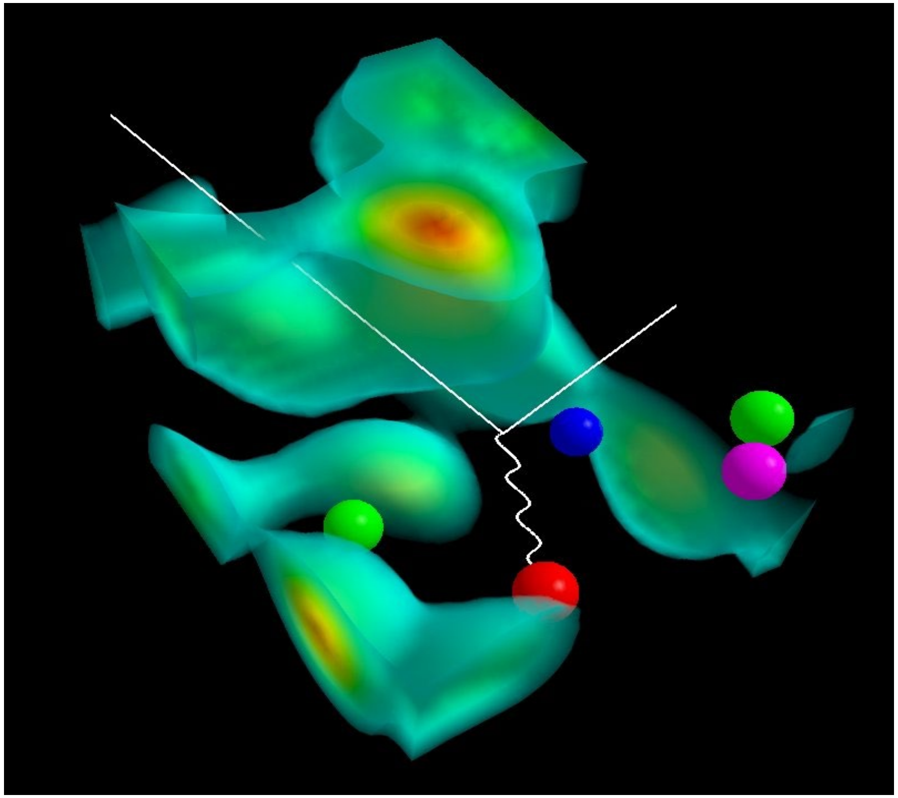
Recent results using the EDF derived from the quark-meson coupling (QMC) model



Anthony W. Thomas

Experimental and Theoretical Aspects of Neutron-Proton Pairing

Workshop : CEA Saclay

8th September 2023

Outline

I. **Nuclei from Quarks**
  - **start from a QCD-inspired model of *hadron* structure**
  - **develop a quantitative theory of nuclear structure**

II. **Search for observable effects of the change in hadron structure in-medium**

III. **Recent results for finite nuclei**

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DARK MATTER

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

I. Insights into nuclear structure

– what is the atomic nucleus?

There are two very different extremes….

Quark Structure matters/doesn't matter

- **Nuclear femtography: the science of mapping the quark and gluon structure of *atomic nuclei* is just beginning (EIC motivation)**

- **"Considering quarks is in contrast to our modern understanding of nuclear physics… the basic degrees of freedom of QCD (quarks and gluons) have to be considered only at higher energies. The *energies relevant for nuclear physics are only a few* MeV"**

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What do we know?

- Since 1970s: Dispersion relations have told us that the intermediate range NN attraction is a strong Lorentz scalar

- In relativistic treatments (RHF, RBHF, QHD…) this leads to mean scalar field on a nucleon ~300 to 500 MeV!!

Very large scalar mean-fields are a fact

1970 R. BROCKMANN AND R. MACHLEIDT

TABLE II. Results of a relativistic Dirac-Brueckner calculation in comparison to the tential B. As a function of the Fermi momentum k_F, it is listed: the energy per nucleon vector potentials U_S and U_V, and the wound integral κ.

	Relativistic				
k_F (fm^{-1})	$\mathcal{E}/A$ (MeV)	$\tilde{M}/M$	U_S (MeV)	U_V (MeV)	κ (%)
0.8	−7.02	0.855	−136.2	104.0	23.1
0.9	−8.58	0.814	−174.2	134.1	18.8
1.0	−10.06	0.774	−212.2	164.2	16.1
1.1	−11.18	0.732	−251.3	195.5	12.7
1.2	−12.35	0.691	−290.4	225.8	11.9
1.3	−13.35	0.646	−332.7	259.3	12.5
1.35	−13.55	0.621	−355.9	278.4	13.0
1.4	−13.53	0.601	−374.3	293.4	13.8
1.5	−12.15	0.559	−413.6	328.4	14.4
1.6	−8.46	0.515	−455.2	371.0	15.8

Brockmann and Machleidt, Phys Rev C52 (1990) 1965

What do we know?

- Since 1970s: Dispersion relations → intermediate range NN attraction is a strong Lorentz scalar

- In relativistic treatments (RHF, RBHF, QHD…) this leads to mean scalar field on a nucleon ~300 to 500 MeV!!

- *This is not small* – up to half the nucleon mass
  - death of "wrong energy scale" arguments

- Largely cancelled by large vector mean field BUT these have totally different dynamics: ω^0 just shifts energies, σ seriously modifies internal hadron dynamics

- Latter not naturally captured by EFT with N and π alone

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Suggests a different approach : QMC Model

(Guichon 1988, Guichon, Saito, Tsushima et al., Rodionov et al., Stone - see Saito *et al*., Prog. Part. Nucl .Phys. 58 (2007) 1 and Guichon *et al*., Prog. Part. Nucl. Phys. 100 (2018) 262-297 for reviews)

- Start with quark model (MIT bag/NJL...) for all hadrons

- Introduce a relativistic Lagrangian with σ, ω and ρ mesons coupling to non-strange quarks

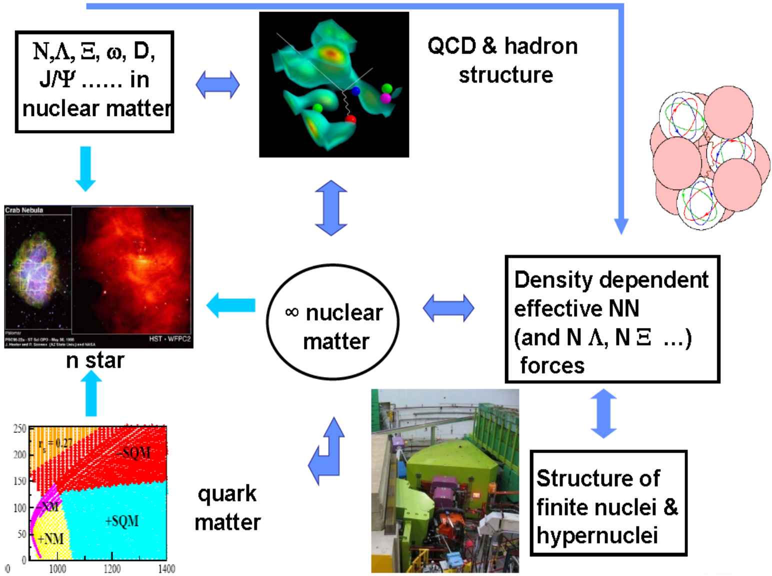
- Hence, initially only 4 parameters

  $(m_\sigma, g^{\sigma,\omega,\rho}{}_q)$

  – determine by fitting to:

  ρ_0, E/A and symmetry energy

  – same in dense matter & finite nuclei



- Must solve self-consistently for the internal structure of baryons in-medium

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Self-consistent solution for confined quarks in a hadron in nuclear matter

$$[i\gamma^\mu \partial_\mu - (m_q - g_\sigma{}^q \bar{\sigma}) - \gamma^0 g_\omega{}^q \bar{\omega}]\psi = 0$$

Source of σ changes: $\int_{Bag} d\vec{r}\,\bar{\psi}(\vec{r})\psi(\vec{r})$

SELF-CONSISTENCY

and hence mean scalar field changes…

and hence quark wave function changes….

THIS PROVIDES A NATURAL SATURATION MECHANISM
(VERY EFFICIENT BECAUSE QUARKS ARE LIGHT) Guichon 1988

source is suppressed as mean scalar field increases (i.e. as density increases)

Quark-Meson Coupling Model (QMC):
Role of the Scalar Polarizability of the Nucleon

The response of the nucleon internal structure to the scalar field is of great interest… and importance

$$M^*(\vec{r}) = M - g_\sigma \sigma(\vec{r}) + \frac{d}{2}\left(g_\sigma \sigma(\vec{r})\right)^2$$

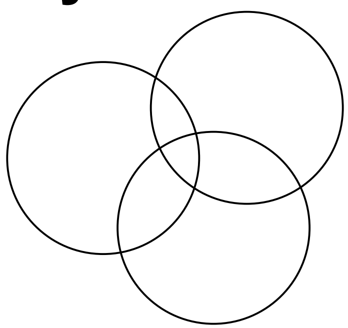
Non-linear dependence through the scalar polarizability

d ~ 0.22 R in original QMC (MIT bag)

Indeed, in nuclear matter at mean-field level, this is the ONLY place the response of the internal structure of the nucleon enters.

Summary : Scalar Polarizability

- Consequence of polarizability in atomic physics is many-body forces:



$$V = V_{12} + V_{23} + V_{13} + V_{123}$$

  - same is true in nuclear physics

- Three-body forces (NNN, HNN, HHN…) generated with NO new parameters
  – critical in neutron stars

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Guichon & Thomas, Phys. Rev. Lett. 93 (2004) 132502

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Application to nuclear structure

Initial Derivation of Density Dependent Effective Force

Physical origin of density dependent forces of Skyrme type within the quark meson coupling model

P.A.M. Guichon [a,*], H.H. Matevosyan [b,c], N. Sandulescu [a,d,e], A.W. Thomas [b]

Nuclear Physics A 772 (2006) 1–19

- **Start with classical theory of MIT-bag nucleons with structure modified in medium to give M_{eff} (σ).**
- **Quantise nucleon motion (non-relativistic), expand in powers of derivatives**
- **Derive equivalent, local energy density functional:**

$$\langle H(\vec{r})\rangle = \rho M + \frac{\tau}{2M} + \mathcal{H}_0 + \mathcal{H}_3 + \mathcal{H}_{\rm eff} + \mathcal{H}_{\rm fin} + \mathcal{H}_{\rm so}$$

Derivation of EDF (cont.)

$$\mathcal{H}_0+\mathcal{H}_3=\rho^2\left[\frac{-3G_\rho}{32}+\boxed{\frac{G_\sigma}{8(1+d\rho G_\sigma)^3}}-\frac{G_\sigma}{2(1+d\rho G_\sigma)}+\frac{3G_\omega}{8}\right]$$
$$+(\rho_n-\rho_p)^2\left[\frac{5G_\rho}{32}+\boxed{\frac{G_\sigma}{8(1+d\rho G_\sigma)^3}}-\frac{G_\omega}{8}\right],$$

$$\mathcal{H}_{\mathrm{eff}}=\left[\left(\frac{G_\rho}{8m_\rho{}^2}-\frac{G_\sigma}{2m_\sigma{}^2}+\frac{G_\omega}{2m_\omega{}^2}+\frac{G_\sigma}{4M_N{}^2}\right)\rho_n+\left(\frac{G_\rho}{4m_\rho^2}+\frac{G_\sigma}{2M_N^2}\right)\rho_p\right]\tau_n$$
$$+p\leftrightarrow n,$$

$$\mathcal{H}_{\mathrm{fin}}=\left[\left(\frac{3G_\rho}{32m_\rho{}^2}-\frac{3G_\sigma}{8m_\sigma{}^2}+\frac{3G_\omega}{8m_\omega{}^2}-\frac{G_\sigma}{8M_N{}^2}\right)\rho_n\right.$$
$$\left.+\left(\frac{-3G_\rho}{16m_\rho{}^2}-\frac{G_\sigma}{2m_\sigma{}^2}+\frac{G_\omega}{2m_\omega{}^2}-\frac{G_\sigma}{4M_N{}^2}\right)\rho_p\right]\nabla^2(\rho_n)+p\leftrightarrow n,$$

$$\mathcal{H}_{\mathrm{so}}=\nabla\cdot J_n\left[\left(\frac{-3G_\sigma}{8M_N{}^2}-\frac{3G_\omega(-1+2\mu_s)}{8M_N{}^2}-\frac{3G_\rho(-1+2\mu_v)}{32M_N{}^2}\right)\rho_n\right.$$
$$\left.+\left(\frac{-G_\sigma}{4M_N{}^2}+\frac{G_\omega(1-2\mu_s)}{4M_N{}^2}\right)\rho_p\right]+p\leftrightarrow n.$$

Spin-orbit force predicted!

Note the totally new, subtle density dependence

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Systematic approach to finite nuclei

J.R. Stone, P.A.M. Guichon, P. G. Reinhard & A.W. Thomas:
(Phys Rev Lett, 116 (2016) 092501)

- Constrain 3 basic quark-meson couplings (g_σ^q, g_ω^q, g_ρ^q) so that nuclear matter properties are reproduced within errors

$-17 < E/A < -15$ MeV

$0.14 < \rho_0 < 0.18$ fm^{-3}

$28 < S_0 < 34$ MeV

$L > 20$ MeV

$250 < K_0 < 350$ MeV

- Fix at overall best description of finite nuclei with 5 parameters (3 for the EDF +2 pairing pars)

- Benchmark comparison: SV-min 16 parameters (11+5 pairing)

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Overview of 106 Nuclei Studied – Across Periodic Table

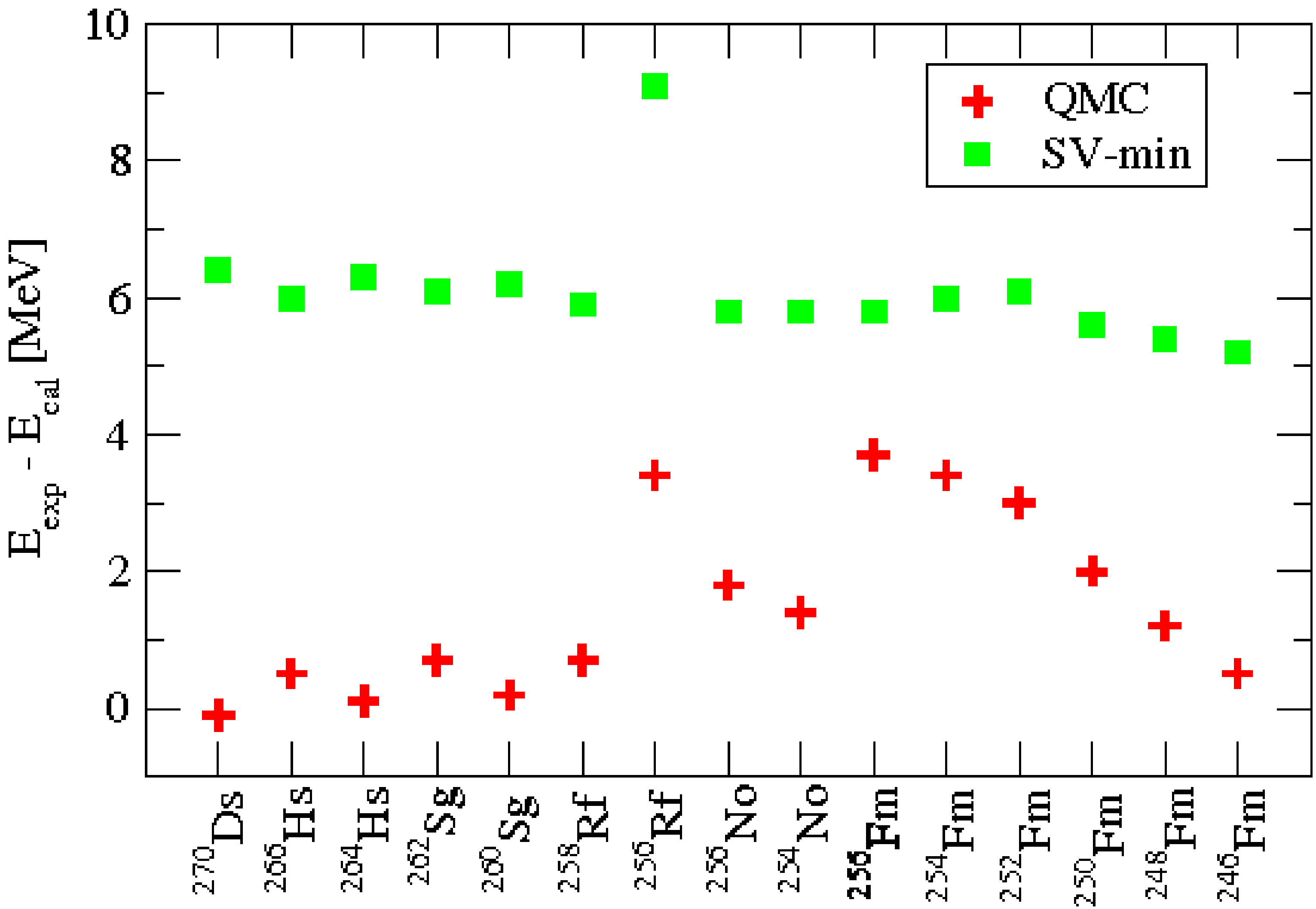
Element	Z	N	Element	Z	N
C	6	6 -16	Pb	82	116 - 132
O	8	4 -20	Pu	94	134 - 154
Ca	20	16 – 32	Fm	100	148 - 156
Ni	28	24 – 50	No	102	152 - 154
Sr	38	36 – 64	Rf	104	152 - 154
Zr	40	44 -64	Sg	106	154 - 156
Sn	50	50 – 86	Hs	108	156 - 158
Sm	62	74 – 98	Ds	110	160
Gd	64	74 -100			

Not fit (Fm – Ds)

N	Z	N	Z
20	10 – 24	64	36 - 58
28	12 – 32	82	46 - 72
40	22 – 40	126	76 - 92
50	28 – 50		

i.e. We look at most challenging cases of p- or n-rich nuclei

Superheavy Binding : 0.1% accuracy



Stone et al., PRL 116 (2016) 092501

For detailed study of SHE see: arXiv:1901.06064

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Overview of Initial Work on Finite Nuclei

- The effective force was *derived* at the quark level *based upon the changing structure of a bound nucleon*
- Has many less parameters but reproduces nuclear properties at a level comparable with the best phenomenological Skyrme forces
- Looks similar to standard nuclear forces
- BUT underlying theory also predicts modified internal structure and hence modified
  - DIS structure functions
  - elastic form factors......

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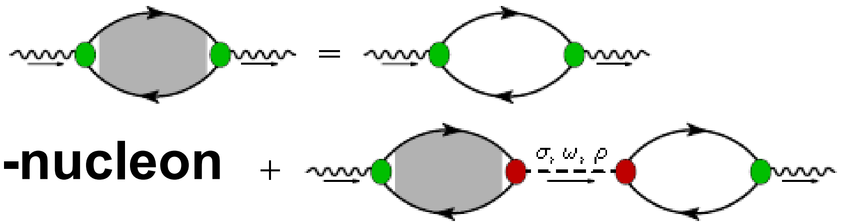
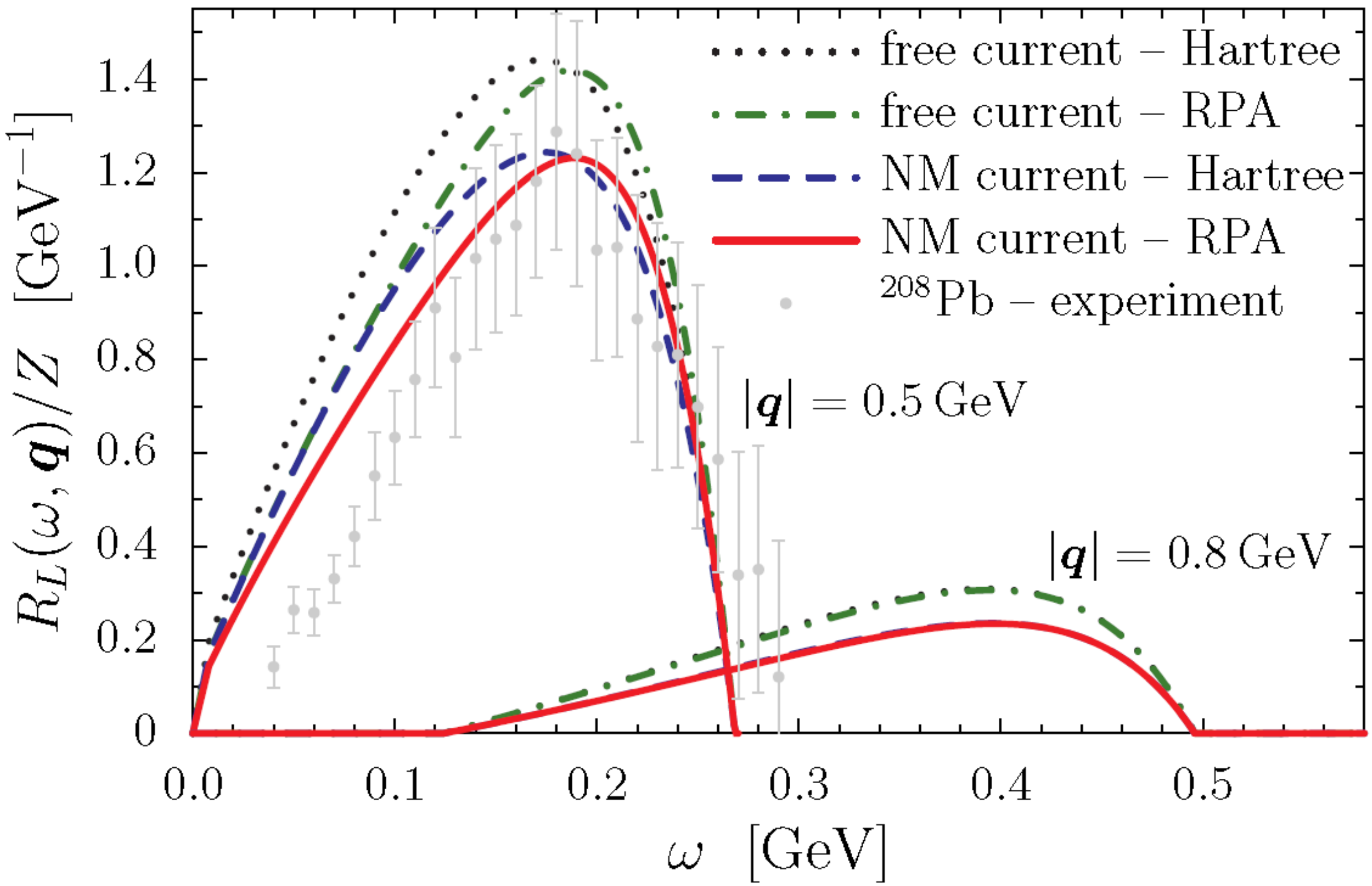
Modified Electromagnetic Form Factors In-Medium

Response Function

$$\frac{d^2\sigma}{d\Omega\, d\omega} = \sigma_{\mathrm{Mott}}\left[\frac{q^4}{|\boldsymbol{q}|^4} R_L(\omega, |\boldsymbol{q}|) + \left(\frac{q^2}{2|\boldsymbol{q}|^2} + \tan^2\frac{\theta}{2}\right) R_T(\omega, |\boldsymbol{q}|)\right]$$

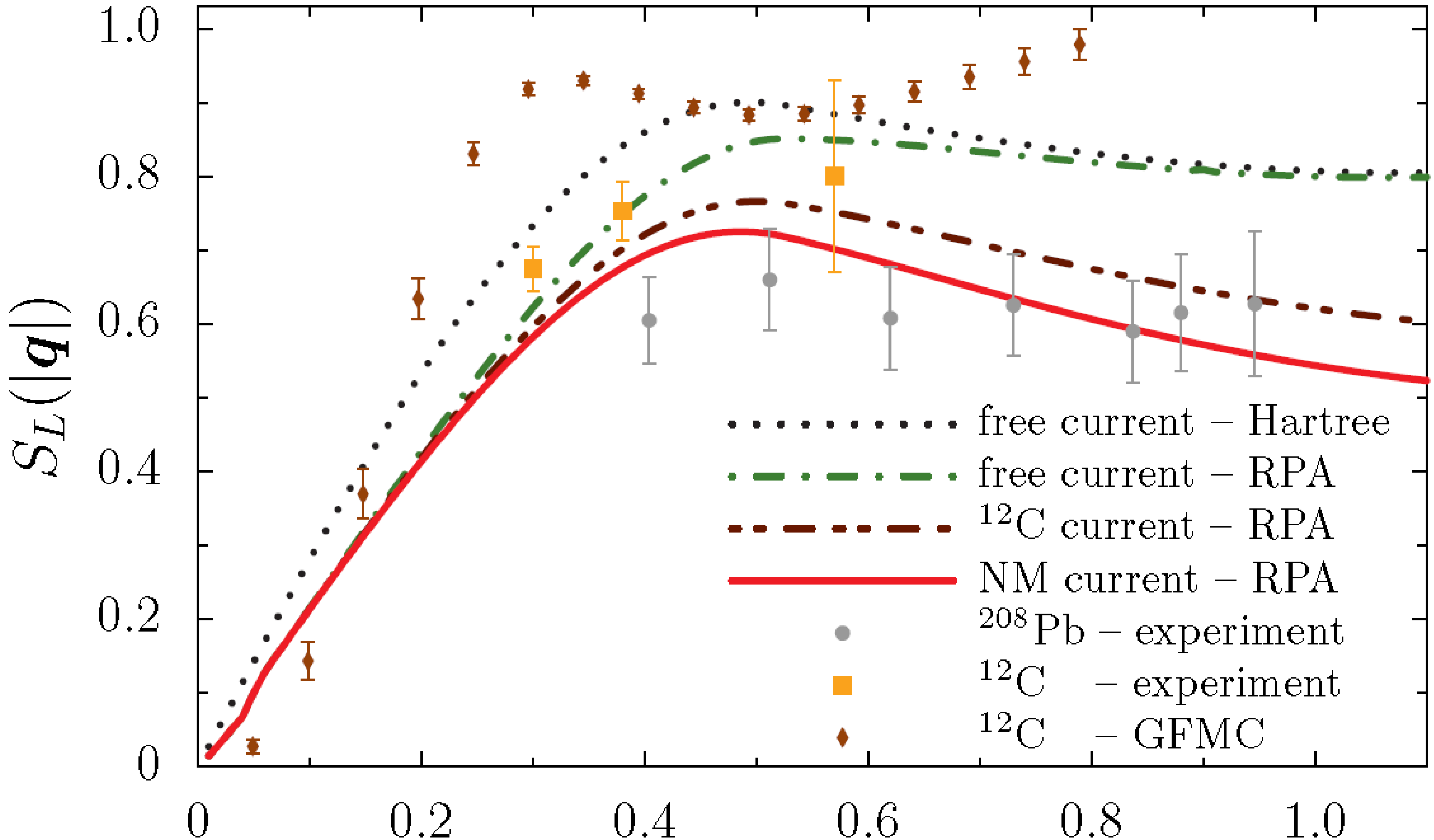
RPA correlations repulsive
Significant reduction in Response Function from the modification of bound-nucleon





Cloët, Bentz & Thomas, PRL 116 (2016) 032701

Comparison with Unmodified Nucleon & Data



$$S_L(|\boldsymbol{q}|) = \int_{\omega+}^{|q|} d\omega \, \frac{R_L(\omega, |\boldsymbol{q}|)}{Z\, G_{Ep}^2(Q^2) + N\, G_{En}^2(Q^2)}$$

Data: Morgenstern & Meziani

Calculations: Cloët, Bentz & Thomas (PRL 116 (2016) 032701)

More Nuclear Structure

Includes some unpublished results for QMC-π III from

PhD thesis of Kay Martinez

- now at Silliman University (Philippines)
(publications in preparation)

QMC-π II and III incorporate a much more accurate evaluation of H^{σ}

Giant Monopole Resonances

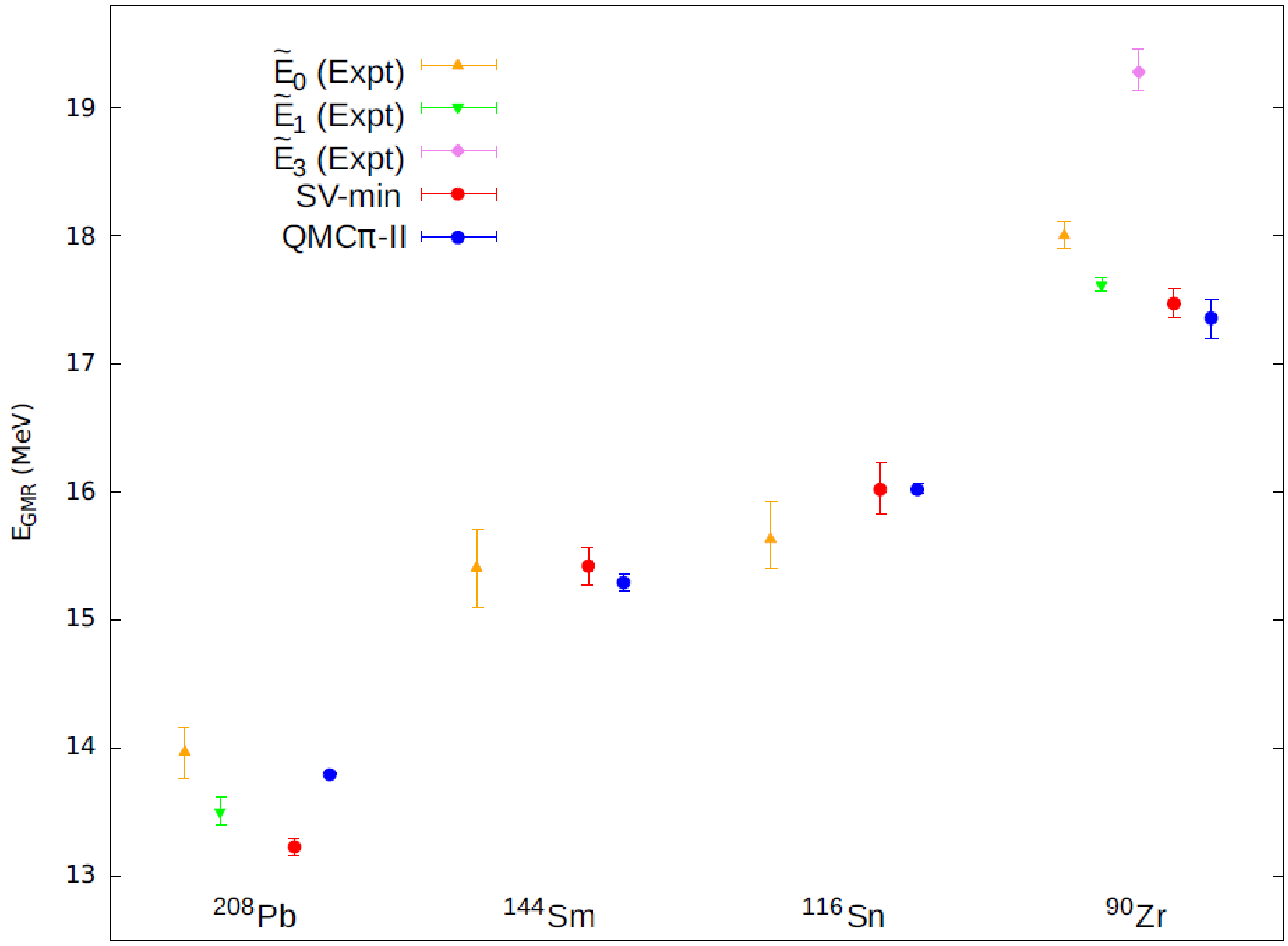
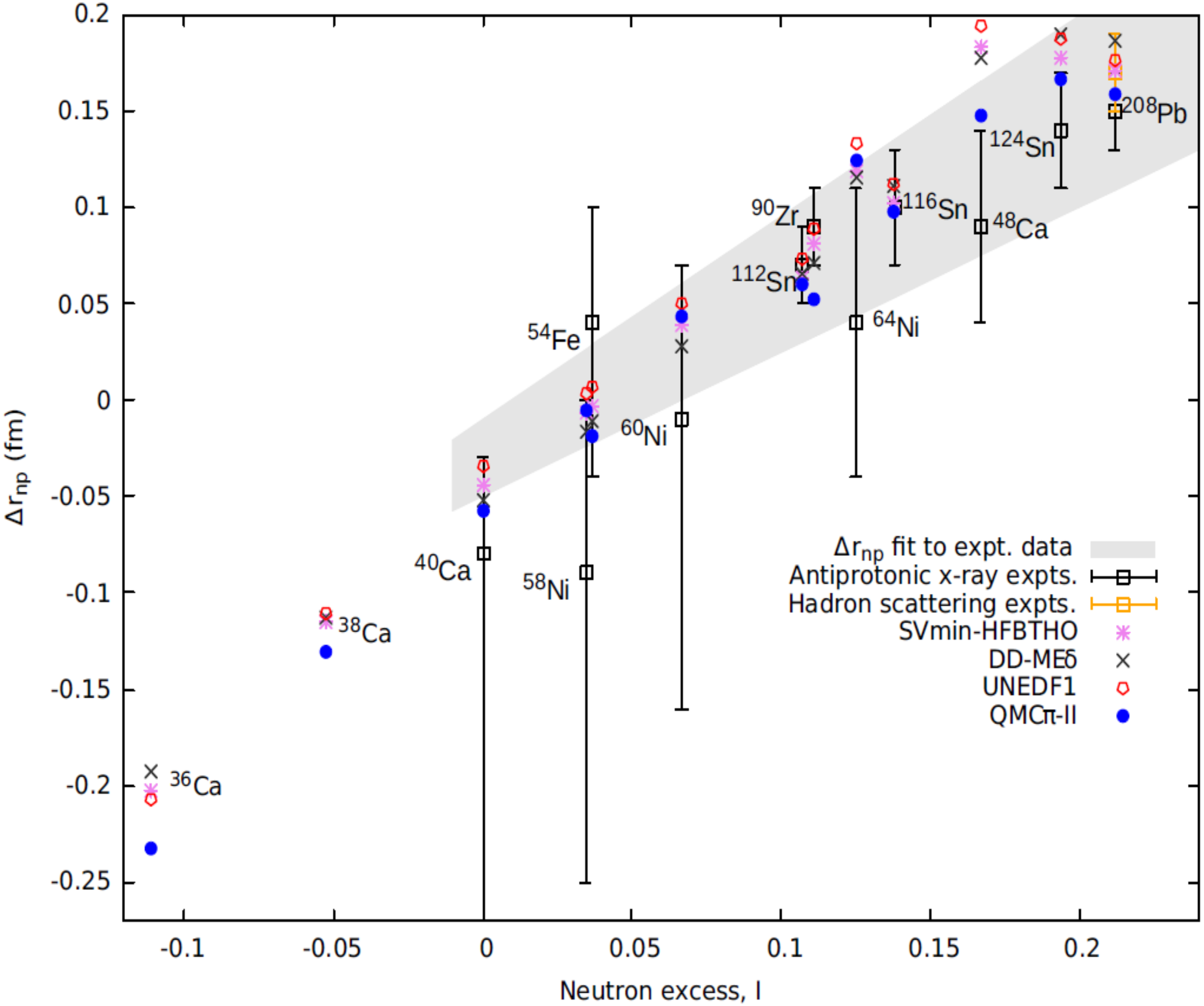


FIG. 13. GMR energies for ^{208}Pb, ^{144}Sm, ^{116}Sn, and ^{90}Zr from experiment and for the QMCπ-II and SVmin models. Experimental data are taken from Table 1 of Ref. [24].

This required the introduction of a term ~$\lambda_3\sigma^3$

Kay Martinez *et al.*, Phys Rev C100 (2019) 024333

Neutron distributions



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Kay Martinez *et al.*, Phys Rev C100 (2019) 024333

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QMC π3

- **Just 5 parameters***: m_σ, quark couplings to σ, ω and ρ mesons and λ_3 - the strength of σ^3 term
- Tensor term included:

$$H^J_{\sigma,\omega,\rho} = \left(\frac{G_\sigma(1-d\,v_0)^2}{4m_\sigma^2} - \frac{G_\omega}{4m_\omega^2}\right)\sum_m \vec{J}_m^2 - \frac{G_\rho}{4m_\rho^2}\sum_{m,m'} S_{m,m'}\vec{J}_m\cdot\vec{J}_{m'},$$

and

$$H^J_S = -\frac{G_\sigma - G_\omega}{16M^2}\sum_m \vec{J}_m^2 + \frac{G_\rho}{16M^2}\sum_{mm'} S_{m,m'}\vec{J}_m\cdot\vec{J}_{m'}.$$

with

$$\vec{J}_m = i\sum_{i\in F_m}\sum_{\sigma\sigma'} \vec{\sigma}_{\sigma'\sigma}\times[\vec{\nabla}\phi^i(\vec{r},\sigma,m)]\phi^{i*}(\vec{r},\sigma',m),\quad \vec{J} = \vec{J}_p + \vec{J}_n,$$

- Pairing interaction (simple BCS) derived in the model

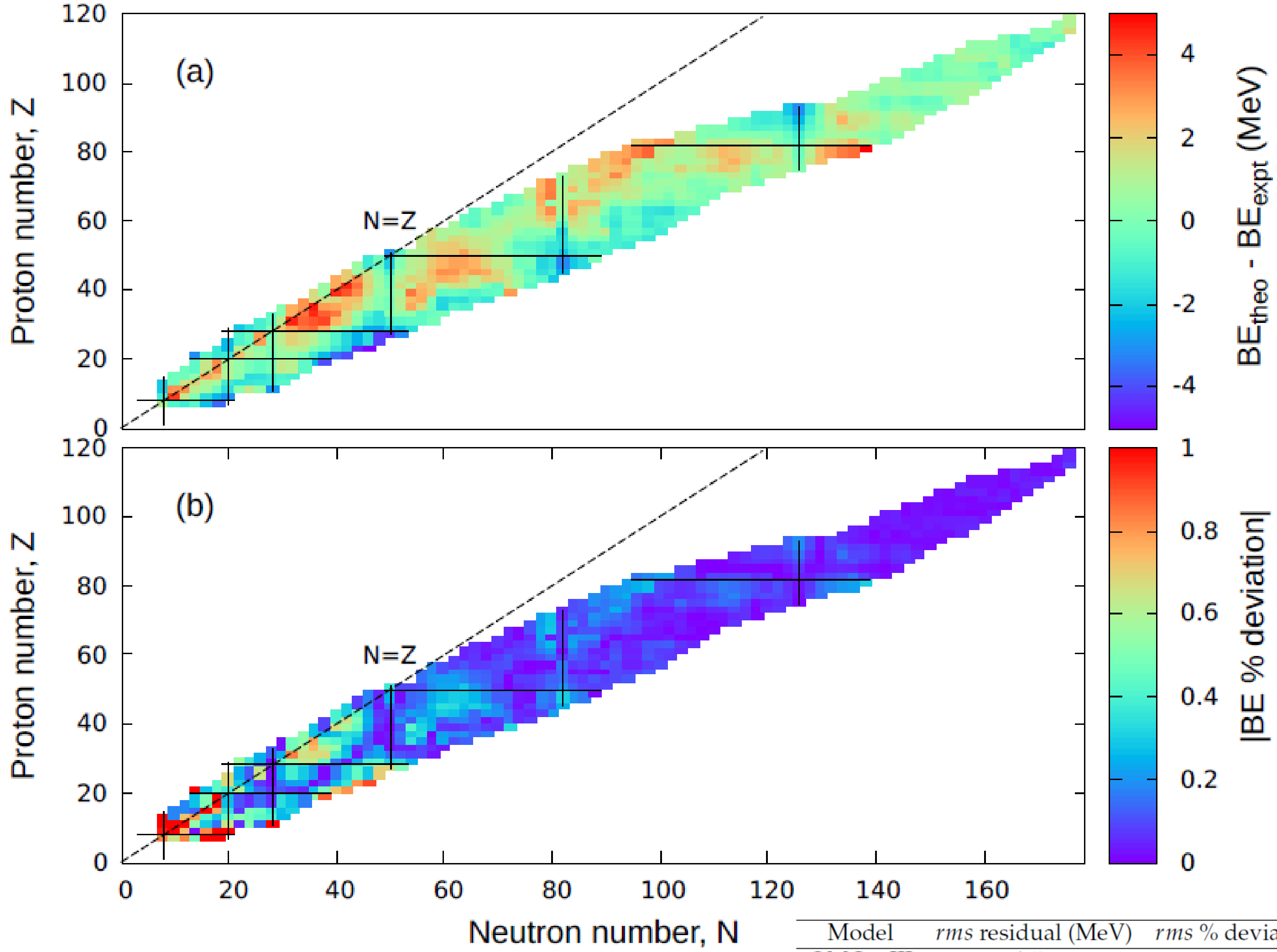
$$V^{\mathrm{QMC}}_{\mathrm{pair}} = -\left(\frac{G_\sigma}{1+d'G_\sigma\rho(\vec{r})} - G_\omega - \frac{G_\rho}{4}\right)\delta(\vec{r}-\vec{r}\,')$$

$$d' = d + \tfrac{1}{3}G_\sigma\lambda_3.$$

*cf. Over 20 in FRDM and typically 16 (11+5) in Skyrme forces

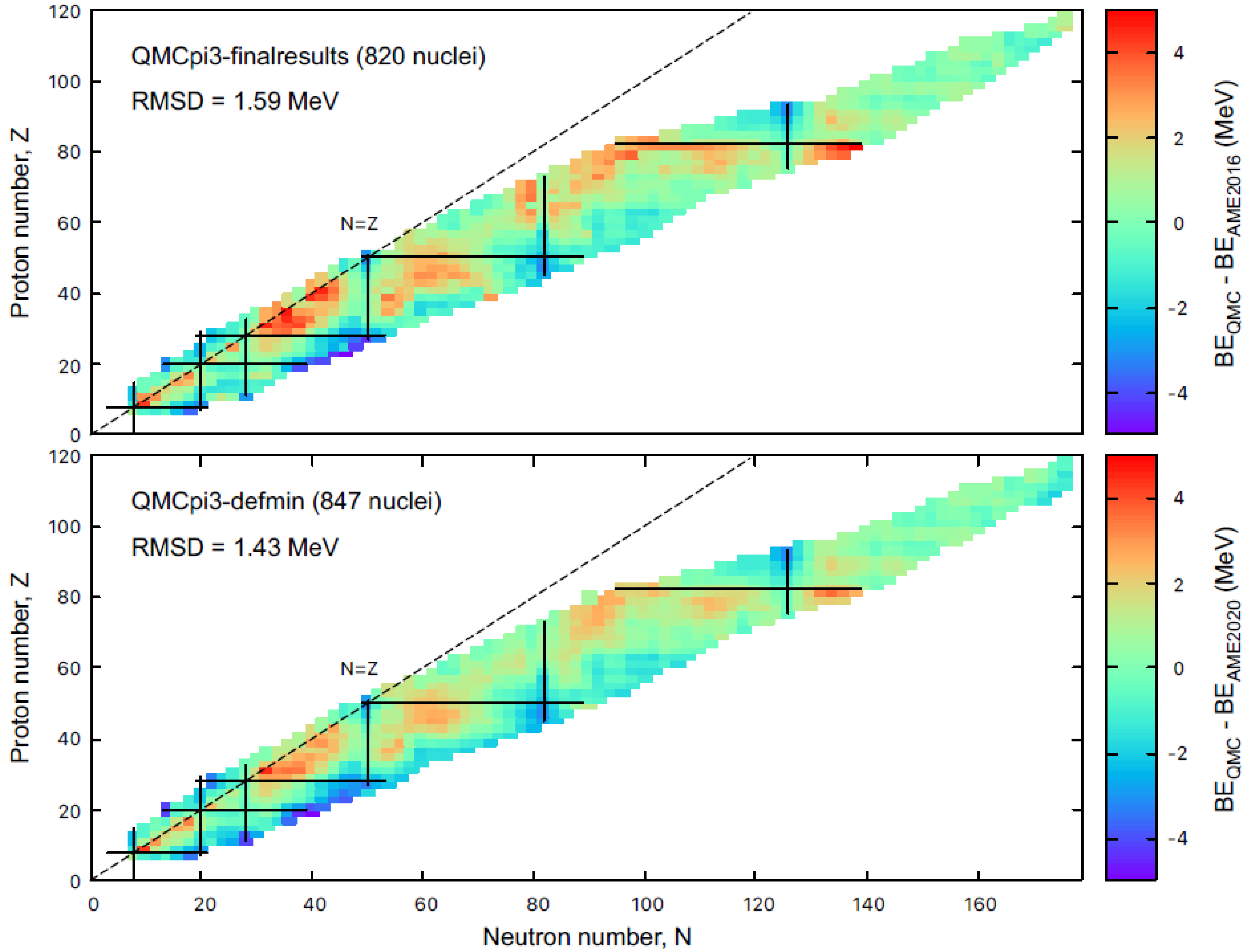
Binding Energies – All Known Even-Even Nuclei

2021



Model	*rms* residual (MeV)	*rms* % deviation
QMCπ-III	1.59	0.29
QMCπ-II	2.34	0.39
QMCπ-I	2.78	0.50
QMC-I	3.84	0.69
SV-min	3.64	0.38
UNEDF1	2.06	0.55
DD-MEδ	2.41	0.42
FRDM	0.89	0.18

Latest analysis: data from Atomic Mass Evaluation 2020



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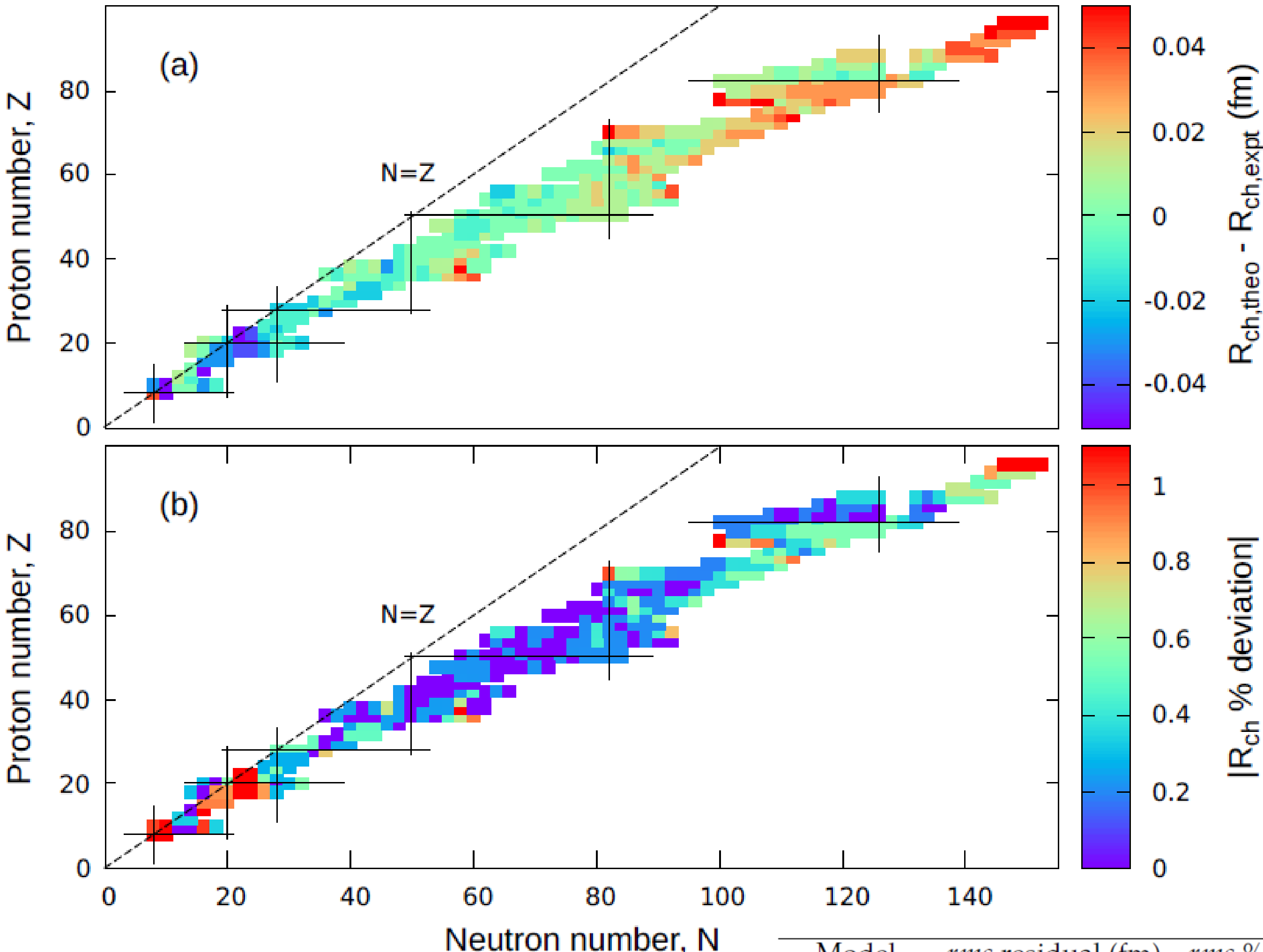
Charge Radii



Model	*rms* residual (fm)	*rms* % deviation
QMCπ-III	0.024	0.50
QMCπ-II	0.029	0.66
QMCπ-I	0.028	0.65
QMC-I	0.030	0.66
SV-min	0.024	0.61
UNEDF1	0.029	0.65
DD-MEδ	0.035	0.78

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Deformation

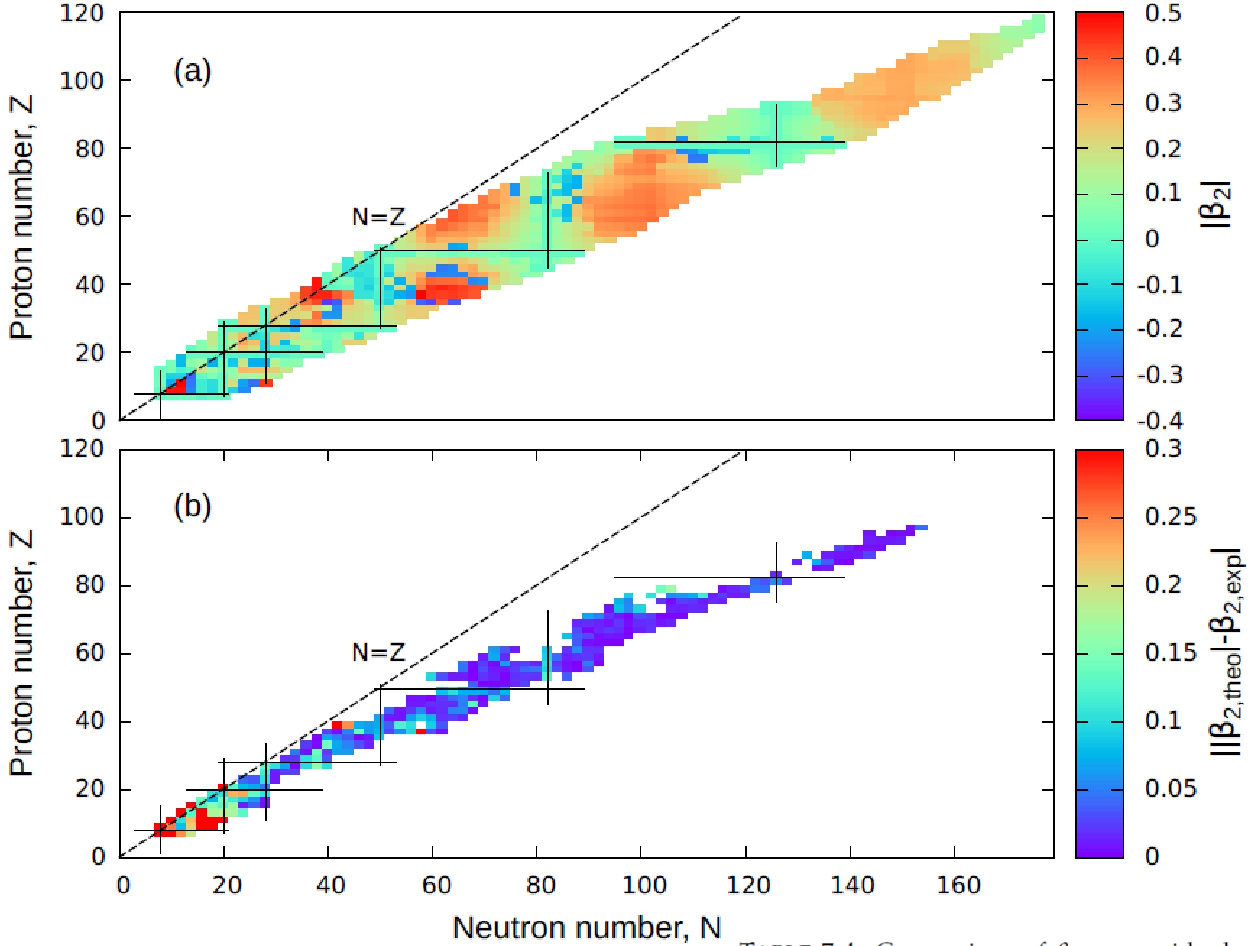

TABLE 7.4: Comparison of β_2 *rms* residuals and *rms* % deviations from QMCπ-III and from other nuclear models. There are a total of 324 even-even nuclei with available data for β_2 included for comparison.

Model	*rms* residual	*rms* % deviation
QMCπ-III	0.11	28
SV-min	0.16	59
UNEDF1	0.15	53
DD-MEδ	0.14	40
FRDM	0.11	30

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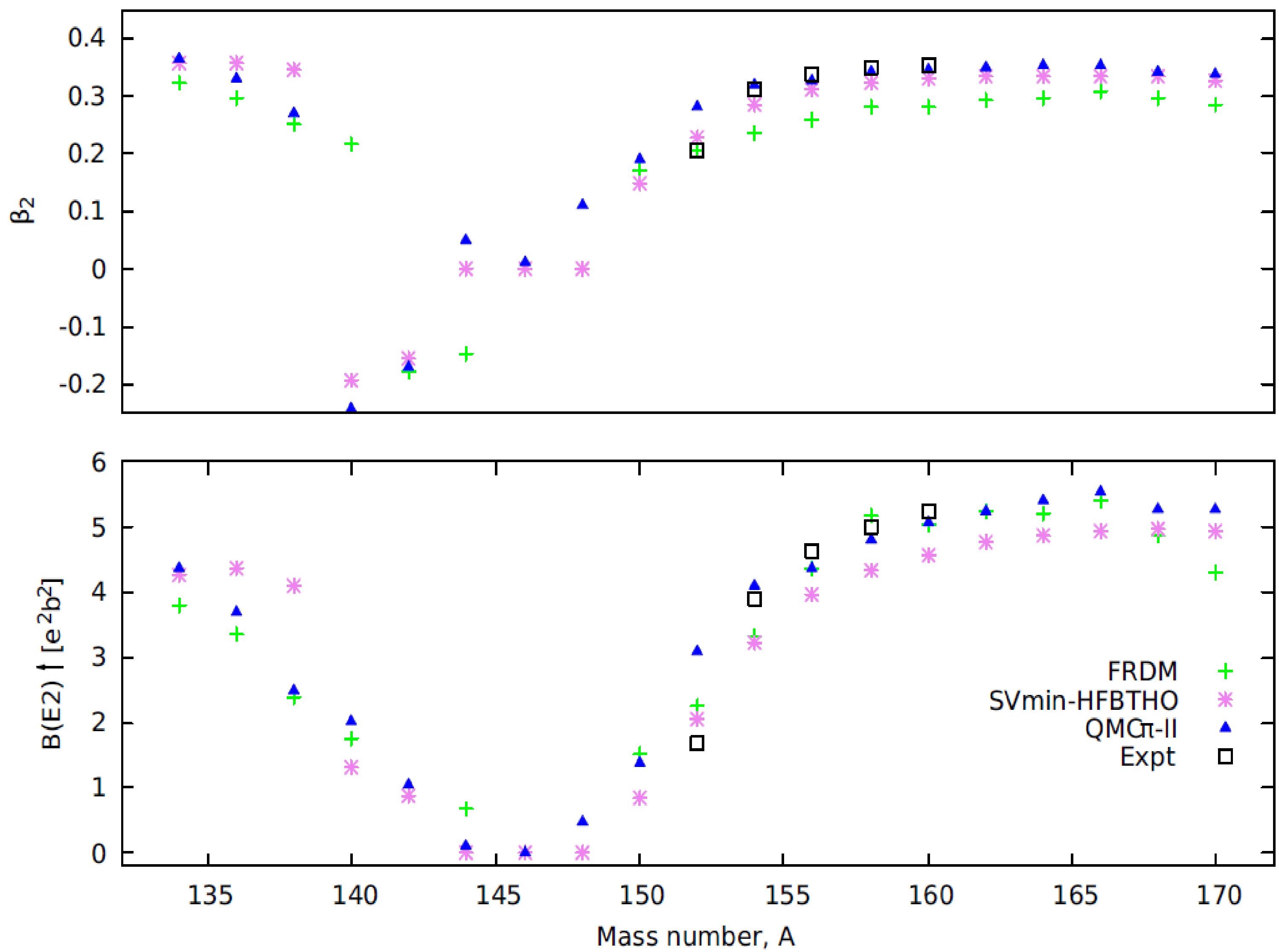
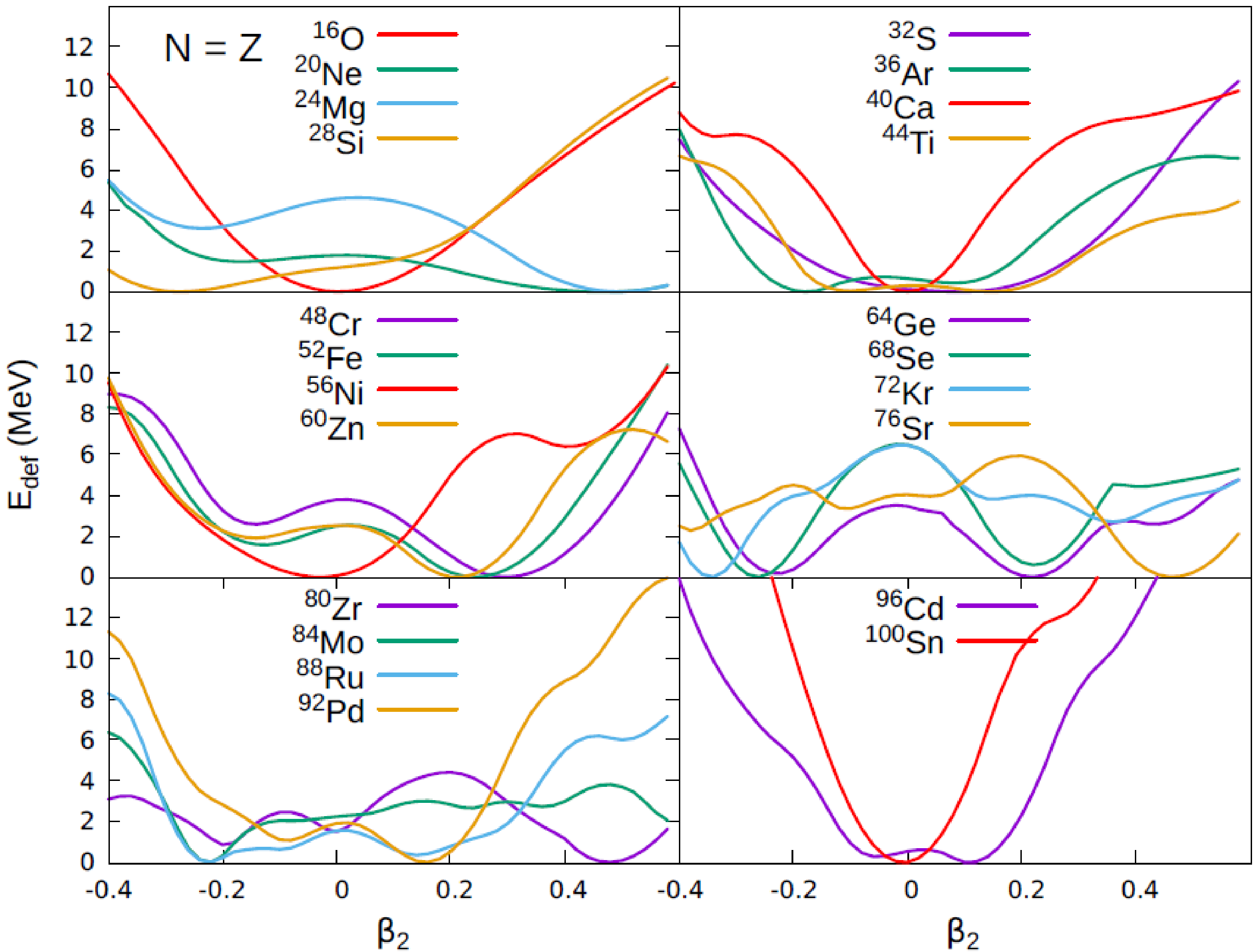
Deformation of Gd isotopes



Kay Martinez *et al.*, Phys Rev C100 (2019) 024333

Shape Co-Existence for Z=N



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Separation energies: Drip Lines

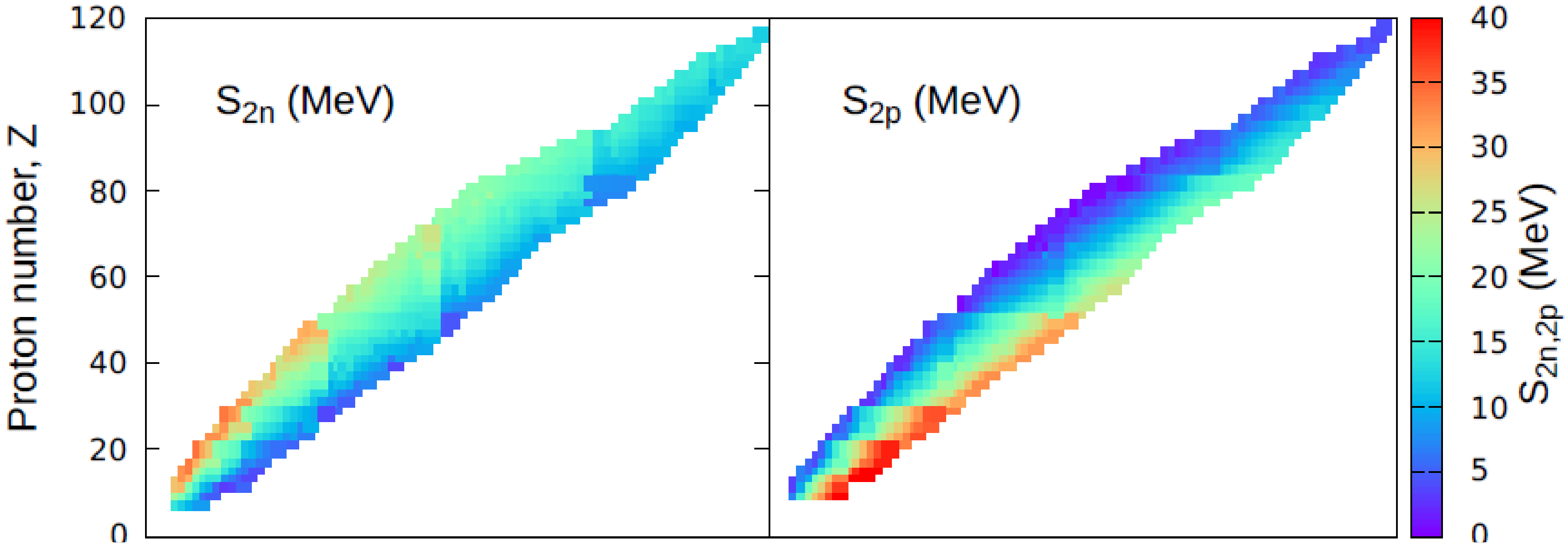

TABLE 7.3: Comparison of *rms* residuals for separation energies (in MeV) from QMC and from other nuclear models.

Model	S_{2n}	S_{2p}	δ_{2n}	δ_{2p}	Q_α
QMCπ-III	0.97	0.95	1.24	1.28	1.07
QMCπ-II	1.03	1.08	1.20	1.25	1.19
SV-min	0.77	0.82	0.87	1.00	0.79
UNEDF1	0.74	0.82	0.85	0.90	0.80
DD-MEδ	1.01	1.05	1.12	1.11	1.30
FRDM	0.50	0.55	0.61	0.75	0.61

The Superheavy Region

First study:

PHYSICAL REVIEW C **100**, 044302 (2019)

Physics of even-even superheavy nuclei with $96 < Z < 110$ in the quark-meson-coupling model

J. R. Stone*
Department of Physics (Astro), University of Oxford, Keble Road OX1 3RH, Oxford, United Kingdom
and Department of Physics and Astronomy, University of Tennessee, Knoxville, Tennessee 37996, USA

K. Morita†
Department of Physics, Kyushu University, Nishi-ku, Fukuoka 819-0395, Japan
and RIKEN Nishina Center, RIKEN, Wako-shi, Saitama 351-0198, Japan

P. A. M. Guichon‡
CEA/IRFU/SPhN Saclay, F91191, France

A. W. Thomas§
CSSM and CoEPP, Department of Physics, University of Adelaide, SA 5005, Australia

Updated and expanded here (Martinez thesis)

Binding Energies

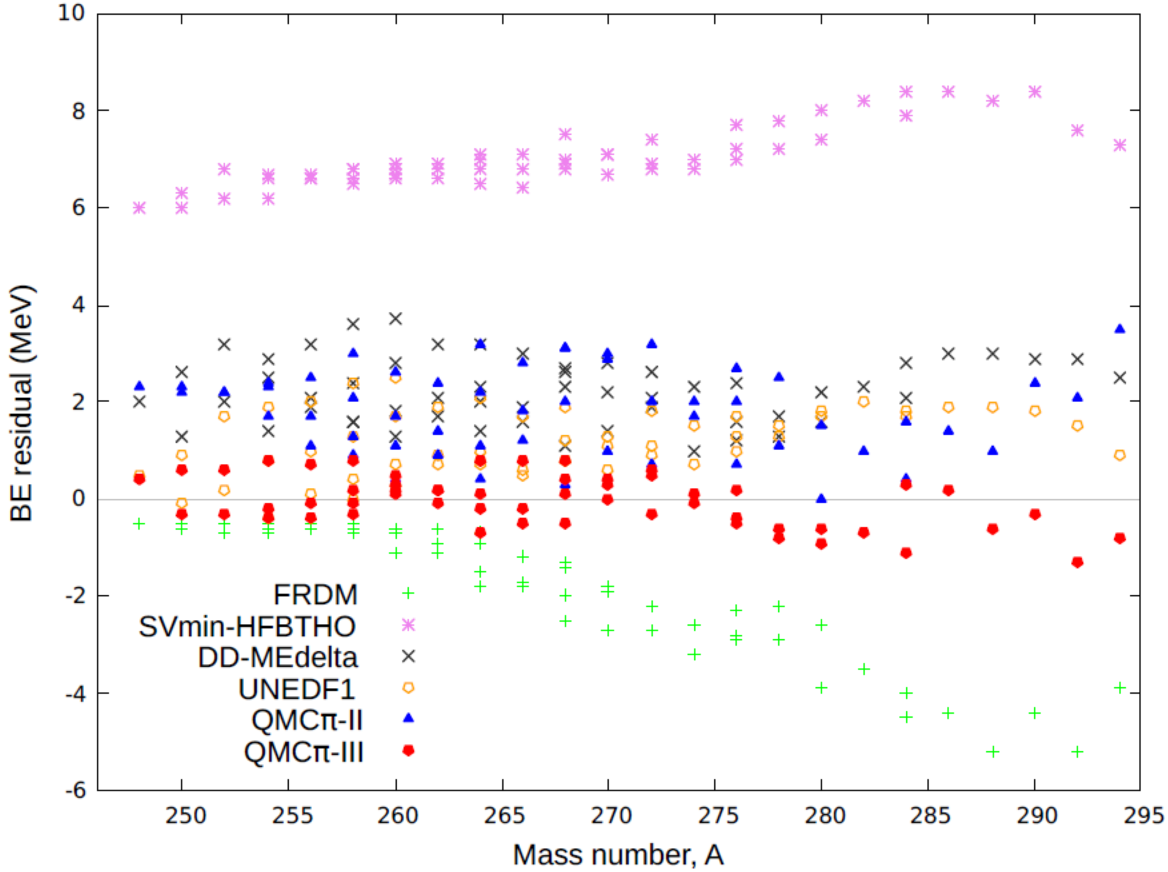

TABLE 6.1: Comparison of *rms* percent deviations and *rms* residuals from QMC and from other nuclear models for SHE with available data.

	rms % deviation	*rms* residual (MeV)
QMCπ-III	0.03	0.52
QMCπ-II [54]	0.11	2.04
QMCπ-I [53]	0.12	2.42
QMC-I [8]	0.08	1.50
FRDM [23]	0.11	2.25
SV-min [24]	0.36	6.99
UNEDF1 [28]	0.07	1.31
DD-MEδ [66]	0.12	2.28

Outstanding agreement

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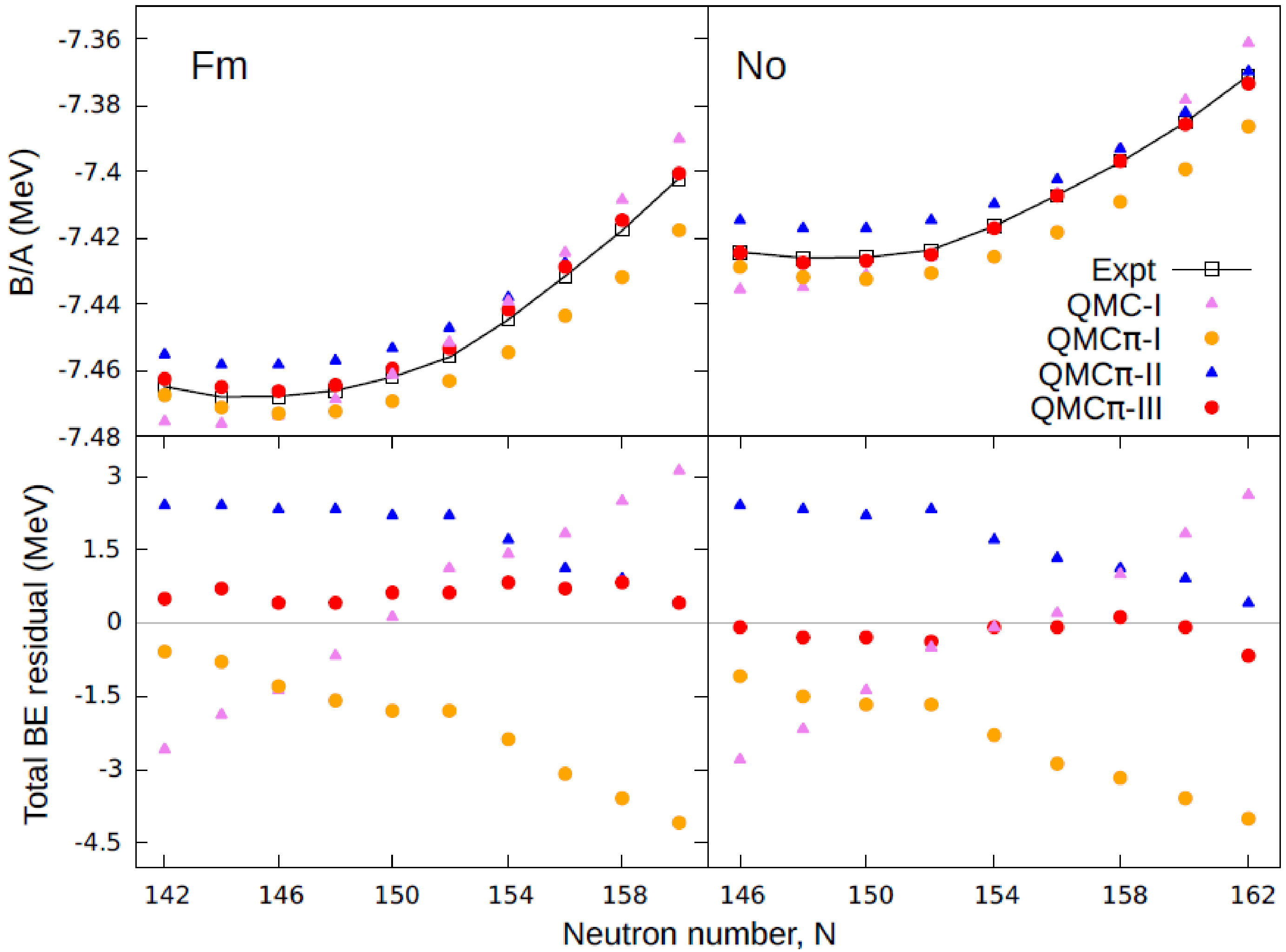
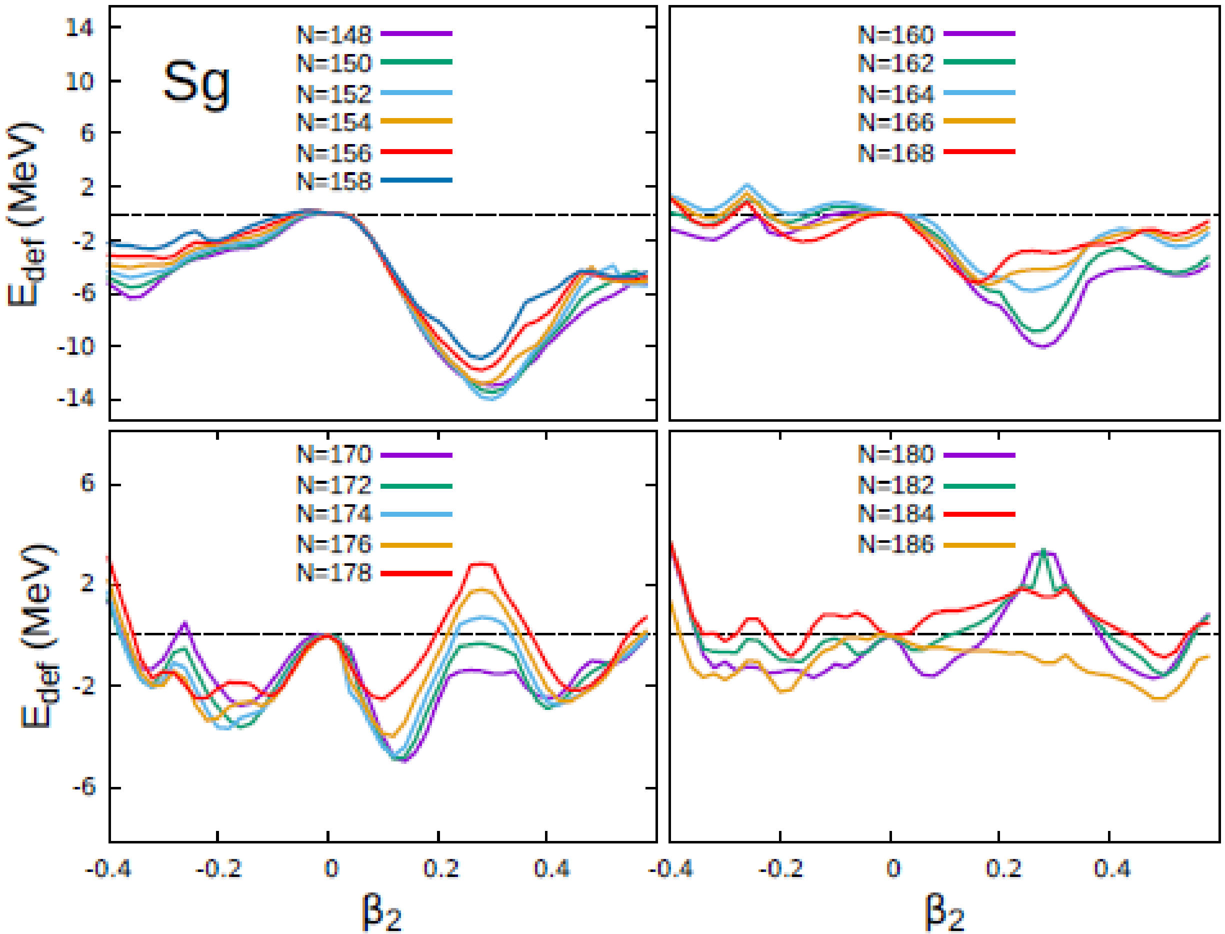
Trends Along Chains: ^{100} Fermium and ^{102} Nobelium



Many Almost Degenerate Minima in Superheavy Region



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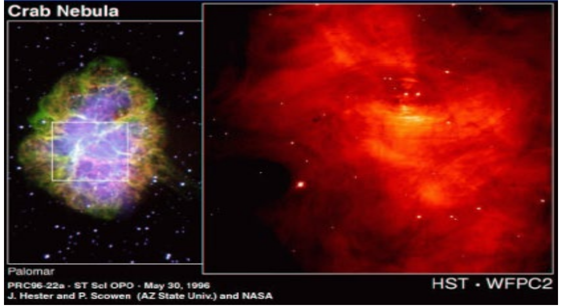
I. Summary



- Intermediate range NN attraction is STRONG Lorentz scalar

- This modifies the intrinsic structure of the bound nucleon

  − profound change in shell model :

  what occupies shell model states are NOT free nucleons

- Scalar polarizability is a natural source of three-body forces (NNN, HNN, HHN…)

  − clear physical interpretation

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II. Summary

- Need empirical confirmation of changing baryon structure:
  - Response Functions & Coulomb sum rule
  - EMC effect; spin EMC (not too long…)
  - Change in Λ decay rate in nuclei?
- Initial systematic study of finite nuclei very promising

  With just 5 parameters:

  - Binding energies typically within 0.29% across periodic table
  - Super-heavies (Z > 100) especially good: 0.03%
  - Systematics of charge radii, deformations, shell and subshell closures pretty good

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Special Mentions……

Guichon

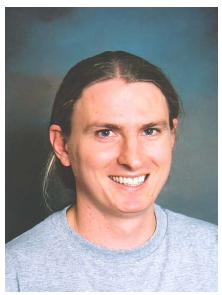
Tsushima

Saito

Stone

Krein

Matevosyan



Cloët

Whittenbury

Simenel

Bentz

Martinez

Motta

Antic

Kalaitzis

P. G. Reinhard
Skyax

Latest papers

- **QMC π3;**
  Martinez et al., Phys Rev C102 (2020) 034304

- **Review:**
  Guichon *et al.*, PPNP 100 (2018) 262

- **SHE:**
  Stone *et al.*, arXiv: 1901.06064

- **Systematic application to finite nuclei:**
  Stone *et al.*, Phys Rev Lett 116 (2016) 092501

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Key papers on QMC

- **Many-body forces:**
  1. Guichon, Matevosyan, Sandulescu, Thomas, Nucl. Phys. A772 (2006) 1.
  2. Guichon and Thomas, Phys. Rev. Lett. 93 (2004) 132502

- **Built on earlier work on QMC: e.g.**
  3. Guichon, Phys. Lett. B200 (1988) 235
  4. Guichon, Saito, Rodionov, Thomas, Nucl. Phys. A601 (1996) 349

- **Major review of applications of QMC to many nuclear systems:**
  5. Saito, Tsushima, Thomas, Prog. Part. Nucl. Phys. 58 (2007) 1-167 (hep-ph/0506314)
  6. Guichon et al., Prog. Part. Nucl. Phys. 100 (2018) 262

References to: Covariant Version of QMC

- **Basic Model: (Covariant, chiral, confining version of NJL)**
- Bentz & Thomas, Nucl. Phys. A696 (2001) 138
- Bentz, Horikawa, Ishii, Thomas, Nucl. Phys. A720 (2003) 95
- **Applications to DIS:**
- Cloet, Bentz, Thomas, Phys. Rev. Lett. 95 (2005) 052302
- Cloet, Bentz, Thomas, Phys. Lett. B642 (2006) 210
- **Applications to neutron stars – including SQM:**
- Lawley, Bentz, Thomas, Phys. Lett. B632 (2006) 495
- Lawley, Bentz, Thomas, J. Phys. G32 (2006) 667

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Shape Co-Existence

TABLE 7.5: β_2 values corresponding to the locations of the first and second deformed minima for symmetric nuclei obtained from QMCπ-III. Also added for comparison are experimental data which are only available in absolute values [52], as well as FRDM results [23].

Z or N	Expt.	QMCπ-III 1st	QMCπ-III 2nd	FRDM	Z or N	Expt.	QMCπ-III 1st	QMCπ-III 2nd	FRDM
8	0.36	0.00	-	-0.01	30	-	0.22	-0.14	0.16
10	0.73	0.46	-0.16	0.36	32	-	0.22	-0.22	0.21
12	0.61	0.50	-0.24	0.39	34	-	-0.26	0.22	0.23
14	0.41	-0.28	-	-0.36	36	-	-0.34	-	-0.37
16	0.31	0.10	-	0.22	38	-	0.46	-	0.40
18	0.26	-0.18	0.08	-0.26	40	-	0.48	-0.20	0.43
20	0.12	0.00	-	0.00	42	-	-0.22	-	-0.23
22	0.27	0.14	-	0.00	44	-	-0.22	0.14	-0.24
24	0.34	0.30	-0.14	0.23	46	-	0.16	-0.16	0.00
26	-	0.24	-0.12	0.12	48	-	0.10	-0.06	-0.02
28	0.17	-0.02	-	0.00	50	-	0.00	-	0.00

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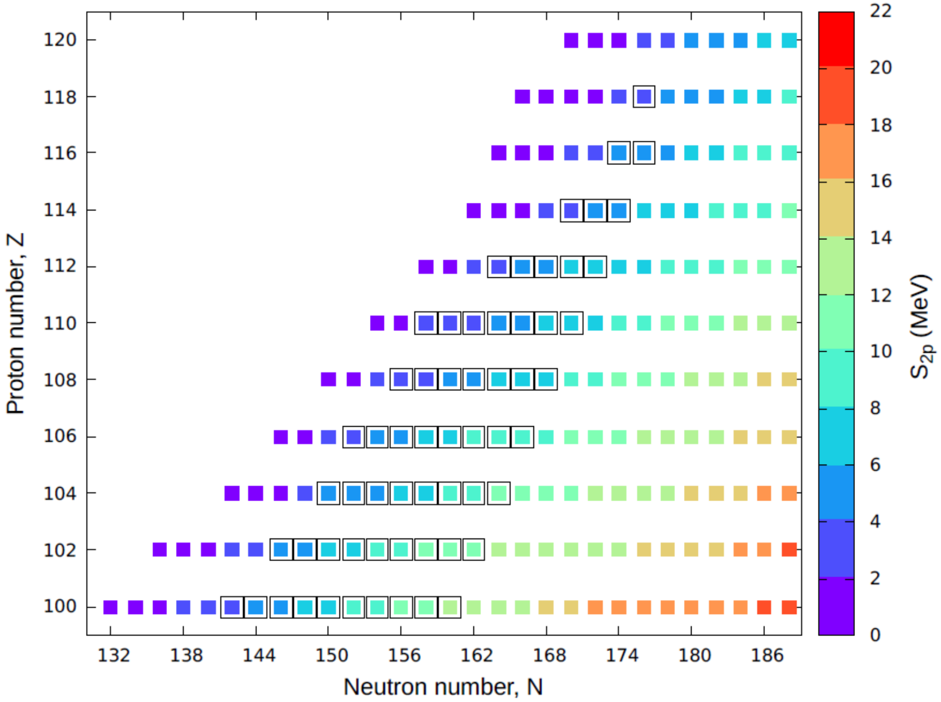
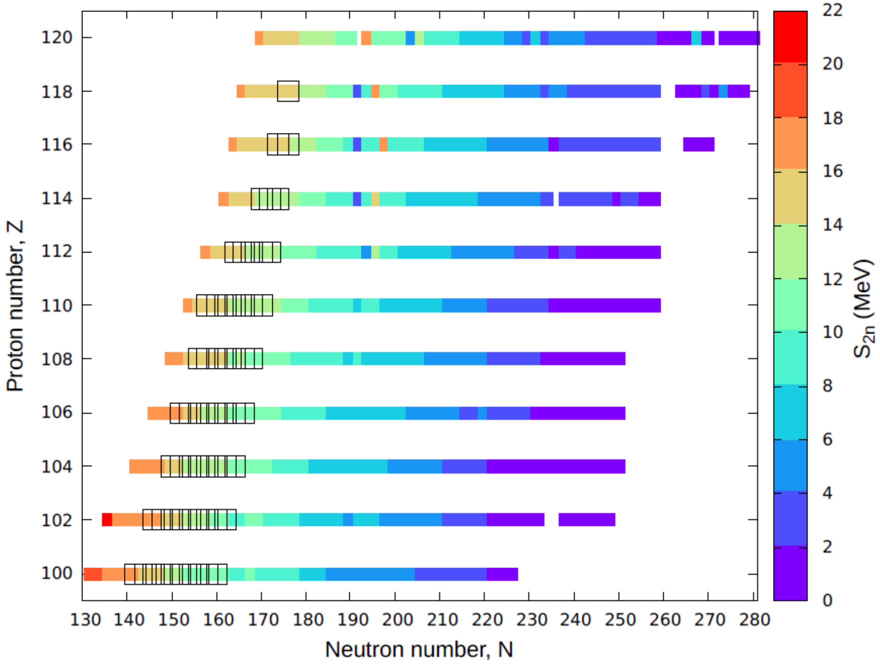
Proton and Neutron Drip Lines





α Decay Half-Lives

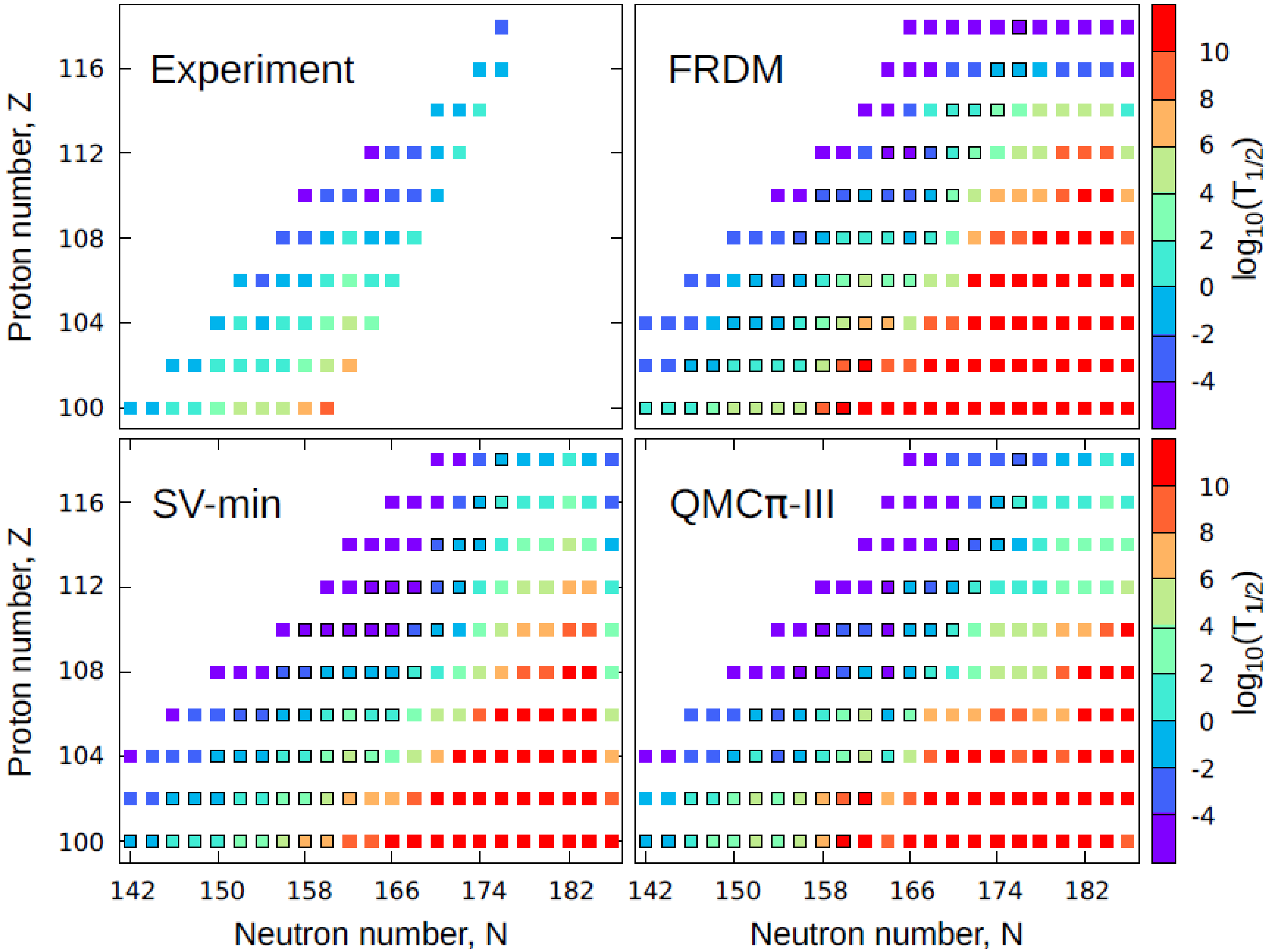


FIGURE 6.10: Comparison of $\log_{10}(T_{1/2})$ predictions from FRDM, SV-min and QMCπ-III along with values obtained from available data.

Not as good as UNDEF1 and SV-min comparable to FRDM

Recall that in QMCπ-II we write the σ field as $\sigma = \bar{\sigma} + \delta\sigma$, which naturally leads to a classical mean part of the σ field Hamiltonian, H^{σ}_{mean} and a fluctuation part H^{σ}_{fluc}. The effective QMC nucleon mass is expressed as before, as $M_{\text{QMC}}(\bar{\sigma}) = M - g_{\sigma}\bar{\sigma} + \frac{d}{2}(g_{\sigma}\bar{\sigma})^2$, where g_{σ} is the coupling of the nucleon to the σ meson in free space, d is the scalar polarizability, and the classical σ field satisfies the wave equation,

$$-\nabla^2\bar{\sigma} + \frac{dV(\bar{\sigma})}{d\bar{\sigma}} = -\left\langle \frac{\partial K}{\partial\bar{\sigma}} \right\rangle,$$

where K is the relativistic nucleon kinetic energy, including its mass. The potential $V(\bar{\sigma})$ is expressed as in QMCπ-II, where it adds an additional parameter λ_3 to account for the self-coupling of the σ meson. One of the main improvements in this new version is that we employ the full expansion for the σ field solution, $g_{\sigma}\bar{\sigma}$, instead of using a Padé approximant. This solution can be explicitly written in terms of the particle density ρ and the kinetic energy density τ as

$$g_{\sigma}\bar{\sigma} = v(\rho, \tau, \nabla^2\rho, (\vec{\nabla}\rho)^2) = v_0(\rho) + v_1(\rho)\tau + v_2(\rho)\nabla^2\rho + v_3(\rho)(\vec{\nabla}\rho)^2, \tag{1}$$

where

$$v_0 = \frac{-(1 + G_{\sigma}d\rho) + \sqrt{(1 + G_{\sigma}d\rho)^2 + 2G_{\sigma}^2\lambda_3\rho}}{\lambda_3 G_{\sigma}},$$
$$v_1 = \frac{-v_0'(\rho)}{2M^2_{\text{QMC}}(v_0(\rho))},$$
$$v_2 = \frac{1}{\lambda_3 G_{\sigma} v_0(\rho) + (1 + dG_{\sigma}\rho)}\frac{v_0'(\rho)}{m_{\sigma}^2} + \frac{v_0'(\rho)}{4M^2_{\text{QMC}}(v_0(\rho))},$$
$$v_3 = \frac{1}{\lambda_3 G_{\sigma} v_0(\rho) + (1 + dG_{\sigma}\rho)}\frac{v_0''(\rho)}{m_{\sigma}^2}. \tag{2}$$

As before, the coupling parameter is defined as $G_{\sigma} = g_{\sigma}^2/m_{\sigma}^2$ where the σ meson mass m_{σ} is taken as a free parameter in the model. Using the expressions for H^{σ}_{mean} and H^{σ}_{fluc} in Ref. [9] and upon simplification using the new expressions for $g_{\sigma}\bar{\sigma}$ and $M_{\text{QMC}}(\bar{\sigma})$, we then solve for the expectation value of the σ Hamiltonian.

The new σ contribution to the total QMC Hamiltonian is now expressed as

$$\left\langle H^{\sigma}_{\mathrm{QMC}\pi-III}\right\rangle = h_0(\rho) + h_4(\rho)\left(J_p^2 + J_n^2\right) + \sum_{f=p,n} h_1^f(\rho_p, \rho_n)\tau_f + \sum_{f=p,n} h_2^f(\rho_p, \rho_n)\nabla^2\rho_f + \sum_{f,g=p,n} h_3^{fg}(\rho_p, \rho_n)\vec{\nabla}\rho_f \cdot \vec{\nabla}\rho_g,$$

where the coeffients are defined as

$$h_0(\rho) = M_{\mathrm{QMC}}(v_0)\rho + \frac{1}{2G_\sigma}v_0^2 + \frac{\lambda_3}{3!}v_0^3 + \frac{1}{4}G_\sigma(1 - dv_0)^2\left(\rho_p^2 + \rho_n^2\right),$$

$$h_1^f(\rho_p, \rho_n) = \frac{1}{2M_{\mathrm{QMC}}(v_0)} - \frac{1}{4}\left[\frac{2dv_1G_\sigma(1 - dv_0)^2}{1 - dv_0}\right](\rho_p^2 + \rho_n^2) - \frac{1}{2}q(\rho)\rho_f,$$

$$h_2^f(\rho_p, \rho_n) = -\frac{1}{4M_{\mathrm{QMC}}(v_0)} - \frac{1}{4}\left[\frac{2dv_2G_\sigma(1 - dv_0)^2}{1 - dv_0}\right](\rho_p^2 + \rho_n^2) + \frac{1}{4}q(\rho)\rho_f,$$

$$h_3^{fg}(\rho_p, \rho_n) = \frac{v_0'^2}{2m_\sigma^2 G_\sigma} - \frac{1}{4}\left[\frac{2dv_3G_\sigma(1 - dv_0)^2}{1 - dv_0} + p'^2\right](\rho_p^2 + \rho_n^2) + \delta(f, g)\frac{1}{8}p^2,$$

$$h_4(\rho) = \frac{1}{4}p^2,$$

with $p(\rho) = \frac{-\sqrt{G_\sigma}(1-dv_0)}{m_\sigma}$ and $q(\rho) = (1 + \frac{m_\sigma^2}{2M^2{}_{\mathrm{QMC}}(v_0)})p^2$.