Impact of a Large-Scale Magnetic Field on Stellar Structure

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Abstract. We present the derivation of non force-free magneto-hydrostatic (MHS) equilibria in spherical geometry, supposing any prescription for the toroidal current. This allows us to study the influence on the stellar structure of a large-scale magnetic field, both on the mechanical and on the energetical balances. Two cases illustrate this approach : (i) the field is buried below a given radius, in order to model deep fossil magnetic fields in solar-like stars; (ii) the internal field matches at the surface with an external potential magnetic field that corresponds to fossil fields in more massive stars. The stellar structure perturbations are semi-analytically computed in both cases. This allows us to establish a hierarchy between the orders of magnitude of the different terms. Finally, the limit of validity of the linear perturbation is discussed.

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1. INTRODUCTION

Considering stellar magnetic fields as one of the essential ingredients of stellar evolution is today a necessity, when looking for answers to unsolved problems such as the flatness of the rotation profile in the solar radiation zone, or the discrepancy between the sound speed deduced from helioseismology observations and the one found using existing stellar evolution codes [1, 2]. Hence more and more observations are one hand devoted to the external topology of the magnetic field for very different types of stars thanks to the development of spectropolarimetry [3, 4], whereas on the other hand, neutrinos, gravity waves or gravity modes become very promising probes to improve our knowledge on internal fields [5, 6]. In this context, a tremendous work is today being undertaken to implement the macroscopic processes associated with the magnetic field in 2D models [7], but none of these works treat at the present date the transport mechanism including both rotation and magnetisms in a self-consistent way, i.e. by solving the angular momentum transport and the induction equations.
An alternative promising approach resids in the implementation of these 2D equations (in the axisymmetric case) in a unidimensionnal code, looking then at their projections on the spherical harmonics of low order [8, 9], which leads to a spectral model including all the refinements of the microscopic physics contained in 1D stellar evolution codes. This approach differs from what has been proposed so far in this domain [10, 11], where the impact of magnetic fields on the stellar structure along the evolution appeared through a modification of the thermodynamics of the model, with a magnetic field generally treated as an "effective magnetic pressure" and with some ad-hoc approximations concerning its geometrical nature.
In this perspective, it is relevant to quantify the impact on the stellar structure of a large-scale magnetic field, likely to act over evolutionary timescales.
To model these magnetic fields, we derive the magneto-hydrostatic (MHS) configurations for any given azimuthal current, by integrating a Grad-Shafranov-Poisson equation using a method based on Green's functions. Two cases of equilibria are analytically investigated for illustration purpose : (i) the case of a solar-like star, perturbed by a fossil field buried below its convection zone and with a strength of 7 MG [12]; (ii) the case of an A_p-type star, perturbed by a field spanning across the star, and matching at its surface with a potential, dipolar field with a 10 kG strength [13].
The physical quantities likely to modify the stellar structure are then semi-analytically derived and computed in the two cases of interest.

2. THE MODIFIED STELLAR STRUCTURE EQUATIONS

In this section we depict briefly how is the set of stellar structure equations modified when taking into account the effects of the magnetic field. These ones are given by

$$\begin{aligned}
\frac{\partial P}{\partial M} &= -\frac{GM}{4\pi R^4} + \frac{1}{4\pi\rho R^2} < \boldsymbol{F}_{\mathscr{T};r} >_\theta; & (1)\\
\frac{\partial T}{\partial M} &= \frac{\partial P}{\partial M}\frac{T}{P}\nabla; & (2)\\
P &= P_{\rm mag} + P_{\rm gas}; & (3)\\
\frac{\partial R}{\partial M} &= \frac{1}{4\pi R^2 \rho}; & (4)\\
\frac{\partial L}{\partial M} &= \left\langle \varepsilon - \frac{\partial U}{\partial t} + \frac{P}{\rho^2}\frac{\partial \rho}{\partial t} + \frac{1}{\rho} Q_{\rm Ohm} + \frac{1}{\rho} F_{\rm Poynt} \right\rangle_\theta; & (5)\\
\frac{\partial X_i}{\partial t} &= -\frac{\partial F_i}{\partial M} + \Psi_i(P_{\rm gas}, T; \chi) \quad (1 \le i \le n_{\rm elem}); & (6)
\end{aligned}$$

$\boldsymbol{B}$ being the magnetic field, $\boldsymbol{j}$ the current density, $||\eta||$ the magnetic diffusivity tensor, μ_0 the plasma magnetic permeability, ρ the density, $P_{\rm gas}$ the gas pressure and $\chi = \{X_i\}$ the chemical composition vector. We introduced the magnetic pressure $P_{\rm mag} = \boldsymbol{B}^2/2\mu_0$, the magnetic tension $\boldsymbol{F}_{\mathscr{T}} = -(\boldsymbol{B}\cdot\nabla)\boldsymbol{B}/2\mu_0$, the Ohmic heating $Q_{\rm Ohm} = (1/\mu_0)||\eta|| \otimes (\nabla\times\boldsymbol{B})\cdot(\nabla\times\boldsymbol{B})$, and the Poynting's flux $F_{\rm Poynt} = (1/\mu_0)\nabla\cdot(\boldsymbol{E}\times\boldsymbol{B})$. The other quantities are denoted by their traditional abbreviations. The MHS equilibrium and the energy equations are considered radially, the two-dimensional quantities $Z(r,\theta)$ have therefore been latitudinally averaged according to $< Z >_\theta (r) = (1/2)\int_0^\pi Z(r,\theta)\sin\theta d\theta$.

3. THE NON FORCE-FREE MHS EQUILIBRIUM

Let us express the magnetic field $\boldsymbol{B}(r,\theta)$ in the axisymmetric case as a function of a poloidal flux $\Psi(r,\theta)$ and a toroidal potential $F(r,\theta)$ such that it remains by construction divergence-free :

$$\boldsymbol{B} = \frac{1}{r\sin\theta}\nabla\Psi\times\hat{\mathbf{e}}_\varphi + \frac{1}{r\sin\theta}F\,\hat{\mathbf{e}}_\varphi. \tag{7}$$

where in spherical coordinates the poloidal direction is in the meridional plane $(\hat{\mathbf{e}}_{\mathbf{r}}, \hat{\mathbf{e}}_\theta)$ and the toroidal direction is along the azimuthal one $\hat{\mathbf{e}}_\varphi$. Notice that $\boldsymbol{B}_{\rm P}\cdot\nabla\Psi = 0$, so the poloidal field $\boldsymbol{B}_{\rm P} = B_r\hat{\mathbf{e}}_{\mathbf{r}} + B_\theta\hat{\mathbf{e}}_\theta$ belongs to iso-Ψ surfaces. Let us write the magneto-hydrostatic (MHS) equilibrium as follows:

$$\rho\,\boldsymbol{g} - \nabla P_{\rm gas} + \boldsymbol{F}_{\mathscr{L}} = \boldsymbol{0}, \tag{8}$$

where $\boldsymbol{F}_{\mathscr{L}} = \boldsymbol{j}\times\boldsymbol{B}$ is the Lorentz force. We have to verify that its toroidal component $F_{\mathscr{L}_\varphi}$ vanishes everywhere, since in lack of rotation there is no other force in this direction to compensate for the equilibrium deviation. This condition writes as $\partial_r\Psi\partial_\theta F - \partial_\theta\Psi\partial_r F = 0$ where the notation $\partial_x = \partial/\partial_x$ has been used. The non trivial values for F are therefore obtained by setting $F(r,\theta) = F(\Psi)$, which for a regular function, can be written at first order as $F(\Psi) = \alpha_1\Psi$. Therefore, the Lorentz force can be expressed concisely as

$$\boldsymbol{F}_{\mathscr{L}} = \mathscr{A}(r,\theta)\nabla\Psi \quad \text{where} \quad \mathscr{A}(r,\theta) = -\frac{1}{\mu_0 r^2 \sin^2\theta}\left(\alpha_1^2\Psi + \Delta^*\Psi\right). \tag{9}$$

Its poloidal component is non-zero a priori for any given Ψ; the pressure gradient and the density perturbation adjust themselves in order to relax towards equilibrium. In the azimuthal direction the field is force-free.
The azimuthal component of Ampere's law in the MHD classical approximation, together with the expression for the poloidal component of the magnetic field leads to the following relation between j_φ and Ψ:

$$\Delta^*\Psi = -\mu_0 r\sin\theta\ j_\varphi, \tag{10}$$

where we introduce the so-called Grad-Shafranov operator in spherical coordinates

$$\Delta^*\Psi \equiv \frac{\partial^2\Psi}{\partial r^2} + \frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial\Psi}{\partial\theta}\right). \tag{11}$$

This a Grad-Shafranov-Poisson equation, to which we will refer hereafter as the GSP equation. Let us now prescribe an azimuthal current j_φ to study the effects of a magnetic field localized in a chosen region. Here, to illustrate our purpose we choose a current density $j_\varphi(r,\theta) = j_{\varphi_0} j_{\varphi_r}(r) j_{\varphi_\theta}(\theta)$ where the radial function $j_{\varphi_r}(r) = \sin_c^3\left[(2r/R-1)\pi\right]$ is regular and vanishes at the center and at $r = R$, the angular dependence $j_{\varphi_\theta}(\theta) = \sin\theta$ being dipolar while j_{φ_0} is calibrated for the field to have the imposed strength, i.e. at the center in the case of the Sun or at the surface in the case of the A_p star.

3.1. Solutions to the GSP equation

We solve the equation (10) using the Green's functions method (cf. Morse and Feshbach [14], Payne and Melatos [15]). The general expression for the flux function Ψ is then given by

$$\Psi(r,\theta) \;=\; -\mu_0 \sum_{l=0}^{\infty} \mathscr{N}_l^{-1} \sin^2\theta\, C_l^{3/2}(\cos\theta) \int_0^R g_l(r',r) \left[\int_0^\pi j_\varphi(r',\theta')\, C_l^{3/2}(\cos\theta') \sin^3\theta' d\theta'\right] r'^3 dr' \tag{12}$$

where $g_l(r,r')$ is solution of the equation $r^2\partial_{r^2} g_l(r,r') - (l+1)(l+2)\, g_l(r,r') = \delta(r-r')$, for the given boundary conditions at $r = R$, $C_l^{3/2}(\cos\theta)$ is the Gegenbauer polynomial of order $3/2$ and the normalization coefficient is defined by $\mathscr{N}_l = 2(l+1)(l+2)/(2l+3)$. For boundary conditions modeling a field buried in an inner radiation zone (below the confinement radius $R = R_c$) we set $g_l(0,r') = 0$ and $g_l(R_c,r') = 0$; we thus get the radial Green's function $g_{l_<}(r,r') = \frac{1}{(2l+3)}\left[\frac{r'^{l+2}}{R_c^{2l+3}} - \frac{1}{r'^{l+1}}\right] r^l$ if $r < r'$ or $g_{l_>}(r,r') = \frac{1}{(2l+3)}\left[\frac{r^l}{R_c^{2l+3}} - \frac{1}{r^{l+3}}\right] r'^{l+2}$ if $r > r'$, verifying the continuity at $r = r'$. For a field matching at the stellar surface (at $R = R_*$) with a potential magnetic field we set $g_l(0,r') = 0$ and $g_l(R_*,r') = g_{l;\mathrm{pot}}$, $g_{l;\mathrm{pot}}$ being such that the magnetic field verifies $\nabla \times \boldsymbol{B} = 0$. The corresponding radial Green's functions are then $g_{l_<}(r,r') = -\frac{1}{(2l+3)}\frac{r^l}{r'^{l+1}}$ if $r < r'$ or $g_{l_>}(r,r') = -\frac{1}{(2l+3)}\frac{r'^{l+2}}{r^{l+3}}$ if $r > r'$.

4. INFLUENCE ON STELLAR STRUCTURE

4.1. Physical Quantities Modifying The Mechanical Balance

4.1.1. Lorentz Force

Lorentz force influences the stellar structure through modification of the hydrostatic balance in the meridional plane (eq. 8). In Fig. 1 (left) is drawn the radial dependence of the averaged Lorentz force for both cases studied here. It appears that the Lorentz force is centrifugal within the internal third of the radius whereas it is centripetal in the external part. We can then predict that its impact on the density redistribution throughout the evolution is to counteract the effect of the gravity, owing to the fact that the mechanical balance is biased by the density distribution which favours what happens near the center. Furthermore, it stems from the latitudinal profile that the component of the Lorentz force along this direction is directed towards the equator, so it will increase the density in the equatorial latitudes. Therefore the sphere deformation is prolate.

4.1.2. Magnetic Pressure Force vs. Magnetic Tension Force

We can write the Lorentz force as the sum of the gradient of a magnetic pressure and of a magnetic tension force:

$$\boldsymbol{F}_{\mathscr{L}} = \boldsymbol{F}_{\mathscr{T}} - \nabla P_{\mathrm{mag}}. \tag{13}$$

The magnetic pressure gradient has a predominant role in the internal part of the star over the magnetic tension. However, the latter's strength is of the order of the former in particular on the symmetry axis and in the vicinity of the surface, where both ones counterbalance each other. This leads to a force-free state, that cannot be achieved by considering the magnetic pressure as the only effect.

4.1.3. Lorentz Force Perturbations on the Stellar Structure

Let us then project the Lorentz force components on the Legendre polynomials $P_l(\cos\theta)$ (of order $l = 0$ and $l = 2$ in the case of a dipolar field), assuming it is a perturbation around the stellar non-magnetic state:

$$F_{\mathscr{L},r}(r,\theta) = \sum_l \mathscr{X}_{\boldsymbol{F}_{\mathscr{L}};l}(r)\, P_l(\cos\theta)\,, \qquad F_{\mathscr{L},\theta}(r,\theta) = -\sum_l \mathscr{Y}_{\boldsymbol{F}_{\mathscr{L}};l}(r)\, \partial_\theta P_l(\cos\theta) \tag{14}$$

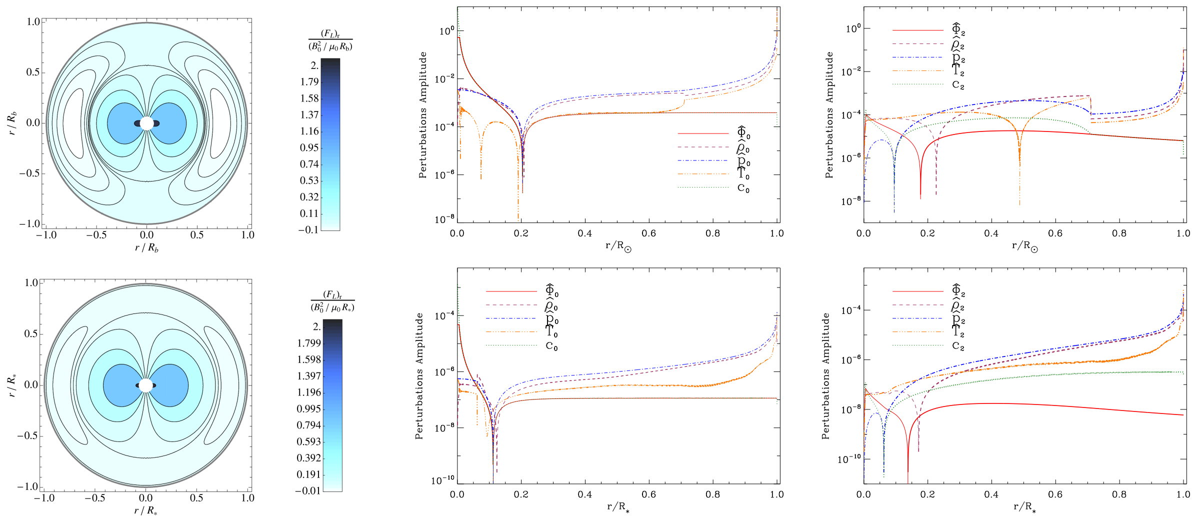

FIGURE 1. Left: radial Lorentz force isocontours in the meridional plane, (up) for a 7 MG field buried below the convection zone of the Sun; (down) for a 10 kG field in an Ap star. Middle and Right : perturbations (in log. scale) of mode $l=0$ (middle) and $l=2$ (right) respectively for both cases above. Bold lines represent positive values whereas thin lines represent negative ones. The spikes corresponds to the vanishing of source terms for the equations (15), (16) and (17).

which gives us at the surface the gravitational moments $J_l = (R_*/GM_*)\,\widehat{\phi_l}\,(r=R_*)$. We can then deduce the gravitational potential perturbation $\widehat{\phi_l}(r)$ to the non-magnetic state $\phi_0(r)$, from Sweet's equation[1]

$$\frac{1}{r}\frac{\mathrm{d}^2}{\mathrm{d}r^2}\left(r\widehat{\phi_l}\right)-\frac{l(l+1)}{r^2}\widehat{\phi_l}-\frac{4\pi G}{g_0}\frac{\mathrm{d}\rho_0}{\mathrm{d}r}\widehat{\phi_l}=\frac{4\pi G}{g_0}\left[\mathscr{X}_{\boldsymbol{F}_{\mathscr{L}};l}+\frac{\mathrm{d}}{\mathrm{d}r}\left(r\mathscr{Y}_{\boldsymbol{F}_{\mathscr{L}};l}\right)\right], \tag{15}$$

where g_0 is the equilibrium gravity and where we have $\phi\,(r,\theta)=\phi_0(r)+\sum_l\widehat{\phi_l}\,(r)\,P_l\,(\cos\theta)$. After numerical integration, the density perturbation $\widehat{\rho_l}$ and the pressure one $\widehat{P_l}$ (where the density and the pressure have been expanded on the Legendre polynomials as done for the gravitational potential) for the mode l, can respectively be computed according to

$$\widehat{\rho_l}=\frac{1}{g_0}\left[\frac{\mathrm{d}\rho_0}{\mathrm{d}r}\widehat{\phi_l}+\mathscr{X}_{\boldsymbol{F}_{\mathscr{L}};l}+\frac{\mathrm{d}}{\mathrm{d}r}\left(r\mathscr{Y}_{\boldsymbol{F}_{\mathscr{L}};l}\right)\right]\quad\text{and}\quad\widehat{P_l}=-\rho_0\widehat{\phi_l}-r\mathscr{Y}_{\boldsymbol{F}_{\mathscr{L}};l}. \tag{16}$$

Diagnosis from the stellar radius variation induced by the magnetic field can be established. The radius of an isobar is given by

$$r_P(r,\theta)=r\left[1+\sum_{l\geq 0}c_l(r)P_l(\cos\theta)\right]\quad\text{with}\quad c_l=-\frac{1}{r}\frac{\widehat{P_l}}{\mathrm{d}P_0/\mathrm{d}r}=\frac{\rho_0}{\mathrm{d}P_0/\mathrm{d}r}\left(\frac{1}{r}\widehat{\phi_l}+\frac{\mathscr{Y}_{\boldsymbol{F}_{\mathscr{L}};l}}{\rho_0}\right). \tag{17}$$

Finally, it can be interesting to look for temperature perturbations. Following [18], we introduce the general equation of state for the stellar plasma $\mathrm{d}\rho/\rho=\alpha_s\mathrm{d}P/P-\delta_s\mathrm{d}T/T$, where $\alpha_s=(\partial\ln\rho/\partial\ln P)_{T,\mu_s}$, $\delta_s=-(\partial\ln\rho/\partial\ln T)_{P,\mu_s}$ and where we neglected the mean molecular weight perturbation. For a perturbative Lorentz force, the stellar temperature (T) can be expanded like P, ρ and ϕ according to $T\,(r,\theta)=T_0\,(r)+\sum_{l\geq 0}\widehat{T_l}\,(r)\,P_l\,(\cos\theta)$. Linearizing the equation of state, we finally obtain

$$\widehat{T_l}=\frac{T_0}{\delta_s}\left[\alpha_s\frac{\widehat{P_l}}{P_0}-\frac{\widehat{\rho_l}}{\rho_0}\right]. \tag{18}$$

Results for the normalized perturbations of gravitational potential $\tilde{\Phi}_l$, density $\tilde{\rho}_l$, pressure $\tilde{P}_l$, temperature $\tilde{T}_l$ and radius c_l are shown in Fig. 1 for the modes $l=0$ and $l=2$ (resp. middle and right panel). The surface values are given in Table 1. It shows that the perturbative approach is suitable in the case of the $\mathrm{A_p}$ star for a 10 kG field strength at the surface, while in the solar case, it cannot cope with fields whose maximum amplitude are higher than $B_{0,\max}=240$kG. For $B_0=7$MG, we are hence much above the perturbative limit and no conclusion can be drawn in this case.

[1] Let us recall that Sweet (1950) [16] was the first to derive this result for the most general perturbing force, Moss (1974) [17] having introduced the special case of the Lorentz force in the case of a poloidal field, while later Mathis & Zahn (2005) [9] treated the general axisymmetric case.

TABLE 1. (Left): normalized surface perturbations in solar case, for a field buried below the convection zone with a maximum field strength of 7MG; (right): in the case of the A_p type star for a field with a 10 kG amplitude at the surface.

	Sun		A_p star	
	$l=0$	$l=2$	$l=0$	$l=2$
J_l	3.85×10^{-4}	-6.31×10^{-6}	1.14×10^{-7}	-5.95×10^{-9}
$\tilde{\rho}_l$	(-8.58)	(0.14)	-8.35×10^{-6}	-2.41×10^{-4}
$\tilde{P}_l$	(-4.04×10^{-1})	6.62×10^{-3}	1.40×10^{-4}	4.40×10^{-4}
$\tilde{T}_l$	(8.18)	(-1.34×10^{-1})	-1.25×10^{-4}	6.80×10^{-4}
c_l	-7.95×10^{-5}	1.30×10^{-6}	6.86×10^{-8}	2.25×10^{-7}

4.2. Perturbation of the Energetic Balance

Here again a perturbative approach is adopted. The luminosity is expanded as

$$L(r)=L_0(r)+\widehat{L}_{\rm tot}(r). \tag{19}$$

$\widehat{L}_{\rm tot}$ is the luminosity perturbation due to the magnetic terms : $\widehat{L}_{\rm tot}(r)=L_{\rm Ohm}(r)+L_{\rm Poynt}(r)+\widehat{L}_{\rm nuc}(r)$, which are respectively the Ohmic heating contribution, the Poynting's flux one, and the one related to the induced modification of the specific energy production rate.

First, we integrate the Ohmic heating and the Poynting's flux over the spherical volume delimited by r

$$L_{\rm Ohm}(r)=\int_0^r\int_\Omega Q_{\rm Ohm}(r',\theta')\,{\rm d}\Omega\,r'^2{\rm d}r';\; L_{\rm Poynt}(r)=\int_0^r\int_\Omega F_{\rm Poynt}(r',\theta')\,{\rm d}\Omega\,r'^2{\rm d}r', \tag{20}$$

where ${\rm d}\Omega=\sin\theta'{\rm d}\theta'{\rm d}\phi'$, r' thus ranging from 0 to r, θ' from 0 to π and ϕ' from 0 to 2π.

The magnetic diffusivity η has been evaluated with the temperature-dependent law from [19] $\eta=5.2\times10^{11}\log\Lambda\,T^{-3/2}\ {\rm cm^2s^{-1}}$ where we took for the coulombian logarithm $\log\Lambda\approx10$. Then, to be able to conclude we finally consider the modification of the specific energy production rate (ε), which depends on ρ and T, due to magnetic field. First, the logarithmic derivative of ε is expanded like the one of ρ (cf. the equation of state; see Mathis & Zahn 2004 [8] and references therein): ${\rm d}\ln\varepsilon=\lambda\,{\rm d}\ln\rho+\nu\,{\rm d}\ln T$, where $\lambda=(\partial\ln\varepsilon/\partial\ln\rho)_T$ and $\nu=(\partial\ln\varepsilon/\partial\ln T)_\rho$. Then, like ρ and T, we expand ε on the Legendre polynomials so that we finally end up with

$$\varepsilon(r,\theta)=\varepsilon_0(r)+\sum_{l\geq0}\widehat{\varepsilon}_l(r)P_l(\cos\theta)\quad\text{where}\quad\widehat{\varepsilon}_l=\varepsilon_0\left[\lambda\frac{\widehat{\rho}_l}{\rho_0}+\nu\frac{\widehat{T}_l}{T_0}\right]. \tag{21}$$

Then, the luminosity perturbation induced by the MHS equilibrium over the nuclear reaction rates is

$$\widehat{L}_{\rm nuc}(r)=\int_0^r\int_\Omega\widehat{\varepsilon}_0\,\rho_0\,r'^2{\rm d}r'{\rm d}\Omega=4\pi\int_0^r\left\{\varepsilon_0\left[\lambda\frac{\widehat{\rho}_0}{\rho_0}+\nu\frac{\widehat{T}_0}{T_0}\right]\right\}\rho_0\,r'^2{\rm d}r'. \tag{22}$$

The values found at the stellar surface are given in Table 2 for both cases here considered.

5. CONCLUSION AND PERSPECTIVES

First, we emphasize the fact that one has to take into account both effects of magnetic pressure gradient and magnetic tension when the magnetic field is included in the hydrostatic balance through the Lorentz force. In particular, in the vicinity of the stellar surface (or near the confinement radius), the magnetic tension has an important qualitative role since it allows to compensate the magnetic pressure gradient that leads to a force-free state; this is also the case near the magnetic field axis where the two contributions are similar.

Then, a first order-perturbative treatment has been performed, which puts in evidence the perturbation in structural quantities as a function of the radius. This approach is valid in high-β regimes and it has been applied to both cases considered here. The modal fluctuations in structural quantities such as the gravific potential, the density, the pressure, the temperature, the gravitational multipole moments and the isobar radius are computed. Nevertheless, this approach fails to derive the perturbations associated with a strong magnetic field; in the case of the Sun it is suitable for fields

TABLE 2. Contribution of Ohmic heating $(L_{\rm Ohm})$ and contribution of Poynting's flux $\left(L_{\rm Poynt}\right)$ on the luminosity perturbation for the mode $l = 0$ (in erg.s); contribution for the perturbation induced by the deviation to the hydrostatic equilibrium over the nuclear efficiency $\widehat{L}_{\rm nuc}$; total luminosity $L_{\rm tot}$.

	Sun	A_p star
$L_{\rm Ohm}$	5.81×10^{24}	2.15×10^{27}
$L_{\rm Poynt}$	-1.01×10^{23}	-2.90×10^{25}
$\widehat{L}_{\rm nuc}$	1.49×10^{28}	5.05×10^{30}
$L_{\rm tot}$	1.59×10^{35}	3.85×10^{33}

whose strengths are not higher than 240 kG. This justifies to solve the complete set of evolution equations taking into account all the magnetic modifications directly. It is directly explained by the fact that the Lorentz force is a volumetric quantity so when the density decreases, the relative magnetic effects are increased and become of the same order than the gravity.
Next, the Poynting's flux and the Ohmic heating contribute to the modification of the energetic balance and influence it with approximately the same order of magnitude. Nevertheless, we notice that their contributions remain weak in the studied cases. Nevertheless, it has been shown straightforwardly that the energetic balance can be modified more significantly by the nuclear production rates indirect changing through the modification of pressure and density when the magnetic field modifies the hydro-static balance.
Finally, we have to emphasize that this study is the first step in the implementation of global magnetic fields in stellar models and the introduction of the different terms along the evolution in a 1D stellar evolution code. They may have a greater impact especially in the early and late stages of the evolution.

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