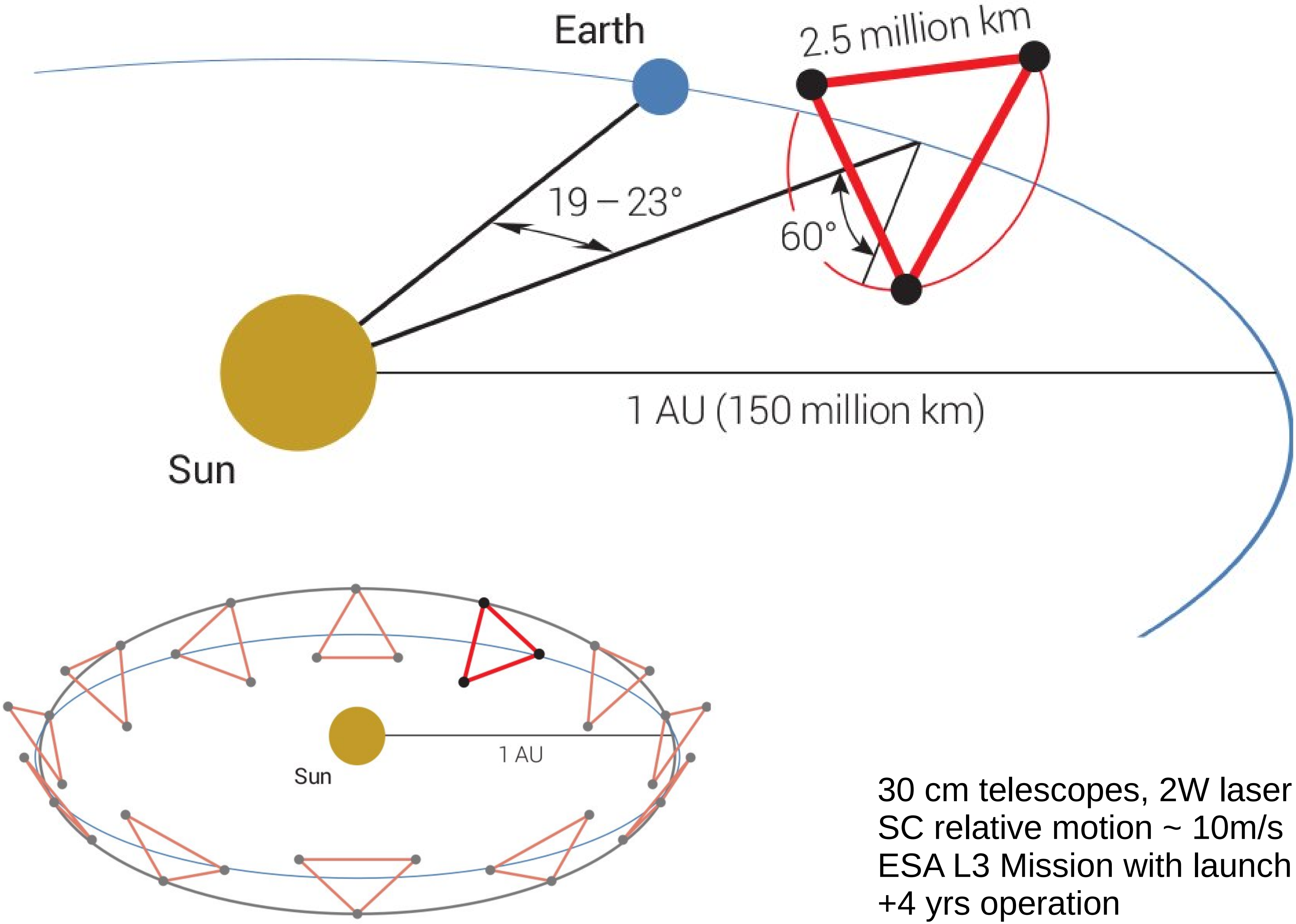
Gravitational Wave memory

One example of Fundamental Physics with LISA

Marc Besancon (DPhP) 22.09.2020

Based on : K. Islo, J. Simon, S. Burke-Spolaor, X. Siemens,
Prospects for Memory Detection with Low-Frequency Gravitational Wave Detectors
arXiv:1906.11936

LISA constellation

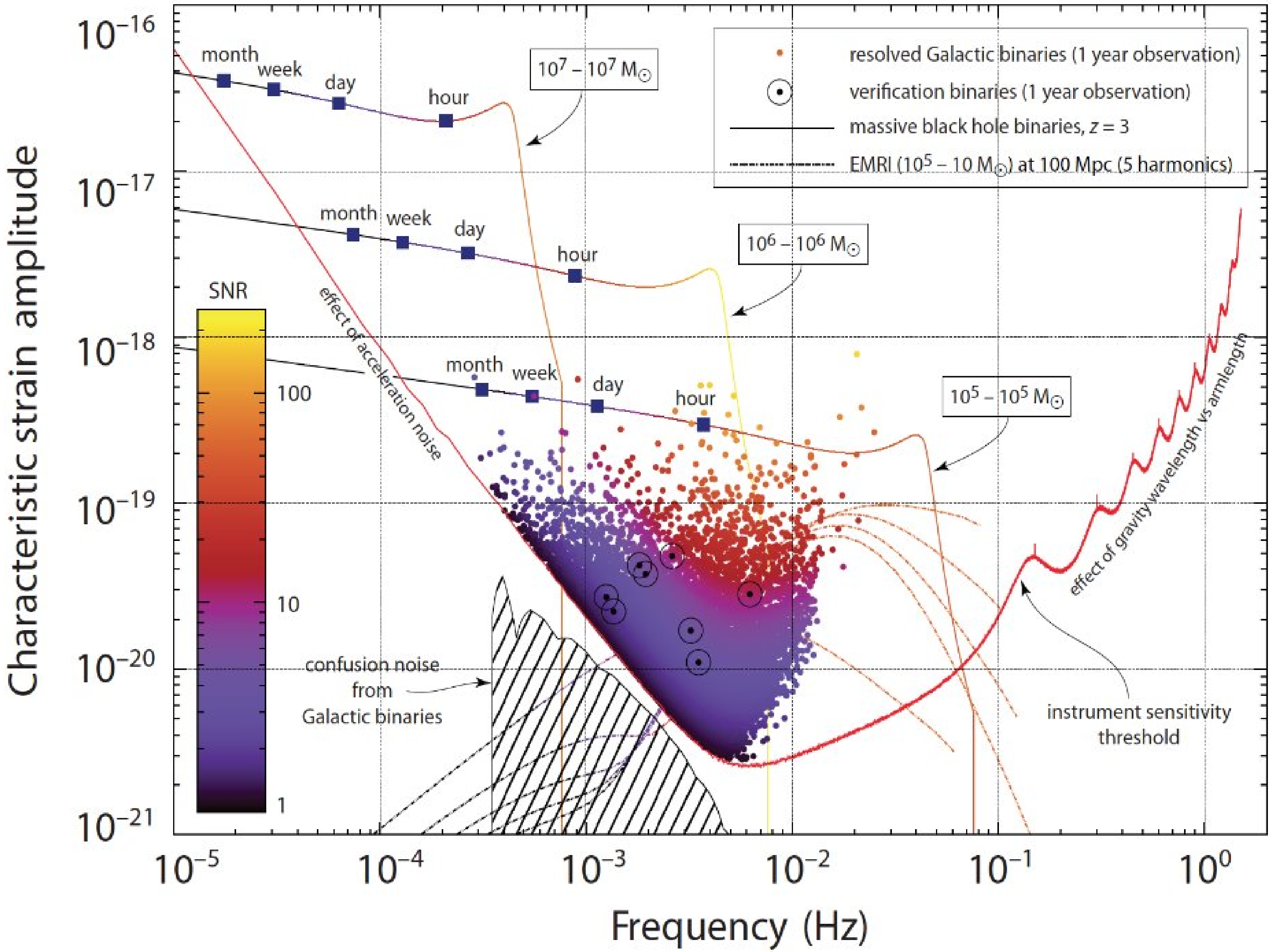


30 cm telescopes, 2W laser beams
SC relative motion ~ 10m/s
ESA L3 Mission with launch in 2034
+4 yrs operation

Sources

- **Supermassive Black Hole Binaries (SMBHBs): ~ 10^3**
  Coalescences with mass ratio larger than 10^{-1} and total masses in $(10^5, 10^7)\ M_\odot$
- **Intermediate-Mass Black Hole Binaries (IMBHBs):**
  Coalescences with mass ratio larger than 10^{-1} and total masses in $(10^2, 10^5)\ M_\odot$
- **Extreme mass-ratio and intermediate mass-ratio inspirals (EMRIs and IMRIs):**
  Coalescences with mass ratios in $(10^{-6}, 10^{-3})$ and $(10^{-3}, 10^{-1})$
  and total masses in $(10^3, 10^7)\ M_\odot$: **~10^3 EMRIs**
- **Stellar origin BH binaries (SOBHBs):**
  Inspirals with sufficiently low total mass e.g. in (50, 500) $M_\odot$
  such that they could be detected both by LISA and 2nd or 3rd generation ground-based detectors
- **Galactic Binaries: ~10^5**
  White dwarf or neutron star binary inspirals within the Milky Way that produce nearly monochromatic signals
- **Stochastic Backgrounds:**
  Cosmological sources of GWs that produce a stochastic background
- **…………………………………..**

Sources


Characteristic strain amplitude
Frequency (Hz)
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}
resolved Galactic binaries (1 year observation)
verification binaries (1 year observation)
massive black hole binaries, $z = 3$
EMRI ($10^5 - 10$ $M_\odot$) at 100 Mpc (5 harmonics)
$10^7 - 10^7$ $M_\odot$
$10^6 - 10^6$ $M_\odot$
$10^5 - 10^5$ $M_\odot$
month
week
day
hour
SNR
100
10
1
effect of acceleration noise
confusion noise from Galactic binaries
instrument sensitivity threshold
effect of gravity wavelength vs armlength

Gravitational Wave memory

- GW passing through a system of 2 isolated free-falling test masses would **permanently stretch or compress the comoving distance** between them

- memory is sourced by a changing time derivative of the system's mass multipoles (like the oscillatory component of a GW)

- it grows through the cumulative history of GW emission

- memory signal inherits the radiating system's evolving past: its strength at any time is the result of the integrated history of the system

- can be generalized i.e.not only displacement memory effect but also (subdominant) :

  - spin memory
  - center of mass memory

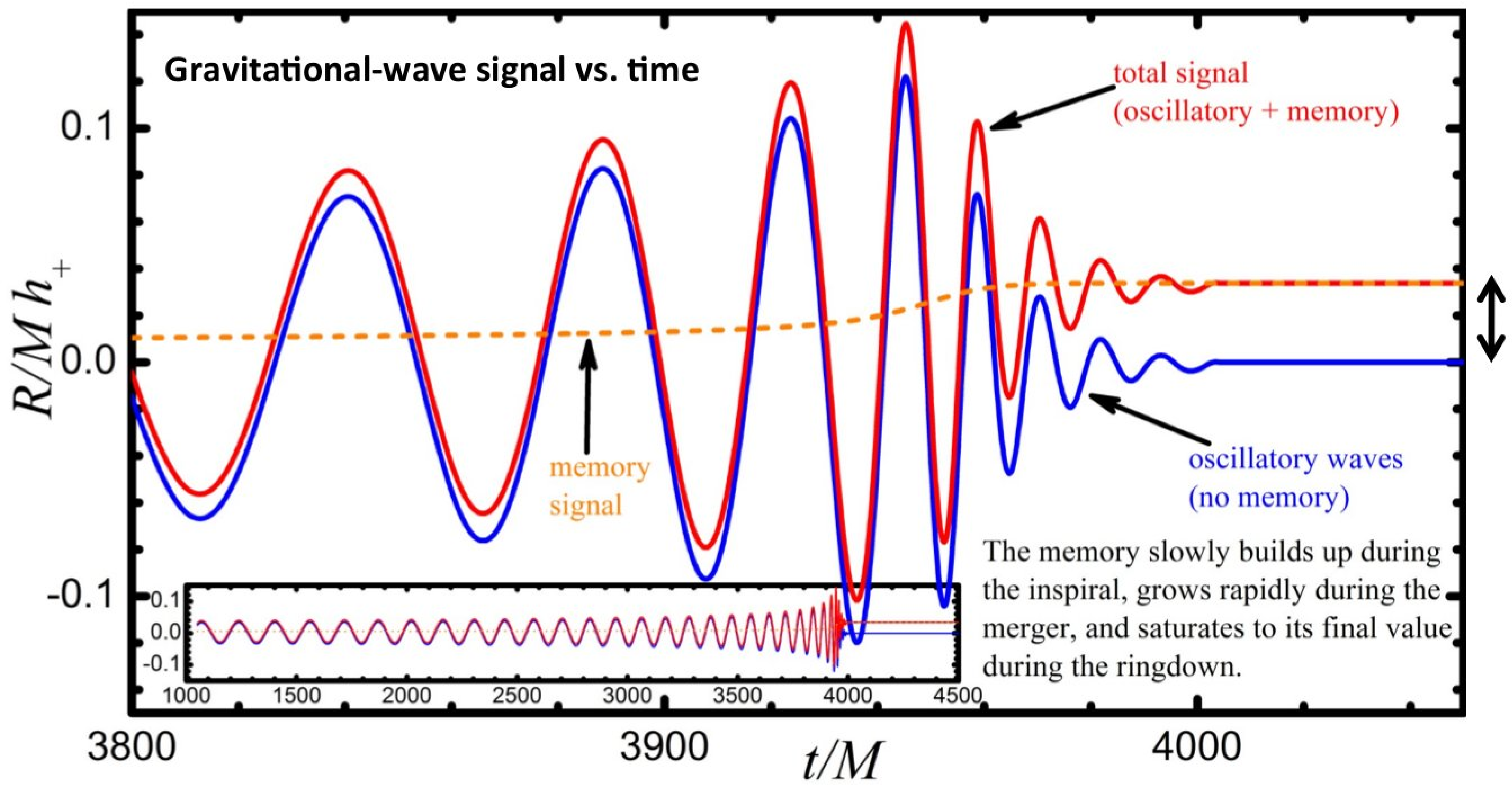
  } motivated by / associated to a symmetry (as for the displacement memory effect)

  - relative proper time, relative velocity, relative rotation memories

- focus here on the permanent displacement memory effect

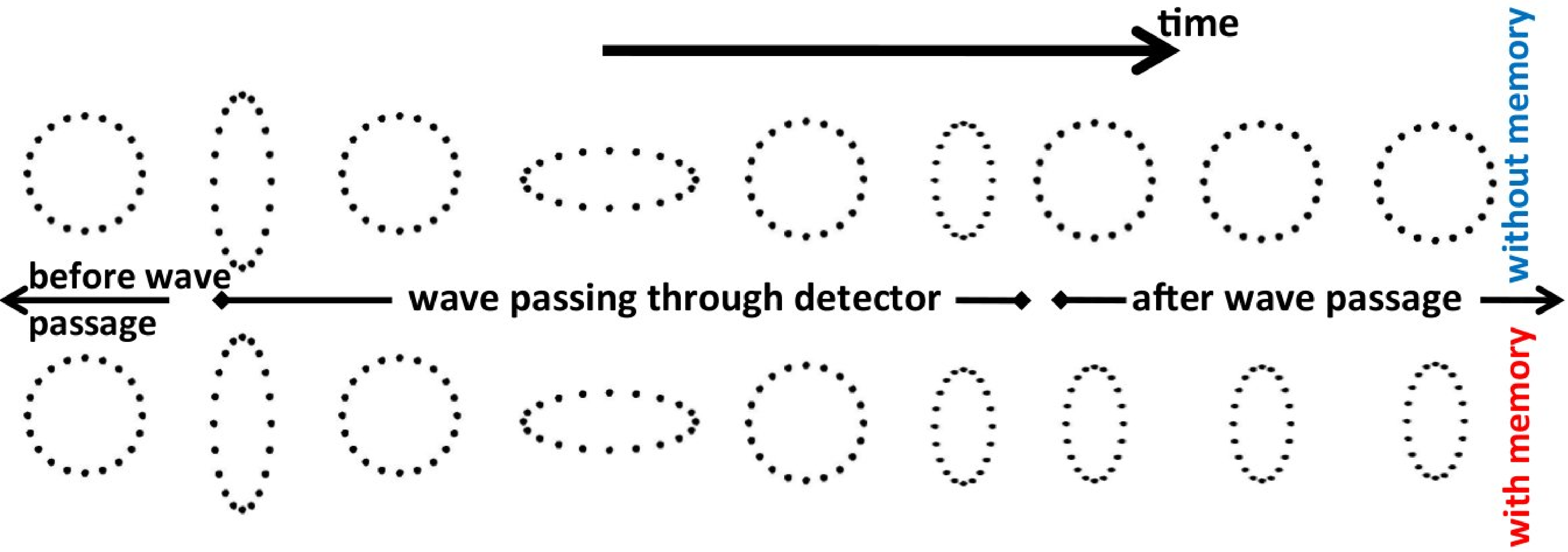
Gravitational Wave memory

example: nonlinear memory from binary black-hole mergers



from M. Favata

Gravitational Wave memory



(gravitational-waves propagating into the screen)

from M. Favata

GW memory with SMBHBs

- for a supermassive black hole binary (SMBHB) undergoing coalescence :

  - memory signal initially displays negligible growth corresponding to the slow time evolution of the binary's inspiral

  - during binary coalescence (most dynamic phase), system emits a burst of memory signal

- observations of SMBHB coalescence memory events would :

  - shed light on strong-field effects of General Relativity (GR)

  - provide information about SMBHB properties augmenting that obtained from the oscillatory components

  - provide hints for **fundamental symmetries in GR** such as **BMS symmetries**

Asymptotic symmetries : BMS group

asymptotically flat spacetimes → metric becoming flat as one approaches ∞

asymptotic symmetries :

- ordinary 4-dimensional Minkowski spacetime has a 10-parameter group of isometries
  → Poincaré group

  this isometry group plays an important role in the analysis of the behavior of physical fields on Minkowski spacetime, in particular in the proof of conservation laws

  In a general curved spacetime one would not expect any exact isometries to be present

- possible to define the notion of an asymptotic symmetry

  but group of asymptotic symmetries is not the Poincaré group

  it is a much larger group containing an infinite-dimensional subgroup of "angle dependent translations" called supertranslations → BMS group

  BMS from Bondi, Van der Burg, Metzner, Sachs (1962)

Asymptotic symmetries : BMS group

- supertranslations → angle dependent translations

→ associated conserved charges are the supermomenta

→ non-trivial diffeomorphisms acting on the asymptotically flat phase space
transforming a geometry into another one - physically inequivalent

→ supertranslations have a relationship with gravitational radiation

- supertranslations commute with the time translation

→ their associated charges will commute with the Hamiltonian

→ **all these degenerate states have the same energy**

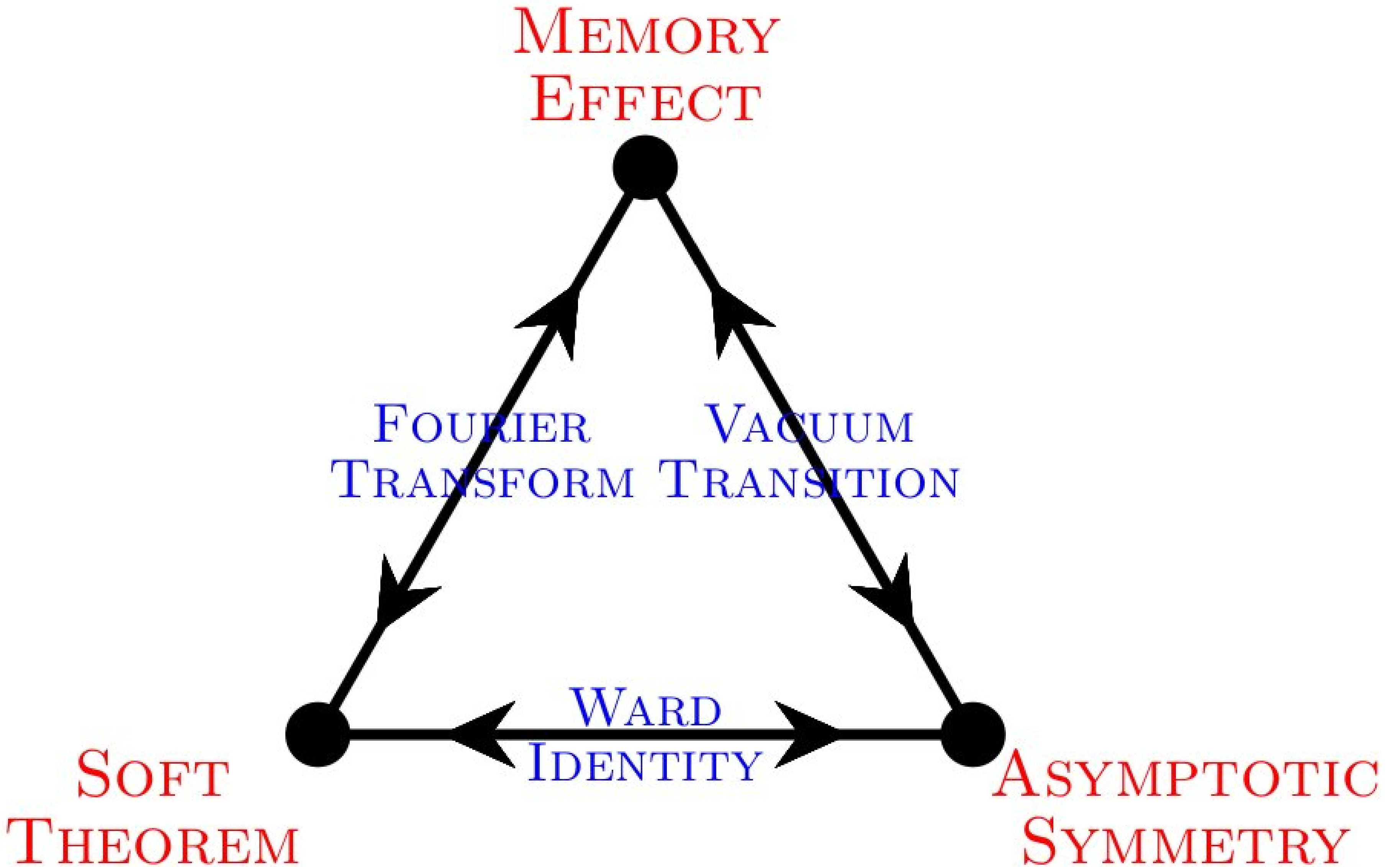
- BMS group : BMS_4 = Lorentz x Supertranslations

→ reproducing the semi-direct structure of the Poincaré group

→ only difference is that the translational part is enhanced,
implying **degeneracy of the gravitational Poincaré vacua**

→ **change in the vacuum state is detected by a net permanent displacement**
i.e. passage of GW radiation changes the vacuum by a BMS transformation

Memory effect, asymptotic symmetries and soft theorems



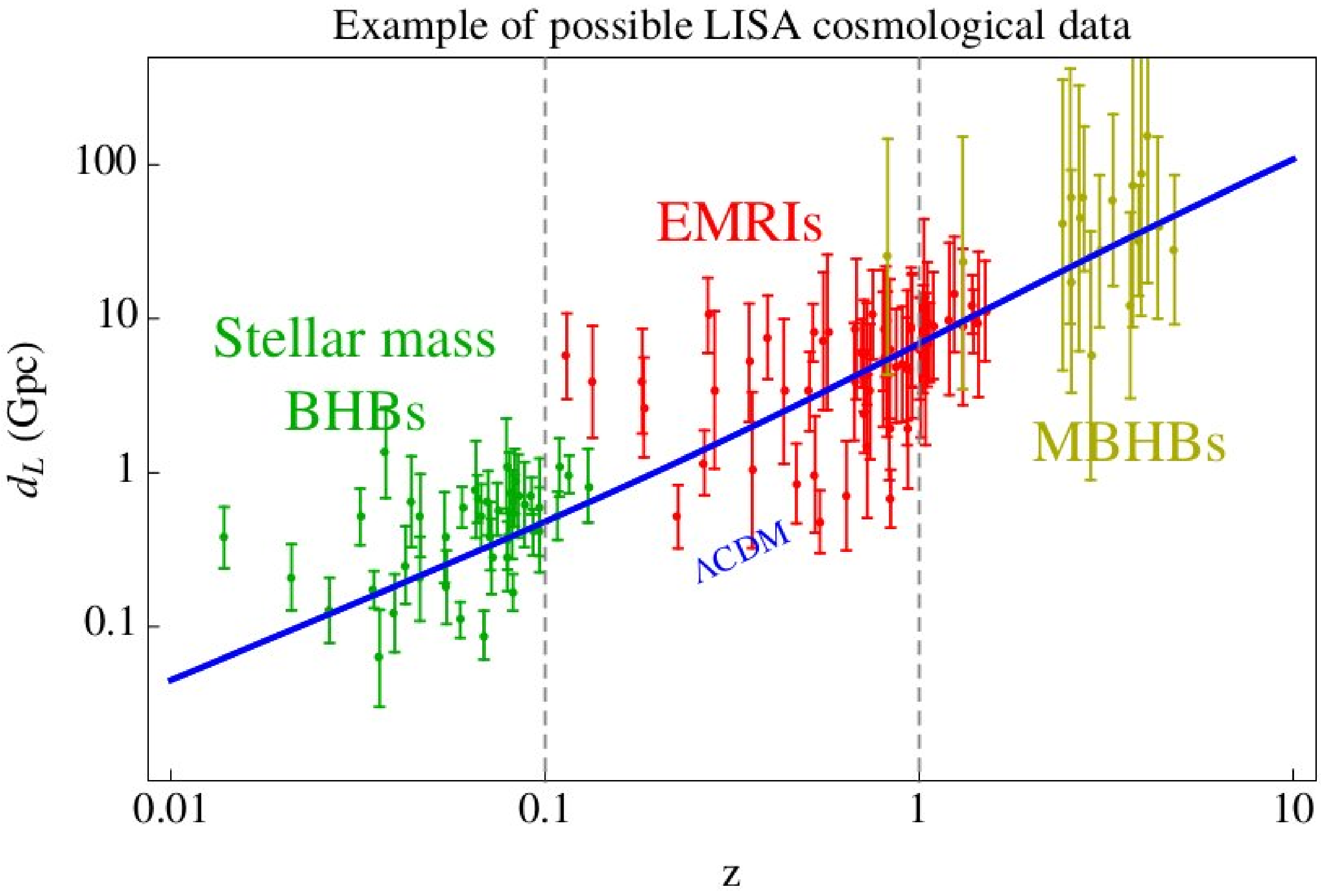
S. Weinberg Phys. Rev. 140, B516 (1965)

A. Strominger, arXiv:1703.05448

An aside on SMBHB events

- observations of SMBHB coalescence events in general would provide more informations:

  - e.g : **together with observable e.m. counterpart** →
    could provide information on the distance-redshift relation for redshifts up to 8



Prospects for Memory Detection with Low-Frequency GW Detectors

K. Islo, J. Simon, S. Burke-Spolaor, X. Siemens

arXiv:1906.11936

In this paper :

→ estimate the current and future potential to detect GW memory from SMBHB coalescence using simulation based on semi-analytic models for the SMBHB population

→ models are based on local observables for SMBHBs
encompass only uncertainties from local mass functions, galaxy merger timescales …

→ expand models to include « lower » black hole masses
i.e. down to $M_{BH} \gtrsim 10^5 M_\odot$ and higher redshifts
bands relevant to both Pulsar Timing Arrays (PTAs) and the Laser Interferometer Space Antenna (LISA)

→ try to take into account the unknown decoupling radius for binary-host interactions

SMBHBs creation and GW

- SMBHB created by hierarchical evolutionary processes involving the mergers of increasingly massive galaxies

  → SMBHBs form during major galaxy mergers

  → grow more tightly bound through repeated interactions with their galactic environment

  → interaction drives orbital evolution to smaller separations

- **effectiveness of the mechanisms by which these black hole systems are driven to coalescence is an open question in astrophysics**

- SMBHBs that eventually coalesce are candidates for producing strong GW memory bursts

SMBHBs creation and GW

- for equal-mass SMBHBs → energy available for the GW memory burst ranges from 5% to 10% of the total binary energy

- precise value depends on binary inclination and the degree of black hole spin-alignment

- **for example, an optimally-oriented binary consisting of two 10^9 $M_\odot$ black holes coalescing 1 Gpc away from Earth will emit a GW memory burst with amplitude**

→ $h_{mem} \sim 10^{-15}$

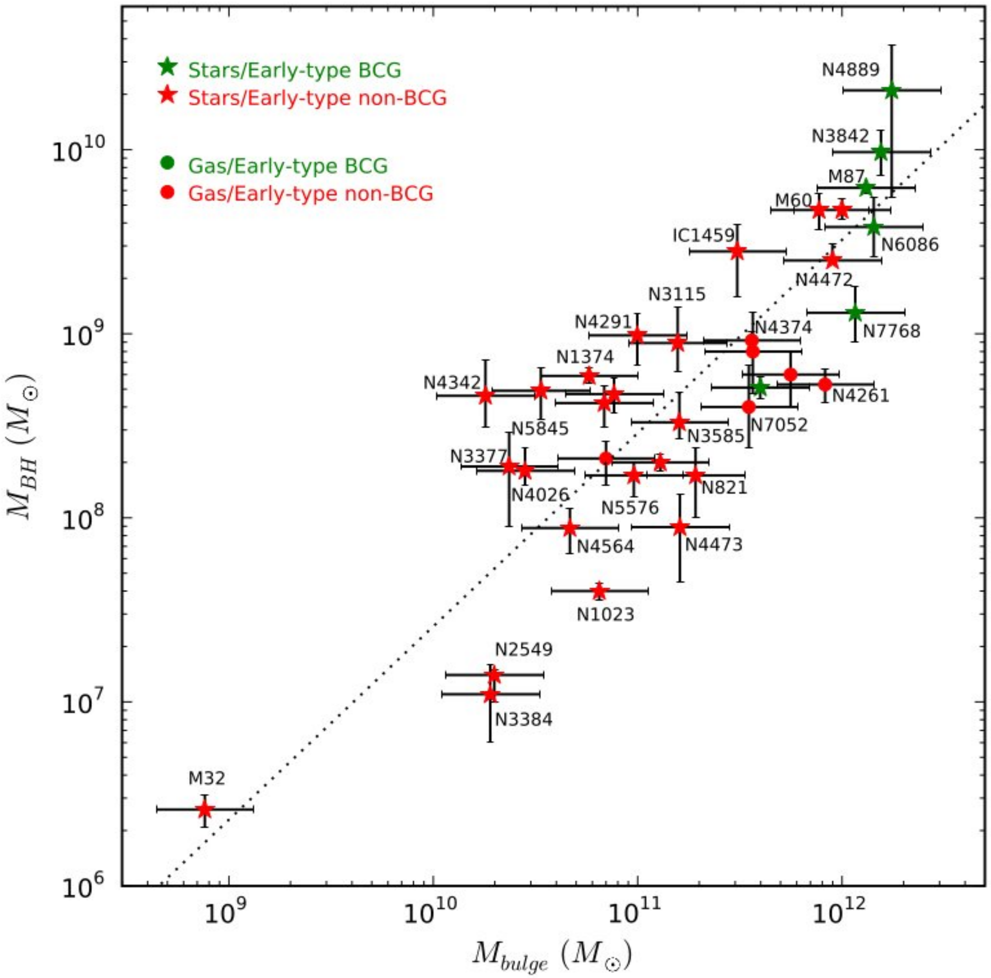
SMBHB population

- need to estimate the **SMBHB coalescence rates → model dependent**

  population model → simulation of semi-analytic models for the SMBHB population
  (Simon, Burke-Spolaor 2016)

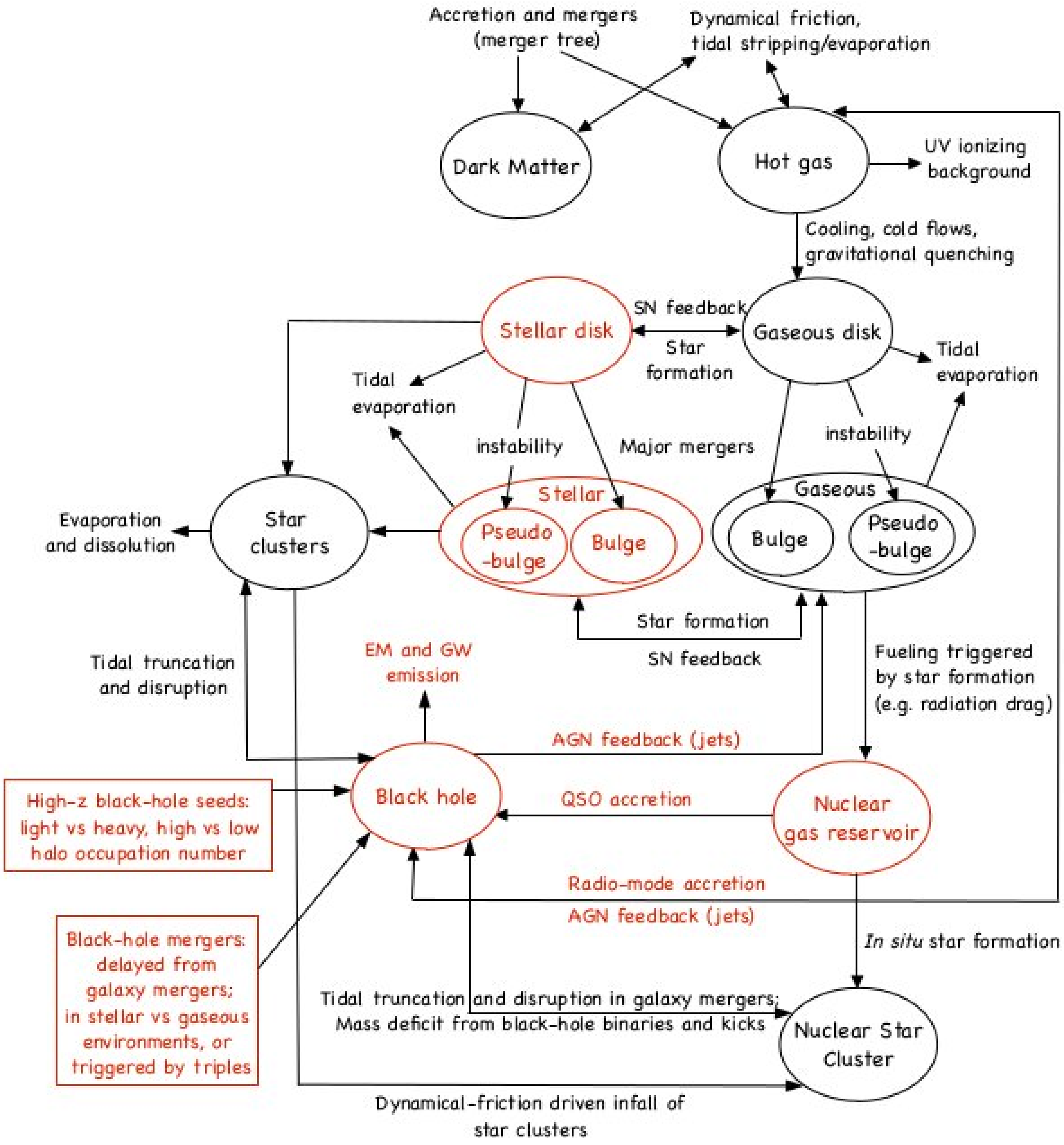
- number density of SMBHB coalescences occurring at different time interval depends on redshift, mass and mass ratio of galaxy pairs

- cast from galaxy pairs into inferred SMBHBs using the empirical relationship found between host galaxy bulge mass and black hole mass
 (McConnell & Ma 2013; Shankar et al. 2016)



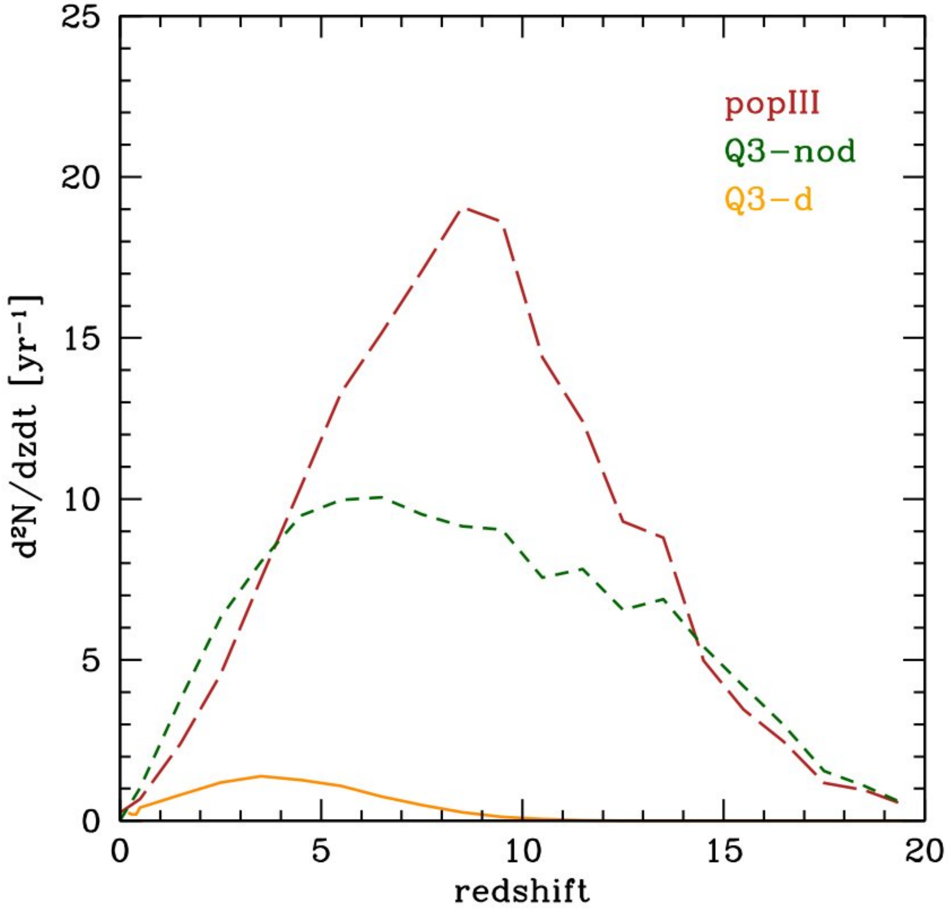
McConnell & Ma, ApJ, 764 (2013) 184

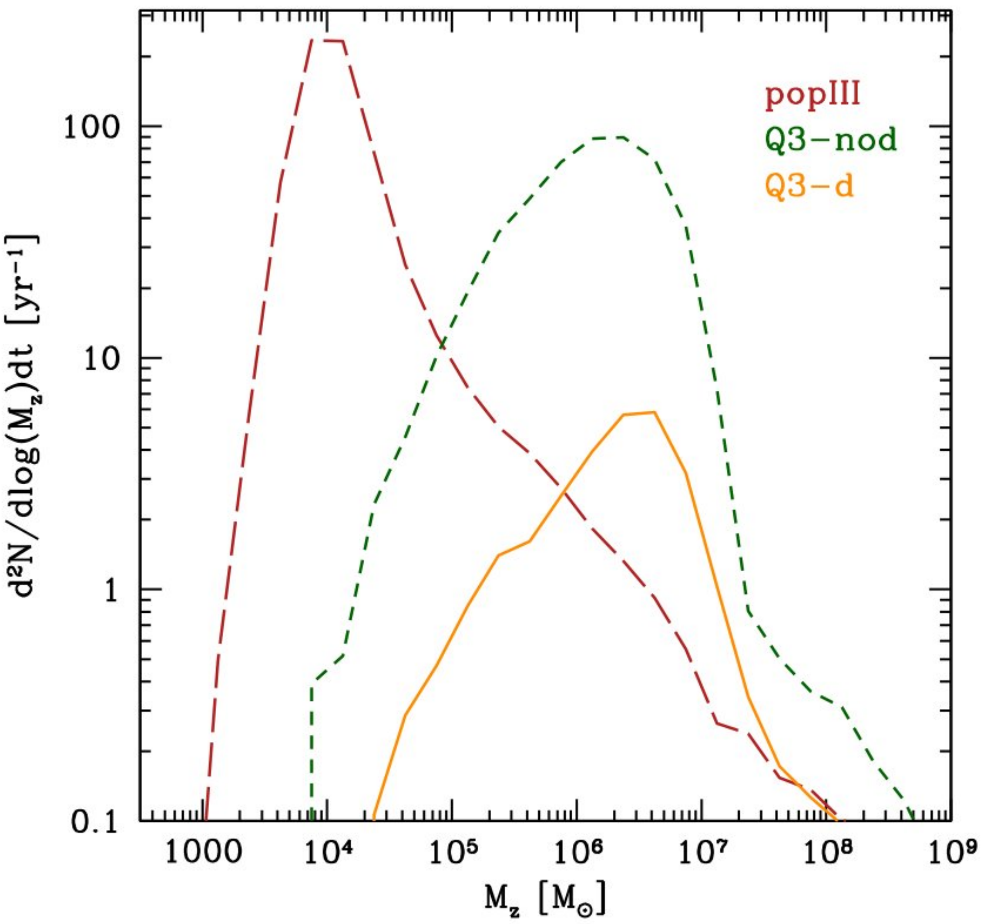
Example of Galaxy/BH co-evolution



Klein et al. PRD 93, 024003 (2016)

SMBHB event rates for LISA are uncertain





Klein et al. PRD 93, 024003 (2016)

SMBHBs population

- restrict the parameter space (to include only what is known observationally?)

  redshift : $z < 3$

  primary galaxy mass : $10^8\ M_\odot \leq M \leq 10^{12}\ M_\odot$

  mass ratio $0.25 < q < 1$ (to be consistent with ‘major mergers?)

- addition of lower-mass black holes binaries in the study i.e. $10^5 - 10^7\ M_\odot$

  Galaxy And Mass Assembly survey (Wright et al. 2017)
  to estimate the distribution of these lower mass binaries at $z < 0.1$

  ULTRAVista survey (Ilbert et al. 2013) for higher z

Evolution of SMBHBs

- at early stages of galaxy merger

  → dynamical friction to reduce the orbital angular momentum of the individual black holes until they sink to the center of the merger remnant forming a SMBHB

- **dominant mode of energy loss below ~10 pc binary separation is not yet understood**
  many environmental interactions potentially contribute (Merritt & Milosavljević 2005)

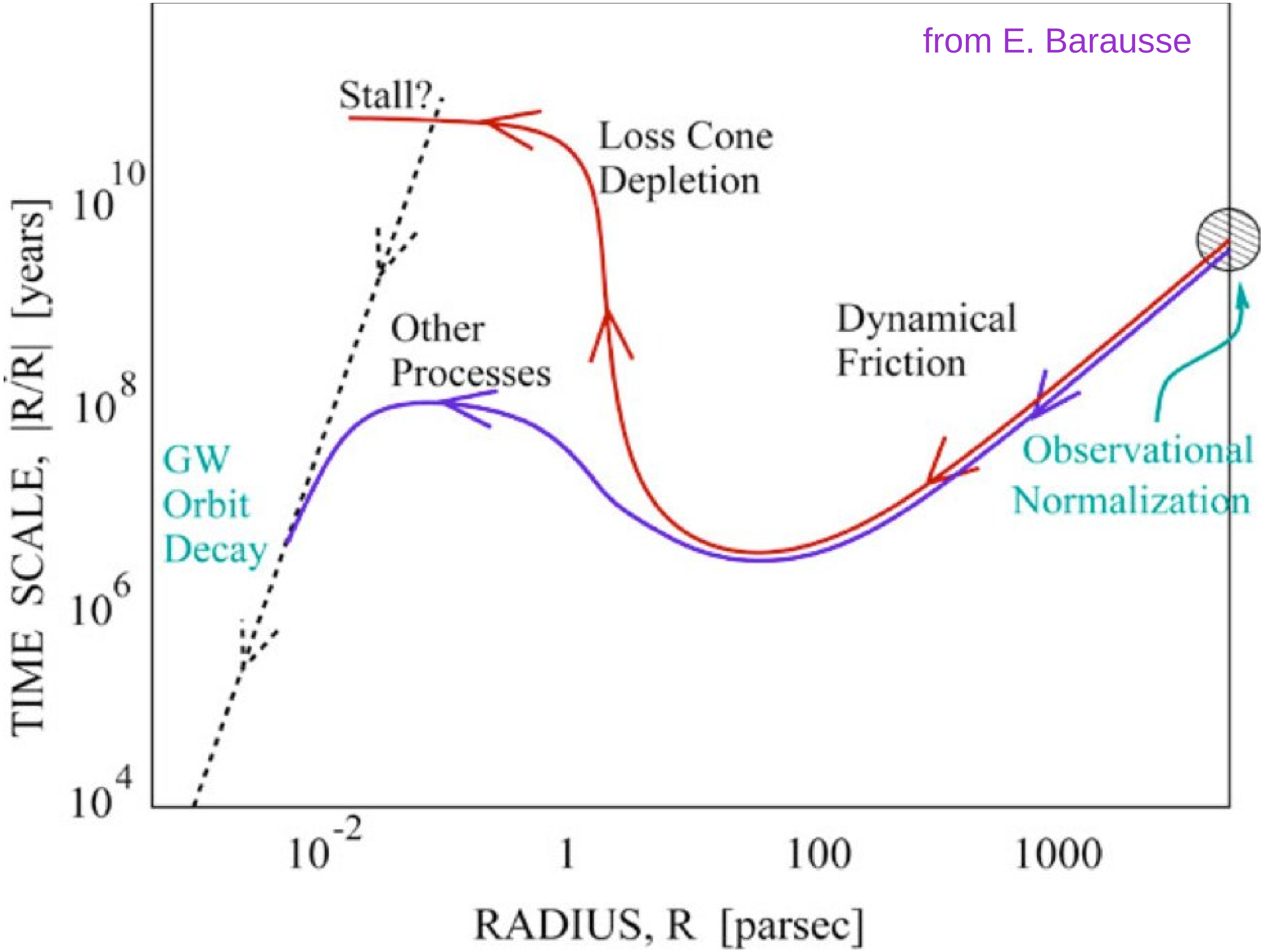
- **unclear when the environment decouples from the binary after which GW emission dominates**

- what is the upper limit to how fast the binary BH can merge ?

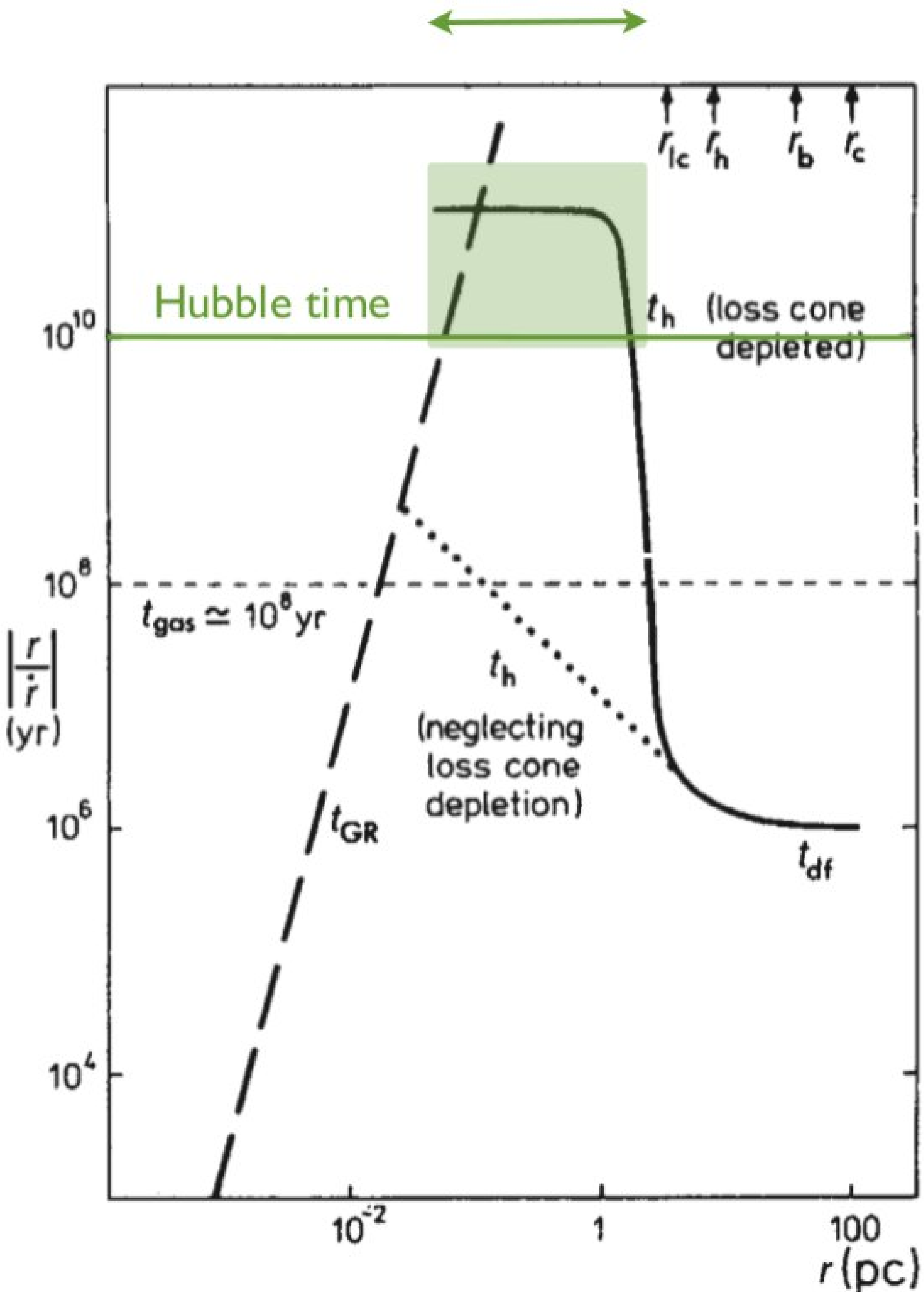
  → final parsec question ?

  interactions with stars can lead to binary BH merger
  but only over times exceeding 1 Gyr and only if all conditions are favorable (Ostriker) ?

The final parsec problem ?



there are ~ 1-2 orders of magnitude in radius between 10 pc and 0.01 pc in which orbital decay time exceeds the Hubble time (the “bottleneck”) the “bottleneck”



does the binary inspiral stall in the bottleneck?

there are ~ 1-2 orders of magnitude in radius between 10 pc and 0.01 pc in which orbital decay time exceeds the Hubble time (the “bottleneck”)

Begelman, Blandford & Rees (1980)

from E. Ostriker

Evolution of SMBHBs

- consider two redshifts (assuming coalescence does not immediately follow binary formation)

→ redshift at which a galaxy forms a binary : z_{gal}

→ redshift of the memory burst upon SMBHB coalescence : z_{burst}

- offset between z_{gal} and z_{burst} (trying to incorporate varying environmental influence)

→ introduce a parameter : **decoupling radius a_d**
to be the orbital separation at which the binary's evolution typically becomes GW-dominated

$a_d \gtrsim 1$ pc : SMBHBs embedded within sparse environments exhaust their environmental interactions earlier and have the potential to stall before reaching a regime where GW-radiation can drive the binary to coalesce

$a_d \to 0$: opposite scenario involving a binary strongly coupled to its environment, undergoing extremely efficient orbital shrinking and reaching coalescence quickly

Evolution of SMBHBs

- **adopt a simple power-law model** relating total binary mass and decoupling radius to emulate any environmental interaction (more common among smaller SMBHBs than larger ones):

$$a_d = a_8 \left(\frac{M_{tot}}{10^8 M_\odot} \right)^\alpha$$

- **least efficient environments** consist of persistently depleted loss-cone and sparse gas in the galaxy merger core

  → find the orbital separation at which the binary will stall for a given galaxy-merger-bulge mass (following Begelman, Blandford, Rees, Nature 1980)

  → assume the ratio between bulge radius and bulge mass to be linear with M87 serving as the fiducial ratio

  → best-fit parameters in this scenario : a_8 = 1.3 pc and **α = 1.0**

- **maximally-efficient environment** allows for even the most massive binaries to reach sub-parsec separations through continual loss-cone refilling and a ready supply of in-flowing gas

  → in this context : choose a_8 = 0.01 pc (fig. 1 of Begelman, Blanford Rees 1980))
  and consider **$1.0 \leq \alpha \leq 3.0$**

GW memory signal model

CEA - Saclay

→ **in the time domain** the signature of a memory signal from a SMBHB can be **approximated by a step-function** centered at the moment of coalescence

$$h_{\times +}^{(\mathrm{mem})}(t) = \Delta h_{\times +}^{(\mathrm{mem})} \; \Theta(t)$$

where Θ (t) is the Heaviside-step function

→ in the frequency domain including a minor correction for LISA

(since LISA may be able to resolve the time varying features of the memory signal between onset of coalescence and ringdown) :

$$h_{+}^{(\mathrm{mem})}(t) \simeq i \, \frac{\Delta h_{+}^{(\mathrm{mem})}}{2\pi f} \left[1 - \frac{\pi^2}{6} (\tau f)^2 \right] \qquad \text{for} \quad 0 < f < f_c$$

$$h_{+}^{(\mathrm{mem})}(t) = 0 \qquad \text{for} \quad f \geq f_c$$

where f_c is the cut-off frequency corresponding to twice the orbital frequency at coalescence

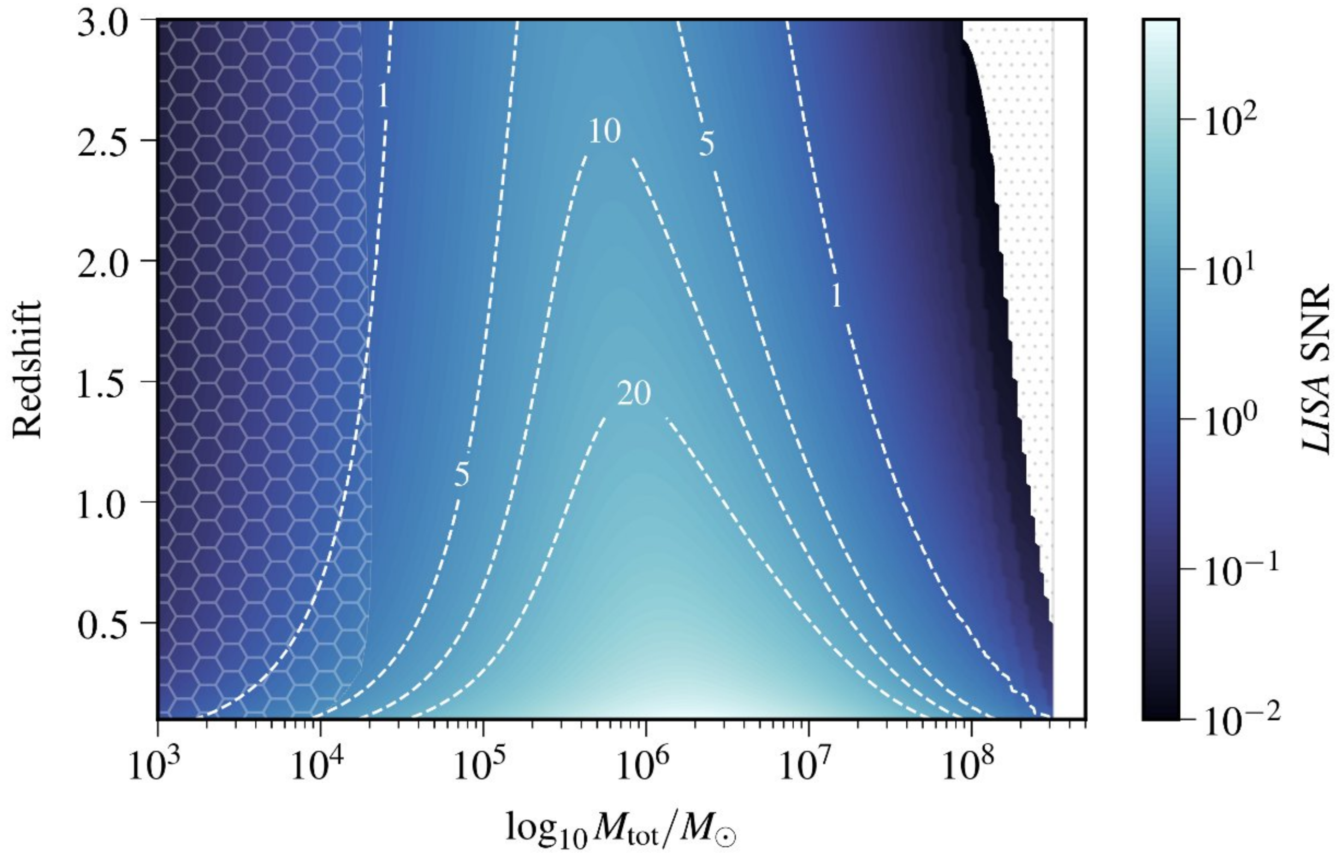
frequencies larger than f_c do not contribute to the GW signal.

can approximate $f_c \sim \tau -1$ where τ is the light crossing time of the merger remnant

τ is also the timescale for the rise of the memory signal during the merger

SNR estimates

SNR for a memory burst produced by SMBHBs
of total binary mass M_{tot} coalescing at redshift z
(optimally beamed i.e. $\iota = \pm\pi/2$)



- expected SNR ranges from 100 to 10000
  → highest SNR event from binaries at $z < 0.5$ and with $10^5\, M_\odot < M_{tot} < 10^7\, M_\odot$

- $M_{tot} < 10^{4.2}\, M_\odot$ coalescences will occur beyond the LISA frequency band (≥ 1 Hz)
  → in which case, the memory signal will be the dominant coalescence signature
  i.e. "Orphan memory" signals

Memory event rates

- LISA prospects for SNR ≥ 5 events :

→ occurring 0.3 − 2.8 times / year in the most optimistic model

→ less than 1 per million years in the most pessimistic

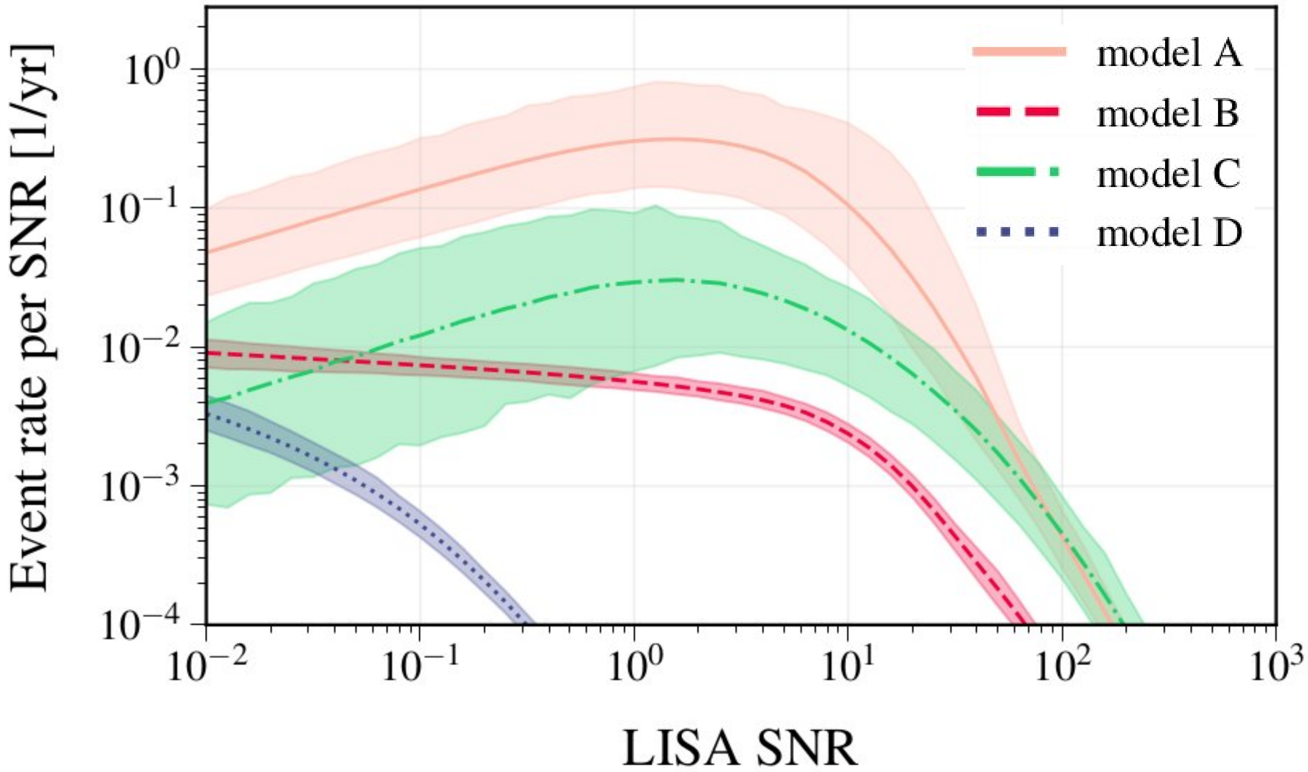


Table 1. Glossary of decoupling models

Model	GSMF	$M - M_{\rm bulge}$	Orbital Decay	Power-law parameters
A	Ilbert+Baldry	McConnell & Ma	replenished loss-cone; gas-driven	$1.0 \leq \alpha \leq 3.0$, $a_8 = 0.01\,$pc
B	Ilbert+Baldry	Shankar	replenished loss-cone; gas-driven	$1.0 \leq \alpha \leq 3.0$, $a_8 = 0.01\,$pc
C	Ilbert+Baldry	McConnell & Ma	no loss-cone refilling, no gas	$\alpha = 1.0$, $a_8 = 1.3\,$pc
D	Ilbert+Baldry	Shankar	no loss-cone refilling, no gas	$\alpha = 1.0$, $a_8 = 1.3\,$pc

Memory event rates

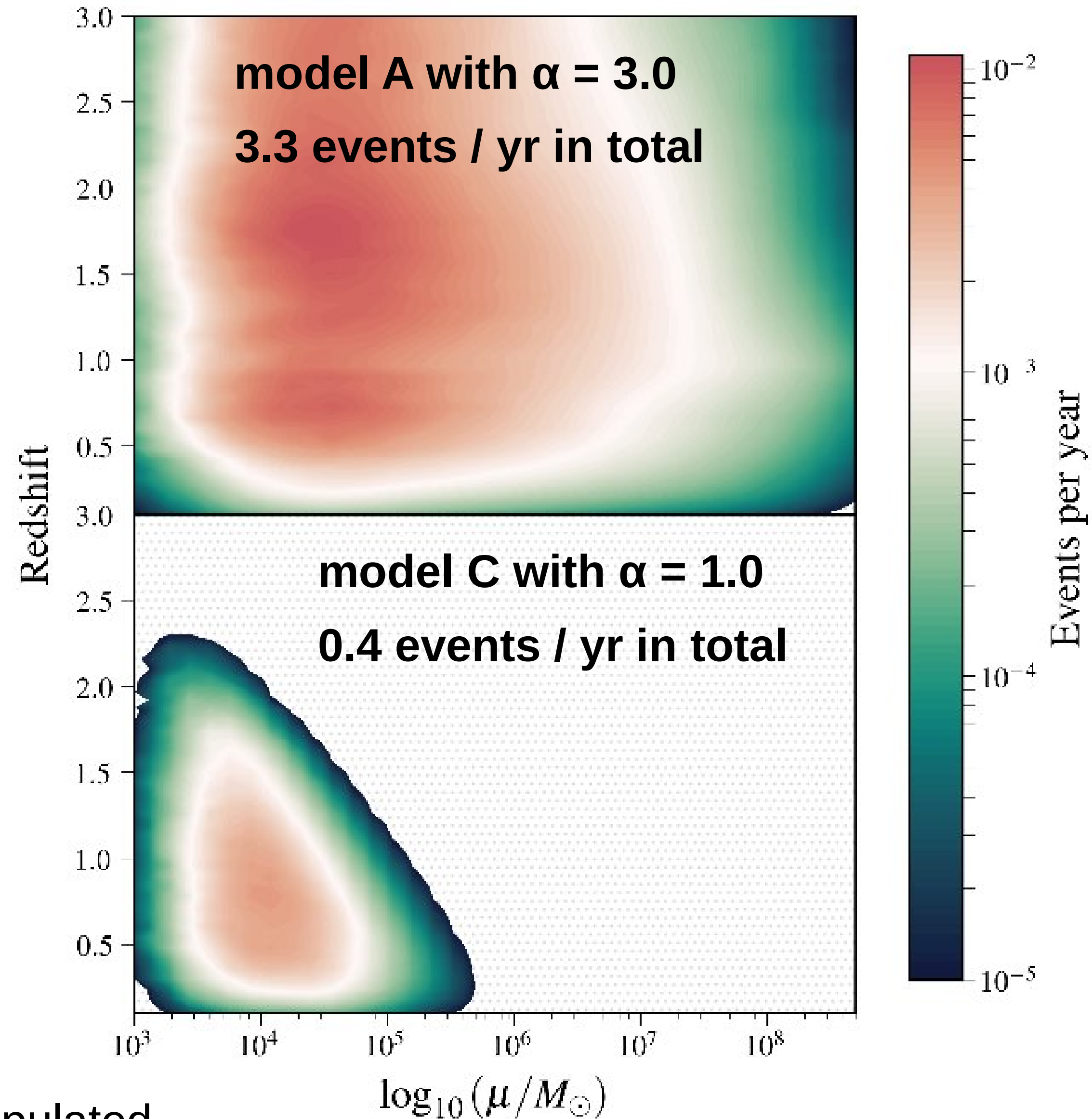
- frequent binary coalescences to occur among reduced masses μ between 10^3 $M_\odot$ and 10^6 $M_\odot$ constituting 99% of all memory events

- from model A (optimistic) to C (pessimistic)

→ decreasing environmental efficiency results in higher-mass binaries stalled at significant rates

→ total number of bursts across parameter space drops from 3.3 to 0.4 times / yr

→ lower-mass binaries initiated near the limits of parameter space evolve to closer redshifts, making up for those which may have stalled and keeping 0.1 < z < 1.5 consistently populated



outlook

- prospects for detecting a GW memory burst from SMBHB sources with LISA using simulation based on semi-analytic models for the SMBHB population

- memory effects associated to GR fundamental symetries → BMS asymptotic symmetries

- strong dependence on astrophysical inputs

  - SMBHB population

  - SMBHB coalescence mechanisms, coalescence rates

  - try to take environmental effects into account → decoupling radius (coalescence time, last parsec ?)

  - add lower-mass black holes binaries i.e. $10^5 - 10^7\ M_\odot$, in the study

- up to 3 to 4 memory burst events with SNR > 5 per year in LISA in optimistic scenarii

Reading suggestions (1)

CEA - Saclay

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BACKUP

Short formulary on GW

Consider Einstein equation

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Consider a small perturbation of the flat Cartesian metric weak field (far from source)

$$g_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu} \qquad \text{with} \qquad |\bar{h}_{\mu\nu}| \ll 1$$

Define trace reverse tensor $h^{\mu\nu} \equiv \bar{h}^{\mu\nu} - \frac{1}{2}\eta^{\mu\nu}\bar{h}$ with $\bar{h} = \eta_{\alpha\beta}\bar{h}^{\alpha\beta}$ and $\bar{h} = -h$

Equation of wave is

$$\Box h = -\frac{16\pi G}{c^4} T_{\mu\nu} = 0$$

in the Lorentz gauge (also denoted as harmonic or De Donder gauge) $\partial_\nu h^{\mu\nu} = 0$

with $\Box = \eta_{\rho\sigma}\partial^\rho \partial^\sigma$

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

propagation of GWs once they have been generated → wave equation in vacuum → $T_{\mu\nu} = 0$

$$\Box\, h = 0$$

GWs propagate at the speed of light

denote the field h^{ij} satisfying transverse and traceless gauge conditions

$$h^{00} = 0 \,,\quad h^{0i} = 0 \,,\quad \partial_i h^{ij} = 0 \,,\quad h^{ii} = 0$$

i.e. the transverse-traceless (TT) tensor h^{ij}_{TT}

assume a GW plane wave propagating along the z-axis

$$h^{ij}_{TT}(t,z) = \begin{pmatrix} h_+ & h_\times & 0 \\ h_\times & -h_+ & 0 \\ 0 & 0 & 0 \end{pmatrix} \cos\left[\omega\left(t - \frac{z}{c}\right)\right]$$

where h_+ and $h_\times$ are the two independent polarization states

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

By integrating the wave equation one gets :

$$h^{ij}(t, \vec{x}) = \frac{2G}{r c^4} \frac{d^2}{dt^2} I^{ij}\left(t - \frac{r}{c}\right)$$

Distance source-observer

Moment of inertia, or second mass moment, or quadrupole moment of mass

Retarded time

$$I^{ij} = \frac{1}{c^2} \int dx^3 \, T^{00}(t, \vec{x}) \, x^i x^j$$

not all accelerating masses generate GW but only those QUADRUPOLE

monopole and dipole disappear

for a gravitational wave to form, there must be an ASYMMETRY IN MASS DISTRIBUTION

$G/c^4 \approx 8.24 \times 10^{-45}\ s^2 . m^{-1} . kg^{-1}$ (→ space-time « rigidity »)

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

momenta of the mass density (quantity T^{00}/c^2 is a mass density)

Mass energy (conserved)

$$I = \frac{1}{c^2} \int dx^3 \, T^{00}(t, \vec{x})$$

centre of mass energy (conserved)

$$I^i = \frac{1}{c^2} \int dx^3 \, T^{00}(t, \vec{x}) \, x^i$$

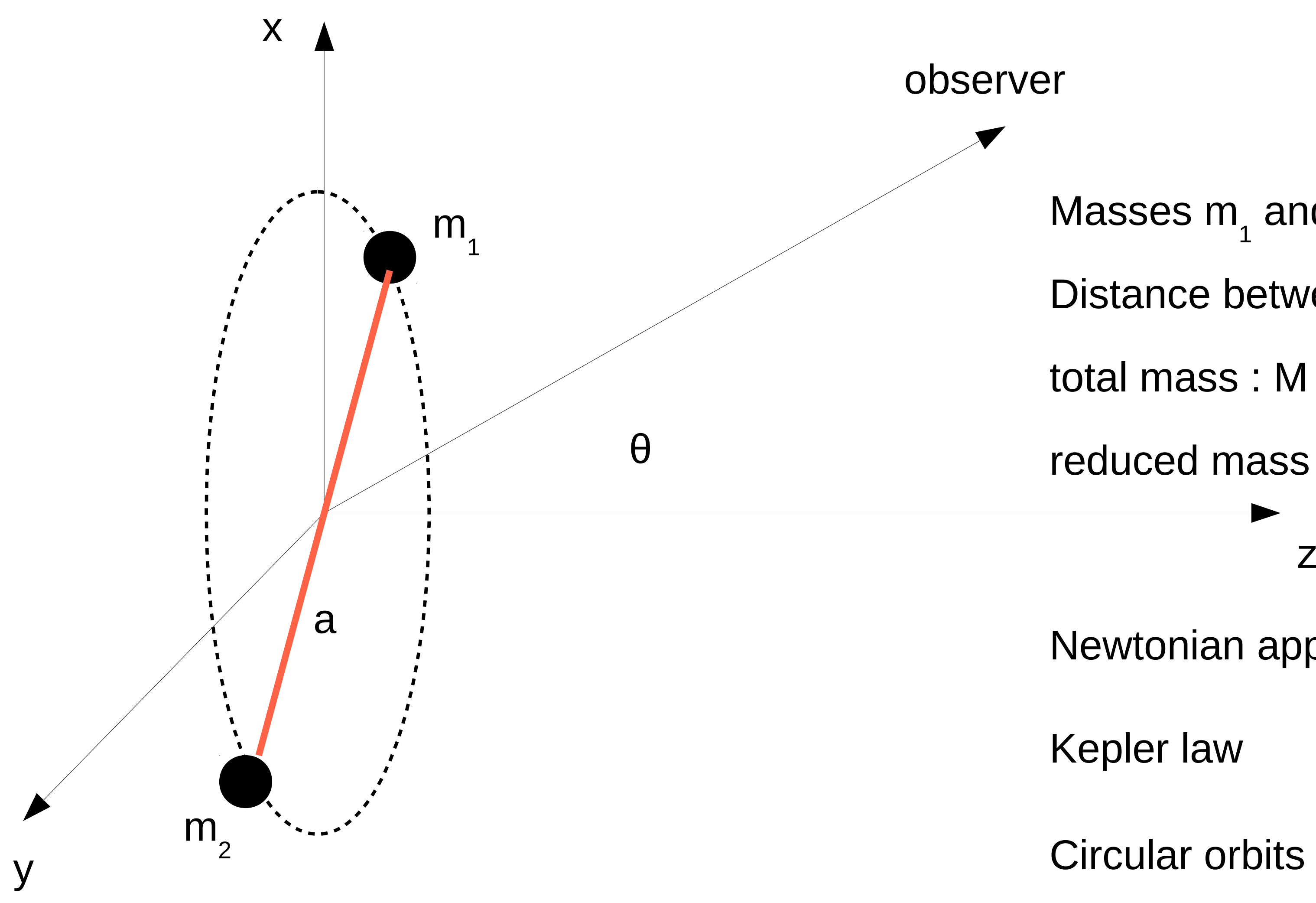
Moment of inertia (not conserved)

$$I^{ij} = \frac{1}{c^2} \int dx^3 \, T^{00}(t, \vec{x}) \, x^i x^j$$

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

GWs from BINARIES



Masses m_1 and m_2

Distance between compact object : a

total mass : $M = m_1 + m_2$

reduced mass : $M = m_1 m_2 / m_1 + m_2$

Newtonian approximation

Kepler law $\omega = \sqrt{\frac{G\,m}{a^3}}$

Circular orbits

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

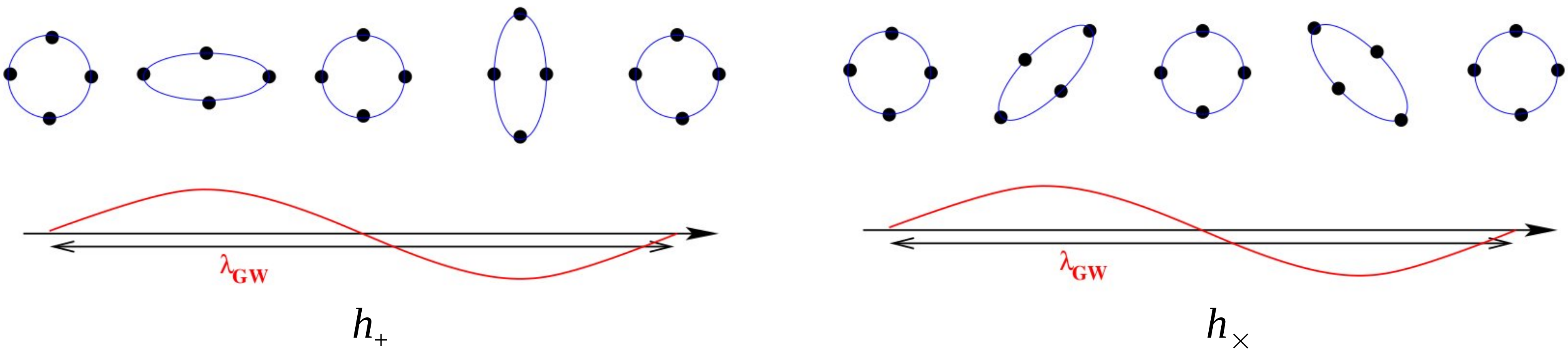
For a binary system, one can show that $h^{ij}(t, \vec{x}) = \frac{2G}{r c^4} \frac{d^2}{dt^2} I^{ij}\left(t - \frac{r}{c}\right)$ can be put as

(in spherical coordinates (r, θ, Φ) and for eccentricty e = 0) :

$$h_{+}(t, \theta, \phi, r) = \frac{1}{r} \frac{4 G \mu}{c^4} \omega_{orb}^2 a^2 \left(\frac{1 + \cos^2 \theta}{2}\right) \cos\left(2 \omega_{orb} t_{ret} + \phi\right)$$

$$h_{\times}(t, \theta, \phi, r) = \frac{1}{r} \frac{4 G \mu}{c^4} \omega_{orb}^2 a^2 \cos\theta \sin\left(2 \omega_{orb} t_{ret} + \phi\right)$$

where $t_{ret} = t - r/c$ $\qquad \omega_{orb}^2 = \frac{G (m_1 + m_2)}{a^3}$



From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

Frequency term depends only on 2 ω_{orb}

Frequency of GW : $\omega_{GW} = 2\,\omega_{orb}$ (true for most of evolution)

Amplitude of GW (dimensionless strain) :

$$h = \frac{1}{2}\sqrt{h_+^2 + h_\times^2} = \frac{2\,G^2 m_1 m_2}{a\,c^4}\,\frac{1}{r}\,\sqrt{\left(\frac{1 + \cos^2\theta}{2}\right)^2 + \cos^2\theta}$$

→ the bigger the amplitude (strain), the easier the detection

→ the farther the binary, the smaller the amplitude

→ the larger the masses, the larger the amplitude

→ the smaller the semi-major axis (a) , the larger the amplitude

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

emission of GWs implies loss of orbital energy : $E_{orb} = -\frac{G\left(m_1 + m_2\right)}{2a}$

the binary shrinks while emitting Gws till it merges

If the binary shrinks (a → 0), frequency becomes higher : $\omega_{GW} = 2\,\omega_{orb} = 2\sqrt{\frac{G\left(m_1 + m_2\right)}{a^3}}$

If the binary shrinks the amplitude increases : $h \propto \frac{1}{a}$

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

emission of GWs implies loss of orbital energy → Power radiated by GWs :

$$P_{GW} = \frac{32}{5}\frac{G^4}{c^5}\frac{1}{a^5} m_1^2 m_2^2 \left(m_1 + m_2 \right)$$

From GR

$$P_{GW} = \frac{d E_{orb}}{dt} = \frac{G m_1 m_2}{2 a^2} \frac{da}{dt}$$

From Kepler
and Newton

$$\longrightarrow \quad \frac{da}{dt} = \frac{64}{5}\frac{G^3}{c^5} a^{-3} m_1 m_2 \left(m_1 + m_2 \right)$$

Integrating the differential equation → one gets the timescale for a system to merge by GW emission :

$$t_{GW} = \frac{5}{256}\frac{c^5}{G^3}\frac{a^4}{m_1 m_2 \left(m_1 + m_2 \right)}$$

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

For binaries with general eccentricity e

$$t_{GW} = \frac{5}{256} \frac{c^5}{G^3} \frac{a^4 \left(1 - e\right)^{7/2}}{m_1 m_2 \left(m_1 + m_2\right)}$$

Timescale depends on semi-major axis (a) , eccentricity, masses

Timescale extremely long :

for 2 neutron stars with mass equal to the Sun mass $m_1 = m_2 = M_{sun}$
orbiting at the distance between Sun and Earth i.e. a = 1 AU and with eccentricity e = 0
→ $t_{GW} \sim 2 \times 10^{17}$ yr
Life of the universe GW ~ 13×10^9 yr

$$a\left(t\right) = a_0 \left[1 - \frac{256/5\, G^3 m_1 m_2 \left(m_1 + m_2\right) t}{c^5 \left(1 - e^2\right)^{7/2} a_0^4}\right]^{1/4}$$

Previous equations are only true before merger when binary can be considered Keplerian
i.e. only during inspiral

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

the orbital frequency and GW frequency change in time

$$\omega_{orb}^2 = \frac{G\left(m_1 + m_2\right)}{a^3} = \frac{GM}{a^3} \qquad E_{orb} = -\frac{G\left(m_1 + m_2\right)}{2\,a} \longrightarrow \dot{a} = -\frac{2}{3}\left(a\,\dot{\omega}_{orb}\right)\left(\frac{\dot{\omega}_{orb}}{\omega_{orb}^2}\right)$$

as long as $\dot{\omega}_{orb}/\omega_{orb}^2 \ll 1$

→ the radial velocity is smaller than the tangential velocity and
the binary's motion is well approximated by an adiabatic sequence of quasi-circular orbits

the orbital frequency varies as (with $\nu = \mu / M$) :

$$\frac{\dot{\omega}_{orb}}{\omega_{orb}^2} = \frac{96}{5}\ \nu\left(\frac{G\ M\ \omega_{orb}}{c^3}\right)^{5/3}$$

and the GW frequency $\omega_{GW} = 2\,\omega_{orb}$

$$\dot{\omega}_{GW} = \frac{96}{5}\ \pi^{8/3}\left(\frac{G\ M_{chirp}}{c^3}\right)^{5/3}\ \omega_{GW}^{11/3}$$

with $M_{chirp} = \mu^{3/5} M$

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

Introducing the time to coalescence $\tau = t_{coal} - t$

and integrating $\dot{\omega}_{GW} = \frac{96}{5} \pi^{8/3} \left(\frac{G\ M_{chirp}}{c^3} \right)^{5/3} \omega_{GW}^{11/3}$ one gets :

$$\omega_{GW} \simeq 130 \left(\frac{1.21\, M_{\odot}}{M_{chirp}} \right)^{5/8} \left(\frac{1\ \mathrm{sec}}{\tau} \right)^{3/8} \ \mathrm{Hz}$$

coalescence times of ~ 17min, 2sec, 1msec, for ω_{GW} ~ 10, 100, 10^3 Hz

relation between the radial separation and the GW frequency

$$a \simeq 300 \left(\frac{M}{2.8\, M_{\odot}} \right)^{1/3} \left(\frac{100\ \mathrm{Hz}}{\omega_{GW}} \right)^{2/3} \ \mathrm{km}$$

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

a useful quantity is the number of GW cycles, defined by :

$$N_{GW} = \frac{1}{\pi} \int_{t_{\rm in}}^{t_{\rm fin}} \omega(t)\, dt = \frac{1}{\pi} \int_{\omega_{\rm in}}^{\omega_{\rm fin}} \frac{\omega}{\dot{\omega}} d\,\omega$$

Assuming $\omega_{\rm fin} \gg \omega_{\rm in}$, we get

$$N_{GW} \simeq 10^4 \left(\frac{M_{chirp}}{1.21\, M_\odot} \right)^{-5/3} \left(\frac{\omega_{\rm in}}{10\ \ {\rm Hz}} \right)^{-5/3}$$

From M. Mapelli, A. Buonanno and D. Buskulic

Short formulary on GW

displacements ΔL induced by a passing GW $\Delta L / L \sim h$

For a GW strain of 10^{-20} and typical LISA arm length of $2.5\ 10^{9}$ m
→ displacement $\Delta L \sim 2.5\ 10^{-11}$ m = 10 pm

From M. Mapelli, A. Buonanno and D. Buskulic

Dark energy and the ΛCDM model

- test models of dark energy **through the distance-redshift relation**

- GW sources at cosmological distances as reliable and independent distance indicators

  → yield a direct measurement of the luminosity distance
  (which does not need to be calibrated with the cosmic distance ladder)

  → for cosmological applications they need a corresponding redshift measurement

  joint detection of an EM counterpart to infer the GW source redshift

  without any EM counterpart identification use “statistical method” on galaxy catalogues
  to infer redshift information

- LISA will detect mainly 3 types of GW sources at cosmological distances : SMBHBs, EMRIs, and SOBHBs

  → these sources to be observed at different redshift ranges:

  SOBHBs at $z < 0.1$
  EMRIs at $0.1 < z < 1$
  SMBHBs at $1 < z < 10$

  → **only SMBHBs are expected to provide observable EM counterparts**

modeling of the expected sources

CEA - Saclay

- use the results of semi-analytical simulations of the evolution of the BH masses and spins during galaxy formation and evolution :

→ predict the rate and redshift distribution of MBHB merger events

→ produce several variants of semi-analytical model by considering :

- competing scenarios for the initial conditions for the massive BH population at high z

1) "**light-seed**" scenario : first massive BHs form from remnants of population III stars (popIII)

2) "**heavy-seed**" scenario : massive BHs form from the collapse of protogalactic disks

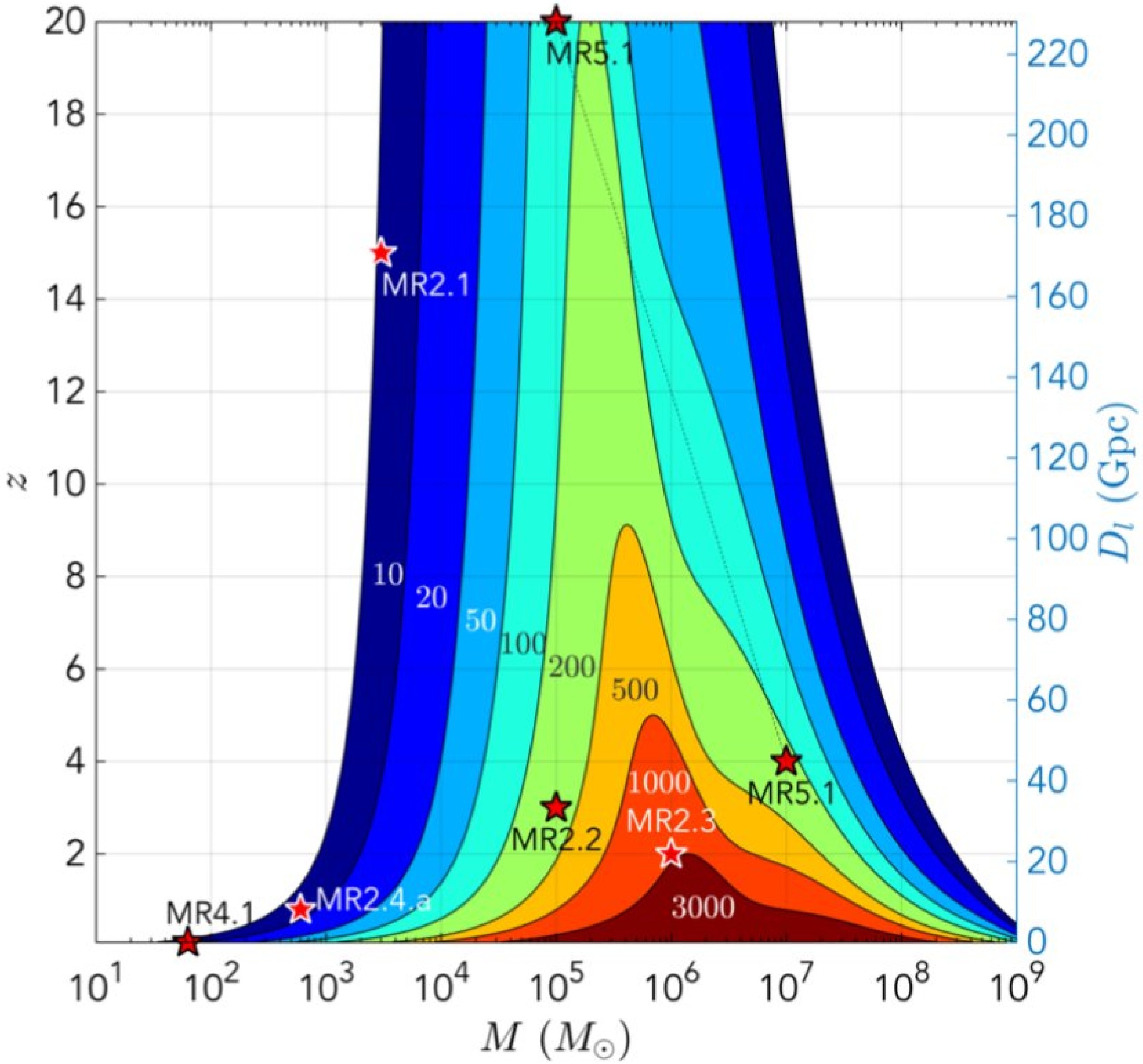
- delays with which massive BHs merge after their host galaxies coalesce

→ for each variant produce synthetic catalogues of :

- MBHB merger events including all information about MBHBs (masses, spins, z, ...)

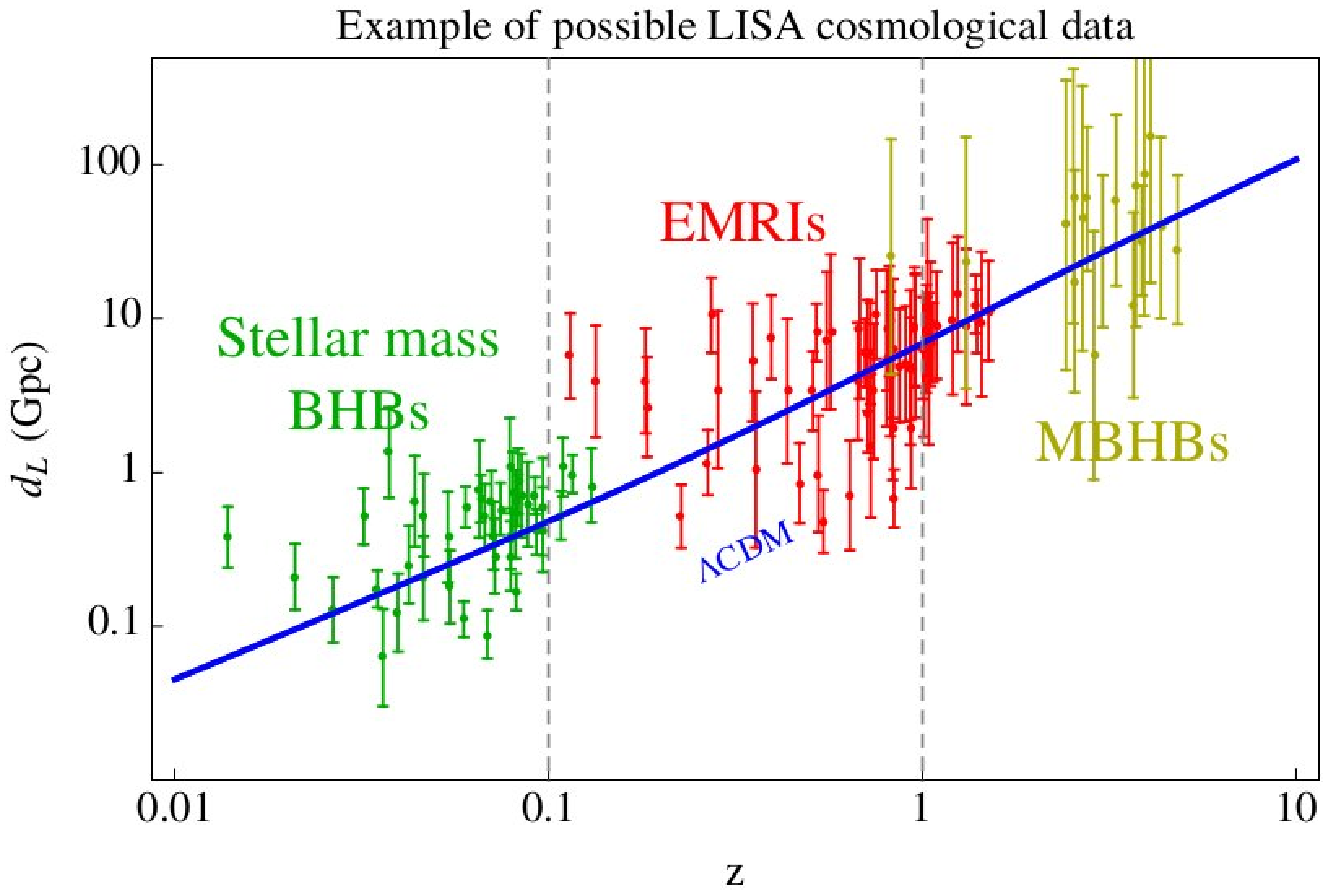
- and their host galaxies (mass in gas, mass in stars, ...)

Dark energy and the ΛCDM model

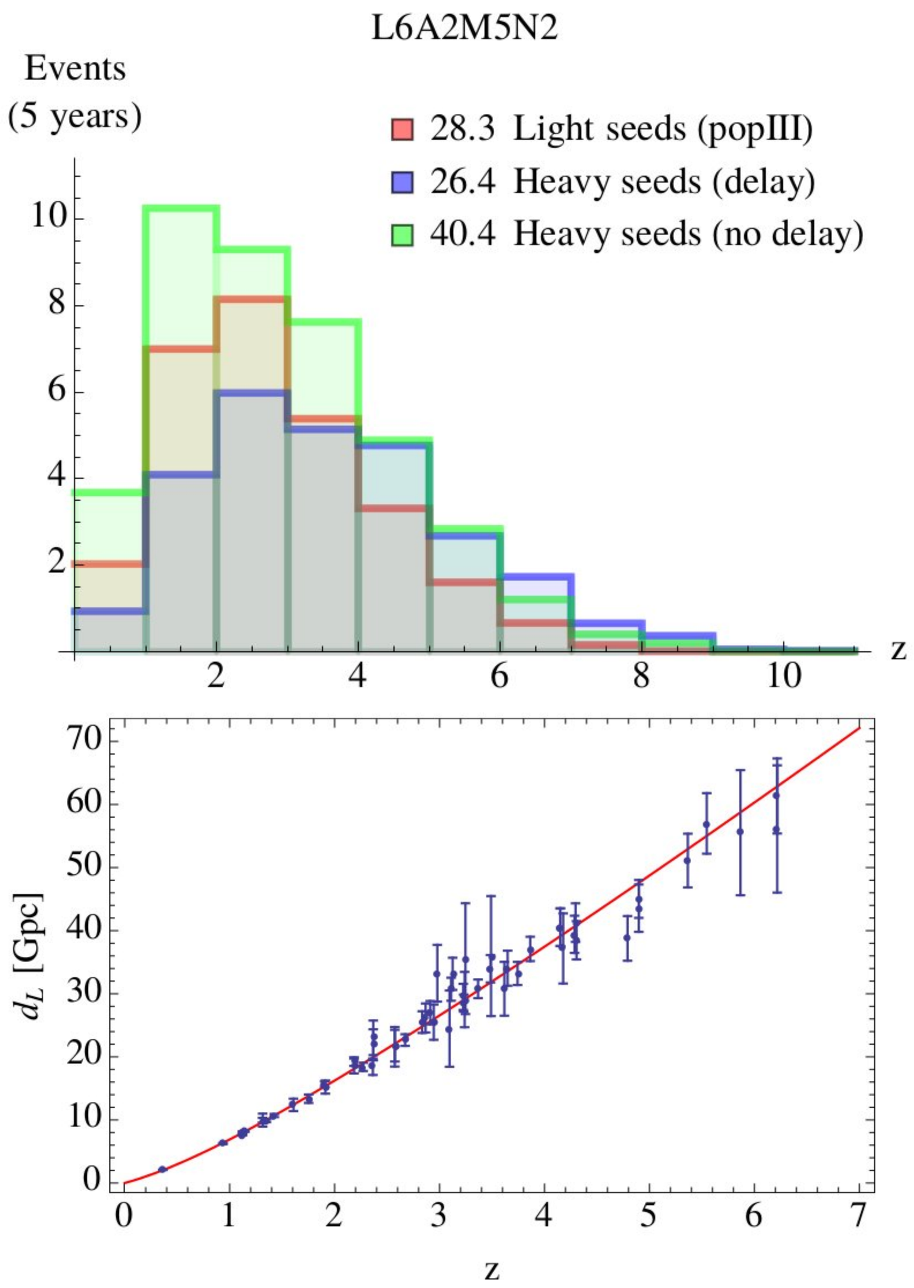


S/N levels as a function or redshift (left scale) and luminosity distance (right scale) and of total source frame mass for the baseline configuration of LISA, for a fixed mass ratio of 0.2 (the stars identify threshold cases to define mission requirements)

Dark energy and the ΛCDM model



Dark energy and the ΛCDM model



- redshift range: 1 < z < 8

- method: with counterparts

- expected detections: 10 − 100 /yr

- average LISA errors:

  $\Delta d_L / d_L$ ~ few % (inc. lensing)

  $\Delta\Omega < 10\ deg^2$

- useful standard sirens

  ∼ 6 /yr (with counterpart)

- expected results:

  H_0 to ∼1%

  w_0 to ∼ 15%

Definitions reminder

CEA - Saclay

- from Friedmann equations → **Hubble rate** $H = \dot{a}/a$ in the late universe can be expressed in terms of the **redshift** $z = a_0/a - 1$ as :

$$H(z) = H_o \sqrt{\Omega_M (z+1)^3 + (1 - \Omega_\Lambda - \Omega_M)(z+1)^2 + \Omega_\Lambda \exp\left[-\frac{3 w_a z}{z+1}\right](z+1)^{3(1+w_o+w_a)}}$$

$H_0 = h \times 100$ km/(s Mpc) → Hubble constant today

$\Omega_M = 8\pi G \rho^0_M / (3H^2_o)$ → relative energy density of matter today (dark + baryonic)

$\Omega_\Lambda = \Lambda c^2 / (3 H^2_o)$ or $\Omega_\Lambda = 8\pi G\rho^0_{DE} / (3H^2_o)$ → cosmological constant or dark energy (DE) energy density today

$w(z) = w_0 + (1-a)\, w_a = w_0 + w_a\, z / (z+1)$ → model for DE equation of state

$\Omega_k = -k c^2 / (a_o H_o)^2$ → effective relative energy density for the curvature and we have $\Omega_k + \Omega_M + \Omega_\Lambda = 1$

adopt fiducial cosmological model with parameter values :
$\Omega_M = 0.3$, $\Omega_\Lambda = 0.7$, $h = 0.67$ ($H_0 = 67$ km/s/Mpc), $w_0 = -1$, $w_a = 0$

Definitions reminder

- luminosity distance $d_L = \sqrt{L/(4\pi F)}$

  L → intrinsic luminosity of a source

  F → the flux received by the observer

- accounting for the redshift and expansion effects one gets the distance-redshift relation :

$$d_L(z) = \frac{c}{H_0}\frac{1+z}{\sqrt{|\Omega_k|}} \sin\left[\sqrt{|\Omega_k|}\int_0^z \frac{H_o}{H(z')}dz'\right] \quad \text{if} \quad \Omega_k = 1 - \Omega_M - \Omega_\Lambda > 0$$

$$d_L(z) = c\,(1+z)\int_0^z \frac{1}{H(z')}dz' \quad \text{if} \quad \Omega_k = 1 - \Omega_M - \Omega_\Lambda = 0$$

$$d_L(z) = \frac{c}{H_0}\frac{1+z}{\sqrt{|\Omega_k|}} \sinh\left[\sqrt{|\Omega_k|}\int_0^z \frac{H_o}{H(z')}dz'\right] \quad \text{if} \quad \Omega_k = 1 - \Omega_M - \Omega_\Lambda < 0$$

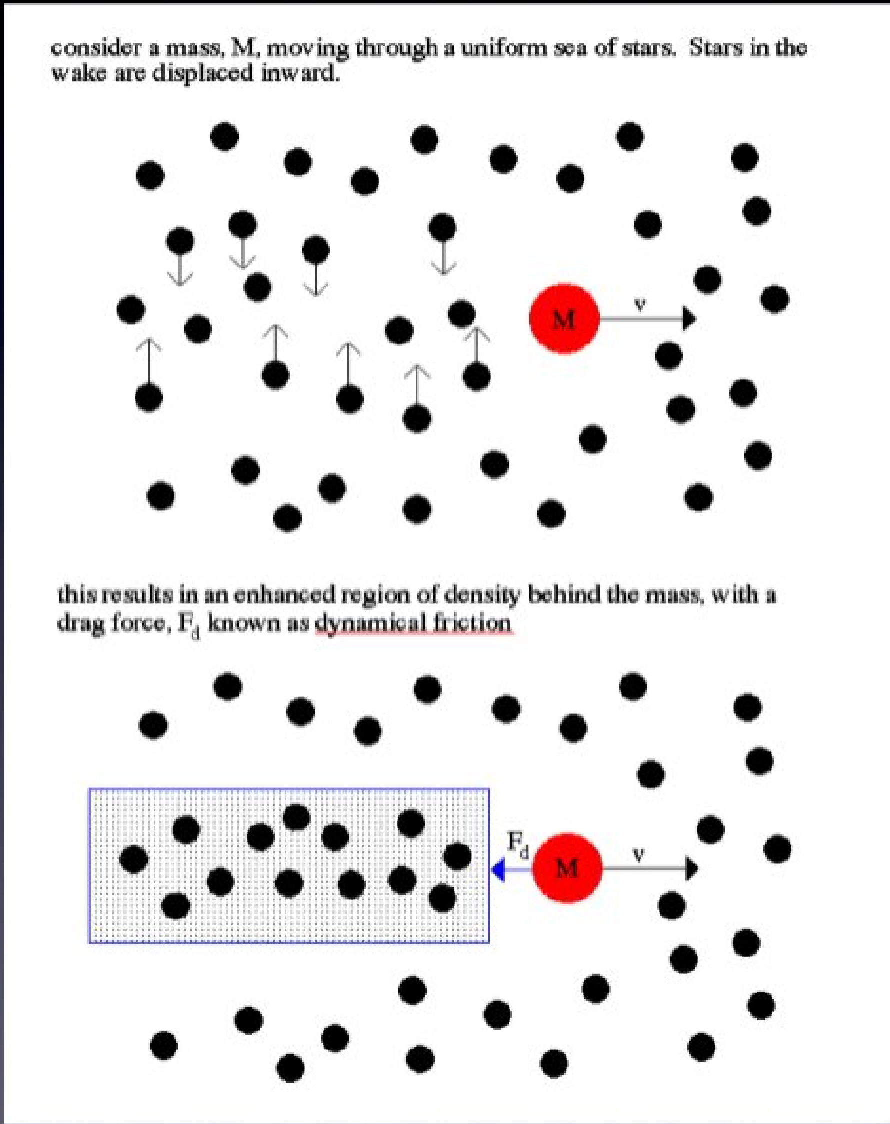
Definitions reminder

- when **measuring the distance-redshift relation** $d_L(z)$ with observations one can in principle constrain the values of all the five parameters Ω_M , Ω_Λ , h , w_o , w_a

- however there is a strong degeneracy between the parameters Ω_M , Ω_Λ , h and the dark energy equation of state parameters w_0 , w_a

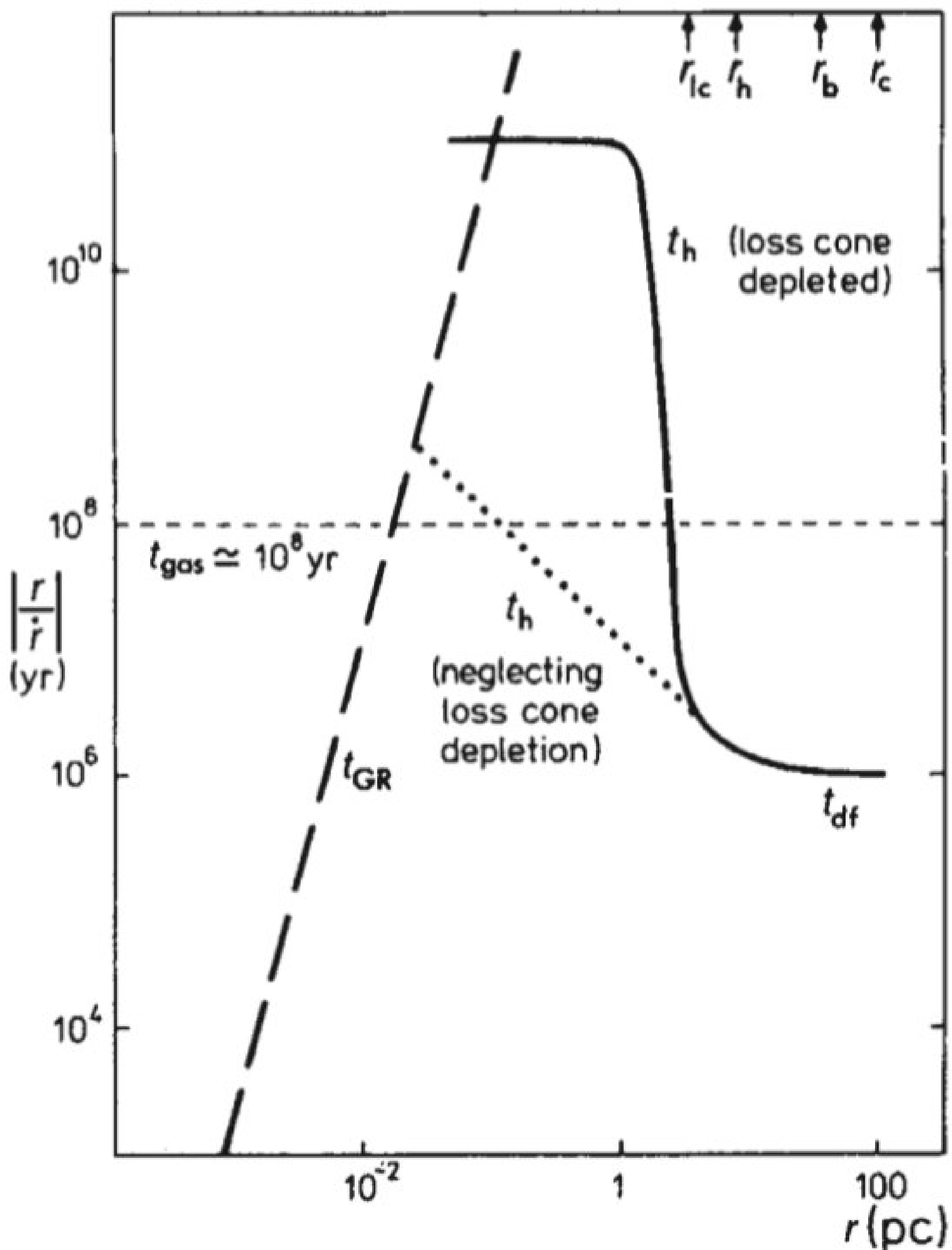
→ makes the simultaneous determination of the five parameters very difficult in practice

- most galaxies contain black holes at their centers
- black-hole mass is 10^6 - 10^{10} solar masses or roughly 0.2-0.5% of the stellar mass of the host galaxy
- galaxies form by hierarchical merging
- if two galaxies with black holes merge, then after the merger, the black holes are left orbiting in the body of the merged galaxy
  - dynamical friction causes the orbits of the black holes to decay, so they spiral to the center



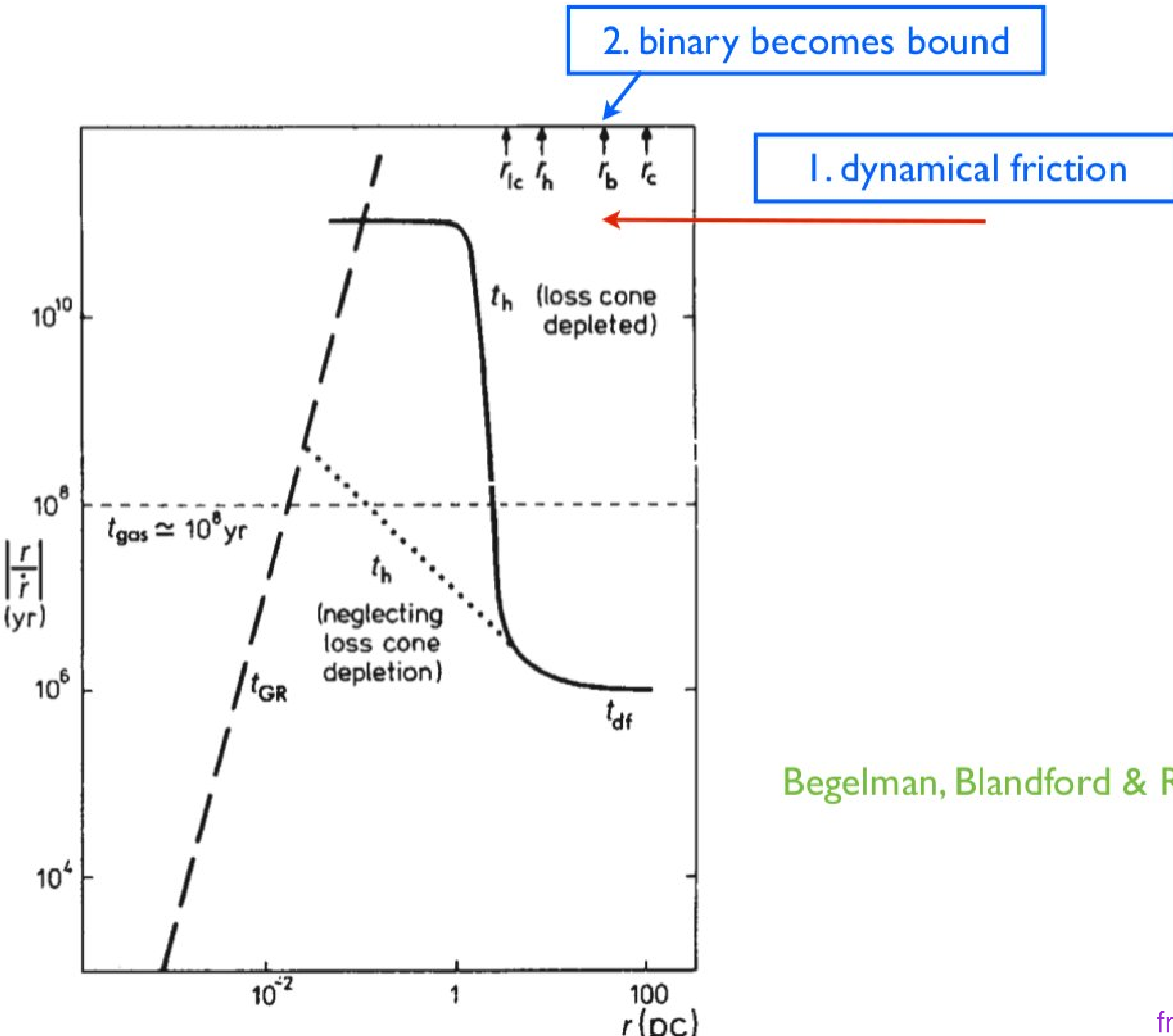
J. Schombert

The final parsec problem



Begelman, Blandford & Rees (1980)

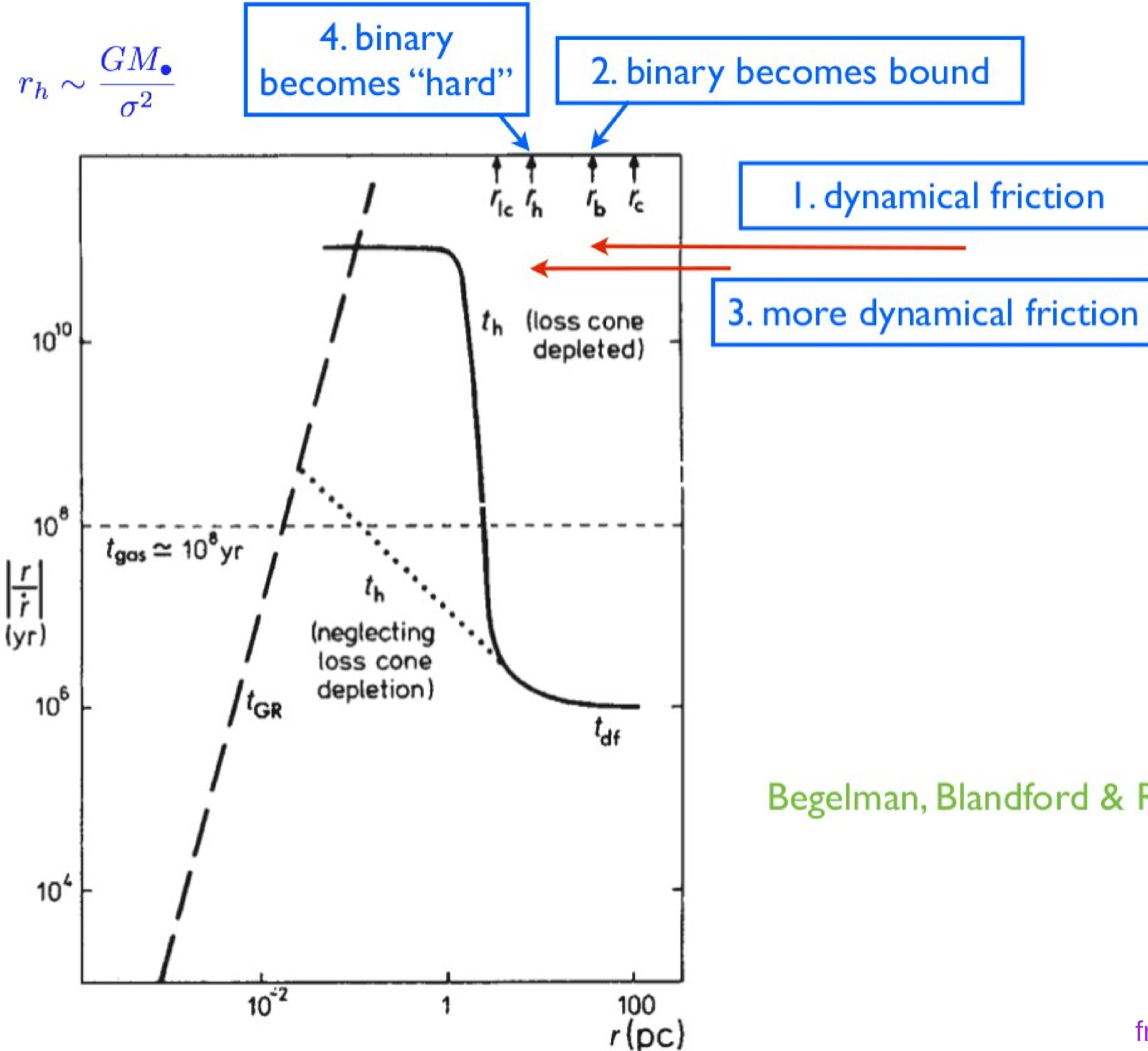
from E. Ostriker



Begelman, Blandford & Rees (1980)

from E. Ostriker

$r_h \sim \frac{GM_\bullet}{\sigma^2}$



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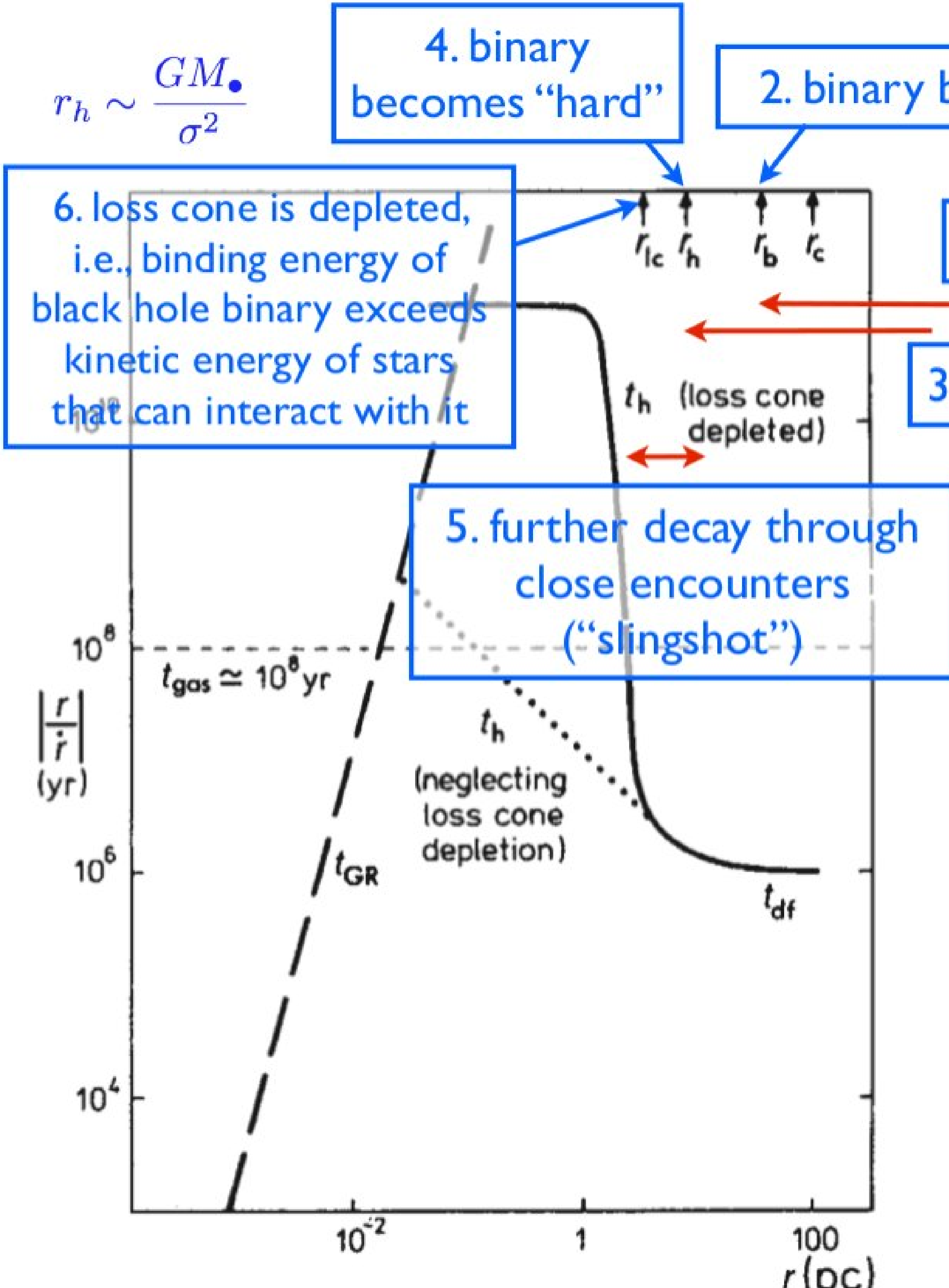
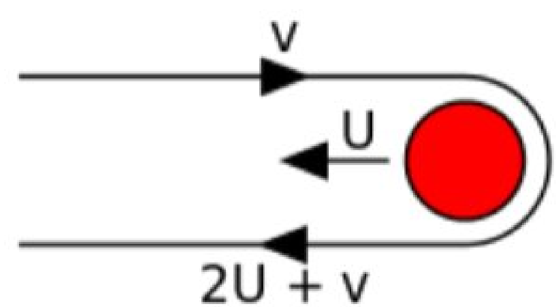


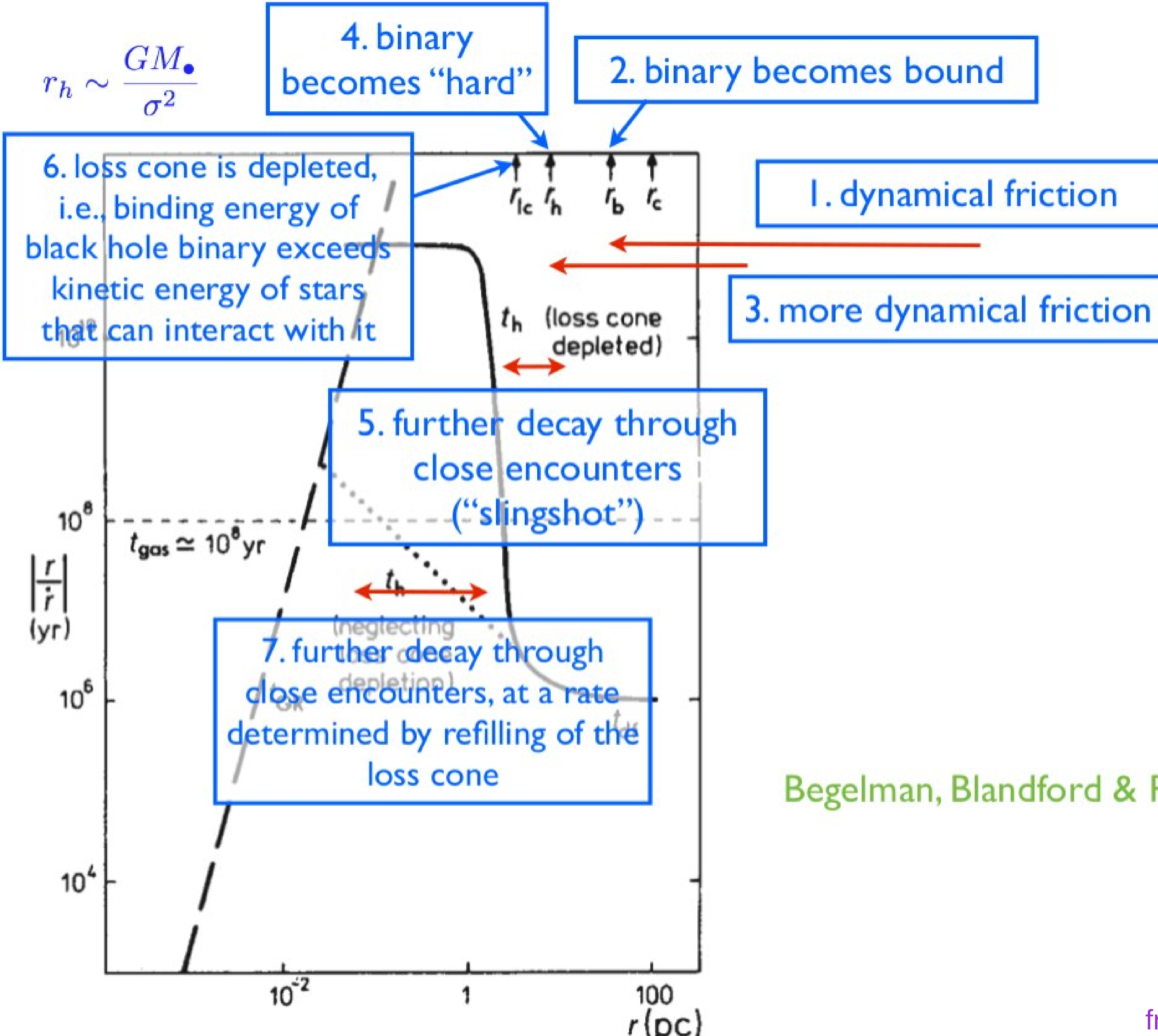
slingshot: encounters in which ejection velocity is comparable to the binary orbital velocity



loss cone: region of phase space in which stars have close encounters with the binary black hole

from E. Ostriker

$$r_h \sim \frac{GM_\bullet}{\sigma^2}$$



Begelman, Blandford & Rees (1980)

from E. Ostriker

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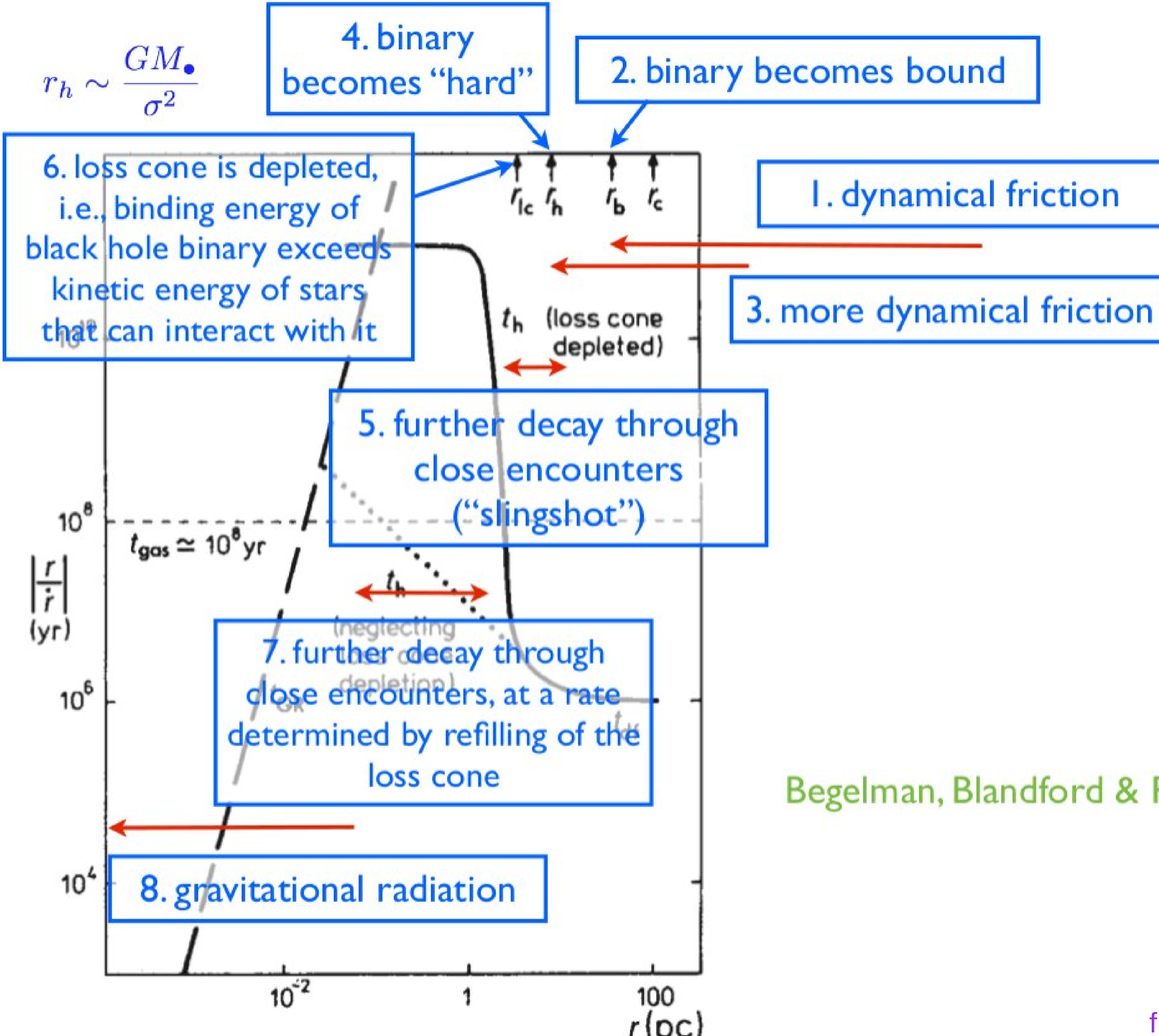
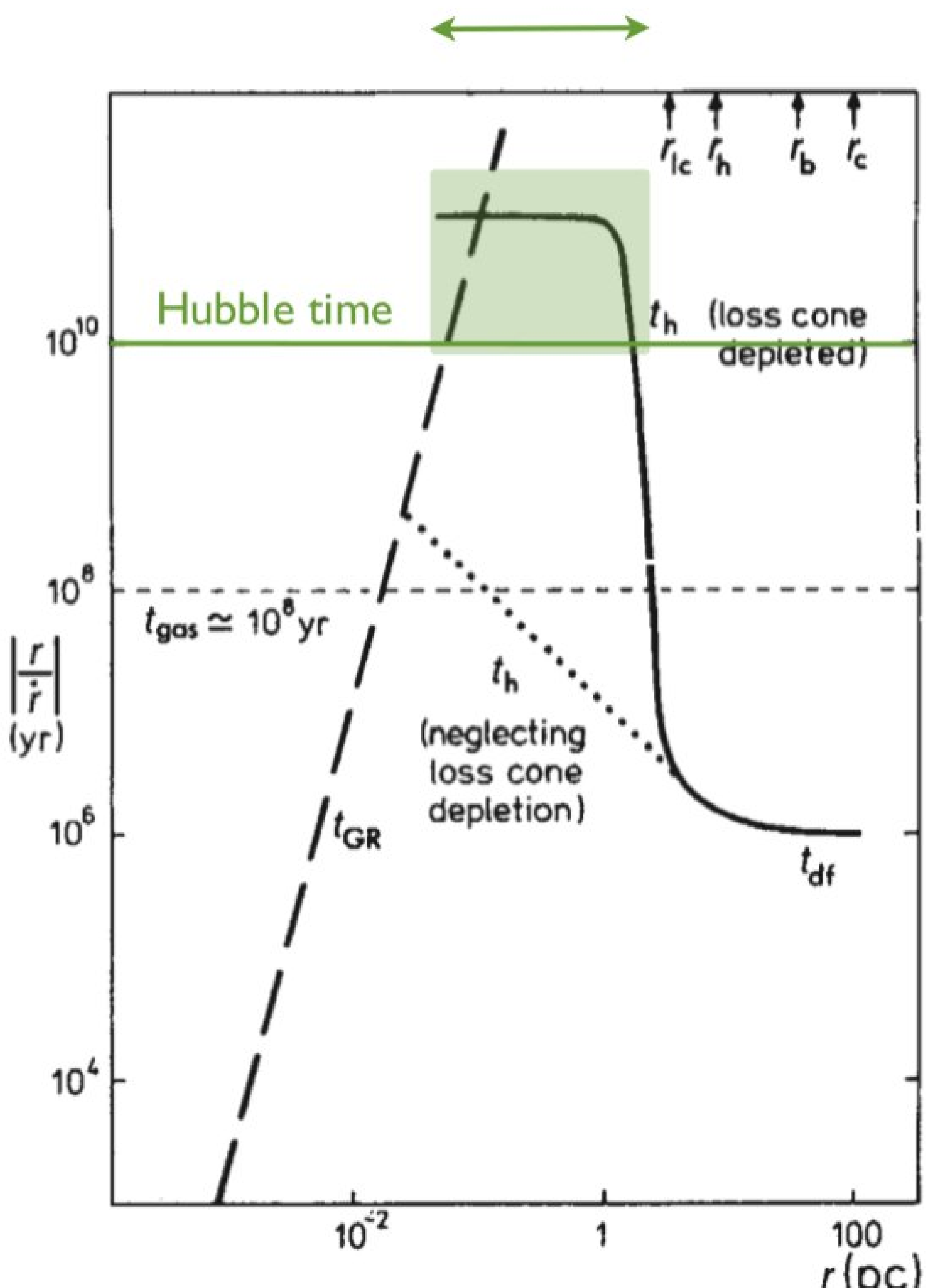


Begelman, Blandford & Rees (1980)

from E. Ostriker

the "bottleneck"

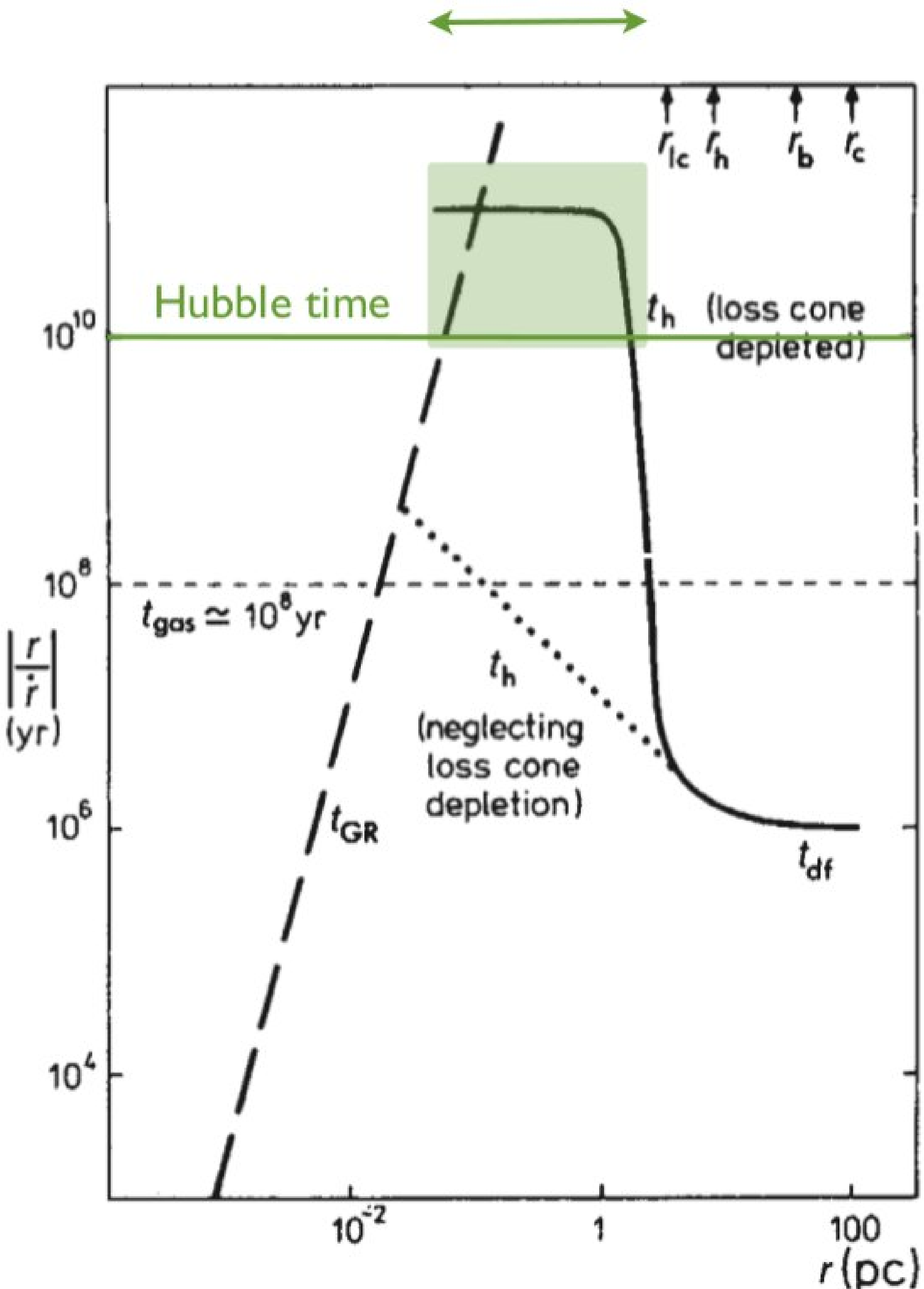


there are ~ 1-2 orders of magnitude in radius between 10 pc and 0.01 pc in which orbital decay time exceeds the Hubble time (the "bottleneck")

Begelman, Blandford & Rees (1980)

from E. Ostriker

the "bottleneck"



does the binary inspiral stall in the bottleneck?

Begelman, Blandford & Rees (1980)

from E. Ostriker

1. Rapid refilling of the loss cone:

- bottleneck arises because energy loss required to shrink the BH binary exceeds the energy needed to eject all the stars that can interact with it ("the loss cone is emptied")
  - in standard model, loss cone is replenished only by two-body relaxation so this is the rate-limiting step
  - other processes may refill the loss cone more rapidly
    - Brownian motion of the BH (Quinlan & Hernquist 1997, Yu 2002, Milosavljević & Merritt 2003)
    - tidal forces, if the galaxy is non-spherical (Yu 2002, Gualandris + 2017)
    - massive perturbers such as star clusters, stellar-mass black holes, molecular clouds, etc. (Perets + 2007)

1. Rapid refilling of the loss cone:

- what is the upper limit to how fast the binary BH can merge (Gould & Rix 2000)?
  - for a binary BH of total mass $M_\bullet$ to decay by n_e e-folds in semi-major axis a it needs to eject a mass of stars $m \sim n_e M_\bullet$
  - if the mass m is enclosed in radius r >> a then the minimum time needed to eject it is ~ (orbital period at r) X (ratio of phase-space volume inside r to phase-space volume in the loss cone)
  - m and r are related to the dispersion in the galaxy by $Gm \sim \sigma^2 r$
  - then $t_{merge} \geq n_e^{8/5}\, GM_\bullet c/\sigma^4$
  - moreover $M_\bullet \sim 10^8\, M_\odot\, (\sigma/\ 200\ \mathrm{km/s})^4$
  - therefore $t_{merge} \geq 1\ \mathrm{Gyr}\ (n_e/5)^{1.6}$ independent of the BH mass, binary mass ratio, galaxy properties, etc.

Interactions with stars can lead to binary BH merger, but *only* over times exceeding 1 Gyr and *only* if all conditions are favorable

2. Gas accretion:

- a large fraction of black hole growth occurs through gas accretion (Sołtan 1982) so over the Hubble time a large gas mass must flow into the central parsec
- standard model (Ivanov + 1999, Armitage & Natarajan 2002, MacFadyen & Milosavljević 2008, Cuadra + 2009, Haiman + 2009, Roedig + 2012, Rafikov 2013, Tang + 2017, Miranda + 2017)
  - gas forms a circumbinary disk around the two BHs
  - smaller "mini-disks" form around each BH
  - most of accretion is onto smaller BH
  - torques on binary orbit are both positive and negative so the sign of the evolution is still controversial
  - if gas accretion leads to inspiral, $\dot{a}/a \sim -k\,\dot{M}/M$ with $k \sim 1$
  - if bottleneck is n_e e-folds in semi-major axis then $M_{final} = M_{initial} \exp(n_e/k) \sim 150$ for $n_e=5$ and $k=1$

The final parsec problem

- the long-term evolution of massive black hole binaries at the centers of galaxies is studied in a variety of physical regimes, with the aim of resolving the ‘‘final parsec problem,’’ i.e., how black hole binaries manage to shrink to separations at which emission of GW becomes efficient… (Milosavljević & Merritt 2003)

The final parsec problem

- there is no final parsec “problem” since there’s no evidence that supermassive black holes actually do merge

- in the simplest models (no gas, spherically symmetric) mergers take :
  10^9 - 10^{10} yr in low-luminosity galaxies
  10^{10} - 10^{13} yr in high-luminosity galaxies (OK for LISA, bad for PTAs)

- other collisionless effects (e.g., triaxiality) can shorten merger time, but not below 10^9 yr

- interactions with gas or a third black hole can accelerate or hinder the merger

- perhaps the black holes don’t make it to the final parsec?

GW memory signal model

- over the binary's lifetime memory undergoes :

1) slow growth prior to merger

2) rapid accumulation of power during coalescence

3) eventual saturation to a constant value at ringdown

GW memory signal model

- magnitude of the spacetime offset Δh_{mem} is affected by BH spin-alignment

→ maximally aligned spinning case exhibiting the strongest signal

→ higher-order spin effects should be incorporated in future simulations to properly reflect the saturated memory amplitude

SNR estimates

- average SNR given by

$$< \rho^2 > = 4 \ \mathrm{Re} \int_0^\infty \frac{< \tilde{h}_+^{(\mathrm{mem})}(f) | \tilde{h}^*_{(\mathrm{mem})+}(f) >}{S_f} \, df$$

$$= \frac{\left(\Delta h_+^{(\mathrm{mem})} \right)^2}{\pi^2} \int_0^{f_C} \frac{df}{f^2} \left(1 - \frac{\pi^2}{6} (\tau f)^2 \right) \frac{1}{S_n(f)}$$

S_f is the power spectral density of strain noise

Gravity in Bondi gauge

- introduce gravity with metric $g^{\mu\nu}$

- define a notion of asymptotic flatness using an adapted coordinate system :

→ Bondi-Sachs coordinate system : $x^\mu = (u, r, x^A)$ with $x^A = (\theta, \varphi)$

→ in the Bondi gauge : $g^{rr} = 0$, $g^{rA} = 0$

→ select the radial coordinate r to be the luminosity distance

→ specifying fall-off conditions as $r \to \infty$ (far from « sources »)

→ would like to obtain Minkowski spacetime in the limit $r \to \infty$ at constant u, x^A

→ asymptotically flat spacetimes which approach a notion of future null infinity I^+

→ physically this describes the so-called "radiation zone" where for example gravitational waves leave their imprint on spacetime far from the sources

Gravity in Bondi gauge

- class of allowed metrics :

$$ds^2 = -du^2 - 2du\,dr + r^2\gamma_{AB}\,dx^A dx^B \qquad \text{(Minkowski)}$$

$$+ \frac{2m}{r}du^2 + r\,C_{AB}\,dx^A dx^B + D^B C_{AB}\,du\,dx^A$$

$$+ \frac{1}{16r^2}C_{AB}C^{AB}du\,dr + \frac{1}{r}\left[\ \frac{4}{3}\left(N_A + u\,\partial_A m_B\right) - \frac{1}{8}\partial_A\left(C_{BC}C^{BC}\right)\ \right] du\,dx^A$$

$$+ \frac{1}{4}\gamma_{AB}C_{CD}C^{CD}dx^A dx^B$$

$$+ \text{(Subleading terms)}$$

all indices are raised with γ^{AB} and we have also $\gamma^{AB}\,C_{AB} = 0$

Gravity in Bondi gauge

- **$m \equiv m(u, x^A)$** is the **Bondi mass aspect**

→ gives the angular density of energy of the spacetime as measured from a point at I^+ labeled by u and in the direction pointed out by the angles x^A

→ physically, radiation carried by gravitational waves (or e.m. fields) escapes through I^+

- **$C^{AB}(u, x^A)$** which is traceless and symmetric (i.e. contains two polarization modes)

→ contains all the information about the gravitational radiation around I^+

→ its retarded time variation is the **Bondi news tensor $N_{AB} = \partial_u C_{AB}$**

→ this is the analog of the Maxwell field for gravitational radiation and its square is proportional to the energy flux across I^+

- **$N_A(u, x^A)$** is the **angular momentum aspect**

→ closely related to the angular density of angular momentum with respect to the origin defined as the zero luminosity distance r = 0

Gravity in Bondi gauge

- metric as written so far not yet obeying Einstein's equations

- **two additional constraints** upon pluging following 2 ansätze into Einstein's equations :

$$\partial_u m = \frac{1}{4} D^A D^B N_{AB} - T_{uu}$$

$$\partial_u N_A = -\frac{1}{4} D^B \left(D_B D^C C_{AC} - D_A D^C C_{BC} \right) + u \partial_A \left(T_{uu} - \frac{1}{4} D^B D^C N_{BC} \right) - T_{uA}$$

with $T_{uu} = \frac{1}{8} N_{AB} N^{AB} + 4\pi \lim_{r \to \infty} \left(r^2 T^M_{uu} \right)$

and $T_{uA} = 8\pi \lim_{r \to \infty} \left(r^2 T^M_{uA} \right) - \frac{1}{4} \partial_A \left(C_{BC} N^{BC} \right) + \frac{1}{4} D_B \left(C^{BC} N_{CA} \right) - \frac{1}{2} C_{AB} D_C N^{BC}$

$T^M_{\mu\nu}$ is the stress tensor of matter and D_A is the covariant derivative associated to γ_{AB}

- because of these constraints generic initial data on I^+ is specified by m, C_{AB} and N_A at initial retarded time and N_{AB} at all retarded times

in addition of course with all the subleading fields that we ignored so far

Asymptotic symmetries : BMS_4 group

- to find which are the (BMS) asymptotic symmetries one looks for vector fields ξ which generate infinitesimal diffeomorphisms and which satisfy Killing equations on the asymptotic metric

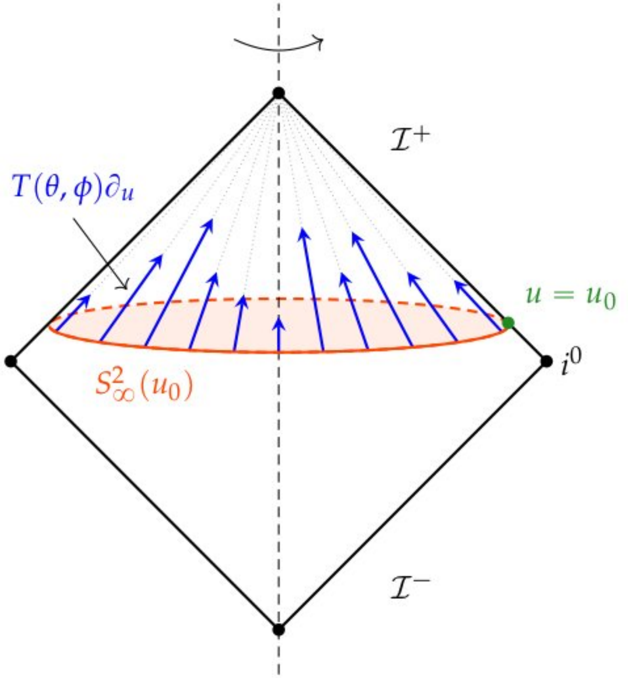
  i.e. looking for infinitesimal diffeomorphisms that preserve the Bondi gauge and the boundary conditions (Lie derivative $L_\xi g_{rr} = 0$, etc ….)

- **the asymptotic algebra is larger than the Poincaré algebra**

- the generators can be divided into 2 categories :

1) supertranslations (vectors generated by T)

2) superrotations (vectors generated by R^A)
   → asymptotic Lorentz transformations



Asymptotic symmetries : BMS_4 group

- supertranslations → angle dependent translations

→ associated conserved charges are the supermomenta

→ non-trivial diffeomorphisms acting on the asymptotically flat phase space **transforming a geometry into another one physically inequivalent**

→ supertranslations have a relationship with gravitational radiation

- supertranslations commute with the time translation

→ their associated charges will commute with the Hamiltonian

→ **all these degenerate states have the same energy**

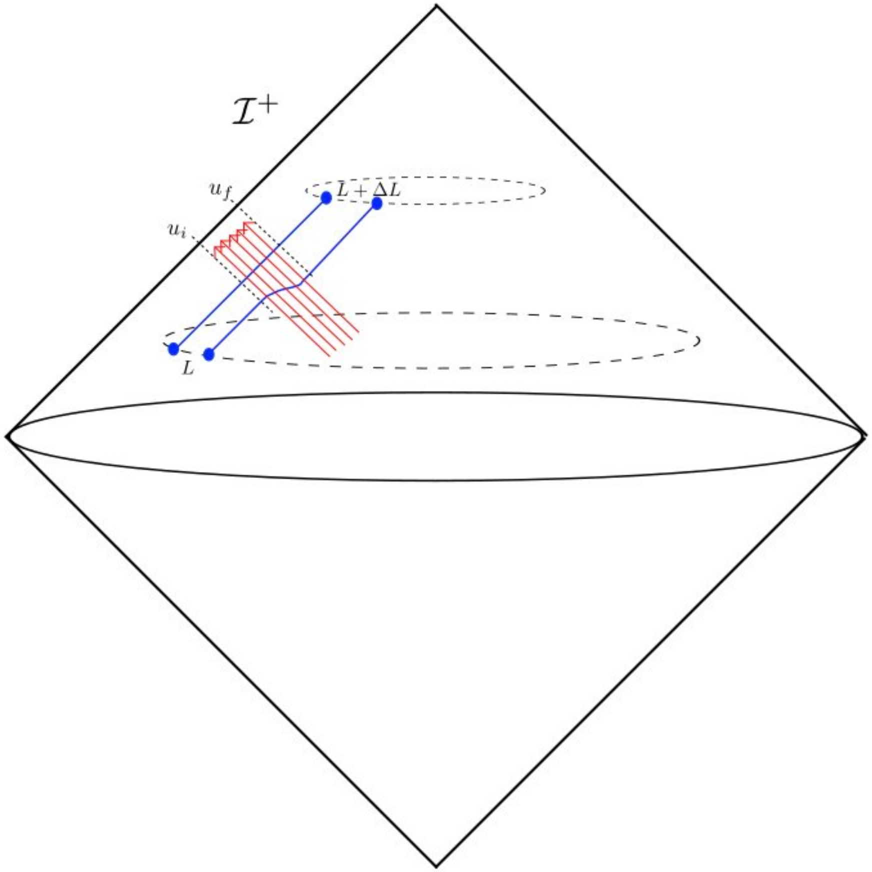
- one then gets at the end BMS_4 = Lorentz x Supertranslations

→ reproducing the semi-direct structure of the Poincaré group

→ only difference is that the translational part is enhanced, implying **degeneracy of the gravitational Poincaré vacua** !

Gravitational memory and BMS

- transit of GW radiation through a set of detectors in the vicinity of the future null infinity I^+



- detectors are located at large r_0 and inserted at different points on the sphere S^2 separated by distance L

- change in the vacuum state is detected by the net **permanent** displacement ΔL

- the new vacuum is related to the old one by a supertranslation

- to summarize the passage of GW radiation through I^+ changes the vacuum by a BMS transformation

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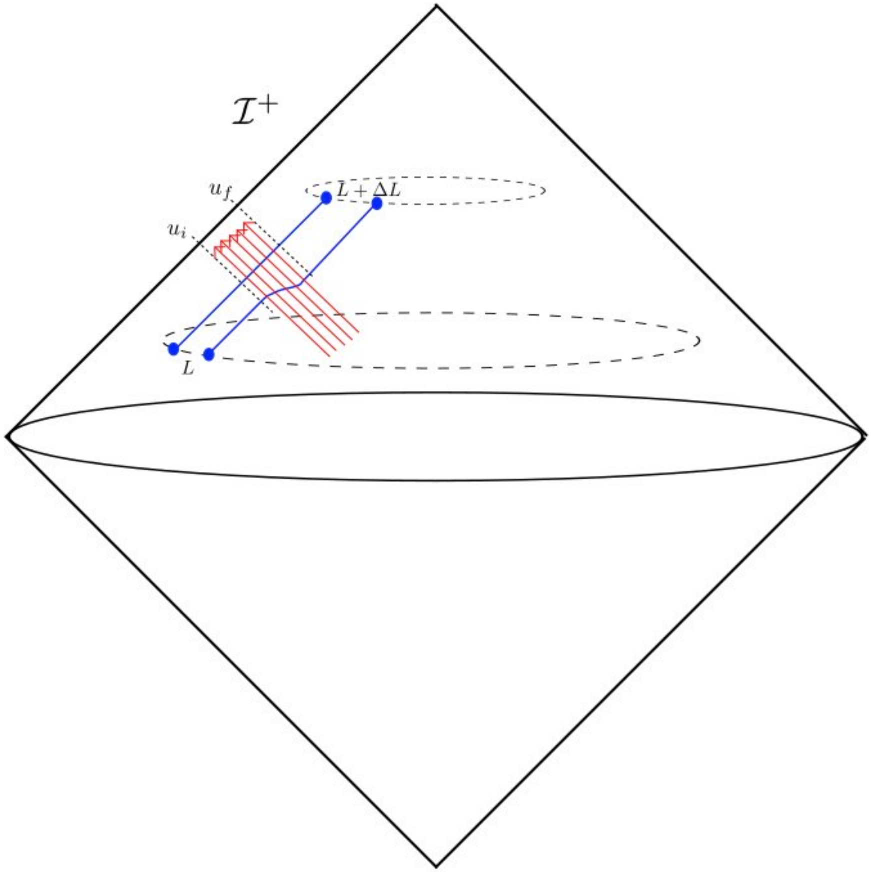


Persistent observables

Observable	Reference	Definition (Sec.)	Result (Eq.)	Number of time integrals of the Riemann tensor	Scaling near $\mathscr{I}$ (if known) Linearized GR	Full GR	Associated with a known symmetry
Displacement	[1]	II A	(2.1)	2	$1/r$	$1/r$	Yes
Relative velocity	[38]	II A	(2.3)	1	$1/r^2$	⋯	No
Relative rotation	[32]	II A	(2.5)	1	$1/r^2$	⋯	No
Relative proper time	[39]	II A	(2.6)	1	$1/r^2$	⋯	No
Subleading displacement [a]	[40, 41]	II A	(2.2)	3	$1/r$	$1/r$	Yes
Curve deviation	⋯	II C	(2.11), (2.12)	1–3 [b]	⋯	⋯	No
Holonomy	⋯	II D	(2.21), (2.22)	1–3 [b]	⋯	⋯	no
Spinning test particle	⋯	II E	(2.25), (2.26)	1–2	⋯	⋯	No

[a] Subleading displacement memory near null infinity includes the spin memory [40] and center of mass memory [41].
[b] With acceleration, the number of time integrals is 4 and higher.

E.E. Flanagan, A.M. Grant, A.I. Harte, D.A. Nichols, Persistent gravitational wave observables: general framework, Phys.Rev.D 99 (2019) 8, 084044

Evolution of SMBHBs

- assuming circular binaries → time to coalescence can be expressed in terms of Keplerian parameters and chirp mass $M = (M_1 M_2)^{3/5} / (M_1 + M_2)^{1/5}$

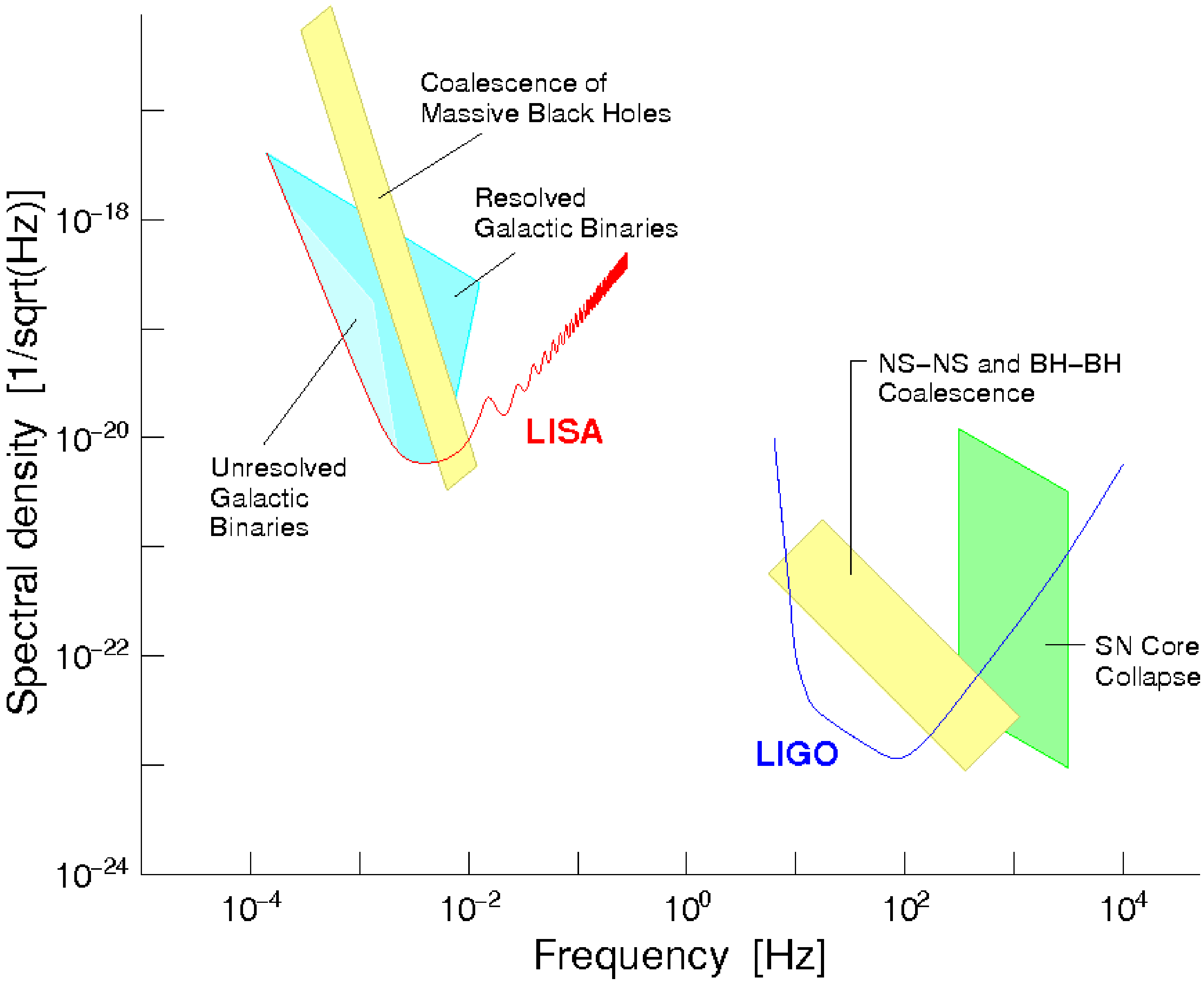
$$\tau_{GW} = \frac{5c^5}{256G} \frac{a_d^4}{M^{5/3} M_{tot}^{4/3}}$$

- establishing a_d is therefore akin to specifying the total time to binary coalescence :

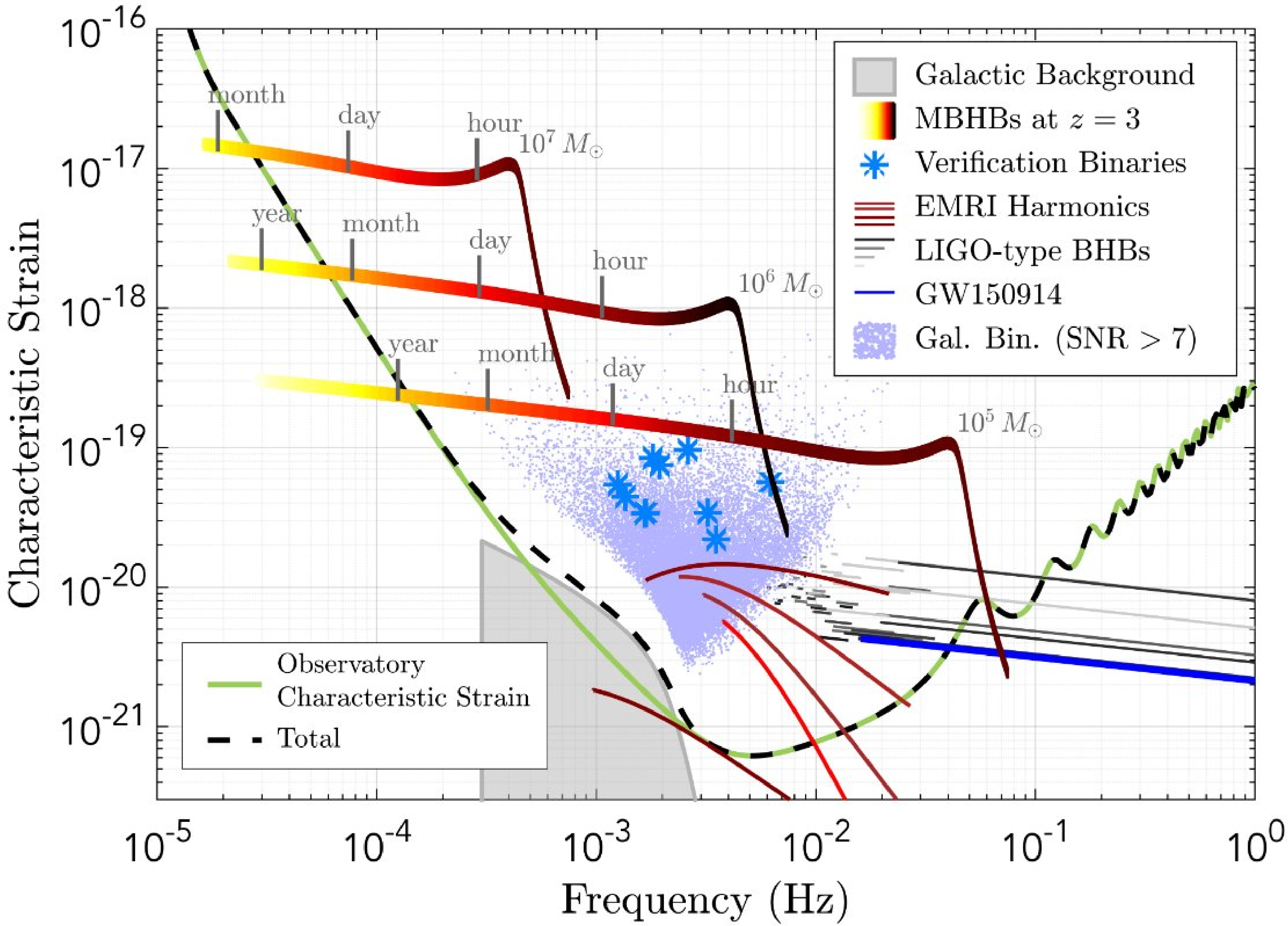
$$t_{burst} = t_{gal} + \tau_{GW}$$

where t_{gal} is the time between galaxy merger and SMBHB formation

GW spectrum

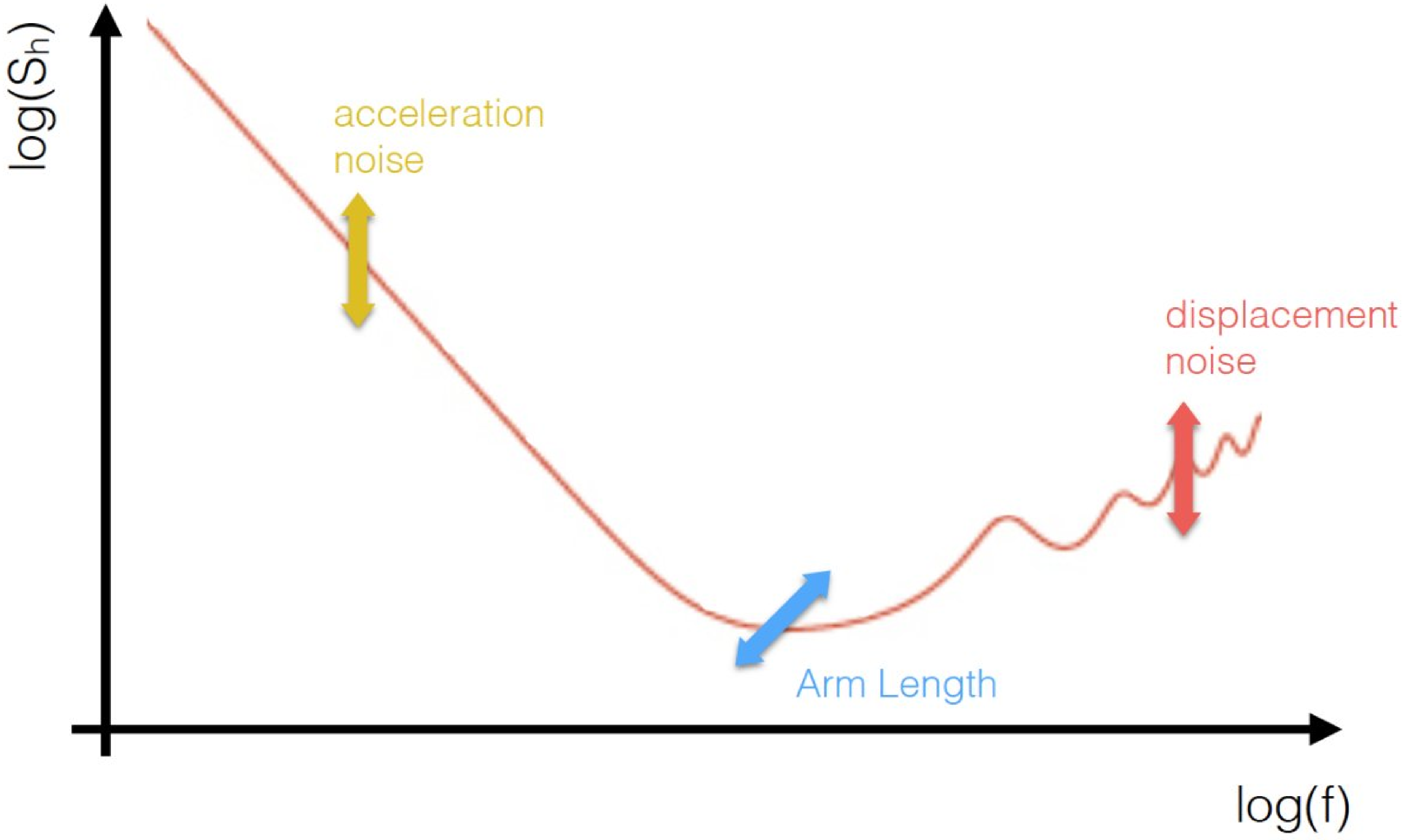

Coalescence of
Massive Black Holes
Resolved
Galactic Binaries
Unresolved
Galactic
Binaries
LISA
NS–NS and BH–BH
Coalescence
SN Core
Collapse
LIGO
10^{-18}
10^{-20}
10^{-22}
10^{-24}
10^{-4}
10^{-2}
10^{0}
10^{2}
10^{4}
Spectral density [1/sqrt(Hz)]
Frequency [Hz]

GW spectrum



Galactic Background
MBHBs at $z = 3$
Verification Binaries
EMRI Harmonics
LIGO-type BHBs
GW150914
Gal. Bin. (SNR > 7)
Observatory Characteristic Strain
Total
month
day
hour
$10^7\,M_\odot$
year
month
day
hour
$10^6\,M_\odot$
year
month
day
hour
$10^5\,M_\odot$
Characteristic Strain
Frequency (Hz)
10^{-16}
10^{-17}
10^{-18}
10^{-19}
10^{-20}
10^{-21}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}

GW spectrum



log(Sh)
acceleration noise
displacement noise
Arm Length
log(f)

GW spectrum