GRAVITATIONAL MICROLENSING*

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Abstract

Ten years after Paczyński's proposal to use gravitational microlensing as a tool for discovering dark stars and four years after the identification of the first candidate events, searches for microlensing events have opened a new window in astronomy and started to yield quantitative information on the contribution of dark compact objects to the dark halo of the Galaxy.

1. Introduction

For the past 70 years, evidence has been accumulated for a large amount of Dark Matter on all scales, from individual galaxies to the whole Universe. The most compelling piece of evidence comes from the observation of the velocities at which matter rotates about galaxies. The rotation curves of most spiral galaxies show the following trend: after an initial rise, they flatten and tend to remain more or less constant as one goes to larger and larger radii, instead of falling with the expected $1/\sqrt{R}$ dependence beyond the visible component. This implies a mass distribution extending far beyond the optical radius of the galaxy. To make up for this discrepancy, a dark halo component is added to the galactic structure.

*Talk given by L. Moscoso at the Fifth School on Non-Accelerator Particle Astrophysics, Trieste, Italy, June 29 – July 7, 1998.

This situation is quite universal, and the Milky Way is no exception: its flat rotation curve out to about 20 kpc from the Galactic Center implies a mass at least three times as large as that derived from the study of its visible components, including stars and gas.

There is a wide range of candidates to explain this dark matter, both baryonic — molecular Hydrogen clouds, stellar remnants such as white dwarfs (~ 1 $M_\odot$), neutron stars or black holes (which could be very massive, up to several solar masses), stars below the Hydrogen burning limit called brown dwarfs (< 0.08 $M_\odot$) or faint stars such as red dwarfs (~ 0.1 $M_\odot$) — and non-baryonic — WIMPS, axions or massive neutrinos. The comparison of the limits on the content of the Universe in baryonic dark matter derived from Big Bang Nucleosynthesis, with the amount of Dark Matter required from dynamical arguments imply that both categories should exist (see figure 1). However, it is remarkable that the predicted range for baryonic dark matter and the additional mass needed on the scale of galaxies coincide exactly, making it reasonable to fill the halos of galaxies with baryonic dark matter. This prompted interest in looking for dark compact objects wandering in the halo of the Milky Way, using a new detection technique first proposed by [1]: microlensing. As one considers the Dark Matter issue on larger scales, however, it becomes unavoidable to invoke non-baryonic dark matter, and different detection techniques are then used.

2. Gravitational microlensing

According to the principles of general relativity, the light rays from a source star are deflected by the presence of a massive deflector located near the line of sight between the star and the observer. This forms two distorted images of the source, as illustrated in figure 2.

In the particular configuration where all three objects are perfectly aligned, the two images merge into a ring, whose radius is called the Einstein radius R_E

$$R_E = \sqrt{\frac{4GM}{c^2}\, D_{OS}\, x(1-x)} \tag{1}$$

where $x = D_{OD}/D_{OS}$ is the ratio of the distance observer-deflector to the distance observer-source and M the mass of the deflector. When probing the dark matter content of the Galactic halo, the source star is typically 60 kpc away (located in one of the Magellanic Clouds) from the observer and the lens typically a solar-mass object or lighter located half-way between the source and the observer, so the angular separation between the images ($\sim 2R_E/D_{OD}$) is only of the order of the milliarcsecond. Given the limited resolution of optical telescopes with current technology, only the combined light intensity can therefore be recorded. The total magnification, however, is always greater than what the observer would receive from the source in the absence of the lens, which makes the latter detectable. If u denotes the impact parameter in units of the Einstein radius, the maximum magnification of the source star is given

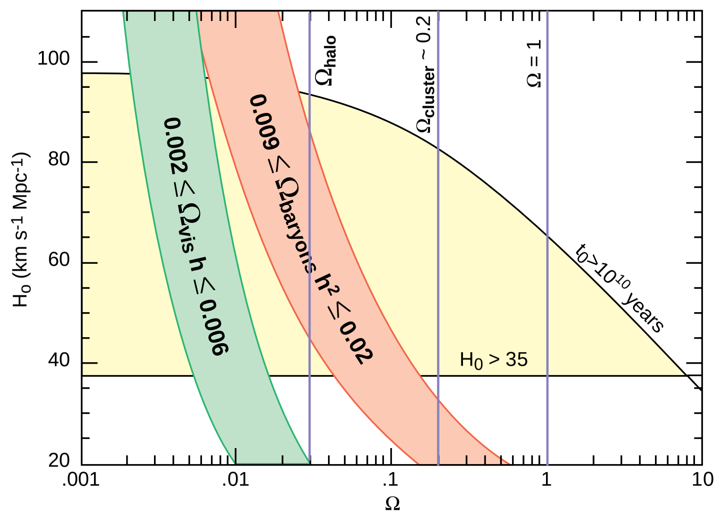

Figure 1: $H_0 - \Omega$ plot (assuming $\Lambda = 0$), indicating allowed (shaded) and excluded regions. The fraction of visible matter in the Universe, Ω_{vis} is shown, along with the fraction of baryonic matter Ω_B resulting from nucleosynthesis, the average value of the fraction of matter in halos of galaxies $\Omega_{\rm halo}$ and the average value of the fraction of matter in clusters of galaxies $\Omega_{\rm clusters}$. Two more limits are indicated: the lower bound $H_0 \geq 35\,{\rm km\,s^{-1}\,Mpc^{-1}}$ obtained from white dwarf stars and supernovae, and the lower bound on the age of the Universe $t_0 \geq 10\,{\rm Gyr}$ obtained essentially from the age of globular clusters, under the reasonable assumption that the Universe can hardly be younger than its components (adapted from Copi and Schramm 96).

by

$$A = \frac{u^2+2}{\sqrt{u^2(u^2+4)}} \tag{2}$$

The evolution of the magnification with the impact parameter is illustrated in figure 3.

As the lens moves in the halo with respect to the line-of-sight, the typical time scale Δt of a microlensing event is given by

$$\Delta t = \frac{R_E}{v_t} \simeq 90\sqrt{M/{\rm M}_\odot} \tag{3}$$

where v_t is the transverse velocity of the lens which to first order can be taken as the rotation velocity of the galactic halo, ie 220 km/s.

When searching for microlensing events, one can thus require the following properties of the light curve:

- **Achromatism** since the phenomenon is purely geometrical
- **Symmetry** if we assume that the transverse velocity of the lens is constant

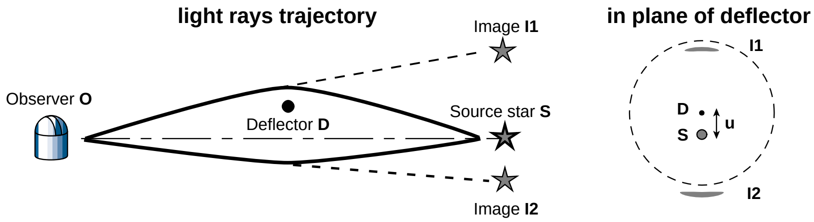

Figure 2: Deflection of light by a massive body D located near the line of sight between the observer O and the source star S. The dotted circle is the Einstein ring.

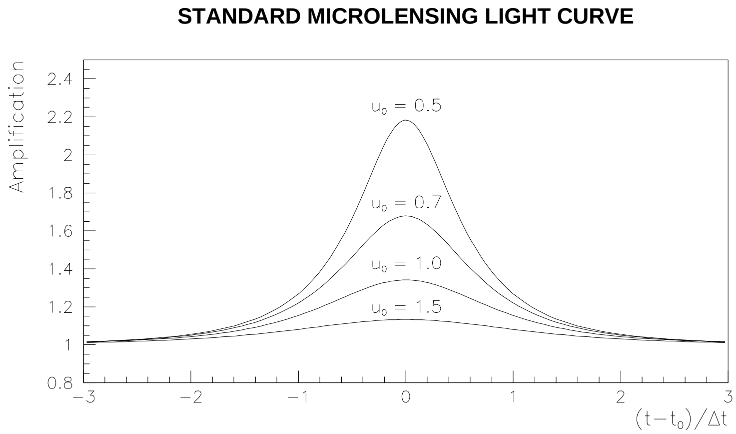

Figure 3: Magnification as a function of time during a microlensing event. Curves are shown for four different impact parameters: $u_0 = 1.5$, $u_0 = 1$, $u_0 = 0.7$ and $u_0 = 0.5$. Obviously, the smaller the impact parameter, the higher the magnification at $t = t_0$.

- that the magnification occur on a **normal star** since any star is as likely to undergo a microlensing event. This characteristic will be of great help to reject variable stars.

The probability that a given star is amplified at a given time is very low, typically $5\,10^{-7}$. This implies that an additional criterion can be applied to the set of light curves in order to select microlensing events:

- **Unicity** of the magnification on a given light curve. This criterion is the major source of rejection of variable stars which outnumber microlensing events by about 1000:1.

This low probability also implies that millions of stars have to be monitored for years in order to ever be able to detect such a rare event. More details on the general principles of microlensing can be found in [2,3].

The only targets far enough to probe a large fraction of the galactic halo but close enough to resolve millions of stars are the large and the small Magellanic Clouds (LMC and SMC respectively), observable from the southern hemisphere. Various experiments are involved in the survey of stars located in these two satellite galaxies: EROS 2 (French experiment observing in Chile), MACHO (American experiment observing in Australia) and OGLE 2 (Polish experiment observing in Chile). The following sections present the latest results obtained by these experiments on the contribution of dark compact objects to the mass of the Galactic halo.

Because the mass range to probe extends from planetary objects ($\sim 10^{-7}$ $M_\odot$) to stellar dark objects (a few solar masses), the event time-scales that the experiments have to be sensitive to vary from a few hours to a few months (see equation 3). Dedicated experiments and analyses are therefore performed separately to search for either small or large mass deflectors.

3. Limits on contribution of small mass objects

To search for planetary mass dark matter in the galactic halo, the EROS 1 CCD experiment, on the one hand, monitored 150 thousand stars in the LMC, with a high efficiency of $\sim 80\%$, thanks to a very good time sampling of roughly one point every half hour. On the other hand, the MACHO experiment monitored 8.6 million stars thanks to a large coverage of the LMC, but with only a $\sim 1\%$ efficiency to short time-scale events since their observational strategy was optimized for long time-scale events, covering a large field but with at most 2 points per night on a given light curve. Neither of the two experiments found any such event, which allowed them to set quite stringent limits on the maximum contribution of small mass objects to the dark mass of the Galactic halo [4,5]. Figure 4 shows the limits obtained by either project, under the hypothesis of a "standard" isotropic and isothermal spherical halo entirely filled of dark compact objects all having the same mass M which is indicated as the abscissa of the graph. The vertical axis represents the maximum halo mass fraction of the halo that could be composed of objects of a given mass M.

As explained above, EROS and MACHO have chosen very different analysis techniques, and there is little overlap in exposure for the two projects. Combining both sets of data after removal of this small overlap thus yields even stronger limits [6]. The new exclusion diagram obtained is also illustrated in figure 4. For instance, if one assumes a halo fully composed of $10^{-5} M_\odot$ objects, the EROS (resp. MACHO) experiment alone excludes that more than $\sim 15\%$ (resp. $\sim 18\%$) of the mass of the halo be in the form of such objects, while the combined analysis excludes a contribution of more than $\sim 8\%$. From the exclusion limit obtained, it can be concluded that not more than $\sim$10% of the halo can be composed of objects in the mass range $[10^{-7} - 0.02]$ $M_\odot$, at the 95% confidence level. Because we are using δ-function mass distributions to calculate this exclusion plot and since the limit is quite flat in the mass interval mentioned above, any mass function that peaks in this range is also

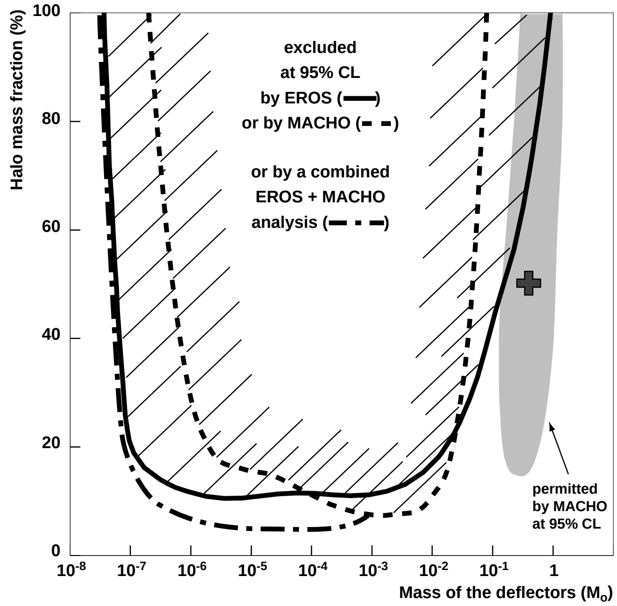

Figure 4: Exclusion diagram showing the 95% CL limits as obtained by EROS (full curve), by MACHO (dotted curve) and by a combined analysis of EROS + MACHO data (dot-dashed curve). Also shown is the 95% CL region compatible with the 6 events detected by the MACHO two year analysis.

excluded at the same confidence level.

4. Contribution of high mass objects

4.1. Present results toward the LMC

Both the EROS 1 experiment (using photographic plates) and the MACHO experiment (using CCD's) have searched and found long time-scale microlensing candidates. A total of 10 events have been detected, two by EROS [7] and eight by MACHO [8], although one of them is slightly asymmetric and thus often disregarded as a microlensing candidate and another is an LMC binary event where the lens is most probably located in the LMC itself.

The typical Einstein radius crossing time associated to the LMC events is of the order of 40 days, which implies a surprisingly high most probable mass for the lenses: $\sim 0.5\ M_\odot$. This mass is much larger than the upper limit of $0.08\ M_\odot$ for brown dwarfs, and the lenses could be interpreted as, instead, white dwarfs or black holes. The optical depth implied by the mean duration of the events is compatible with about half that required to account for the dynamical mass of the dark halo. Such an interpretation, however, is not quite accepted among astronomers: by observing younger galaxies where we could detect the light due to a significant white dwarf component in the halo, a limit of 10% has been set on their contribution [9]; furthermore, if indeed half of the dark halos of galaxies consisted of such stellar remnants, we should observe an enrichment of the interstellar medium in Helium, which we do not. There is thus no consensus yet as to the nature of the deflectors causing the observed events.

The 95% CL region allowed by the MACHO experiment due to the detection of 6 events (*ie* disregarding the LMC binary and the asymmetric event) is illustrated in figure 4. Note that the present exclusion limit and detections are compatible with one another.

4.2. Future data

The interpretation of the present data is ambiguous, and huge error bars remain on both the most probable mass of the deflectors (about a factor of 3 either way) and the halo fraction in compact objects (compatible with both $\sim 10\%$ and way over 100%). More statistics is thus required, and several experiments are accumulating data to answer the questions of the presence or not and the nature of dark compact objects in the halo of the Galaxy:

- The EROS 2 experiment is now taking data with a completely redesigned setup and a new strategy. Using a wide field CCD camera (data taken in two colors simultaneously, with in each color a 1 square degree mosaic consisting of eight 2048×2048 CCD's), EROS covered 66 deg^2 on the LMC during the first year of observation (August 1996 - May 1997) and a total of 88 deg^2 the second year. Data is also taken toward the SMC (10 deg^2), the galactic center ($\sim$100 deg^2) and the spiral arms of the Milky Way ($\sim$30 deg^2). The exposure times and time sampling are now adapted to a search for long time-scale events.

- The MACHO experiment is presently analyzing four years of data on 15 deg^2 in the LMC, which means an increase of a factor of 2 in time scale and 1.4 in area. Preliminary results indicate 6 new events with time-scales ranging between 15 and 110 days with an average of about 50 days. This would confirm a high mass for halo deflectors if the lenses are indeed located in the halo of the Galaxy, with roughly the same rate as that estimated from their 2-year analysis presented in the previous section.

- The OGLE 2 experiment uses an upgraded setup and started taking data in summer 1997. They now also cover fields in the Large and the Small Magellanic Clouds (their previous strategy concerned only fields toward the Galactic Center, thus probing disk Dark Matter and not halo Dark Matter).

5. Highlights toward the SMC

The Small Magellanic Cloud is highly valuable as a new line-of-sight through the Milky Way halo and a new population of source stars. The use of various lines-of-sight is very important since a comparison of the event rates is a powerful tool for discriminating between various shapes for the dark halo [10,11]. In addition, this allows for discrimination between various theories for the populations responsible for the LMC lenses [12].

The EROS and MACHO experiments (and more recently the OGLE experiment) thus monitor stars in the SMC to search for microlensing events. EROS recently published the first analysis on SMC data, whose results are presented hereafter.

5.1. First analysis of SMC data

The EROS 2 experiment covers the densest 10 square degrees of the Small Magellanic Cloud. On these 10 fields, a total of 5.3 millions light curves were built and subjected to a series of selection criteria and rejection cuts to isolate microlensing candidates [13]. Ten light curves passed all cuts and were inspected individually. Several correspond to physical processes other than microlensing (one of them, for instance, is the light curve of a nova that exploded in the SMC), and only one of the candidates passes a final visual inspection. Its light curve is shown in figure 5.

Once corrected for blending — because of the high stellar density of the fields monitored in microlensing surveys, the flux of each reconstructed star generally results from the superposition of the fluxes of many source stars — the event light curve is well fitted by that of a microlensing event with an Einstein radius crossing time of 129 days, a maximum magnification of 2.6 occurring on January 11, 1997 and a χ^2/d.o.f. = 332/217 = 1.5. The best microlensing fit is for 70% of the monitored flux being amplified and 30% being the contribution from blending unamplified light.

The source star being very bright, the value of the reduced χ^2 of the fit is surprisingly high (the photometric resolution on such bright stars is $\sim 2\%$). A search for periodicity was therefore performed on the fit residuals, and a modulation was detected, with an amplitude of 2.5% and a period of 5.12 days. Fitting again the candidate light curve for microlensing allowing for a periodic modulation yields much more satisfactory residuals than before: χ^2/d.o.f. = 199/214. This strongly supports the microlensing interpretation of the observed magnification. The modulation detected was later confirmed by the OGLE experiment, on their own data taken after the event occurred (first points in June 1997). They also confirm the value of the blending coefficient of the source star since their new camera allows the separation of the two components of the blend and thus the individual measure of each of the two fluxes. This first candidate towards the SMC is therefore very convincing.

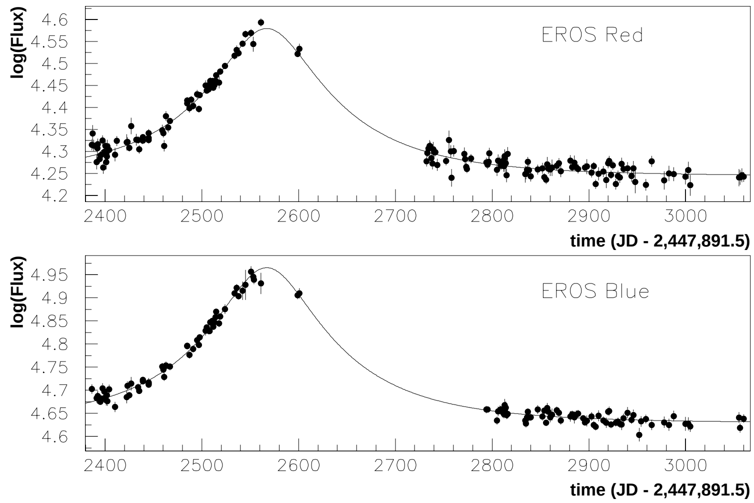

Figure 5: Light curve of microlensing candidate SMC #1, with a microlensing fit including blending superimposed. Time is in days since Jan. 0, 1990 (Julian date 2,447,891.5). Red light curve on top, blue on the bottom.

5.2. Estimate of Halo fraction in compact objects and lens mass

The time-scale of the observed event allows one to estimate the fraction of the halo that can be composed of dark compact objects generating microlensing events, independently of their mass. Assuming that the deflector is in the halo of the Milky Way, and considering a standard halo model (*ie* an isotropic and isothermal spherical halo), the EROS experiment estimated that the detected event is compatible with about 50% of the mass of the halo in dark compact objects. This fraction can vary by as much as a factor of two when considering other halo models (flattened halos for instance, or thinner halos and thicker disks so as to reproduce the rotation curve of the Galaxy but which have less mass in the dark halo component).

Using a likelihood analysis based on the time-scale of the detected event, the most probable mass of the deflector generating the event can be estimated. Under the assumption of a standard halo composed of dark compact objects having a single mass M (ie the mass function is supposed to be a Dirac-function), the most probable mass of the Halo deflector, given with 1σ error bars, is:

$$M = 2.6^{+8.2}_{-2.3}\ \mathrm{M}_{\odot} \tag{4}$$

The event has the highest time-scale observed so far, and consequently the highest most probable mass. Only a neutron star or a black hole could be that massive and yet be dark. It is even harder than for the LMC events to explain how the halo of the Galaxy could be filled (even partially) with such heavy dark objects. Other interpretations therefore have to be looked at seriously.

5.3. Interpretation as SMC self lensing

The very long time-scale of the observed event suggests that it could show measurable distortions in its light curve due to the motion of the Earth around the Sun (a detailed analysis of the parallax effect can be found in [14]), provided that the Einstein radius projected onto the plane of the Earth is not much larger than the Earth orbital radius. The first detection of parallax in a gravitational microlensing event was observed by [15].

No evidence for distortion due to parallax is detectable on the light curve, implying either a deflector near the source or a very massive deflector with thus a very large Einstein radius (see equation 1) making the projected radius of the Earth's orbit negligible as compared to the Einstein radius. The absence of parallax detection can be expressed as a constraint on the mass of the deflector and its distance to the observer, through the following relation, at the 95% CL:

$$\frac{M}{\mathrm{M}_\odot} \times \frac{x}{1-x} > 0.7 \tag{5}$$

where, as previously, $x = D_{OD}/D_{OS}$.

If the deflector is in the **halo of the Galaxy** (assuming a standard halo, $x < 0.66$ at the 95% CL) this yields a lower limit on the mass of the deflector: $M > 0.6\ \mathrm{M}_\odot$.

If the deflector is located in the **SMC itself**, $1 - x \sim 1/10$ and the mass of the deflector is then $M \sim 0.1\mathrm{M}_\odot$, typical of a brown dwarf or faint star in the SMC.

To validate a possible SMC self lensing interpretation of the first event detected toward this new line-of-sight, it is necessary to check whether the SMC stellar population could provide such an event, in terms of duration and optical depth (probability that at a given time, a given star be magnified).

Various authors have suggested that the SMC is quite elongated along the line-of-sight, with a depth varying from a few kpc to as much as 20 kpc, depending on the region under study. We will approximate the SMC density profile by a prolate ellipsoid:

$$\rho = \frac{\Sigma_0}{2h}\, e^{-|z|/h}\, e^{-r/r_d} \tag{6}$$

where z is along the line-of-sight and r is transverse to the line-of-sight. The depth h will be a free parameter, allowed to vary between 2.5 and 7.5 kpc. The values of the various parameters of the model are fit to the isophote levels of the cloud [16] (which yields $\Sigma_0 \simeq 400\ \mathrm{M}_\odot \mathrm{pc}^{-2}$ and $r_d = 0.5$ kpc) and considering a mass-to-light ratio of 3 $\mathrm{M}_\odot/\mathrm{L}_\odot$ (typical of the values measured in the disk of the Milky Way). For $h = 2.5$, 5.0 or 7.5 kpc, the predicted optical depths are $\tau = 1.0\ 10^{-7}$, $1.7\ 10^{-7}$ or $1.8\ 10^{-7}$ respectively, to be compared with the experimental optical depth of

$3.3\ 10^{-7}$. Considering the very limited statistics we have, the model is consistent with the observations.

Finally, considering a velocity dispersion of 30 km/s in the SMC (and the 129 days time-scale of the event), the mass M of the deflector can be estimated according to an assumed distance between the source star and the lens. If the deflector is 5 kpc (resp. 2.5 kpc) from the source, its mass is $M \sim 0.1\ \mathrm{M}_\odot$ (resp. $0.2\ \mathrm{M}_\odot$), compatible with the results obtained from the parallax analysis.

An SMC self-lensing interpretation of the first microlensing event detected toward this new line-of-sight is thus quite plausible.

5.4. A Binary Lens towards the SMC

After the detection of this first SMC event, the MACHO collaboration alerted the microlensing community in June 1998 of an ongoing microlensing event (IAU circular 6935), which was later identified as a binary source event. In that case, the variation of the amplification is no longer simple: the gravitational potential of the double lens gives rise to caustic lines. When the source star crosses such a caustic, the amplification becomes singular, making it possible, given an appropriate time sampling of the light curve, to resolve the finite size of the source star by measuring the duration of the caustic crossing. This duration is related to the proper motion of the source, ie the ratio of the transverse velocity of the source projected onto the plane of the deflector by the distance to the deflector. Its estimate thus helps raising the three-fold degeneracy intrinsic to the sole measure of the crossing time Δt, since this parameter involves simultaneously the mass of the deflector, its transverse velocity and its distance to the observer.

The measurements obtained on this microlensing event allowed to predict precisely the date of the second caustic crossing, June 18, 1998. All microlensing collaborations took data this night. Among them, the PLANET collaboration obtained well sampled data at the time of the maximum magnification, and the EROS collaboration equivalent data at the end of the caustic crossing (see figure 6).

Both data sets allowed to put constraints on the duration of the caustic crossing [17,18]. Combining this result with an estimate of the size of the source star, it was then possible to put limits on the proper motion of the lens. The most plausible interpretation for this event is that the lens lies in the SMC itself: only 7% of the halo population has a proper motion compatible with the one measured.

6. Conclusions

Two targets have been explored so far, in the search for dark matter in the halo of the Milky Way. They are the Large and the Small Magellanic Clouds.

The LMC data collected by the MACHO and the EROS experiments have allowed them to exclude any major contribution to the dark mass of the halo from compact objects in the mass range $10^{-7}\,\mathrm{M}_\odot - 0.02\,\mathrm{M}_\odot$. Eight events compatible with mi-

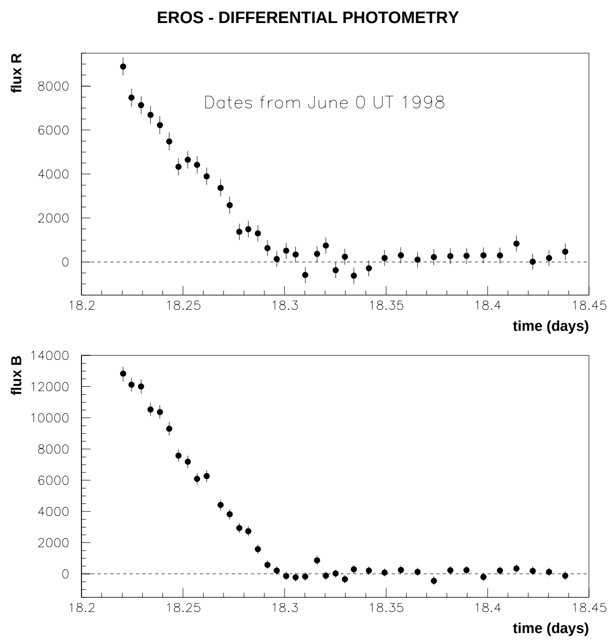

Figure 6: Differential photometry of EROS data taken on 18 June 1998. R data on top, B data on bottom, in ADU.

crolensing by halo lenses were detected, with an average time-scale of 40 days, which could be interpreted as about 50% of the halo dark matter in the form of $\sim 0.5\mathrm{M}_\odot$ objects. A huge controversy remains as to the nature of these objects.

The SMC data has yielded one event found during the analysis of the first year of data, and one binary event detected online by the MACHO group. The first event has the longest time-scale observed so far: 129 days. If the lens causing the event is assumed to be in the halo of the Galaxy then its most probable mass is $2.6\mathrm{M}_\odot$, with a lower limit of $0.6\mathrm{M}_\odot$ coming from parallax analysis. Such a high mass is very hard to explain. A more plausible explanation is to assume that the lens and the source star are both located in the SMC. The mass of the deflectors would then be typical of that of a faint star in the SMC, and the experimental optical depth compatible with a "thick disk" model of the SMC. For the second event, the caustic crossing duration indicates clearly that the lens is in the SMC.

The status of microlensing experiments and their implications on the galactic structure can be summarized in a few words. With about 100 microlensing events detected toward the Galactic Center, nearly 10 toward the LMC, 1 or 2 toward the SMC and none yet toward the Andromeda Galaxy (M31), there is strong evidence for the existence of a bar in the bulge and for lenses residing either in the halo of our Galaxy or in the LMC/SMC themselves. The main question now raised by these results is to determine where the lenses generating the detected events belong. Are they halo objects or intrinsic to either cloud? More statistics is still being accumulated. The MACHO experiment will run until 1999, EROS 2 plans to run until 2002 and OGLE 2 is just starting to take new data. The answer to this problem can then come from at least three possible studies:

- The comparison of the time-scales of the events toward the LMC and the SMC. They are expected to be similar if the deflectors are in the Galactic halo but different (due to different velocity dispersions) if they are intrinsic to each cloud.
- The analysis of the spatial distribution of LMC events. The events are expected to be distributed evenly over the entire cloud if the lenses belong to the Galactic halo, while they should follow the stellar density of the LMC if they are LMC stars themselves.
- Finally, because the disk of the Andromeda Galaxy is slanted with respect to the line-of-sight, different fractions of its halo will be probed according to which end of the disk is being monitored; this will yield a larger number of events on the far side than on the near side, for microlensing events produced by deflectors in the halo of M31. Several experiments are exploring this line of sight and the first results are expected soon.

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