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NON CURRENT-FREE CORONAL CLOSURE OF SUBPHOTOSPHERIC MHD MODELS

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ABSTRACT

We propose a method which allows the matching of two classes of models which have been well developed so far, but largely independently from each other: (i) Convection zone (CZ) models, which generally either end up below the photosphere or are matched with an external potential field. (ii) Coronal models of eruptive processes and heating, which usually consider the evolution of current carrying magnetic fields driven by given photospheric changes. In our approach, the thin turbulent photospheric layer between the two large regions is modelled by a resistive layer across which the physical quantities suffer stiff variations. We show that this layer enables the transport of an electric current into the corona through the tangential component of the electric field (continuous across the various interfaces), as well as a good conservation of the global magnetic helicity. To illustrate our general approach, we present in details the model problem in which the rising of an initially twisted flux rope throughout the CZ is described kinematically while the physics inside the corona is described by a full MHD model. We show that the evolution leads to the emergence of magnetic flux and electric current into the corona, with the creation of a flux rope suffering eventually a dynamical transition towards a fast expansion.

Subject headings: MHD – stars: corona – stars: magnetic fields – stars: flares

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1. INTRODUCTION

It is now well admitted that, in the solar corona, the heating processes and the big eruptive phenomena are powered by the energy of the magnetic field generated by both global and localized dynamo actions at the bottom and inside the convection zone (CZ; see, e.g., Fan (2004)). Actually, there are subtle interactions between the field and turbulent convection and rotation which lead to many interesting features. Various patterns are produced, such as granulation, supergranulation, and magnetic field concentrations in the photospheric layer. Large scale flux tubes escape occasionally from the strong toroidal field production zone and find their way through the CZ, eventually piercing the photosphere and emerging into the corona as twisted ropes. More generally, emerging tubes feed the network outside active regions where they contribute via reconnection processes to the coronal heating.

Most of the MHD CZ models are largely disconnected from the solar atmosphere. For instance, in standard dynamo or convection simulations, the vertical component of the velocity is imposed to vanish at the top of the numerical box, while the field is assumed either to be confined by that boundary or to match thereon with an external potential field (e.g., Brun et al. (2004)). Smaller scales are considered in another class of models in which it is the emergence through the stiff photosphere of an individual flux rope launched not too deep in the CZ which is tentatively tackled (Fan 2001; Magara & Longcope 2003; Abbett & Fisher 2003; Magara 2004; Manchester IV et al. 2004). In these works, there is a large horizontal component of the velocity field and then a cell-like structure of the motions. Finally, there are also recent alternative attempts in which it is the role of purely vertical motions in the transfer of energy and helicity which is investigated (Fan & Gibson 2003, 2004; Amari et al. 2004), but of course such an approach neglects the cell-like character of the convection structure. On the other hand, most of the coronal models presented so far (e.g., Amari et al. (2003b,a)) qualify to be members of the class of *tangential electric field boundary value problems*, as they try to determine the evolution of a current carrying (at least force-free) magnetic field driven by given flows or magnetic flux variations occurring at the photospheric level (see Zhang & Low (2005) for a review). In particular, they have focussed in that framework on the question of the storage and release of energy and helicity.

It thus appears that an important task is to make a large scale CZ model to match with a realistic coronal model, which would ensure in particular the consistent transfer of helicity from the CZ into the corona and would make the CZ state being dependent on the coronal one. The main purpose of this Letter is to make a step towards such a goal by introducing the Resistive Layer Model (RLM). Precisely, we would like with the latter to address the following issue: *How to "close" a subphotospheric MHD model in order to naturally allow the transfer of magnetic energy and helicity into the solar corona through non current-free fields.*

This problem represents a serious numerical challenge as we have to solve a global MHD problem including both the large scale CZ and atmosphere and their small scale boundary layer interface through which the physical quantities suffer very stiff changes.

The key quantity controlling the transfer of energy and helicity between two regions has been abundantly shown in our previous studies to be the parallel component $\mathbf{E}_s$ of the electric field. Thus our approach here is based on following up the value of the latter throughout a turbulent photospheric boundary layer in which the effective resistivity is larger than in the CZ just below and in the corona above, and that we modelize by a resistive layer. Basically, $\mathbf{E}_s$ is continuous during this crossing, but its expression changes from an inductive form (related to the convection flow) near the top of the CZ to a resistive form inside the photospheric layer and again to an inductive form at the basis of the corona, where it acts as the driver of an evolution in which force-free magnetic fields are naturally produced.

The RLM is described in general terms in Sect. 2. In Sect. 3, it is illustrated by the computation of a case in which an initially twisted flux rope is kinematically raised by a convection cell in the CZ and evolves in a full MHD way after its emergence in the corona.

2. THE RESISTIVE LAYER MODEL

We take the solar corona, the CZ, and their interface to be represented, respectively, by the upper half-space $\Omega_+ = \{z > 0\}$, the lower one $\Omega_- = \{z < 0\}$, and the plane $\Gamma = \{z = 0\}$. We assume that two MHD models are available for determining for all $t > 0$ the evolution in Ω_+ and Ω_-, respectively, of a state $\{\mathbf{B}, \mathbf{v}, p, \rho\}$ (with standard notations). Moreover, we suppose that the normal component of the velocity vanishes on Γ ($v_z = 0$). We recall that, from Ohm's law, we have for the horizontal component of the electric field at any point

$$\mathbf{E}_s = \hat{\mathbf{z}} \times (B_z \mathbf{v}_s - v_z \mathbf{B}_s) + \eta \mathbf{j}_s, \tag{1}$$

where η is the magnetic diffusivity and $\mathbf{X}_s = X_x \hat{\mathbf{x}} + X_y \hat{\mathbf{y}}$. This quantity which is continuous accross any interface will play a basic role hereafter.

Let us first take the plasma to be perfectly conducting in both domains, whence $\mathbf{E}_s = \hat{\mathbf{z}} \times \mathbf{v}_s^+ B_z = \hat{\mathbf{z}} \times \mathbf{v}_s^- B_z$ on Γ by Eq. (1) (the $+/-$ indicate a value just above/under an interface). Therefore $\mathbf{v}_s^+ = \mathbf{v}_s^-$ and we just get a generic shearing-like boundary condition for the field in Ω_+, with the associated injection of magnetic helicity depending on the only component B_z of $\mathbf{B}$ (the component $\mathbf{B}_s$ has disappeared from the expression of $\mathbf{E}_s$, in contrast with the situation considered in Amari et al. (2004)). But $\mathbf{B}_s$ contributes in general to the creation of a nonzero normal component of the electric current on $\Gamma_{-,+}$.

Let us now introduce at the top of Ω_- a thin resistive layer Ω_{RBL} of width z_{RBL} – i.e., a region of nonzero resistivity which may be thought to mimic the turbulent photosphere at the top of the CZ in which an effective resistivity enhancement is expected. Then Ω_{RBL} is comprised between the upper plane $\Gamma^u_{RBL} = \Gamma$ and the new lower interface $\Gamma^l_{RBL} = \{z = -z_{RBL}\}$, and $\mathbf{E}_s$ is continuous across both of them. On Γ^u_{RBL}, we have now $\mathbf{E}_s = \hat{\mathbf{z}} \times \mathbf{v}_s^- B_z + \eta \mathbf{j}_s^- = \hat{\mathbf{z}} \times \mathbf{v}_s^+ B_z$. The second term in the middle member is crucial since it shows that the MHD state in Ω_- depends on the one in Ω_+ through $\mathbf{B}$. In principle, the kernel in Ω_+ could be : (i) Potential : $\mathbf{B} = \nabla\Phi$, with $\Delta\Phi = 0$, $\partial_z\Phi(x, y, 0, t) = g(x, y, t)$. (ii) Force-free: $\nabla \times \mathbf{B} = \alpha\mathbf{B}$, $B_z(x, y, 0, t) = g(x, y, t)$ and $\alpha(x, y, 0, t) = h(x, y, t)$ on $\{\mathbf{r} \in \Gamma \mid B_z(\mathbf{r}, t) > 0\}$, with h being determined by $\mathbf{B}_s$ in Ω_{RBL}. (iii) MHD: $\mathbf{B}$ is obtained by solving the full set of MHD equations. As noted in the Introduction, option (i) is often adopted for closing dynamo models (e.g., Brun et al. (2004)); option (ii) seems to have never been implemented so far, while option (iii) is the one considered here.

In order to fix an adequate value of z_{RBL}, we consider the various length and time scales involved in the problem. We first define the *return layer* in Ω_- to be the layer of thickness Δ_{RL} in which $\mathbf{v}$ switches from an almost vertical direction to an horizontal one in order to match the condition $v_z \mid_\Gamma = 0$, and assume that $\Delta_{RL} << L_C$, with L_C the global length scale of the domain. As shown in Amari et al. (2003b,a, 2004), the key quantity to transfer magnetic helicity into Ω_+ is $\mathbf{E}_s$, and the latter was efficiently determined in our previous kinematic model (Amari et al. 2004) by the imposed purely vertical velocity field $\mathbf{v}_- = v_z(x, y)\hat{\mathbf{z}}$. We therefore choose as a first condition (C_1): $\Delta_{RL} \approx z_{RBL}$. Accross Γ^l_{RBL}, the expression of $\mathbf{E}_s$ switches from the ideal form $\mathbf{E}_s = -\hat{\mathbf{z}} \times v_z \mathbf{B}_s^-$ valid below (we neglect here the weak horizontal velocity) to a resistive form, and it switches back from a resistive form to an ideal one on Γ^u_{RBL}. The net effect may be described as a transfer through Ω_{RBL} of the parallel component $\mathbf{B}_s$ of the field from the region in Ω_- where v_z is non zero into Ω_+. Next we introduce the diffusion time τ_D associated to z_{RBL}, the characteristic convection time τ_C associated to L_C and the characteristic time τ_{MHD} associated to the coronal MHD evolution (typically $\tau_{MHD} = \tau_{\mathrm{Alfven}}$). In order to have the magnetic field convected in Ω_- not diffusing before being transferred upwards, we impose $\tau_D \gg \tau_C$. As $z_{RBL} \ll L_C$ (C_1), this is obtained by requiring (C_2) η being not too large. As a last condition (C_3), we demand that $\tau_C \geqslant \tau_{MHD}$ so that the coronal evolution should adapt to subphotospheric changes (either quasi-statically or at most on the wave crossing time scale). This is fullfiled by taking $v_C \leqslant v_{MHD}$ ($v_C \ll v_{\mathrm{Alfven}}$ in the case considered below).

3. RESULTS AND DISCUSSION

We consider in this section a version of the RLM in which the model in Ω_- describes kinematically the convection of a flux rope. All the quantities used below are written in nondimensionalized form as in Amari et al. (1999). We first fix the conditions in Ω_-, represented in our simulations by the domain $\Omega_-^h = [0, 200] \times [0, 200] \times [-10, 0]$. Following Fan (2001) and Amari et al. (2004), we take the initial magnetic field to be given by

$$\mathbf{B}_0 = B_0 e^{-[y^2+(z-z_0)^2]/a^2}[\hat{\mathbf{x}} - q(z - z_0)\hat{\mathbf{y}} + qy\hat{\mathbf{z}}], \tag{2}$$

with $q = -2$, $z_0 = -5$, $a = 2$, and $B_0 = 2$. $\mathbf{B}_0$ describes a rope which is cylindrically symmetric about the axis $\{y = 0, z = z_0\}$ and has a twist controlled by the parameter q. The shape of the field lines are shown in Figure 1.

The velocity field advecting $\mathbf{B}$ for $t \geq 0$ is chosen to be of the form $\mathbf{v}(x, y, z) = v_0 f(x)\nabla\Psi(y, z) \times \hat{x}$, with $f(x) = e^{-\frac{(x-x_c)^2}{\sigma_x^2}}$ and

$$\begin{aligned}\Psi(y,z) &= (y-y_c)^{2n+1}\left(1-\frac{(y-y_c)^{2p}}{R_1{}^{2p}}\right)\left(1-\frac{(y-y_c)^{2q}}{R_2{}^{2q}}\right)\times \\ &\quad \tanh\left(\frac{z}{d}\right)\tanh\left(\frac{z+L_z}{d}\right)\left(1+\frac{(y-y_c)^{2s}}{R_3{}^{2s}}\right)^{-1}.\end{aligned} \tag{3}$$

The following values of the parameters are selected: $x_c = y_c = 100$, $\sigma_x = 10$, $R_1 = 10$, $R_2 = 100$, $R_3 = 10$, $p = 1$, $q = 1$, $s = 4$, $d = 1$, $L_z = 10$, and $v_0 = 10^{-2}$ (as $v_A = 1$ in our units, condition C_3 is fulfilled). $\mathbf{v}$ is clearly incompressible, contrarily to the velocity used in Amari et al. (2004), and it may be shown to describe the motion taking place in: (i) A simple generic convection cell forcing the central part of the rope to rise, with a return layer of width $\Delta_{RL} = 2$ in which matching with the tangential boundary condition is achieved – see Figure 2 representing the contours of the Ψ function; note that the invariance along the x-axis is broken by the introduction of the function f. (ii) Two weak external half-cells in which the flow decreases very rapidly near the boundaries. In the upper part of Ω_-, we impose a resistivity of the form $\eta(x, y, z) = \eta_0 \exp[-(z - z_c)^2/\sigma_z^2]$, with $\eta_0 = 10^{-3}$ and $z_c = -1$. This defines our resistive layer Ω_{RBL}^h, which has $z_{RBL} = \sigma_z = 0.5$ – hence C_1 is fulfilled.

In the representation Ω_+^h of the coronal part Ω_+, there is initially no background field, and the evolution of the field and the plasma for $t \geq 0$ is controlled by only imposing on the boundary the tangential electric field defined by equation (1) with $v_z = 0$.

In Ω_-^h, the convection-diffusion equation for **B** is solved numerically on a nonuniform mesh of size 160x158x110 nodes. In Ω_+^h, the evolution is governed by the usual system of MHD equations with zero resistivity (see Sect. 4.1 of Amari et al. (2003a)). Small values are used for the kinematic viscosity ($\nu = 10^{-2} - 10^{-3}$), the plasma β is either set to zero or to 10^{-3} – the actual coronal value – with no differences appearing, and the initial density ρ is set to 1. The MHD equations are discretized on a nonuniform mesh of size 160x158x110 nodes, and solved by using our semi-implicit scheme (Amari et al. 1999). Note that due to the definition of the RBL through a resistivity profile, the transition between Ω_+^h and Ω_-^h is sharp but much less than in the idealized case of two non-overlapping domains.

We now describe the main features of the evolution of **B**: (i) While in the case $\eta = 0$ (which was run for a test) the twisted flux rope in Ω_- is strongly deformed, but no flux is transferred through Γ, emergence occurs when $\eta \neq 0$ as clearly shown by panels (b)-(d) of Figure 1. (ii) As seen on Figure 1, there is emergence of both magnetic flux (see also panel (b) of Fig. 3) and electric current (shear appears to be present in Ω_+). This shows that a closure of the MHD model in Ω_- by a current carrying solution in Ω_+ can be performed only in the presence of resistivity when the condition $v_z = 0$ is imposed on Γ. (iii) As magnetic flux and electric current emerge, there exists a critical time $t_{fl} > 0$ at which the magnetic topology switches from an arcade type to a flux rope type (see panel (b) of Fig. 1). This transition occurs while the rate of increase of the magnetic flux on Γ starts decreasing. (iv) As t increases the configuration inflates much more rapidly as shown by the variation of the kinetic energy (panel (a) of Fig. 3). Eventually, it reaches the top of the domain, exhibiting a dynamic transition at some critical time t_c, $0 < t_{fl} < t_c$ (panel (c) of Fig. 1). (v) The energy W_{pot} of the potential field having a distribution of B_z on $\{z = 0\}$ identical to that of **B**, first increases and thus decreases at a finite rate as shown on panel (1) of Figure 3. It is worth noticing that the energy W of the configuration decreases at a much smaller rate than W_{pot}. Unlike the case of the purely vertical CZ flow considered in Amari et al. (2004), the total relative magnetic helicity (panel (b) of Fig. 3) does not keep increasing at the same rate, but seems to saturate at a value outside the limits of the simulations.

We conclude with a brief summary of the main characteristics of the RLM: (i) By introducing a resistive layer of width equal to that of the return layer where the convection flow matches the photosphere, it is possible to transfer a part of the transverse component of **B** from the region where v_z is strong to the corona. This allows the closure of the CZ model by a current carrying coronal solution. (ii) An important role is played by the resistivity inside the photosphere even if it is not too large: the absence of resistivity inhibits indeed the transfer of the parallel component of **B**. (iii) In the case where the rising of a twisted flux rope is described by a kinematical convection model, the RLM exhibits several features observed in full MHD simulations of the transition between the CZ and the chromosphere, such as concentration of magnetic flux and transfer of magnetic helicity. However, it shows that a divergence-free velocity field closing up at the boundary implies a tendency for magnetic energy and helicity to saturate, a feature not seen when imposing vertical compressible nonuniform motions (Amari et al. 2004). As in several previous studies (Amari et al. 2000, 2003a, 2004), a twisted flux rope is produced during an equilibrium phase of the evolution rather than during a major disruption as in Amari et al. (2003b) and Antiochos et al. (1999). (iv) The RLM could be used to couple coronal models with more elaborate large scale CZ models such as anelastic spherical harmonics or compressible models.

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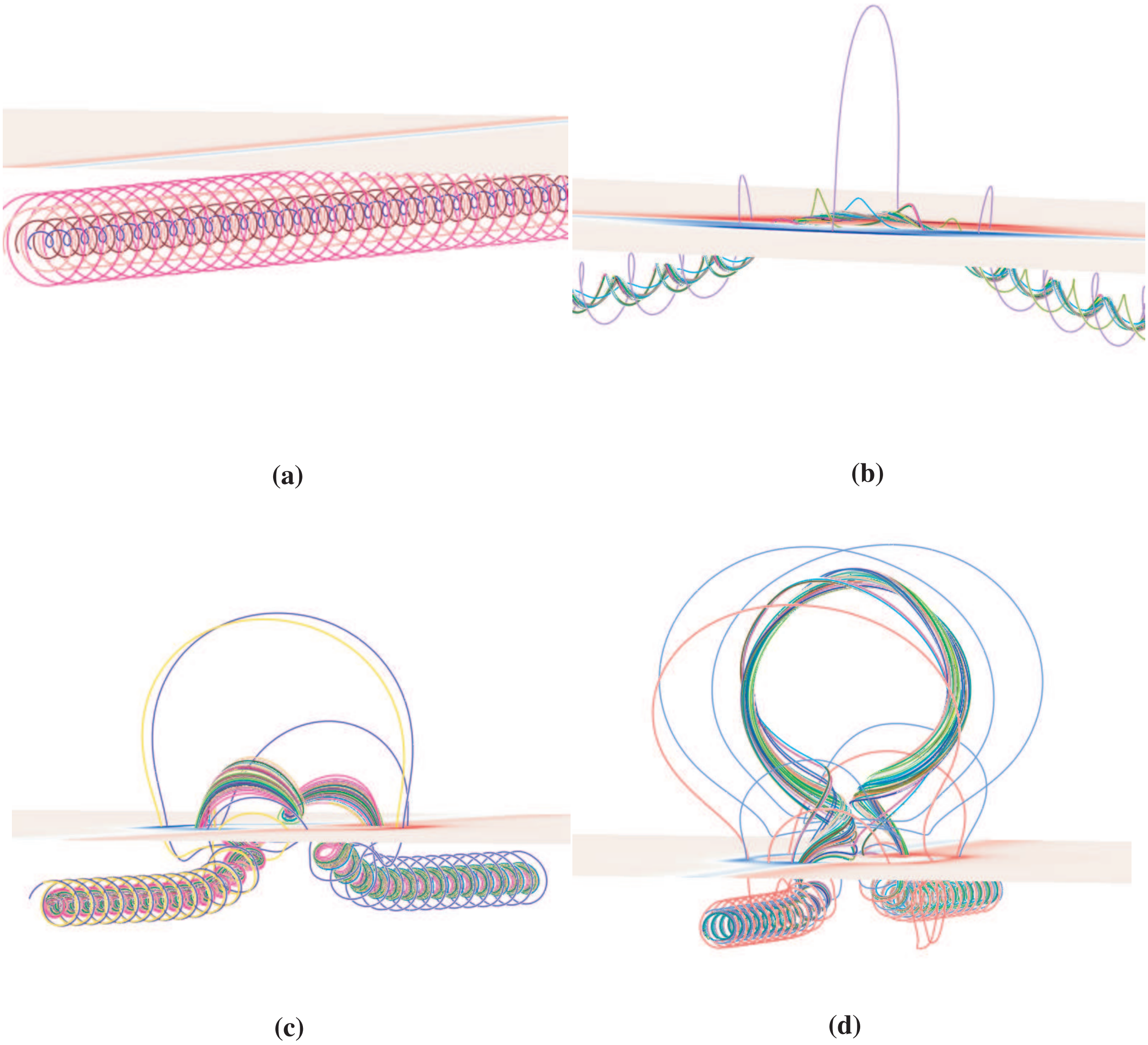

Fig. 1.— Selected field lines of the global configuration obtained at different stages of the evolution: (a) Initial flux rope sitting in the CZ at $t = 0$, at a depth equal to -5. (b) At $t = 800$, emergence of flux and electric currents leads to an arcade like current carrying configuration. (c) At $t = 1000$, further emergence leads to a change of magnetic topology from the previous arcade type to the new flux rope type (this transition occurs after the magnetic axis has emerged). (d) At $t = 3000$, the configuration evolves more rapidly and experiences strong inflation.

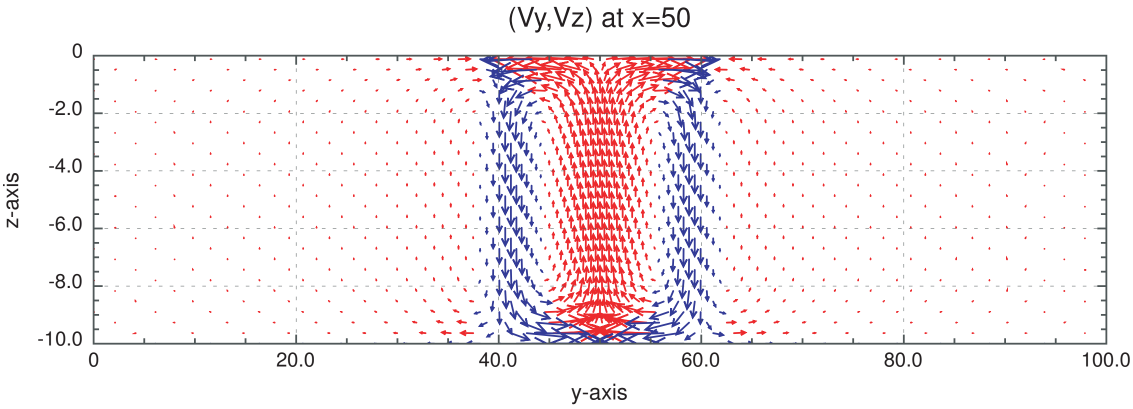

Fig. 2.— Velocity field used for the kinematic MHD model in Ω_-. The velocity field is divergence-free and describes the motion in a main central convection cell and two closing external weak half-cells.

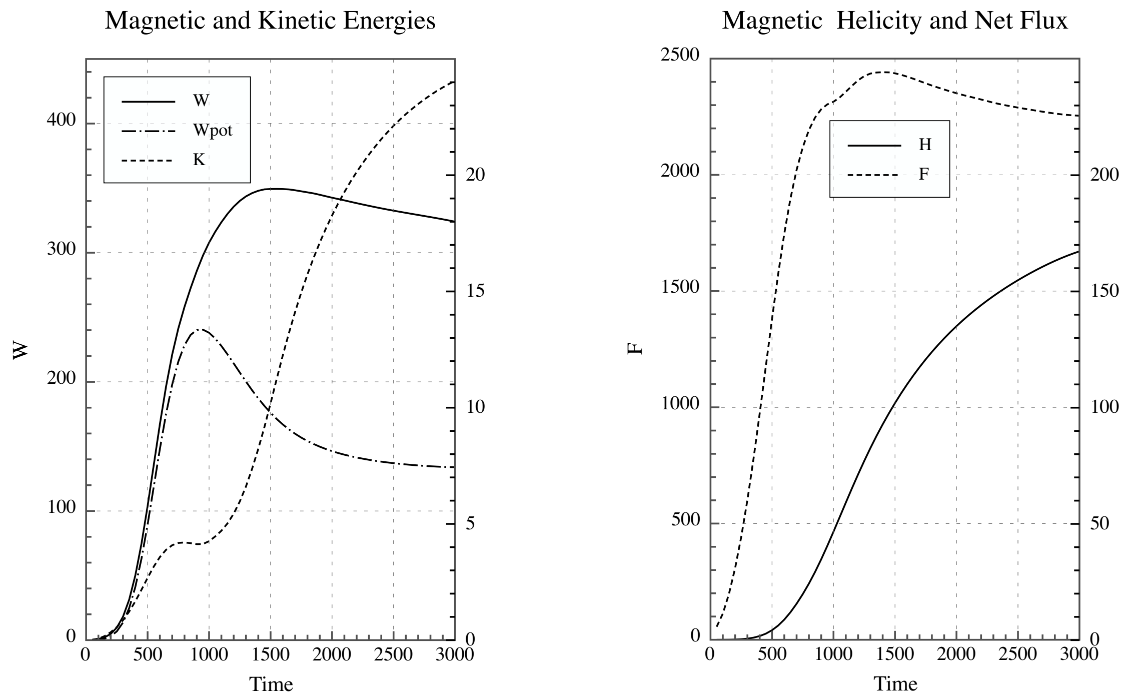

Fig. 3.— Variations of some relevant global quantities during the evolution. (a) Magnetic energy W in Ω_+, showing an injection from Ω_-. The magnetic energy $W_{\rm pot}$ of the corresponding potential field is shown as a reference. At some time, $W_{\rm pot}$ starts suffering a net decrease related to the magnetic flux decrease shown on panel (b), unlike W which saturates before suffering a slight decrease associated to an increase of the kinetic energy K and later slow diffusion. (b) Total net magnetic flux on $\Gamma_{-,+}$ and relative helicity in Ω_+.