Making the new International System of units compatible with

$$c = \hbar = \mu_\circ = \epsilon_\circ = 1$$

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The INTERNATIONAL SYSTEM of UNITS

7 base units :

time	second	s
length	metre	m
mass	kilogram	kg
electric current	ampere	A
thermodynamic temperature	kelvin	K
amount of substance	mole	mol
(luminous intensity	candela	cd)

Unit of time: 1967 $\rightarrow$ 2018

"The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium 133 atom"

Essentially unchanged; new wording:

[The second] is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{\mathrm{Cs}}$, the unperturbed ground-state hyperfine transition frequency of the caesium-133 atom, to be 9 192 631 770 when expressed in the unit Hz, which is equal to s^{-1}

Unit of length: the METRE

Since 1983, the metre is defined from the second by fixing the value of the speed of light in vacuum, c

"The metre is the length of the path travelled by light in vacuum during a time interval of 1/299 792 458 of a second."

now reformulated as follows:

"[The metre] is defined by taking the fixed numerical value of the speed of light in vacuum c to be 299 792 458 when expressed in the unit m s^{-1},

where the second is defined in terms of the caesium frequency $\Delta\nu_{\mathrm{Cs}}$

i.e. c fixed to $\boxed{c \;=\; \mathbf{299\,792\,458\ \ m/s}\,,}$

(as a result of definition of metre from second)

(it would have been nicer to fix the speed of light to $c = 1$, as suggested by relativity, but this would lead to unit of space $\approx$ 300 000 km, generally considered as ununpractical)

(see later, however)

Unit of mass: the KILOGRAM

Since 1901 $\rightarrow$ 20 May 2019 :

"The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram."

(yes indeed ...)

But quantity of matter in International Prototype of the Kilogram (IPK) does not remain constant

it varies very slighly over the years,

although its mass must remain constant by definition

The differences in mass between IPK and supposedly identical copies

average to a few tens of μg per century

Could we avoid using such a unique physical object (artefact) to define the unit of mass,

and obtain it from reproducible universal phenomena ?

(as for second and metre)

This is what the recent redefinition of the SI,

in effect from 20 May 2019, has allowed to achieve

Use QUANTUM PHYSICS

Angular momenta and actions are quantized in units of $\hbar = h/2\pi$, or h

with $h \simeq 6.626 \times 10^{-34}$ J s, or $\hbar \simeq 1.054... \times 10^{-34}$ J s

The new SI provides a quantum definition of the kilogram, by fixing the value of h to be, exactly,

$$\boldsymbol{h = 6.626\,070\,15 \times 10^{-34}\ \mathrm{J\,s}}$$

$\rightarrow$ **new quantum definition of kilogram:**

"[The kilogram] is defined by taking the fixed numerical value of the Planck constant h to be $6.626\,070\,15 \times 10^{-34}$ when expressed in the unit J s ,

which is equal to kg m^2 s^{-1}, where the metre and the second are defined in terms of c and $\Delta\nu_{\mathrm{Cs}}$"

Great definition; joule and kilogram now defined universally, from quantum physics

(We shall see later how this may be practically realized, making use of electrical phenomena, and units)

Still it would have been nicer, maybe, to choose $\hbar = 1$

... but would lead to unit of energy $\approx 10^{-34}$ J, unit of mass $\approx 10^{-51}$ kg

usually rejected as unpractical

(we shall come back to this later)

After s, m, and kg:

What about ELECTRICAL UNITS ?

were traditionally defined **from mechanical ones** through old definition of the ampere:

Since 1946 ($\rightarrow$ 2019)

"The ampere [A_{old}] is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to 2×10^{-7} newton per metre of length"

in agreement with Ampère's law $\frac{F}{L} = \frac{\mu_\circ}{2\pi} \frac{I^2}{r}$

where $\mu_\circ = 4\pi \times 10^{-7}$ $\mathrm{N/A^2_{(old)}}$

defines the coulomb as 1 C = 1 A s, etc.

Electrical units (were) defined from mechanical ones, and strongly tied to them

numerical value of $\mu_\circ$ defined as fixed quantity, conventionally taken as $4\pi \times 10^{-7}$ for historical reasons

in some sense the ampere appeared as $\propto \sqrt{\mathrm{N}}$ with a fixed proportionality coefficient

$$\boxed{1\ \mathrm{A_{old}} \propto \sqrt{\mathrm{N}}}$$

such electrical units are well defined

and constant in time, once the newton is defined from a really constant kilogram, as may be obtained by fixing h

The VACUUM IMPEDANCE and the OHM

Another characteristic of the vacuum is the so-called “vacuum impedance” $Z_\circ$

it has also, traditionally, a fixed value

From $\mu_\circ$ and $\epsilon_\circ$ one gets the speed of light in vacuum c, and vacuum impedance $Z_\circ$

measured in SI units, **m/s** and Ω, respectively

$$\begin{cases} c \;=\; \dfrac{1}{\sqrt{\mu_\circ \epsilon_\circ}} \;=\; 299\,792\,458 \text{ m/s} \\[2ex] Z_\circ \;=\; \sqrt{\dfrac{\mu_\circ}{\epsilon_\circ}} \;=\; \mu_\circ\, c \;=\; (4\pi \times 10^{-7}\,\text{N/A}^2_{\text{old}}) \times (299\,792\,458 \text{ m/s}) \;=\; 376,730\,313\,461\ldots\ \Omega_{\text{old}} \end{cases}$$

$\uparrow$

exactly known

The ohm Ω_{old}, as previously defined until 2019, is a fixed known fraction of the vacuum impedance $Z_\circ$

just as fixing c to a fixed number of m/s defines the metre

fixing $Z_\circ$ to a fixed number of ohms defines (or may define) the ohm

fixing $Z_\circ$ $\Longleftrightarrow$ fixing $\mu_\circ$ (or $\epsilon_\circ$)

and this may well be the natural thing to do

but ... can it still be done ?

... things have now changed

as new SI chooses to $\boxed{\textbf{FIX ALSO THE VALUE OF } e}$ in addition to c and h

Redefining the kilogram by fixing $h = 6.626\,070\,15 \times 10^{-34}$ J s

is done by fixing also value of elementary charge to

$$e = 1.602\,176\,634 \times 10^{-19} \text{ C}$$

so that the Josephson and von Klitzing constant (used in quantum electrical measurements)

get numerically exactly fixed to
$$\begin{cases} K_J = 2e/h = 483\,597.848\,416\,983\ldots \text{ GHz/V} \\ R_K = h/e^2 = 25\,812.807\,459\,304\ldots \ \Omega \end{cases}$$

Motivation: ***this allows for electrical units to** **return within the SI***

They were practically defined since 1990 by fixing conventional values for K_J and R_K :

$$\begin{cases} K_{J-90} = 483\,597.9 \text{ GHz/V}, \\ R_{K-90} = 25\,812.807 \ \Omega, \end{cases}$$
which amounted to fixing the value of e

But this was ***outside the SI***

in which the value of e, in coulombs, was, until recently, not fixed, but measured experimentally

(with 1 C = 1 A s, the ampere being defined through Ampère's law)

However: $\boxed{\textbf{Fixing the value of elementary charge } e\textbf{, instead of measuring it, has a price}}$

"fix h": $h = 6.626\,070\,150\,(69) \times 10^{-34}$ J s in 2017 $\rightarrow$ $h = 6.626\,070\,15 \times 10^{-34}$ J s

$\Rightarrow$ **abandon old definition of the kilogram from IPK**

("The kilogram is the unit of mass; it is equal to the mass of the international prototype of the kilogram.")

Very good ! We no longer need to rely on the IPK to measure masses, energies, ...

IPK can now be used to measure its mass (no longer 1 kg by definition) and how it varies within time

* * *

"fix e": $e = 1.602\,176\,634\,1\,(83) \times 10^{-19}$ C in 2017 $\rightarrow$ $e = 1.602\,176\,634 \times 10^{-19}$ C now

Maybe, not so good:

$\Rightarrow$ **abandon previous definitions of coulomb and ampere**

We have to **adjust the sizes of the coulomb and ampere** so that e be exactly $1.602\,176\,634 \times 10^{-19}$ C

$$\mathrm{C_{old}} \rightarrow \mathrm{C_{new}} = \mathrm{C}$$

$$\boxed{\eta = \frac{\mathrm{C_{new}}}{\mathrm{C_{old}}} \simeq 1 \quad (\text{up to } \lesssim \text{ a few } 10^{-10})}$$

This **adjustement depends on value of fine structure constant α**, as we shall see ...

The COULOMB and AMPERE have to be redefined

$$\boldsymbol{e} \;=\; \boldsymbol{e}_{\mathbf{old}}\, \mathbf{C}_{\mathbf{old}} \;=\; \mathbf{e}_{\circ}\, (\mathbf{C}_{\mathbf{new}} = \mathbf{C})$$
$$\uparrow$$
$$e_{\circ} = 1.602\,176\,634 \times 10^{-19}$$

the coulomb, and thus the ampere, have to be slightly adjusted

$$\boxed{\Rightarrow \begin{cases} \boldsymbol{e}_{\circ} \;=\; \boldsymbol{\eta}^{-1}\, \boldsymbol{e}_{\mathbf{old}} = \mathbf{1.602\,176\,634 \times 10^{-19}} \\ 1\,\mathrm{C} \;=\; \boldsymbol{\eta}\;\, \mathrm{C}_{\mathbf{old}} \\ 1\,\mathrm{A} \;=\; \boldsymbol{\eta}\;\, \mathrm{A}_{\mathbf{old}} \end{cases}}$$

changes in electrical units expressed by $\eta \;\approx\; 1$ (up to $\approx$ a few 10^{-10})

nearly invisible, but ***conceptually very important***

η has to be measured experimentally

new ampere A "larger" (or possibly smaller) than earlier one $\mathrm{A}_{\mathrm{old}}$ by factor η

force/m between conductors at 1 m : $F/L = 2 \times 10^{-7}$ N/m $\;\rightarrow\;$ $\boldsymbol{F/L = 2 \times 10^{-7}\; \eta^2}$ N/m

still given by Ampère's law $F/L = \dfrac{\mu_{\circ}}{2\pi}\, \dfrac{I^2}{r}$,

$$\boxed{\text{New expression of } \boldsymbol{\mu}_{\circ} : \;\; \boldsymbol{\mu}_{\circ} = \mathbf{4}\boldsymbol{\pi} \times \mathbf{10^{-7}}\, \boldsymbol{\eta}^{\mathbf{2}}\; \mathbf{N/A^2}}$$

But ... can we say that $\mu_{\circ}$ "has changed" ??

$\mu_\circ$ is a physical quantity, the vacuum magnetic permeability

$\mu_\circ$ has not changed, it stays the same.

but sizes of SI units have changed $\Rightarrow$ **expression of $\mu_\circ$ in new SI units is modified**:

$$\boxed{\text{same} \quad \mu_\circ \; = \; \underbrace{4\pi \times 10^{-7}}_{\text{conventionally fixed}} \;\; \text{N/A}^2_{\text{old}} \; = \; \underbrace{4\pi \times 10^{-7}\;\eta^2}_{\text{to be measured experimentally}} \;\; \text{N/A}^2}$$

(check consistency, as 1 A = η A_{old})

numerical value of $\mu_\circ$, in new SI units (N/A^2 or H/m), now has to be measured experimentally

(according to this reasoning, what we shall measure is the actual size of the new electrical SI units themselves !)

$\mu_\circ$ now appears (somewhat artificially) promoted to a new "fundamental constant of nature"

whose value should be measured

although nothing has changed in physics

This change for the expression of $\mu_\circ$ when the value of e is fixed is not surprising:

finite structure constant ***α, dimensionless, should stay unchanged***

$$\alpha \; = \; \frac{e^2}{4\pi\epsilon_\circ \hbar c} \; = \; \frac{\mu_\circ c\, e^2}{4\pi\hbar} \; \simeq \; \frac{1}{137.035\;999\;...}$$

$$\text{size of coulomb gets multiplied by } \eta \;\; \Rightarrow \left\{ \begin{array}{l} \text{num. value of } e \text{ multiplied by } \eta^{-1} \\ \text{num. value of } \mu_\circ \text{ multiplied by } \eta^2 \end{array} \right\}$$

$$\Rightarrow \;\; \alpha \; = \; \frac{\mu_\circ c\, e^2}{4\pi\hbar} \;\; \text{unchanged}$$

$$\alpha \;=\; \frac{e^2}{4\pi\epsilon_\circ \hbar c} \;=\; \frac{\mu_\circ c\, e^2}{4\pi\hbar} \;\simeq\; \frac{1}{137.035\,999\,...} \quad \text{unchanged}$$

$$\text{but now} \quad \alpha \;=\; \frac{(\mathbf{4\pi \times 10^{-7}\, \eta^2\, N/A^2}) \times (299\,792\,458\ \mathrm{m/s}) \times (1.602\,176\,634 \times 10^{-19}\ \mathrm{C})^2}{2 \times 6.626\,070\,15 \times 10^{-34}\ \mathrm{J\,s}}$$

$$\alpha = \eta^2 \times \underbrace{\left[\alpha_\circ = \frac{1}{137.035\,999\,158\,...}\right]}_{\text{fixed}} \propto\ \eta^2$$

$$\boxed{\boldsymbol{\eta \;=\; \frac{\mathrm{C}}{\mathrm{C_{old}}} \;=\; \sqrt{\frac{\alpha}{\alpha_\circ}} \;\propto\; \sqrt{\alpha}}}$$

Electrical units are now *DECOUPLED FROM MECHANICAL ONES*

Sizes of new electrical units ***(as compared to what old ones would have been)***

now fixed by η i.e. by experimental measurements of α

they may even, potentially, depend on time, should α depend on time ...

This raises the question of the ***possible dependence of electrical units with time***

(in connection with how they are defined)

This also provides, even within the new SI, ***two equivalent pictures***:

– usually one considers the new electrical units $\mathrm{C} = \mathrm{C_{new}}$, ... as fixed (official point of view)

– but if one considers instead $\mathrm{C_{old}}$, ... as fixed (using the old definition of the ampere)

then the new electrical units may be viewed as ***potentially depending on time*** return to **VACUUM IMPEDANCE**

previously exactly fixed to

$$Z_\circ = \mu_\circ c = (4\pi \times 10^{-7}\ \mathrm{N/A^2}) \times (299\,792\,458\ \mathrm{m/s}) = \underset{\substack{\uparrow \\ \text{exactly known}}}{376.730\,313\,461\ \ldots}\ \Omega_{\rm old}$$

$$\Rightarrow\ \boldsymbol{Z_\circ = \mu_\circ c = (4\pi \times 10^{-7}\, \eta^2\ \mathrm{N/A^2}) \times (299\,792\,458\ \mathrm{m/s})} = \underbrace{\mathbf{376.730\,313\,461\ \ldots\ \times \eta^2}}_{\text{to be measured experimentally}}\ \boldsymbol{\Omega}$$

It is the same $Z_\circ$, but size of ohm has changed

Ohm adjusts so that $\eta\,\Omega \equiv \Omega_{\rm old}$ $\Rightarrow$ $Z_\circ$ unchanged

What is the value of η ? **A measurement of α is a measurement of η^2**

$$\boldsymbol{\eta^2 = \frac{\alpha}{\alpha_\circ} = \frac{\mu_\circ}{4\pi \times 10^{-7}\ \mathrm{N/A^2}}} = \begin{cases} 1 + 2.0\ (2.3) \times 10^{-10} & \text{with 2017 adjustment (compatible with 1)} \\ \mathbf{1 + 5.4\,(1.5) \times 10^{-10}} & \text{with 2018 CODATA recommended value of } \alpha \end{cases}$$

shows (at $\approx 3.6\,\sigma$) small discontinuity in definition of new electrical units, as compared to older ones.

Effects of fixing value of e (rather than $\mu_\circ$) are already visible !

$$\begin{cases} \text{vacuum magnetic permeability } \mu_\circ = 4\pi \times 10^{-7} \, \dfrac{\alpha}{\alpha_\circ} \ \text{N/A}^2, \\ \text{vacuum impedance } Z_\circ = \mu_\circ c = 376.730\,313\,461\ldots \, \dfrac{\alpha}{\alpha_\circ} \ \Omega\,, \end{cases}$$

now depend on measurement of α ***(i.e. on what was the value of e before it got fixed)***

Price to pay for "fixing e" rather than $\mu_\circ$:

new-SI measures for vacuum properties (magnetic permeability, electric permittivity, impedance)

now depend on α i.e. on strength of interaction

i.e. on what was, previously, the value of the elementary charge

This is somewhat disturbing

(even if it does not imply any inconsistency in the new SI)

The new values of $Z_\circ$ and $\mu_\circ$ may even depend on time, should α depend on time

(as occasionally considered)

(This does not necessarily mean, however, that the physical quantities $\mu_\circ$ and $Z_\circ$ depend on time,
as the new units themselves may then be viewed as potentially depending on time)

Now, a new proposal: $\boxed{\textbf{CAN WE ALSO FIX } \boldsymbol{c = \hbar = 1}\ \textbf{?}}$ (to begin with)

This does not seem to make much sense, at first sight

Still, relativity allowed for defining metre from second, by fixing $c = 299\ 792\ 458\ \mathrm{m/s}$

If we impose, *"en même temps"*, $c = 1$, we can consider the second as being also a unit of length

and **identify the metre as a fixed fraction of the second**

$$\boldsymbol{c = c_\circ\ \mathrm{m/s} = 299\ 792\ 458\ \mathrm{m/s} = 1 \quad \Longleftrightarrow \quad 1\,\mathrm{m}\ \equiv\ (1/c_\circ)\ \mathrm{s} = (1/299\ 792\ 458)\ \mathrm{s}}$$

Similarly, kg defined by fixing $h = 6.626\ 070\ 15 \times 10^{-34}\ \mathrm{J\,s}$.

If we impose, *"en même temps"*, $\hbar = h/2\pi = 1$, we can consider the second^{-1} as being, also, a unit of energy,

and **identify the joule as a fixed number of s^{-1}**

$$\boxed{\left\{\begin{array}{lllll} \boldsymbol{c} & = \boldsymbol{c_\circ\ \mathrm{m/s}} & = & \boldsymbol{299\ 792\ 458\ \mathrm{m/s}} & = \boldsymbol{1} \\ \boldsymbol{\hbar} & = \boldsymbol{\hbar_\circ\ \mathrm{J\,s}} & = & \boldsymbol{(6.626\ 070\ 15 \times 10^{-34}/2\pi)\ \mathrm{J\,s}} & = \boldsymbol{1} \end{array}\right.}$$

$$\left\{\begin{array}{llll} \boldsymbol{1\,\mathrm{m}} & \boldsymbol{\equiv\ (1/c_\circ)\ \mathrm{s}} & = & \boldsymbol{(1/299\ 792\ 458)\ \mathrm{s}} \\ \boldsymbol{1\,\mathrm{J}} & \boldsymbol{\equiv\ (1/\hbar_\circ)\ s^{-1}} & = & \boldsymbol{0.948\ 252\ 156\ 246 \ldots \times 10^{34}\ \mathrm{s}^{-1}} \end{array}\right.$$

The newton and the kilogram can then be identified as

$$\left\{\begin{array}{lllll} \boldsymbol{1\,\mathrm{N}} & = \boldsymbol{1\,\mathrm{J/m}} & = \boldsymbol{c_\circ/\hbar_\circ\ \mathrm{s}^{-2}} & = & \boldsymbol{2.842\ 788\ 447\ 250 \ldots \times 10^{42}\ \mathrm{s}^{-2}} \\ \boldsymbol{1\,\mathrm{kg}} & = \boldsymbol{1\,\mathrm{J\,s^2/m^2}} & = \boldsymbol{c_\circ^2/\hbar_\circ\ \mathrm{s}^{-2}} & = & \boldsymbol{0.852\ 246\ 536\ 175 \ldots \times 10^{51}\ \mathrm{s}^{-1}} \end{array}\right. \tag{1}$$

ELECTRICAL UNITS

Impose also $\underline{\mu_\circ = 1}$, with $\mu_\circ = \underbrace{4\pi \times 10^{-7}\eta^2}_{\mu_{\circ\circ}}$ N/A^2

Just as $c = c_\circ$ m/s = 1 and $\hbar = \hbar_\circ$ J s $= 1$ fix the metre and joule

$$\mu_\circ = \underbrace{4\pi \times 10^{-7}\eta^2}_{\mu_{\circ\circ}} \ \mathrm{N/A^2} = 1 \quad \textbf{fixes the ampere}$$

$$\boxed{1\,\mathrm{A} = \sqrt{\mu_{\circ\circ}\,\mathrm{N}} = \eta\ \underbrace{\sqrt{4\pi \times 10^{-7}\ \mathrm{N}}}_{A_{\mathrm{old}}}}$$

The ampere is proportional to a square root of the Newton

$$\Rightarrow \left\{ \begin{array}{lllll} 1\,\mathrm{N} = 1\,\mathrm{J/m} & = \dfrac{c_\circ}{\hbar_\circ}\,\mathrm{s}^{-2} & = 2.842\ldots \times 10^{42}\ \mathrm{s}^{-2} & & \\ 1\,\mathrm{A} = \sqrt{\mu_{\circ\circ}\,\mathrm{N}} & = \sqrt{\dfrac{\mu_{\circ\circ}c_\circ}{\hbar_\circ}}\,\mathrm{s}^{-1} & = \eta \times 1.890\ldots \times 10^{18}\ \mathrm{s}^{-1} & = \eta\ \mathrm{A_{old}} \\ 1\,\mathrm{C} = 1\,\mathrm{A\,s} & = \sqrt{\dfrac{\mu_{\circ\circ}c_\circ}{\hbar_\circ}} & = \eta \times 1.890\ldots \times 10^{18} & = \eta\ \mathrm{C_{old}} \end{array} \right.$$

Coulomb is dimensionless

$$\boxed{1\,\mathrm{C} = \eta \times 1.890\,067\,015\ldots \times 10^{18} = \eta\ \mathrm{C_{old}}}$$

express all other units, V, Wb, T, F, H, Ω in similar way (table later)

$$1\ \mathrm{V} = 1\ \mathrm{J/C} = \frac{1}{\sqrt{\mu_{\circ\circ}c_\circ\,\hbar_\circ}}\ \mathrm{s}^{-1} = \eta^{-1} \times 5.017\,029\,284\,119\ldots \times 10^{15}\ \mathrm{s}^{-1}, \quad \text{etc.}$$

Ohm and weber are dimensionless

$$\boxed{1\,\Omega = 1\,\mathrm{V/A} = \frac{1}{\mu_{\circ\circ}c_\circ} = \eta^{-2} \times (1/376\,730\,313\,461\ldots)}$$

ELEMENTARY CHARGE and VACUUM IMPEDANCE

ELEMENTARY CHARGE is dimensionless:

$$e = e_\circ\,\mathrm{C} = (1.602\,176\,634 \times 10^{-19}) \times (1.890\,067\,015\ldots \times 10^{18} \times \eta) = .302\,822\,120\,871\ldots \times \eta$$

What is this number ?

$$e = e_\circ\,\mathrm{C} = e_\circ\sqrt{\frac{\mu_{\circ\circ}\,c_\circ}{\hbar_\circ}} = \sqrt{\frac{\mu_{\circ\circ}c_\circ\,e_\circ^2}{\hbar_\circ}} = \sqrt{\frac{\mu_\circ c\,e^2}{\hbar}} = \sqrt{\frac{e^2}{\epsilon_\circ\hbar c}} = \sqrt{4\pi\alpha} = .3028\ldots \times \eta$$

When $\boldsymbol{\hbar = c = \mu_\circ = \epsilon_\circ = 1}$,

$$\boxed{e = \mathbf{1.602\,176\,634 \times 10^{-19}\ C} = \sqrt{4\pi\alpha} = \eta\,\sqrt{4\pi\alpha_\circ} = \mathbf{.302\,822\,120\,871\ldots} \times \eta}$$

(the latter expression of e depends on η i.e. on α, and ought to be measured experimentally)

VACUUM IMPEDANCE

$$\left.\begin{aligned} Z_\circ = \mu_\circ c &= \mu_{\circ\circ}\,c_\circ\,\Omega = 376.730\ldots\,\eta^2\,\Omega \\ \Omega &= \frac{1}{\mu_{\circ\circ}c_\circ} = \eta^{-2} \times (1/376\,730\ldots) \end{aligned}\right\} \Rightarrow \boxed{Z_\circ = 376.730\ldots\,\eta^2\,\Omega \equiv 1}$$

$Z_\circ = 1$ is the natural unit of impedance, as it should with $\boldsymbol{\mu_\circ = \epsilon_\circ = c = 1}$
still remaining **close to 376.730 Ω as in the SI**

(just as $c = 1/\sqrt{\mu_\circ\epsilon_\circ} = 1 = c_\circ$ m/s is natural unit of speed)

[Still many things would have been simpler here, at least conceptually,
if the SI had chosen to keep the value of $\mu_\circ$ fixed, instead of fixing e]

$1\ \mathrm{m}$	$=$	$\dfrac{1}{c_\circ}\ \mathrm{s}$	$=$	$\dfrac{1}{299\ 792\ 458}\ \mathrm{s}$
$1\ \mathrm{J}$	$=$	$\dfrac{1}{\hbar_\circ}\ \mathrm{s}^{-1}$	$=$	$.948\ 252\ 156\ 246\ldots \times 10^{34}\ \ \mathrm{s}^{-1}$
$1\ \mathrm{kg}$	$=$	$\dfrac{c_\circ^2}{\hbar_\circ}\ \mathrm{s}^{-1}$	$=$	$.852\ 246\ 536\ 175\ldots \times 10^{51}\ \ \mathrm{s}^{-1}$
$1\ \mathrm{N}$	$=$	$\dfrac{c_\circ}{\hbar_\circ}\ \mathrm{s}^{-2}$	$=$	$2.842\ 788\ 447\ 250\ldots \times 10^{42}\ \ \mathrm{s}^{-2}$
$1\ \mathrm{A} = \sqrt{\mu_{\circ\circ}\,\mathrm{N}}$	$=$	$\sqrt{\dfrac{\mu_{\circ\circ}c_\circ}{\hbar_\circ}}\ \mathrm{s}^{-1}$	$=$	$1.890\ 067\ 014\ 853\ldots \times 10^{18}\ \mathrm{s}^{-1}$
$1\ \mathrm{C} = \sqrt{\mu_{\circ\circ}\,\mathrm{kg\,m}}$	$=$	$\sqrt{\dfrac{\mu_{\circ\circ}c_\circ}{\hbar_\circ}}$	$=$	$1.890\ 067\ 014\ 853\ldots \times 10^{18}$
$1\ \mathrm{V} = 1\ \mathrm{J/C}$	$=$	$\dfrac{1}{\sqrt{\mu_{\circ\circ}c_\circ\hbar_\circ}}\ \mathrm{s}^{-1}$	$=$	$5.017\ 029\ 284\ 119\ldots \times 10^{15}\ \mathrm{s}^{-1}$
$1\ \mathrm{V/m} = 1\ \mathrm{N/C} = \dfrac{1}{\sqrt{\mu_{\circ\circ}}}\sqrt{\mathrm{J/m^3}}\ \mathrm{m/s}$	$=$	$\sqrt{\dfrac{c_\circ}{\mu_{\circ\circ}\hbar_\circ}}\ \mathrm{s}^{-2}$	$=$	$1.504\ 067\ 540\ 944\ldots \times 10^{24}\ \mathrm{s}^{-2}$
$1\ \mathrm{T} = 1\ \mathrm{N/(A\,m)} = \dfrac{1}{\sqrt{\mu_{\circ\circ}}}\sqrt{\mathrm{J/m^3}}$	$=$	$\sqrt{\dfrac{c_\circ^3}{\mu_{\circ\circ}\hbar_\circ}}\ \mathrm{s}^{-2}$	$=$	$4.509\ 081\ 050\ 976\ldots \times 10^{32}\ \mathrm{s}^{-2}$
for $\eta = 1$ $1\ \mathrm{Wb} = 1\ \mathrm{V\,s}$	$=$	$\dfrac{1}{\sqrt{\mu_{\circ\circ}c_\circ\hbar_\circ}}$	$=$	$5.017\ 029\ 284\ 119\ldots \times 10^{15}$
$1\ \mathrm{F} = 1\ \mathrm{C/V} = 1\ \mathrm{s}/\Omega = \mu_{\circ\circ}\,\mathrm{s^2/m}$	$=$	$\mu_{\circ\circ}c_\circ\ \mathrm{s}$	$=$	$376.730\ 313\ 461\ldots\ \mathrm{s}$
$1\ \mathrm{H} = 1\ \mathrm{J/A^2} = 1\ \Omega\,\mathrm{s} = \dfrac{1}{\mu_{\circ\circ}}\ \mathrm{m}$	$=$	$\dfrac{1}{\mu_{\circ\circ}c_\circ}\ \mathrm{s}$	$=$	$1/376.730\ 313\ 461\ldots\ \mathrm{s}$
$1\ \Omega = 1\ \mathrm{V/A} = 1\ \mathrm{W/A^2} = \dfrac{1}{\mu_{\circ\circ}}\ \mathrm{m/s}$	$=$	$\dfrac{1}{\mu_{\circ\circ}c_\circ}$	$=$	$1/376.\,730\ 313\ 461\ldots$
$Z_\circ = \mu_{\circ\circ}c_\circ\ \Omega = 376.730\ 313\ 461\ldots\ \Omega$	$=$	$\mu_\circ\,c$	$=$	1
$e = 1.602\ 176\ 634 \times 10^{-19}\ \mathrm{C}$	$=$	$\sqrt{4\pi\alpha}$	$=$	$.302\ 822\ 120\ 789\ldots$
$1\ \mathrm{eV} = 1.602\ 176\ 634 \times 10^{-19}\ \mathrm{J}$	$=$	$\dfrac{e_\circ}{\hbar_\circ}\ \mathrm{s}^{-1}$	$=$	$1.519\ 267\ 447\ 878\ldots \times 10^{15}\ \mathrm{s}^{-1}$
$\Phi_\circ = h/2e = 2.067\ 833\ 848\ 461\ldots \times 10^{-15}\ \mathrm{Wb}$	$=$	$\sqrt{\dfrac{\pi}{4\alpha}} = \dfrac{\pi}{e}$	$=$	$10.374\ 382\ 972\ldots$
$K_J = 2e/h = 483\ 597.\,848\ 416\ 983\ldots\ \mathrm{GHz/V}$	$=$	$\sqrt{\dfrac{4\alpha}{\pi}} = \dfrac{e}{\pi}$	$=$	$.0963\ 912\ 748\ 023\ldots$
$R_K = \dfrac{h}{e^2} = \dfrac{\mu_{\circ\circ}c_\circ\,\Omega}{2\alpha} = 25\ 812.\,807\ 459\ 304\ldots\ \Omega$	$=$	$\dfrac{\mu_\circ c}{2\alpha} = \dfrac{1}{2\alpha}$	$=$	$68.517\ 999\ 579\ldots$

CONCLUSIONS

New quantum definition of kilogram a **huge progress**

By "fixing e" rather than $\mu_\circ$, new SI requires to ***adjust sizes of electrical units, no longer tied to mechanical ones***

redefined in terms of $\eta = \dfrac{\mathrm{C}}{\mathrm{C_{old}}} \propto \sqrt{\alpha}$

$\Rightarrow$ values of vacuum magnetic permeability and impedance

$\mu_\circ = 4\pi\, 10^{-7}\, \eta^2\, \mathrm{N/A^2}$, $Z_\circ = \mu_\circ c = 376.730 \ldots \eta\, \Omega$ ***now depend on*** α, to be measured experimentally

* * *

One can also fix, ***at the same time***, $\hbar = c = \mu_\circ = \epsilon_\circ = k_B = N_A = 1$

$\mathbf{m} \propto \mathbf{s}$, $\mathbf{J} \propto \mathbf{s}^{-1}$, $\mathbf{kg} \propto \mathbf{s}^{-1}$, $\mathrm{N} \propto \mathrm{s}^{-2}$, ... , $\mathbf{A} \propto \sqrt{\mathbf{N}} \propto \mathbf{s}^{-1}$, Ω, C and Wb dimensionless

ohm adjusts so that $Z_\circ = 376.730 \ldots \eta\, \Omega \equiv 1$, natural unit of impedance

($Z_\circ = \sqrt{\mu_\circ \epsilon_\circ} = 1$, just as $c = 1/\sqrt{\mu_\circ \epsilon_\circ} = 1$ natural unit of speed)

$$e = 1.602 \ldots \times 10^{-19}\ \mathrm{C} = \sqrt{4\pi\alpha} = 0.3028 \ldots \times \eta$$

mole equal to fixed Avogadro number

Allies practicality of SI units with elegance of symmetric system with $\hbar = c = \ldots = 1$.

* * *

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Making the new International System of units compatible
with $\boldsymbol{c = \hbar = \mu_\circ = \epsilon_\circ = 1}$

Pierre FAYET

The new SI brought a huge progress by defining the kilogram from quantum physics. But, by fixing e, it requires to adjust the sizes of the coulomb and ampere, changing $\mu_\circ$ and the vacuum impedance $Z_\circ = \mu_\circ c$ into unfixed quantities, to be measured experimentally. Electrical units, now decoupled from mechanical ones, get modified in a way which depends on α.

We show how the SI may be embedded in a new framework in which the "fundamental constants of nature" are fixed and equal to 1, $c = \hbar = \mu_\circ = \epsilon_\circ = k_B = N_A = 1$. All SI units can be defined in terms of the second, with the coulomb, ohm and weber dimensionless, and the mole identified as the Avogadro number.

The m, J and kg are identified as fixed numbers of s or s^{-1}. The elementary charge, fixed to $e = 1.602\,176\,634$ $\times 10^{-19}$ C, is also equal to $\sqrt{4\pi\alpha} \simeq .3028$. The ohm, now dimensionless and $\simeq 1/376.730$, is such that the vacuum impedance, still $Z_\circ \simeq\ 376.730\ \Omega$, is the unit of impedance, identical to 1.

This reconciles the practicality of SI units with the elegance of having $c = \hbar = \mu_\circ = \epsilon_\circ = 1$, as suggested by relativity and quantum physics.