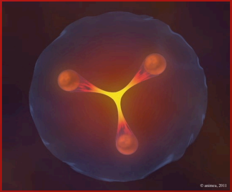
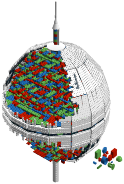
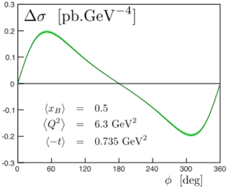
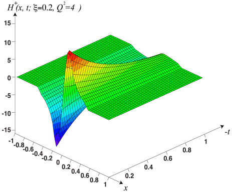
DE LA RECHERCHE À L'INDUSTRIE



Hadron Reverse Engineering: From 3D Hadron Structure to QCD Dynamics







From 3D Hadron Structure to QCD Dynamics | Hervé MOUTARDE

May. 12$^{\text{th}}$, 2015

Reverse engineering

Reverse engineering is the process of discovering the technological principles of a device, object, or system through analysis of its structure, function, and operation.

Eilam and Chikofsky, *Reversing: secrets of reverse engineering*, John Wiley & Sons, 2007.

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Motivation.

Study hadron structure to shed new light on nonperturbative QCD.

Reverse engineering

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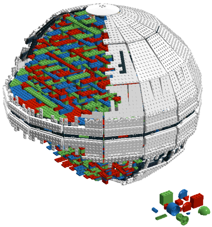
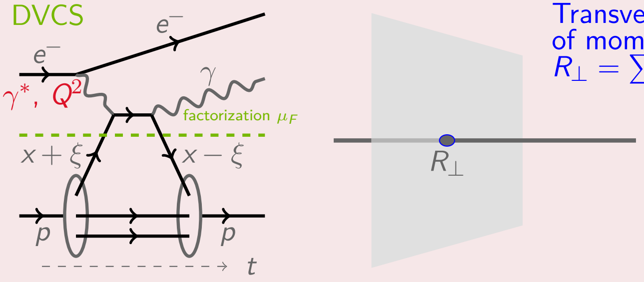
Eilam and Chikofsky, *Reversing: secrets of reverse engineering*, John Wiley & Sons, 2007.

- Interplay between **perturbative** and **non-perturbative** QCD.
- Interacting colored degrees of freedom confined in **colorless** hadrons.
- **Emergence** of hadron characteristics from **fundamental building blocks**.



- Correlation of the **longitudinal momentum** and the **transverse position** of a parton in a hadron.
- DVCS recognized as the cleanest channel to access GPDs.

Deeply Virtual Compton Scattering (DVCS)



Transverse center of momentum $R_\perp$

$R_\perp = \sum_i x_i r_{\perp i}$

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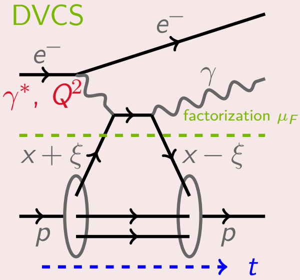

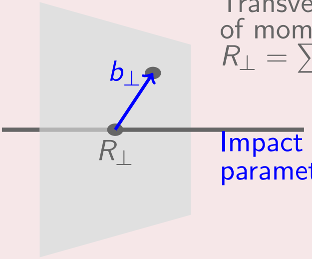

Transverse center of momentum $R_\perp$
$R_\perp = \sum_i x_i r_{\perp i}$

Impact parameter $b_\perp$

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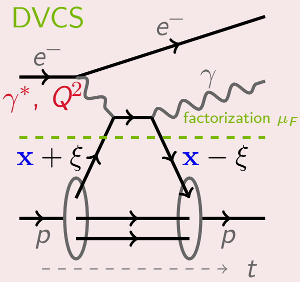

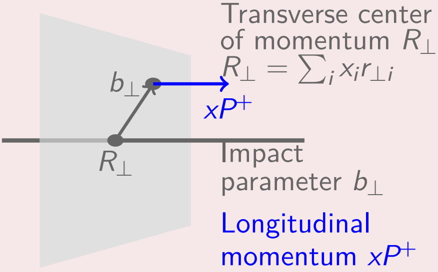

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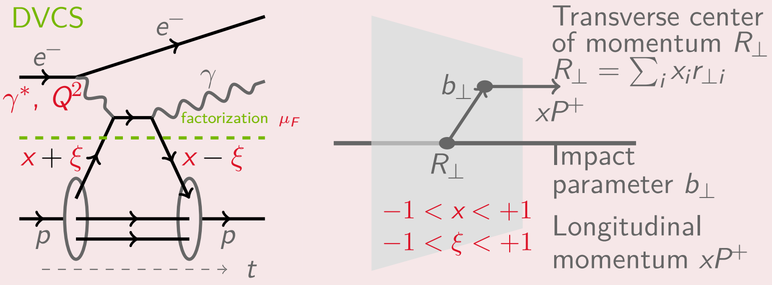


- **24 GPDs** $F^i(x, \xi, t, \mu_F)$ for each parton type $i = g, u, d, \ldots$ for leading and sub-leading twists.

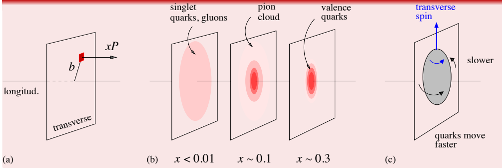
- **Probabilistic interpretation** of Fourier transform of $\mathrm{GPD}(x, \xi = 0, t)$ in **transverse plane**.

$$\rho(x, b_\perp, \lambda, \lambda_N) = \frac{1}{2}\left[H(x, 0, b_\perp^2) + \frac{b_\perp^j \epsilon_{ji} S_\perp^i}{M} \frac{\partial E}{\partial b_\perp^2}(x, 0, b_\perp^2) + \lambda \lambda_N \tilde{H}(x, 0, b_\perp^2) \right]$$

- Notations : quark helicity λ, nucleon longitudinal polarization λ_N and nucleon transverse spin $S_\perp$.

Burkardt, Phys. Rev. **D62**, 071503 (2000)

Can we obtain this picture from exclusive measurements?



Weiss, AIP Conf. Proc. **1149**, 150 (2009)

- Most general structure of matrix element of energy momentum tensor between nucleon states:

$$\left\langle N, P+\frac{\Delta}{2} \right| T^{\mu\nu} \left| N, P-\frac{\Delta}{2} \right\rangle = \bar{u}\left(P+\frac{\Delta}{2}\right)\left[A(t)\gamma^{(\mu}P^{\nu)} + B(t)P^{(\mu}i\sigma^{\nu)\lambda}\frac{\Delta_\lambda}{2M} + \frac{C(t)}{M}(\Delta^\mu\Delta^\nu - \Delta^2\eta^{\mu\nu})\right]u\left(P-\frac{\Delta}{2}\right)$$

with $t = \Delta^2$.

- Key observation: **link between GPDs and gravitational form factors**

$$\int \mathrm{d}x\, x H^q(x,\xi,t) = A^q(t) + 4\xi^2 C^q(t)$$

$$\int \mathrm{d}x\, x E^q(x,\xi,t) = B^q(t) - 4\xi^2 C^q(t)$$

Ji, Phys. Rev. Lett. **78**, 610 (1997)

- Spin sum rule:

$$\int \mathrm{d}x\, x\big(H^q(x,\xi,0) + E^q(x,\xi,0)\big) = A^q(0) + B^q(0) = 2J^q$$

Ji, Phys. Rev. Lett. **78**, 610 (1997)

- **Shear** and **pressure** of a hadron considered as a continuous medium:

$$\langle N \,|\, T^{ij}(\vec{r}) \,|\, N \rangle\, N = s(r)\left(\frac{r^i r^j}{\vec{r}^2} - \frac{1}{3}\delta^{ij}\right) + p(r)\delta^{ij}$$

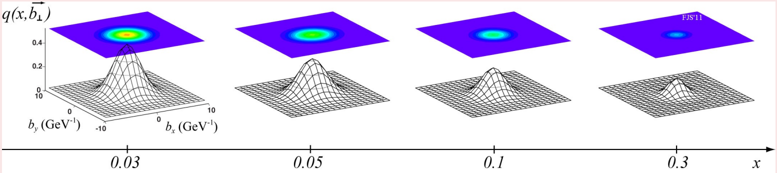
Polyakov and Shuvaev, hep-ph/0207153

Anatomy of hadrons.

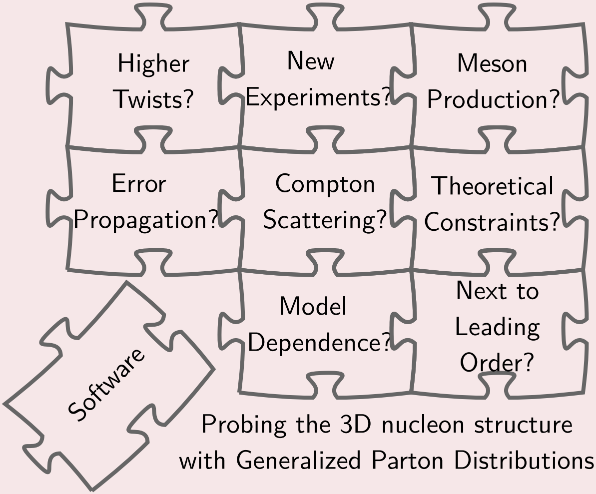
Different questions and different tools to answer them.

1. Study of exclusive processes.
2. Metrology of Generalized Parton Distributions.
3. Understanding of QCD mechanisms and modeling of Generalized Parton Distributions.

What do we learn from this picture?



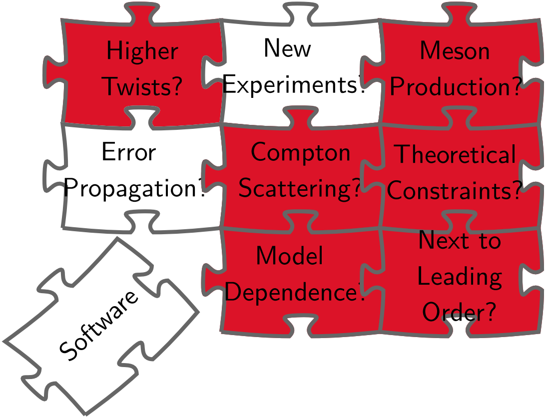
Common tools for different technical questions.

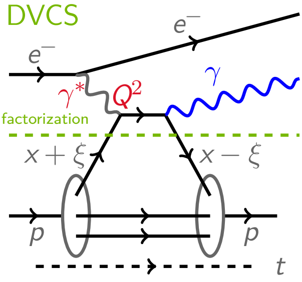


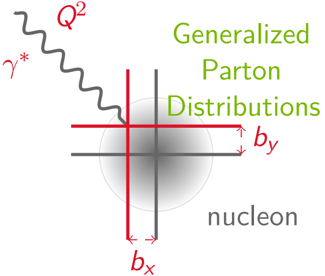
Probing the 3D nucleon structure with Generalized Parton Distributions

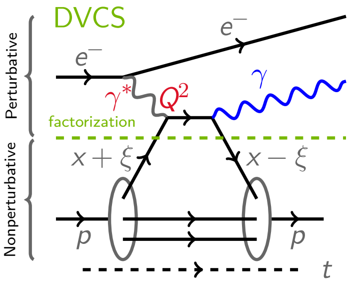
Needs assessment

Needs assessment




DVCS
e^-
e^-
γ^*
Q^2
γ
factorization
$x+\xi$
$x-\xi$
p
p
t


Q^2
γ^*
Generalized Parton Distributions
b_y
nucleon
b_x



DVCS
Perturbative
Nonperturbative
e^-
e^-
γ^*
Q^2
γ
factorization
$x+\xi$
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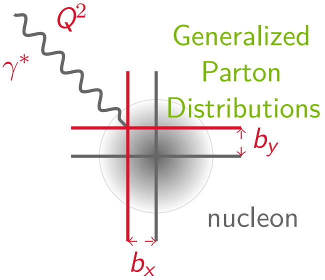
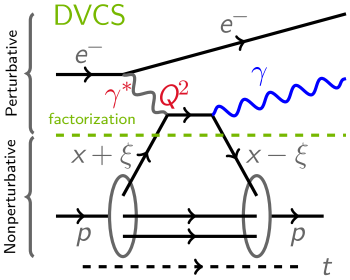
p
p
t

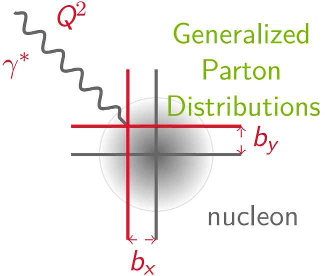


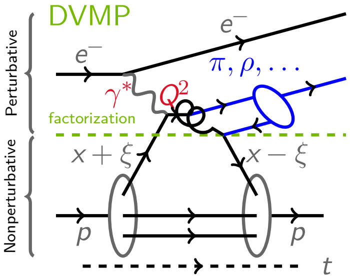
Q^2
γ^*
Generalized Parton Distributions
b_y
b_x
nucleon

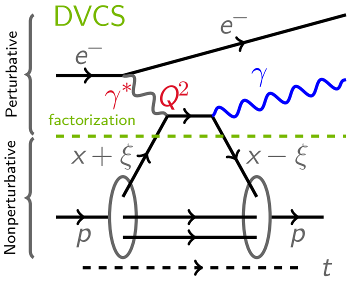
Exclusive processes of current interest (1/2).

Factorization and universality.

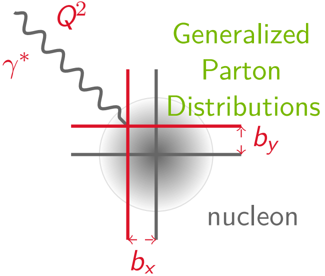




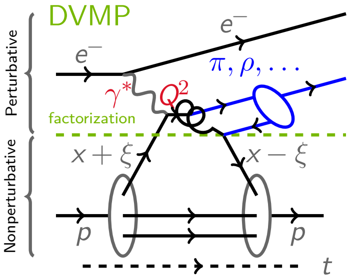




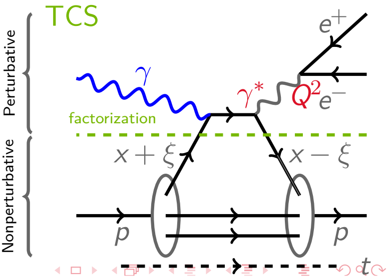
DVCS
Perturbative
Nonperturbative
e^-
e^-
γ^*
Q^2
γ
factorization
$x+\xi$
$x-\xi$
p
p
t



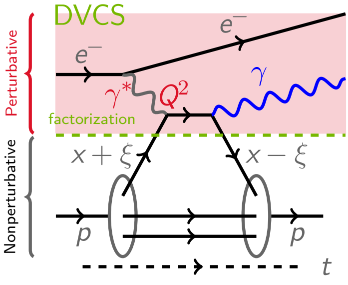
Q^2
γ^*
Generalized
Parton
Distributions
b_y
nucleon
b_x



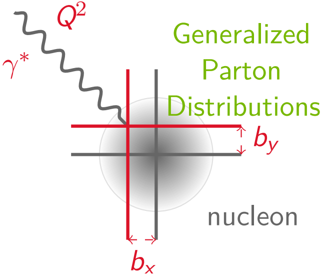
DVMP
Perturbative
Nonperturbative
e^-
e^-
$\pi, \rho, \ldots$
γ^*
Q^2
factorization
$x+\xi$
$x-\xi$
p
p
t



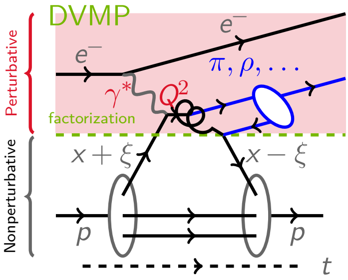
TCS
Perturbative
Nonperturbative
e^+
γ
γ^*
Q^2
e^-
factorization
$x+\xi$
$x-\xi$
p
p
t



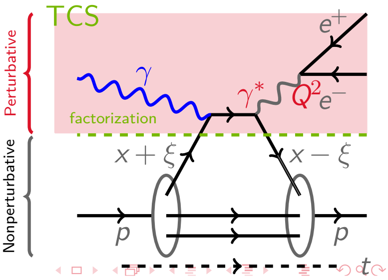
DVCS
Perturbative
Nonperturbative
e−
e−
γ*
Q2
γ
factorization
x + ξ
x − ξ
p
p
t



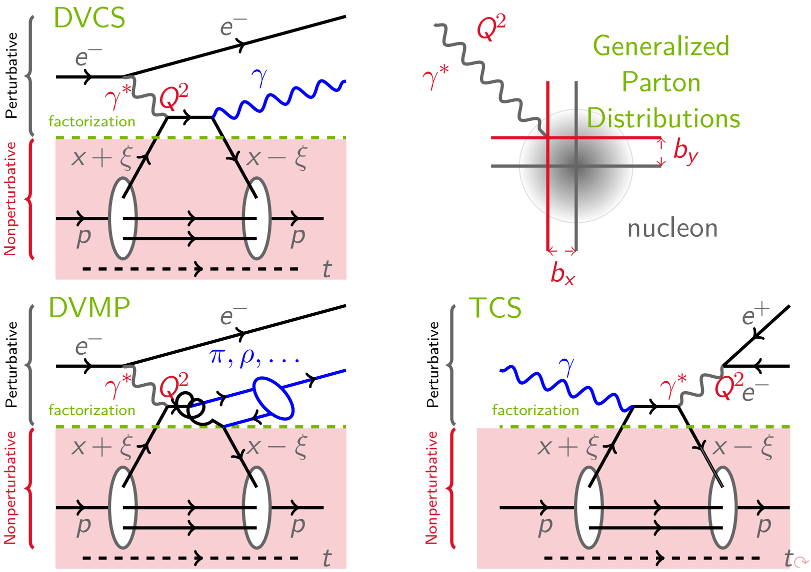
Q2
γ*
Generalized Parton Distributions
by
nucleon
bx



DVMP
Perturbative
Nonperturbative
e−
e−
γ*
Q2
π, ρ, . . .
factorization
x + ξ
x − ξ
p
p
t



TCS
Perturbative
Nonperturbative
e+
γ
γ*
Q2
e−
factorization
x + ξ
x − ξ
p
p
t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Generalized Parton Distributions
b_y
b_x
nucleon
DVMP
$\pi, \rho, \ldots$
TCS
e^+
e^-

Bjorken regime : large Q^2 and fixed $xB \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on **factorization theorems**.
- All-order proofs for DVCS, TCS and some DVMP.
- GPDs depend on a (arbitrary) factorization scale μ_F.
- **Consistency** requires the study of **different channels**.

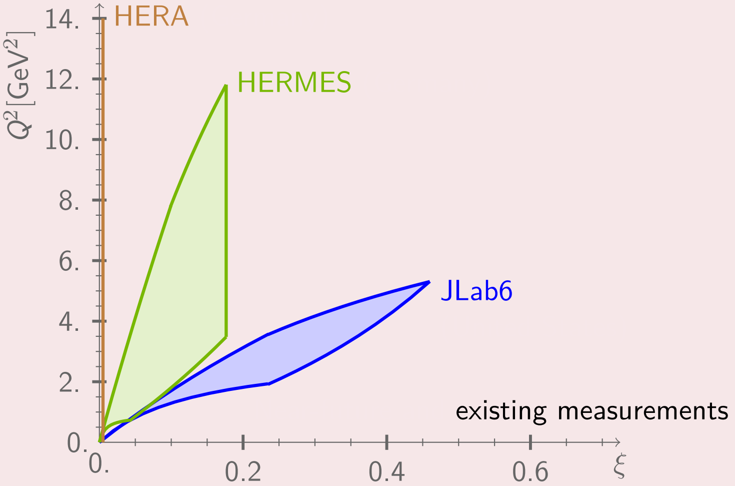
- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^{1} dx\, C\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F)$$

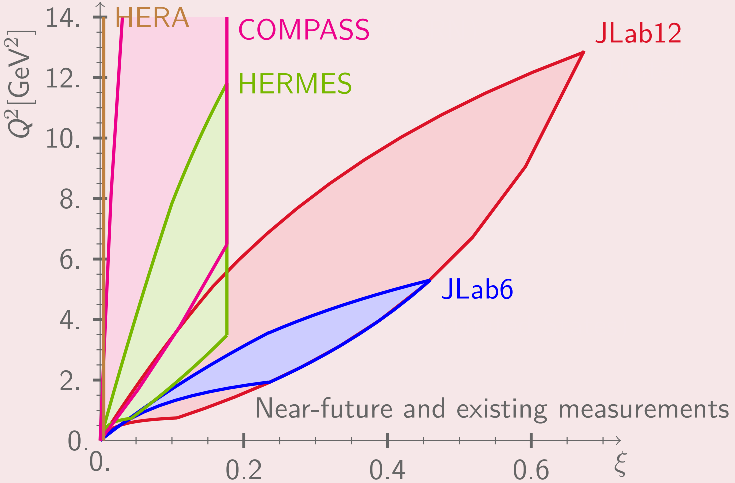
for a given GPD F.

- CFF $\mathcal{F}$ is a **complex function**.

Kinematic reach of existing or near-future DVCS measurements



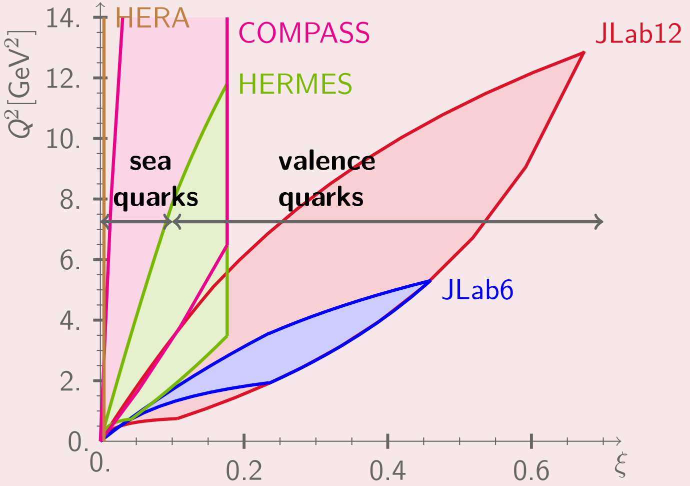
Kinematic reach of existing or near-future DVCS measurements



Need for global fits of world data.

Different facilities will probe different kinematic domains.

Kinematic reach of existing or near-future DVCS measurements



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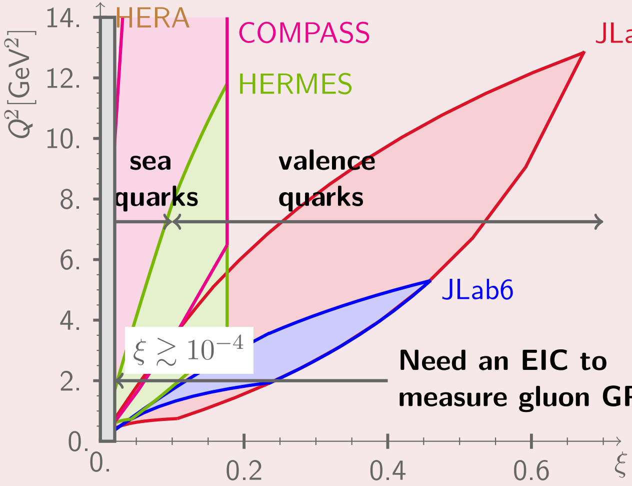
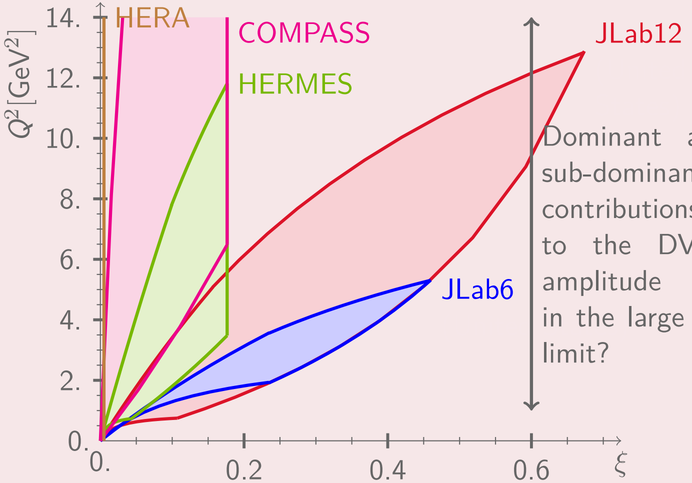
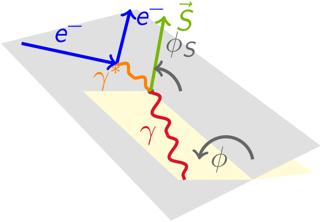


Need an EIC to measure gluon GPDs

Kinematic reach of existing or near-future DVCS measurements



Dominant and sub-dominant contributions to the DVCS amplitude in the large Q^2 limit?

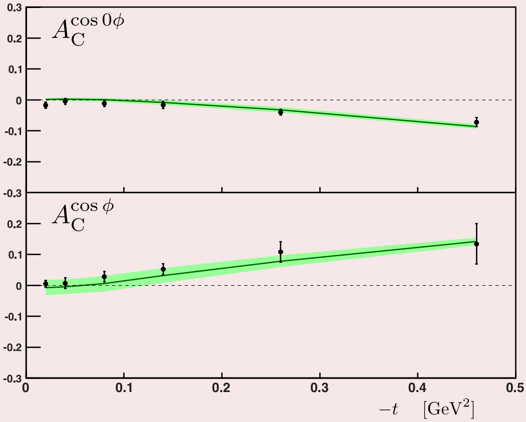


- Study the **harmonic structure** of $ep \to ep\gamma$ amplitude.

Diehl *et al.*, Phys. Lett. **B411**, 193 (1997)

Experiment	Kinematics		
	x_B	Q^2 [GeV^2]	t [GeV^2]
HERA	0.001	8.00	-0.30
COMPASS	0.05	2.00	-0.20
HERMES	0.09	2.50	-0.12
CLAS	0.19	1.25	-0.19
HALL A	0.36	2.30	-0.23

Beam Charge Asymmetry, HERMES

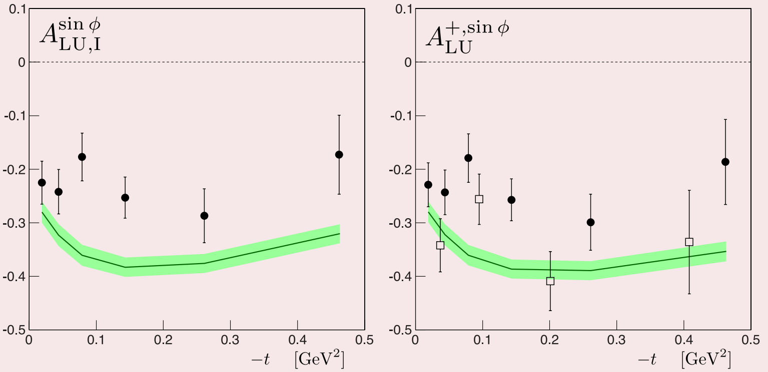


Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

Goloskokov-Kroll (GK) model on DVCS.

No parameter of the GK model **was tuned** to analyse DVCS.

Beam Spin Asymmetry, HERMES



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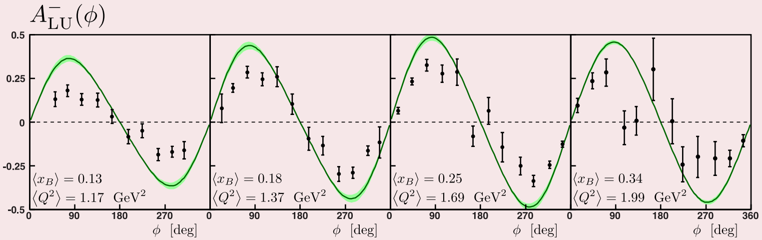
Beam Spin Asymmetry, CLAS



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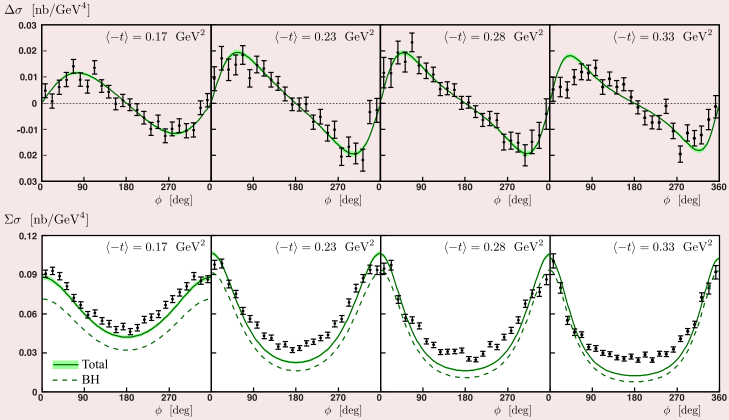
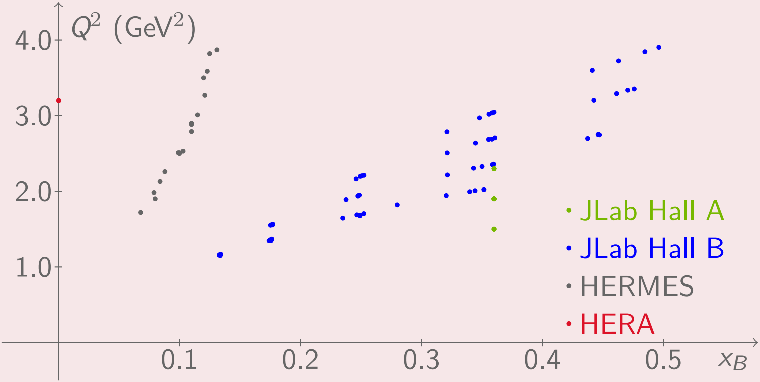
Helicity-dependent and independent cross sections, JLab Hall A



Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

- **Dominance** of twist 2 and **validity** of a GPD analysis of DVCS data.
- $Im\mathcal{H}$ **best determined**. Large uncertainties on $Re\mathcal{H}$.
- However sizable **higher twist contamination** for DVCS measurements.
- Already some indications about the **invalidity** of the H-dominance hypothesis with **unpolarized data**.

What is large Q^2 ?



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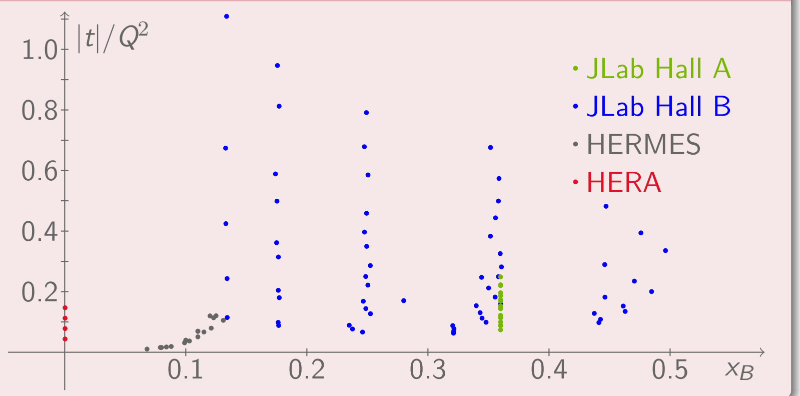
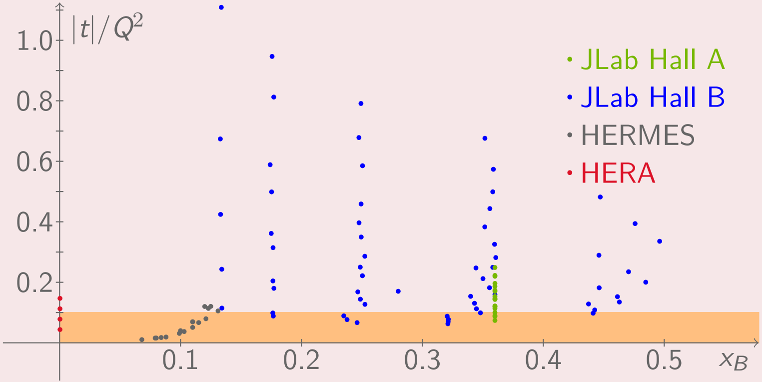


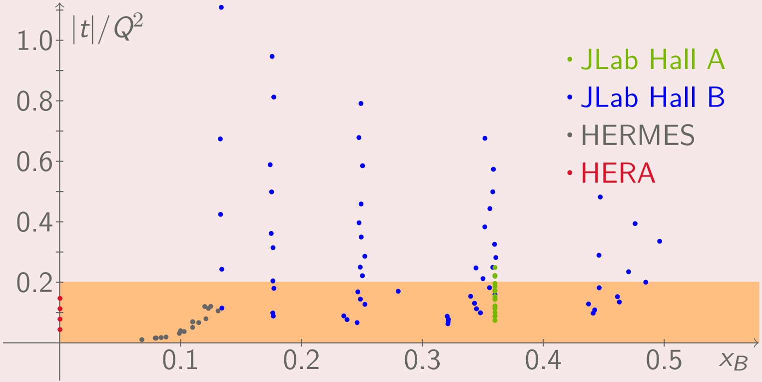
- Q^2 is **not so large** for most of the data.

What is large Q^2 ?



- Q^2 is **not so large** for most of the data.
- **Higher twists**, finite-t and target mass corrections?

What is large Q^2 ?

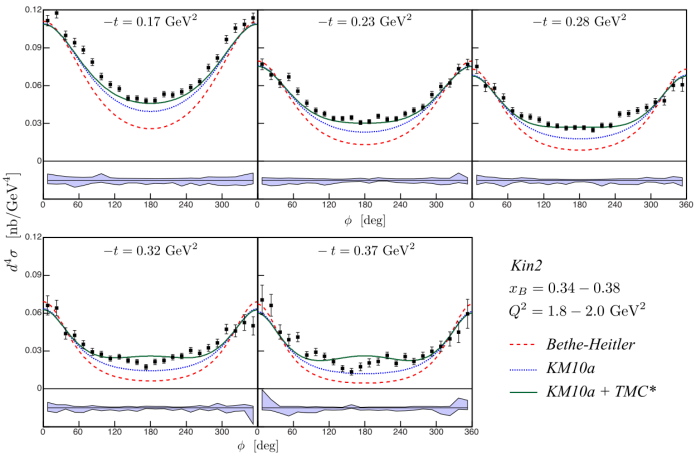


- Q^2 is **not so large** for most of the data.
- **Higher twists**, finite-t and target mass corrections?
- **Consistent modeling** of GPDs beyond leading twist?

The challenges brought by JLab.

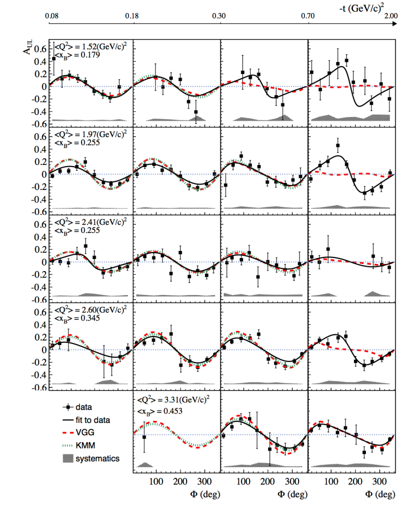
Hints of target mass corrections from recent DVCS analysis.

Improved analysis of 2004 Hall A data (unpolarized target)



Defurne *et al.*, arXiv:1504:05453 [nucl-ex]

Analysis of 2009 Hall B data (polarized target)



Pisano *et al.*, Phys. Rev. **D91**, 052014 (2015)

Hadron Reverse Engineering

Introduction

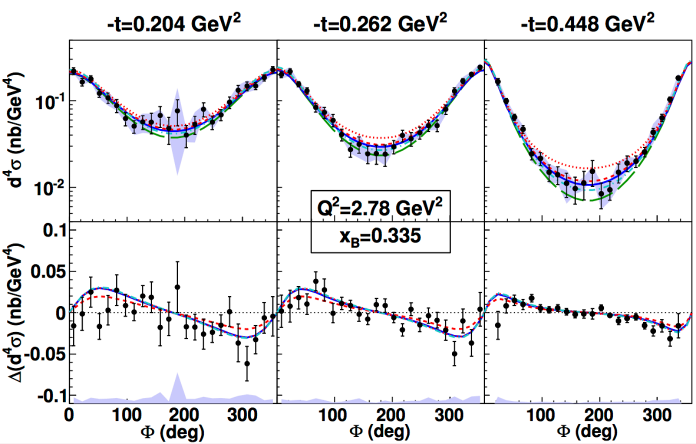
Needs assessment
- Experimental access
- DVCS Kinematics
- First universality tests
- **Towards precision studies**

PARTONS Project
- Computing chain
- Example
- Automation
- Team

GPD modeling
- Formalism
- About Polynomiality
- Survey of models
- Dyson-Schwinger

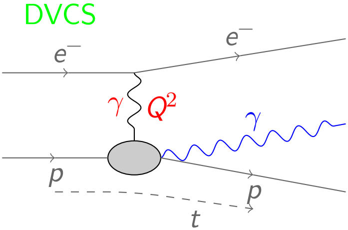
Conclusion

Final analysis of 2005 Hall B data (unpolarized target)



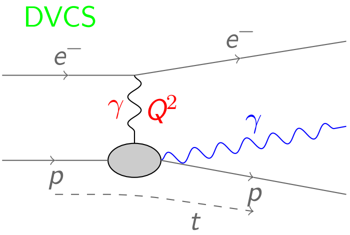
Jo *et al.*, arXiv:1504:02009 [hep-ex]

solid: VGG dash-dotted: KMS dotted / dashed:KM10 / KM10a



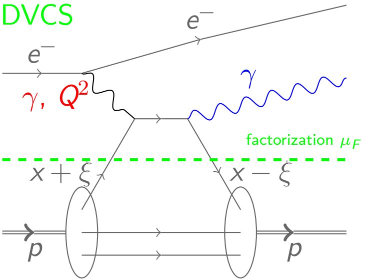
Compton Form Factors (CFF)

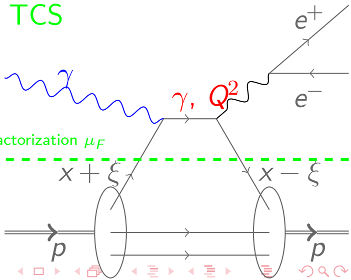
- Parametrize amplitudes.

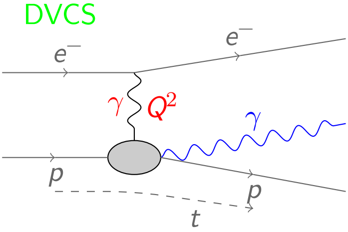


Compton Form Factors (CFF)

- Parametrize amplitudes.
- Evaluation at LO.

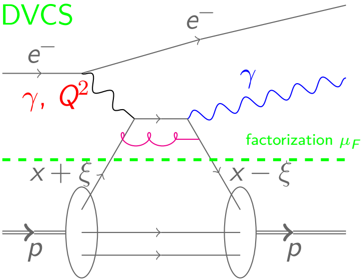


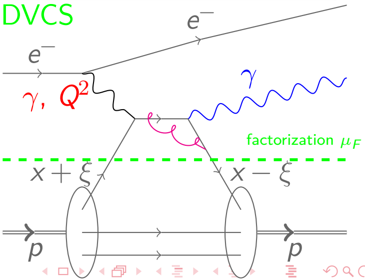


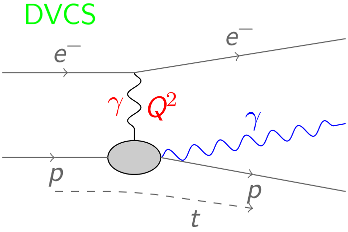


Compton Form Factors (CFF)

- Parametrize amplitudes.
- Evaluation at LO.
- Evaluation at NLO.

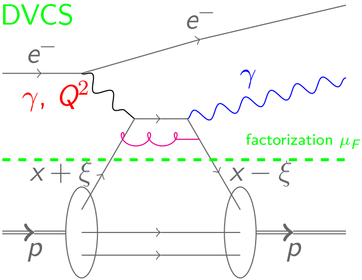


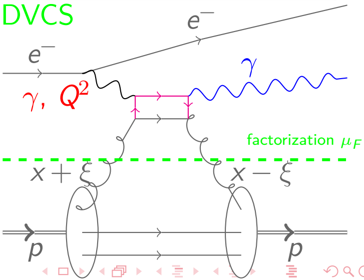




Compton Form Factors (CFF)

- Parametrize amplitudes.
- Evaluation at LO.
- Evaluation at NLO.
- Other diagrams at NLO, including gluon GPDs.





- Convolution of singlet GPD $H_q^+(x) \equiv H_q(x) - H_q(-x)$:

$$\mathcal{H}_q(\xi, Q^2) = \int_{-1}^{+1} dx\, H_q^+(x, \xi, \mu_F)\ T_q\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) + \int_{-1}^{+1} dx\, H_g(x, \xi, \mu_F)\ T_g\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right)$$

Belistky and Müller, Phys. Lett. **B417**, 129 (1998)
Pire *et al*, Phys. Rev. **D83**, 034009 (2011)

- Convolution of singlet GPD $H_q^+(x) \equiv H_q(x) - H_q(-x)$:

$$\mathcal{H}_q(\xi, Q^2) \stackrel{\mathrm{LO}}{=} \int_{-1}^{+1} dx\, H_q^+(x, \xi, \mu_F)\; C_0^q(x, \xi) + \int_{-1}^{+1} dx\, H_g(x, \xi, \mu_F)\; 0$$

Belistky and Müller, Phys. Lett. **B417**, 129 (1998)
Pire *et al*, Phys. Rev. **D83**, 034009 (2011)

- Integration yields **imaginary** parts to $\mathcal{H}$:

$$Im\mathcal{H}_q(\xi, Q^2) \stackrel{\mathrm{LO}}{=} \pi H_q^+(\xi, \xi, \mu_F)$$

- Convolution of singlet GPD $H_q^+(x) \equiv H_q(x) - H_q(-x)$:

$$\mathcal{H}_q(\xi, Q^2) \overset{\text{NLO}}{=} \int_{-1}^{+1} dx\, H_q^+(x, \xi, \mu_F) \left[C_0^q + C_1^q + \frac{1}{2} \ln \frac{|Q^2|}{\mu_F^2} C_{\text{Coll}}^q \right] + \int_{-1}^{+1} dx\, H_g(x, \xi, \mu_F) \left(0 + C_1^g + \frac{1}{2} \ln \frac{|Q^2|}{\mu_F^2} C_{\text{Coll}}^g \right)$$

Belistky and Müller, Phys. Lett. **B417**, 129 (1998)
Pire *et al*, Phys. Rev. **D83**, 034009 (2011)

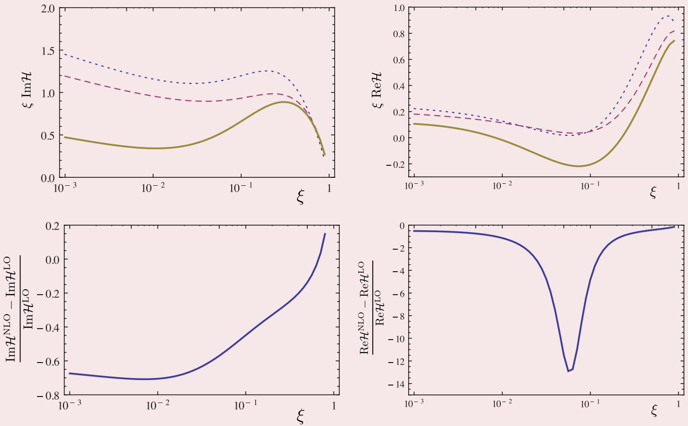
- Integration yields **imaginary** parts to $\mathcal{H}$:

$$Im\mathcal{H}_q(\xi, Q^2) \overset{\text{NLO}}{=} \mathcal{I}(\xi) H_q^+(\xi, \xi, \mu_F) + \int_{-1}^{+1} dx\, \mathcal{T}^q(x) \Big(H_q^+(x, \xi, \mu_F) - H_q^+(\xi, \xi, \mu_F) \Big) + \text{gluon contributions.}$$

Large NLO corrections.

Mostly due to gluons, maximum in HERMES / COMPASS region.

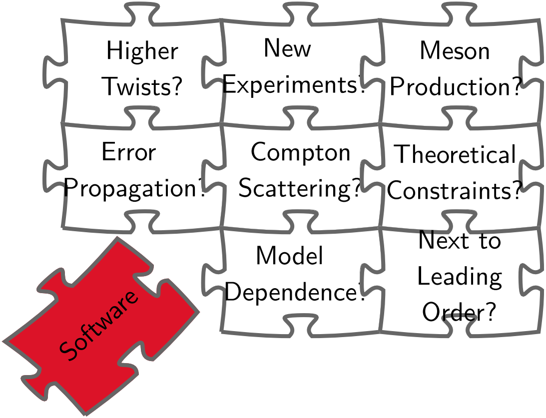
$\mathcal{H}$ at LO and NLO ($t = -0.1$ GeV^2, $Q^2 = \mu_F^2 = 4.$ GeV^2)



Moutarde *et al.*, Phys. Rev. **D87**, 054029 (2013)

dotted: LO dashed: NLO quark corrections solid: full NLO

PARTONS Project



PARTONS Project

PARtonic
Tomography
Of
Nucleon
Software

Computing chain design.

Differential studies: physical models and numerical methods.

Experimental data and phenomenology	Full processes
Computation of amplitudes	Small distance contributions
First principles and fundamental parameters	Large distance contributions

Computing chain design.

Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Full processes

Computation of amplitudes

Small distance contributions

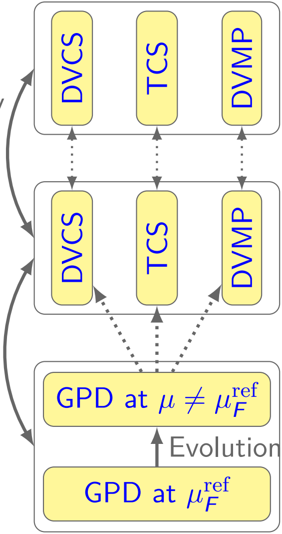
First principles and fundamental parameters

Large distance contributions

Experimental data and phenomenology

Computation of amplitudes

First principles and fundamental parameters



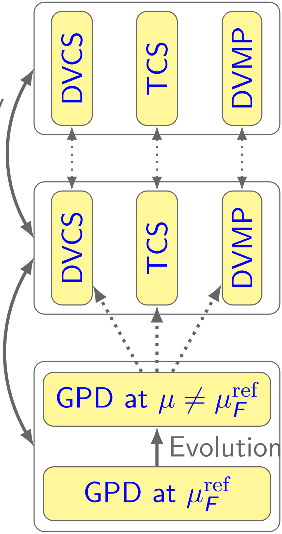
Computing chain design.

Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

Computing chain design.

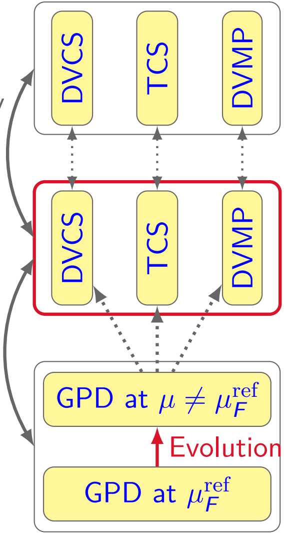
Differential studies: physical models and numerical methods.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

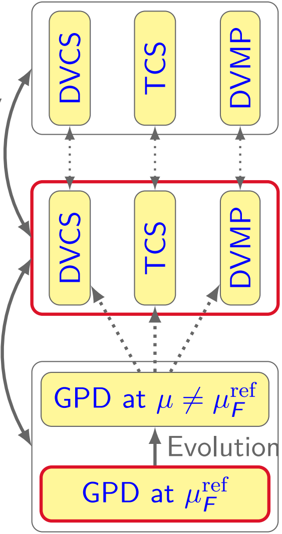
- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



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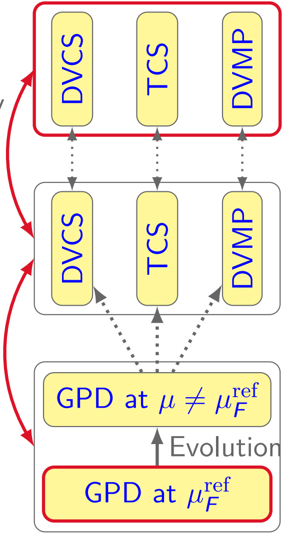
Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



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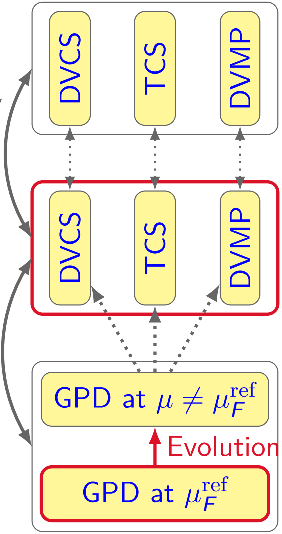

- Many observables.
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Computing chain design.

Differential studies: physical models and numerical methods.

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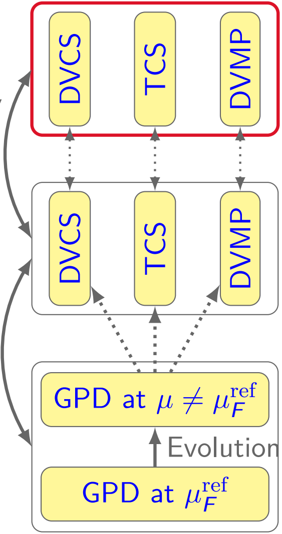

- Many observables.
- Kinematic reach.

- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

Status.

Currently: integration, tests, validation.

- 3 stages:
  1. Design.
  2. Integration and validation.
  3. Production.
- Flexible software architecture.
- 1 new physical development = 1 new module.
- What *can* be automated *will be* automated.
- Get ready for 12 GeV!

GPD computing made simple.

Each line of code corresponds to a physical hypothesis.

gpdEvolutionExample()

```
// Load QCD evolution module
EvolQCDModule* pEvolQCDModule = pModuleObjectFactory->
getEvolQCDModule( VinnikovEvolQCDModel::moduleID ) ;

// Configure QCD evolution module
pEvolQCDModule->setQcdOrderType( QCDOrderType::LO ) ;

// Load GPD module
GPDModule* pGK11Module =
pModuleObjectFactory->getGPDModule( GK11Model::moduleID ) ;

// Create kinematic configuration ( x, xi, t, MuF, MuR )
GPDKinematic gpdKinematic( 0.25, 0.29, -0.28, 1.82, 1.82 ) ;

// Compute GPD and store results
GPDOutputData results = pGPDService->
computeGPDModelWithEvolution( gpdKinematic, pGK11Module,
pEvolQCDModule, GPDComputeType::H ) ;

// Print results
std::cout << results.toString() << std::endl ;
```

GPD computing made simple.

Each line of code corresponds to a physical hypothesis.

gpdEvolutionExample()

```
// Load QCD evolution module
EvolQCDModule* pEvolQCDModule = pModul
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pEvolQCDModule−>setQcdOrderType( QCDO

// Load GPD module
GPDModule* pGK11Module =
pModuleObjectFactory−>getGPDModule( GK1

// Create kinematic configuration ( x, xi , t, M
GPDKinematic gpdKinematic( 0.25, 0.29, −0.2

// Compute GPD and store results
GPDOutputData results = pGPDService−>
computeGPDModelWithEvolution( gpdKinemat
pEvolQCDModule, GPDComputeType::H ) ;

// Print results
std :: cout << results.toString() << std::endl ;
```

Preliminary

Hu = 1.5435
Hu(-) = 2.04736
Hu(+) = 1.03964

Hd = 0.524068
Hd(-) = 1.00457
Hd(+) = 0.0435651

Hs = -0.539675
Hs(-) = 0
Hs(+) = -1.07935

Hg = -0.3086

GPD computing made systematic.

What can be automated is automated.

scenario_01.xml

```
<?xml version="1.0" encoding="UTF-8" standalone="yes" ?>
<scenario id="01" date="" description="Test␣using␣GPD␣service">
    <!-- Select type of computation -->
        <operation service="GPDService" method="computeGPDModel" >
            <!-- Specify kinematics -->
            <GPDKinematic x="-0.99" xB="0.33" t="-0.1" MuF2="2" MuR2="2">
            </GPDKinematic>
            <!-- Choose GPD model and set parameters -->
            <GPDModule id="GK11Model">
                <param name="" value="" />
                <param name="" value="" />
            </GPDModule>
        </operation>
</scenario>
```

void playScenarioExample()

```
ScenarioManager* pScenarioManager = ScenarioManager::getInstance();
// Compute without compiling
pScenarioManager->playScenario(PropertiesManager::getInstance()
    ->getString("scenario.directory") + "scenario_01.xml");
```

GPD computing made systematic.

What can be automated is automated.

scenario_01.xml

Work in progress: database storage

```
bryan@phnpcg46: ~
Fichier  Édition  Affichage  Rechercher  Terminal  Aide
200 rows in set (0.00 sec)

mysql> select * from gpd_kinematic;
+----+-------------+-------------------+-------------------------+------+------+------+------+
| id | scenario_id | kinematic_type_id | x                       | xi   | t    | MuF  | MuR  |
+----+-------------+-------------------+-------------------------+------+------+------+------+
|  1 |           0 |                 1 |                      -1 |  0.2 | -0.1 |    2 |    2 |
|  2 |           0 |                 1 |                   -0.99 |  0.2 | -0.1 |    2 |    2 |
|  3 |           0 |                 1 |                   -0.98 |  0.2 | -0.1 |    2 |    2 |
|  4 |           0 |                 1 |                   -0.97 |  0.2 | -0.1 |    2 |    2 |
|  5 |           0 |                 1 |                   -0.96 |  0.2 | -0.1 |    2 |    2 |
|  6 |           0 |                 1 |                   -0.95 |  0.2 | -0.1 |    2 |    2 |
```

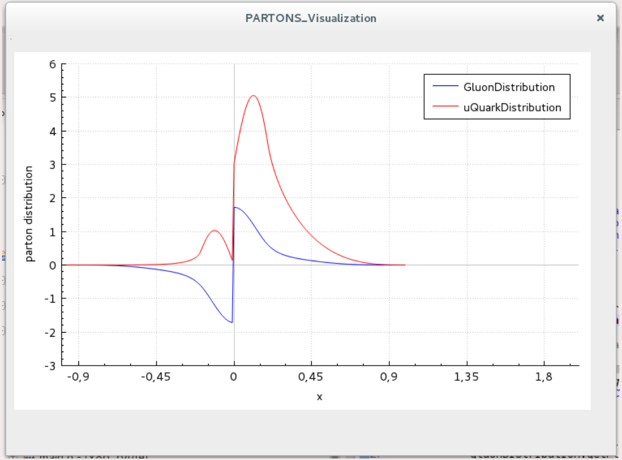
```
        </GPDModule>
    </operation>
</scenario>
```

void playScenarioExample()

```
ScenarioManager* pScenarioManager = ScenarioManager::getInstance();
// Compute without compiling
pScenarioManager->playScenario(PropertiesManager::getInstance()
    ->getString("scenario.directory") + "scenario_01.xml");
```

scenario_01.xml

Work in progress: Visualization



Members and areas of expertise.

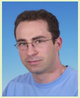
Collaborations at the national and international levels.

Development team


B. Berthou


C. Mezrag


H. Moutarde


F. Sabatié


J. Wagner

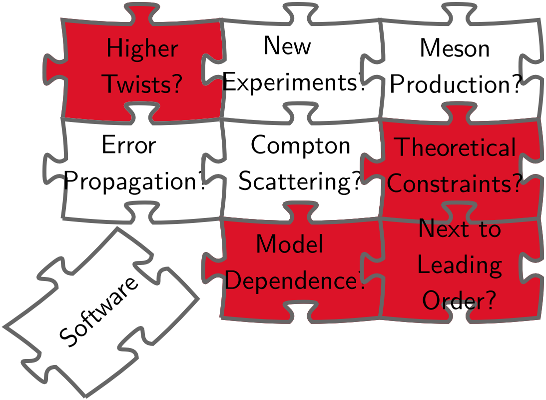
IPN and LPT (Orsay), Irfu (Saclay) and CPhT (Polytechnique)

ANR

Experimental data analysis | Perturbative QCD
World data fits | GPD modeling

GPD Modeling

GPD Modeling

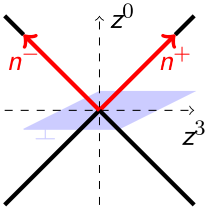


$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' \left| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \right| p\rangle_{z^+=0, z_\perp=0}$$

$$= \frac{1}{2P^+}\left[H^q \bar{u}(p')\gamma^+ u(p) + E^q \bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2M} u(p)\right]$$

$$\tilde{F}^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' \left| \bar{q}\left(-\frac{z}{2}\right)\gamma^+\gamma_5 q\left(\frac{z}{2}\right) \right| p\rangle_{z^+=0, z_\perp=0}$$

$$= \frac{1}{2P^+}\left[\tilde{H}^q \bar{u}(p')\gamma^+\gamma_5 u(p) + \tilde{E}^q \bar{u}(p')\frac{\gamma^5\Delta^+}{2M} u(p)\right]$$



References

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)

Ji, Phys. Rev. Lett. **78**, 610 (1997)

Radyushkin, Phys. Lett. **B380**, 417 (1996)

$$F^q = \frac{1}{2}\int \frac{dz^-}{2\pi} e^{ixP^+z^-} \langle p' \left| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right)\right| p\rangle_{z^+=0, z_\perp=0}$$

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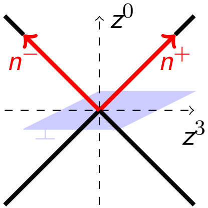
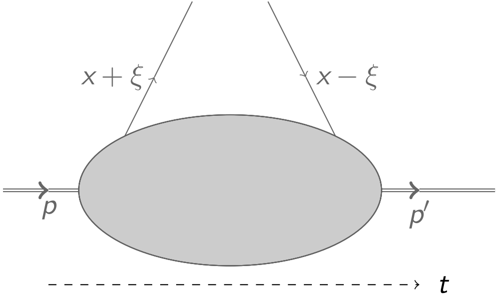
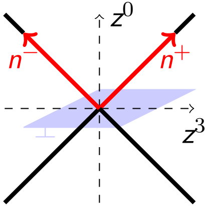


8 GPDs per parton type at twist 2

- Partons with a **light-like** separation.
- **Quarks**, **gluon** and **transversity** GPDs.
- $\mathrm{GPD}^{q,g} = \mathrm{GPD}^{q,g}(x, \xi, t, \mu_F)$.





Interpretation

- $x \in [\xi, 1]$: q emitted + q absorbed.
- $x \in [-\xi, +\xi]$: $\bar{q}$ emitted + q absorbed.
- $x \in [-1, -\xi]$: $\bar{q}$ emitted + $\bar{q}$ absorbed.

$$H^q_\pi(x,\xi,t) \quad = \quad \frac{1}{2}\int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \left\langle \pi, P+\frac{\Delta}{2} \middle| \bar{q}\left(-\frac{z}{2}\right)\gamma^+ q\left(\frac{z}{2}\right) \middle| \pi, P-\frac{\Delta}{2} \right\rangle_{\substack{z^+=0\\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.

- From **isospin symmetry**, all the information about pion GPDs is encoded in $H^u_{\pi^+}$ and $H^d_{\pi^+}$.
- Further constraint from **charge conjugation**: $H^u_{\pi^+}(x,\xi,t) = -H^d_{\pi^+}(-x,\xi,t)$.

- PDF **forward limit**

$$H^q(x, 0, 0) = q(x)$$

- PDF **forward limit**
- Form factor **sum rule**

$$\int_{-1}^{+1} dx\, H^q(x\xi, t) = F_1^q(t)$$

- PDF **forward limit**
- Form factor **sum rule**
- **Polynomiality**

$$\int_{-1}^{+1} dx\, x^n H^q(x, \xi, t) = \text{polynomial in } \xi$$

Properties.

Generalization of form factors and Parton Distribution Functions.

- PDF **forward limit**
- Form factor **sum rule**
- **Polynomiality**
- **Positivity**

$$H^q(x, \xi, t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

- PDF **forward limit**
- Form factor **sum rule**
- **Polynomiality**
- **Positivity**
- H^q is an **even function** of ξ from time-reversal invariance.

Properties.

Generalization of form factors and Parton Distribution Functions.

- PDF **forward limit**
- Form factor **sum rule**
- **Polynomiality**
- **Positivity**
- H^q is an **even function** of ξ from time-reversal invariance.
- H^q is **real** from hermiticity and time-reversal invariance.

- PDF **forward limit**
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- H^q is an **even function** of ξ from time-reversal invariance.
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- H^q has support $x \in [-1, +1]$.

- PDF **forward limit**
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- H^q is an **even function** of ξ from time-reversal invariance.
- H^q is **real** from hermiticity and time-reversal invariance.
- H^q has support $x \in [-1, +1]$.
- **Soft pion theorem** (pion target)

$$H^{I=1}(x, \xi = 1, t = 0) = \phi_\pi \left(\frac{1+x}{2} \right)$$

- PDF **forward limit**
- Form factor **sum rule**
- **Polynomiality**
- **Positivity**
- H^q is an **even function** of ξ from time-reversal invariance.
- H^q is **real** from hermiticity and time-reversal invariance.
- H^q has support $x \in [-1, +1]$.
- **Soft pion theorem** (pion target)

Numerous theoretical constraints on GPDs.

- There is no known GPD parameterization **relying only on first principles**.
- Modeling becomes a key issue.

- Introduce **isovector** and **isoscalar** GPDs:

$$H^{I=0}(x,\xi,t) = H^u_{\pi^+}(x,\xi,t) + H^d_{\pi^+}(x,\xi,t)$$
$$H^{I=1}(x,\xi,t) = H^u_{\pi^+}(x,\xi,t) - H^d_{\pi^+}(x,\xi,t)$$

- Compute Mellin moments of GPDs:

$$\int_{-1}^{1} \mathrm{d}x\, x^m H^{I=0}(x,\xi) = 0 \ (m \text{ even})$$
$$\int_{-1}^{1} \mathrm{d}x\, x^m H^{I=0}(x,\xi) = \sum_{\substack{i=0\\ \text{even}}}^{m} (2\xi)^i C^{I=0}_{mi} + (2\xi)^{m+1} C^{I=0}_{m\,m+1} \ (m \text{ odd})$$
$$\int_{-1}^{1} \mathrm{d}x\, x^m H^{I=1}(x,\xi) = \sum_{\substack{i=0\\ \text{even}}}^{m} (2\xi)^i C^{I=1}_{mi} \ (m \text{ even})$$
$$\int_{-1}^{1} \mathrm{d}x\, x^m H^{I=1}(x,\xi) = 0 \ (m \text{ odd})$$

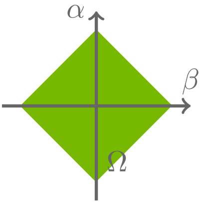
Double Distributions.

A convenient tool to encode GPD properties.

- Define Double Distributions F^q and G^q as matrix elements of **twist-2 quark operators**:

$$\left\langle P+\frac{\Delta}{2}\middle|\bar{q}(0)\gamma^{\{\mu}i\overleftrightarrow{D}^{\mu_1}\ldots i\overleftrightarrow{D}^{\mu_m\}}q(0)\middle|P-\frac{\Delta}{2}\right\rangle = \sum_{k=0}^{m}\binom{m}{k}$$

$$\left[F^q_{mk}(t)2P^{\{\mu}-G^q_{mk}(t)\Delta^{\{\mu}\right]P^{\mu_1}\ldots P^{\mu_{m-k}}\left(-\frac{\Delta}{2}\right)^{\mu_{m-k+1}}\ldots\left(-\frac{\Delta}{2}\right)^{\mu_m\}}$$



with

$$F^q_{mk} = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\,\alpha^k\beta^{m-k}F^q(\beta,\alpha)$$

$$G^q_{mk} = \int_\Omega \mathrm{d}\beta\mathrm{d}\alpha\,\alpha^k\beta^{m-k}G^q(\beta,\alpha)$$

Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
Radyushkin, Phys. Rev. **D59**, 014030 (1999)
Radysuhkin, Phys. Lett. **B449**, 81 (1999)

- Representation of GPD:

$$H^q(x,\xi,t) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha\, \delta(x-\beta-\alpha\xi)\big(F^q(\beta,\alpha,t)+\xi G^q(\beta,\alpha,t)\big)$$

- Support property: $x \in [-1,+1]$.
- Discrete symmetries: F^q is α-even and G^q is α-odd.

- Choose $F^q(\beta, \alpha) = 3\beta\theta(\beta)$ ad $G^q(\beta, \alpha) = 3\alpha\theta(\beta)$:

$$H^q(x, \xi) = 3x \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \, \delta(x - \beta - \alpha\xi)$$

- Simple analytic expressions for the GPD:

$$\begin{aligned} H(x, \xi) &= \frac{6x(1-x)}{1-\xi^2} \text{ if } 0 < |\xi| < x < 1, \\ H(x, \xi) &= \frac{3x(x+|\xi|)}{|\xi|(1+|\xi|)} \text{ if } -|\xi| < x < |\xi| < 1. \end{aligned}$$

- Compute first Mellin moments.

n	$\int_{-\xi}^{+\xi} \mathrm{d}x\, x^n H(x, \xi)$	$\int_{+\xi}^{+1} \mathrm{d}x\, x^n H(x, \xi)$	$\int_{-\xi}^{+1} \mathrm{d}x\, x^n H(x, \xi)$
0	$\frac{1+\xi-2\xi^2}{1+\xi}$	$\frac{2\xi^2}{1+\xi}$	1
1	$\frac{1+\xi+\xi^2-3\xi^3}{2(1+\xi)}$	$\frac{2\xi^3}{1+\xi}$	$\frac{1+\xi^2}{2}$
2	$\frac{3(1-\xi)(1+2\xi+3\xi^2+4\xi^3)}{10(1+\xi)}$	$\frac{6\xi^4}{5(1+\xi)}$	$\frac{3(1+\xi^2)}{10}$
3	$\frac{1+\xi+\xi^2+\xi^3+\xi^4-5\xi^5}{5(1+\xi)}$	$\frac{6\xi^5}{5(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{5}$

- Expressions get more complicated as n increases... But they always yield polynomials!

- **Factorized Ansatz.**

$$
\begin{aligned}
H(x,\xi,t) &= \int_{|\alpha|+|\beta|\leq 1} d\beta d\alpha\, \delta(\beta+\xi\alpha-x) f(\beta,\alpha,t) \\
f(\beta,\alpha,t) &= \frac{1}{|\beta|^{\alpha'(1-\beta)t}} h(\beta)\pi_n(\beta,\alpha) \\
\pi_n(\beta,\alpha) &= \frac{\Gamma(2n+2)}{2^{2n+1}\Gamma^2(n+1)} \frac{(1-|\beta|)^2-\alpha^2]^n}{(1-|\beta|)^{2n+1}}
\end{aligned}
$$

- Expressions for h and n :

$$
\begin{aligned}
h_g(\beta) &= |\beta| g(|\beta|) & n_g &= 1 \\
h^q_{\mathrm{sea}}(\beta) &= q_{\mathrm{sea}}(|\beta|)\mathrm{sign}(\beta) & n_{\mathrm{sea}} &= 1 \\
h^q_{\mathrm{val}}(\beta) &= q_{\mathrm{val}}(\beta)\Theta(\beta) & n_{\mathrm{val}} &= 1
\end{aligned}
$$

- Add D-term at $z = x/\xi$:

$$
D(z) \simeq (1-z^2)\Big(-4.C_1^{3/2}(z) - 1.2 C_3^{3/2}(z) - 0.4 C_5^{3/2}(z)\Big)
$$

Guidal *et al.*, Phys. Rev. **D72**, 054013 (2005)

- **Factorized Ansatz**. For $i = g$, sea or val :

$$H_i(x, \xi, t) = \int_{|\alpha|+|\beta|\leq 1} d\beta d\alpha\, \delta(\beta + \xi\alpha - x) f_i(\beta, \alpha, t)$$

$$f_i(\beta, \alpha, t) = e^{b_i t} \frac{1}{|\beta|^{\alpha' t}} h_i(\beta) \pi_{n_i}(\beta, \alpha)$$

$$\pi_{n_i}(\beta, \alpha) = \frac{\Gamma(2n_i + 2)}{2^{2n_i+1}\Gamma^2(n_i + 1)} \frac{(1 - |\beta|)^2 - \alpha^2]^{n_i}}{(1 - |\beta|)^{2n_i+1}}$$

- Expressions for h_i and n_i :

$$\begin{aligned} h_g(\beta) &= |\beta| g(|\beta|) & n_g &= 2 \\ h^q_{\text{sea}}(\beta) &= q_{\text{sea}}(|\beta|)\text{sign}(\beta) & n_{\text{sea}} &= 2 \\ h^q_{\text{val}}(\beta) &= q_{\text{val}}(\beta)\Theta(\beta) & n_{\text{val}} &= 1 \end{aligned}$$

- Designed to study DVMP. Expect better comparison to data at small x_B.

Goloskokov and Kroll, Eur. Phys. J. **C42**, 281 (2005)

- DGLAP region ($|x| > \xi$): Reggeized quark-diquark model.
- ERBL region ($|x| < \xi$): Extension with polynomials of degree 2 or 3.
- Chiral-even and odd GPDs.

Goldstein *et al.*, Phys. Rev. **D84**, 034007 (2011) and arXiv:1311.0483 [hep-ph]

- Recently proved equivalent to **dual models**.

  Müller *et al.*, JHEP **1503**, 052 (2015)

- Start from t-channel **partial-wave expansion**:

$$H_+(x,\xi) = 2 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{nl} \theta\left(1 - \frac{x^2}{\xi^2}\right)\left(1 - \frac{x^2}{\xi^2}\right) C_n^{3/2}\left(\frac{x}{\xi}\right) P_l\left(\frac{1}{\xi}\right)$$

- From $C_n^{3/2}$ define rescaled polynomials $c_n(x,\xi)$ to **recover Mellin moments** when $\xi \to 0$.
- Define **orthogonal polynomials** $p_n(x,\xi)$ such that:

$$\int_{-1}^{+1} dx\, c_n(x,\xi) p_m(x,\xi) = (-1)^n \delta_{nm}$$

- Write partial-wave expansion:

$$H_+(x,\xi) = \sum_{n=0}^{\infty} (-1)^n p_n(x,\xi) H_n(\xi)$$

- Start from partial-wave expansion:

$$H_+(x,\xi) = \sum_{n=0}^{\infty} (-1)^n p_n(x,\xi) H_n(\xi)$$

- Resum by means of **Sommerfeld - Watson transform**:

$$H_+(x,\xi) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \frac{1}{\sin \pi j} p_j(x,\xi) H_j(\xi)$$

Müller and Schäfer, Nucl. Phys. **B739**, 1 (2006)

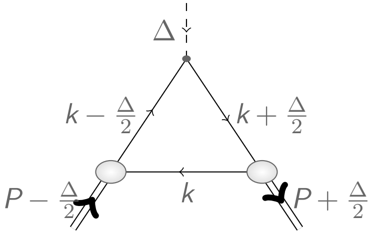
- Express CFF $\mathcal{H}$ in terms of moments H_j:

$$\mathcal{H}(\xi) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \frac{1}{\xi^{j+1}} \left[i + \tan\left(\frac{\pi j}{2}\right)\right] [C_j^0 + \ldots] H_j(\xi)$$

- Regge modeling of $H_j(\xi)$ moments.

Kumericki and Müller, Nucl. Phys. **B841**, 1 (2009)

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$

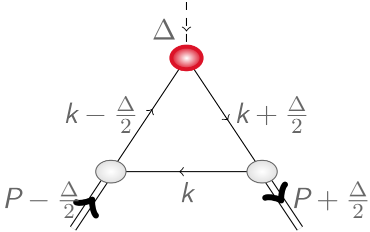


- Compute **Mellin moments** of the pion GPD H.

GPDs in the rainbow ladder approximation.

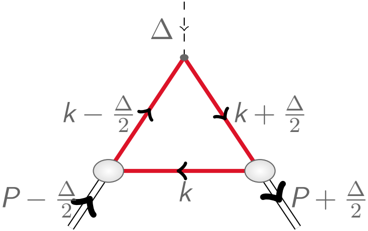
Evaluation of triangle diagrams.

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$



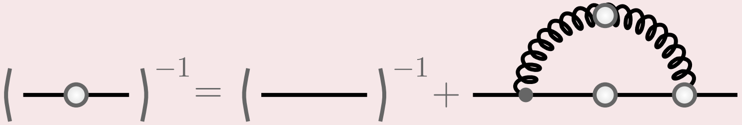
- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0)\gamma^+ (i\overleftrightarrow{D}^+)^m q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$



- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.

Dyson - Schwinger equation



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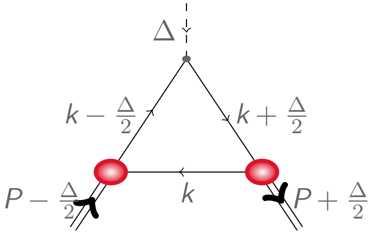
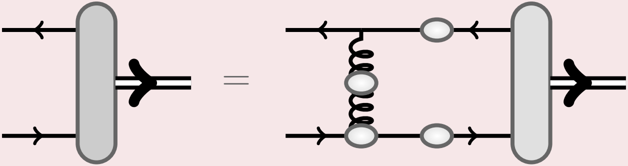
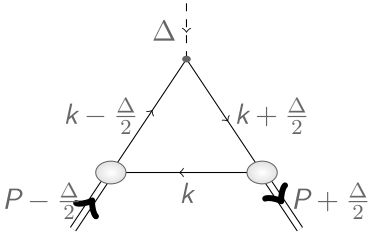


- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.

Bethe - Salpeter equation



$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \left| \bar{q}(0) \gamma^+ (i \overleftrightarrow{D}^+)^m q(0) \right| \pi, P - \frac{\Delta}{2} \right\rangle$$



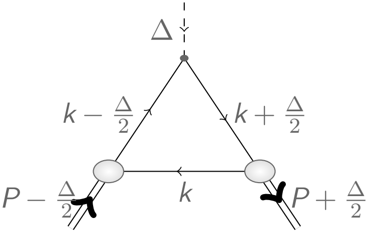
- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.
- **Nonperturbative** modeling.

- Most GPD properties **satisfied by construction**.

GPDs in the rainbow ladder approximation.

Evaluation of triangle diagrams.

$$\langle x^m \rangle^q = \frac{1}{2(P^+)^{n+1}} \left\langle \pi, P + \frac{\Delta}{2} \middle| \bar{q}(0)\gamma^+(i\overleftrightarrow{D}^+)^m q(0) \middle| \pi, P - \frac{\Delta}{2} \right\rangle$$



- Compute **Mellin moments** of the pion GPD H.
- Triangle diagram approx.
- Resum **infinitely many** contributions.
- **Nonperturbative** modeling.
- Most GPD properties **satisfied by construction**.
- Also compute crossed triangle diagram.

Mezrag *et al.*, arXiv:1406.7425 [hep-ph] and Phys. Lett. **B741**, 190 (2015)

Algebraic model.

Intermediate step before using numerical solutions of Dyson-Schwinger and Bethe-Salpeter equations.

- Expression for GPD Mellin moments:

$$2(P^+)^{m+1}\langle x^m\rangle^u = \mathrm{tr}_{CFD}\int\frac{\mathrm{d}^4k}{(2\pi)^4}(k^+)^m\, i\Gamma_\pi\left(k-\frac{\Delta}{2},P-\frac{\Delta}{2}\right) \times S(k-\frac{\Delta}{2})i\gamma^+\, S(k+\frac{\Delta}{2})i\bar{\Gamma}_\pi\left(k+\frac{\Delta}{2},P+\frac{\Delta}{2}\right)S(k-P)$$

- Expressions for vertices and propagators:

$$S(p) = \left[-i\gamma\cdot p+M\right]\Delta_M(p^2)$$

$$\Delta_M(s) = \frac{1}{s+M^2}$$

$$\Gamma_\pi(k,p) = i\gamma_5\frac{M}{f_\pi}M^{2\nu}\int_{-1}^{+1}\mathrm{d}z\,\rho_\nu(z)\ \left[\Delta_M(k^2_{+z})\right]^\nu$$

$$\rho_\nu(z) = R_\nu(1-z^2)^\nu$$

with R_ν a normalization factor and $k_{+z}=k-p(1-z)/2$.

Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

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Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

- Only two parameters:

Algebraic model.

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Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

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Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

- Only two parameters:
  - Dimensionful parameter M.
  - Dimensionless parameter ν

- Expressions for vertices and propagators:

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with R_ν a normalization factor and $k_{+z} = k - p(1-z)/2$.

Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

- Only two parameters:
  - Dimensionful parameter M.
  - Dimensionless parameter ν. **Fixed to 1** to recover asymptotic pion DA.

Realistic model.

Implementing vertices and propagators coming from the numerical resolution of the Dyson-Schwinger and Bethe-Salpeter equations.

- Numerical solutions of equations are taken into account by a fit with the following Ansätze:
  - Ansatz for quark propagator:

$$S(p) = \sum_{j=1}^{j_m} \left(\frac{z_j}{i\not{p} + m_j} + \frac{z_j^*}{i\not{p} + m_j^*} \right)$$

  - Ansatz for scalar functions in Bethe Salpeter amplitude:

$$F(k;P) = c \int_{-1}^{+1} \mathrm{d}z\, \rho_\nu(z) \Lambda k^2 \Delta_\Lambda^2(k_z^2) + \text{ other similar terms}$$

Chang *et al.*, Phys. Rev. Lett. **110**, 132001 (2013)

- Use experience from algebraic model.
- In principle slightly more complex. In practice many more terms. *Work in progress.*

Analytic expression in the DGLAP region.

$$
\begin{aligned}
H^u_{x\geq\xi}(x,\xi,0) \quad = \quad & \frac{48}{5}\left\{ \frac{3\left(-2(x-1)^4\left(2x^2-5\xi^2+3\right)\log(1-x)\right)}{20\left(\xi^2-1\right)^3} \right. \\
& \frac{3\left(+4\xi\left(15x^2(x+3)+(19x+29)\xi^4+5(x(x(x+11)+21)+3)\xi^2\right)\tanh^{-1}\left(\frac{(x-1)}{x-\xi^2}\right.\right.}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(x^3(x(2(x-4)x+15)-30)-15(2x(x+5)+5)\xi^4\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(-5x(x(x(x+2)+36)+18)\xi^2-15\xi^6\right)\log\left(x^2-\xi^2\right)}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(2(x-1)\left((23x+58)\xi^4+(x(x(x+67)+112)+6)\xi^2+x(x((5-2x)x+15)+3\right.\right.}{20\left(\xi^2-1\right)^3} \\
& +\frac{3\left(\left(15(2x(x+5)+5)\xi^4+10x(3x(x+5)+11)\xi^2\right)\log\left(1-\xi^2\right)\right)}{20\left(\xi^2-1\right)^3} \\
& \left. +\frac{3\left(2x(5x(x+2)-6)+15\xi^6-5\xi^2+3\right)\log\left(1-\xi^2\right)}{20\left(\xi^2-1\right)^3} \right\}
\end{aligned}
$$

- **Analytic expression** in the DGLAP region.
- Similar expression in the ERBL region.

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- Similar expression in the ERBL region.
- **Explicit check** of **support property** and **polynomiality** with correct powers of ξ.

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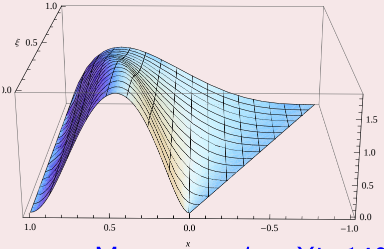
- Similar expression in the ERBL region.
- **Explicit check** of **support property** and **polynomiality** with correct powers of ξ.
- Also direct verification using Mellin moments of H.

Valence $H^u(x, \xi, t)$ as a function of x and ξ at vanishing t.

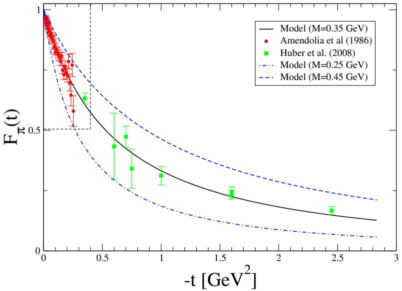


Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

- Pion form factor obtained from isovector GPD:

$$\int_{-1}^{+1} \mathrm{d}x \, H^{I=1}(x, \xi, t) = 2F_\pi(t)$$

- Single dimensionful parameter $M \simeq 350$ MeV.



Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

Pion form factor.

Determination of the model dimensionful parameter M.

- Pion form factor obtained from isovector GPD:

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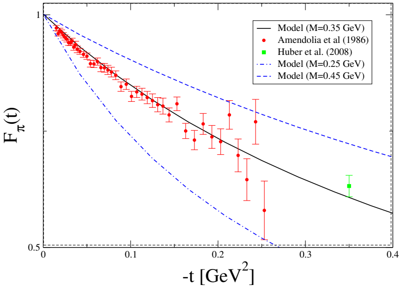

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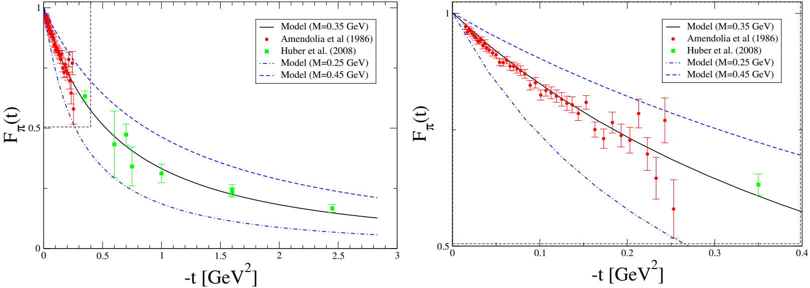


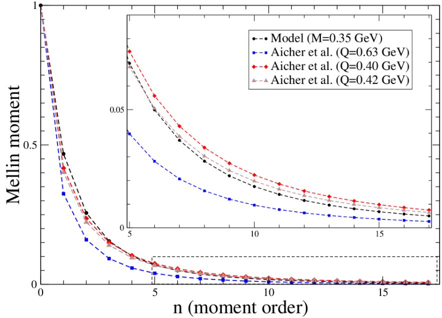
Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

- Pion PDF obtained from forward limit of GPD:

$$q(x) = H^q(x, 0, 0)$$

- Use LO DGLAP equation and compare to PDF extraction.

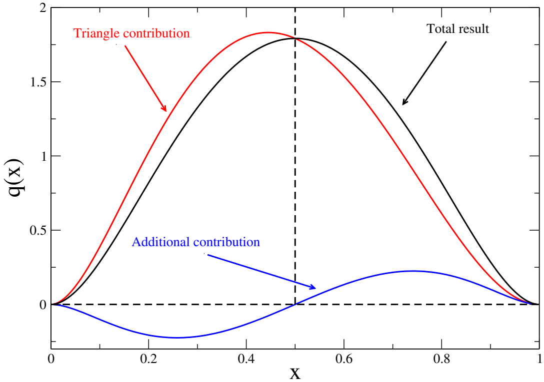
Aicher *et al.*, Phys. Rev. Lett. **105**, 252003 (2010)



Mezrag *et al.*, arXiv:1406.7425 [hep-ph]

- Find model initial scale $\mu \simeq 400$ MeV.

- In a symmetric 2-body problem, the PDF should be symmetric with respect to $x \leftrightarrow 1 - x$.



Chang *et al.*, Phys. Lett. **B737**, 23 (2014)

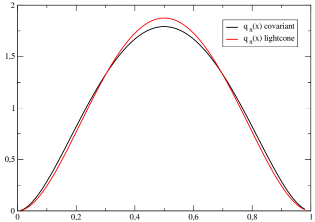
- A triangle diagram calculation neglects part of the gluon exchanges implementing this property.

- Overlap of light-front wave functions:

$$q(x) = 30x^2(1-x)^2$$

- "Improved" triangle diagram calculation:

$$\begin{aligned} q(x) \quad = \quad & \frac{72}{25}\Big(2x(1-x)[6-x(1-x)] \\ & +(1-x)^3[2(1-x)^3-5(1-x)+15]\log(1-x) \\ & +x^3[2x^3-5x+15]\log x\Big) \end{aligned}$$



Conclusion

- Last decade demonstrated **maturity of GPD phenomenology**.
- **Good theoretical control** on the path between GPD models and experimental data.
- **Challenging constraints** expected from JLab in the valence region.
- Building of **QCD-inspired models** to make progress.
- Development of a **platform dedicated to global GPD analysis** to perform precision GPD studies.

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