DE LA RECHERCHE À L'INDUSTRIE

Multichannel Compressed Sensing and its application on radioastronomy

Journées des Thésards 2016 - 7 et 8 Juillet

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http://www.cosmostat.org/

www.cea.fr

Who am I?

Ming JIANG

- **Education**

  2010 - 2014: Télécom Bretagne (Master of Engineering)

  • Information Processing Systems (Option: Signal/Image processing) on 3rd year

  2013 - 2014: Université de Rennes 1 (Master of research)

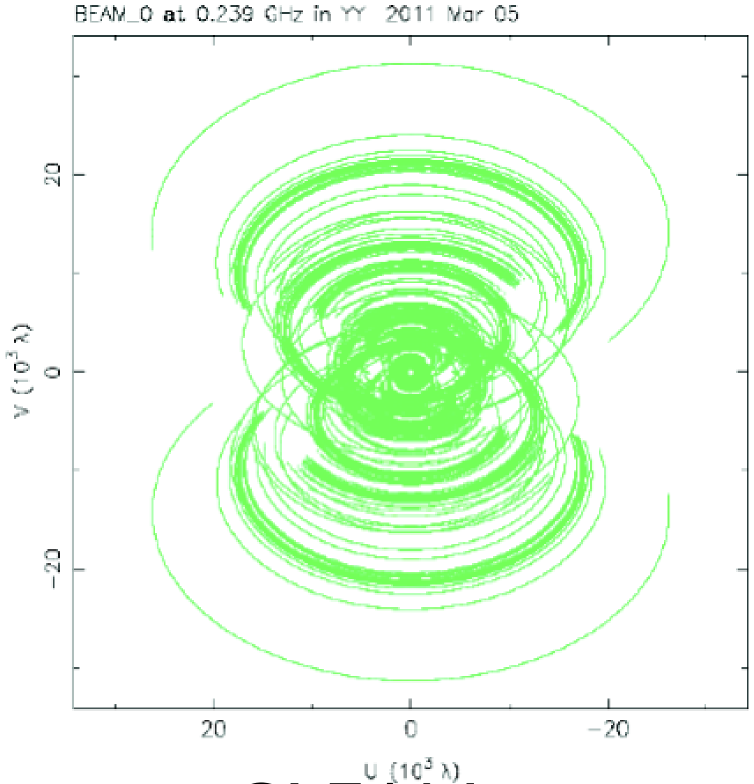
  • SISEA - I(Image processing)

- **Ph.D candidate**

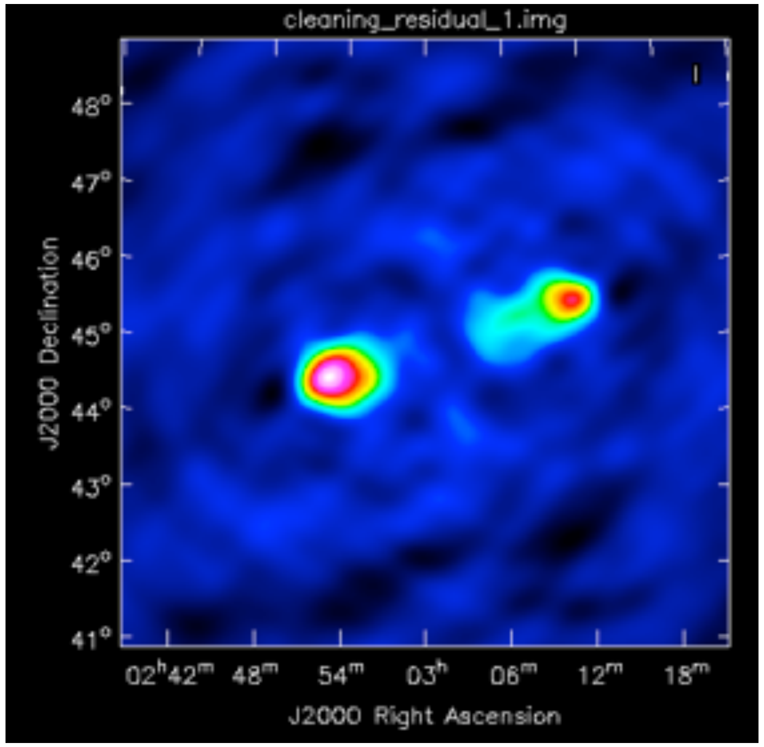
  • Motivation: Interdisciplinary research on signal processing algorithms, applied mathematics, applications in practice, etc.

Compressed Sensing & LOFAR Cygnus A Data

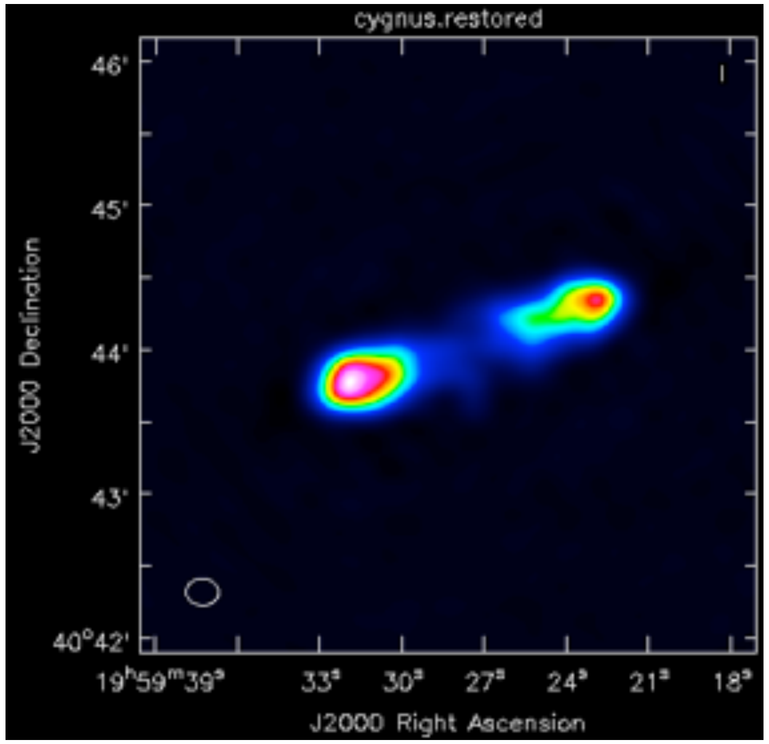
Data



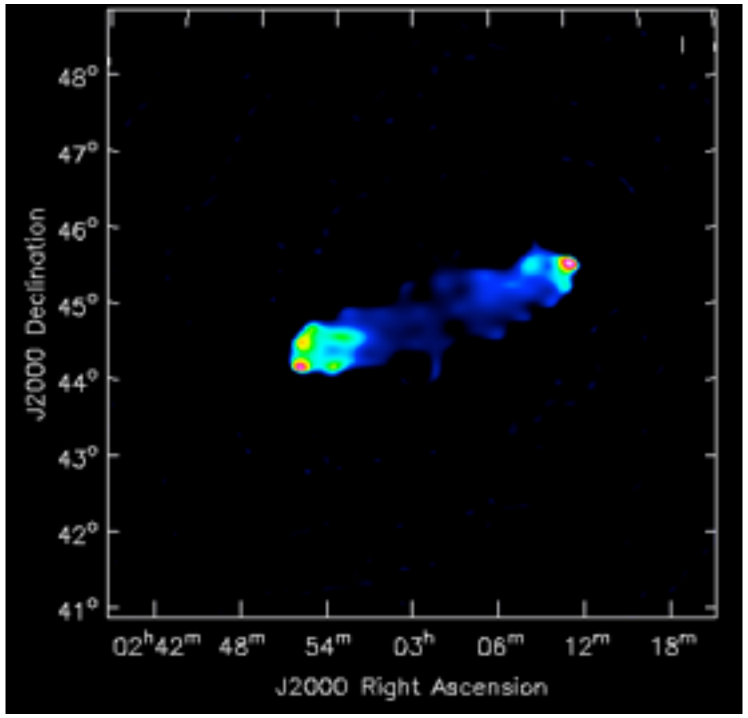
Dirty map



CLEAN



Compressed Sensing



Garsden et al, "**LOFAR Image Sparse Reconstruction**", A&A, 575, A90, 2015. http://arxiv.org/abs/1406.7242

Compressed Sensing & LOFAR Cygnus A Data

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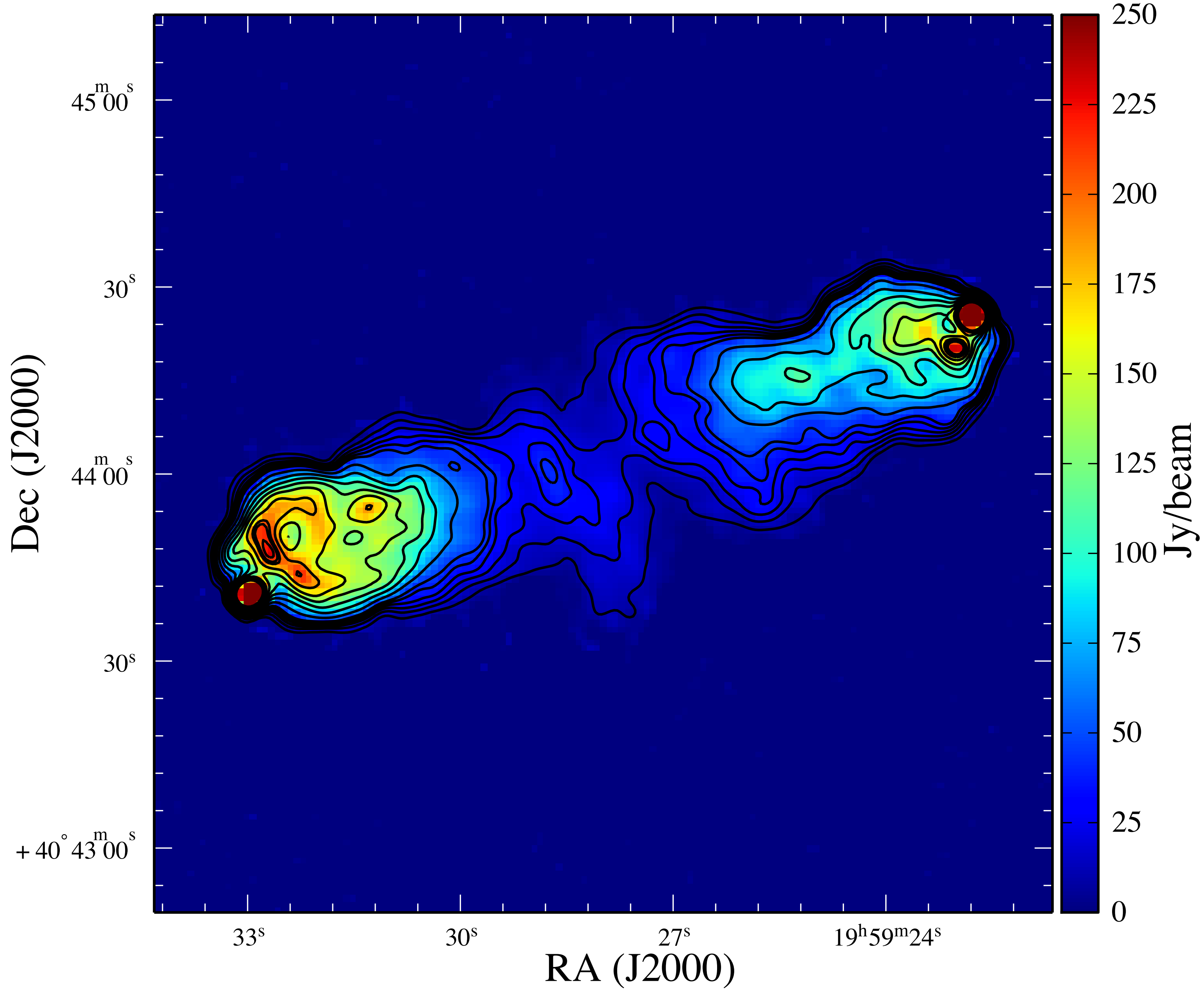
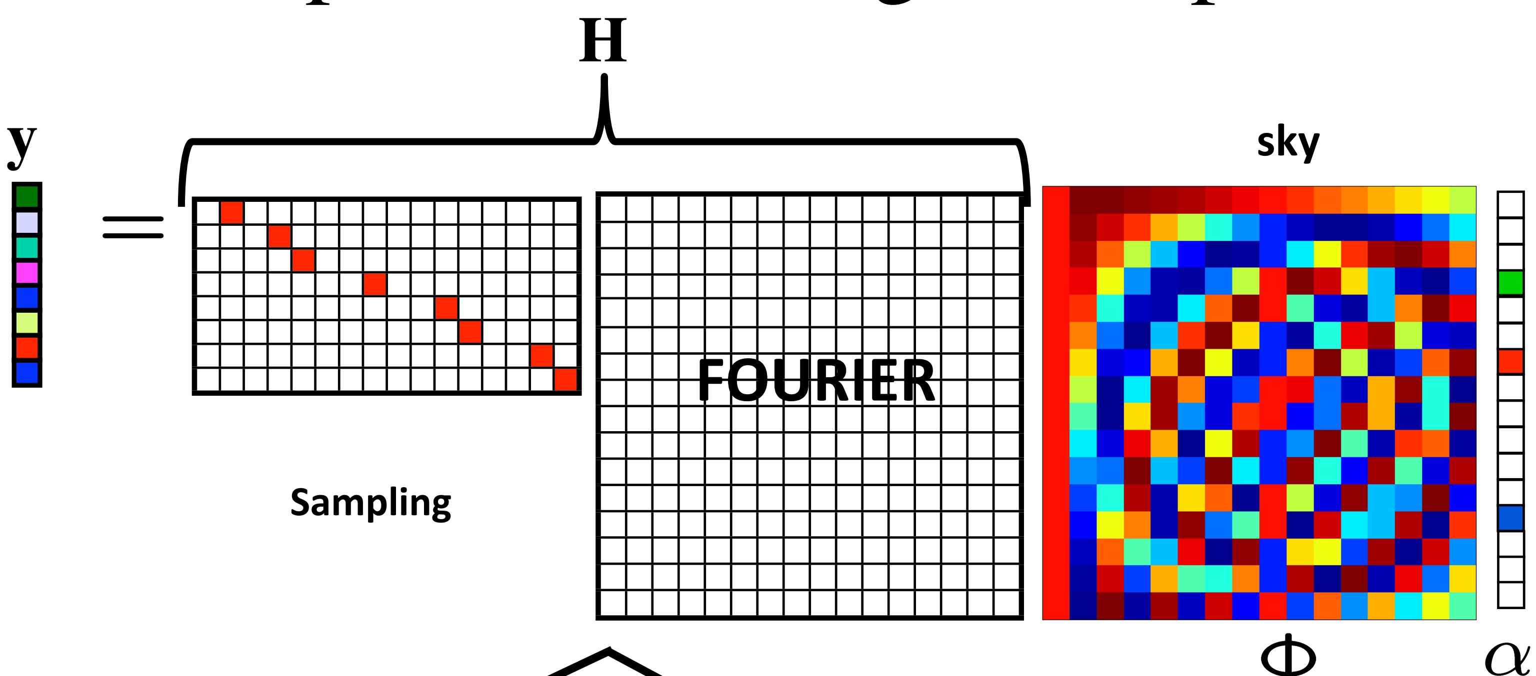


Compressed Sensing Concept



$$X = \Phi\alpha$$

sparse

2D+time

2D-1D sparse reconstruction and transient detection

1st year

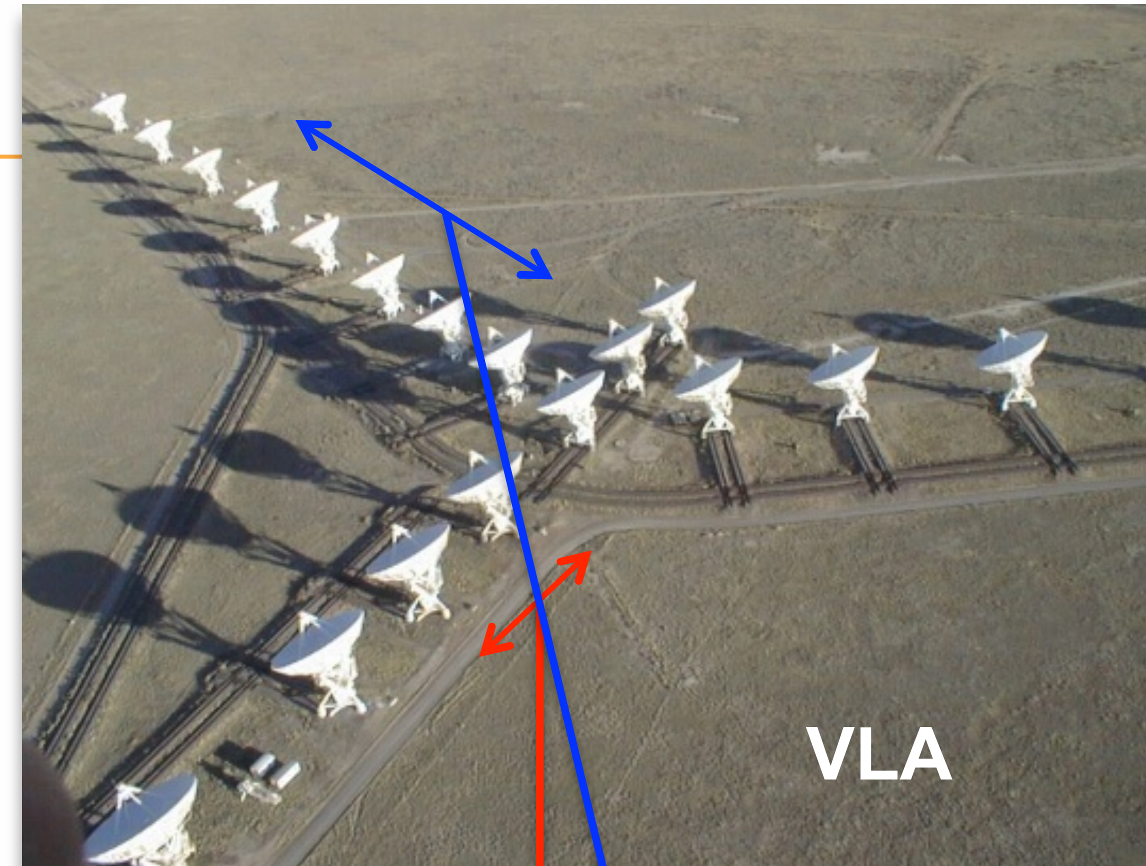
2D+λ

Multi/Hyper spectral image restoration

2nd year

Applications

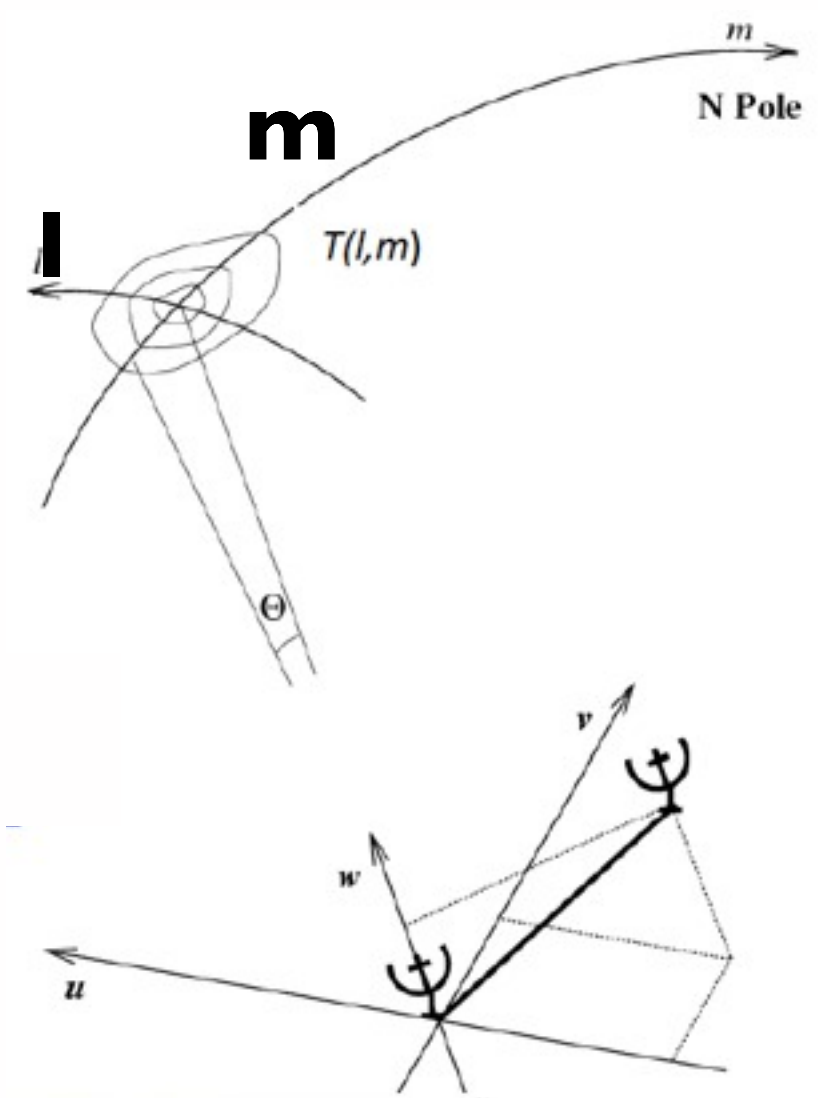
Imaging with interferometry

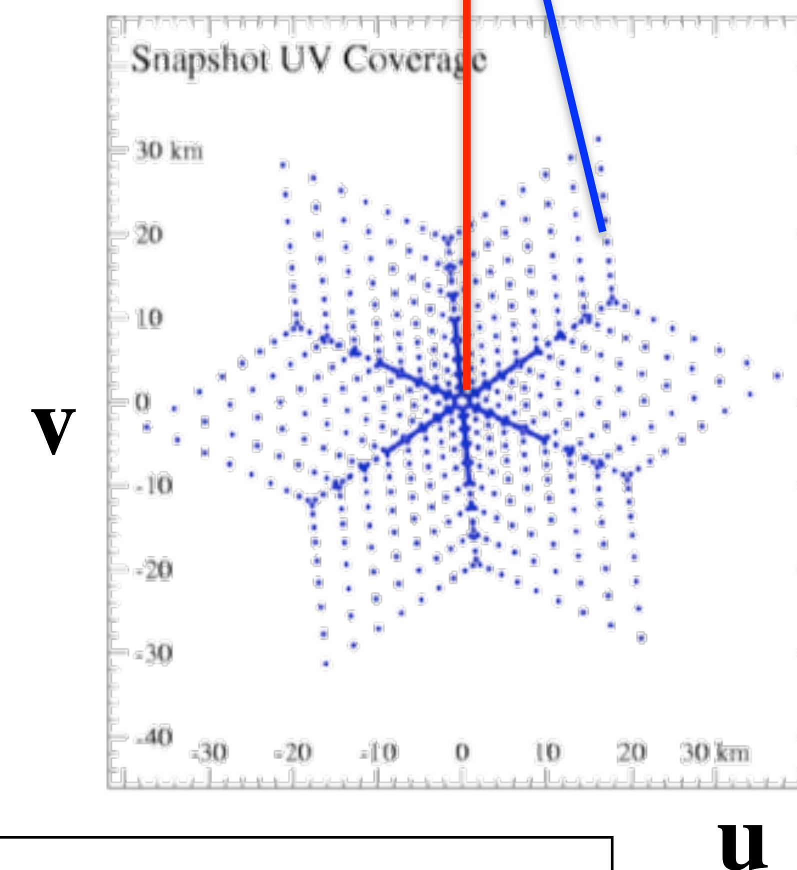


N **antennas/telescopes**

$\frac{N(N-1)}{2}$ **independent baselines**

1 projected baseline
= 1 sample in the Fourier « u,v » plane





(u,v) plane sampling

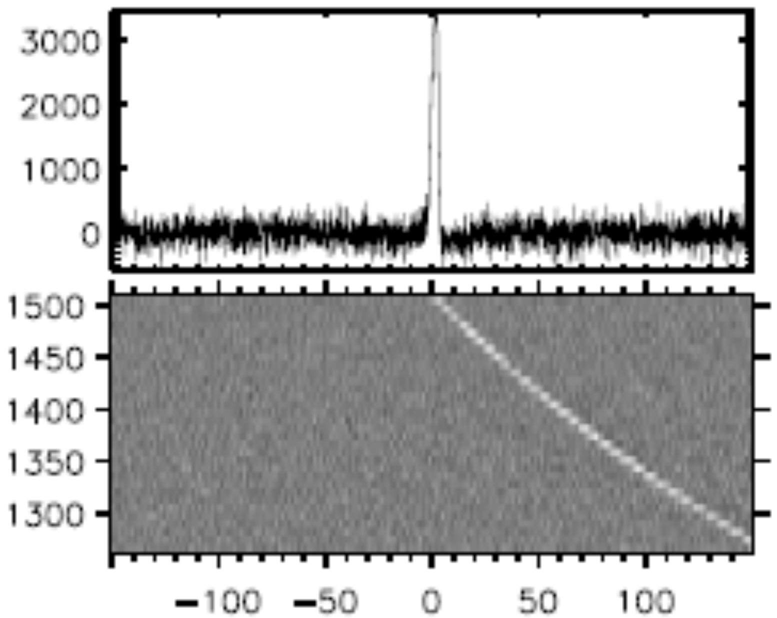
$$\boxed{V(u,v) = \int\int T(l,m)e^{-i2\pi(ul+vm)}dldm} \longrightarrow T(l,m)$$

The Transient Universe in radio

<<1ms 1s 10s 1m 10 m H Days Weeks M Y

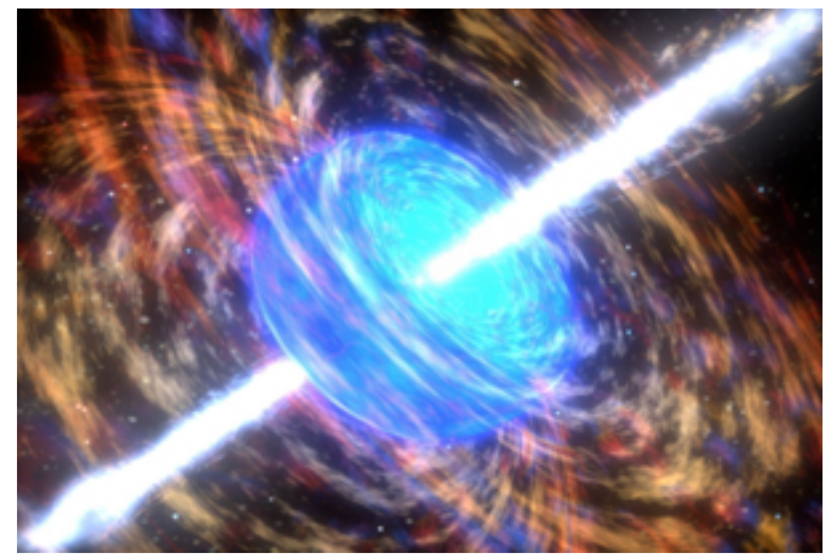
Variability timescale

- **Fast transient sources**



☺ **Good time resolution**
☹ **No/low angular res.**
☹ **Low instantaneous SNR**
☹ **few samples for imaging**

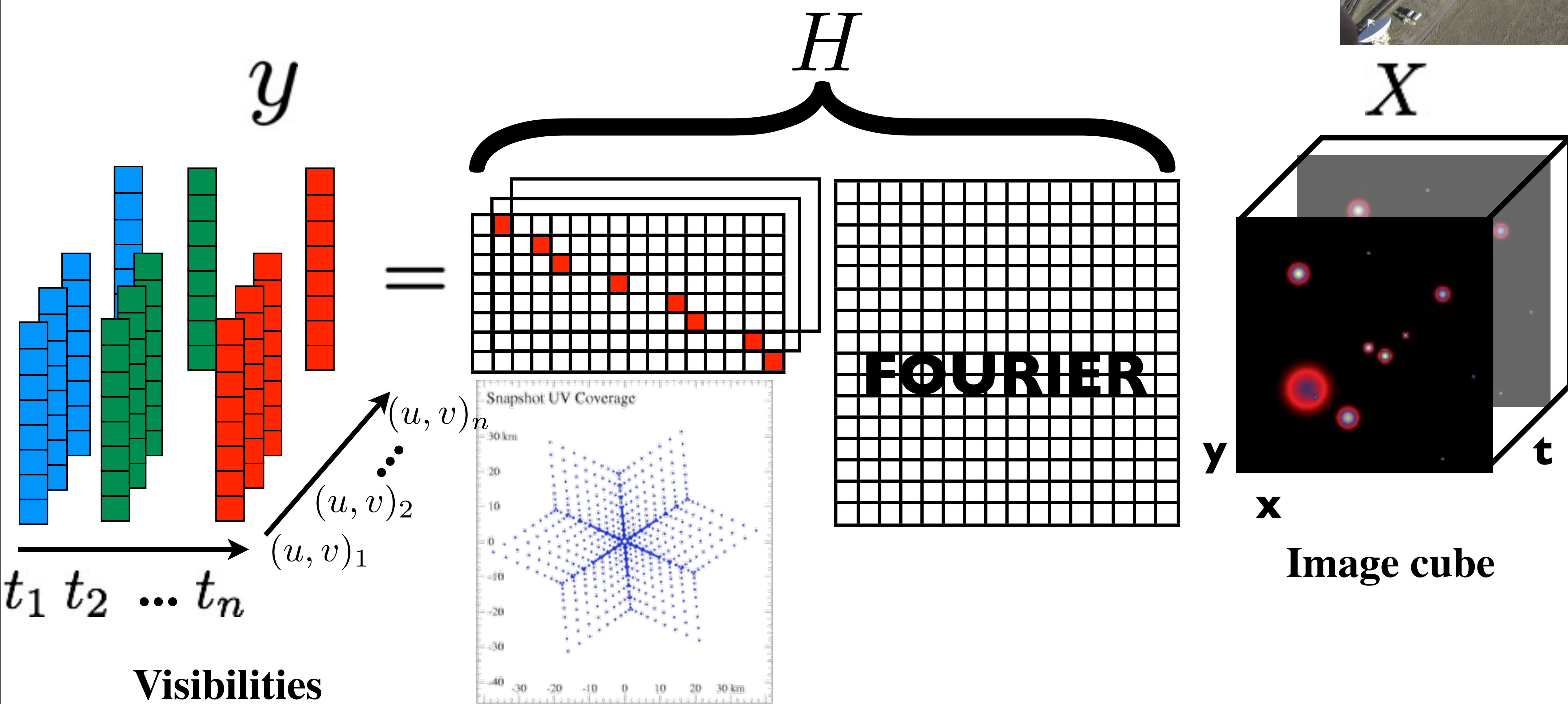
- **Slow transient sources**



☹ **Poor time resolution**
☺ **Good angular resolution**
☺ **Good integrated SNR**
☺ **enough samples for imaging**

+ others problems (instrument stability, ionosphere...)

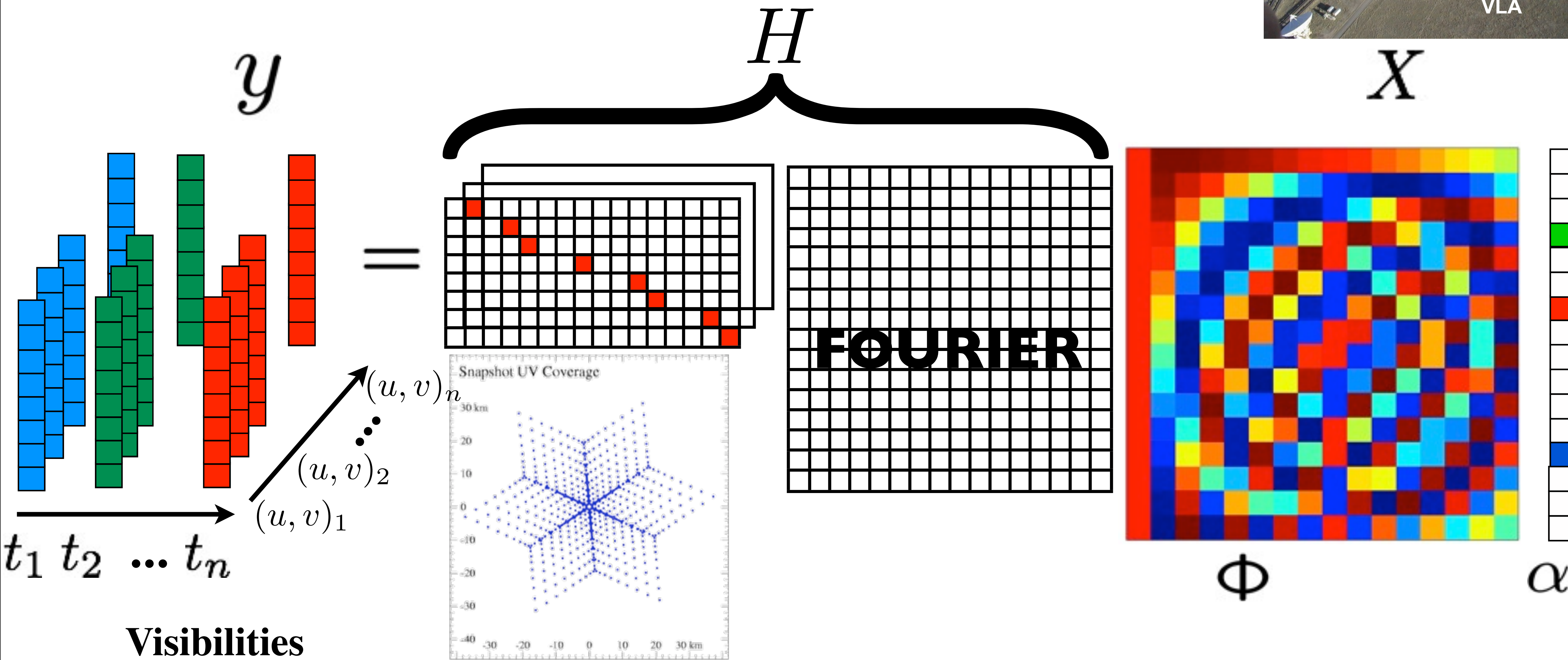
Extension to 2D-1D sparse reconstruction



Measurement matrix
(Fourier + Sampling)

- **The masking operator will also be time-dependent**

Extension to 2D-1D sparse reconstruction



Measurement matrix (Fourier + Sampling)

$$X = \Phi\alpha$$

e.g. Wavelets Tr. Sparse

- **The masking operator will also be time-dependent**

Extension to 2D-1D sparse reconstruction

- **Inverse problem formulation**

$$\mathbf{V} = \mathbf{MFx} + \mathbf{N}$$

- M: 2D-1D mask
- F: Fourier transform
- N: Gaussian noise

$$\min \|\mathbf{\Phi}^t \mathbf{x}\|_1 \quad s.t. \quad \|\mathbf{V} - \mathbf{MFx}\|_2^2 < \epsilon$$

- **Analysis framework**

Data fidelity

Positivity

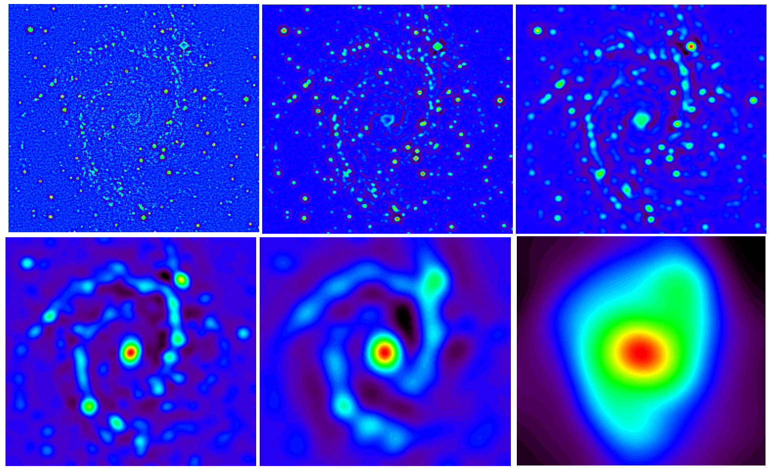
$$\min_x \| V - MFx \|_2^2 + \| W \odot \lambda \odot \Phi^t x \|_1 + i_{P_+}(x)$$

Sparsity constraint

We need **dictionaries** - space and time are independent

$$\psi(x,y,t) = \psi^{(xy)}(x,y)\psi^{(t)}(t)$$

- **for the 2D spatial signal**



Starlets [Starck et al. 2011]

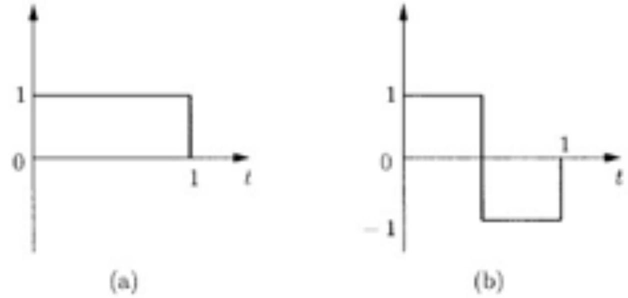
(Isotropic Undecimated Wavelet Transform)

$$\varphi = B_3 - \text{spline}, \quad \frac{1}{2}\psi(\frac{x}{2}) = \frac{1}{2}\varphi(\frac{x}{2}) - \varphi(x)$$

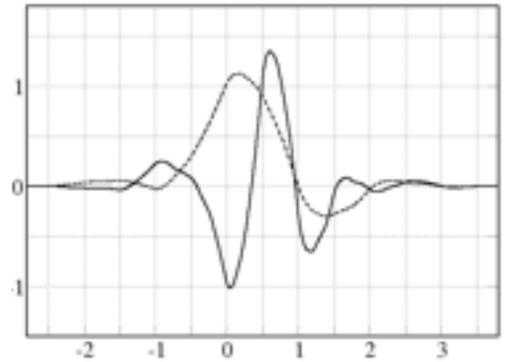
$$h = [1,4,6,4,1]/16, \quad g = \delta - h, \quad \tilde{h} = \tilde{g} = \delta$$

$$I(k,l) = c_{J,k,l} + \sum_{j=1}^{J} w_{j,k,l}$$

- **for the 1D temporal signal**



Haar wavelets Quantified signals



7/9 wavelets Semi-continuous

...

We need Optimization methods

- **Condat-Vu Splitting Method(Condat 2013; Vu, 2013)**

$$\min_x \| V - MFx \|_2^2 + \| W \odot \lambda \odot \Phi^t x \|_1 + i_{P_+}(x) \qquad (1)$$

- Initialize $x^{(0)}, u^{(0)}$
- Iterate i=1,...,Niter

  - Gradient step to update x

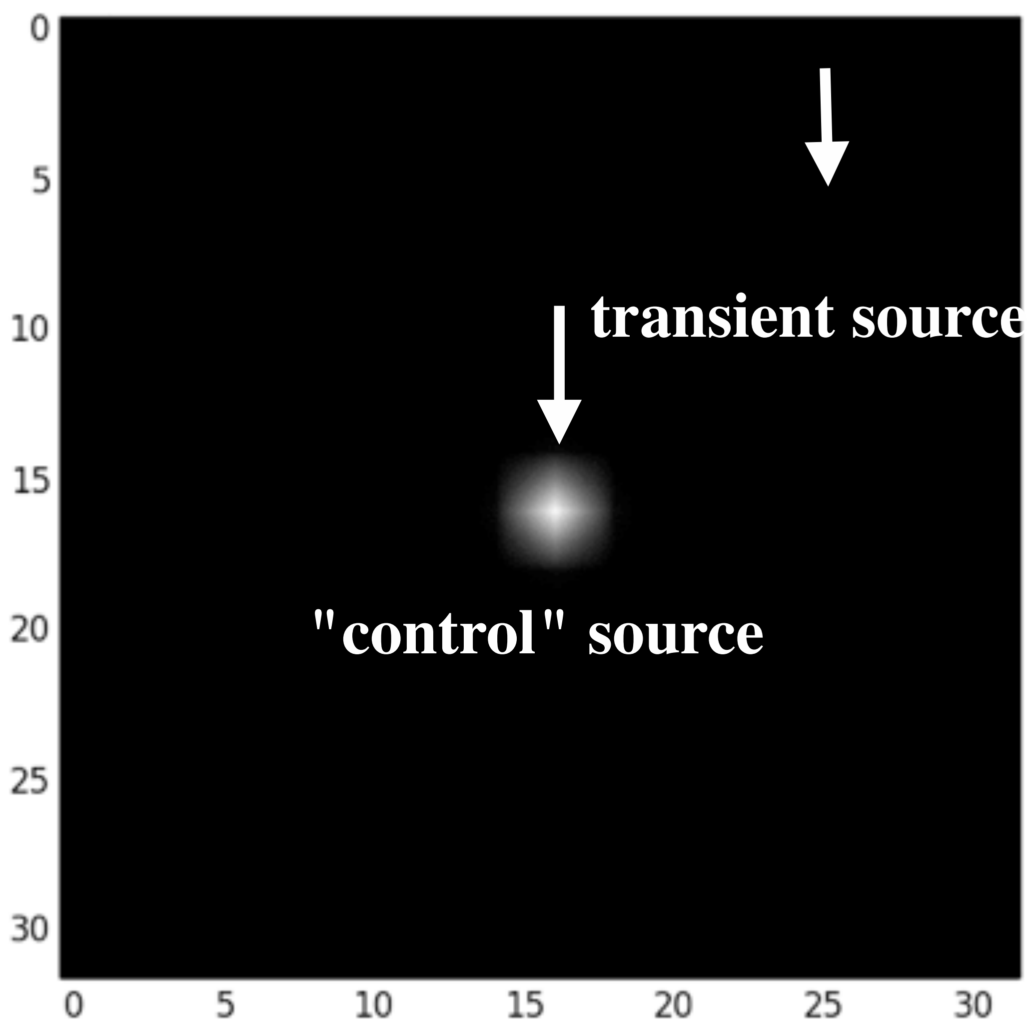
$$x^{(n+1)} = \mathrm{Proj}_{P^+}(x^{(n+1)} - \tau\Phi u^{(n)} + \tau(MF)^*(V - MFx^{(n)}))$$

  - Proximity operation to update wavelet coefficient u

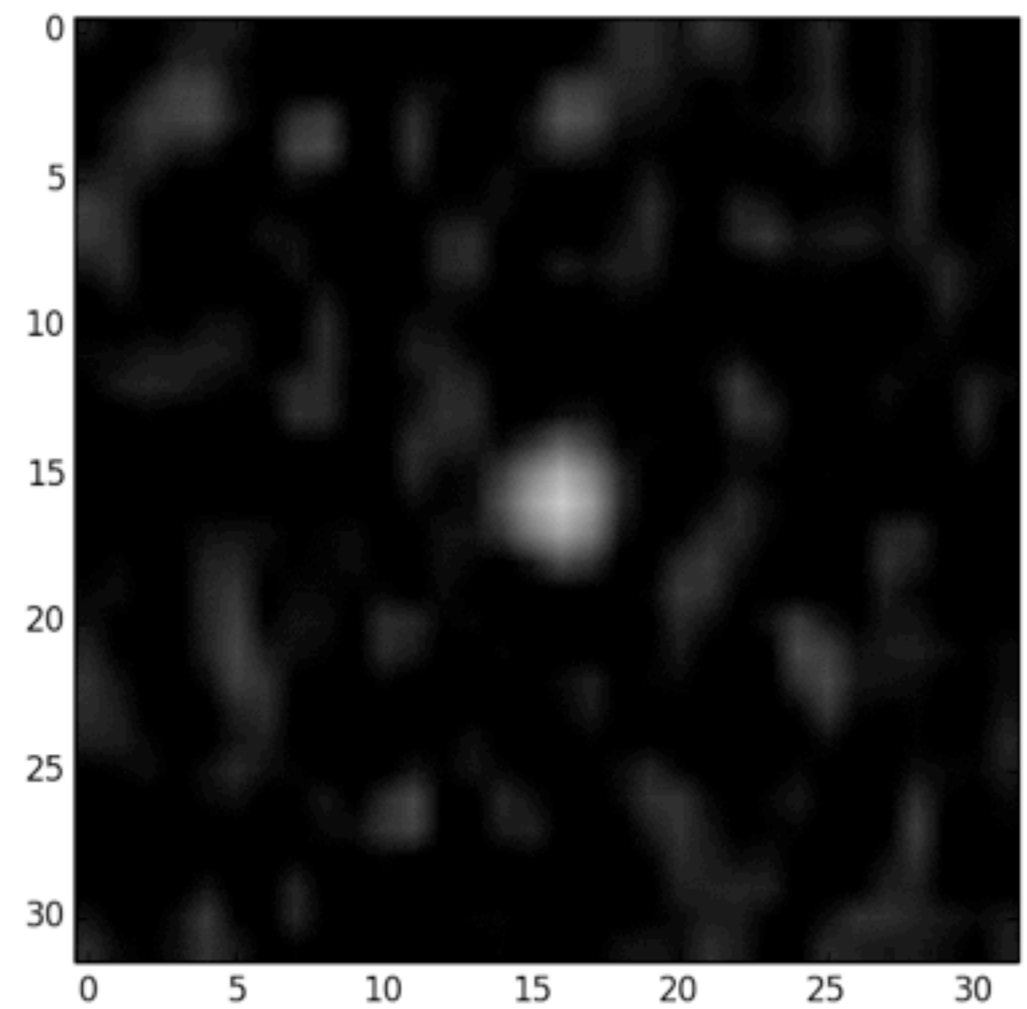
$$u^{(n+1)} = (\mathrm{Id} - \mathrm{ST}_\lambda)(u^{(n)} + \eta\Phi^T(2p^{(n+1)} - x^{(n)}))$$

1) Simulating a transient sky and a radio observation

Sky model



Dirty map



- "control" source: steady

Total flux: 40 FU Max Amp:10 FU

- transient gaussian source:

Flux: 40 FU maximum Max Amp: 10 FU

*FU = arbitrary flux unit

A new source appeared but side lobes as well !

Need a time-agile deconvolution algorithm

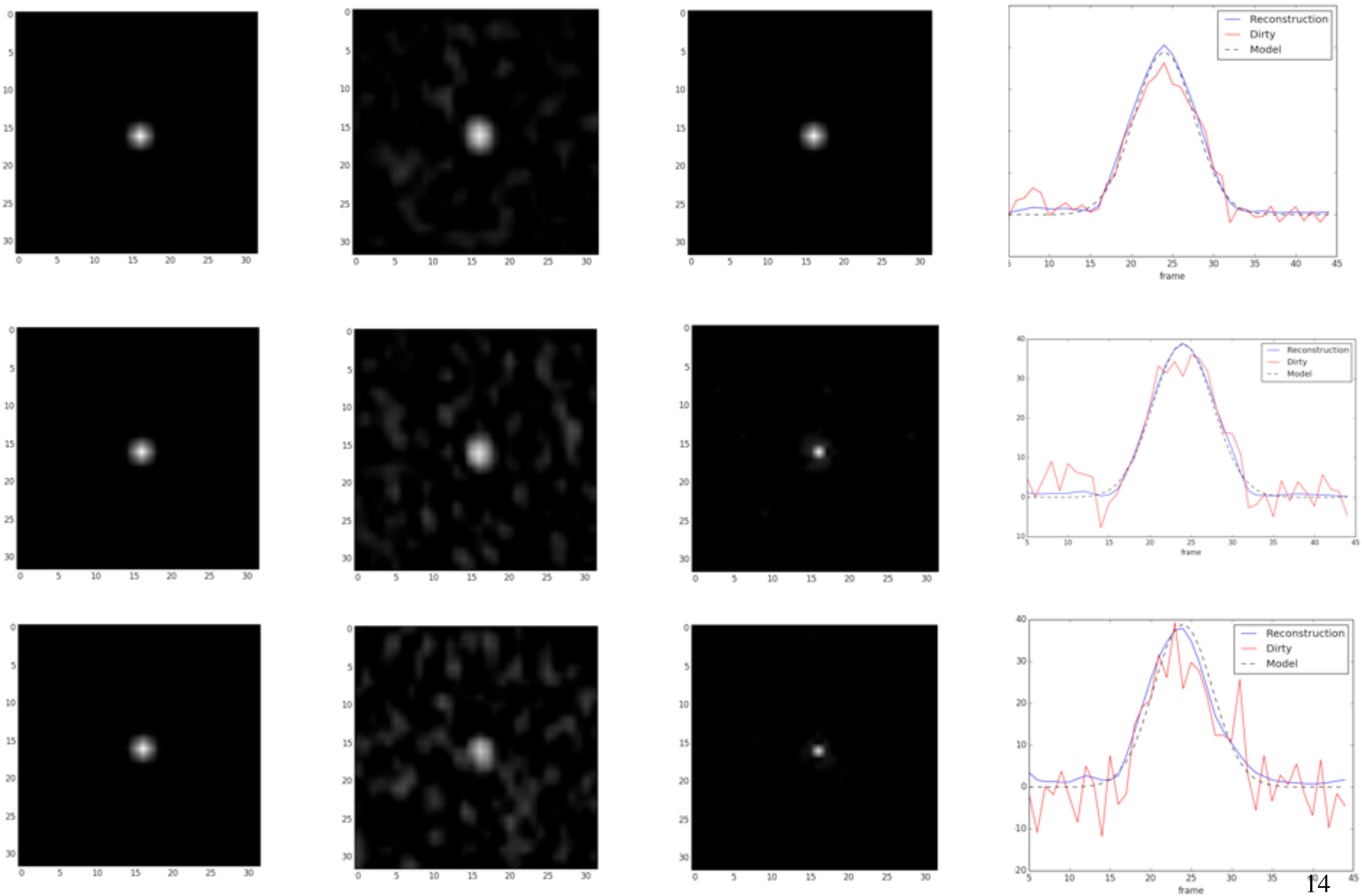
2) Reconstruction for different noisy datasets

Sky model | Dirty map | Reconstruction | Time profile of transient

σ=0.5

σ =1.0

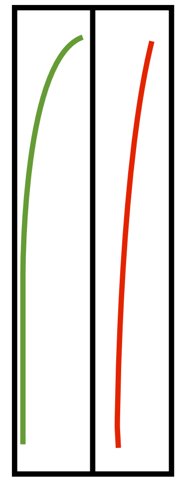
σ =1.5



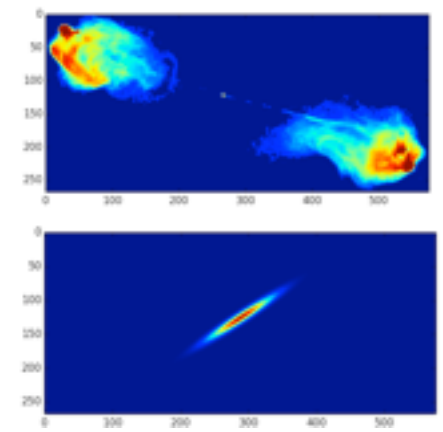
Hyperspectral Data with Source Mixture Model

Ground Truth

A

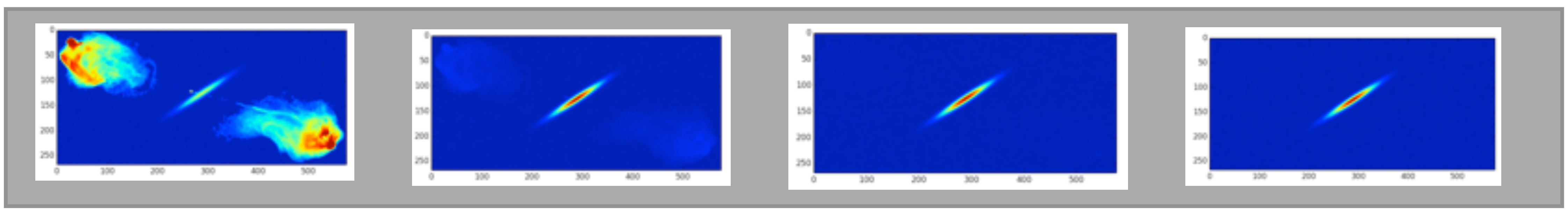


S



Mixtures

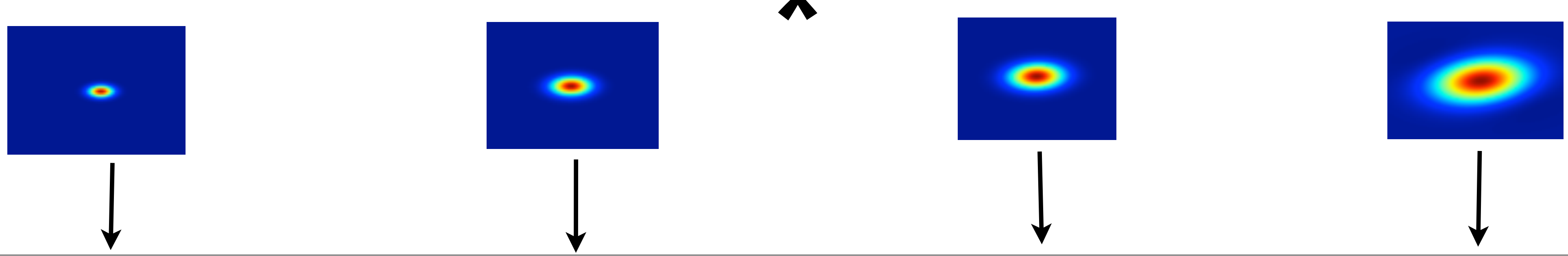
$X = AS$



chan 1 chan 4 chan 7 chan 10

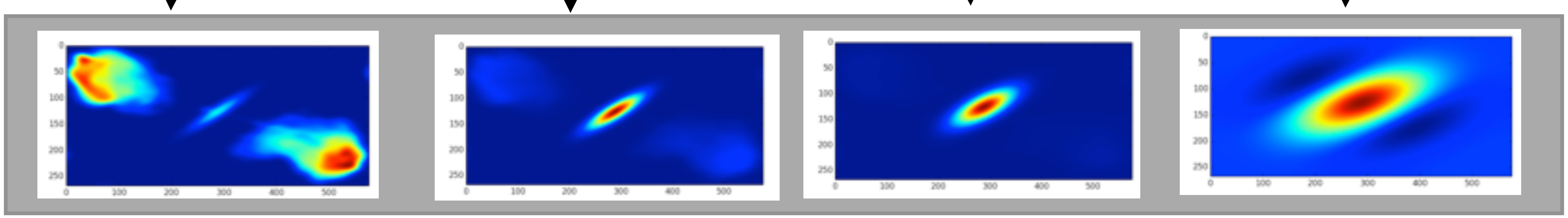
$*$

PSF H



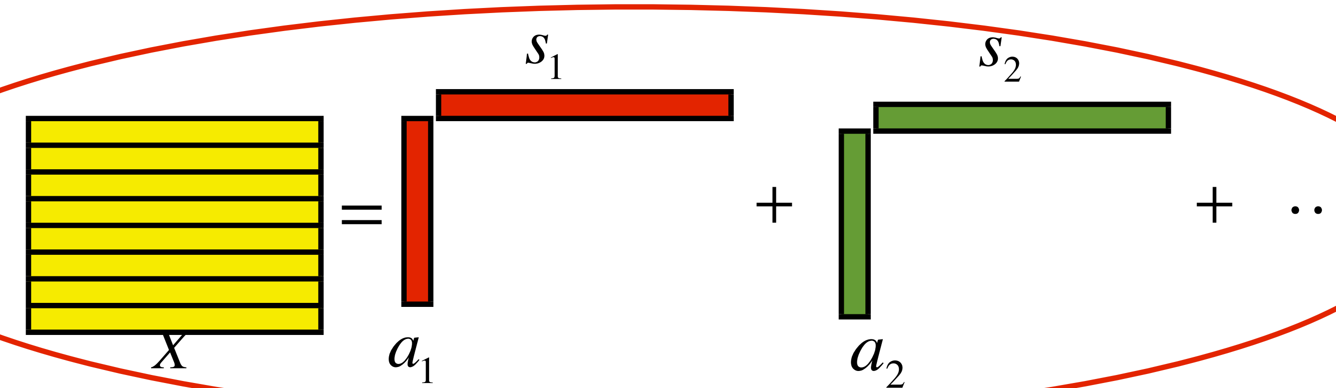
Data

$Y = HX + N$



chan 1 chan 4 chan 7 chan 10

State of the art



- **BSS(Blind Source Separation) problem**

  Statistical approach: ICA (FastICA*(A. HYVARINEN et al.)***), etc.**

  Methods based on morphological diversity: GMCA*(J. BOBIN et al.)*** and its variations**

- **Deconvolution**

  $$Y = H(X) \Rightarrow X = H^{-1}(Y)$$

  e.g. ForWaRD*(R.N. NEELAMANI et al.)*

Joint BSS and Deconvolution?

Very few literatures!

Our method: ForWaRD+GMCA = fGMCA

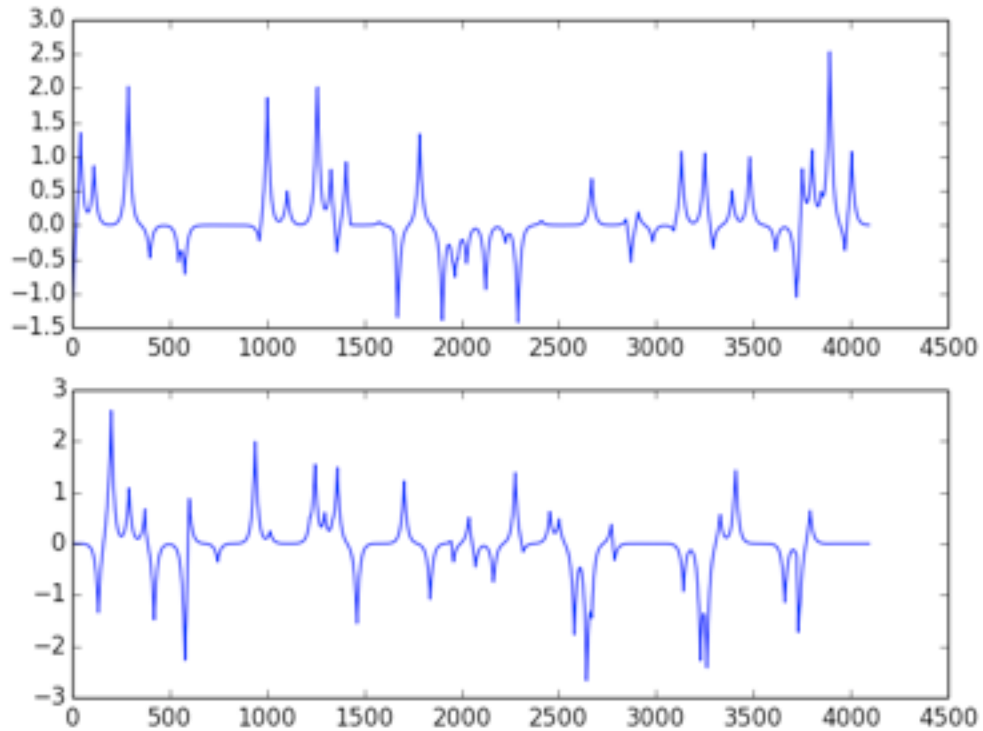
GMCA and ForWaRD

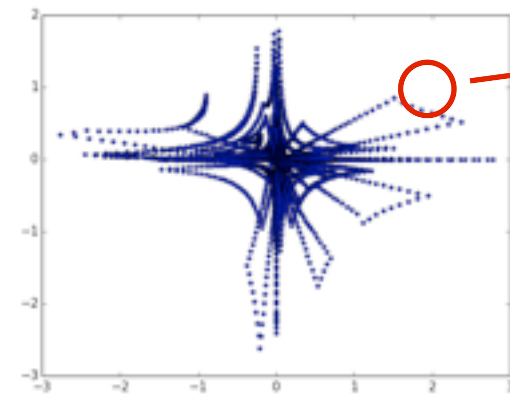
- **GMCA: Generalized Morphological Component Analysis**

1-D signals

S1

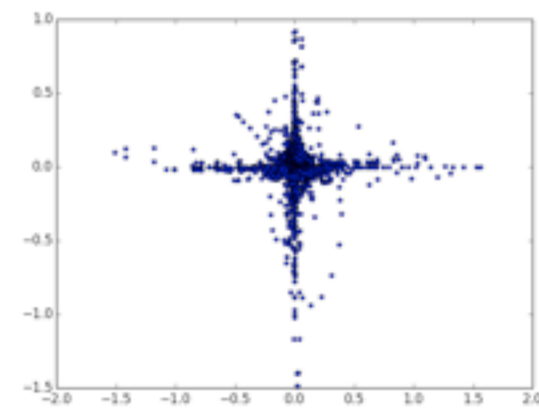
S2





Correlation

S1 v.s. S2 in direct domain

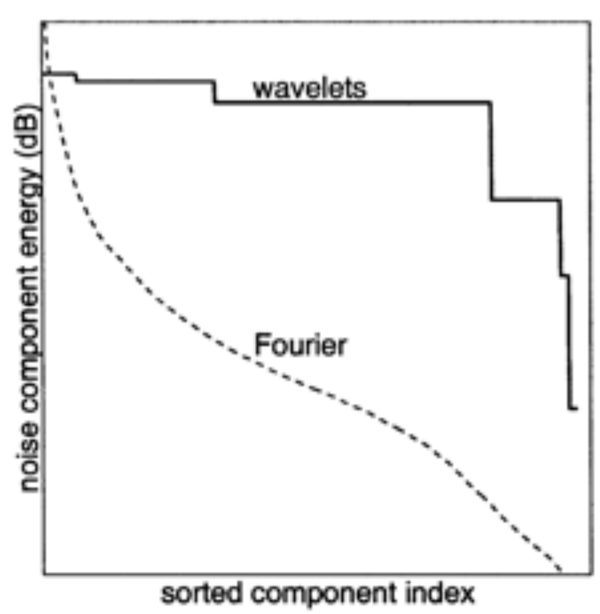


Easier to distinguish sources!

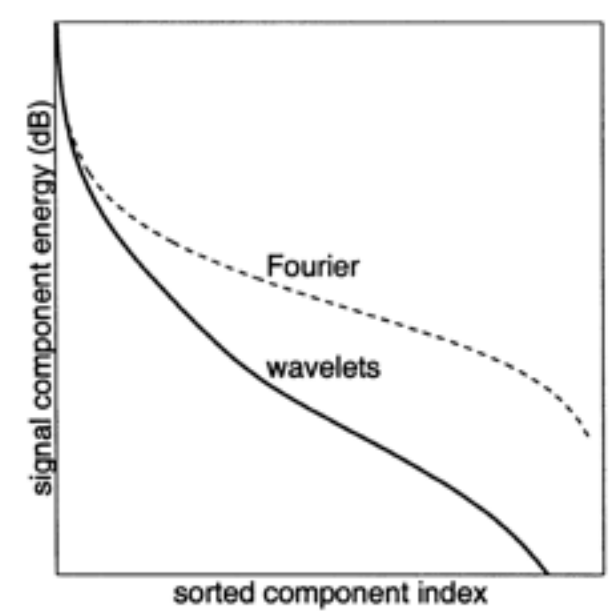
S1 v.s. S2 in wavelet domain

- **ForWaRD: Fourier-Wavelet Regularized Deconvolution**

Fourier domain:
Fourier shrinkage and regularize the ill-conditioned system



Wavelet domain:
Wavelet shrinkage searches for smooth signals and images



[R.N. NEELAMANI et al.]

- **ForWaRD-GMCA algorithm**

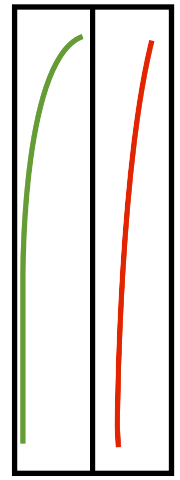
$$\min_{\{S_{j,\bullet}\},\{A_\upsilon\}} \frac{1}{2}\sum_{\upsilon,k} \| V_{\upsilon,k} - H_{\upsilon,k}\mathbf{A}_\upsilon \hat{\mathbf{S}}_k \|_2^2 + \sum_j \lambda_j \| \mathbf{S}_{j,\bullet} \mathbf{\Phi}^t \|_0$$

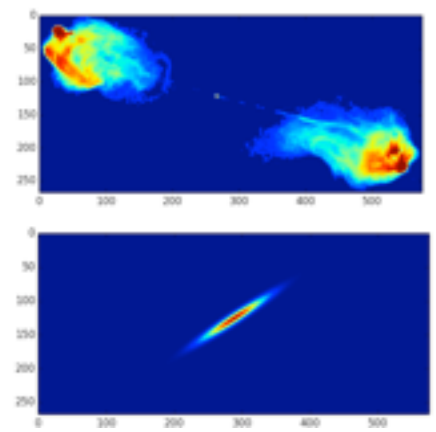
- Initialize $A^{(0)}$
- Iterate i=1,...,Niter
  - Update S knowing A $\min_{\{S_{j,\bullet}\}} \frac{1}{2}\sum_{\upsilon,k} \| V_{\upsilon,k} - H_{\upsilon,k}\mathbf{A}_\upsilon \hat{\mathbf{S}}_k \|_2^2 + \sum_j \lambda_j \| \mathbf{S}_{j,\bullet} \mathbf{\Phi}^t \|_1$
  - Update A knowing S $\min_{\{A_\upsilon\}} \frac{1}{2}\sum_{\upsilon,k} \| V_{\upsilon,k} - H_{\upsilon,k}\mathbf{A}_\upsilon \hat{\mathbf{S}}_k \|_2^2$
  - Decrease the thresholding λ
  - Decrease the Tikhonov parameter ε

Experiments

A S

Ground Truth





Mixtures

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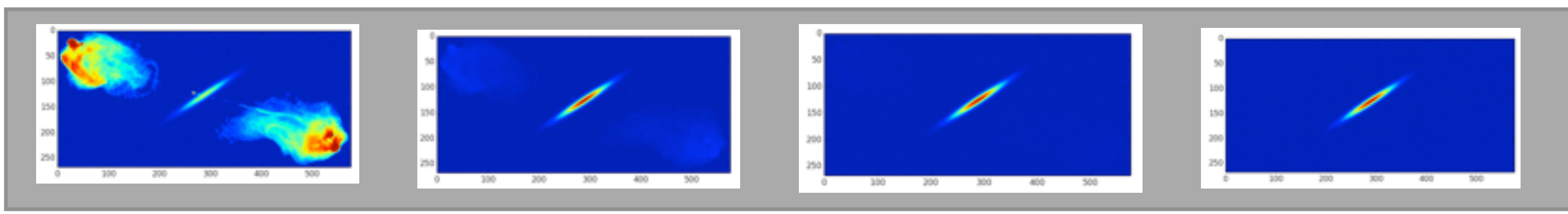
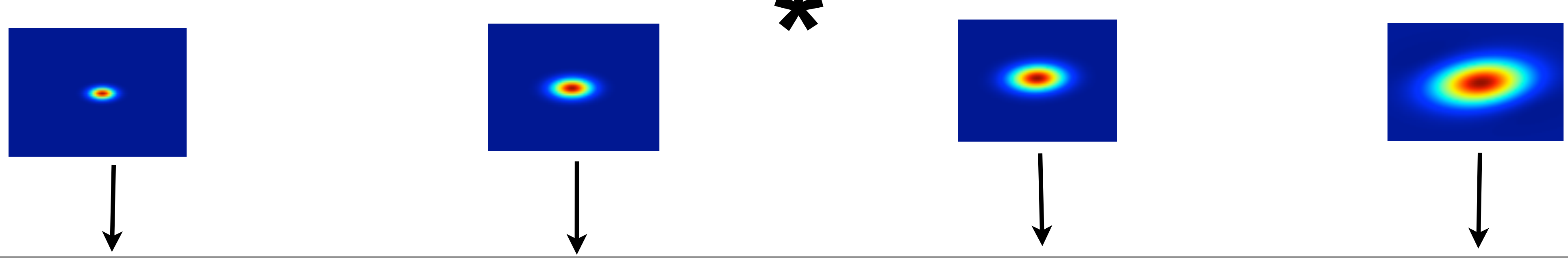


chan 1 chan 4 chan 7 chan 10

$*$

PSF H



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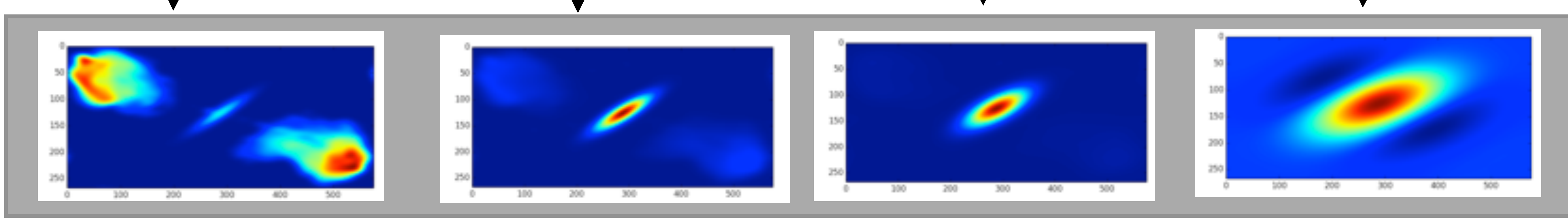
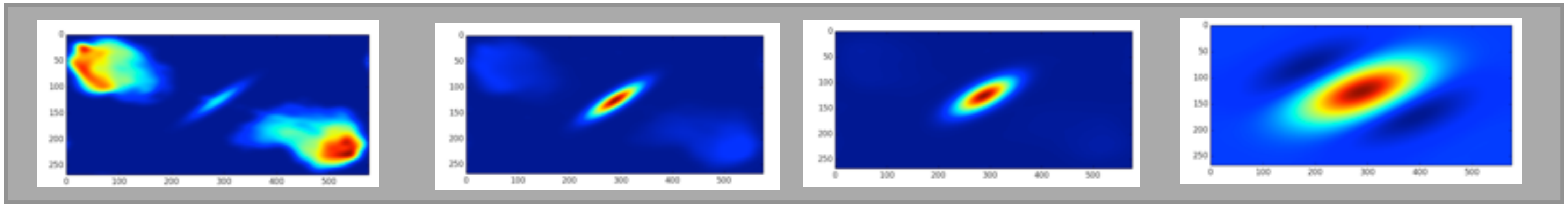


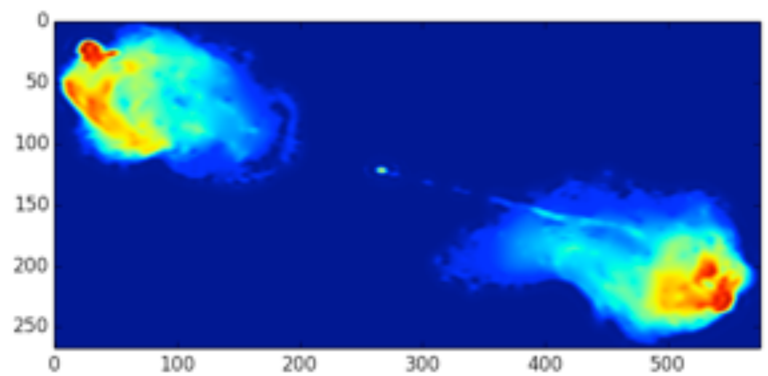
chan 1 chan 4 chan 7 chan 10

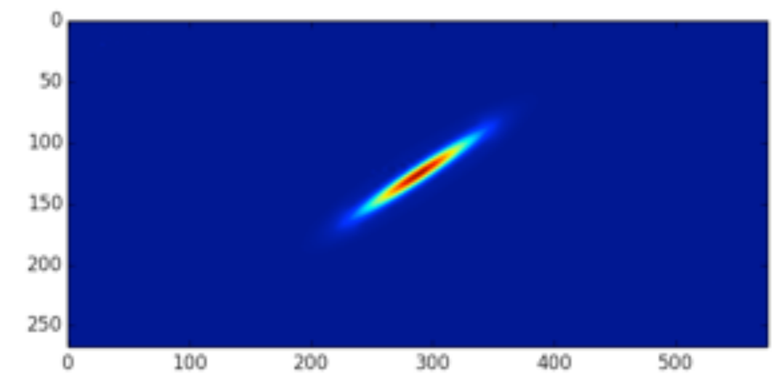
Experiments(Source reconstruction)

Data

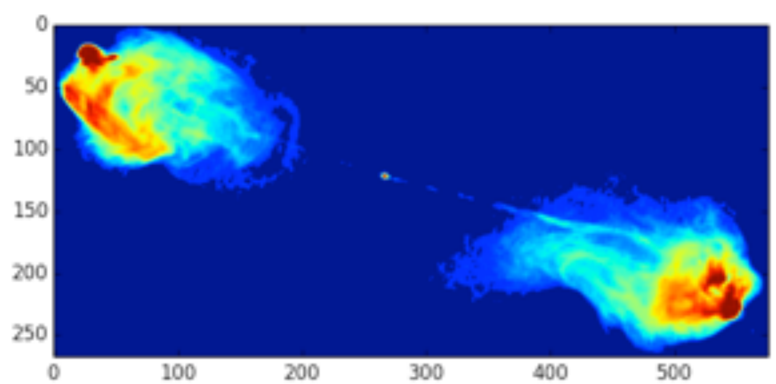


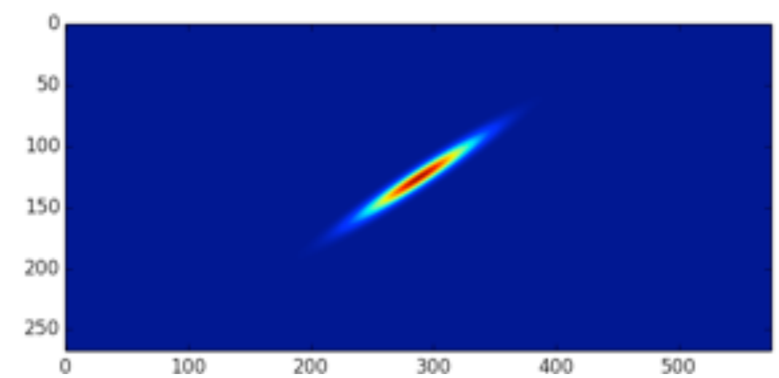
Reconstruction



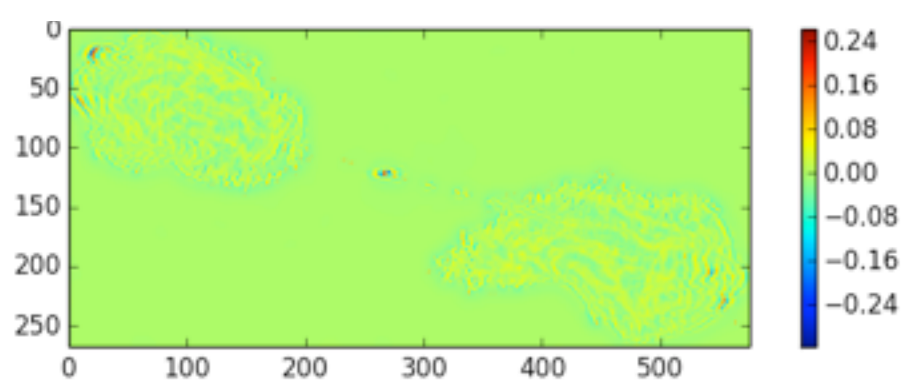


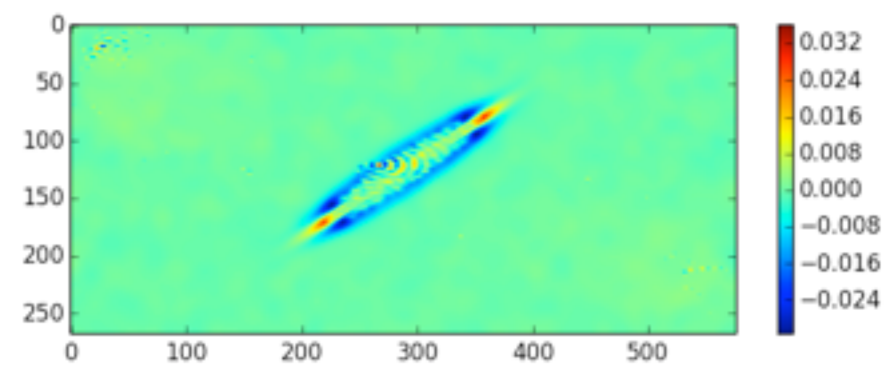
Truth





Error



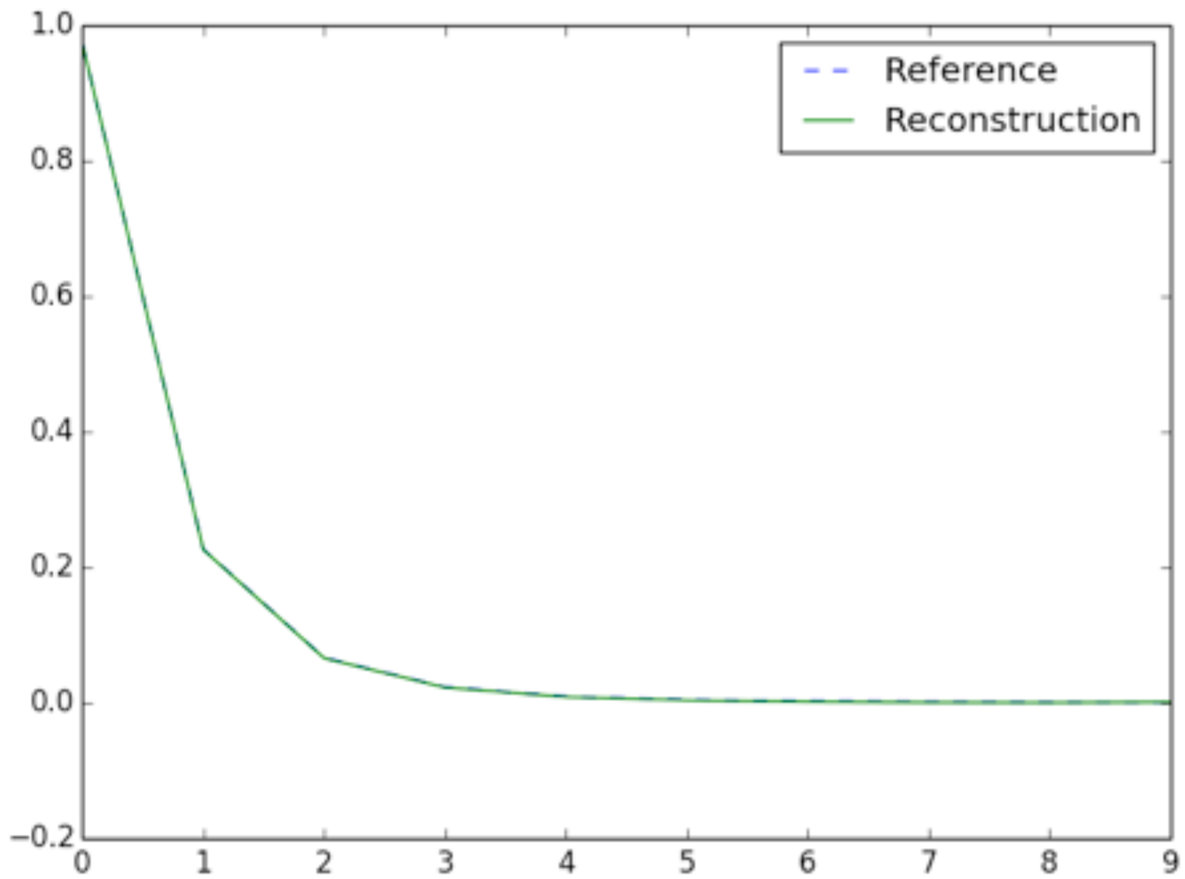


Relative error

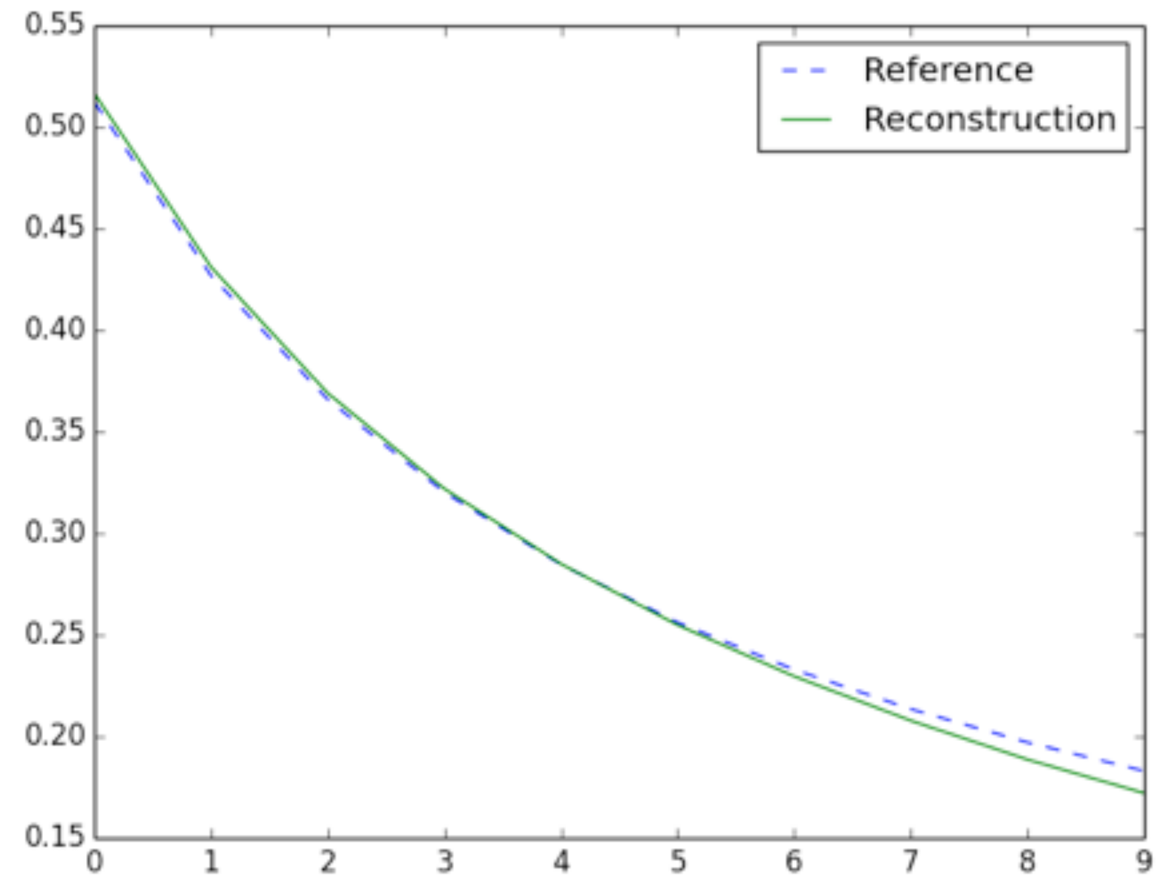
0.42%

0.19%

Experiments(Spectra reconstruction)

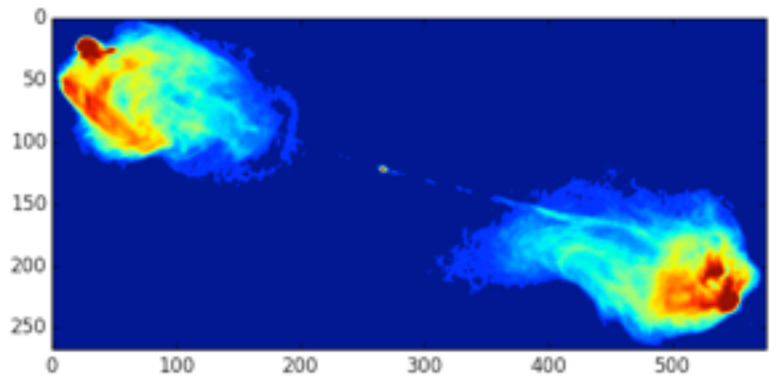


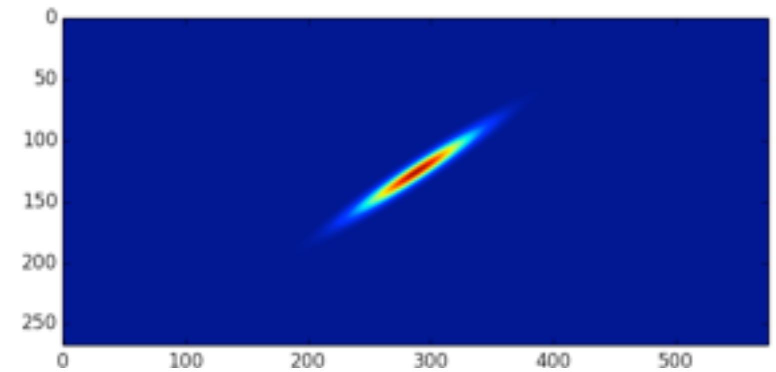
Reconstructed spectrum of S_0 v.s reference



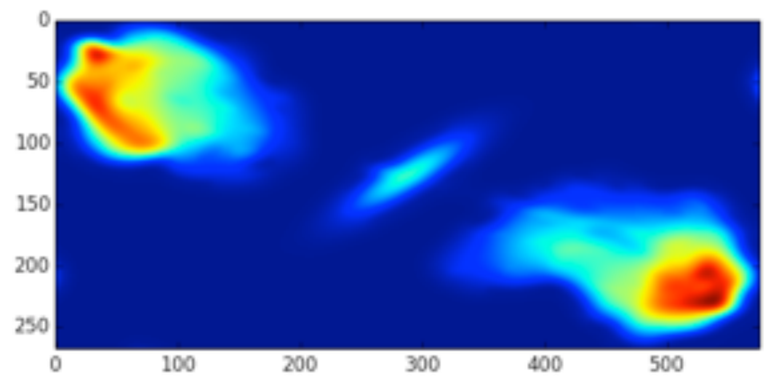
Reconstructed spectrum of S_1 v.s reference

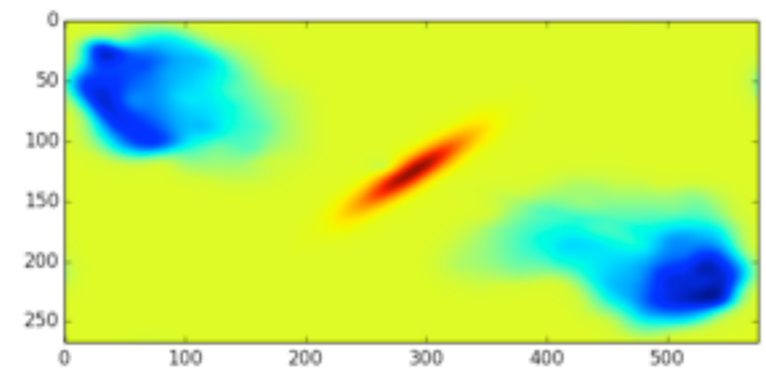
Model sources



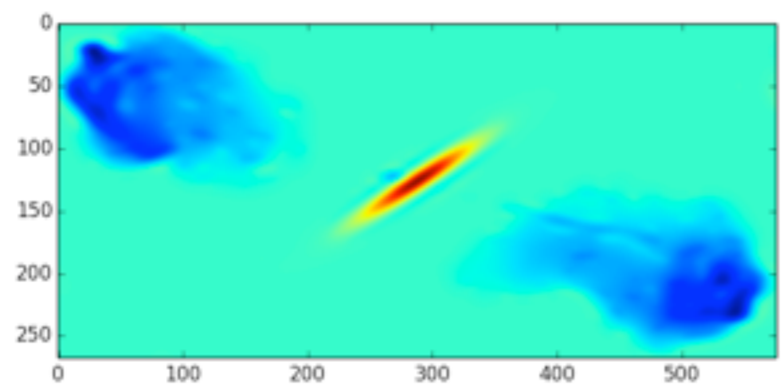


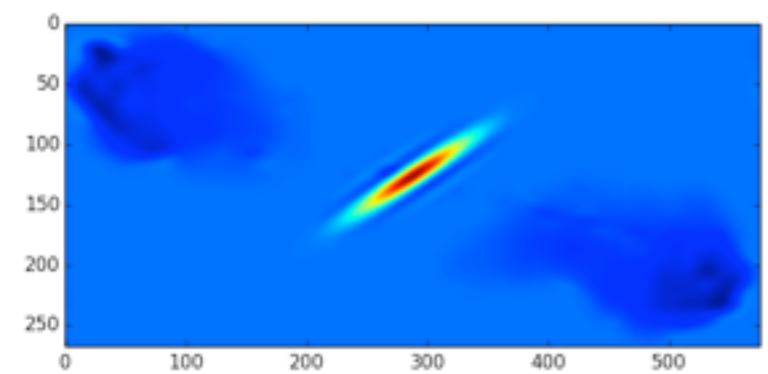
BSS only (GMCA), no deconvolution



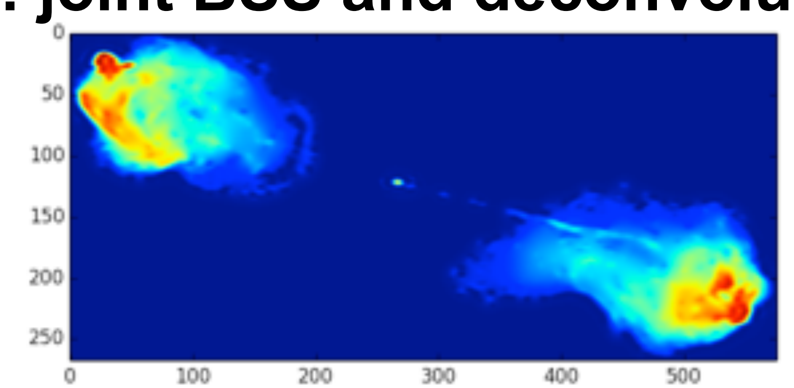


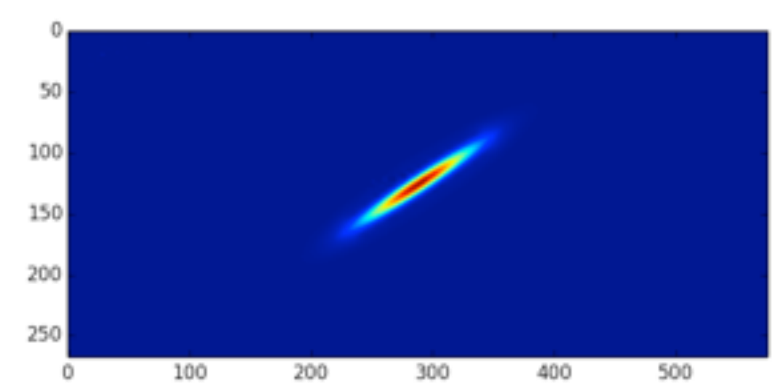
Channel by channel deconvolution (ForWaRD) followed by a BSS (GMCA)





Our method fGMCA : joint BSS and deconvolution





Conclusions and perspectives

Radio sources detection

- Transient study is active in radio astronomy, for the fast transient, detection tool is important
- Proof of compressed sensing concept on 2D image
- Extension to 2D-1D: applied to fast transients search with good angular resolution

Hyperspectral image restoration

- Multi or hyperspectral data generally present channels at different resolution. A rigorous Blind Source Separation method should take into account the different channel resolutions.
- fGMCA is an efficient method to solve jointly the BSS and the deconvolution problems.
- It is shown that taking into account joint BSS and deconvolution gives much better results than applying only a BSS or a channel per channel Deconvolution followed by a BSS.
- Application on radio images(LOFAR, SKA) on 3rd year, study of spectra of radio sources, etc.