DeLLight (*Deflection of Light by Light*) with LASERIX

Modification of the vacuum refractive index in intense electromagnetic fields

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Xavier Sarazin (LAL) Séminaire IRFU 19 Novembre 2018

Is the vacuum optical index constant?

Maxwell's equations are « linear » in vacuum

$$\begin{cases} \mathbf{D} = \varepsilon_0 \mathbf{E} \\ \mathbf{B} = \mu_0 \mathbf{H} \end{cases} \qquad \mathbf{c} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \qquad \blacksquare$$

 ε_0 and μ_0 are CONSTANT Optical index (*n*=1) is constant Do not depend on external fields

> Maxwell's equations are not linear in medium

$$\begin{cases} \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}(\mathbf{E}, \mathbf{B}) = \varepsilon (\mathbf{E}, \mathbf{B}) \cdot \mathbf{E} \\ \mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}(\mathbf{E}, \mathbf{B}) = \mu (\mathbf{E}, \mathbf{B}) \cdot \mathbf{H} \end{cases}$$

$$v = \frac{1}{\sqrt{\varepsilon(E,B)\mu(E,B)}}$$

- Optical index is not constant but depends on external fields $E,B \Rightarrow n(E,B)$ There is a non linear interaction between the electromagnetic fields, through the medium
- n(B): Birefringence induced by an external magnetic field, first measured by **Faraday** (1845) n(E): Refractive index increased by an electric field, first measured by **Kerr** (1875)

Is the vacuum optical index constant?

Is the vacuum a non linear optical medium as other material mediums ?

Can the vacuum optical index be modified by an external field ?

This question has been studied for the first time in 1911 in the case of gravitaion...

Is the vacuum optical index modified by gravitation?

Einstein is the first one to note that *n* and *c* are affected by the gravitation:

- ✓ Einstein, A. 'Über den Einfluss der Schwerkraft auf die Ausbreitung des Lichtes', Annalen der Physik 35, 898-908 (1911)
- ✓ "The constancy of the velocity of light can be maintained only insofar as one restricts oneself to spatio-temporal regions of constant gravitational potential" (Einstein A., Ann. Physik 38 (1912) 1059)

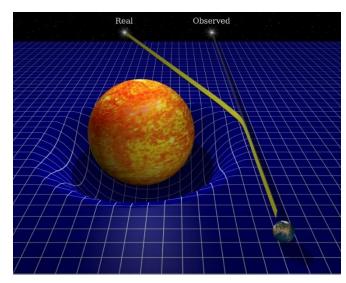
Einstein generalized the « *c* = constante » relativity principle thanks to the introduction of a *curved spacetime metric*

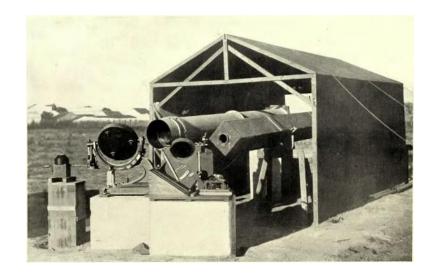
 \Rightarrow The General Relativity is a « *geo-metric* » theory

 \Rightarrow Vacuum has no physical role anymore



Deflection of light first observed by Eddington in 1919





Is the vacuum optical index modified by gravitation?

Another empirical approach initially proposed by Wilson (1921) and Dicke (1957)

- ✓ Euclidean flat metric
- ✓ Spatial change of ε_0 and μ_0 by the gravitational potential

 \Rightarrow Modification of the vacuum optical index and the inertial masses

n(r) formally identical to g_{00} in General Relativity

 \Rightarrow See Landau & Lifshitz (1975) : "A static gravitational field plays the role of a medium with electric and magnetic permeabilities $\varepsilon_0 = \mu_0 = 1/\sqrt{g_{00}}$ "

Exemple : Static spherical gravitational field (Wilson-Dicke Analogy)

$$\begin{cases} n(r) = 1 + \frac{2GM}{rc_{\infty}^2} \\ m(r) = m_{\infty} \times n^{3/2}(r) \end{cases}$$
 (to preserve the equivalence principle)

Wilson, Phys. Rev. 17, 54 (1921) Dicke, Rev. Mod. Phys. 29, 363 (1957)

Cosmology with a vacuum index increasing with time

$$n(r) = 1 + \frac{2GM}{rc_{\infty}^2} ?$$

Dicke's idea:
$$1 = n(t = 0) = \int \frac{2G(r)4\pi\rho r^2}{rc^2(r)} dr$$

 \Rightarrow *n*(*t*) increases with time

 \Rightarrow Hubble cosmological redshift due to a time variation of both n(t) and the atomic energy levels

Recent article: XS et al. Eur. Phys. J. C 78, 444 (2018); arXiv:1805.03503

- ✓ Euclidean static metric
- ✓ Relative variation dn(t)/n(t) is time invariant:

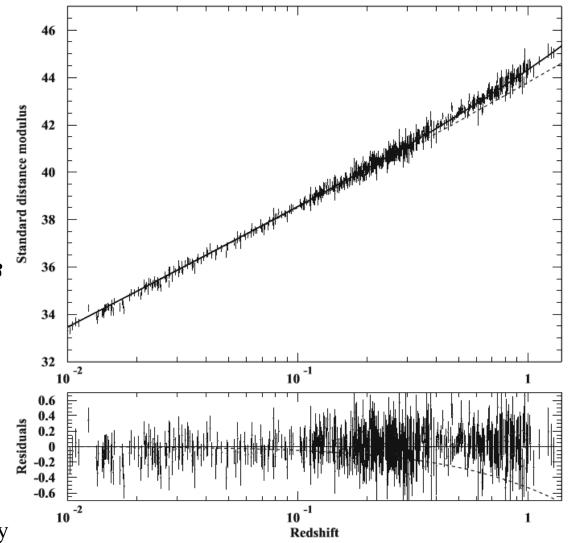
$$n(t) = e^{t/\tau_0}$$

 \Rightarrow Cosmological redshift SN-Ia well fitted (without Λ)

$$\tau_0 = 8.0^{+0.2}_{-0.8}$$
 Gyr

 \Rightarrow Time dilatation of the SN-Ia

 \Rightarrow Evolution of the CMB consistent with standard cosmology



Is the vacuum optical index modified by electromagnetic fields ?

« Born-Infeld » non linear electrodynamics

A crucial problem in physics:

Electromagnetic mass of the electron = self-energy of a point charge... which is infinite !

(By the way, this problem is still unsolved in quantum field theory !...)

How to regularize an electromagnetic field ?

Born and Infeld, in 1934, proposed to introduce non linear interactions between electromagnetic fields by assuming an absolute field E_{abs}

$$\mathcal{L}_{Born} = \epsilon_0 E_{abs}^2 \left(-\sqrt{1 - \frac{\epsilon_0 E^2 - B^2/\mu_0}{\epsilon_0 E_{abs}^2}} - \frac{(\mathbf{E} \cdot \mathbf{B})^2}{\mu_0 E_{abs}^2} + 1 \right) \qquad \begin{array}{l} Born \ and \ I \\ Fouché \ et \ advectories \\ Fouché \ et \ advectories \\ \mathcal{L}_{Born} \cong \mathcal{L}_{Maxwell} + \delta \mathcal{L}_{NL} \\ \left\{ \begin{array}{l} \mathcal{L}_{Maxwell} = \frac{1}{2} \left(\epsilon_0 E^2 - \frac{1}{\mu_0} B^2 \right) \\ \delta \mathcal{L}_{NL} = \frac{1}{8\epsilon_0 E_{abs}^2} \left(\epsilon_0 E^2 - \frac{1}{\mu_0} B^2 \right)^2 + \frac{1}{2\epsilon_0 E_{abs}^2} (\mathbf{E} \cdot \mathbf{B})^2 \end{array} \right.$$

Born and Infeld, Proc. R. Soc. A 144, 425 (1934) Fouché et al., Phys. Rev. D 93, 093020 (2016)

 E_{abs} is a free parameter of the Born-Infeld theory



Born-Infeld theory predicts no birefringence



« Euler-Heisenberg Lagrangian » & non linear QED

Euler-Heisenberg (1935) : nonlinearity induced by the coupling of the field with the e⁺/e⁻ virtual pairs in vacuum *Heisenberg and Euler, Z. Phys.* 98, 714 (1936)

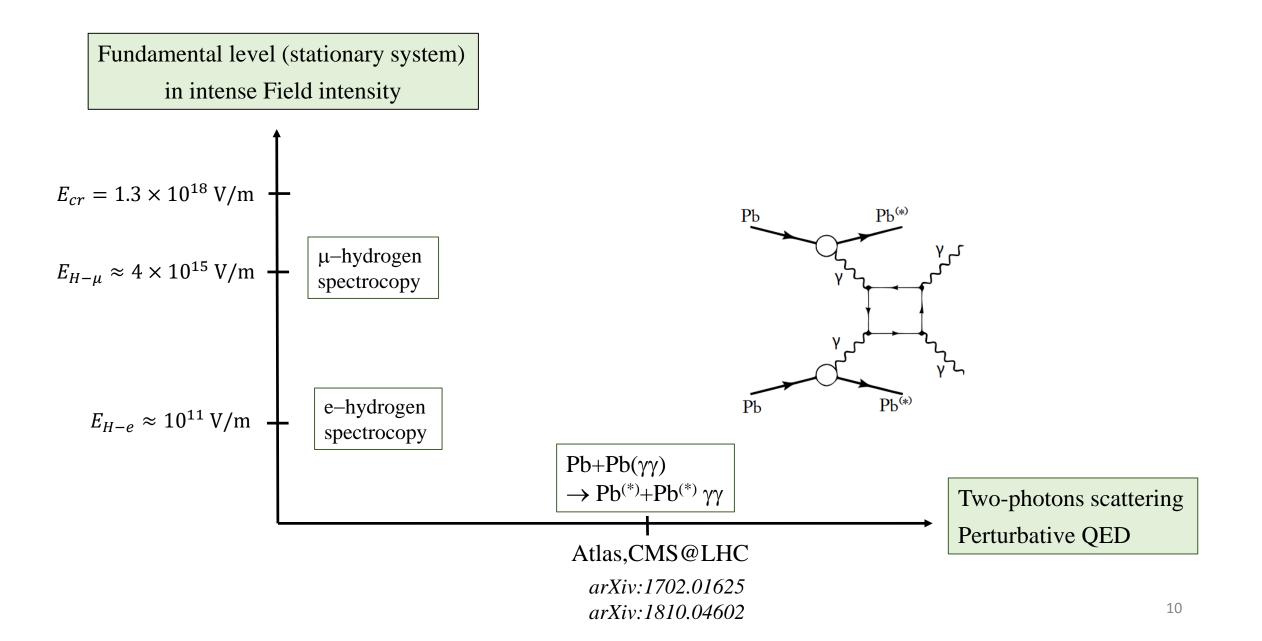
$$= \begin{cases} \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}(\mathbf{E}, \mathbf{B}) \\ \mathbf{B} = \mu_0 \mathbf{H} + \mu_0 \mathbf{M}(\mathbf{E}, \mathbf{B}) \end{cases} \quad \begin{cases} \mathbf{P} = \xi \varepsilon_0^2 [2(E^2 - c^2 B^2) \mathbf{E} + 7c^2 (\mathbf{E}, \mathbf{B}) \mathbf{B}] \\ \mathbf{M} = -\xi \varepsilon_0^2 [2(E^2 - c^2 B^2) \mathbf{B} - 7(\mathbf{E}, \mathbf{B}) \mathbf{E}] \end{cases} \quad \xi^{-1} = \frac{45m_e^4 c^5}{4\alpha^2 \hbar^3} \approx 3 \ 10^{29} \ \text{J/m}^3 \end{cases}$$

 \Rightarrow Modification of the Maxwell's equations in vacuum \Rightarrow Vacuum is a non linear medium

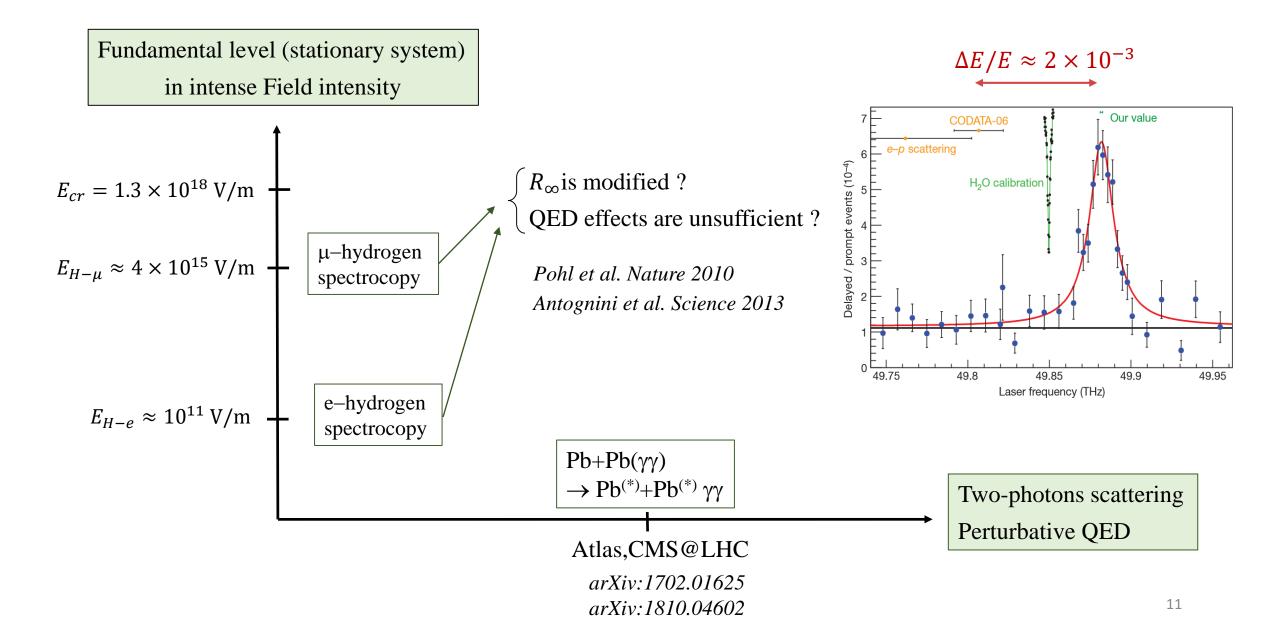
The vacuum refractive index is not an absolute constant $n \neq 1$ It can be modified on large scale (low energy) when it is stressed by intense e.m. fields

This result has been derived later by Schwinger with the QED frame J. Schwinger, Phys. Rev. 82, 664 (1951) Schwinger critical field : $\begin{cases}
E_{cr} = \frac{m_e^2 c^3}{e\hbar} = 1.3 \times 10^{18} \text{ V/m} \\
B_{cr} = E_{cr}/c = 4.4 \times 10^9 \text{ T}
\end{cases}$

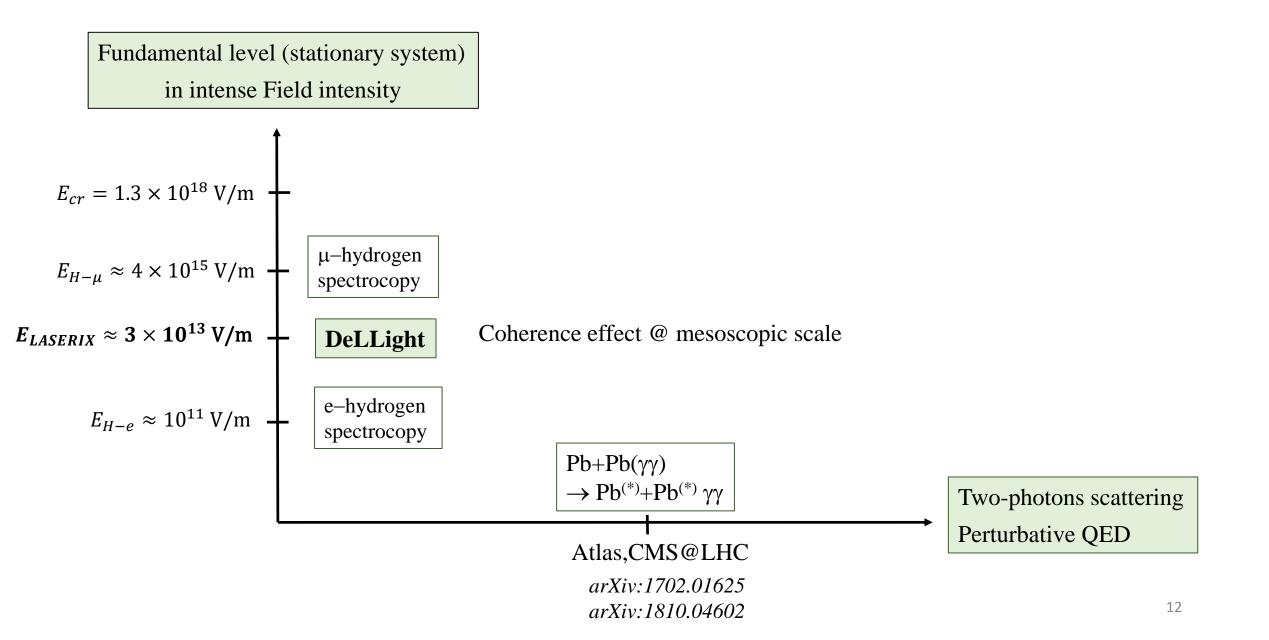
Two-photons scattering v.s. Intense fields



Two-photons scattering v.s. Intense fields



Two-photons scattering v.s. Intense fields



Current experimental tests

Search for birefringence with the PVLAS + BMV experiments

$$\Delta n_{\rm QED} = 4 \times 10^{-24} \ \rm T^{-2}$$

Fabry-Perrot laser cavity with an external B field



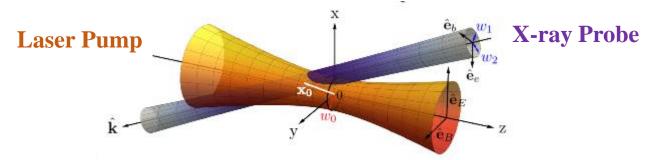
PVLAS: Rotating field B=2.5 T Eur. Phys. J. C (2016) 76:2



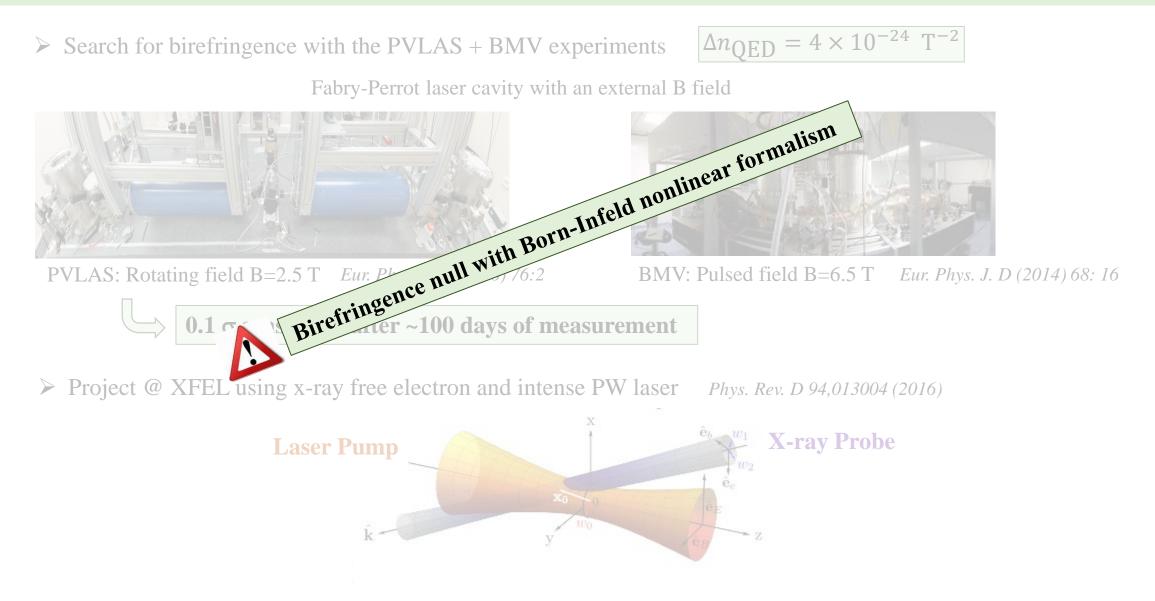
BMV: Pulsed field B=6.5 T Eur. Phys. J. D (2014) 68: 16

 \Rightarrow 0.1 σ sensitivity after ~100 days of measurement

Project @ XFEL using x-ray free electron and intense PW laser Phys. Rev. D 94,013004 (2016)



Current experimental tests



Jone's experiment in 1960...

- Variation of the vacuum refractive index, independentely of the polarization, has been tested only once, by R.V. Jones in ... 1960 !
- > Jones's experiment (1960) : Magnetic prism in vacuum with a static external field B = 1 Tesla Theoretical expected signal Δθ_{QED} ≅ 10⁻²³ rad Sensitivity ≅ 0.5 picorad (!) $\Delta \theta_{QED} \propto B^2$ θ ?

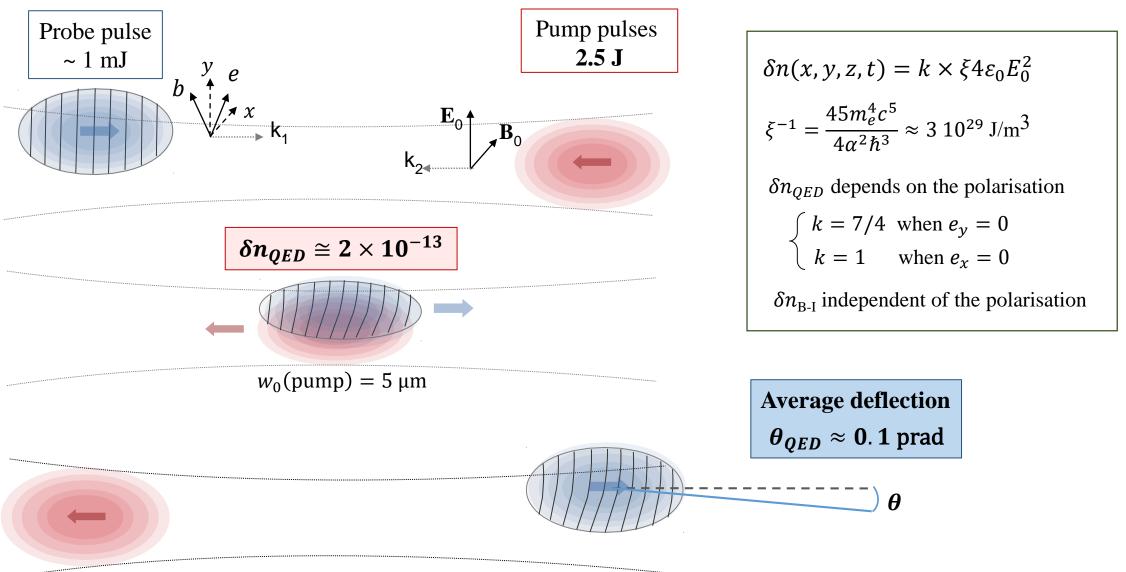


DeLLight with intense laser field produced by LASERIX 2.5 J, 30 fs, $w_0=5\mu m \Rightarrow \sim 3 \times 10^{20} \text{ W/cm}^2 \Rightarrow E \sim 3 \times 10^{13} \text{V/m}, B \sim 10^5 \text{ T}$

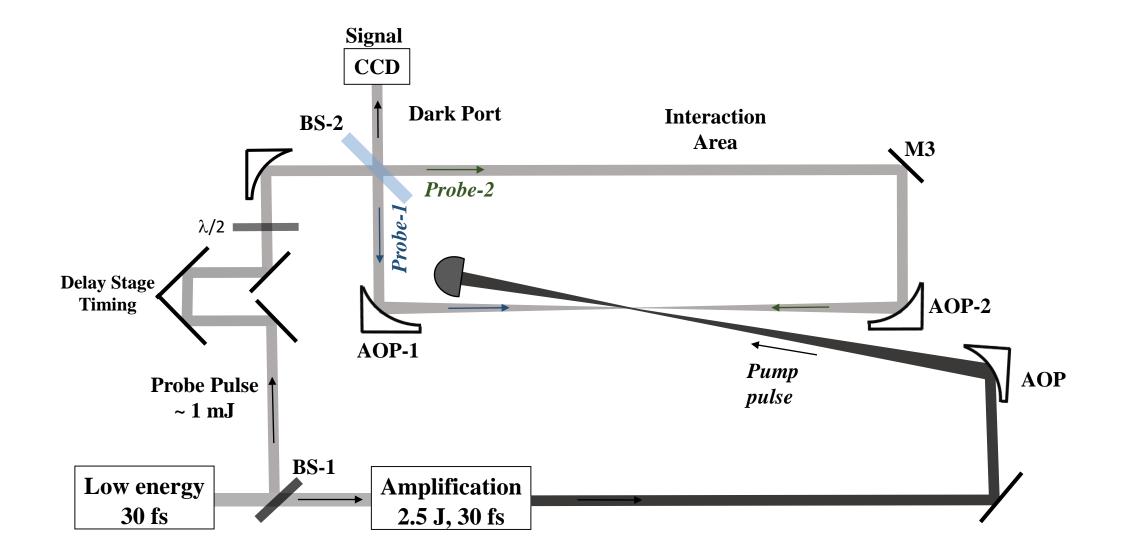
The DeLLight experiment

Pump-Probe interaction

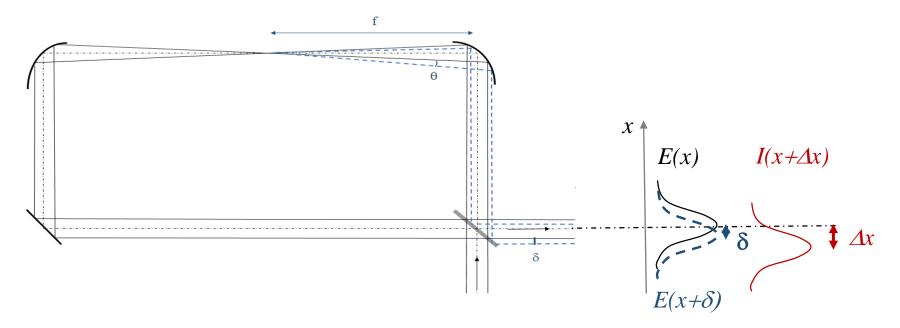
Recent calculations done by Scott Robertson, post-doc LAL & LPT (R. Parentani)



Refraction of the probe pulse \Rightarrow **Transversal shift** Δx of the interference intensity profile



> Refraction of the probe pulse \Rightarrow Transversal shift Δx of the interference intensity profile



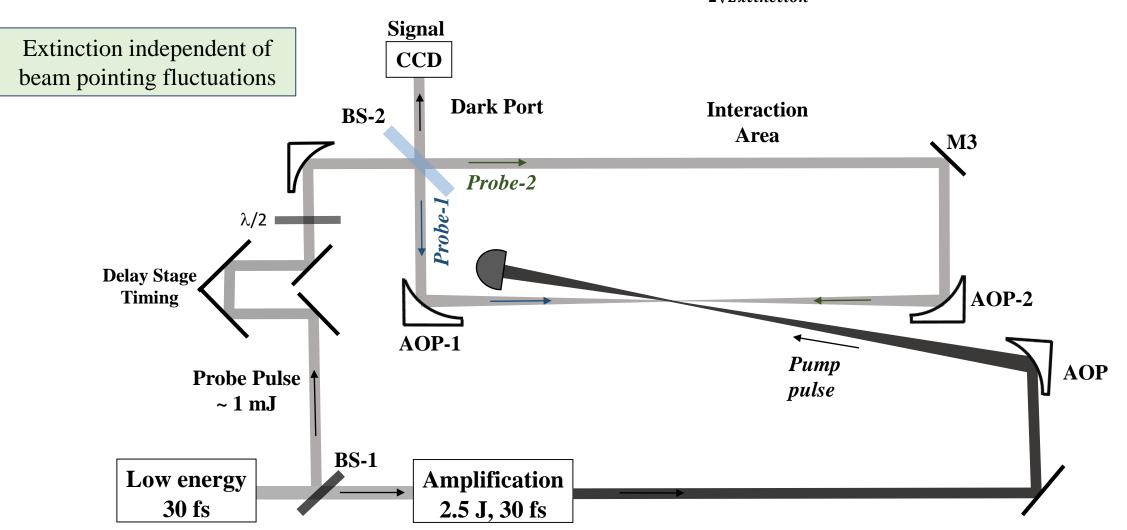
> Interference \Rightarrow Amplification factor \mathcal{F} compared to standard pointing method (with transversal shift δ)

$$\mathcal{F} = \frac{\Delta x}{\delta} = \frac{1}{2\sqrt{Extinction}} \quad \text{where } Extinction = \frac{I_{out}}{I_{in}} = 4\epsilon^2 \quad \text{and } \epsilon = \text{asymetry in intensity of the beam splitter})$$

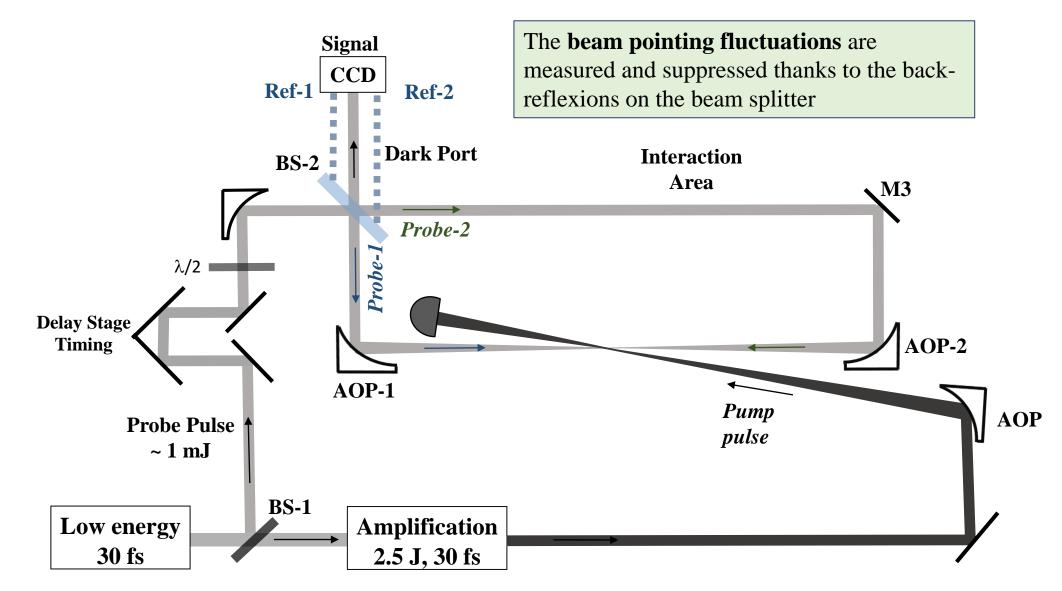
$$\mathbf{\mathcal{F}} \quad \mathbf{\mathcal{F}} = \mathbf{10^{-3}} \Rightarrow Extinction = \mathbf{0.4} \mathbf{10^{-5}} \Rightarrow \mathbf{\mathcal{F}} = \mathbf{250}$$

Refraction of the probe pulse \Rightarrow **Transversal shift** Δx of the interference intensity profile

Amplification factor \mathcal{F} compared to standard pointing method $\mathcal{F} = \frac{1}{2\sqrt{Extinction}} = 250$ when *Extinction* = 0.4 10⁻⁵



Refraction of the probe pulse \Rightarrow **Transversal shift** Δx of the interference intensity profile



Numerical Simulations

Preliminary 2-d (x,z) numerical simulation:

Two pulses (30 fs, 800 nm) with ortogonal polarisation are counter-propagating (along z) and focused

> Transversal profiles of the beams are gaussian: $\mathcal{E}(x, z) = A_0 e^{(-x^2/w_0^2)}$

- Energy pump pulse E=2.5 J; Energy probe pulse is negligible (1 mJ)
- > Minimum waist at focus: w_o (probe) = 2 × w_o (pump)
- \succ Probe beam is shifted transversally by a distance δ_p
- ► Vacuum refractive index is calculated in the interaction : $\delta n_{QED}(x, z, t) = 7\xi \varepsilon_0 c^2 E^2(x, z, t)$

 \Rightarrow After interaction, the probe pulse is refracted by a phase $\varphi_{QED}(x,z) = \int \frac{2\pi c}{\lambda} \delta n_{QED}(x,z,t) dt$

Solution $\mathcal{F} = 4\epsilon^2$ (ϵ = assymetry of the beam splitter)

Numerical Simulations

20

10

0

-10

-20

20

10

0

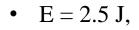
-10

-20

-0.1

-20

(mπ) z



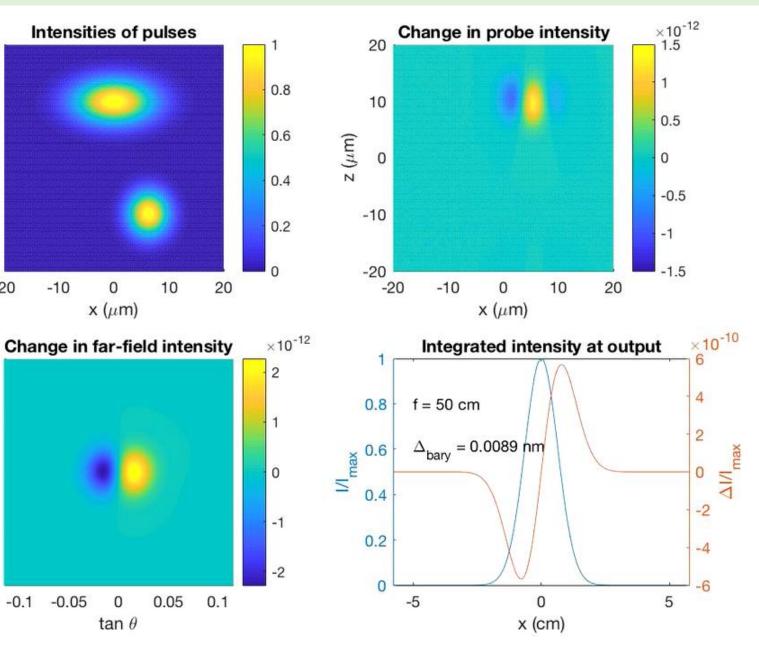
- Extinction = 0.4 10^{-5} ($\epsilon = 10^{-3}$) •
- D = 50 cm (limited by the beam divergence) •
- $w_0(\text{pump}) = 5 \ \mu\text{m}, \ w_0(\text{probe}) = 10 \ \mu\text{m}$

• $\delta_p = w_0/2$

 $\Delta x \approx 0.01 nm$

Signal Δx reduced by ~20% if jitter pump $\pm 2.5 \mu m$

$$\Delta x \approx 6.10^{-10} \text{ m } \times \frac{E(Joule) \times D(m)}{(w_0(\mu m))^3 \times \sqrt{\mathcal{F}/10^{-5}}} \qquad \stackrel{\text{fr}}{\longrightarrow}$$



Expected sensitivity

Switch ON & OFF alternatively the pump beam (laser repetition rate = 10 Hz): \Rightarrow Barycenters of the intensity profile : \bar{x}_k^{ON} and \bar{x}_k^{OFF} \Rightarrow Signal (ON-OFF) for the measurement $k : \Delta x_k = \bar{x}_k^{ON} - \bar{x}_k^{OFF}$

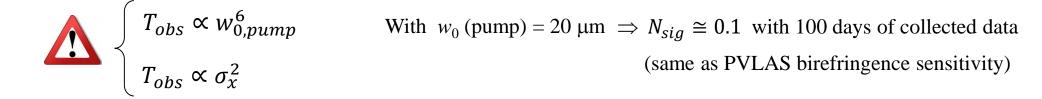
> N_{mes} measurements collected \Rightarrow Average signal = $\overline{\Delta x} \pm \sigma_x / \sqrt{N_{mes}}$ where σ_x is the ON-OFF spatial resolution, including systematics

> The sensitivity (number of standard deviations N_{sig}) is :

$$N_{sig} = \frac{\overline{\Delta x}}{\sigma_x / \sqrt{N_{mes}}} \cong 500 \times \frac{\sqrt{T_{obs}(\text{days})}}{\left(w_{0,pump}(\mu\text{m})\right)^3 \times \sqrt{\mathcal{F}/10^{-5}} \times \sigma_x(\text{nm})}$$

$$\begin{cases} \text{Extinction } \mathcal{F} = 0.4 \ 10^{-5} \ (\epsilon = 10^{-3}) \\ \sigma_x = 10 \text{ nm} \\ w_0 \text{ (pump)} = 5 \text{ } \mu\text{m} \end{cases} \Rightarrow N_{sig} \cong$$

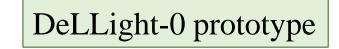
$$N_{sig} \cong 0.6 \sqrt{T_{obs}(\text{days})} \Rightarrow 3 \text{ sigma discovery in 25 days}$$



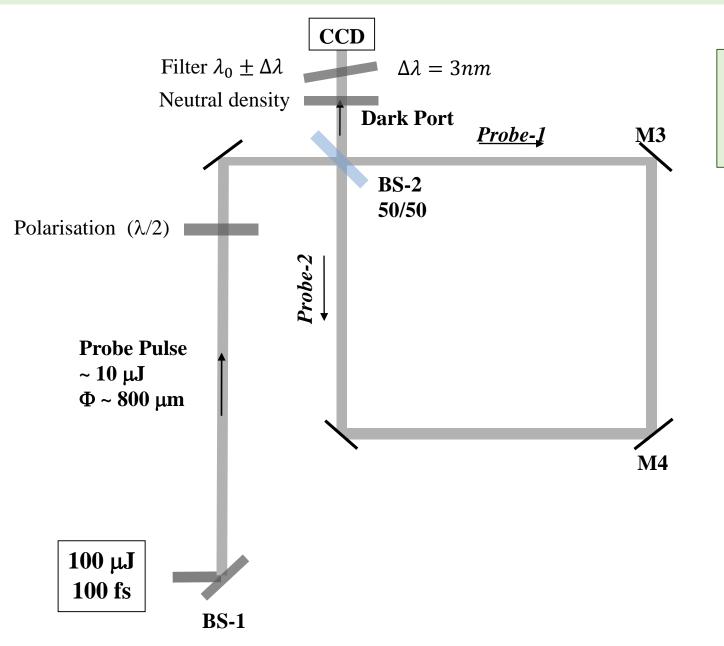
Experimental challenges

✓ Extinction:
$$\mathcal{F} = 0.4 \ 10^{-5} \ (\epsilon = 10^{-3})$$

- ✓ Spatial resolution: $\sigma_x = 10 \text{ nm}$
- ✓ Waist at focus as low as possible
 + stability of the pump-probe overlap

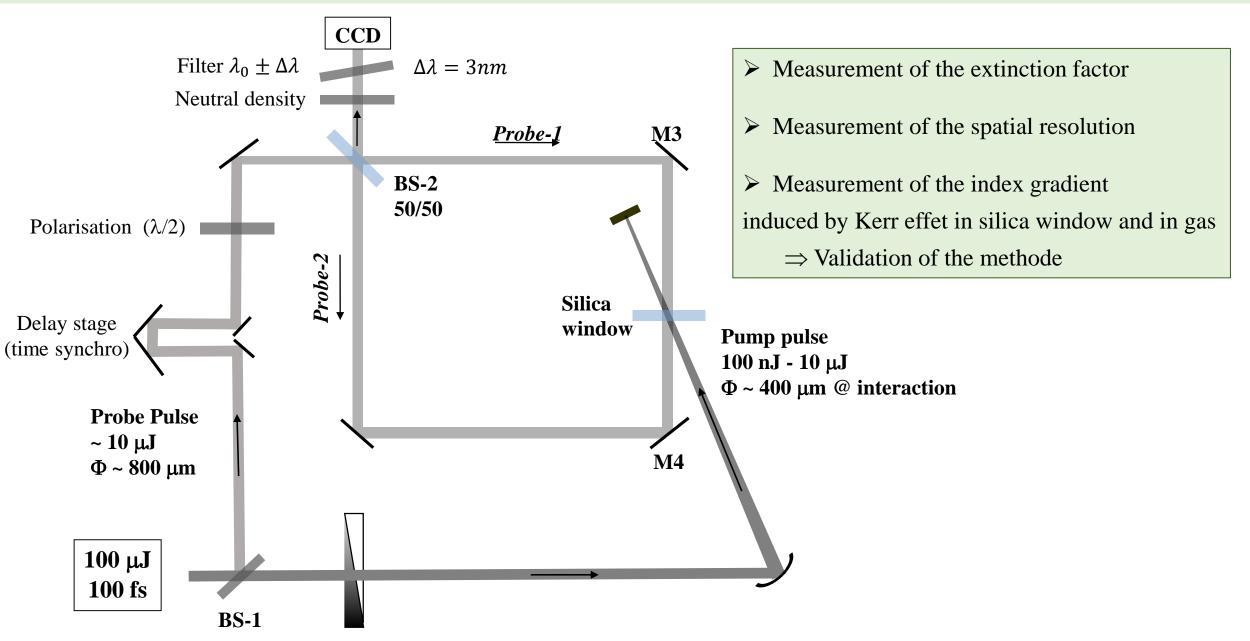


DeLLight-0 prototype

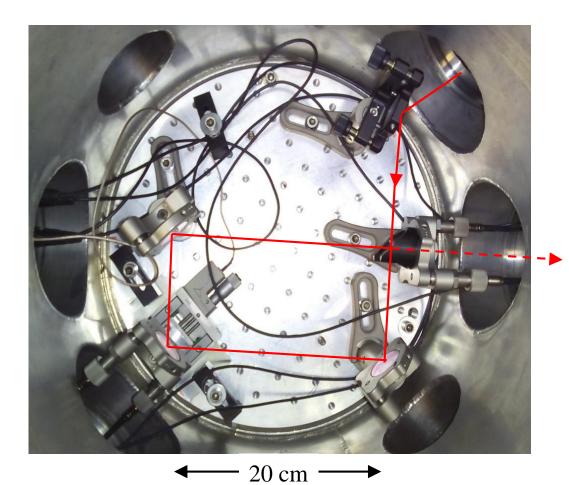


Measurement of the extinction factorMeasurement of the spatial resolution

DeLLight-0 prototype

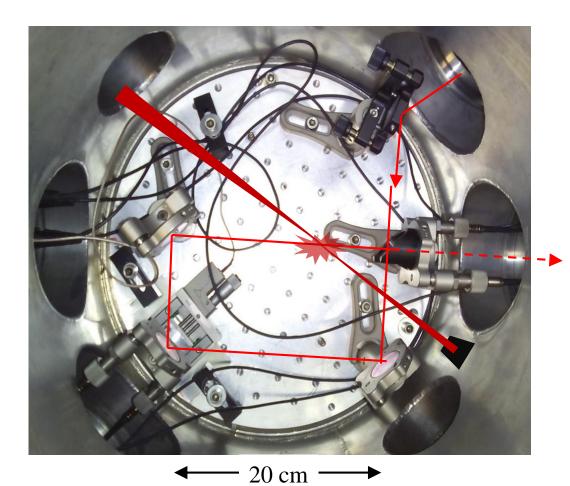


DeLLight-0 demonstrator



- ✓ Beam Splitter 50/50 *Semrock*[™] (thickness=3mm)
- ✓ Flat silver mirror standard (λ /10)
- ✓ BS and opposite mirror controled with piezo adjuster
 POLARIS® *K1S2P* 5 nrad/mV
- ✓ Dark Output:
 - Filter $\Delta \lambda = 3 \text{ nm} @ 800 \text{ nm}$
 - CCD camera BASLERTM acA1300-60gm 1260x1080 pixels pixel size = $5.3 \mu m$ saturation $\approx 10^4$ electrons/pixel

DeLLight-0 demonstrator



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- ✓ Fused silica window (6mm thick)

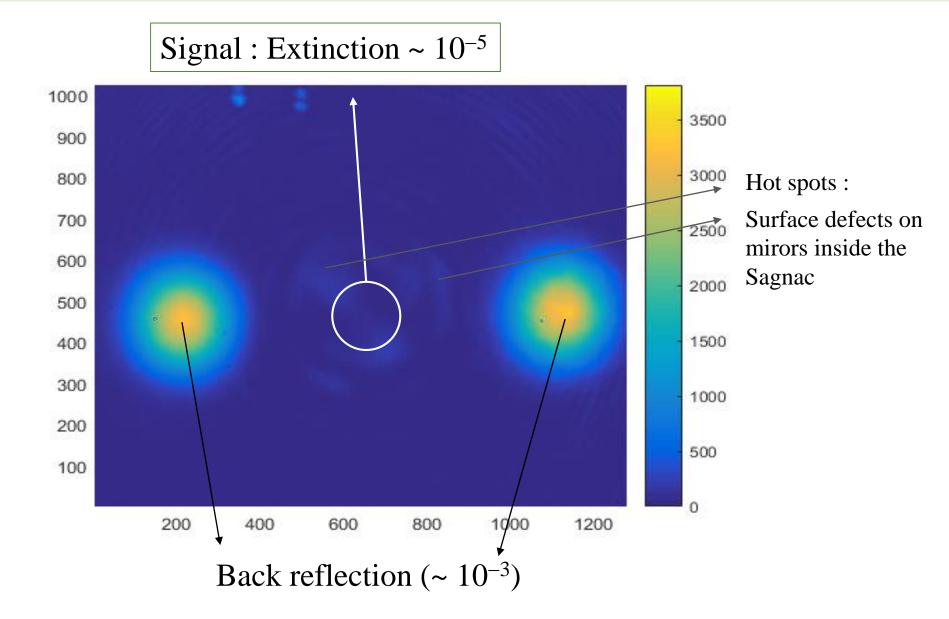
Experimental challenges

✓ Extinction: $\mathcal{F} = 4\epsilon^2$ ($\epsilon = 10^{-3}$)

✓ Spatial resolution: σ_x

✓ Waist at focus as low as possible
+ stability of the pump-probe overlap

Extinction of the interferometer



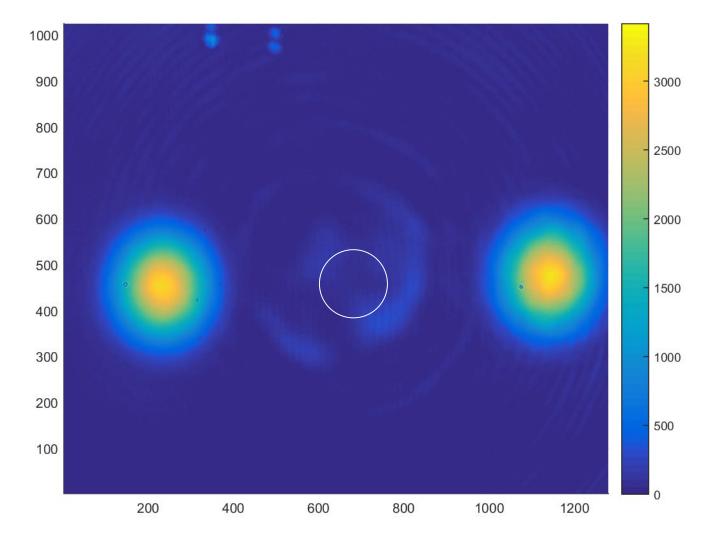
Extinction of the interferometer

Polarization of the probe beam

Extinction = $4\varepsilon^2$

 $\varepsilon = I_t/I_r$ = Asymetry (intensity) of the beam splitter

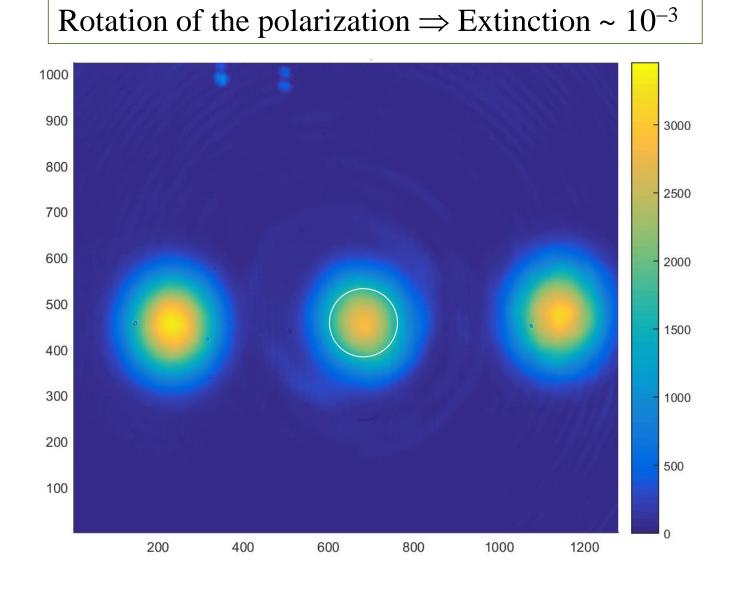
 ε depends upon the polarization



Extinction of the interferometer

Extinction = $4\varepsilon^2$ $\varepsilon = I_t/I_r$ = Asymetry (intensity) of the beam splitter

 ε depends upon the polarization



Experimental challenges

$$\checkmark$$
 Extinction: $\mathcal{F} = 4\epsilon^2 \cong 10^{-5}$

- ✓ Spatial resolution: σ_x
- ✓ Waist at focus as low as possible
 + stability of the pump-probe overlap

Spatial resolution

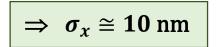
Expected resolution limited by the photon statistic:

$$\Rightarrow \sigma_{\chi} \propto \frac{d_{pix}}{\sqrt{N_{e^{-}}^{max}}}$$

- ➢ Monte-Carlo: CCD (BASLER™ acA1300-60gm)
 - Pixel size d_{pix} : 5.4×5.4 µm²
 - Charge saturation $N_{e-}^{max} \cong 10^4 \text{ e}^{-/\text{pixel}}$

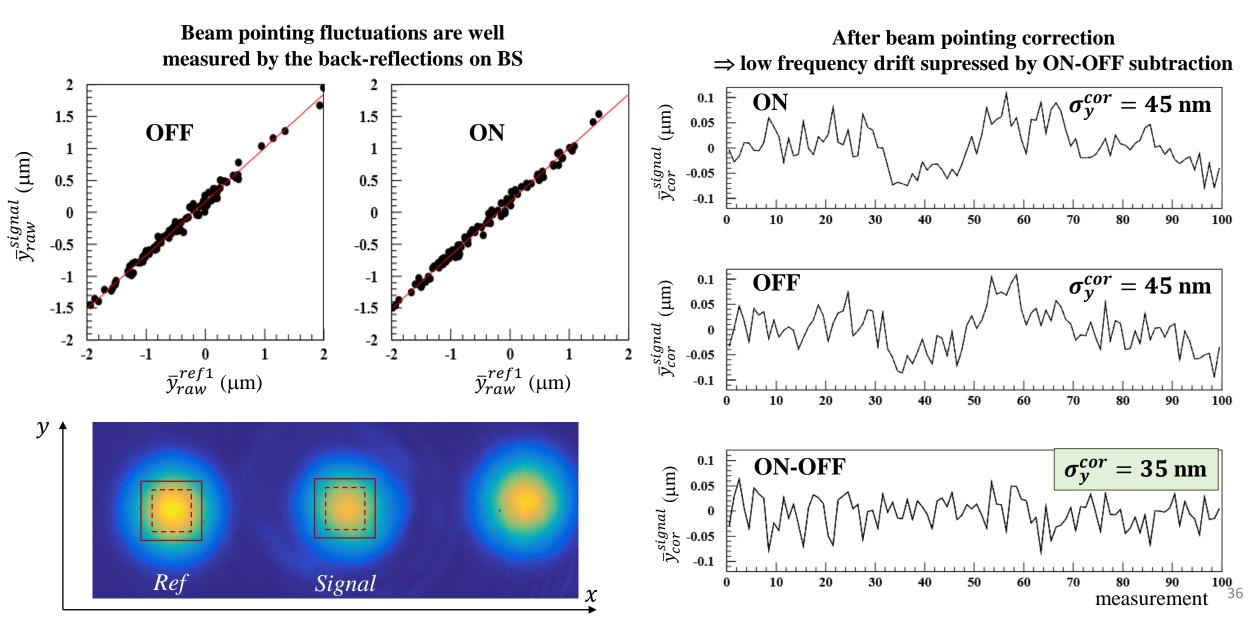
$\Rightarrow \sigma_x \cong 33 \text{ nm}$

- > With better CCD BASLER TM (acA4024-29um):
 - Pixel size d_{pix} : 1.8×1.8 μ m²
 - Charge saturation $N_{e-}^{max} \cong 10^4 \text{ e}^{-/\text{pixel}}$

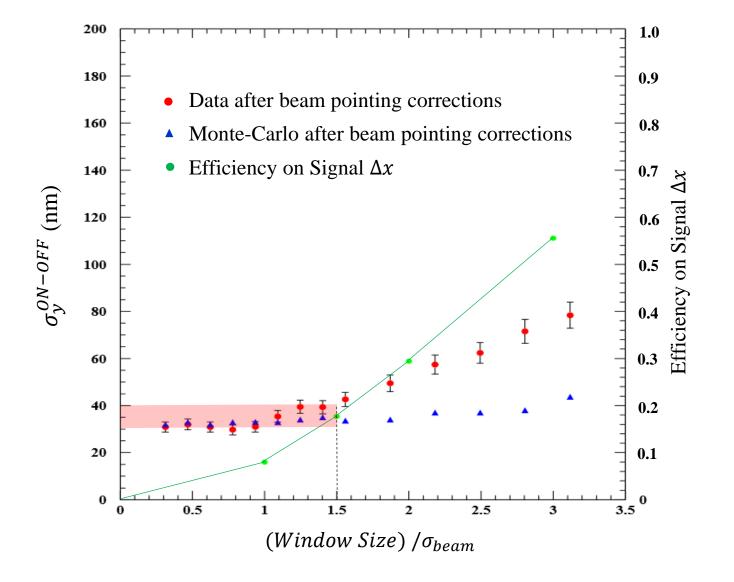


Spatial resolution

Preliminary analysis based on a barycenter calculation in a simple square analysis window (RoI)



Spatial resolution



• RoI $\lesssim 1.5 \times \sigma_{beam}$

 \Rightarrow Data $\sigma_x \cong 30 - 40$ nm

- RoI $\gtrsim 1.5 \times \sigma_{beam}$
 - ⇒ Fluctuations of the interference profile (induced by the hot spots)
 - Work in progress to reduce them:
 - Background mapping & substraction
 - Fit of the profiles
 - Surface quality of the optics
 - CCD uniformity
 - Etc...

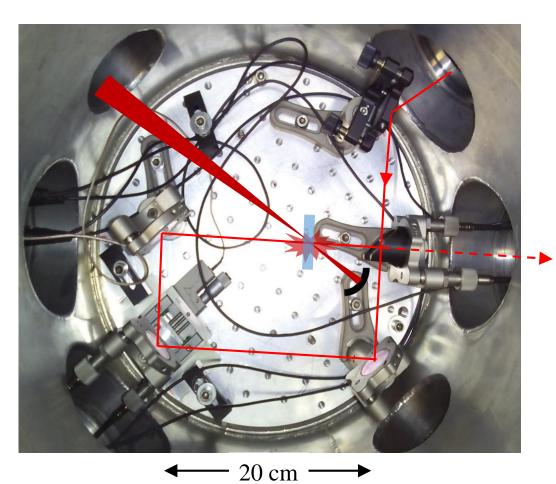
Experimental challenges

- ✓ Extinction: $\mathcal{F} = 4\epsilon^2 \cong 10^{-5}$
- ✓ Spatial resolution: σ_x

 \checkmark Demonstration of the method by observing the non linear Kerr effect

Observation of the non linear Kerr effect

Kerr effect induced in a fused silica window (6mm thick)

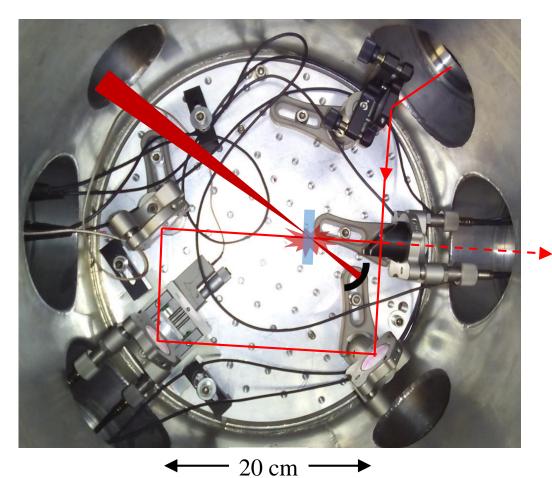


$$n(I) = n_0 + n_2 \times I(W/cm^2)$$
$$n_2(Silica) \approx 3 \times 10^{-16} \text{ cm}^2/W$$

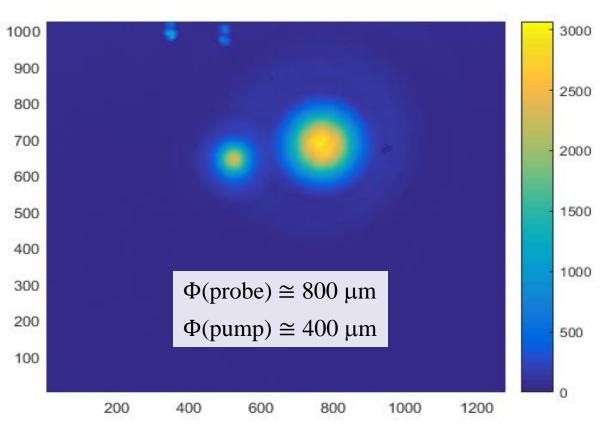
Data taken in June & July 2018

- $\Phi(\text{probe}) \cong 800 \ \mu\text{m} (\text{fwhm})$
- $\Phi(\text{pump}) \cong 400 \ \mu\text{m} (\text{fwhm})$
- Duration of the pulses $\Delta t \sim 50 100$ fs
- Energy Pump varies from $\sim 12 \mu J$ down to $\sim 300 nJ$

Observation of the non linear Kerr effect

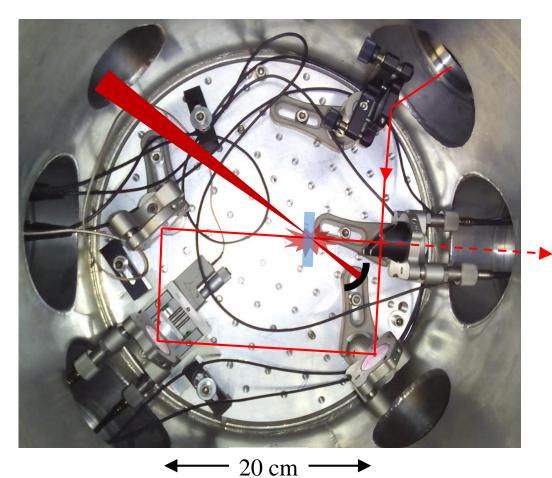


Intensity profiles of the Pump & Probe in the interaction area

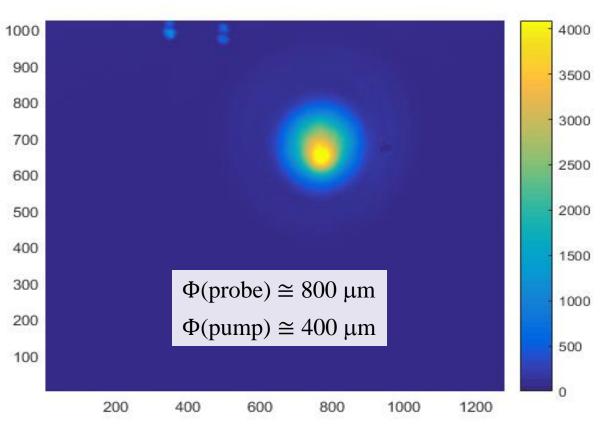


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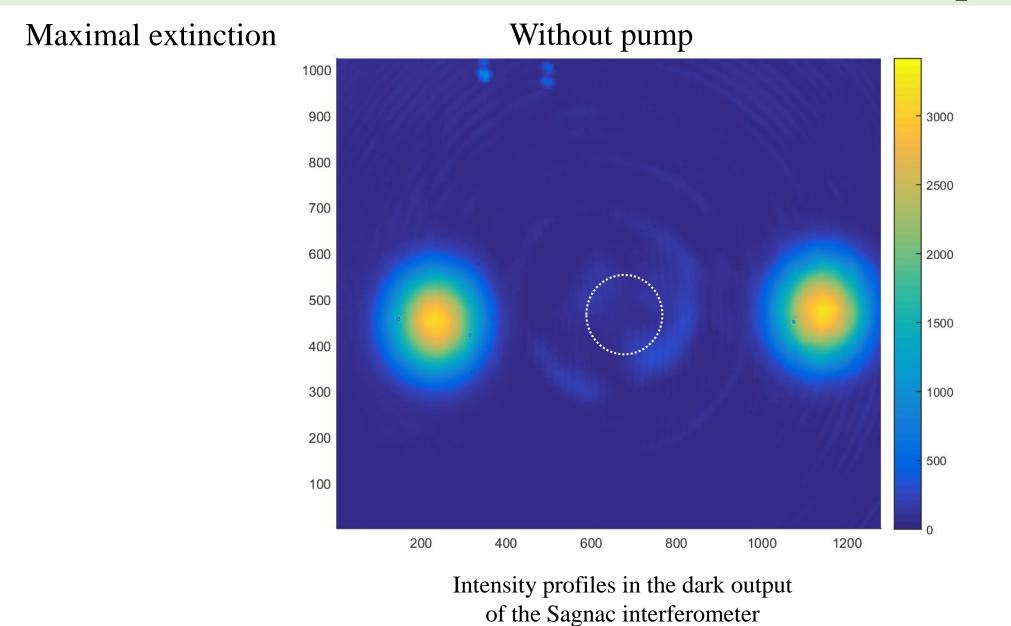
Observation of the non linear Kerr effect

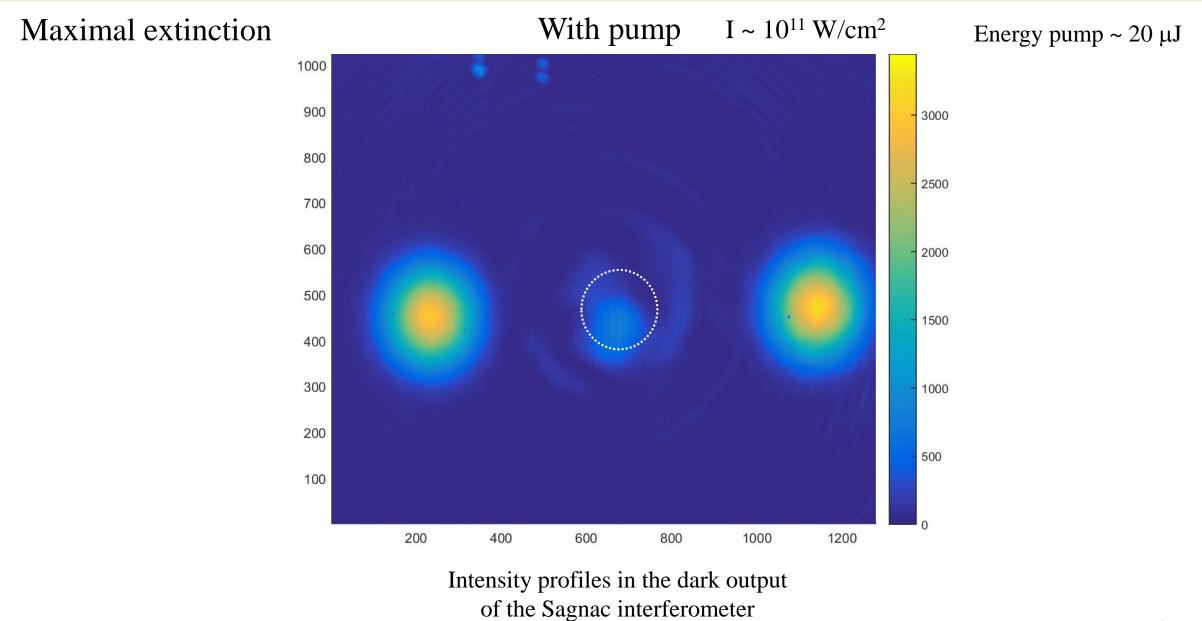


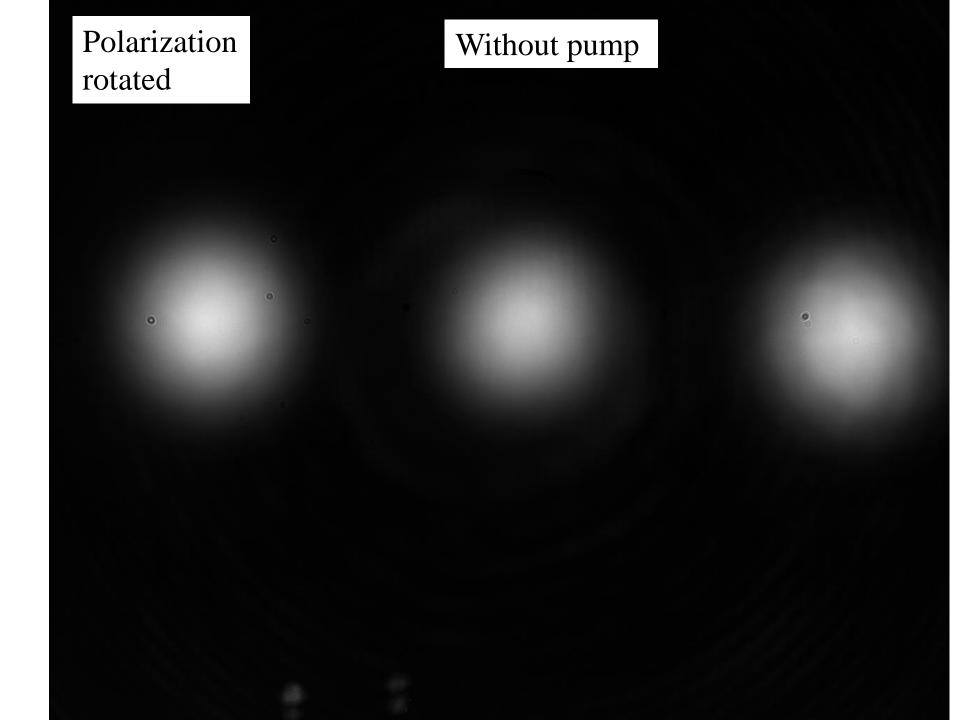
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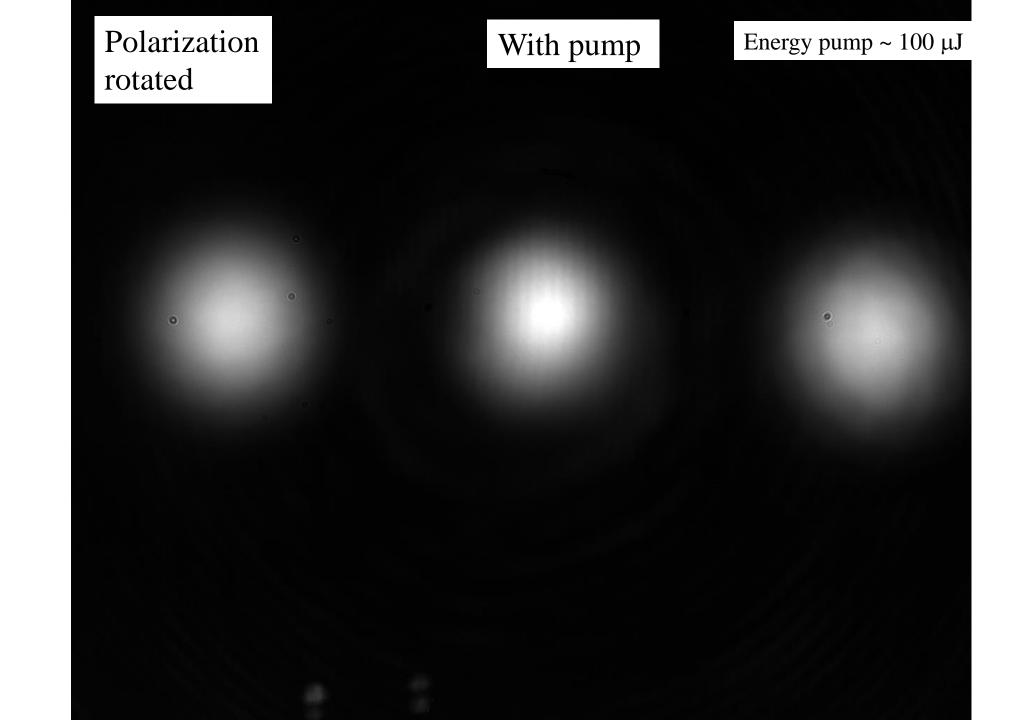


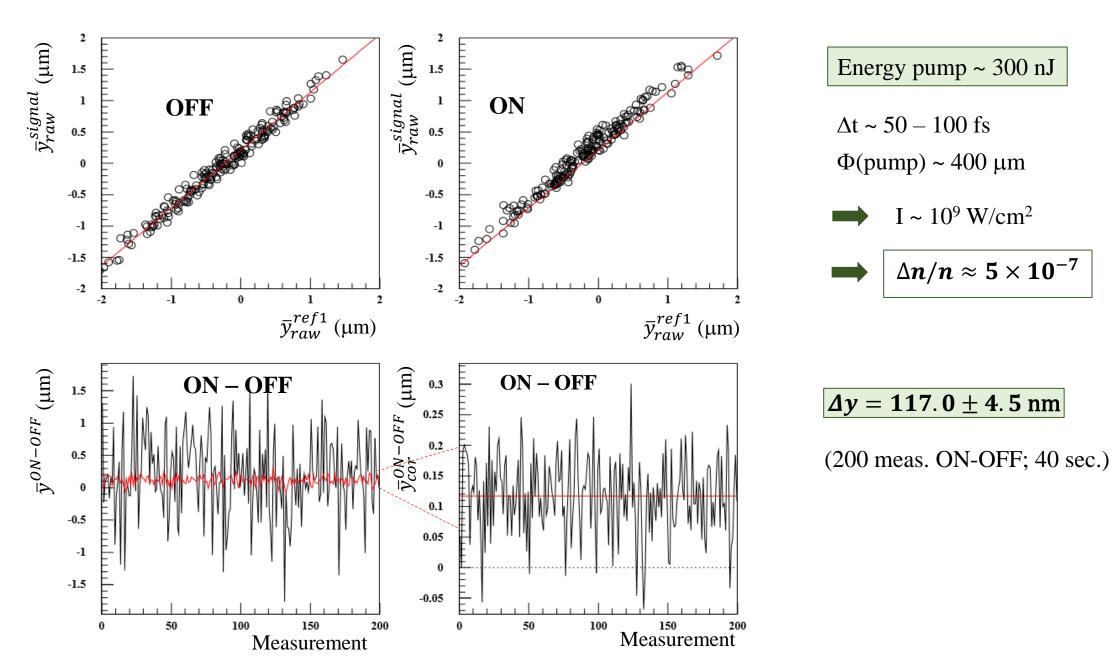
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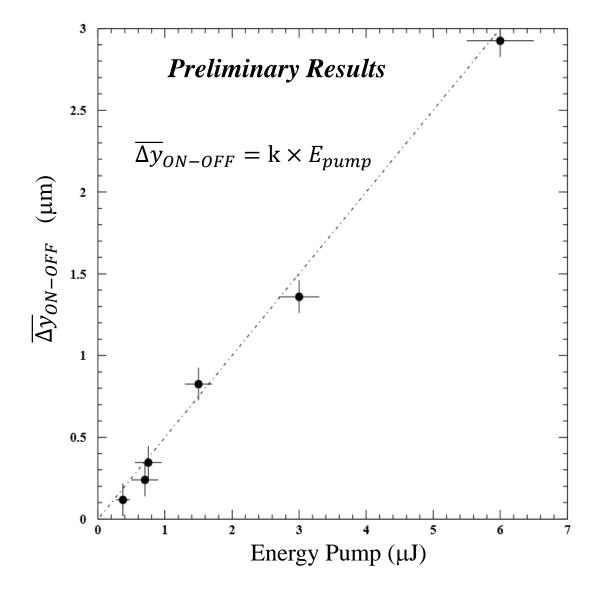






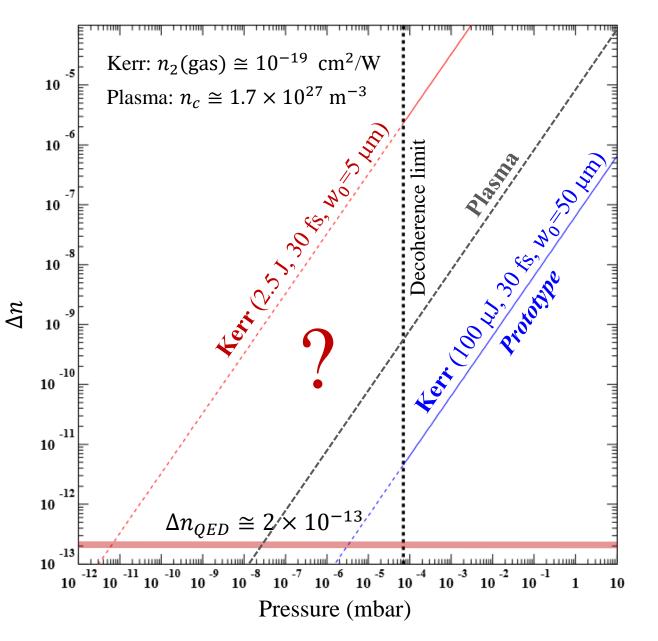




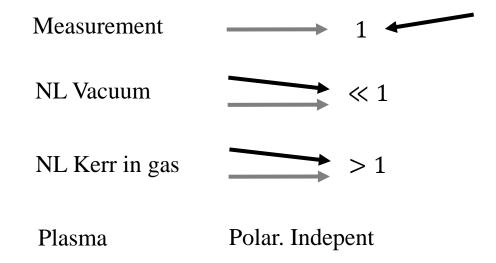


- ✓ Signal $\overline{\Delta y}_{ON-OFF}$ is proportional to the energy of the pump, as expected for the Kerr effect
- ✓ Preliminary results, work in progress...
 - Simulations of the Kerr effect
 - Influence of the polarization
 -
- ✓ Next step: measure Kerr effect in gas

Kerr effect and plasma in residual gas



- ➤ Kerr effect in gas: Decoherence limit ? $p \cong 7 \times 10^{-5} \text{ mbar} \Rightarrow \text{distance between atoms} \cong \lambda_{laser}$
- ▶ **Plasma**: $\Delta n_{plasma} \cong \Delta n_{QED}$ for $p = 2 \times 10^{-8}$ mbar
- Beam polarisation & orientation used to distinguish the processes



DeLLight for the next 3 years

Funded (~310 keuros) by ANR Oct. 2018 – Oct. 2021

2/3 Equipement1/3 2-years post-doc (starting spring 2019)

Partners: LAL, LPGP, LUMAT, APC

Program:

- 1. DeLLight-0 (2019):
 - Kerr effect inside Silica window $\Rightarrow \delta n \approx 10^{-8}$
 - Kerr effect & plasma inside low pressure gas $\Rightarrow \delta n \approx 10^{-11}$
- 2. DeLLight Phase 1 (2019-2020): Measure in vacuum with 2 Joules & focus $w_0 = 10 20 \,\mu\text{m}$
- 3. DeLLight Phase 2 (2020-2021): Measure in vacuum with focus $w_0 = 5\mu m \Rightarrow \delta n \approx 2 \times 10^{-13}$

DeLLight and other intense laser facilities

► **LASERIX** (LAL, Orsay):

running 2J, 30fs $\Rightarrow \sim 70$ TW @ 10 Hz $\Rightarrow \Delta x_{LASERIX} \approx 0.01$ nm

BELLA laser (Berkeley LBNL):

running with 40J, 30fs $\Rightarrow \sim 1$ PW @ 1 Hz

> **APOLLON** laser (Saclay):

2019: 30 J, 30 fs \Rightarrow ~1 PW @ 0.1 Hz

Target: 100 J, 20 fs $\Rightarrow \sim 5$ PW @ 0.1 Hz

> HAPLS laser (developed by LLNL and running @ ELI Beamlines Research Center, Czech Republic)

Diode-pumped petawatt laser in order to reach 10 Hz repetition rate

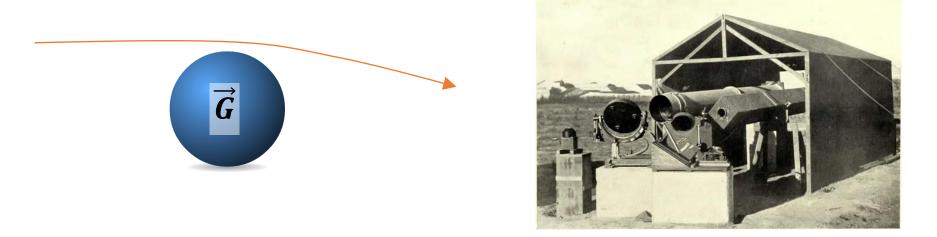
June 2018: 16 joules, 27 femtosecond pulse duration (0.5 PW) @ 3.3Hz

2019 : 30 J, 30 fs \Rightarrow 1 PW @ 10 Hz $\Rightarrow \Delta x_{HAPLS} \approx$ 0.1 nm

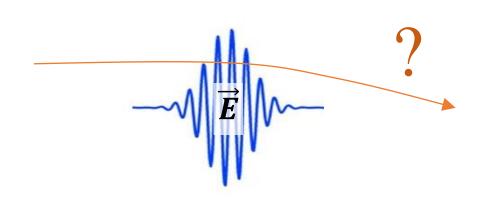
Target: ~200 Joules, 30 fs \Rightarrow ~6 PW @ 10 Hz $\Rightarrow \Delta x_{HAPLS} \approx 1$ nm

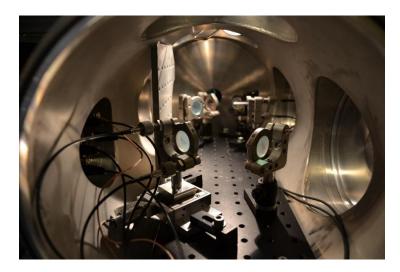
Conclusions

➤ In May 29, 1919 Eddington measured the deflection of light by a gravitational field



➤ In May 29, 20.... DeLLight-LASERIX will measure the deflection of light by an electromagnetic field ?





Backup

Pressure in the interaction area

Phase-1 ($w_0 \cong 10 - 15 \,\mu\text{m}$) P=10⁻⁶ mbar

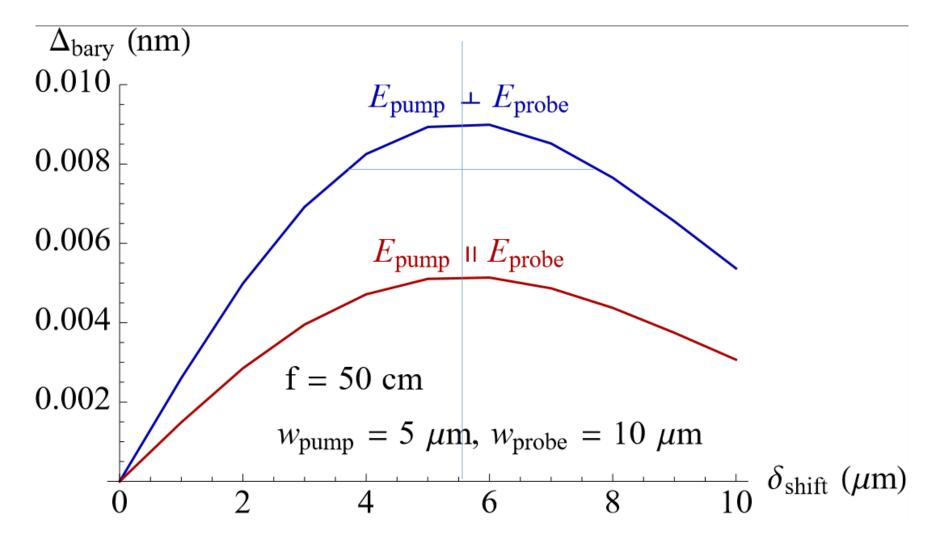
 $\Rightarrow \sim 10$ molecules in the volume $V = w_0^2 \times \Delta t \times c$ $(\Delta t \times c = 10 \mu s)$

Phase-2 ($w_0 \cong 5 \mu m$) P=10⁻⁹ mbar

 $\Rightarrow \sim 1$ molecule in the volume $V = (20 \ \mu m)^2 \times 10 \times \Delta t \times c$

Numerical Simulations

Systematics due to pump-probe jitter



Amplification with a Sagnac Interferometer

$$I(x) = I_0 \left(\left(\frac{1}{2} + \epsilon\right) E(x+\delta) - \left(\frac{1}{2} - \epsilon\right) E(x) \right)^2 \cong 2\delta\epsilon \frac{\partial E}{\partial x} + 4\epsilon^2 E^2(x) \quad (\delta \ll 1)$$

$$E(x) = exp\left(-\frac{x^2}{2\sigma^2}\right) \implies I(x) = \left(\frac{2\epsilon\delta}{\sigma^2}x + 4\epsilon^2\right) exp\left(-\frac{x^2}{\sigma^2}\right) \implies \Delta x = \frac{\int_{-\infty}^{+\infty} xI(x)dx}{\int_{-\infty}^{+\infty} I(x)dx} = \frac{\delta}{4\epsilon}$$
Extinction factor : $\mathcal{F} = \frac{I_{out}}{I_{in}} = 4\epsilon^2 \implies \text{Amplification} = \frac{\Delta x}{\delta} = \frac{1}{2\sqrt{\mathcal{F}}}$

$$\theta$$

Expected sensitivity

Sensitivity depends strongly on the waist of the pump at focus $T_{obs} \propto w_0^6$

