Mon. Not. R. Astron. Soc. **000**, 1–6 (2006) Printed 24 March 2006 (MN LATEX style file v2.2)

Rotation speed and stellar axis inclination from p modes: How CoRoT would see other suns

J. Ballot,[1⋆] R. A. García[2,1] and P. Lambert[2,1]
[1] *AIM – Unité mixte de recherche CEA-CNRS-Université Paris VII – UMR nº 7158, CEA/Saclay, 91191 Gif-sur-Yvette* CEDEX*, France*
[2] *DSM/DAPNIA/Service d'Astrophysique, CEA/Saclay, 91191 Gif-sur-Yvette* CEDEX*, France*

Accepted 2006 March 24. Received 2005 November 18

ABSTRACT
In the context of future space-based asteroseismic missions, we have studied the problem of extracting the rotation speed and the rotation-axis inclination of solar-like stars from the expected data. We have focused on slow rotators (at most twice solar rotation speed), firstly because they constitute the most difficult case and secondly because some of the CoRoT main targets are expected to have slow rotation rates. Our study of the likelihood function has shown a correlation between the estimates of inclination of the rotation axis i and the rotational splitting $\delta\nu$ of the star. By using the parameters, i and $\delta\nu^\star = \delta\nu \sin i$, we propose and discuss new fitting strategies. Monte Carlo simulations have shown that we can extract a mean splitting and the rotation-axis inclination down to solar rotation rates. However, at the solar rotation rate we are not able to correctly recover the angle i although we are still able to measure a correct $\delta\nu^\star$ with a dispersion less than 40 nHz.

Key words: Rotation – Stars: oscillations – Sun: helioseismology – Methods: data analysis – Instrument: CoRoT

1 INTRODUCTION

Understanding dynamical phenomena inside stars is one of the most important current challenges for stellar physics. During the last decades, helioseismology has allowed astrophysicists to constrain the internal structure and dynamics of the Sun. In the same way, asteroseismology will aim to improve our knowledge of stellar dynamics, especially convection and rotation. With future asteroseismic missions like CoRoT (Convection Rotation and planetary Transits, Baglin 2003), it will be possible for example to determine the extent of the convective region in stars and to extract information on rotation. Since 2003 the first Canadian satellite dedicated to asteroseismology, MOST (Microvariability and Oscillations of STars, Walker et al. 2003), has been operational, beginning the space age for asteroseismology.

Asteroseismology has already provided information on the internal rotation of stars (e.g. Aerts et al. 2003, for results on a β Cepheid). However, the most accurate seismic information has been obtained for the Sun. Helioseismology has provided very accurate profiles of the internal rotation (see Thompson et al. 2003, and the references therein) as deep as 0.2 $R_\odot$ (Couvidat et al. 2003; García et al. 2004a), thanks to the Solar Heliospheric Observatory and to ground-based networks. Because of the rotation of stars, modes are not single peaks but multiplets. The splitting of the multiplet components gives information on the rotation speed in the acoustic cavity covered by the mode. Nowadays and in the near future, the asteroseismic observations will be limited to low-degree modes because of the absence of spatial resolution on the stellar surface. Thus, new inversion techniques have been developed and checked to derive, for example, the radial rotation profile (e.g. Goupil et al. 1996; Lochard, Samadi & Goupil 2004) or to infer the latitudinal differential rotation (Gizon & Solanki 2004).

Rotational splittings could be derived from the oscillation spectrum along with the other mode parameters. However, as we have learned from the solar case, the rotational splitting is harder to extract for low-degree modes because of the limited number of components in a multiplet. Moreover, another difficulty appears in the stellar case: the angle of inclination (i) of the rotation axis, which determines the multiplet pattern, is generally unknown.

Gizon & Solanki (2003) (hereafter GS03) have recently studied the simultaneous extraction of the splitting and the angle i from low-degree oscillation modes. We propose here to follow up their analysis by studying the potential of multi-mode fitting for more critical situations and by proposing automatable procedures. Our main objective is to determine the precision and limits in the determination of the rotation

⋆ *Present address:* Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85748 Garching, Germany; E-mail: jballot@mpa-garching.mpg.de

of solar-like stars from a mission like CoRoT. To do so, we have simulated CoRoT-type observations (150-day long) of a Sun spinning at different speeds with different axis orientations. We have considered rather realistic signal-to-noise ratios (S/N) and we have focused on the particular situation of slow rotators (less than twice solar rotation).

Our preliminary results have been outlined in Ballot et al. (2004). The present paper fully develops this work. The layout of the rest of the paper is as follows. In Sect. 2 we describe the main properties of modes for a star under rotational effects. In Sect. 3 we describe the techniques used to extract splittings and angle i from several modes together. In Sect. 4 we present the results of our method applied to several example cases. Finally, we discuss the fitting methods before concluding in the last section.

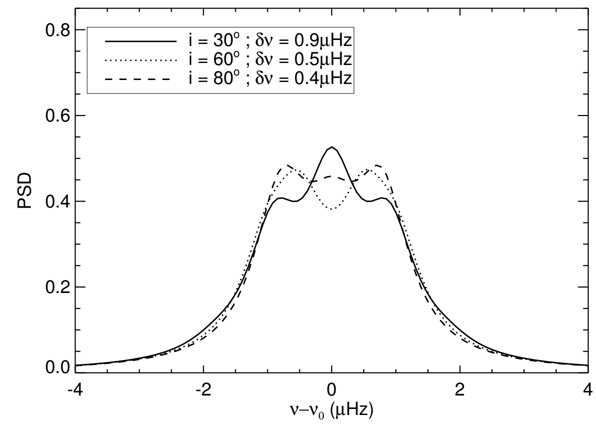

Figure 1. An $\ell = 2$ mode for three different speeds and angles.

2 OSCILLATION SPECTRUM OF A SPINNING STAR

2.1 Mode properties

Acoustic (p) modes in solar-like stars are excited by turbulent convective motions. Oscillations are damped but permanently re-excited (Goldreich, Murray & Kumar 1994). The oscillation power spectrum of such modes can be modelled as a noisy Lorentzian profile. For a power spectrum classically computed with the Fourier transform of a regularly-sampled time series, this noise is a multiplicative exponential. A mode (n, ℓ, m) – see below – is also characterized by its frequency, its amplitude and its FWHM.

In solar-like stars, the width Γ of a p-mode depends only on its frequency ν. For the Sun, the function $\Gamma(\nu)$ shows a S-shape. There is a plateau in the range 2300–3200μHz around a value of 1 μHz. At low frequency widths decrease rapidly and increase at high frequency (e.g. García et al. 2004b).

In the absence of rotation the frequency of a mode depends only on its radial order n and its degree ℓ: we denote it $\nu_{n\ell}$. Modes are $(2\ell+1)$-times degenerate among the azimuthal order m. This degeneracy is removed by breaking the spherical symmetry, especially by rotation. The frequency of mode (n, ℓ, m) is expressed as $\nu_{n\ell m} = \nu_{n\ell} + \delta\nu_{n\ell m}$. The asymptotic first-order approximation, developed for a star spinning as a solid body with an angular velocity Ω, gives $\delta\nu_{n\ell m} = -m\delta\nu$ with $\delta\nu = \Omega/2\pi$ (Ledoux 1951). We call $\delta\nu$ *rotational splitting* (or simply *splitting*).

For geometrical reasons, only low-degree modes have a sufficient amplitude to be visible in an oscillation spectrum due to the integration of the luminosity – or the radial velocity – on the full stellar disk. Mode amplitudes also depend on their azimuthal order m. Calculations are rather straightforward and can be found for example in GS03. Assuming the equipartition of energy between the different components of a multiplet (n, ℓ), their amplitudes can be expressed as

$$A_{n\ell m} = a_{\ell m}(i) V_\ell^2 \alpha_{n\ell} = a_{\ell m}(i) A_{n\ell}. \tag{1}$$

In this expression, the factor V_ℓ is the mode visibility. It depends on the limb-darkening function, i.e. on the atmospheric properties. The visibility V_ℓ decreases strongly when ℓ increases: for $\ell = 1, .., 5$, we have calculated $(V_\ell/V_0)^2 = 1.5, 0.53, 0.027, 0.0039, 0.00067$, assuming an Eddington law for the limb-darkening function. For this reason, we expect to measure only modes $\ell = 0$, 1, 2 and probably a few $\ell = 3$.

The factor $a_{\ell m}(i)$ is the amplitude ratio of modes inside a multiplet. It is a purely geometrical term, depending on i, the angle between the line of sight and the rotation axis. This is true under only one condition, that the contribution of each stellar-surface element to the total flux depends only on its distance to the disk centre. Even if it is not exactly true for velocity-fluctuation observations due to the rotation of the star (e.g. Henney 1999), this assumption stays very good for luminosity observations. The final factor of the mode amplitude $\alpha_{n\ell} \approx \alpha(\nu_{n\ell})$ depends mainly on the frequency and excitation mechanisms. We note $A_{n\ell} = V_\ell^2 \alpha_{n\ell}$. This approach is valid for low rotation rates, when rotation can be interpreted as perturbation.

Thus a mode (n, ℓ) is modelled by a multiplet parametrized by five parameters (only three for $\ell = 0$): the central frequency $\nu_{n\ell}$, the amplitude $A_{n\ell}$, the width $\Gamma_{n\ell}$ common to all the components, the splitting $\delta\nu$ and the angle i.

2.2 Classification depending on $\delta\nu$

We have defined three different scenarios according to $\delta\nu$:

(1) $\delta\nu \gg \delta_{02}\nu$,
(2) $\Gamma < \delta\nu \lesssim \delta_{02}\nu$,
(3) $\delta\nu \lesssim \Gamma$;

where $\delta_{02}\nu$ denotes the small separation $\nu_{n+1,\ell=0} - \nu_{n,\ell=2}$ (around 10 μHz for the Sun in the range 2000–3000 μHz). In the first case, the components of different modes are mixed and it could be difficult to label each peak in a spectrum with the correct values of ℓ, m and relative n. However, when this identification is done, all of the splittings $\delta\nu_{n\ell m}$ are accurately defined. In the second situation, mode identification does not pose any problem in general for good S/N and, as the components of a multiplet are well separated, splittings are easily measured. In the third and last case, the multiplet components are blended. The effect on the amplitude ratio of a multiplet due to a given inclination axis is not always distinguishable from those of the splitting as illustrated by Fig. 1. For three different configurations chosen as an example, the mode profiles are nearly the same; only fine differences appear in the structure of profile tops. When an exponential multiplicative noise is taken into account, these differences are very difficult to catch. We have studied this more challenging situation, corresponding to $\delta\nu \lesssim 1\,\mu$Hz (for suns), i.e. $\Omega \lesssim 2\Omega_\odot$ ($\Omega_\odot/2\pi \approx 0.4\,\mu$Hz).

3 EXTRACTING THE MODE PARAMETERS

3.1 Fitting modes: maximum likelihood

Splittings and inclination angle should be deduced from the oscillation spectra at the same time as all the other mode parameters. For that, we use techniques developed and applied in full-disk-integrated helioseismology. Oscillation spectra are fitted with a maximum likelihood method as described by Appourchaux, Gizon & Rabello-Soares (1998). The power spectrum of a solar-like star is modelled as the sum of modes, modelled by multiplets, and a background noise, mainly due to convective motions (granulation, supergranulation), and instrumental noises. The first step of the analysis is to remove the background, previously fitted following the model of Harvey (1985), to obtain a "flat" background. Then the modes are classically fitted alone or by pairs $(\ell, \ell+2)$ according to the value of the small separation and the mode amplitudes. The residual background is considered as a constant inside the fitting window. As i is a new parameter compared to the classic helioseismic analysis, we have explored its impact, especially on the splitting determination.

3.2 Guessing and assumptions

The fitting method needs guesses for the parameters to fit. This estimate is a starting point of the parameter-space exploration by the algorithm maximising the likelihood. We denote by $\tilde{x}$ the estimate of the parameter x. A crude estimation of the mode central frequency can be obtained, by looking for its centroid. The amplitudes and widths can be first determined on $\ell = 0$ modes, which are insensitive to rotation. As amplitudes $\alpha_{n\ell}$ and widths $\Gamma_{n\ell}$ depend mainly on frequency, initial values for the modes $\ell \geqslant 1$ can be interpolated from those of $\ell = 0$ as follows:

$$\tilde{A}_{n-1,2} = \frac{{V_2}^2}{{V_0}^2}\tilde{A}_{n,0}, \quad \tilde{A}_{n-1,1} = \frac{{V_1}^2}{{V_0}^2}\frac{\tilde{A}_{n-1,0}+\tilde{A}_{n,0}}{2}, \qquad (2)$$

$$\tilde{\Gamma}_{n-1,2} = \tilde{\Gamma}_{n,0}, \qquad \tilde{\Gamma}_{n-1,2} = \frac{\tilde{\Gamma}_{n-1,0}+\tilde{\Gamma}_{n,0}}{2}. \qquad (3)$$

Determining the estimates $\tilde{\imath}$ and $\tilde{\delta\nu}$ is not easy when multiplet components are not well separated. A first possibility is to fit each mode as a single Lorentzian. The comparison between the widths of two neighbouring modes $\ell = 2$ and $\ell = 0$ allows us to detect the presence of rotation (when $i > 0$), but a quantitative interpretation of this broadening is difficult because of the cumulative effects of i and $\delta\nu$.

The sensitivity of the fitting to $\tilde{\imath}$ and $\tilde{\delta\nu}$ has been tested along with the impact of the noise on i and $\delta\nu$ determination. Modes of interest have been fitted as follows:

- Pairs $\ell = 0$ & 2 are fitted with eight parameters $(A_0, A_2, \nu_0, \nu_2, \Gamma, b, \delta\nu, i)$: their amplitudes, their frequencies, a common width, the background level, the splitting and the inclination angle. Assuming $\Gamma_{n-1,2} = \Gamma_{n,0}$ is a good approximation as shown by the solar case (Chaplin et al. 2006). Thus, the parameter space is reduced as well as the computing time and the risk of non convergence.

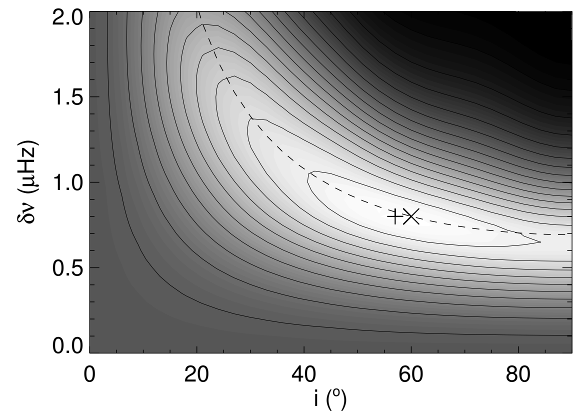

Figure 2. Likelihood function for one simulated spectrum in the plane $(i, \delta\nu)$ of the parameter space. All the other parameters are fixed to their simulated value. The power spectrum is taken in the range 2200–3000μHz. The white colour corresponds to the highest likelihoods and the black to the lowest. The $\times$ is the simulated value $(i_0, \delta\nu_0)$ and the $+$ is the maximum of the likelihood. The dashed line follows $\delta\nu \sin i = \delta\nu_0 \sin i_0$.

- Modes $\ell = 1$ are fitted with six parameters $(A_1, \nu_1, \Gamma_1, b, \delta\nu, i)$. As the expected amplitudes of the $\ell = 3$ modes are very small we do not fit them, although they are present in simulated spectra. Previous results have shown that such a simplification could introduce biases – especially to frequencies and splittings – if neglected modes are not sufficiently small and/or are too close to the fitted ones. We have been careful and we have verified that no significant bias has been introduced in our case.

Different random values for $\tilde{\imath}$ and $\tilde{\delta\nu}$ have been tested on several Monte Carlo realizations of the spectrum (see method in Sect. 4.1). We have seen first that the solution is not unique and a certain dependence upon the first guess parameters is observed. For a given mode in a given realization, the fitting procedure can converge to some different couples $(i, \delta\nu)$ according to the initial values $\tilde{\imath}$ and $\tilde{\delta\nu}$. However the main effect is due to the noise which has a strong impact on the estimation of $(i, \delta\nu)$ and disperses the results. Nevertheless, we observe a clear correlation in the determination of both parameters. Results are organized along the curve: $\delta\nu \sin i \approx \text{constant} = \delta\nu_0 \sin i_0$. We denote with an index 0 $(\delta\nu_0, \sin i_0)$ the real (input) values of the parameters in the simulation. This can be explained by a study of the likelihood function for a simulated spectrum. Figure 2 shows such function in a plane $(i, \delta\nu)$ in the parameter space, with all other parameters fixed to their true value. We observe in such a plane a ridge following the curve $\delta\nu \sin i = \delta\nu_0 \sin i_0$. Thus i and $\delta\nu$ are correlated. A new pair of independent parameters can be built:

$$(i, \delta\nu^\star) \quad \text{with } \delta\nu^\star = \delta\nu \sin i. \qquad (4)$$

Hereafter we use preferentially this new variable $\delta\nu^\star$ which is better suited than $\delta\nu$ for studying fitting issues and discussing results. We did not find major differences between using $\delta\nu^\star$ and $\delta\nu$ for the minimization routine, except for the error bars computed by Hessian-matrix inversion.

3.3 Proposed strategy: multi-mode fitting

With a classical fitting strategy, the determination of i seems very sensitive and tricky. We propose here another strategy aiming to improve the accuracy of the obtained value of i. We have fitted simultaneously several modes, as we can consider – to a first order approximation – that they have the same value of i and $\delta\nu^\star$.

(1) **Choosing initial guesses**. For $\tilde{A}$, $\tilde{\Gamma}$ and $\tilde{\nu}$, see Sect. 3.2. We firstly fit pairs $\ell = 0$ & 2 and single modes $\ell = 1$, using several (typically 20) different random values of $\tilde{\imath}$ and $\tilde{\delta\nu}^\star$. We use the average of all the obtained results as a better guess for these parameters.

(2) **Fitting simultaneously the modes (ℓ=2, $n-1$), (ℓ=0, n) and (ℓ=1, n)**. We used eleven parameters $(A_2, A_0, A_1, \nu_2, \nu_0, \nu_1, \Gamma_{0/2}, \Gamma_1, b, \delta\nu, i)$. We obtained also a series of values for $(i, \delta\nu^\star)$. The mean values are noted (i_m and $\delta\nu^\star_m$) (m for *mean value*). We obtain in this way a first measurement of i and Ω.

(3) **Global fitting on a large range of the spectrum**. Fitting several modes simultaneously, keeping free all the parameters, would be too costly in terms of computing time, and too delicate in terms of convergence. So we have decided to fix all the parameters but $(i, \delta\nu^\star)$ to their values deduced from the previous step. We choose as guesses $\tilde{\imath} = i_m$ and $\tilde{\delta\nu}^\star = \delta\nu^\star_m$. Fitted results are denoted $(i_g, \delta\nu^\star_g)$ (g as *global*).

4 MONTE CARLO SIMULATIONS

4.1 Defining the simulations

The mode characteristics are derived from the observations of the Sun made by the GOLF instrument (Global Oscillations at Low Frequency, Gabriel et al. 1995). However the amplitudes have been adapted to simulate luminosity observations instead of Doppler velocity measurements. We have treated the Sun as it was a main CoRoT target of magnitude 6 observed during 150 days. For such a star, in the frequency range of interest (2200–3000 μHz), stellar noise have to dominate instrumental and photon noise (see discussion in Michel et al. 2005). Thus, S/N (as defined by Libbrecht 1992) of the hightest component of a multiplet varies from 15 to 150 for the $\ell = 1$ modes, from 4 to 45 for $\ell = 2$, and from 0.7 to 7 for $\ell = 3$ (in configurations at 80°). The widths do not vary much (from 0.8 to 1.1 μHz). 150-day (resolution ≈ 77 nHz) power spectra are created including $\ell \leqslant 3$ with a splitting $\delta\nu_0$ and an angle i_0 that we want to simulate. In the chosen frequency range, there are six modes for each degree. This choice of interval results from a compromise: we have rejected modes with too low S/N (i.e. at low frequency) and peaks too broad, useless for our analysis (at higher frequencies). This will give us a lower limit of what we could obtain in the real case, with the hope that CoRoT will reach such modes. To introduce the noise of each realization we follow Fierry Fraillon et al. (1998) by using a random exponential distribution which simulates the stochastic excitation.

To test the analysis method a Monte Carlo simulation is done, i.e., we repeat N times the method on the same theoretical spectrum changing only the realization of noise. As the computing time required in each realization is quite high and we want to do many different cases, we have decided to limit the number of realizations N to 100. The statistical significance of the results is small but it is enough to check the general trends of the solution. In order to verify our results we have increased N to 1000 in some cases, e.g. $i_0 = 60°$, $\delta\nu_0 = 0.8$ μHz. The conclusions remain roughly the same. We have simulated six different configurations: two rotation rates $\Omega = 1$ and $2\,\Omega_\odot$, i.e. $\delta\nu_0 = 0.4$ and $0.8\,\mu$Hz, with three inclination angles $i_0 = 30$, 60 and 80°.

4.2 Star spinning twice as faster as the Sun

This class of stars is the most favourable among those considered. Results obtained with our strategy are satisfactory. A clear improvement is found, relative to classical fitting. The histograms of Fig. 3-a show the distributions of deduced parameters for each considered inclination angle. Both parameter couples $(i_m, \delta\nu^\star_m)$ and $(i_g, \delta\nu^\star_g)$ are plotted for every studied stellar orientation. We make three main comments:

- in the three configurations, determinations of $\delta\nu^\star$ are non-biased and little spread: the dispersion is around 30nHz. Results given by averaging $(\delta\nu^\star_m)$ and by global fitting $(\delta\nu^\star_g)$ are very similar. Global fitting does not lead to a noticeable change in this parameter in this situation.
- On the other hand, the global fit (i.e. i_g) brings, for i, a major improvement at low angle ($i = 30°$) according to averaged results i_m. Although there continue to be several highly spurious results ($i_g \gtrsim 70°$), a large number of realizations lie around 30°.
- There is a slight bias on the i determination for the extreme values, but it remains smaller than the error bar.

4.3 Star spinning as the Sun

Fitting results for the configuration with $\delta\nu = 0.4$ μHz are shown in Fig. 3-b. The study of the distributions of i_m, $\delta\nu^\star_m$, i_g and $\delta\nu^\star_g$ leads to two different conclusions for $\delta\nu^\star$ and i.

- The $\delta\nu^\star$ distributions are quite narrow with dispersions similar to the previous configurations (around 30–40 nHz). However a significant bias appears in the three cases, whereas it is negligible in the simulations at $2\,\Omega_\odot$.
- The angle i is not correctly extracted. The distributions are rather chaotic. However we have remarked that around a fifth of the realizations have given an angle of 90°. For the global fits of these low-splitting cases, this value behaves like an attractor during the likelihood-maximising process.

We wanted to know if it is possible to extract the angle i from the selected modes in a configuration $\Omega = \Omega_\odot$. To do so we have considered an idealized situation: we have performed "ideal" global fits. In such fits all the parameters – except i and $\delta\nu^\star$ – are fixed to their exact values and not to the values deduced from a previous fitting step (cf. step #3 in the strategy § 3.3). Moreover the exact values i_0 and $\delta\nu^\star_0$ are chosen as guesses $\tilde{\imath}$ and $\tilde{\delta\nu}^\star$. Thus all is optimized for fitting: only noise can influence the results. Results of this fitting method are plotted in Fig. 3-b with dashed lines. Thus we can conclude that:

- the bias on $\delta\nu^\star$ disappears. It indicates that this bias was due to errors in the values to which the parameters were fixed. However, the dispersion stays the same: it is mainly generated by the noise.

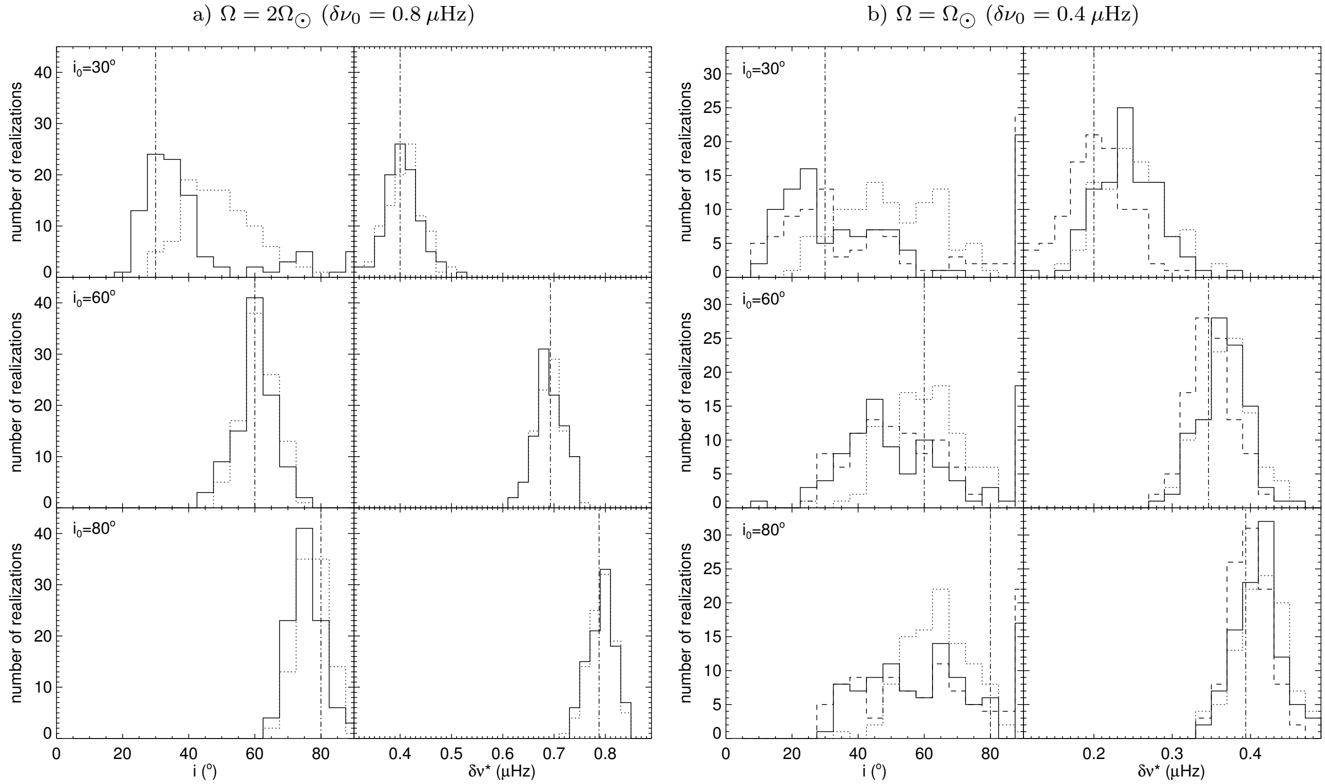

Figure 3. a) Distribution of fitting results in the three configurations $\Omega = 2\,\Omega_\odot$. On the left the angle i; on the right $\delta\nu^\star$. Histograms plotted with solid lines show the results of global fits (i_g and $\delta\nu^\star_g$); Histograms with dotted lines show the averaged results i_m and $\delta\nu^\star_m$. The dot-dash vertical line indicates the input value. b) Same as a) but for $\Omega = \Omega_\odot$. We have added distribution of the "idealized" global fits (cf. text) plotted with dashed lines.

• the determination of i is not changed. Noise dominates above the signature of the angle and that seems inevitable in such data.

5 DISCUSSION

5.1 On the $i/\delta\nu$-correlation and the law $a_{\ell m}(i)$

In the framework of global helioseismology, Chaplin et al. (2001) have observed that changing amplitude ratios fixed inside the multiplets $\ell = 1, 2$ and 3 during the fit of solar spectra introduces a systematic bias on extracted splitting. We can understand the reason by studying the likelihood function shown in Fig. 2. Changing the amplitude ratio is similar to changing the angle i, thus it introduces a bias on splitting determination due to the correlation we have found. Our results generalise this observation. They show perfectly that we should be cautious of bias introduced when parameters are fixed, because of the correlation existing between the different parameters (see also Fierry Fraillon et al. 1998).

This analysis shows that it could be interesting to derive the angle i by other ways, like directly studying the light curve of stars and trying to follow up modulations due to activity spots (e.g. Rucinski et al. 2004). If such an additional constraint is available, the situation would become similar to the solar case and the amplitude ratios $a_{\ell m}$ could be fixed *a priori* and individual splittings fitted. However the measurement of i must be sufficiently accurate (probably $\sim$ 5–8°) otherwise the estimate of $\delta\nu$ will likely be biased.

The results presented in this paper depend on the law we have used to link $a_{\ell m}$ to i. For fitting, this law must be defined *a priori*. As shown once again by solar experience (Chaplin et al. 2004), when multiplet components are blended and not separated – which is the case here – fits are very sensitive to the chosen law $a_{\ell m}(i)$. Luckily, for intensity observations, these ratios depend mainly on well-controlled geometrical considerations (cf. Sect. 2.1).

5.2 Limitation and improvements

The situation can be improved if low-frequency modes are measured. For these modes the splitting can directly be measured because of their finer widths. Then fixing the retrieved splitting can yield to a good estimation of i at higher frequencies where the multiplets are better defined and the influence of the stochastic excitation less important.

In our simulations, we have assumed that the angle and the splitting are the same for all modes. While it is true that i is the same for every mode, $\delta\nu$ can vary for real stars, especially because of the differential rotation that could exist along the radius. However, for the Sun this variation is weak for low-degree modes in the studied frequency range. We could also attempt to extract not a mean splitting but a mean splitting for each degree, as was done for a first stage for the Sun (cf. Lazrek et al. 1996).

If $\ell = 3$ modes have sufficiently high amplitudes in real observations to be correctly fitted, the results shown here would be improved. If they could be observed but with low

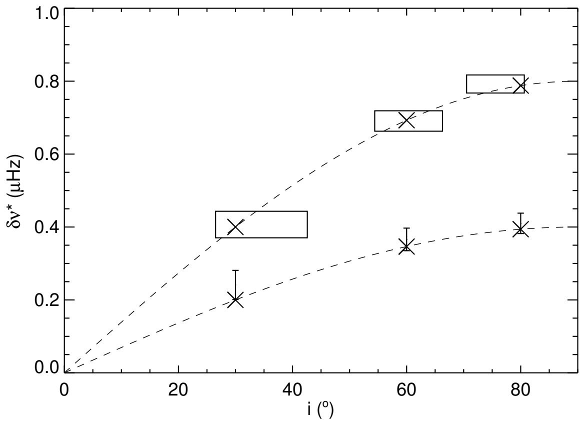

Figure 4. Synthetic representation of biases and error bars for i and $\delta\nu^\star$ deduced from the simulations, in all the studied configurations. The crosses ($\times$) mark the expected values $(i_0, \delta\nu_0^\star)$. For $\Omega = 2\,\Omega_\odot$ cases, the boxes indicate the mean results and their dispersions. For $\Omega = 1\,\Omega_\odot$ cases, only error bars on $\delta\nu^\star$ are plotted because of the absence of good determinations of i. The two dashed lines are isorotations $\delta\nu = \delta\nu_0 = 0.4$ and $0.8\,\mu$Hz.

S/N, we could try to use a so-called n-collapsogram (cf. Ballot et al. 2004) to extract a mean splitting. This technique can be summarized as follows: averaging the spectra of several $\ell = 3$ modes with different orders n, after removing the $\ell = 1$ neighbours, to enhance the S/N and define the multiplet better; and fitting the resulting spectrum. It needs a good determination of the central frequency for every mode, and small variations of $\delta\nu$ and the width (which is the case in the "plateau" frequency range).

We can hope to derive even better results by denoising asteroseismic spectra. Filtering the spectrum and enhancing S/N could improve the contrast of multiplets, guiding their analysis. Lambert et al. (2006) are proposing methods based on curvelet transforms permitting such denoising.

This analysis performed on the Sun can be extended to sufficiently bright CoRoT targets with similar mass and evolutionary state. S/N will depend on the convective-noise level which will be observed in other stars. Some discussions on this topic have taken place after the observations of Procyon by MOST (Matthews et al. 2004; Bedding et al. 2005).

6 CONCLUSION

One of the challenges of present and future asteroseismic space missions is to extract stellar rotation rates and, wherever possible, the internal rotation profile. To do that, mode splittings $\delta\nu$ have to be measured. We have studied the impact of the extra parameter i, appearing in asteroseismology, on the fitting. We have shown a correlation between $\delta\nu$ and i, and defined a new parameter $\delta\nu^\star = \delta\nu \sin i$. Strategies of multi-mode fitting have been developed, tested and validated with Monte Carlo simulations. Figure 4 sums up the results. In agreement with GS03, we find that at $\Omega = 2\,\Omega_\odot$ we can retrieve both parameters in most of the cases, but with error bars improved by the global fitting, especially at low angle. However, at $\Omega = \Omega_\odot$ we have not been able to correctly recover the angle i. This result emphasizes the interest of having an independent measurement of the angle, but it has to be accurate enough to prevent the inclusion of a bias in the splitting determination.

ACKNOWLEDGMENTS

The authors want to thank S. Turck-Chièze and C. Catala for their useful remarks.

REFERENCES

Aerts C., Thoul A., Daszyńska J., Scuflaire R., Waelkens C., Dupret M. A., Niemczura E., Noels A., 2003, Sci., 300, 1926
Appourchaux T., Gizon L., Rabello-Soares M.-C., 1998, A&AS, 132, 107
Baglin A., 2003, Advances in Space Research, 31, 345
Ballot J., García R. A., Lambert P., Teste A., 2004, in Danesy D., ed., ESA SP-559, SOHO14/GONG2004 Workshop, New Haven, p. 309
Bedding T. R. et al., 2005, A&A, 432, L43
Chaplin W. J. et al., 2004, in Danesy D., ed., ESA SP-559, SOHO14/GONG2004 Workshop, New Haven, p. 356
Chaplin W. J., Elsworth Y., Isaak G. R., Marchenkov K. I., Miller B. A., New R., 2001, MNRAS, 327, 1127
Chaplin W. J. et al., 2006, MNRAS, accepted
Couvidat S., García R. A., Turck-Chièze S., Corbard T., Henney C. J., Jiménez-Reyes S., 2003, ApJ, 597, L77
Fierry Fraillon D., Gelly B., Schmider F. X., Hill F., Fossat E., Pantel A., 1998, A&A, 333, 362
Gabriel A. H. et al., 1995, Sol. Phys., 162, 61
García R. A. et al., 2004a, Sol. Phys., 220, 269
García R. A., Jiménez-Reyes S. J., Turck-Chièze S., Ballot J., Henney C. J., 2004b, in Danesy D., ed., ESA SP-559, SOHO14/GONG2004 Workshop, New Haven, p. 436
Gizon L., Solanki S. K., 2003, ApJ, 589, 1009
Gizon L., Solanki S. K., 2004, Sol. Phys., 220, 169
Goldreich P., Murray N., Kumar P., 1994, ApJ, 424, 466
Goupil M.-J., Dziembowski W. A., Goode P. R., Michel E., 1996, A&A, 305, 487
Harvey J., 1985, in Rolfe E., Battrick B., eds, ESA SP-235, Future Missions in Solar, Heliospheric & Space Plasma Physics Workshop, Garmisch-Partenkirschen, p. 199
Henney C. J., 1999, PhD thesis, Univ. California
Lambert P., Pires S., Ballot J., García R. A., Starck J.-L., Turck-Chièze S., 2006, A&A, accepted
Lazrek M. et al., 1996, Sol. Phys., 166, 1
Ledoux P., 1951, ApJ, 114, 373
Libbrecht K. G., 1992, ApJ, 387, 712
Lochard J., Samadi R., Goupil M., 2004, Sol. Phys., 220, 199
Matthews J. M., Kusching R., Guenther D. B., Walker G. A. H., Moffat A. F. J., Rucinski S. M., Sasselov D., Weiss W. W., 2004, Nat., 430, 51
Michel E. et al., 2005, Mem.S.A.It., 75, 282
Rucinski S. M. et al., 2004, PASP, 116, 1093
Thompson M. J., Christensen-Dalsgaard J., Miesch M. S., Toomre J., 2003, ARA&A, 41, 599
Walker G. et al., 2003, PASP, 115, 1023