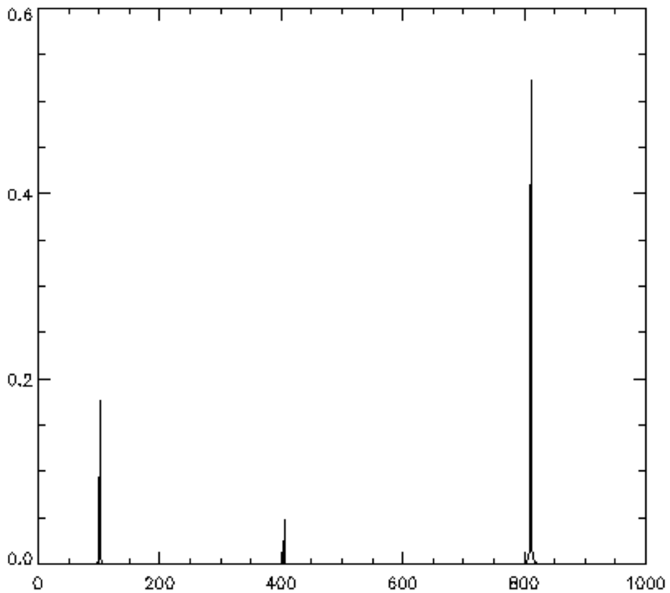
Fourier Transform of a complete sine

Fourier Transform of an incomplete sine

SPARSE INPAINTING ALGORITHM

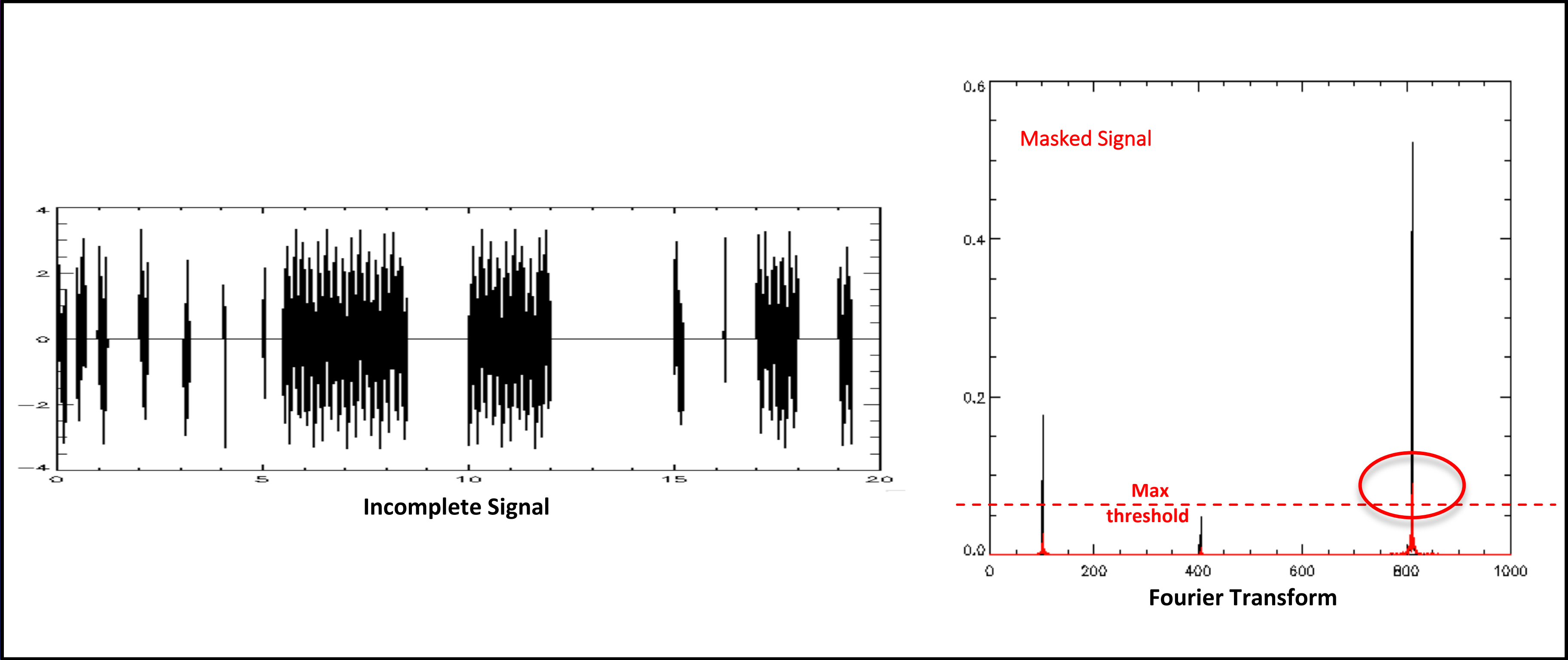


Complete Signal

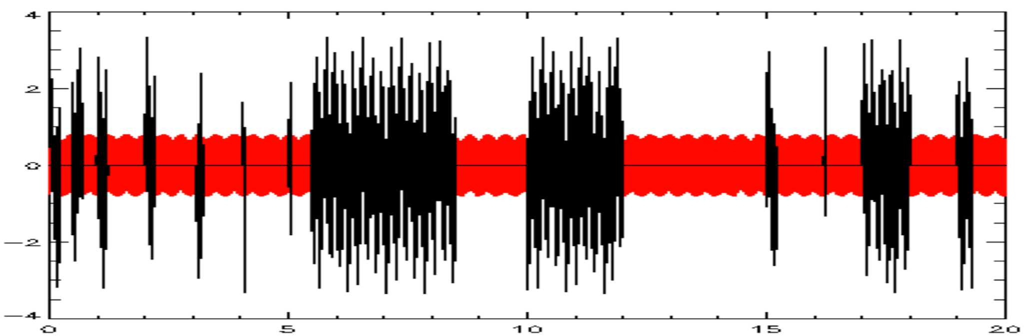


Fourier Transform

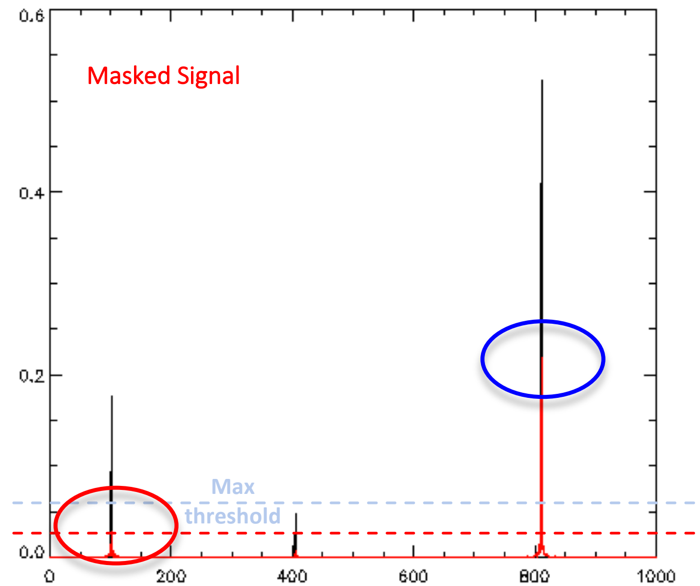
SPARSE INPAINTING ALGORITHM



SPARSE INPAINTING ALGORITHM

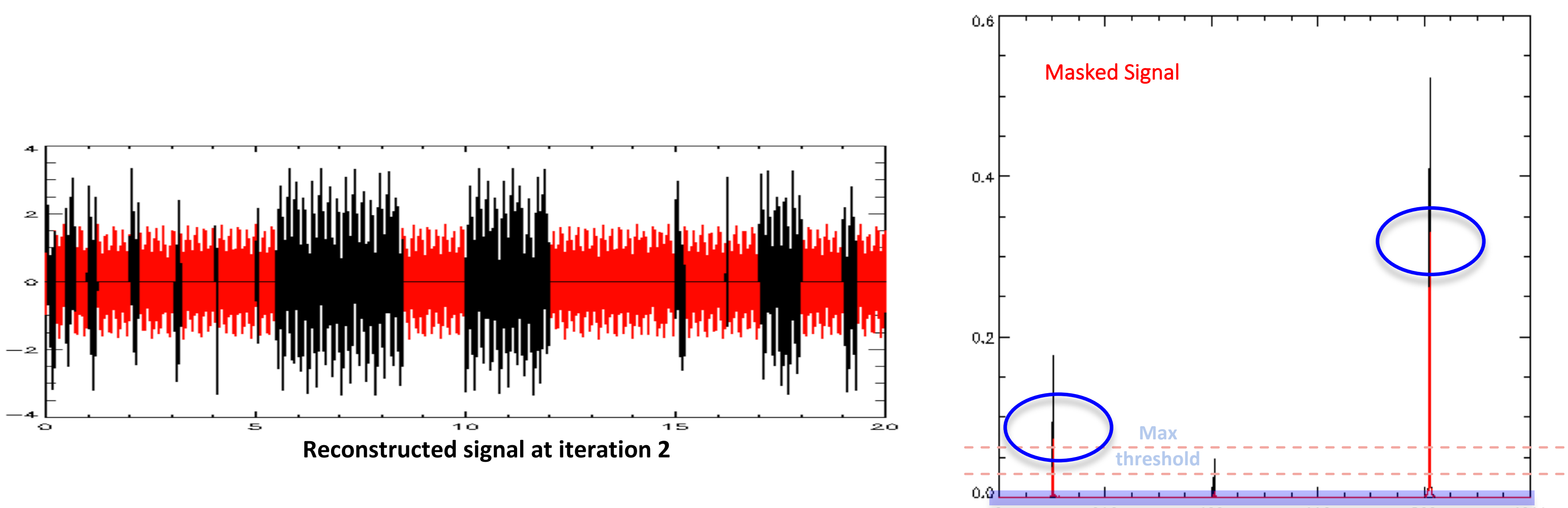


Reconstructed signal at iteration 1



Fourier Transform

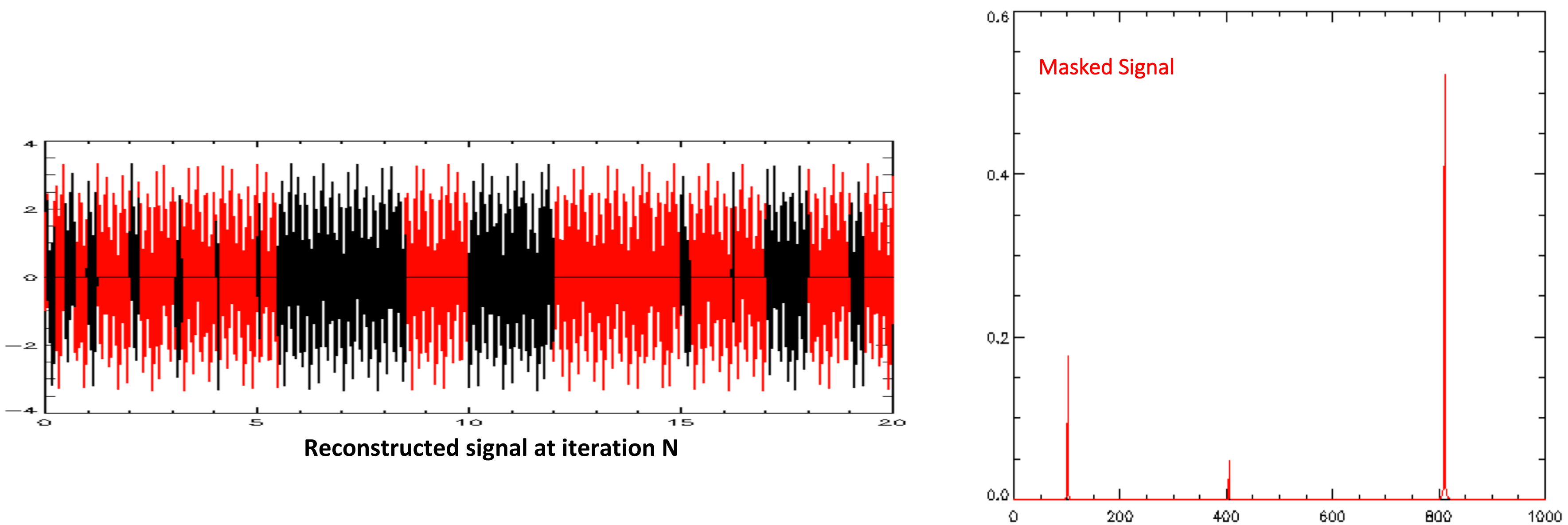
SPARSE INPAINTING ALGORITHM



Reconstructed signal at iteration 2

Fourier Transform

SPARSE INPAINTING ALGORITHM



Reconstructed signal at iteration N

Fourier Transform

SPARSE INPAINTING

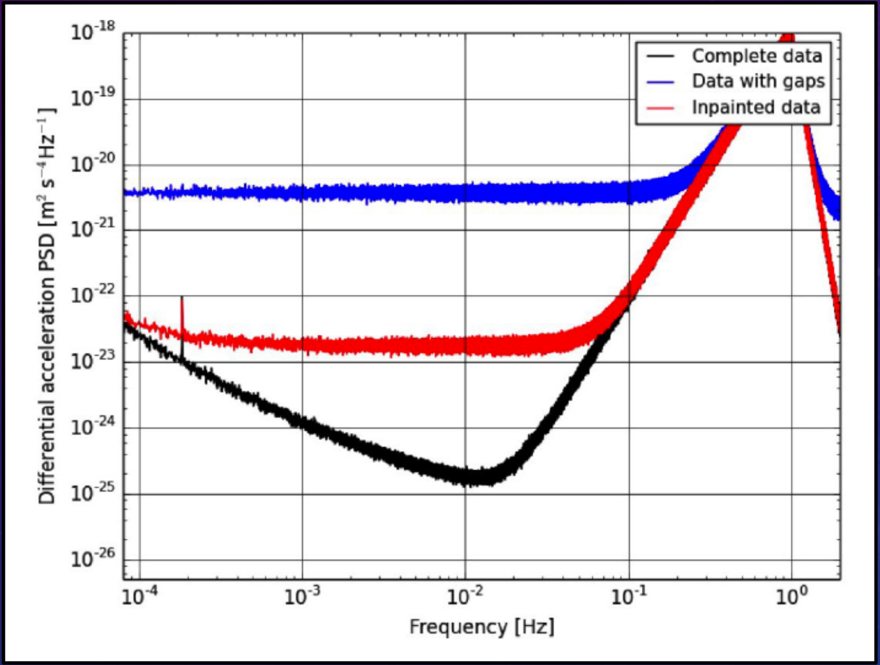
50%

80%

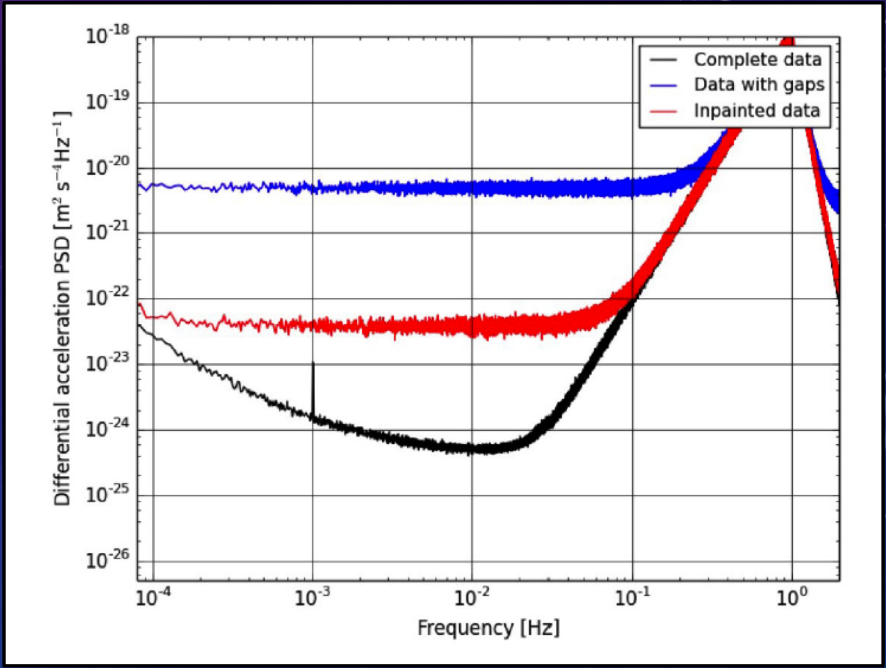
SPARSE INPAINTING

$$\min ||\Phi^T \tilde{\Gamma}_{\text{diff}}||_0 \;\; \text{s.t.} \;\; ||\Gamma_{\text{diff}}^{\text{meas}} - M\tilde{\Gamma}_{\text{diff}}|| \leq \sigma^2$$

Bergé & Pires et al. 2015



Inertial mode ($f_{EP} = f_{orb} = 1.8 \times 10^{-4}$ Hz)



Spin mode ($f_{EP} = 1. \times 10^{-3}$ Hz)

INCLUDING NOISE CONSTRAINT

$$\min ||\Phi^{\mathrm{T}}\tilde{\Gamma}_{\mathrm{diff}}||_0 \text{ s.t. } ||\Gamma^{\mathrm{meas}}_{\mathrm{diff}} - \mathrm{M}\tilde{\Gamma}_{\mathrm{diff}}|| \leq \sigma^2$$

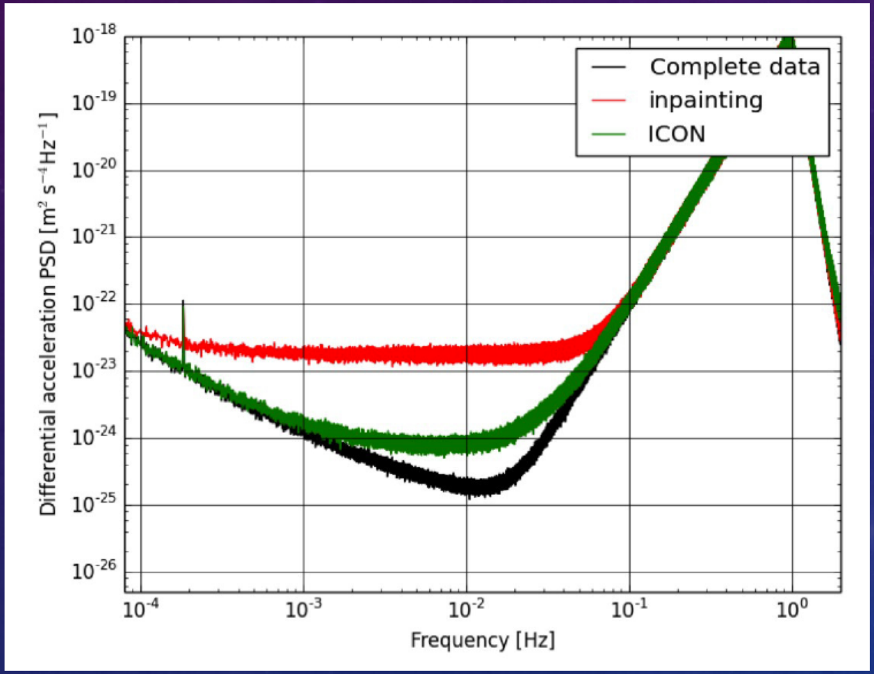
$$\min ||\Phi^{\mathrm{T}}\tilde{\Gamma}_{\mathrm{diff}}||_0 \text{ s.t. } ||\Gamma^{\mathrm{meas}}_{\mathrm{diff}} - \mathrm{M}\mathrm{W}^{\mathrm{T}}\mathrm{Q}\mathrm{W}\tilde{\Gamma}_{\mathrm{diff}}|| \leq \sigma^2$$

Where W is the forward wavelet transform and W^{T} its inverse transform and $\mathrm{Q} = \frac{\sigma_{\mathrm{w}_{1,\mathrm{int}}}}{\sigma_{\mathrm{w}_{1,\mathrm{out}}}}$ is the linear operator used to impose the power spectrum constraint

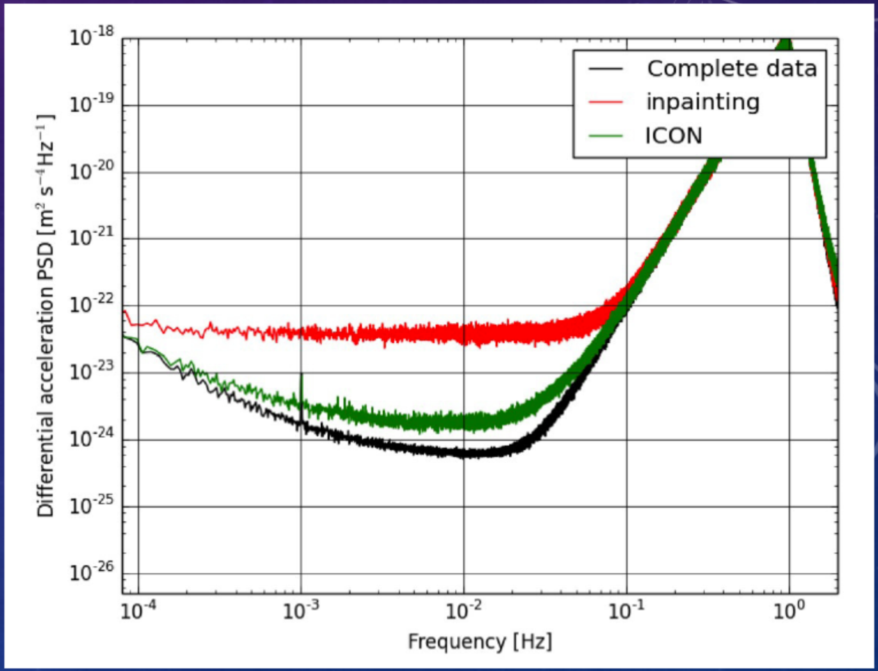
SPARSE INPAINTING FOR COLORED-NOISE

$$\min ||\Phi^T \tilde{\Gamma}_{\rm diff}||_0 \;\; \text{s.t.} \;\; ||\Gamma_{\rm diff}^{\rm meas} - MW^T QW\tilde{\Gamma}_{\rm diff}|| \leq \sigma^2$$

Pires & Bergé et al. 2016



Inertial mode ($f_{EP} = f_{orb} = 1.8 \times 10^{-4}$ Hz)



Spin mode ($f_{EP} = 1. \times 10^{-3}$ Hz)

CONCLUSIONS

- Inpainting allows to correct for missing and masked data and to reach the expected Microscope precision
  - Inertial mode
  - Spin mode
- Inpainting is very useful in processing the data in the calibration phase.
- One tenth of the data have been analysed
  - No violation of the Equivalence Principle
  - Current limit is 1.3×10^{-14} [a factor 10 of improvement]
- The remaining data are currently being processed
  - The goal is to reach a precision of 1×10^{-15}

PUBLICATIONS

- "Dealing with missing data: An inpainting application to the MICROSCOPE space mission", J. Bergé, S. Pires, Q. Baghi, P. Touboul, G. Metris, Physical Review D, 92, 11, December 2015
- "Dealing with missing data in the MICROSCOPE space mission: An adaptation of *inpainting* to handle colored-noise data", S. Pires, J. Bergé, Q. Baghi, P. Touboul, G. Métris, accepted in Physical Review D, December 2016
- Space test of the Equivalence Principle: First results of the MICROSCOPE mission, P. Touboul, …, S.Pires, … et al 2019 Class. Quantum Grav. 36 225006
- MICROSCOPE VIII: Data analysis principle J. Bergé,.., S. Pires, et al (in prep)
- MICROSCOPE IX: Statistics and impact of glitches on the test of the weak equivalence principle J. Bergé, …, S. Pires et al. (in prep)

Fait Marquant de la DRF

Publié le 17 décembre 2019

| Physique théorique | Satellites | Technologies

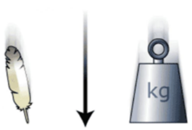
En chute libre dans le vide

Spécialiste de la reconstruction de cartographies de matière noire à partir d'observations astronomiques, une chercheuse de l'Irfu a mis son expertise au service de Microscope, un satellite embarquant une expérience de gravitation. Pour l'instant, une plume et un kilo de plomb chutent dans le vide de la même manière !

Fait Marquant de la L'IRFU

Chute libre

Le satellite Microscope conforte le principe d'équivalence d'Einstein



Tester le principe d'équivalence, un principe de base de la relativité générale élaborée par Albert Einstein dont une conséquence est l'universalité de la chute libre des corps dans le vide, tel est l'enjeu du satellite Microscope. Cette mission spatiale, financée et pilotée par le CNES, conçue par l'ONERA en collaboration avec l'Observatoire de la Cote d'Azur, le CNES et le ZARM (Brème, Allemagne), a été lancée le 25 Avril 2016 avec à son bord l'instrument T-sage développé par l'ONERA. Dans une étude publiée dans la revue Classical and Quantum Gravity, l'équipe Microscope à laquelle participe une chercheuse du DEDIP (Département D'Electronique, des Détecteurs et d'Informatique pour la Physique)/Laboratoire AIM du CEA-Irfu Paris-Saclay vient de vérifier la validité du principe d'équivalence avec une précision inégalée. En prenant minutieusement en compte les différentes sources de bruits, en mettant à profit une meilleure connaissance de l'instrument et en faisant appel à des outils d'analyse également utilisés en astrophysique, l'équipe a pu améliorer d'un facteur 10 la précédente mesure, rendant ainsi particulièrement robuste les résultats publiés antérieurement et confortant par là-même la validité du principe d'équivalence.

END