What is the price of abandoning dark matter ?

(Constraints on alternative gravity theories)

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arXiv:2007.00555

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September, 2020

Alternatives to Dark Matter (e.g. modified gravity) must:

- explain the galaxy rotation curves, distribution of gas in elliptical galaxies and clusters

- match gravitational lensing cosmic shear measurements

- satisfy classical tests of GR (Solar system tests: precession of Mercury, Shapiro time delay, pulsar period decay, ...)

- fit LIGO's gravitational wave signals, which already rule out many models (for DE)

- fit the background expansion (distance-redshift relationship)

- fit the CMB and large-scale structure measurements

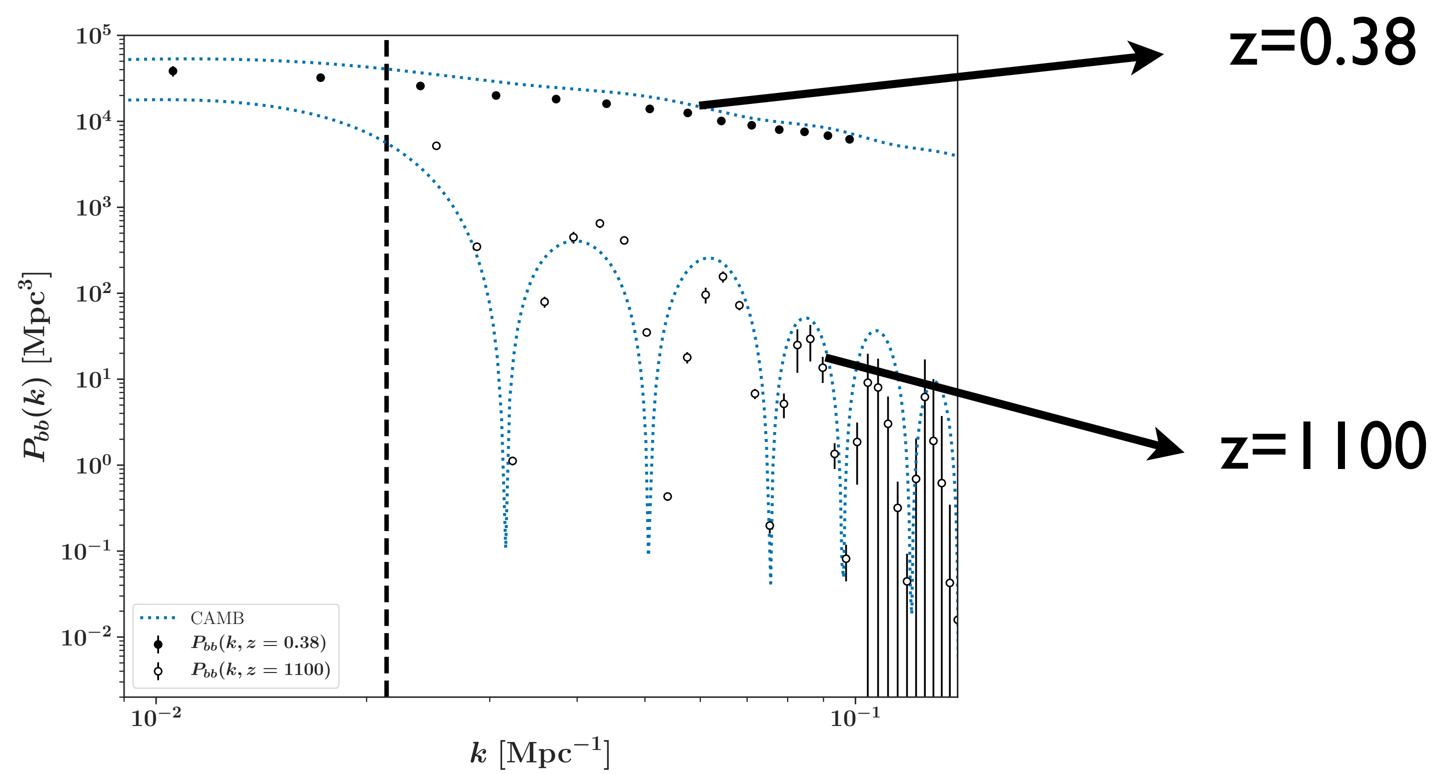
(Some of these features may have different explanations, but this would not be economical. This does not matter for this paper, anyway.)

The point made by this paper:

- at recombination the baryon density power spectrum shows strong BAO oscillations

- at z~0 the baryon density power spectrum shows weak (subdominant) BAO oscillations

It is difficult to imagine how to explain this damping of BAO with time, without DM !



baryon density power spectrum

How this is explained in LCDM:

- the initial fluctuations are adiabatic: overdense regions have an excess of baryons, dark matter, and photons

- baryons & photons are tightly coupled and oscillate like sound waves $c_s \sim c$

- DM has no self/cross interactions. It has no pressure and only feels gravity.
  DM does not show acoustic oscillations,
  its gravitational potential fluctuations keep growing (at a logarithmic rate)

- after recombination, baryons decouple from photons and fall into the DM gravitational potential wells

- baryons catch up with DM, and matter fluctuations now grow as $\delta \propto a(t)$ with small (subdominant) BAO, because baryons are subdominant with respect to DM.

→ The "damping" of BAO is explained by an external component (DM), which has no BAO, is decoupled at z>1100 and dominant at low z.

Problem in baryons-only scenarios:

The damping of the BAO in the baryon density power spectrum must come from the self-dynamics of the baryons themselves, because there is no other component.

Somehow, these new dynamics (modified gravity) must include/generate the BAO scale, to smooth the BAO features.

→ This does not seem natural !

Let’s follow the paper:

The aim:

redshift ($z \sim 0$). Any successful theory for dark matter, whether it invokes particles or alternative theories of gravity, must properly explain how the baryon density field at $z \sim 1100$ evolves into the one at $z \sim 0$. These density fields are typically probed indirectly through fitting the CMB power spectra and the matter power spectrum in tandem [e.g., 9, 10]. This necessarily assumes ΛCDM (or some simple extension), as well as GR. The test we propose here does not invoke GR nor a specific cosmology. Instead it relies solely on small-scale physics – Thomson scattering and the Newtonian continuity equation. Note that while similar tests have been proposed

However, assumes a background expansion close to LCDM. This is assumed to be implied by background data.

small-scale physics on BAO scales, linear regime

Obtaining baryon spectra from observations:

(instead of using a-priori models, with CAMB and LSS software, whose predictions are next fitted to the data)

z~1100:

The polarization of the CMB on small scales is exclusively due to Thomson scattering, which itself only relies on the velocities of the electrons. Because protons and electrons are tightly coupled via Coulomb scattering at early times, we can assume that the velocities of the electrons exactly equals that of the protons. The CMB polarization spectrum then directly measures the velocity of the baryons at $z \sim 1100$. The Newtonian continuity equation, which is valid at small scales, relates the velocities of the baryons to their density field. Thus, the CMB polarization spectrum is a direct measurement of the baryon velocity field at $z \sim 1100$. At $z \sim 0$,

CMB polarization

↓

electron velocities

↓

baryon power spectrum

Why CMB polarization and not CMB temperature anisotropies ?

CMB temperature: mostly depends on the energy density and the gravitational potential

→ not appropriate if we modify gravity !

CMB polarization is a (more) robust measurement of the baryon density power spectrum !

z~0:

of the baryon velocity field at $z \sim 1100$. At $z \sim 0$,
the galaxy-galaxy correlation function traces the baryon density field at large scales. With these two direct mea-

galaxy power spectrum

↓

baryon power spectrum

Green's function (or transfer function):

$$a(k,t) = \hat{F}_1(k)\delta_b(k,t) + \sum_{k'} \hat{F}_2(k,k')\delta_b(k,t)\delta_b(k',t) + \ldots$$

acceleration as a function of density

Since the density field is small, the linear term should dominate the gravitational acceleration in most modified gravity theories. Thus, we focus on linear modifications to GR in this paper. Note that this linear term acts like a transfer function – it has no explicit time dependence and is simply multiplied with a given density configuration in k-space to give the resulting acceleration force.

why no time dependence ?
(extra fields could play the role of a clock ?)
but does not matter for the main argument

If the modified gravity theory has strong nonlinear terms, then the theory will produce significant mode-mode couplings that would be apparent in the large-scale structure. The theory could evade the current strong constraints from Planck on non-Gaussianity [26] if the theory is linear at early times. However, if the theory is nonlinear enough at late times to erase the baryon acoustic oscillations, then these same nonlinearities would induce large non-Gaussian features in the large-scale distribution of structure. These are not seen in the large-scale distribution of structure [27] and they will be further constrained by upcoming missions, such as SphereX[1] [28]. Thus, it is unlikely that a strongly nonlinear theory could produce the correct evolution for the baryons and evade low-redshift non-Gaussianity constraints. Detailed calculations showing this point are left to future work.

- density fluctuations are small

- nonlinear terms would produce significant non-Gaussianities

→ assume linear evolution

Green's function

linear evolution implies (?): $$\delta_b(k, z=0) = T_b(k, z=0)\delta_b(k, z=1100)$$

obtained from the observed power spectra: $$\hat{T}_b^2(k) = \frac{P_{bb}(k, z\sim 0)}{P_{bb}(k, z=1100)} \; .$$

<u>This is the key quantity used in the paper.</u>

This is based on:

- linear evolution

- there is only one relevant component: baryons
  modified-gravity models usually involve other components:
  * OK with this framework if these are slaved to the baryons (e.g., quasi-static approximation) and can be integrated over (no independent degree of freedom).
  * Presumably, the authors consider that truly additional degrees of freedom are equivalent to introducing a disguised DM ?

- the Green's function is multiplicative (Fourier modes are decoupled)

$$\delta_b(k, z=0) \neq \int dk' \; T_b(k, k'; z=0)\delta_b(k', z=1100)$$

Remark: if we had: $\delta_b(k, z=0) = \int dk'\, T_b(k, k'; z=0)\delta_b(k', z=1100)$

Such a convolution could smooth the power spectrum and explain the damping of the BAO

However, this is not allowed by homogeneity and isotropy (?):

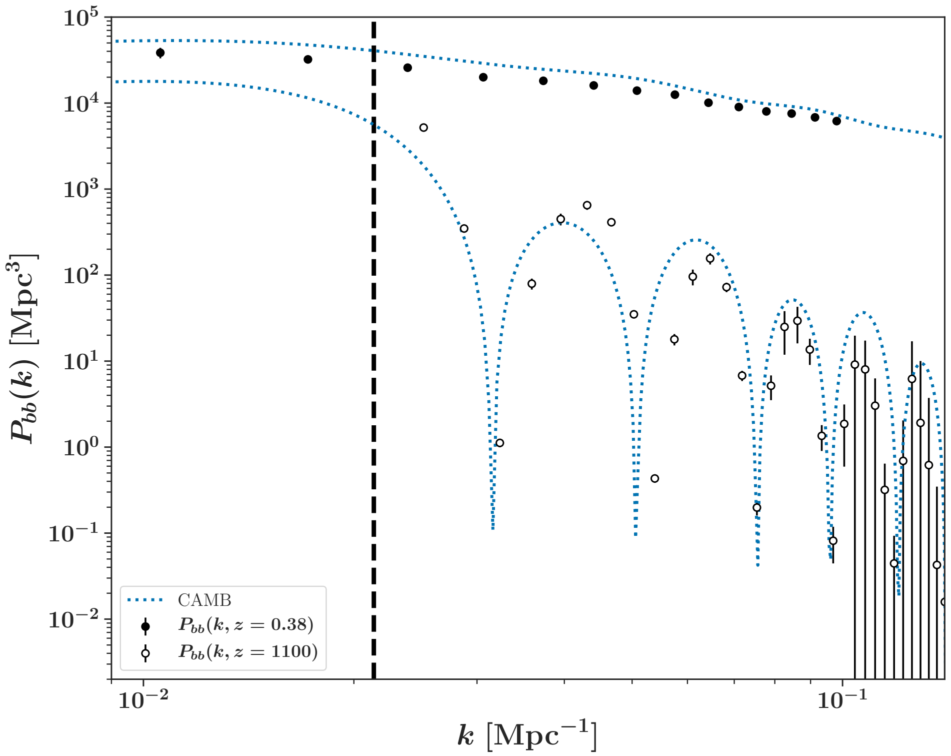
$$\text{convolution} \xleftrightarrow{\text{Fourier transf.}} \text{multiplication}$$

$$\delta_b(x, z=0) = T_b(x)\delta_b(x, z=1100)$$

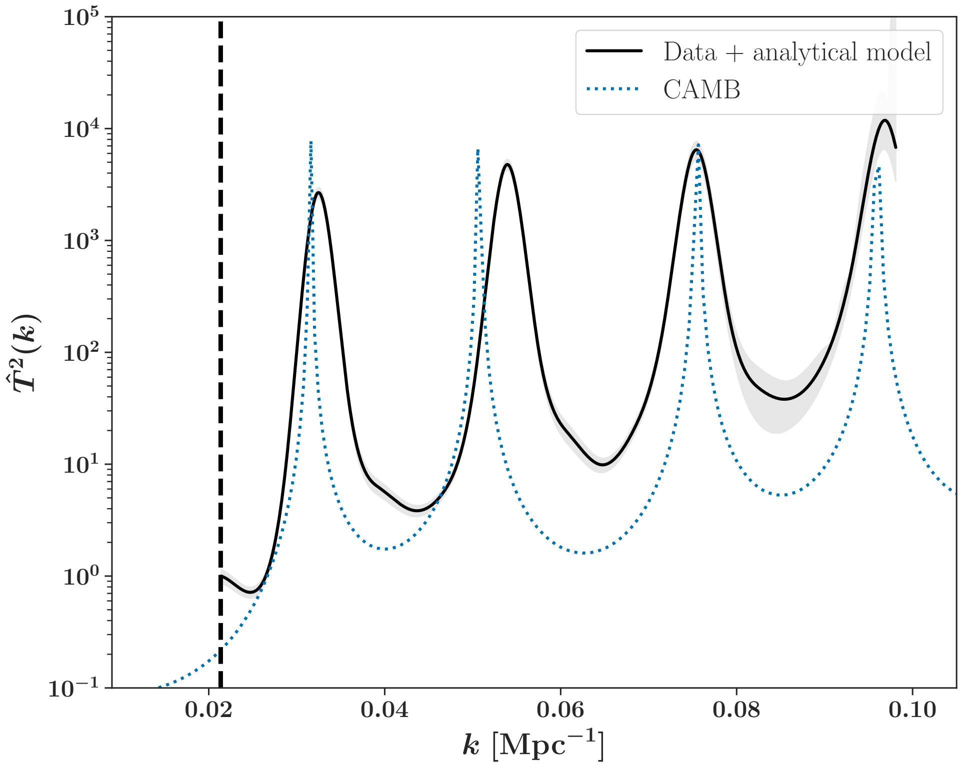
However, because of statistical homogeneity $T_b(x)$ cannot depend on x, more precisely, it should be invariant under $x \to x + a$

$$T_b = \frac{\partial}{\partial x} + \frac{\partial^2}{\partial x^2} + \dots \quad \Longrightarrow \quad T_b = k + k^2 + \dots$$

observed power spectra



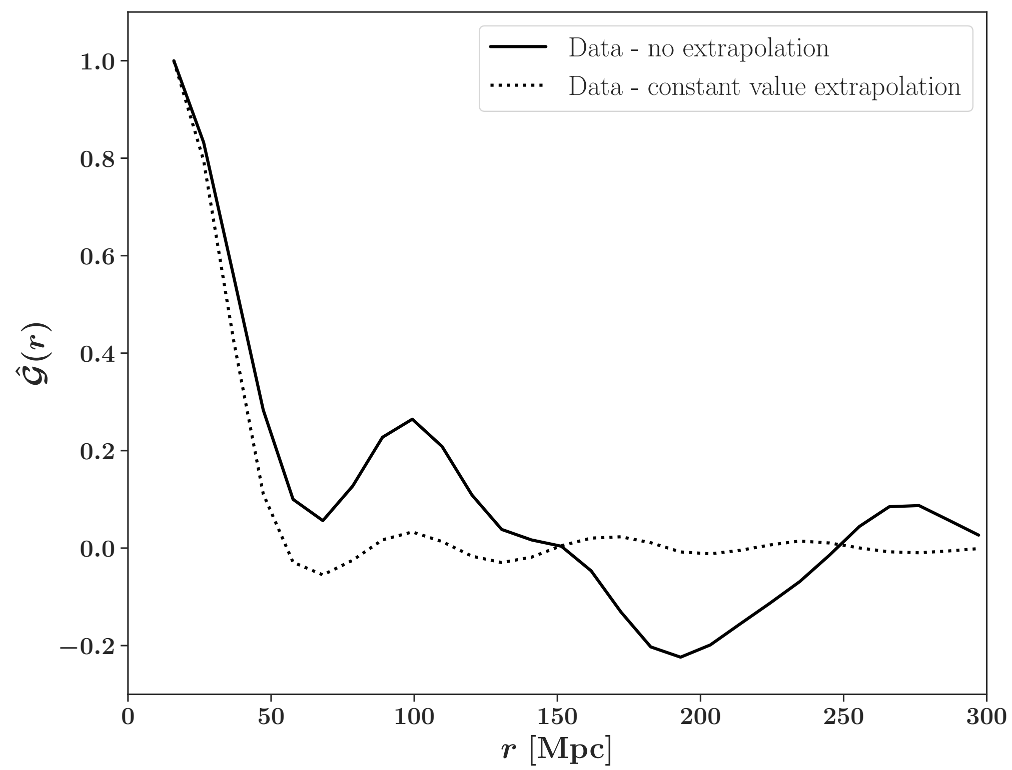
observed transfer function



The transfer function shows a series of "inverted" BAO features.
They are needed to compensate for the initial BAO oscillations, in order to produce a flat power spectrum at low redshift.

Writing the transfer function in real space:

$$\hat{\mathcal{G}}_b(r) = G_0 \int dk \, \frac{k^2}{2\pi^2} \hat{T}_b(k) j_0(kr) \, ,$$



assumptions: 1) $T^2(k > k_{\rm max}) = 0$ (solid, black line); 2) $T^2(k > k_{\rm max}) = T^2(k_{\rm max})$ (dotted, black line). These

Regardless of the assumptions at high-k, the Green's function changes sign near the BAO scale. The Green's

??

Alternative gravity theories (that replace DM) must:

1) contain the BAO scale, to be able to suppress the BAO features

2) have an acceleration law that changes sign around this scale (??)

Conclusion:

MOND, emergent gravity: 1/r force from point sources.
This would give power-law Green's functions.
They do not predict a special scale close to BAO scale nor oscillations.

Perhaps it is possible to use the Green's function of the form found above to find a modified gravity theory that can fit cosmological constraints and all other GR tests. However, given the extreme form of the function, it is not clear that this is possible – in particular, the sign changes would induce quite extreme dynamics within the local volume. CDM remains the simplest explanation for the growth of structure.

Baryon power spectrum at z~0

The baryons at low redshift and large scales ($\gtrsim$ 10 Mpc) are well-traced by galaxies. Thus, we can take the 3D power spectrum of galaxies as the baryon power spectrum. This is given by:

$$\boxed{P_{bb}(k, z \sim 0) = b_{bg}^2 P_{gg}(k, z \sim 0) \ ,} \qquad (5)$$

where b_{bg} is the bias of baryons relative to galaxies and P_{gg} is the 3D galaxy-galaxy power spectrum.

In reality, the galaxies are a biased tracer of the baryons. Most of the baryonic mass in the universe is in gas [29]. However, we expect that for $k < 0.1$ Mpc^{-1} the bias, b_{bg}, approaches some constant value. This is seen in numerical simulations [e.g., 30] and violating this would require moving baryons large distances. Thus, the galaxy-galaxy power spectrum should be a good measure of the shape of the baryonic power spectrum at these large scales.

galaxy power spectrum

$\Downarrow$

baryon power spectrum

approximation: constant bias

Baryon power spectrum at z~1100

Zaldarriaga & Harari (1995)

CMB polarization → electron velocities → baryon density power spectrum

Expand photons temperature and polarization fluctuations over multipoles:

$$\Delta(\hat{n},\vec{k}) = \sum_l (2l+1)\Delta_l P_l(\mu)$$

n: direction of the photon propagation (direction to the sky)

k: wave number of the temperature field

Evolution eqs.:

$$\dot{\Delta}_T + ik\mu(\Delta_T + \Psi) = -\dot{\Phi} - \dot{\kappa}\{\Delta_T - \Delta_{T0} - \mu V_b - \tfrac{1}{2}P_2(\mu)[\Delta_{T2} + \Delta_{P2} - \Delta_{P0}]\} \ ,$$

$$\dot{\Delta}_P + ik\mu\Delta_P = -\dot{\kappa}\{\Delta_P + \tfrac{1}{2}[1 - P_2(\mu)][\Delta_{T2} + \Delta_{P2} - \Delta_{P0}]\} \ ,$$

Eq. of motion of the baryons:

$$\dot{V}_b = -\frac{\dot{a}}{a}V_b - ik\Psi + \frac{\dot{\kappa}}{R}(3\Delta_{T1} - V_b)$$

$$\dot{\kappa} = x_e n_e \sigma_T a/a_0$$ differential optical depth

the photon dipole pushes the electrons (Thomson scattering), hence the baryons (electrons and protons tightly coupled by Coulomb scattering)

Integral form: $$(\Delta_T + \Psi) = \int_0^{\tau_0} d\tau e^{ik\mu(\tau-\tau_0)} e^{-\kappa(\tau_0,\tau)} \times \{\dot\kappa(\Delta_{T0} + \Psi + \mu V_b + \tfrac{1}{2}P_2(\mu)[\Delta_{T2} + \Delta_{P2} - \Delta_{P0}]) - \dot\Phi + \dot\Psi\} ,$$
$$\Delta_P = -\int_0^{\tau_0} d\tau e^{ik\mu(\tau-\tau_0)} \dot\kappa e^{-\kappa(\tau_0,\tau)} \tfrac{1}{2}[1 - P_2(\mu)][\Delta_{T2} + \Delta_{P2} - \Delta_{P0}] ,$$

$\kappa(\tau_0,\tau) = \int_\tau^{\tau_0} x_e n_e \sigma_T \frac{a(\tau)}{a(\tau_0)} d\tau$ is the optical depth to photons emitted at conformal time τ. The combination $\dot\kappa e^{-\kappa}$ is called the conformal time visibility function. It is the probability that photons last scattered within $d\tau$ of τ. For standard recombination this function has a sharp peak at the conformal time of decoupling τ_D [34]. Thus, the integral for Δ_P in Eq. (2.4) is dominated by the value of the integrand around decoupling. In other words, for standard recombination histories, with no reionization, the polarization of the CMB we observe today was produced just before decoupling.

Compute the polarization source term within the tight-coupling approximation: $\dot\kappa \to \infty$

- order 0:

$$\boxed{\Delta_{T1} = \tfrac{1}{3}V_b,}\ \Delta_{Tl} = 0 \quad \text{if} \quad l \geq 2 ,$$
$$\Delta_P = 0 \quad .$$

The interpretation of these formulas is very simple. In the lowest order approximation the photons and baryons are so strongly coupled that the photon distribution is isotropic in the baryon's rest frame. The photon distribution being isotropic, Thomson scattering does not polarize the CMB.

- order 1: to first order in $\tau_C \equiv \dot\kappa^{-1}$,

$$\boxed{\Delta_{P2} = -\tfrac{1}{5}\Delta_{P0} = \tfrac{1}{4}\Delta_{T2}}\quad \boxed{\Delta_{T2} = -\tfrac{8}{15}ik\tau_C\Delta_{T1}}$$
$$\Delta_{T1} = \frac{i}{k}(\dot\Delta_{T0} + \dot\Phi) ,$$
$$\Delta_{Tl} = \Delta_{Pl} = 0 \quad \text{if} \quad l \geq 3 , \qquad \Delta_{P1} = 0 .$$

These equations also have a simple interpretation. The polarization of the CMB is proportional to the quadrupole of the photon distribution function (a dipole does not induce polarization). The quadrupole in the temperature fluctuation, in its turn, is produced by the "free streaming" of the dipole between collisions. We see this from the relation $\Delta_{T2} \propto k\tau_C\Delta_{T1}$. The tight

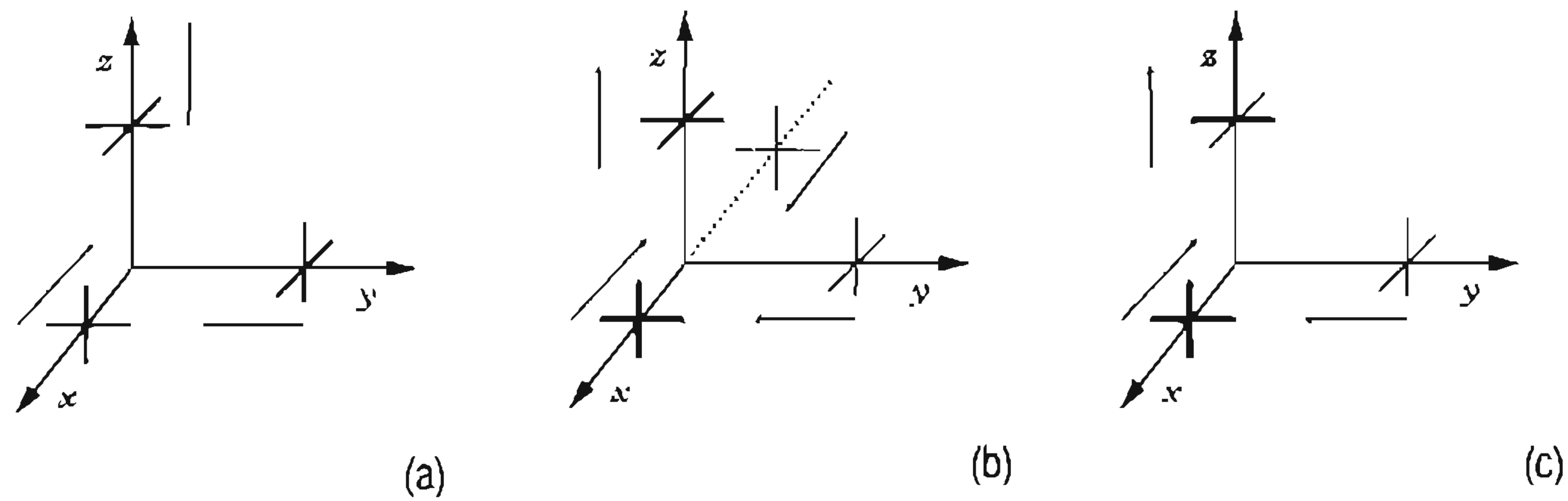


Fig. 6.9 (a) Isotropy implies that the Thomson scattering does not generate any polarization. The polarization amplitude of the waves propagating in the directions x and y are equal. The polarization of the outgoing wave carries away the y-component of the incoming wave that propagates along x and the x-component of the incoming wave propagating along y, which have the same amplitude. (b) Case of dipolar anisotropy: the polarization of the outgoing wave is the sum of the y-components of the incoming waves propagating along x (coming from a hot zone) and $-x$ (coming from a cold zone) that, on average, has the same amplitude as the x-components of the incoming wave propagating along y. It is thus also unpolarized. (c) Case of a quadrupole anisotropy: the outgoing wave has a polarization along y, inherited from the incoming wave along x that comes from a hot region, larger than that along x, inherited from the incoming wave along y that comes from a cold region.

Thus, we had:

$$\Delta_{P2} = -\frac{1}{5}\Delta_{P0} = \frac{1}{4}\Delta_{T2}\,, \quad \Delta_{T2} = -\frac{8}{15} ik\tau_C \Delta_{T1}\,, \qquad \Delta_{T1} = \frac{1}{3}V_b,$$

polarization ← temp. quadrupole ← free streaming of the dipole between collisions   temp. dipole → baryon velocity

Substitute into the integral form. Use that the visibility function is strongly peaked around the time of decoupling,

$$\Delta_P = \frac{3}{4}(1-\mu^2)e^{ik\mu(\tau_D-\tau_0)}\frac{2}{5}ik\Delta_{T1}(\tau_D)\Delta\tau_D \int_0^\infty d\kappa e^{-\frac{7}{10}\kappa}\int_1^\infty \frac{dx}{x}e^{-\frac{3}{10}x\kappa} = (1-\mu^2)e^{ik\mu(\tau_D-\tau_0)}0.51 ik\Delta_{T1}(\tau_D)\Delta\tau_D$$

$$\Rightarrow \quad \Delta_p(\hat{n},\vec{k}) \approx 0.17\Delta\tau_* ikv_b \quad \Rightarrow \quad P_{EE}(k) \approx (0.17\Delta\tau_*)^2 k^2 v_b^2(k)\ .$$

Thus, we obtain the baryon velocity power spectrum from the polarization power spectrum. This is only kinematics (and scatterings), no dependence on gravity nor on DM !

We now need to relate the velocity power spectrum to the density power spectrum.

Prior to recombination, the baryons and photons can be treated as a single fluid. In a universe with no DM, the behavior is simple inside the horizon:

$$\ddot{\delta}_b + c_s^2 k^2 \delta_b = 0 \ ,$$

In ΛCDM, there would be an additional forcing term on the right-hand side, $-3\ddot{\Phi}$, where Φ is the cold dark matter potential.

For adiabatic initial conditions, this admits the solution:

$$\delta_b = A(k)\cos(kr_s) \ ,$$

$$r_s = \int d\eta \ c_s$$

r_s is the sound horizon

The density can be related to the velocity via the continuity equation. At small scales, we can ignore any changes in the potential and simply treat the baryon-photon fluid as a normal Newtonian fluid. Then the continuity equation in Fourier space is:

$$\dot{\delta}_b(k) + ikv_b(k) = 0 \ .$$

$$v_b = \frac{i}{k}\dot{\delta}_b(k) = -ic_s A(k)\sin(kr_s) \ .$$

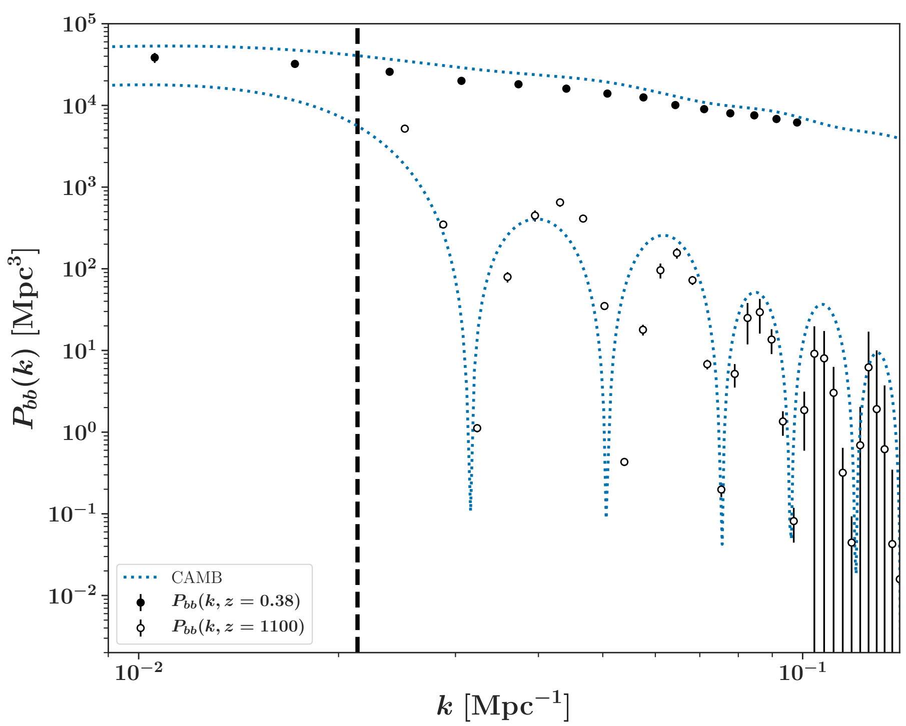
We had: $P_{EE}(k) \approx (0.17\Delta\tau_*)^2 k^2 v_b^2(k) \ .$ $\qquad$ $P_{EE}(k) \approx (0.17\Delta\tau_*)^2 c_s^2 k^2 |A(k)|^2 \sin^2(kr_s)$

Then: from the observed $P_{EE}(k)$ we obtain $A(k)$. This gives $\delta_b(k)$ and next $P_{bb}(k)$.

$P_{EE}(k)$ $\qquad$ $P_{bb}(k)$

For the EE power spectrum, we use the Planck 2018 [35] and the Atacama Cosmology Telescope ACTPol Two Season [38] angular power spectra. We add the data in quadrature. The data is given as multipoles, C_l^{EE}, of the 2D power spectrum. We must convert this to the 3D power spectrum, $P_{EE}(k)$. We approximate $l = k\eta_* - \frac{1}{2}$, where η_* is the conformal distance to the last scattering surface[4] [39]. Then, to order unity, the 3D power spectrum is [39, 40]:

$$P_{EE}(k) \sim \frac{\pi l^2}{k^3} C^{EE}_{l=k\eta_*-\frac{1}{2}}$$



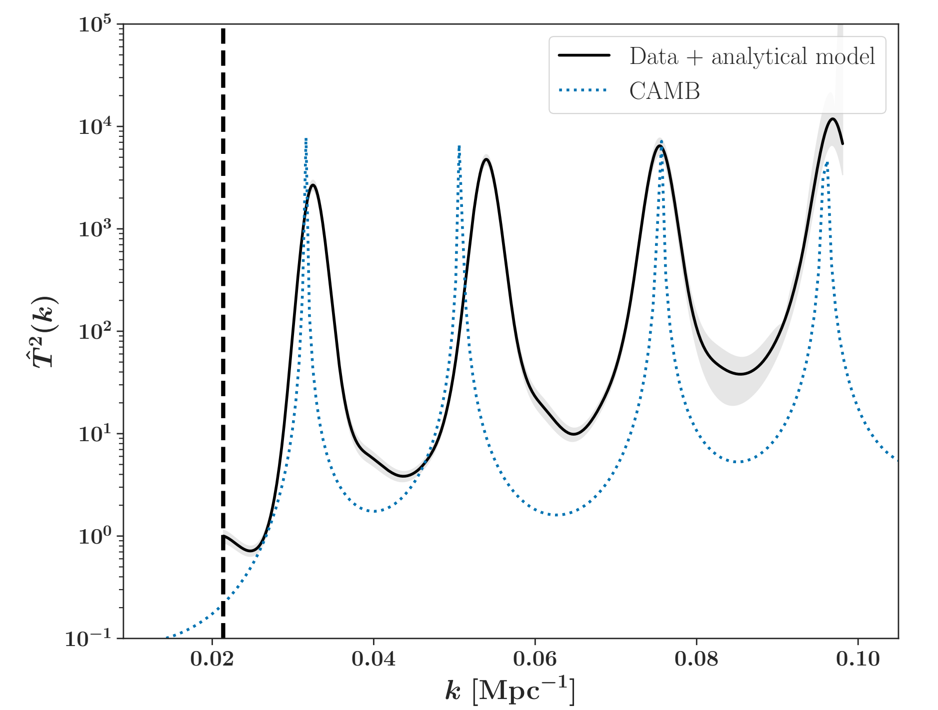


FIG. 2. Baryon transfer function from $z = 1100$ to $z = 0.38$. The black line shows the transfer function computed by applying the analytical model for the EE power spectrum to the Planck data and combining it with the large-scale structure data. The blue, dotted line shows the transfer function computed assuming ΛCDM. The difference between the two shows the limitations of the analytical approximation used to derive the Green's function. The gray region shows the 1-σ error from the data. Any alternative gravity theory must predict something close to this transfer function if it is to explain how the fluctuations in the baryon density traced by the polarization signal at $z \sim 1100$ evolve to the galaxy density field seen at low redshift.

our peaks do not precisely line up with the CAMB[5]-derived peaks at low-k. This occurs because we ignore the cold dark matter driving-term in the continuity equation, which is more prominent at low-k (i.e. velocity overshoot; cf. [21, 41, 42]).

We also indicate the acoustic scale by the dashed, black line on all plots in this paper. We use the Ref. [10] value for the sound horizon size at the drag epoch, $r_d = 147.09$ Mpc and the comoving distance to this time, $\eta_\star$, to set $l_* = \pi\eta_*/r_s$. Finally, we obtain the k value using $k_* = l_*/\eta_* - \frac{1}{2}$.

$$\Theta(k,\mu,\eta_0) = \int_0^{\eta_0} d\eta \tilde{S}(k,\mu,\eta) e^{ik\mu(\eta-\eta_0)-\tau(\eta)}.$$

$$\int_{-1}^{1} \frac{d\mu}{2} \mathcal{P}_l(\mu) e^{ik\mu(\eta-\eta_0)} = \frac{1}{(-i)^l} j_l\left[k(\eta-\eta_0)\right]$$

$$\Theta_l(k,\eta_0) = (-1)^l \int_0^{\eta_0} d\eta \tilde{S}(k,\eta) e^{-\tau(\eta)} j_l\left[k(\eta-\eta_0)\right].$$

$$C_\ell \sim \int \frac{dk}{k} k^3 P(k) j_\ell[k(\eta_0-\eta_s)]^2$$

$$\langle j_l^2(x)\rangle \approx \begin{cases} [2x(x^2-l^2)^{1/2}]^{-1} & \text{if} \quad x > l\ , \\ 0 \quad \text{if} \quad x < l\ . \end{cases}$$

$$C_\ell \sim \frac{k^3}{\ell^2} P(k)$$

with

$$\ell = k(\eta_0-\eta_s) - \frac{1}{2}$$