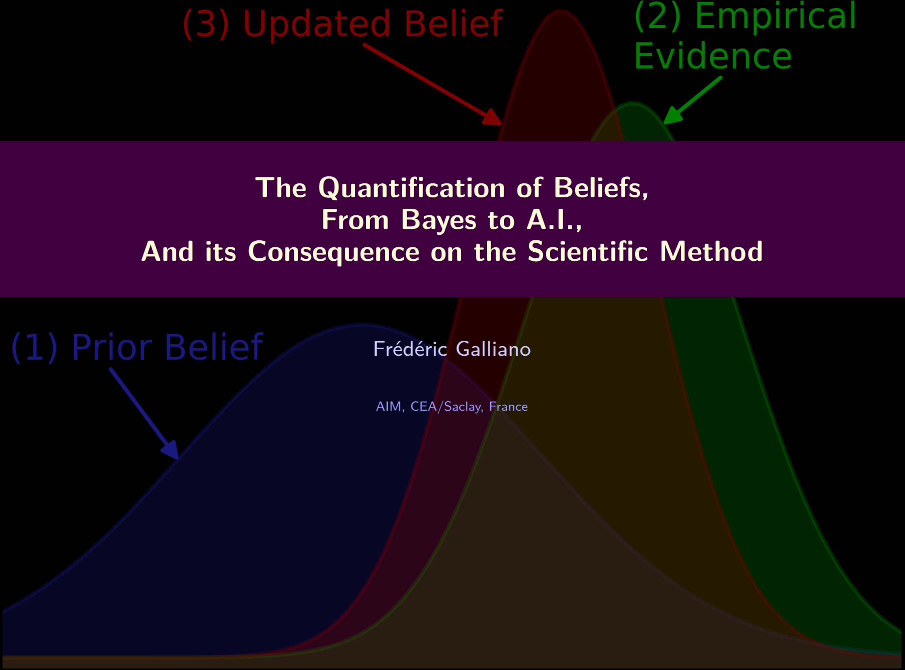

The Quantification of Beliefs,
From Bayes to A.I.,
And its Consequence on the Scientific Method

Frédéric Galliano

AIM, CEA/Saclay, France

Outline of the Talk

1. **BAYESIANS VS FREQUENTISTS**
   - Epistemological Principles & Comparison
   - Demonstration on a Simple Example
   - Limitations of the Frequentist Approach

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   - Bayesian Epistemology
   - How Researchers Actually Work

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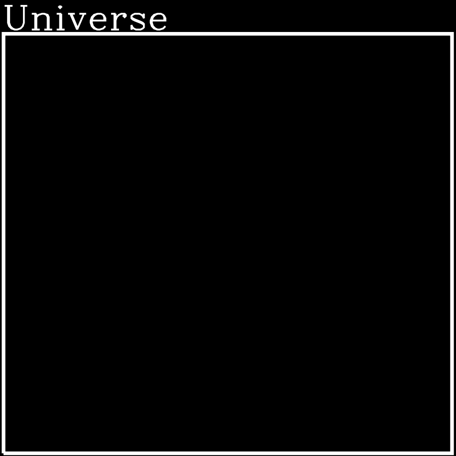

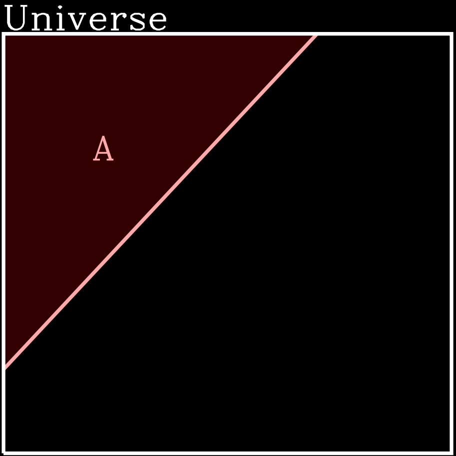
Universe
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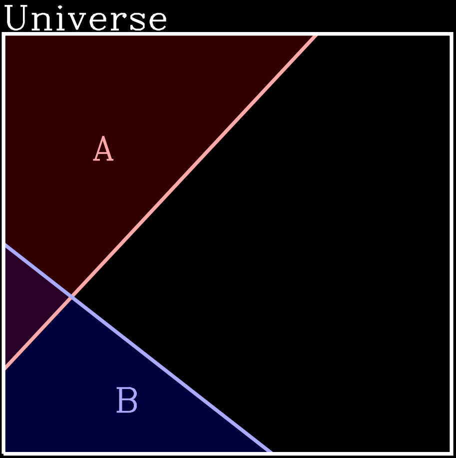

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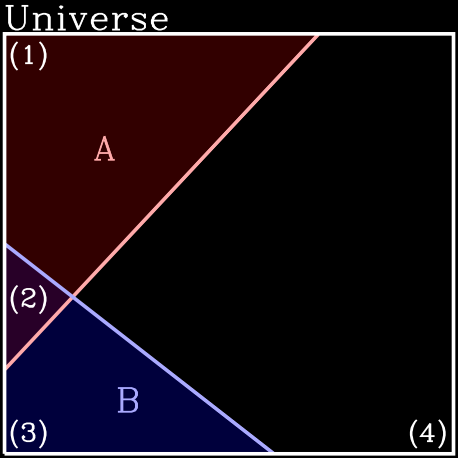

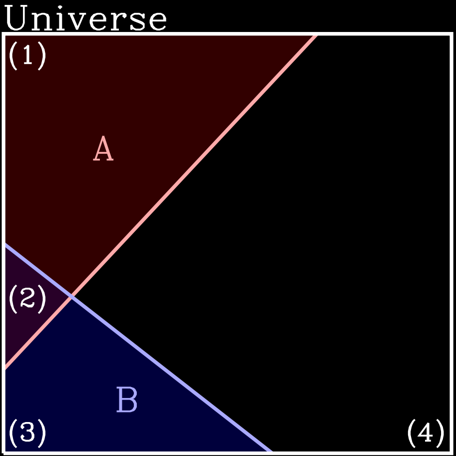

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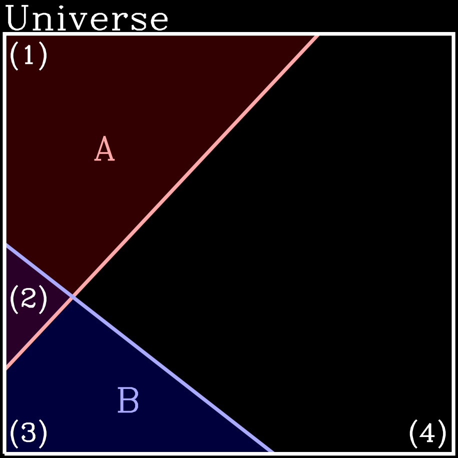

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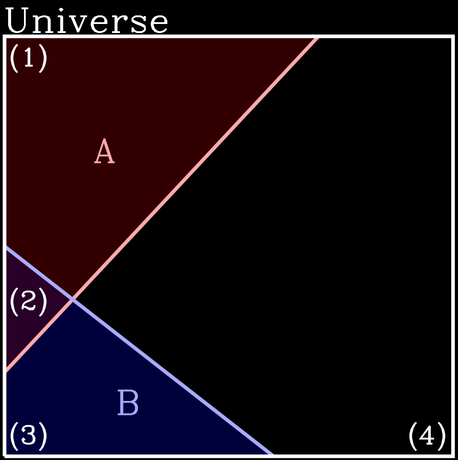

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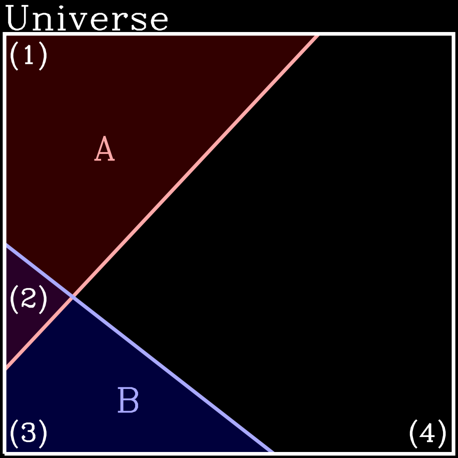

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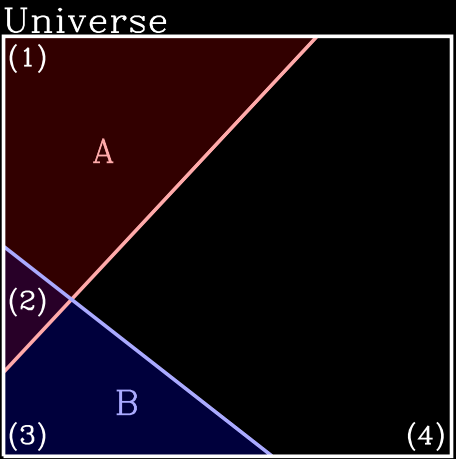

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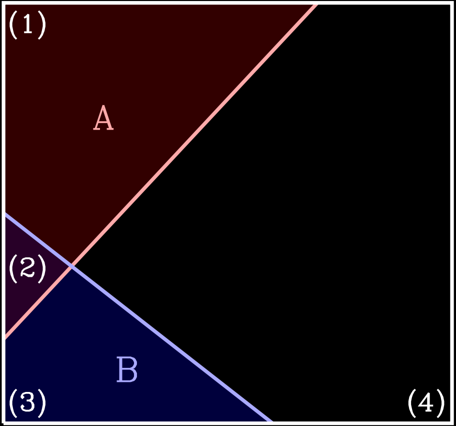

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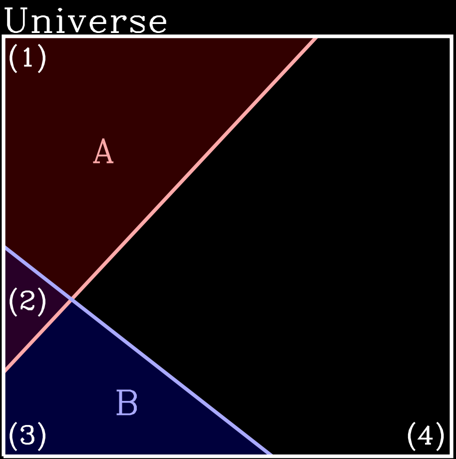

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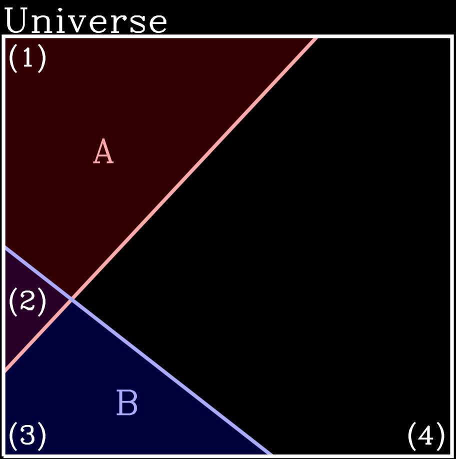

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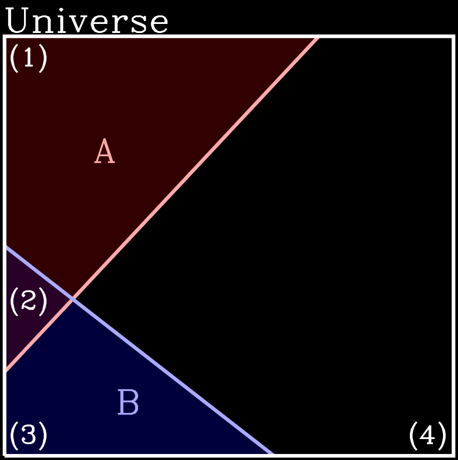

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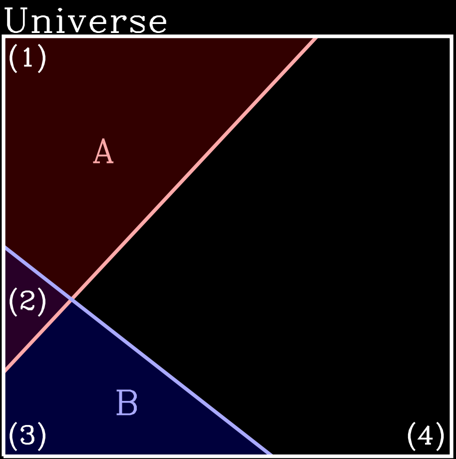

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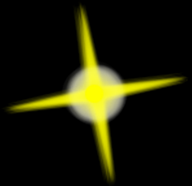

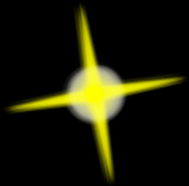

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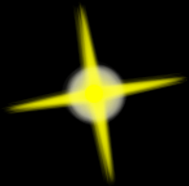

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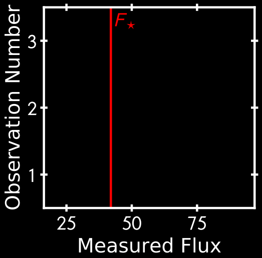

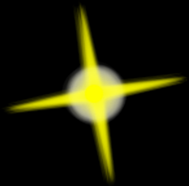

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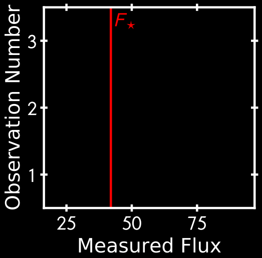

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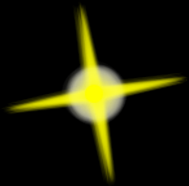

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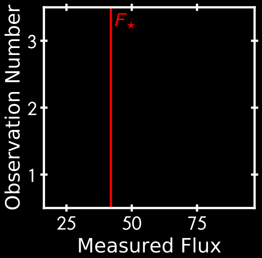

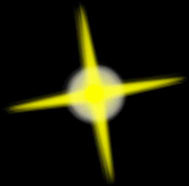

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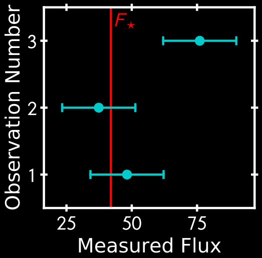

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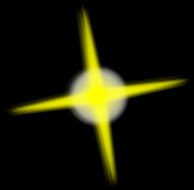

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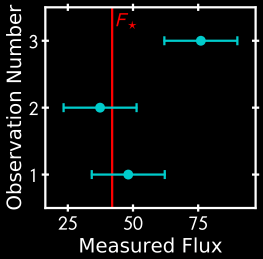

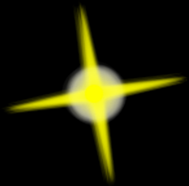

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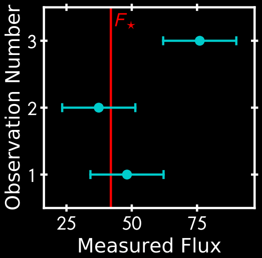

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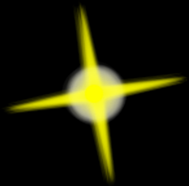

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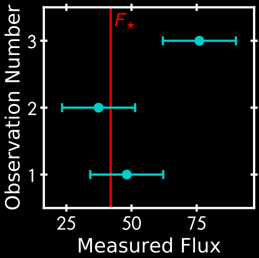

Bayesian Solution:

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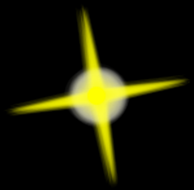

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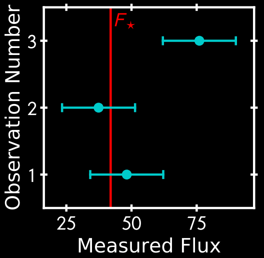

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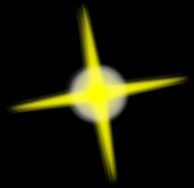

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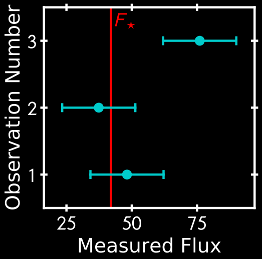

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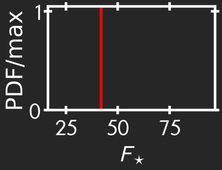

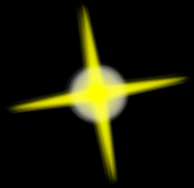

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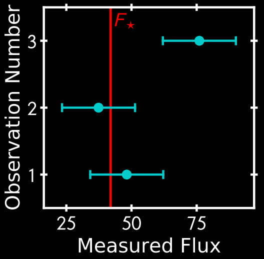

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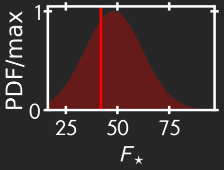

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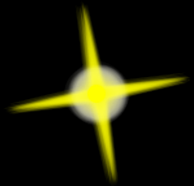

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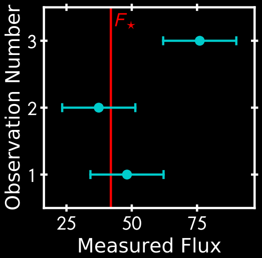

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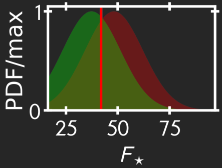

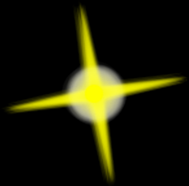

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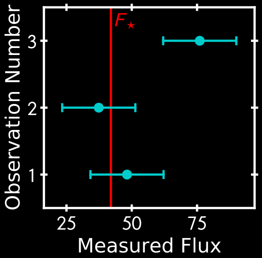

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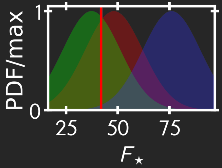

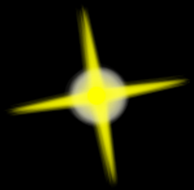

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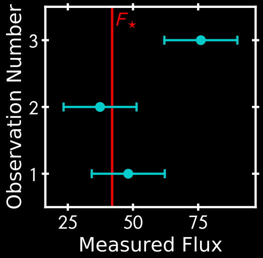

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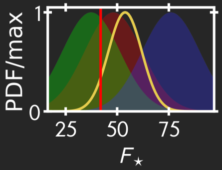

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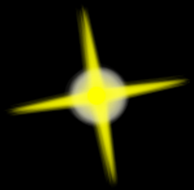

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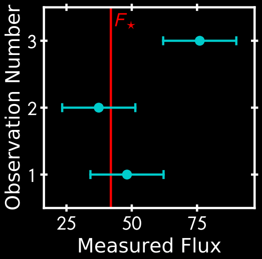

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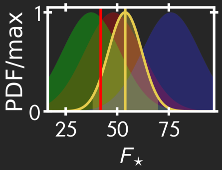

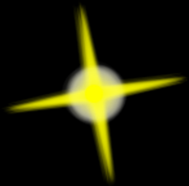

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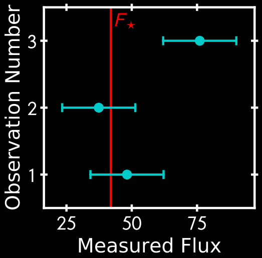

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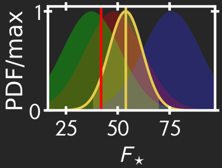

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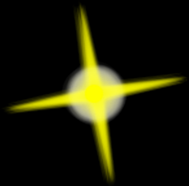

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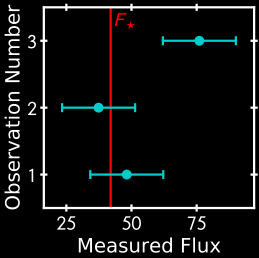

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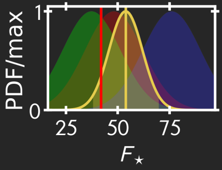

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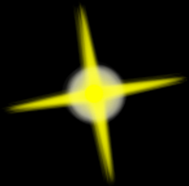

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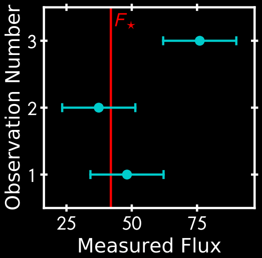

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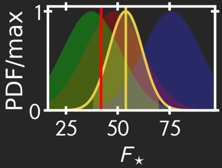

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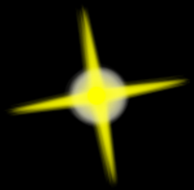

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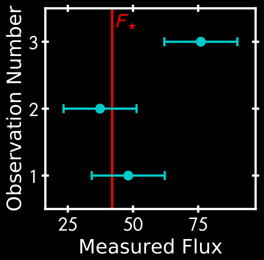

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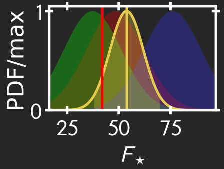

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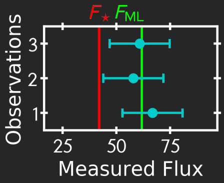

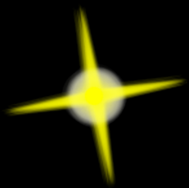

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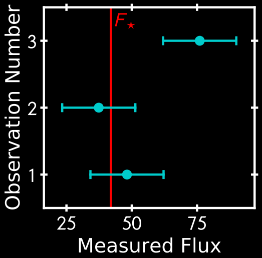

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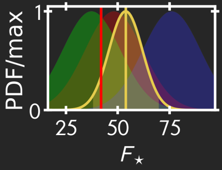

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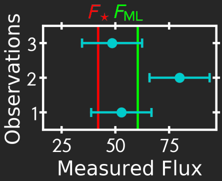

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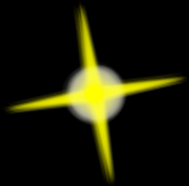

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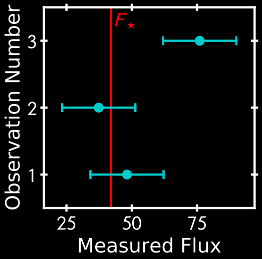

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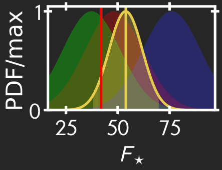

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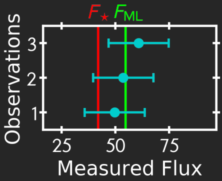

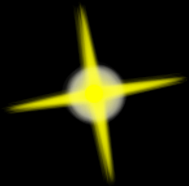

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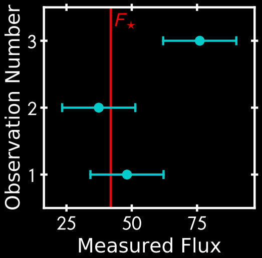

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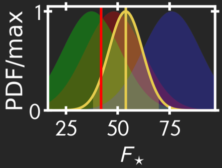

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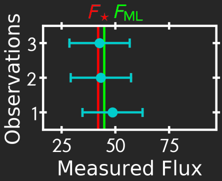

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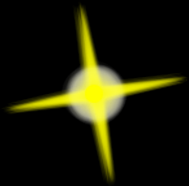

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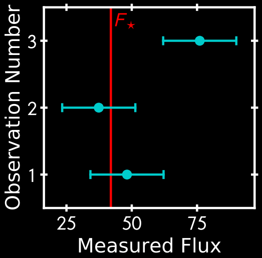

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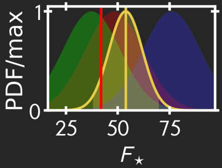

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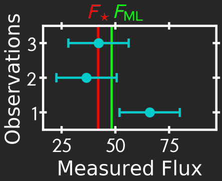

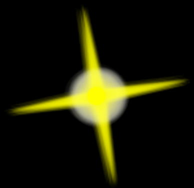

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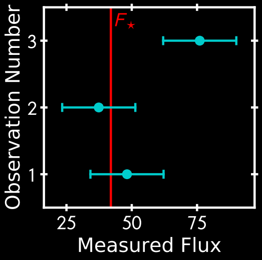
$$F_\star \simeq \frac{\sum_{i=1}^N F_i}{N} \pm \frac{\sigma_F}{\sqrt{N}} = 53.9 \pm 8.1.$$



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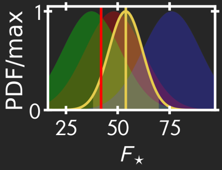

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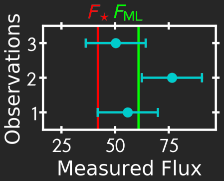

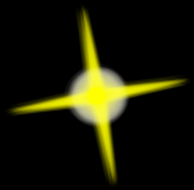

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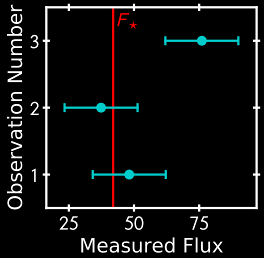

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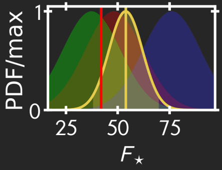

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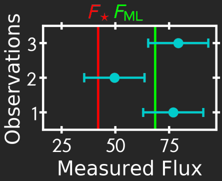

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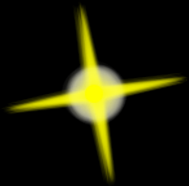

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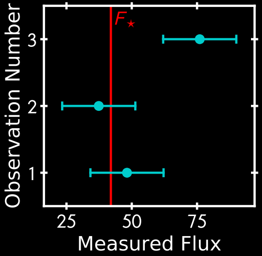

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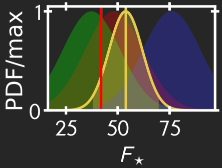

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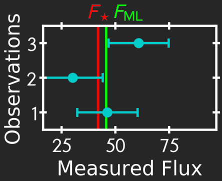

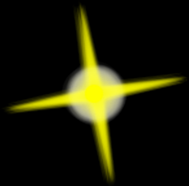

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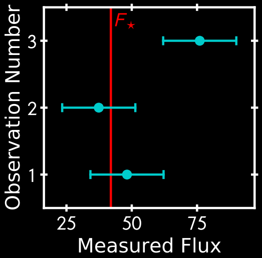

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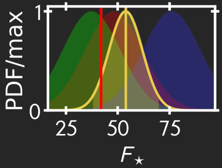

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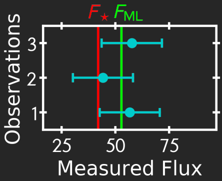

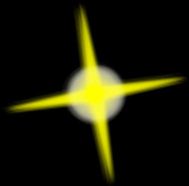

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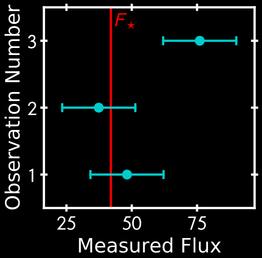

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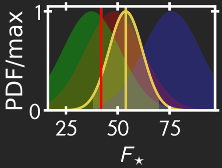

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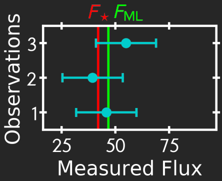

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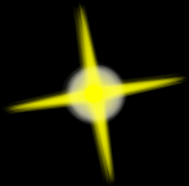

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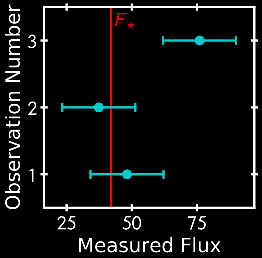

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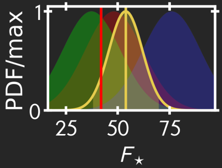

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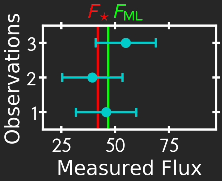

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Difference Between the Two Approaches: Using an Informative Prior

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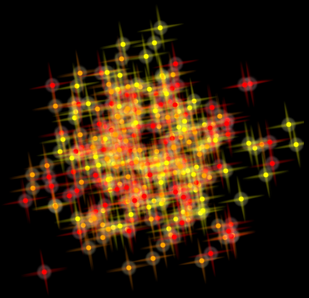

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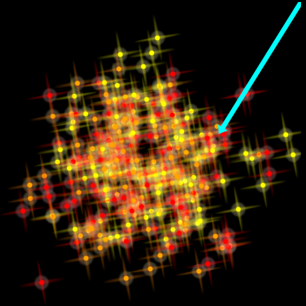

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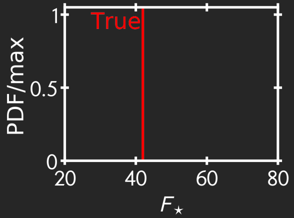

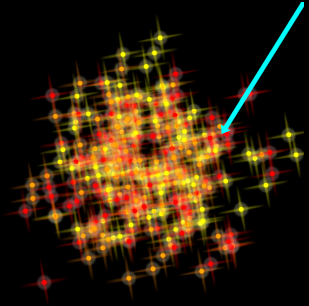

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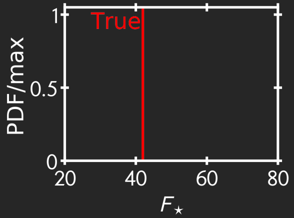

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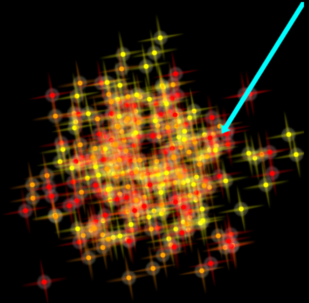

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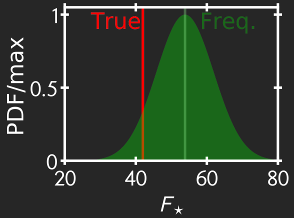

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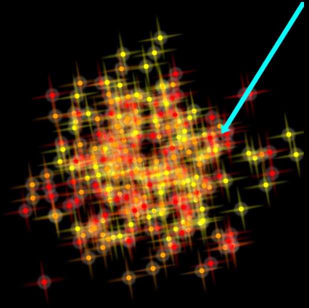

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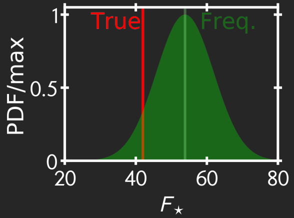

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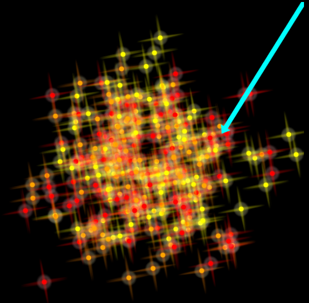

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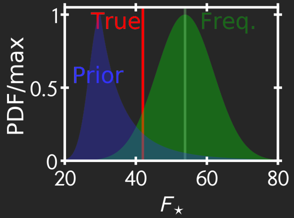

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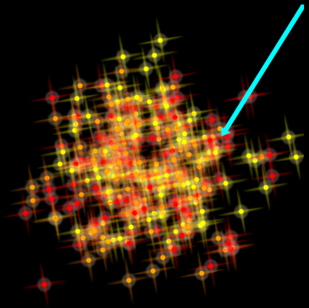

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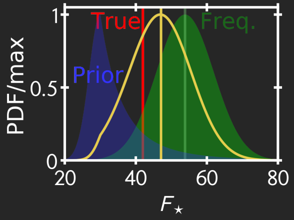

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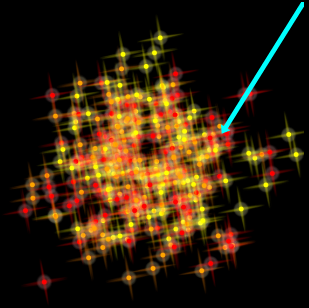

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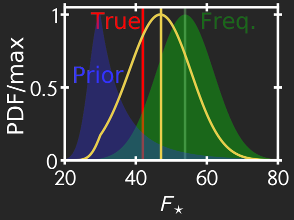

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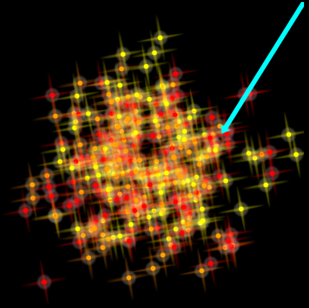

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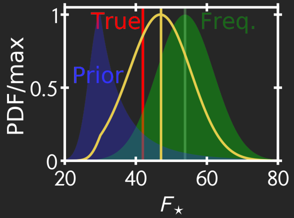

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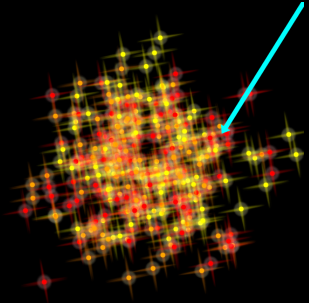

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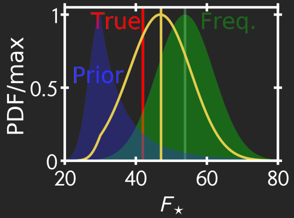

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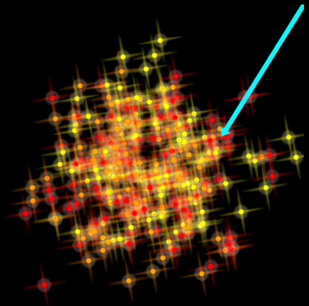

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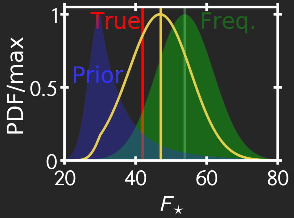

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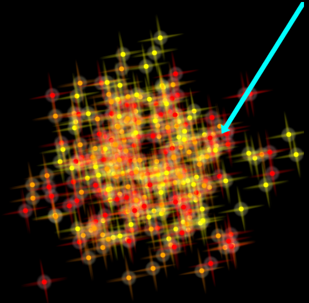

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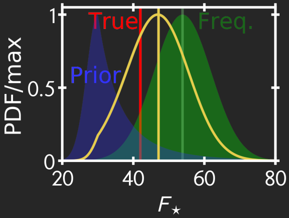

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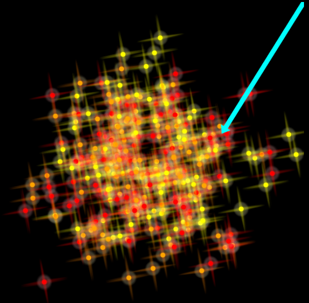

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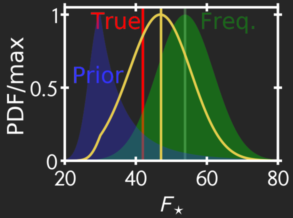

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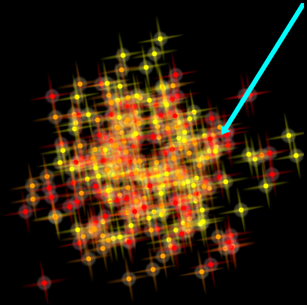


Refining the Prior Based on the Observations:

Cumulation of data: if you perform a series of observations in this cluster:

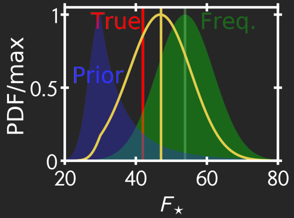
$$\underbrace{p(F_\star)}_{\text{new prior}} \propto \underbrace{1}_{\text{initial prior}} \times \underbrace{p(F_\star^{(1)}|F_\star) \times \ldots \times p(F_\star^{(N_\star)}|F_\star)}_{\text{cumulated data}}$$

Difference Between the Two Approaches: Using an Informative Prior

Accounting for Additional Information:

Example: the star is in a globular cluster & you know the distance.

Prior: cluster mass function & mass/luminosity relation ⇒ luminosity function.



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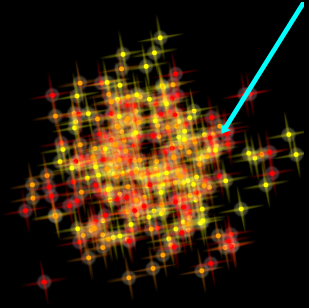

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Hierarchical model: consistently perform this process on all the data, at the same time.

Difference Between the Two Approaches: Lindley's Paradox

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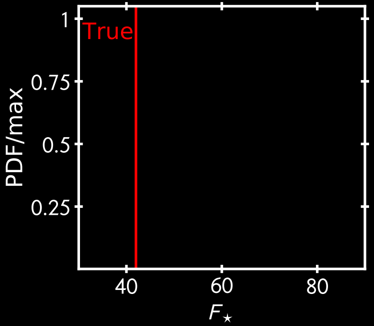

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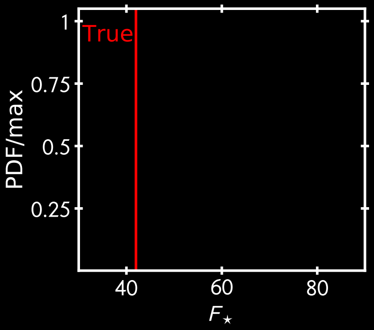

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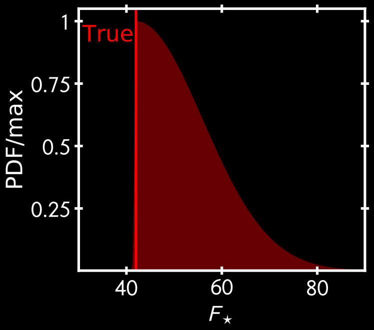

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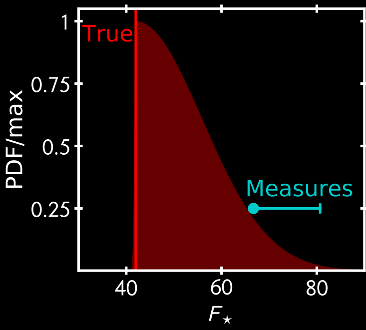

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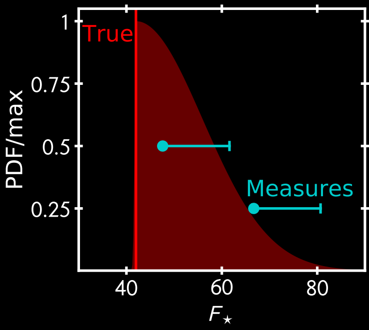

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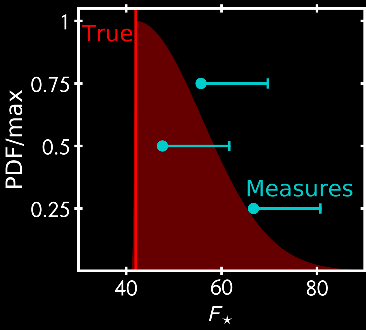

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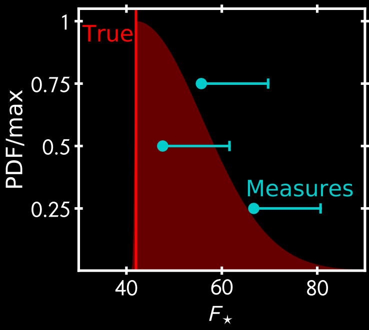

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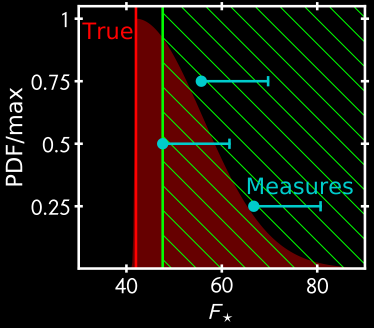

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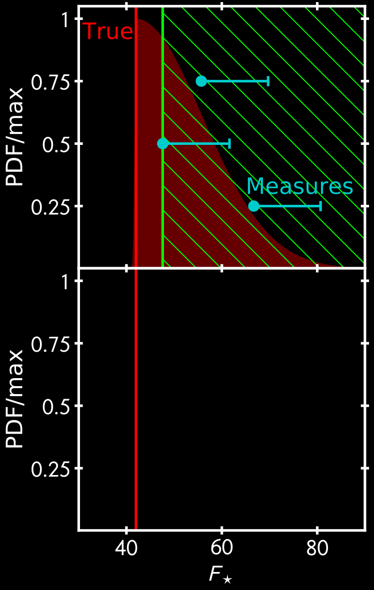

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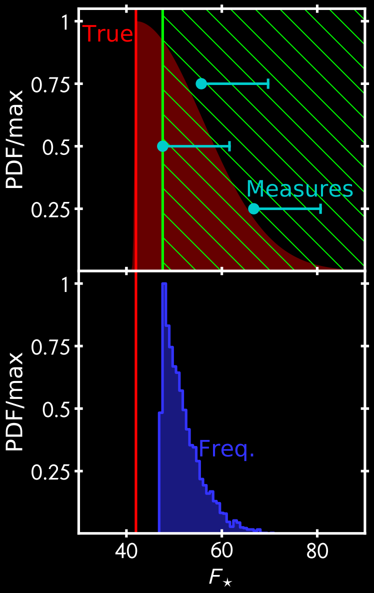

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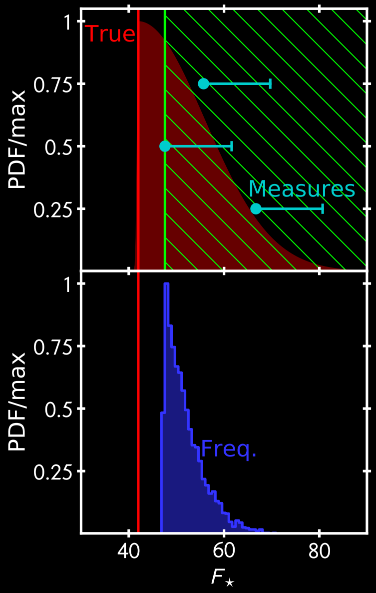

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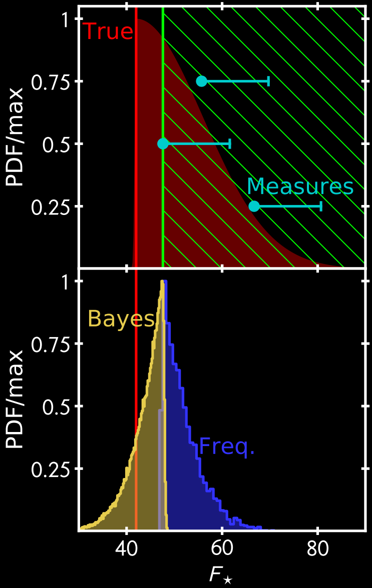

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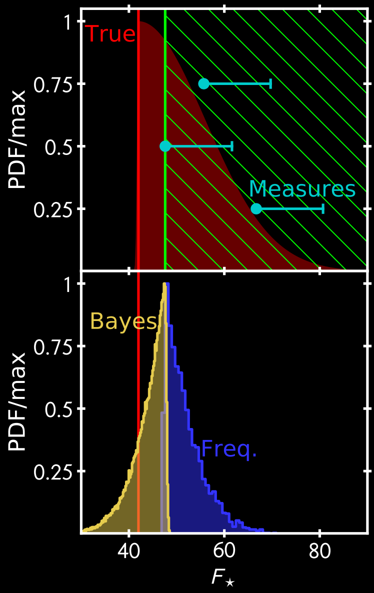

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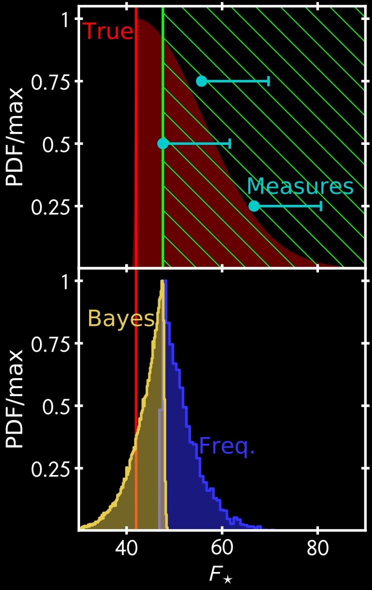

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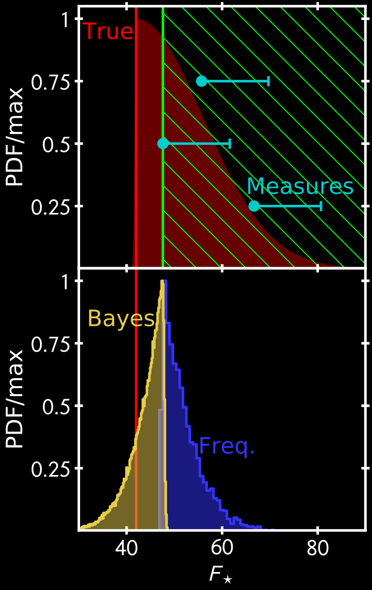

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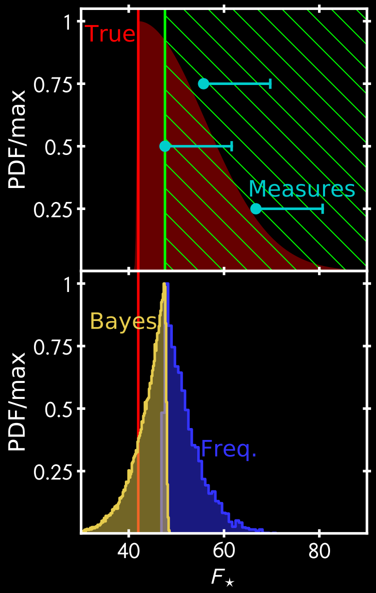

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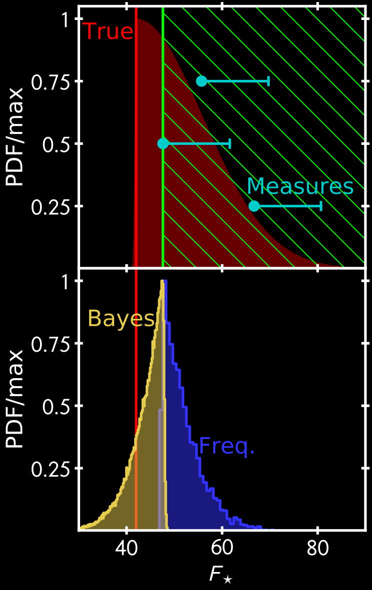
> *Bayesians address the question everyone is interested in by using assumptions no-one believes, while Frequentists use impeccable logic to deal with an issue of no interest to anyone.*
>
> (Lyons, 2013)



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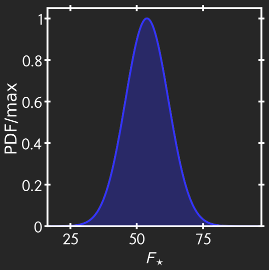

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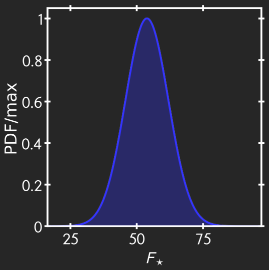

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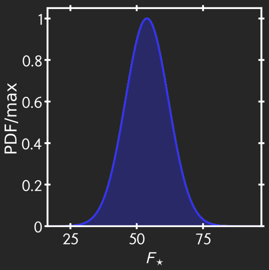

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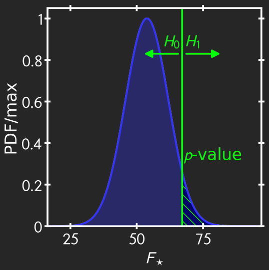

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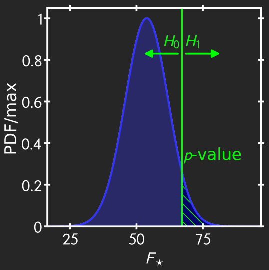

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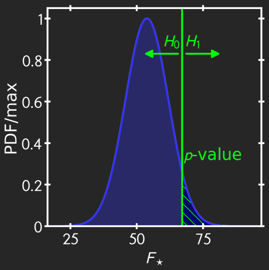

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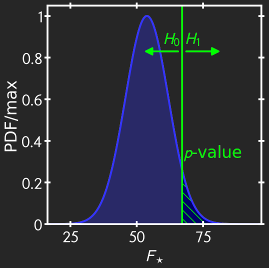

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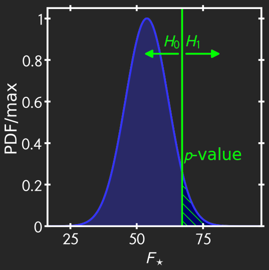

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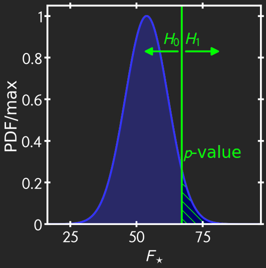

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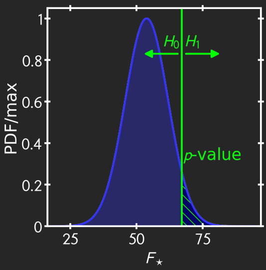

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Hypothesis Testing: the Limitations of Frequentist p-Values

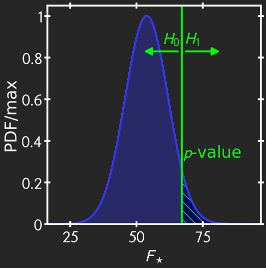
Bayesian Hypothesis Testing:

$$\underbrace{\frac{p(H_1|data)}{p(H_0|data)}}_{\text{posterior odds}} = \underbrace{\frac{p(data|H_1)}{p(data|H_0)}}_{\text{Bayes factor}} \times \underbrace{\frac{p(H_1)}{p(H_0)}}_{\text{prior odds}}$$

Bayes factor	Strength of evidence
1 to 3.2	Barely worth mentioning
3.2 to 10	Substantial
10 to 100	Strong
> 100	Decisive

(Jeffreys, 1961)

Frequentist Hypothesis Testing:



H_0: the *null hypothesis*;

H_1: complement.

Commonly:
$p < 0.05$
$\Rightarrow$ reject H_0.

Recent Controversy About the Interpretation & the Significance of p-Values:

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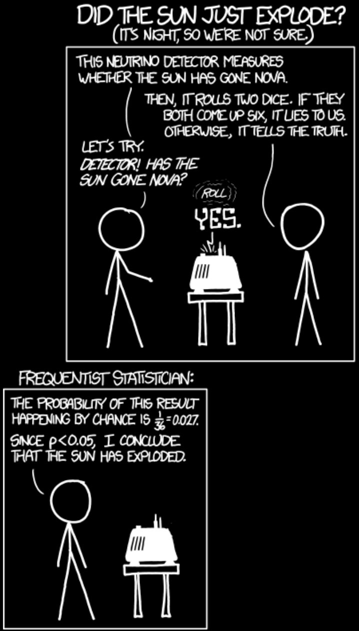
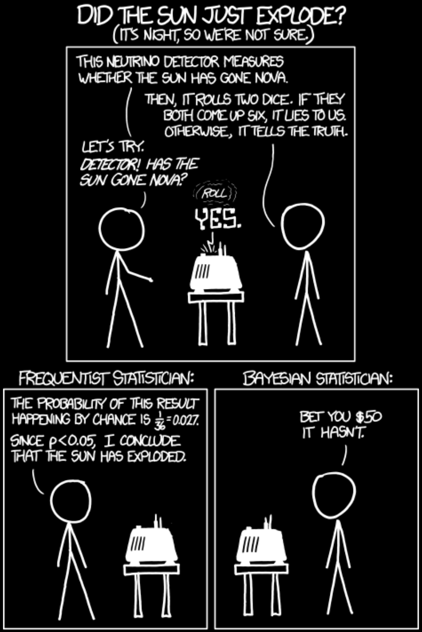
2018: *Political Analysis* *"will no longer be reporting p-values".*

Reductio Ad Absurdum (xkcd)


DID THE SUN JUST EXPLODE?
(IT'S NIGHT, SO WE'RE NOT SURE.)
THIS NEUTRINO DETECTOR MEASURES WHETHER THE SUN HAS GONE NOVA.
THEN, IT ROLLS TWO DICE. IF THEY BOTH COME UP SIX, IT LIES TO US. OTHERWISE, IT TELLS THE TRUTH.
LET'S TRY.
DETECTOR! HAS THE SUN GONE NOVA?
(ROLL)
YES.

Reductio Ad Absurdum (xkcd)

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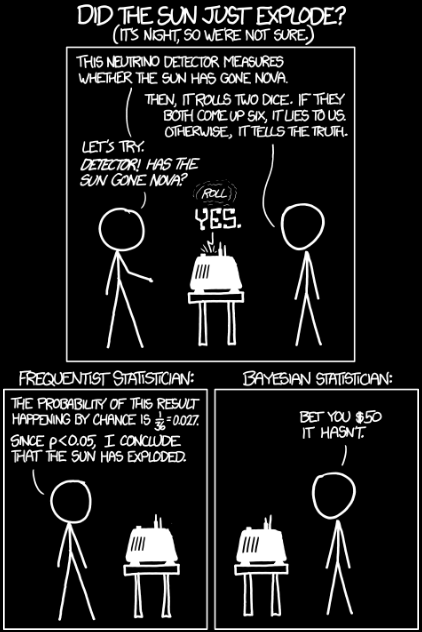
ROLL
YES.
FREQUENTIST STATISTICIAN:
THE PROBABILITY OF THIS RESULT HAPPENING BY CHANCE IS 1/36 = 0.027. SINCE p<0.05, I CONCLUDE THAT THE SUN HAS EXPLODED.
BAYESIAN STATISTICIAN:
BET YOU $50 IT HASN'T.



The Bayesian Point of View:

$$p(\text{nova}|2 \times 6) = \frac{p(2 \times 6|\text{nova}) \times p(\text{nova})}{p(2 \times 6)}$$

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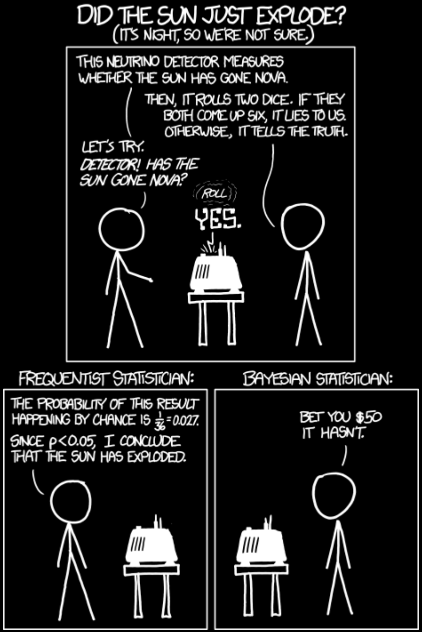

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(Essai philosophique sur les probabilités, 1814)

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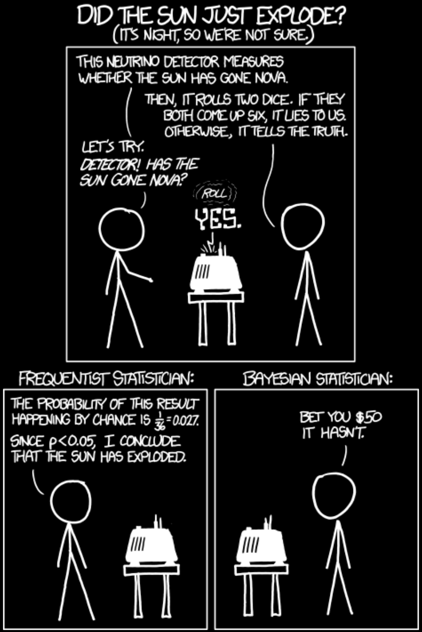

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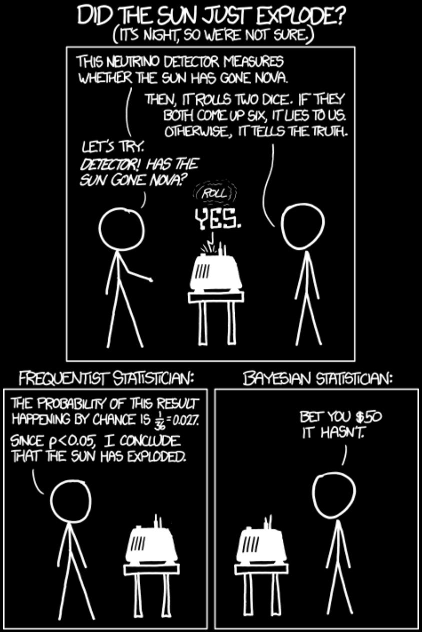

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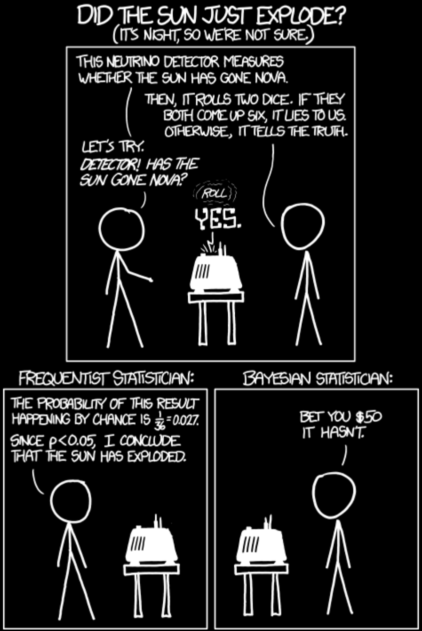

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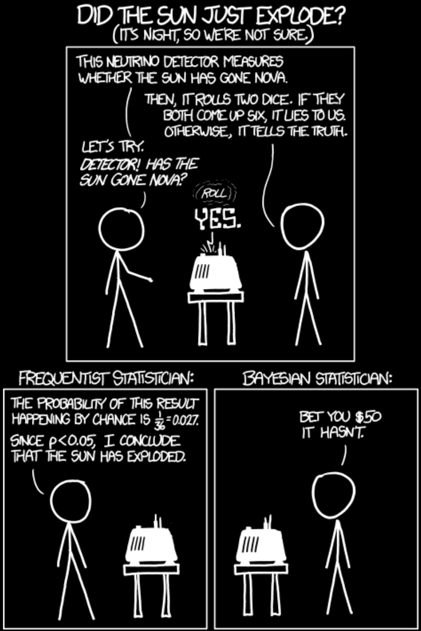

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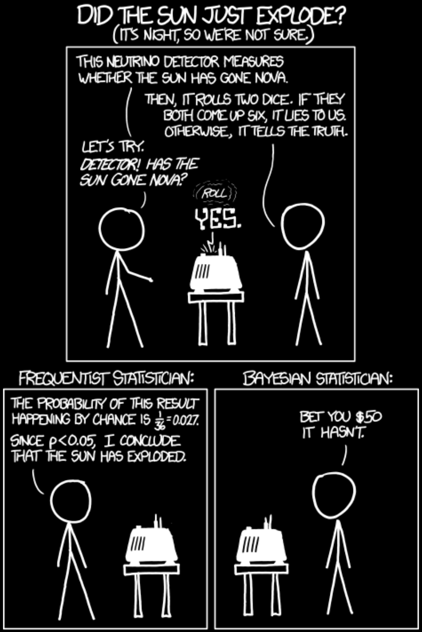

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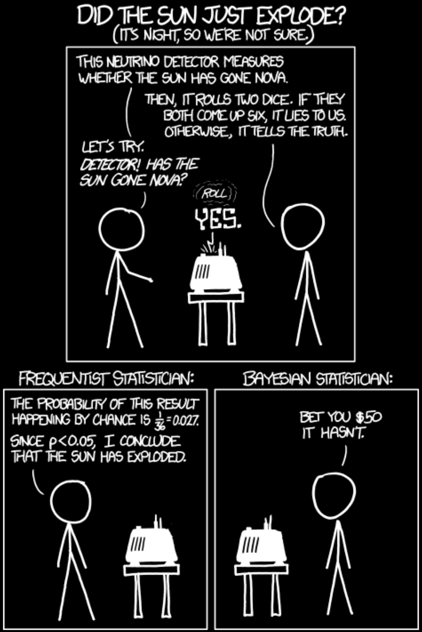

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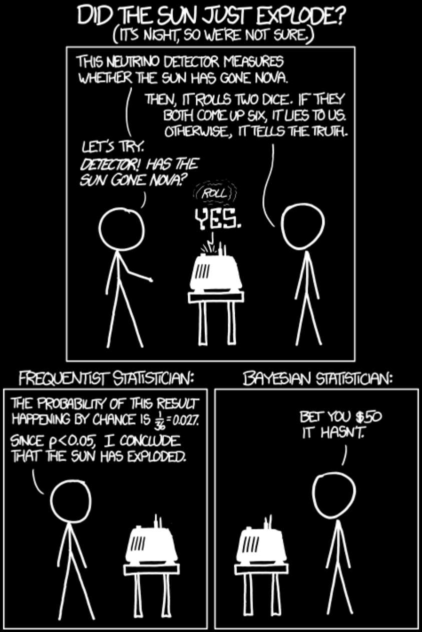

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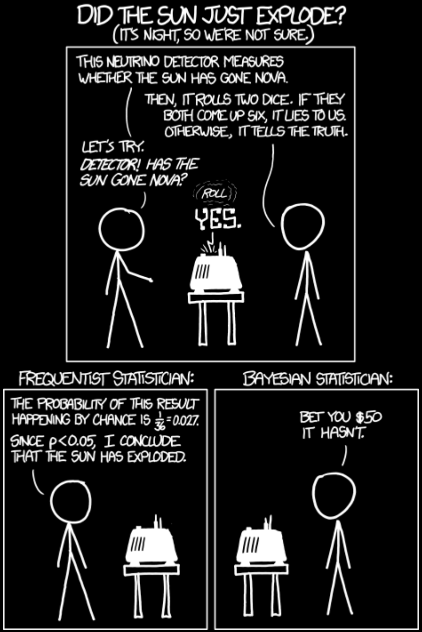

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The Bayesian Approach is Holistic: Stein's Paradox

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Prior Depends on Dust Model Parameters:

Intuitive approach: $p(M_{\text{dust}})$.

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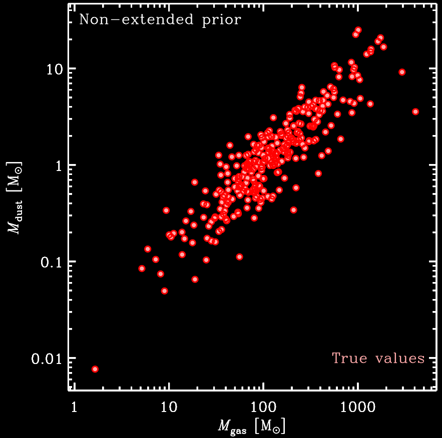

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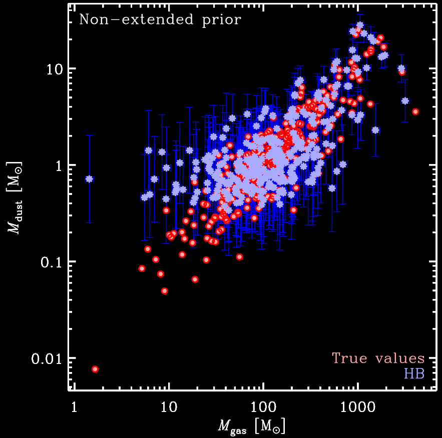

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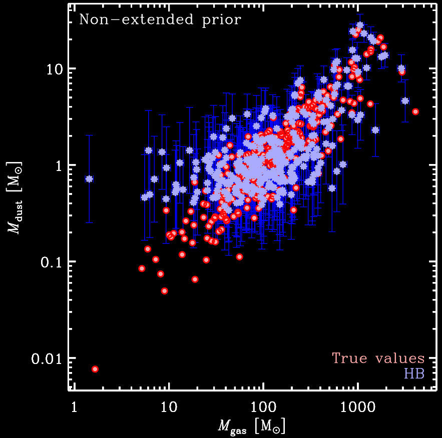

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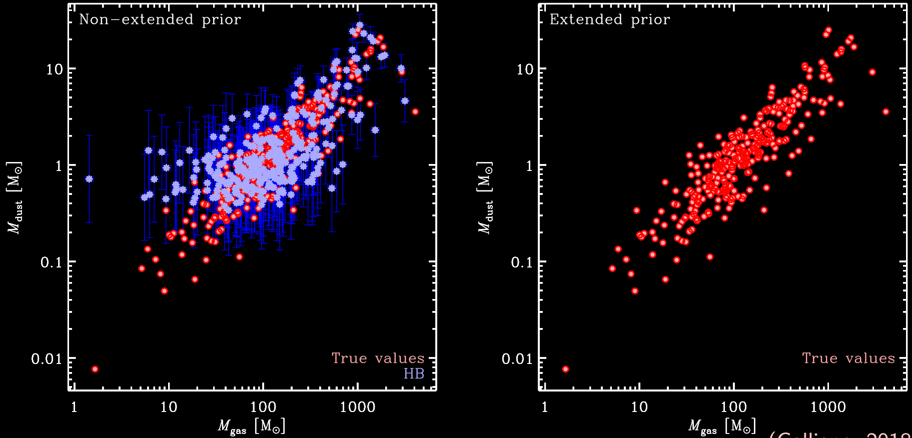

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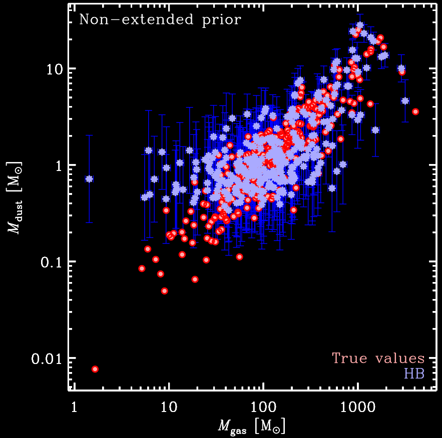

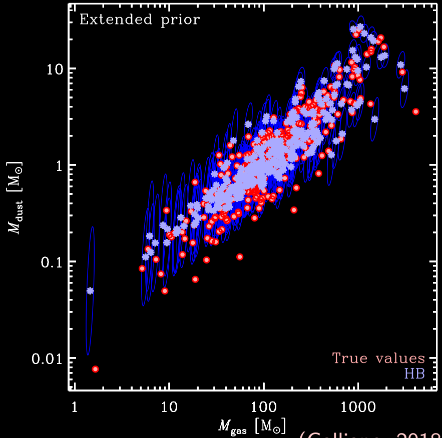

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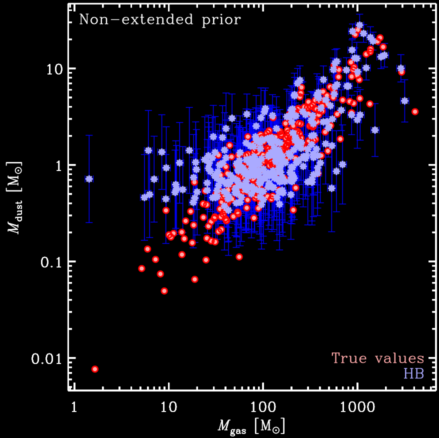

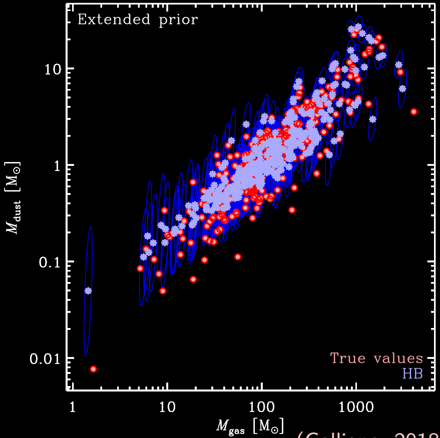

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$\Rightarrow$ partition of knowledge is statistically inadmissible (Bayesian take on *Stein's paradox*).

Pros & Cons of the Two Approaches

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The Bayesian Approach

The Frequentist Approach

(*cf.* reviews by *e.g.* Jaynes, 1976; Loredo, 1990; Lindley, 1999; Wagenmacker, 2008; Lyons, 2013)

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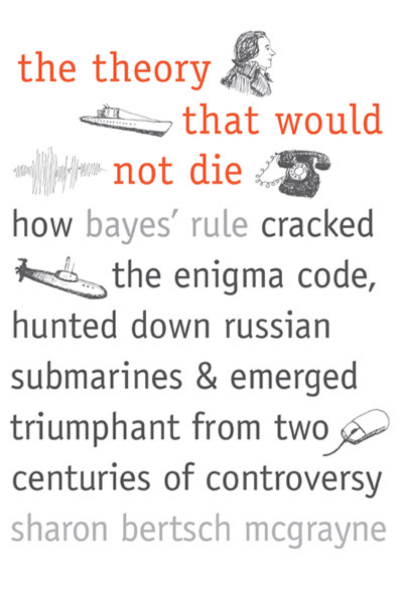
- **CON** samples non-observed data & arbitrary choice of estimator, loss functions or p-value.
- **CON** boolean logic $\Rightarrow$ a proposition is either true or false.
- **PRO** fast computation.
- **CON** does not work well with small samples.
- **CON** difficult to teach (collection of *ad hoc* cooking recipes).
- **CON** strict: can account only for data related to a particular experiment.
- **CON** can give ridiculous answers.

(*cf.* reviews by *e.g.* Jaynes, 1976; Loredo, 1990; Lindley, 1999; Wagenmacker, 2008; Lyons, 2013)

Outline of the Talk

A Comprehensive Historical Perspective: S. McGrayne's Book



Sharon Bertsch McGRAYNE (1942–)

(Published in 2011)

First Formulation by Bayes

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Publication: in 1764, by Richard PRICE: his formula gives the probability of causes ⇒ can be applied to prove God's existence.

Laplace: the Probability of Causes of Events (1/2)

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Pierre Simon LAPLACE (1749–1827)

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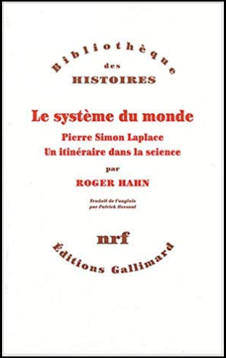

Bibliothèque des HISTOIRES

Le système du monde

Pierre Simon Laplace

Un itinéraire dans la science

par

ROGER HAHN

nrf

Éditions Gallimard

(Hahn, 2004; English version in 2005)

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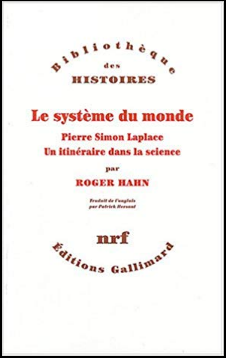

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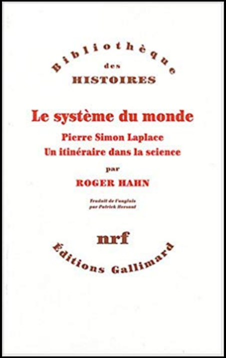

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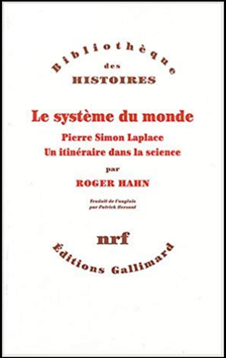

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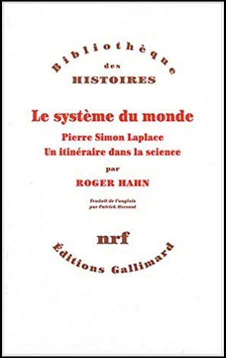

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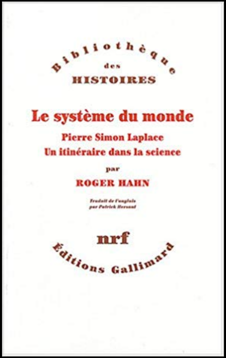

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After Laplace: Early Frequentist Development

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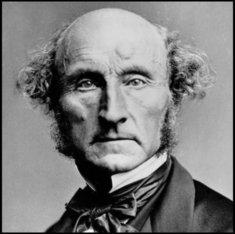

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(1806–1873)

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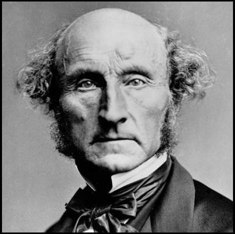

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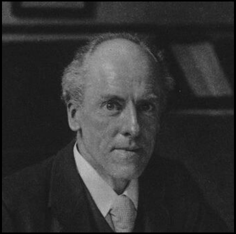

Karl PEARSON
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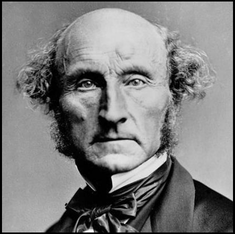

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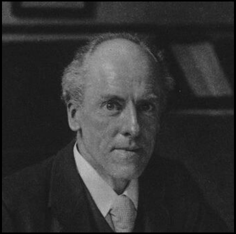

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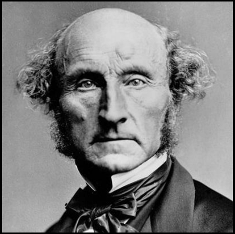

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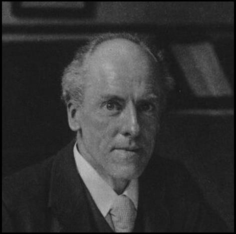

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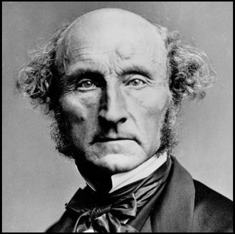

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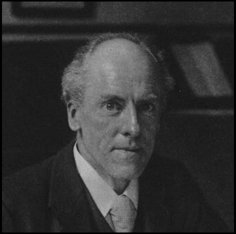

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> *(...) a very slight improvement in the data, by better observations, or by taking into fuller consideration the special circumstances of the case, is of more use than the most elaborate application of the calculus of probabilities founded on the data in their previous state of inferiority. The neglect of this obvious reflection has given rise to misapplications of the calculus of probabilities which have made it the real opprobrium of mathematics.*

(A System of Logic, 1843)

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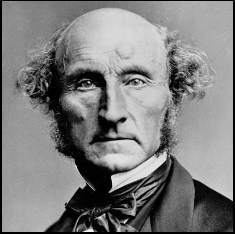

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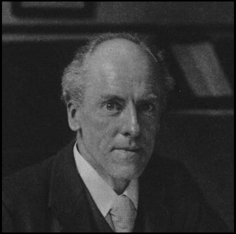

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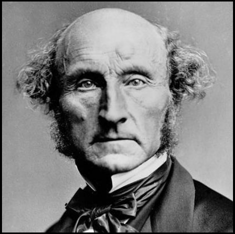

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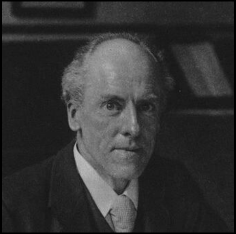

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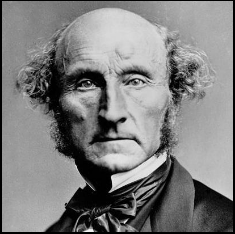

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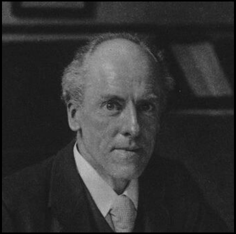

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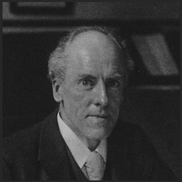

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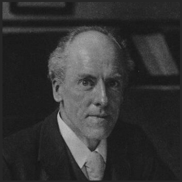

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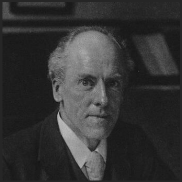

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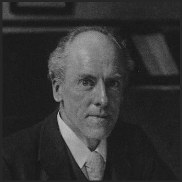

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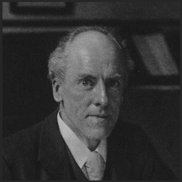

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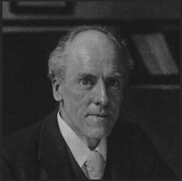

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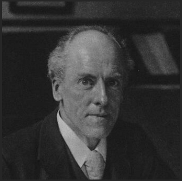

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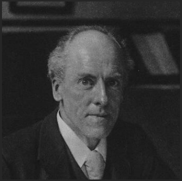

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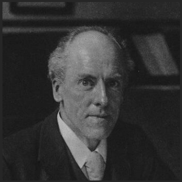

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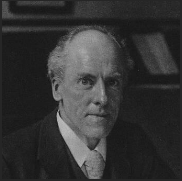

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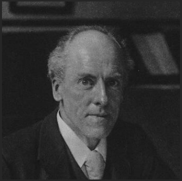

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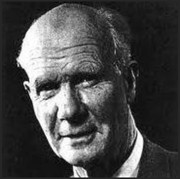

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The Bayesian Resistance

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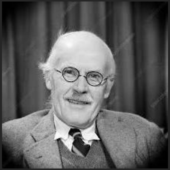

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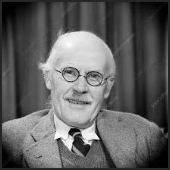

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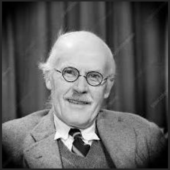

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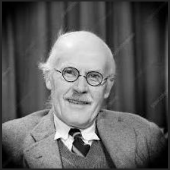

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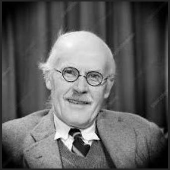

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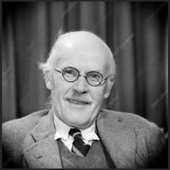

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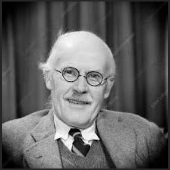

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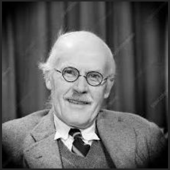

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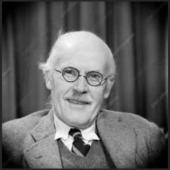

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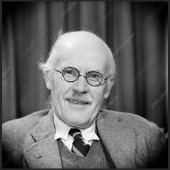

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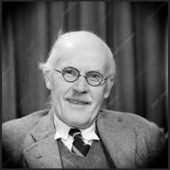

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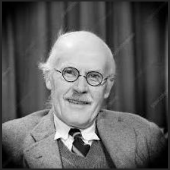

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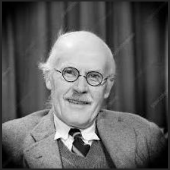

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Dennis LINDLEY & Jimmy SAVAGE popularised Bayes' rule (Savage: *"Fisher is making Bayesian omelet without breaking Bayesian eggs"*).

Jerome CORNFIELD pioneered in applying Bayes' rule to epidemiology ⇒ showed link between smoking & lung cancer (ridiculised Fisher).

Howard RAIFFA & Robert SCHLAIFER taught Bayes' rule for business & decision-making.

Norman RASMUSSEN estimated the risks of a nuclear incident in the 70s ⇒ possible, but not necessarily desastrous (*cf.* Three Mile Island, 1979).

Bayesian Search Algorithm used to find lost nuclear bombs & russian submarines.

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- Training a neural network $\Leftrightarrow$ informing a *prior*.

Importance of Bayes' Rule For Neurosciences

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(Lectures given between 2011 and 2012)

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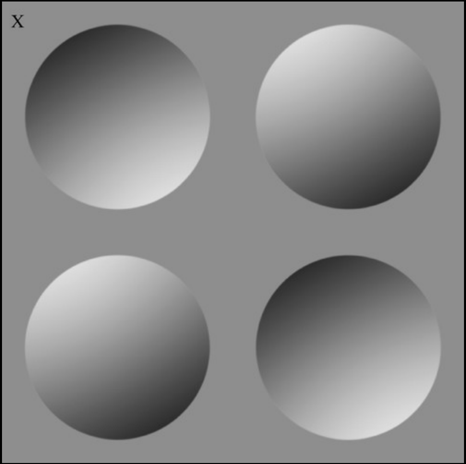

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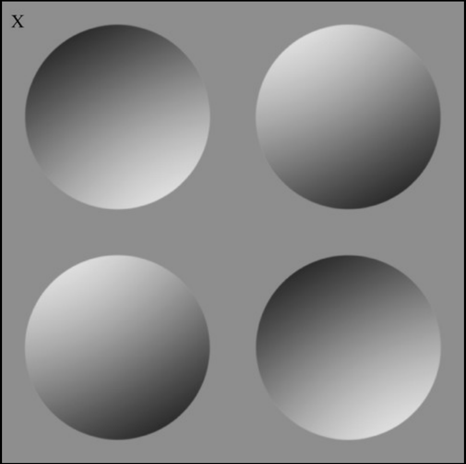

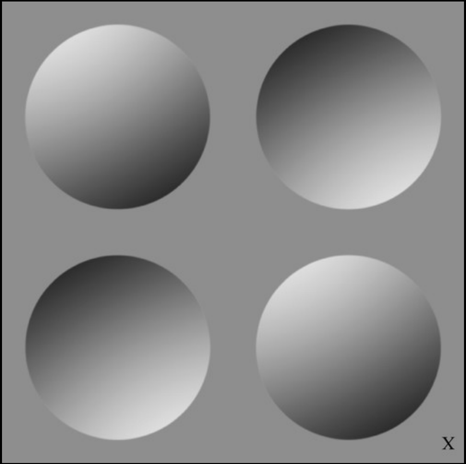

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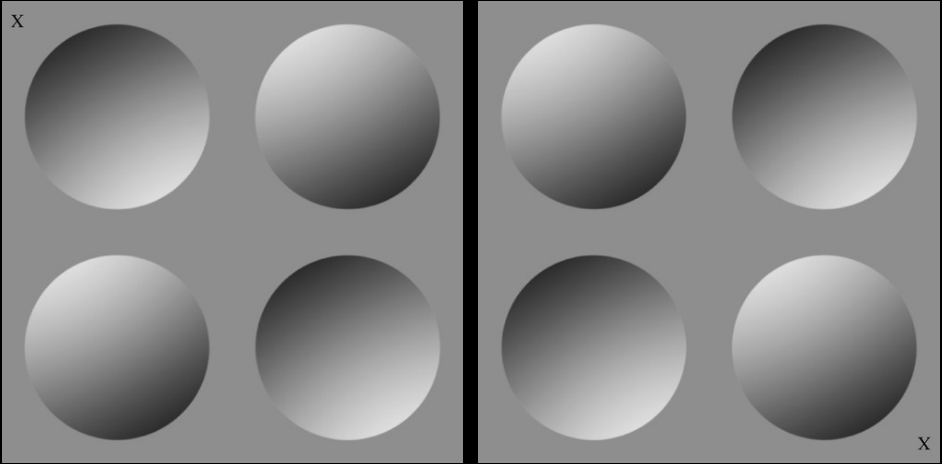

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Most objects are illuminated from above (sunlight, spots, *etc.*)

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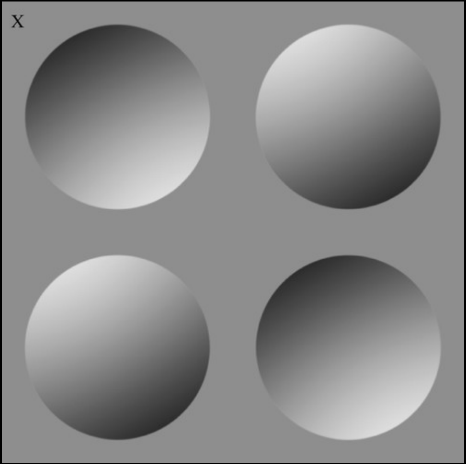

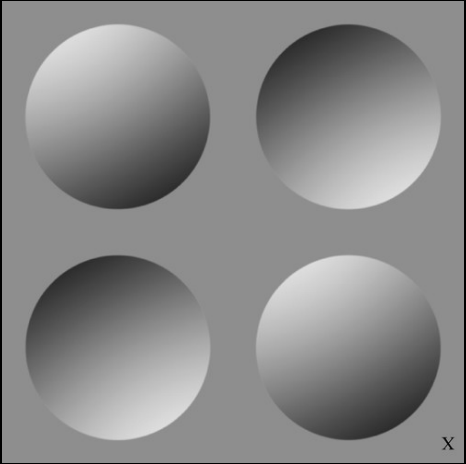

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How Does the Brain Learn New Words?

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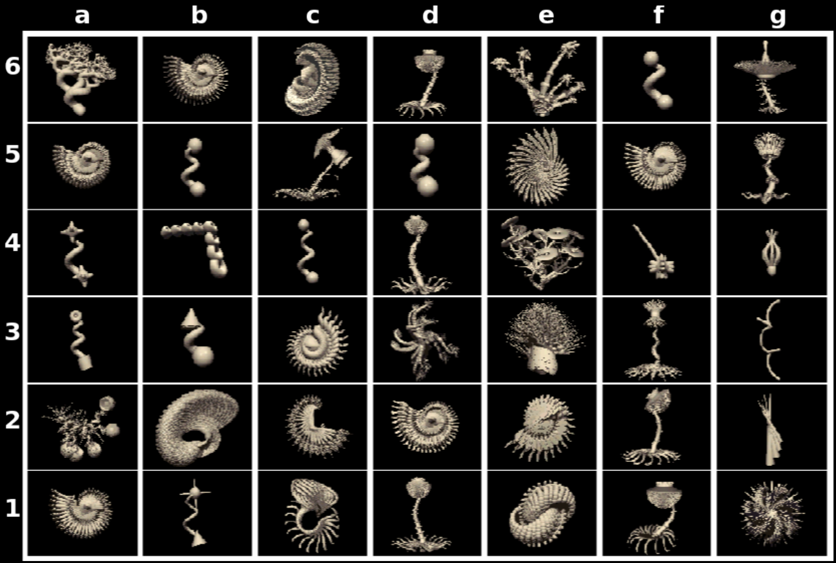

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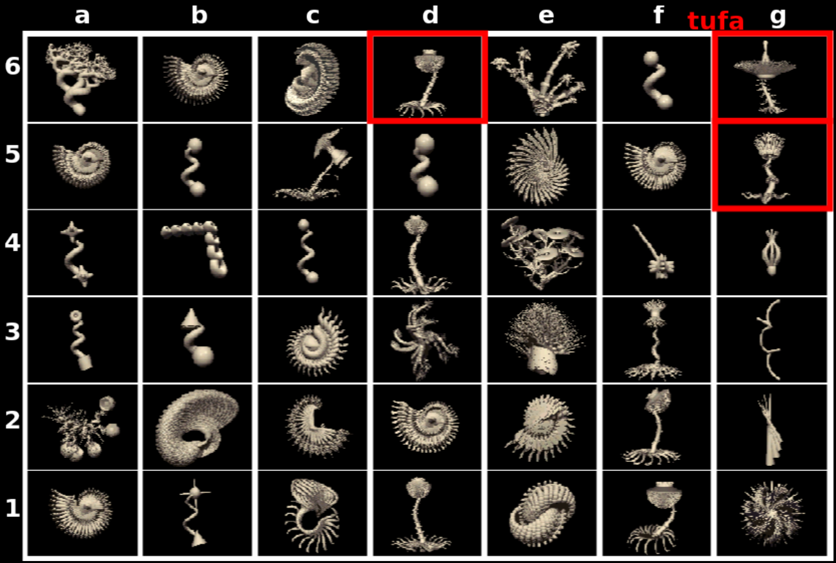

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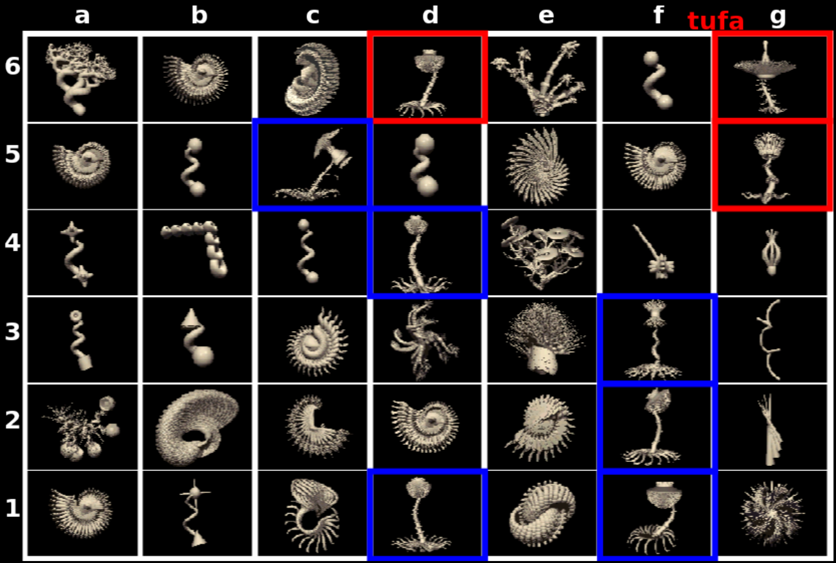

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Outline of the Talk

The Epistemological Debate at the Beginning of the XX$^{\text{th}}$ Century

The unity of science consists alone in its method, not in its material.

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- Abandon rationalism for empiricism $\Rightarrow$ a proposition has a cognitive meaning only if it can be *verified* by experience.

Popper's Criticism of Induction in Empirical Sciences

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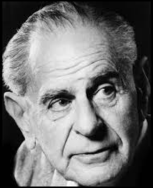

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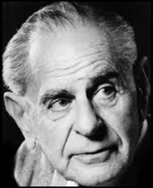

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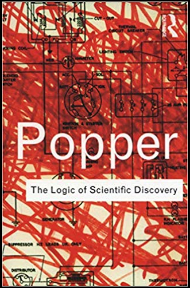

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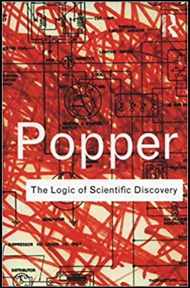

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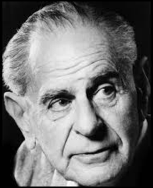

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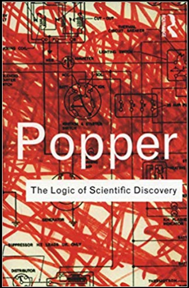

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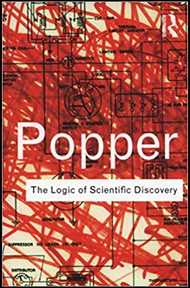

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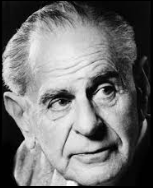

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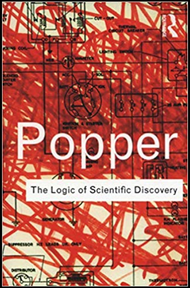

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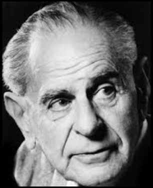

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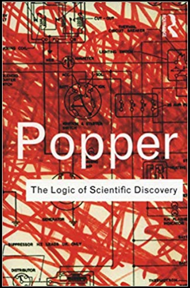

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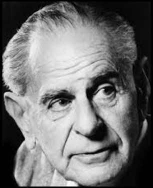

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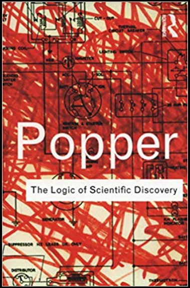

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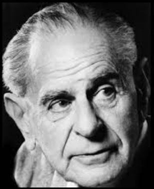

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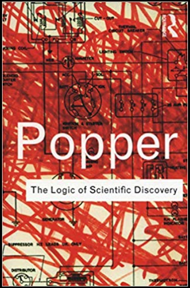

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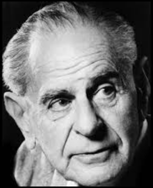

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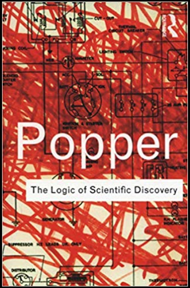

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$\Rightarrow$ Falsifiabilism: deductivism & *modus tollens*.
Modus tollens: $\big((A \Rightarrow B) \wedge \overline{B}\big) \Rightarrow \overline{A}$.

Objectivity & Principle of Parsimony (Occam's Razor)

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- The objectivity of scientific statements lie in the fact that they can be *inter-subjectively tested*.

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- Only repeatable experiments can be tested by anyone $\Rightarrow$ no coincidence.

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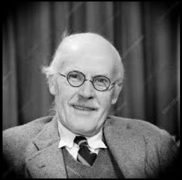
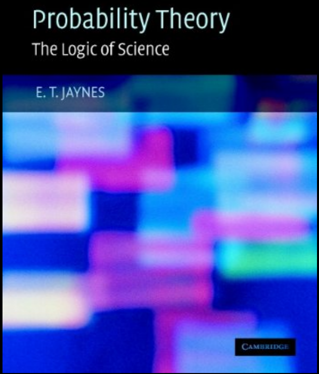


Sir Harold JEFFREYS
(1891–1989) 🇬🇧



(Published in 2003; dedicated to Jeffreys)

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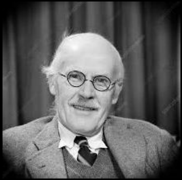

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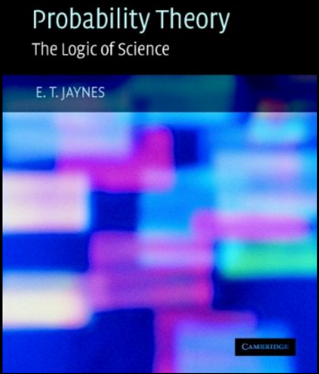

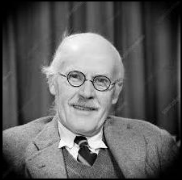

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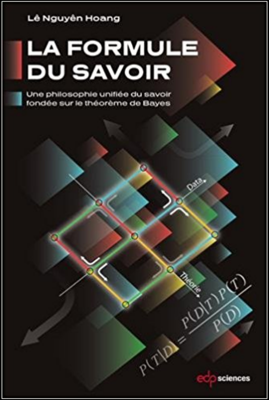

Lê Nguyên HOANG (1987–)

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Falsifiability & the Limits of Platonic Logic

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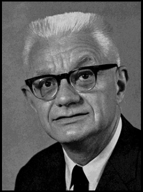

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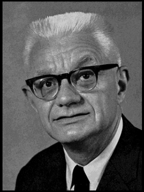

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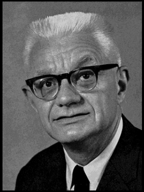

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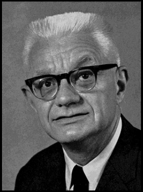

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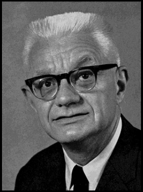

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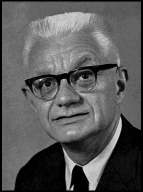

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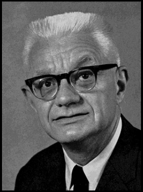

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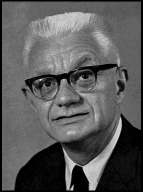

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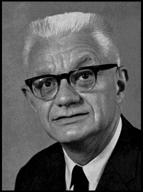

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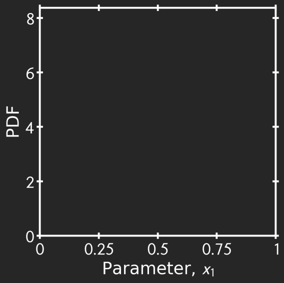

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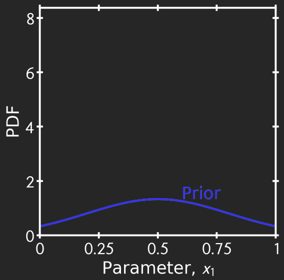

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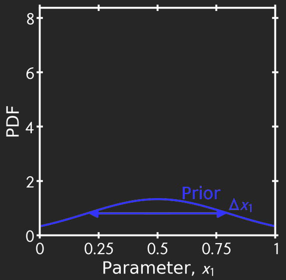

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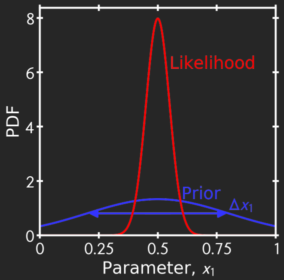

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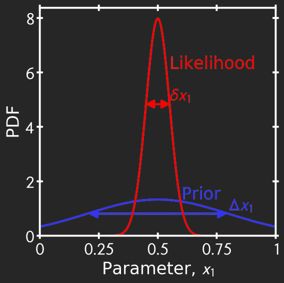

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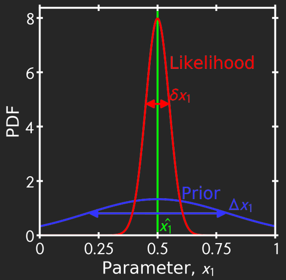

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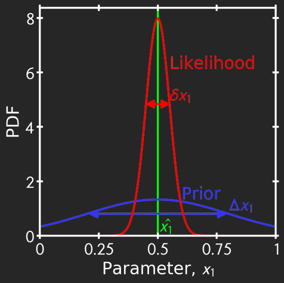

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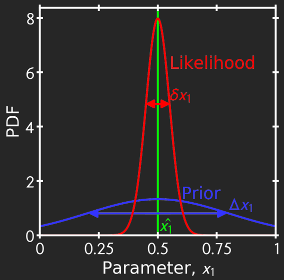

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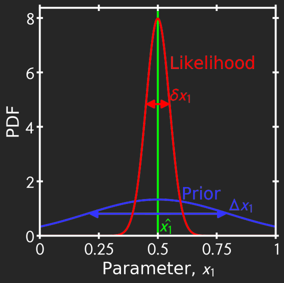

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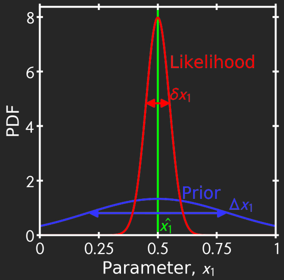

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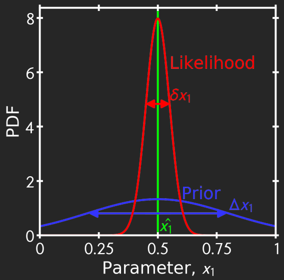

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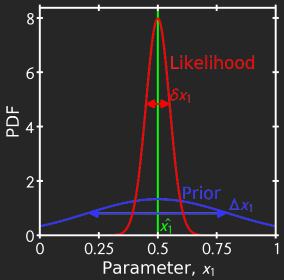

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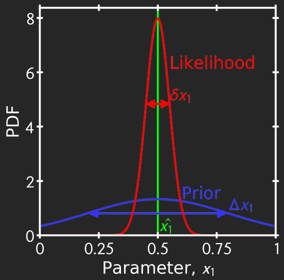

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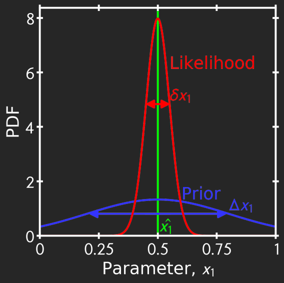

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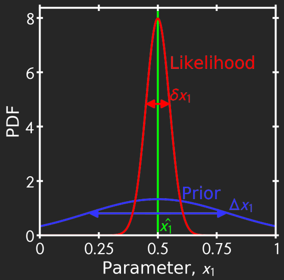

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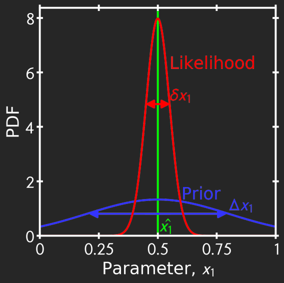

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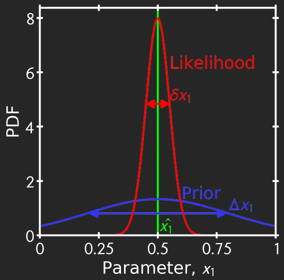

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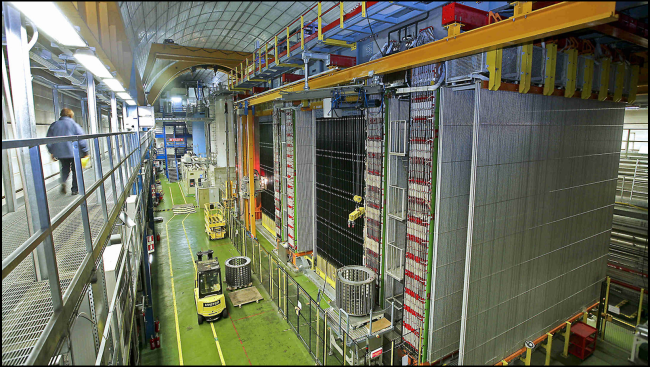

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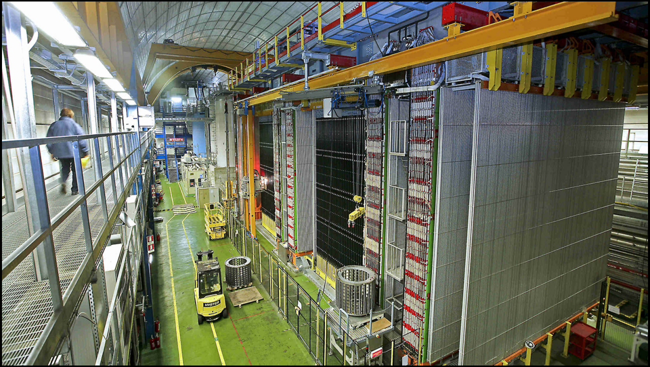

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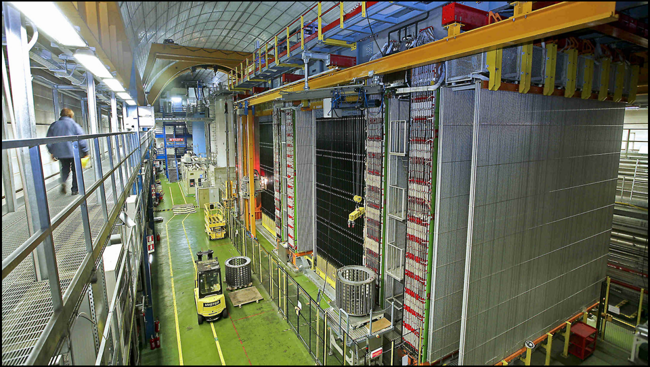

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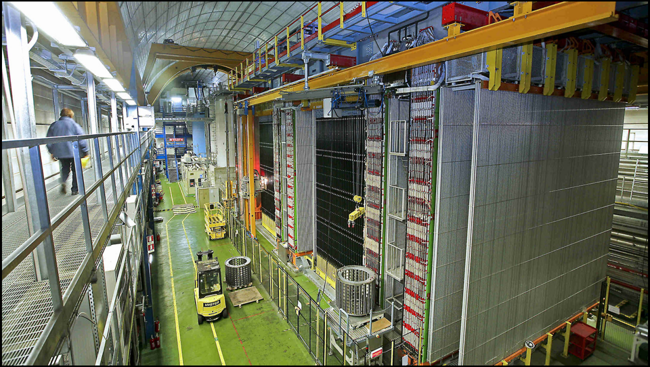

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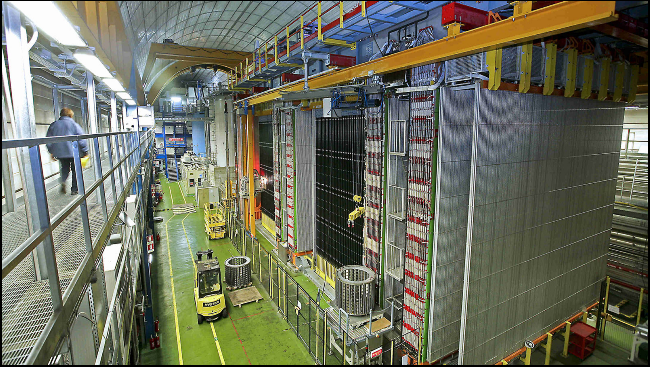

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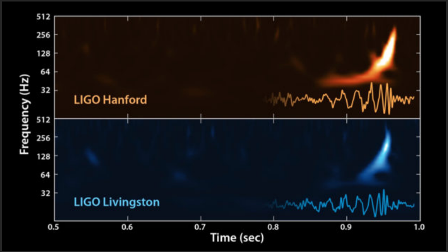

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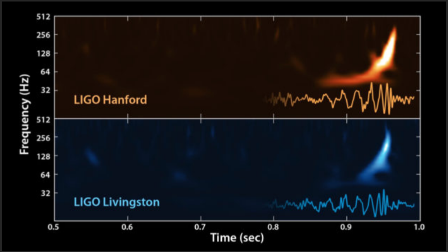

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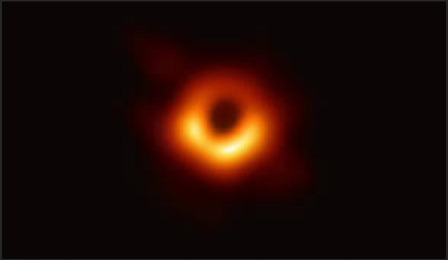

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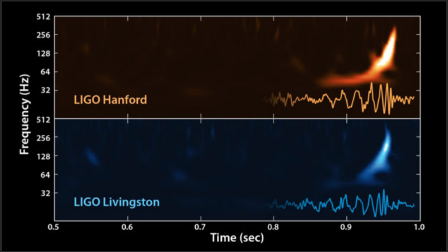

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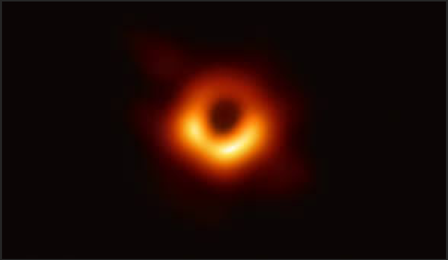

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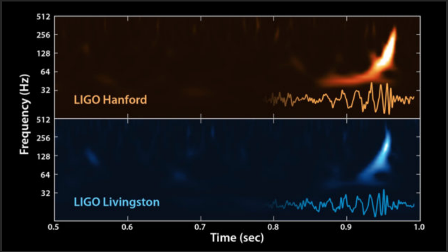

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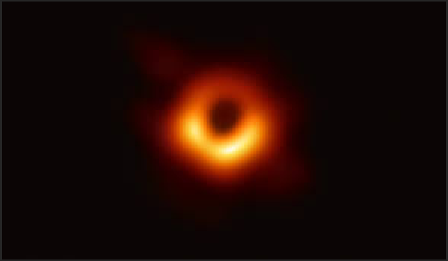

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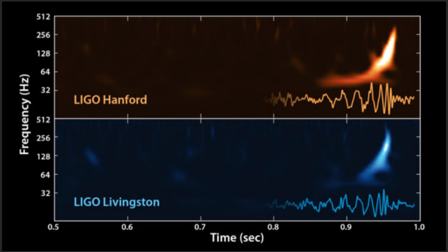

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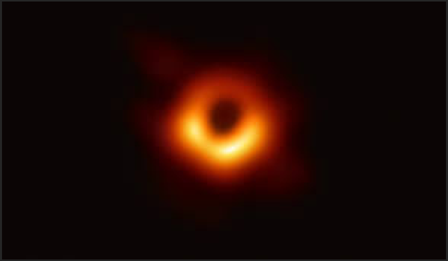

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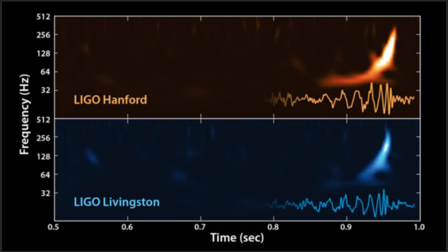

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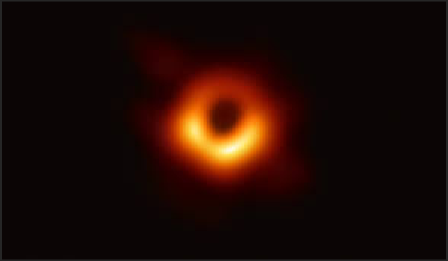

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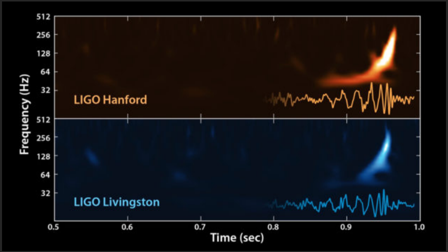

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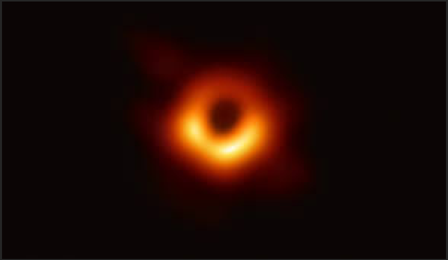

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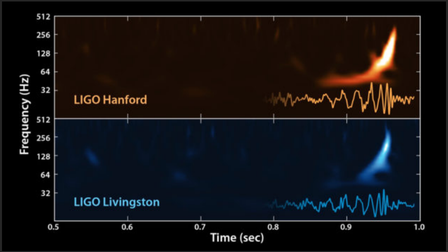

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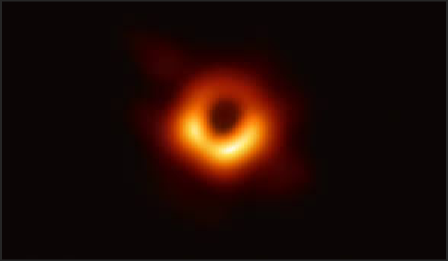

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Those two experiments, combined, bring a large *weight of evidence* in favor of general relativity.

Outline of the Talk

Summary & Conclusion

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The Opposition Between Bayesianism & Frequentism:

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