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precision astrometry:  
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# Two-photon amplitude interferometry for precision astrometry

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Improved quantum sensing of photon wave-functions could provide high resolution observations in the optical benefiting numerous fields, including general relativity, dark matter studies, and cosmology. It has been recently proposed that stations in optical interferometers would not require a phase-stable optical link if instead sources of quantum-mechanically entangled pairs could be provided to them, potentially enabling hitherto prohibitively long baselines. A new refinement of this idea is developed, in which two photons from different sources are interfered at two separate and decoupled stations, requiring only a slow classical information link between them. We rigorously calculate the observables and contrast this new interferometric technique with the Hanbury Brown & Twiss intensity interferometry. We argue this technique could allow robust high-precision measurements of the relative astrometry of the two sources. A basic calculation suggests that angular precision on the order of  $10\mu\text{as}$  could be achieved in a single night's observation of two bright stars.

## I. INTRODUCTION

Quantum phenomena are often strange and non-intuitive effects that happen only in the atomic world. At the core of them is entanglement, which has no counterparts in our classical world, and which is enabling new measurement techniques and devices beyond what can be achieved classically. The next technological frontiers will exploit these quantum phenomena to augment sensitiv-

# Paper brings up fascinating questions

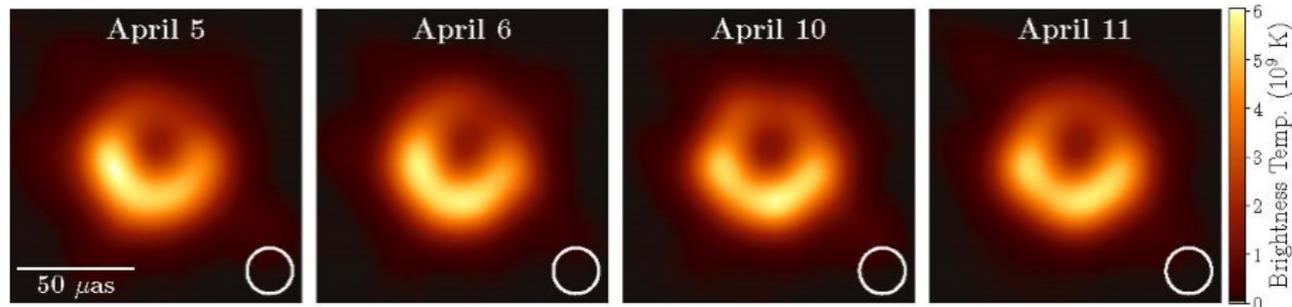
## Conceptual

- Amplitude interferometry vs. Intensity interferometry
- Coherent sources vs incoherent sources
- Classical fields vs. Quantum fields

## Practical applications?

- Black Hole imaging
- Astrometric Microlensing
- Distance Ladder
- Peculiar motions, Galactic potential mapping

# EHT image of black hole in M87



Angular resolution of EHT:

$$\Delta\theta \approx \frac{\lambda}{B} \approx 10^{-10} \text{r} \approx 20 \mu\text{arcsec}$$

where  $\lambda \approx 1 \text{ mm}$  and Baseline,  $B \approx 10000 \text{ km}$

3 order of magnitude improvement if this could be done at optical wavelengths, but current optical interferometers have  $B < 300 \text{ m}$ .

Gaia (not an interferometer):  $\sigma \approx 10 \mu\text{as}$  for  $\text{mag} = 15$

# Amplitude interferometry

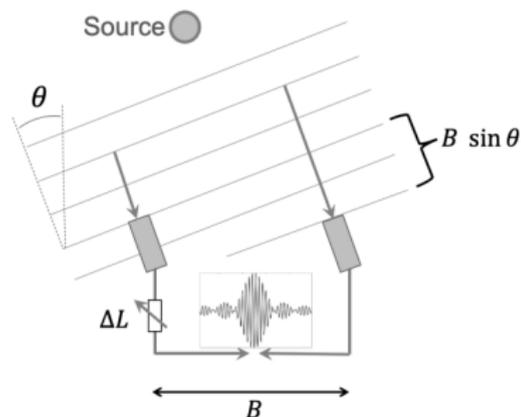


FIG. 1. Traditional stellar interferometry. A single photon from an astronomical source impinges on two detectors nearly simultaneously, with a phase difference determined by the difference in path lengths. The two optical paths are brought together across the baseline, where the photon's interference with itself depends on the path length difference and hence on the direction to the source. Interferometry is generally sensitive to structures with angular scales on the order of  $\Delta\theta \sim \lambda/B$  where  $B$  is the baseline length and  $\lambda$  is the photon wavelength.

Amplitudes of the signals from two receivers are combined coherently. Interference pattern is correlation vs receiver separation,  $B$

# Amplitude interferometry

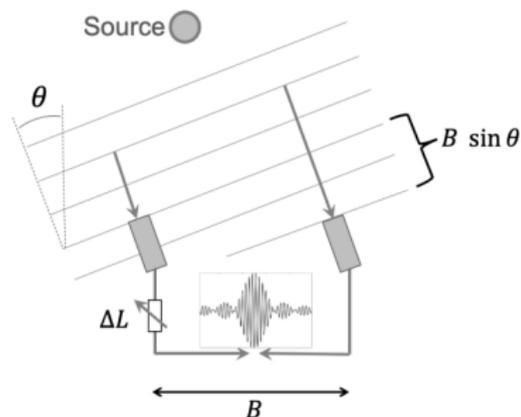


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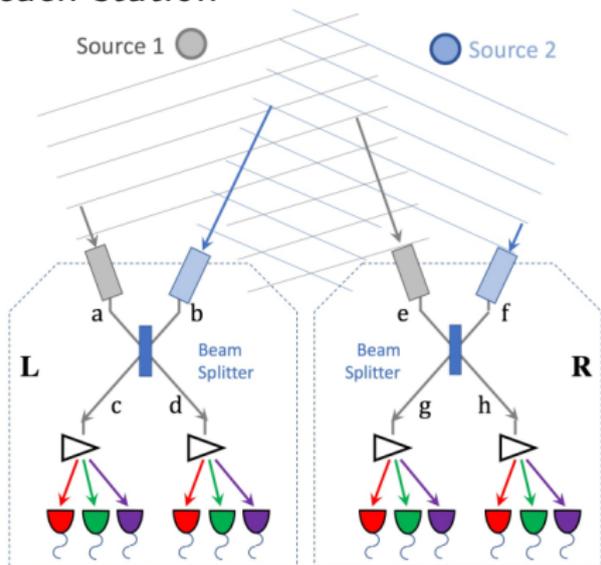
In spite of what is shown and said, the interference pattern here is for an TWO extended sources.

The source separation determines the separation of the maxima and the sizes of the two sources determines the cutoff in the envelope.

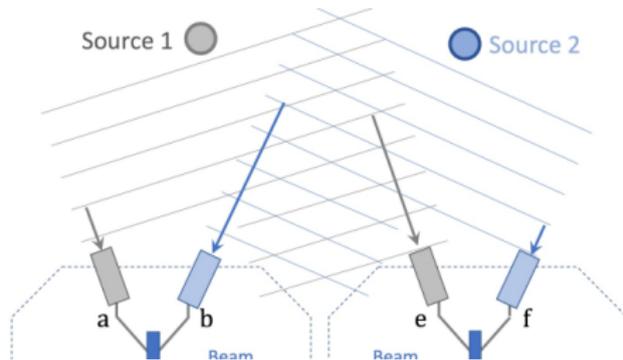
For optical light, requirement of creating a well-definted optical-path limits  $B < 300\text{m}$ .

# Two-source interferometry

2010.09100 proposal:  
Beam splitter and two detectors at  
each station



Traditional setup:  
One detector at each station



Compare amplitudes or intensities  
from 2 stations

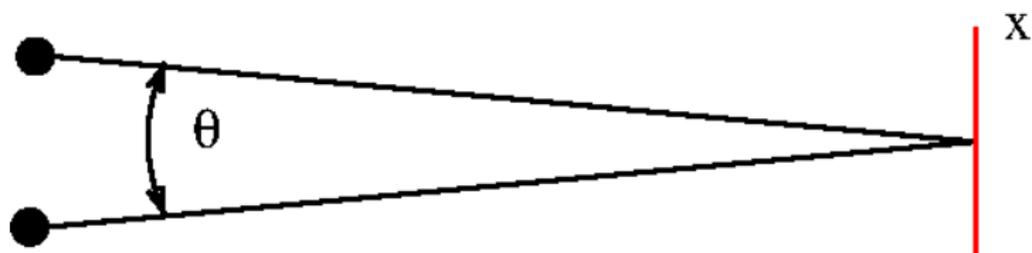
# Classical fields are usually enough

Amplitude and Intensity interferometry can be pretty well understood using classical electromagnetic fields. Quantum mechanics is needed only for calculating the ejection of a photoelectron by an oscillating classical field.

(Because Einstein first used photons to explain the threshold for photoelectron production, physicists are often surprised by this.)

Much of the following comes from consulting my undergraduate textbook [Waves ,Crawford, Berkeley Series, vol 3] which has a brilliant Berkeleysque discription of the Hanbury-Brown-Twiss effect.

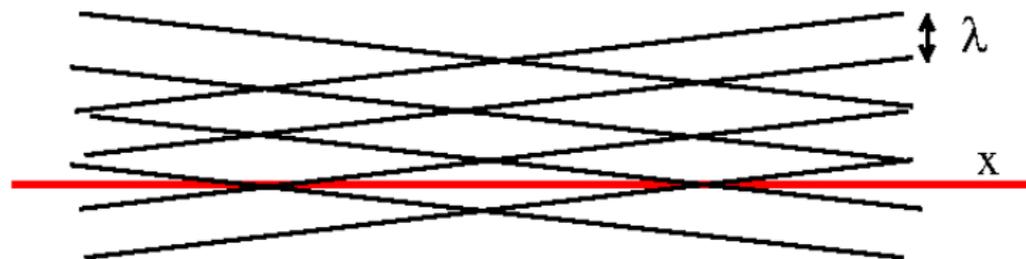
# Interference pattern of 2 sources separated by $\theta$



In astronomy, the two sources are general “incoherent”:

$$t_{coh} \approx 1/\Delta\nu$$

## Interference pattern of 2 sources separated by $\theta$



Sinusoidal interference pattern on detector plane:

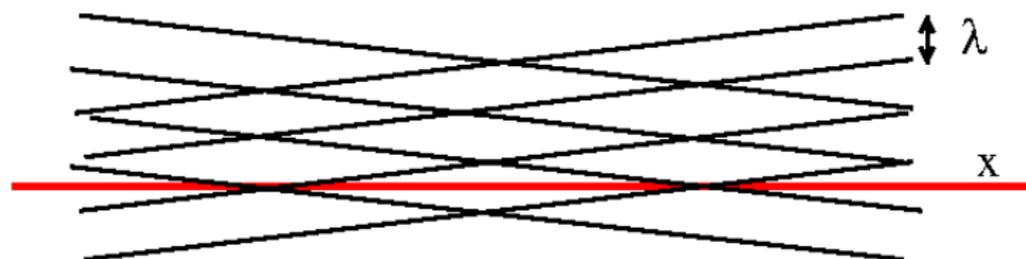
$$E(x) \propto \sin \omega t \sin \left( \frac{x - x_0}{L} \right) \quad L = 2\pi\lambda/\theta$$

$x_0$  depends on relative phases of the two sources: it varies in time chaotically unless the two sources are phase-locked.

Correlation between two receivers separated by  $d$ :

$$E(x)E(x + d) \propto \sin^2 \omega t \sin \left( \frac{x - x_0}{L} \right) \sin \left( \frac{x - x_0 + d}{L} \right)$$

## Interference pattern of 2 sources separated by $\theta$



Correlation between two receivers separated by  $d$ :

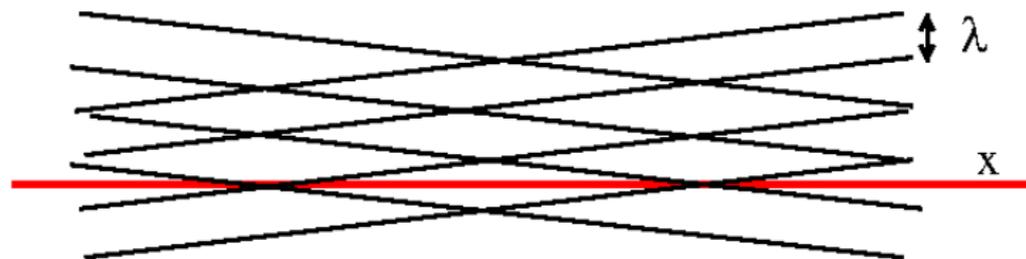
$$E(x)E(x+d) \propto \sin^2 \omega t \left[ \frac{1}{2} \cos \left( \frac{d}{L} \right) - \frac{1}{2} \cos \left( \frac{2x - 2x_0 + d}{L} \right) \right]$$

For optical wavelengths, we necessarily average over  $t$  and  $x_0$ :

$$\langle E(x)E(x+d) \rangle \propto \frac{1}{4} \cos \left( \frac{d}{L} \right) \quad L = 2\pi\lambda/\theta$$

Plotting correlation vs  $d/\lambda \Rightarrow$  angular separation,  $\theta$ .

# Intensity interferometry of 2 sources separated by $\theta$



$$E^2(x)E^2(x+d) \propto \sin^4 \omega t \sin^2 \left( \frac{x-x_0}{L} \right) \sin^2 \left( \frac{x-x_0+d}{L} \right)$$

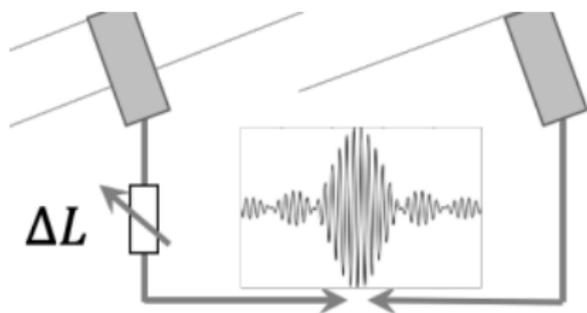
After averaging over  $t$  and  $x_0$ , this becomes

$$\langle E^2(x)E^2(x+d) \rangle \propto \frac{1}{4} \left[ 1 + \frac{1}{2} \cos \left( \frac{2d}{L} \right) \right] \quad L = 2\pi\lambda/\theta$$

Correlation can be done offline if two stations synchronised to a precision of  $t_{coh}$ .

Mean correlation must be subtracted.

# multi-source Amplitude interferometry



In spite of what is shown and said, the interference pattern here is for an TWO extended sources.

The source separation determines the separation of the maxima and the sizes of the two sources determines the cutoff in the envelope.

For one extended source (sum of many point sources), the fine structure disappears, leaving only the envelope, with cutoff at  $d/\lambda \approx 1/\Delta\theta$  where  $\Delta\theta =$  angular size of source.

(Correlation as a function of B is Fourier transform of source)

# Stellar Radii from intensity interferometry

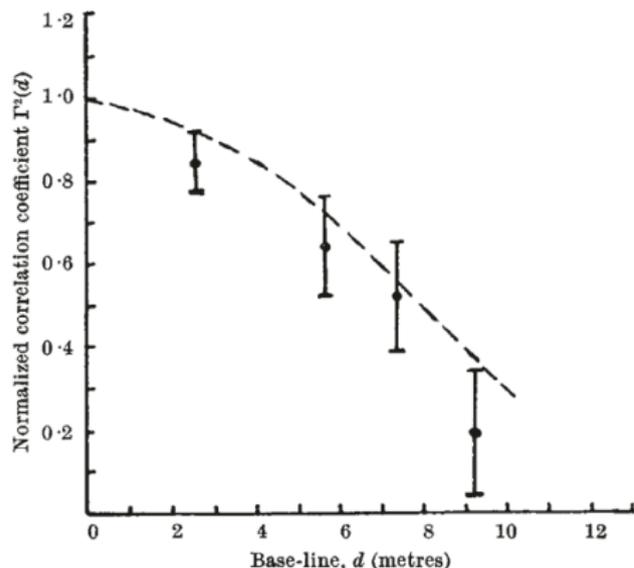


Fig. 2. Comparison between the values of the normalized correlation coefficient  $\Gamma^2(d)$  observed from Sirius and the theoretical values for a star of angular diameter  $0.0063''$ . The errors shown are the probable errors of the observations

Hanbury Brown & Twiss  
Nature (1956)

$$\Rightarrow R_{\text{Sirius}} \approx 6.3 \text{ mas}$$

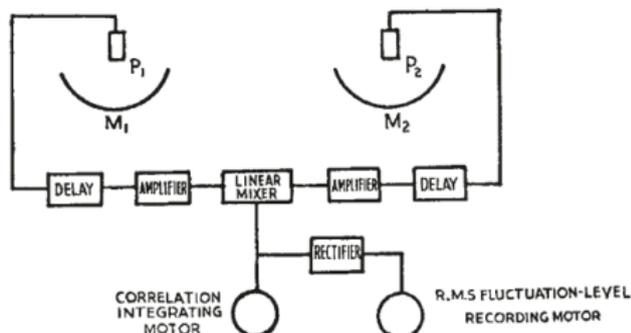


Fig. 1. Simplified diagram of the apparatus

# Stellar Radii from amplitude interferometry

4 T. R. White et al.

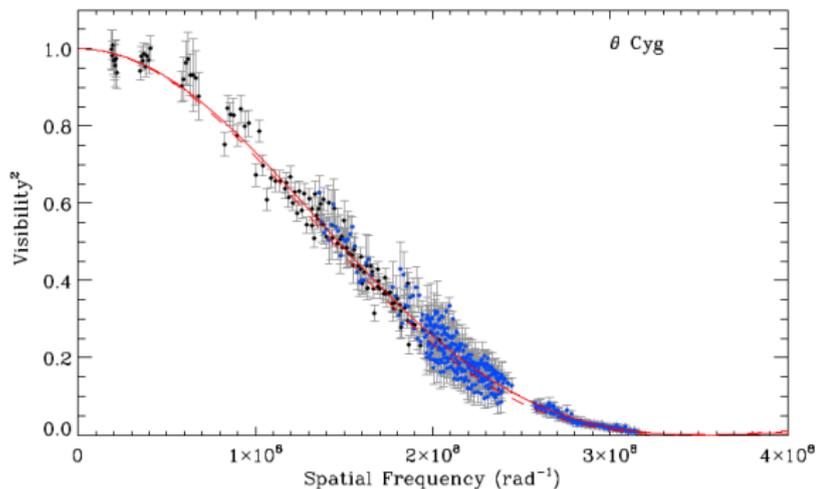


Figure 1. Squared visibility versus spatial frequency for  $\theta$  Cyg for PAVO (blue circles) and MIRC (black diamonds) data. The red lines show the fitted limb-darkened model to the combined data. The solid line is for  $\mu=0.47\pm 0.04$  (PAVO) while the dashed line is for  $\mu=0.21\pm 0.03$  (MIRC).

$\theta$  Cygni:

$$R = 0.753 \pm 0.009 \text{ mas}$$

$$B_{\text{max}} = 300 \text{ m}$$

arXiv:1305.1934

# Quantum calculation gives same answer

CE EFFECTS

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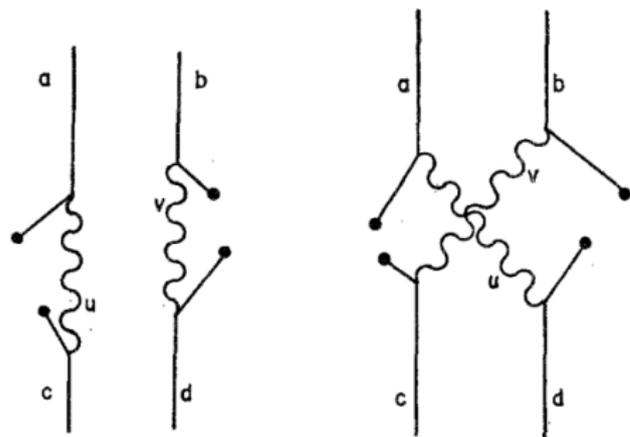


FIG. 1. Lowest-order diagrams representing light emission by a pair of atoms and its absorption by another pair of atoms. The heavy dots indicate ground-state lines.

U. Fano [AmJPh, 1961]

Two interfering amplitudes

Sum amplitudes, square,  
then average over random  
phase between two sources.

⇒ same result as classical  
calculation.

# Quantum or classical: which is better?

People who thought in terms of photons were confused at first.  
Brannen and Fergusson [Nature, September, 1956]

*Note added in proof.* It would appear to the authors, and also to Prof. Jánossy (private communication), that if such a correlation did exist, it would call for a major revision of some fundamental concepts in quantum mechanics. This was, of course, (They had forgotten that photons are bosons)

Purcell [Nature, December, 1956]

ment. Moreover, the Brown-Twiss effect, far from requiring a revision of quantum mechanics, is an instructive illustration of its elementary principles.

The classical EM description automatically includes bosonic effects

# The arXiv:2010.09100 interferometer

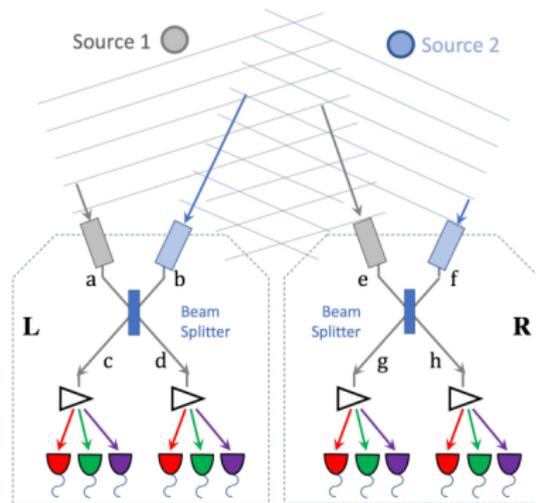


FIG. 3. The two-photon amplitude interferometer. Source 1 sends a photon which arrives as a plane wave at both input detectors “a” and “e”. The path length difference leads to a phase offset of  $\delta_1$ , and the photon is in an entangled state (e.g. we recommend Ref. [14-19] for details

4 counters (c,d,g,h) instead of 2 (L,R)

$\Rightarrow$  4 correlations:

Symmetric: cg and dh

Asymmetric: ch and dg

we can write the expectation value for the total number of each type of coincidence:

$$\langle N(xy) \rangle = \frac{k(S_1 + S_2)^2}{8} [1 \pm V_{2PS} \cos(\delta_1 - \delta_2)]$$

$$V_{2PS} \equiv \frac{2S_1 S_2}{(S_1 + S_2)^2} \quad (13)$$

where the + obtains for the *cg* and *dh* combinations, and the - for *ch* and *dg*; and  $V_{2PS}$  now indicates the two-point-source fringe visibility in the semi-classical approximation.

For  $S_1 = S_2$  we have  $V_{2ps} = 1/2$  (in agreement with elementary formula for intensity interferometry.)

# The arXiv:2010.09100 interferometer

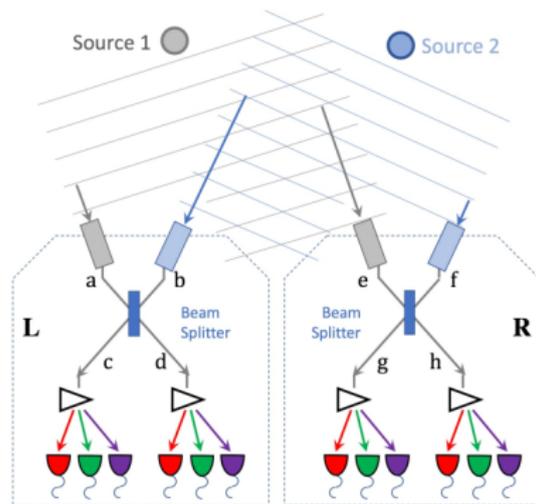


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## Symmetric-Antisymmetric coincidence difference

$$O_{\text{DSI}} \equiv \frac{[N(CG) + N(dh)] - [N(ch) + N(dg)]}{N(CG) + N(dh) + N(ch) + N(dg)} \quad (16)$$

$$\langle O_{\text{DSI}} \rangle = V_{2\text{PS}} \cos \left[ \frac{2\pi B}{\lambda} (\sin \theta_1 - \sin \theta_2) + \frac{2\pi \Delta L}{\lambda} \right]$$

As expected, we can see that the double-source interferometry observables  $\langle N(xy) \rangle$  and  $O_{\text{DSI}}$  are directly sensitive to the *difference* in the sky positions between the two sources, and thus to their relative astrometry.

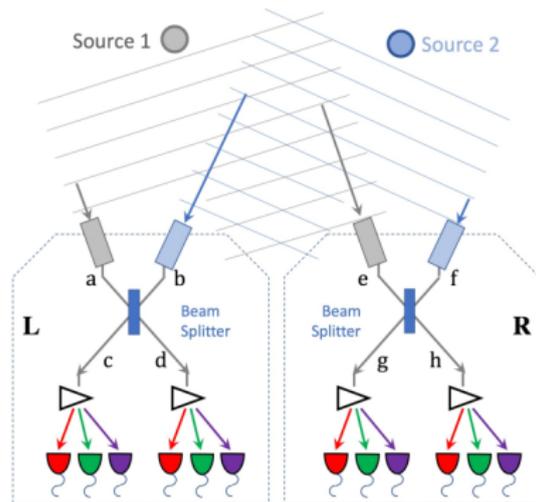


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Equation (30) connects the two-photon count rates after interference to information on the two sources’ relative positions; we can re-cast the observed pair coincidences in the following form:

$$N_c(xy) \propto \left[ 1 \pm V \cos \left( \frac{\omega_0 B (\sin \theta_1 - \sin \theta_2)}{c} + \frac{\omega_0 \Delta L}{c} \right) \right], \quad (32)$$

where  $V$  is the fringe visibility, also discussed in detail later in Section VI. In the case of very narrow frequency filter with  $\Delta\omega \rightarrow 0$ , we can write the fringe visibility  $V$  in accordance with (30), (31) and (28) as follows:

$$V = \frac{2I_1 I_2 \xi_1 \xi_2}{(I_1 + I_2)^2 + (I_1 \xi_1)^2 + (I_2 \xi_2)^2}, \quad (33)$$

where we put  $\xi_j \equiv \xi_j \left( \frac{\omega_0 B}{c} \cos \theta_j \right)$  for simplicity. Equation (33) is now the full visibility including extended sources and all quantum effects, generalizing the semiclassical, two-point-source visibility described earlier in Section IV.

Two extra terms in (33) give  $V = 1/3$  for  $I_1 = I_2$  and  $\xi_1 = \xi_2 = 1!$

# Precision for $\Delta\theta =$ separation of two stars

Strategy: Count fringes as the two stars move across the sky.

Example:

2 stars of magnitude 2

stellar angular diameter=0.5mas

$\lambda = 1\mu\text{m}$

$T_{obs} = 10^4\text{sec}$

200 meter baseline

$1\text{m}^2$  effective collecting area

photon coincidence window 0.15ns

filter bandwidth  $\Delta\nu = 1\text{GHz}$

$4 \times 10^4$  bands

$\Rightarrow \sigma_{\Delta\theta} = 10\mu\text{as}$

# Summary

- Interesting generalization of HBT intensity interferometry  
⇒ Simpler mean correlation subtraction
- Extension of visible photon interferometry to 1000km baselines?
- Is this quantum or classical?