SPECTRAL MULTI-FLAVOUR NEUTRINO TRANSPORT FOR SUPERNOVA SIMULATIONS

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NEUTRINO TRANSPORT

We evolve equations of energy and momentum of the neutrinos in first order $\mathcal{O}(v/c)$:

$$\partial_t E \;+\; \nabla_j F^j + \nabla_j (v^j E) + (\nabla_j v_k) P^{jk} - (\nabla_j v_k)\partial_\varepsilon(\varepsilon P^{jk}) = C^{(0)},$$
$$\partial_t F^i \;+\; c^2 \nabla_j P^{ij} + \nabla_j (v^j F^i) + (F^j \nabla_j v^i) - (\nabla_j v_k)\partial_\varepsilon(\varepsilon Q^{jk}) = C^{(1),i}.$$

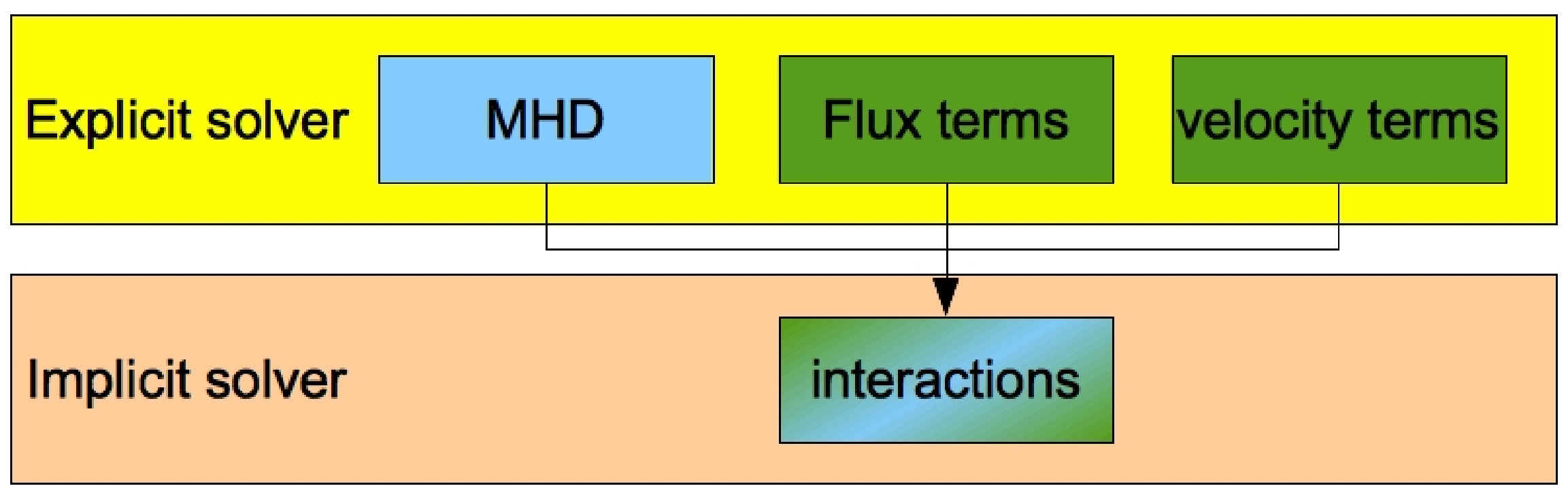
They include terms corresponding to
- radiative fluxes, involving higher moments,
- advection and compression by the flow,
- compression work / aberration,
- spectral redistribution (Doppler shift),
- interaction with matter: absorption, emission, scattering.

Flux terms We close the equations with a local algebraic Eddington factor $\chi^{ij}(\vec{F}/cE, E)$ determining the pressure tensor

$$\partial_t \begin{pmatrix} E \\ F^i \end{pmatrix} + \partial_x \begin{pmatrix} F \\ c^2 \chi^{xx} E \end{pmatrix} = \begin{pmatrix} C^{(0)} \\ C^{(1),x} \end{pmatrix},$$

- *hyperbolic system*
- accurate, yet computationally less expensive than Boltzmann solvers
- valid from diffusion to free streaming
- generically multi-dimensional

Operator splitting We treat the hyperbolic terms of MHD and transport explicitely, and split the potentially stiff interactions between neutrinos and matter off to solve them implicitly.


Explicit solver
MHD
Flux terms
velocity terms
Implicit solver
interactions

Implementation in the Aenus code
- hyperbolic terms solved by high-resolution shock-capturing methods similarly to MHD [5]
- *monotonicity-preserving* reconstruction of $\geq 5^{\text{th}}$ order accuracy
- approximate (multi-stage) Riemann solvers: Lax-Friedrichs, HLL(D)
- Runge-Kutta time up to 4^{th} order
- most expensive part are the reactions, particularly inelastic scattering and pair processes
- parallelised with MPI and OpenMP

PAIR PROCESSES

So far, we had neglected μ/τ neutrinos created by pair processes: electron annihilation [6] and nucleonic bremsstrahlung [2]. They contribute significantly to cooling of the proto-neutron star at late times. Thus, we recently included pair processes; the current set of reactions is (new additions in green):
- $\nu_e + n \leftrightharpoons e^- + p$
- $\bar{\nu}_e + p \leftrightharpoons e^+ + n$
- $\nu_e + A \leftrightharpoons e^- + A'$
- $\nu + n/p \leftrightharpoons \nu + n/p$
- $\nu + A \leftrightharpoons \nu + A$
- $\nu + e^\pm \leftrightharpoons \nu + e^\pm$
- $e^+ + e^- \leftrightharpoons \nu + \bar{\nu}$
- $N + N \leftrightharpoons N + N + \nu + \bar{\nu}$

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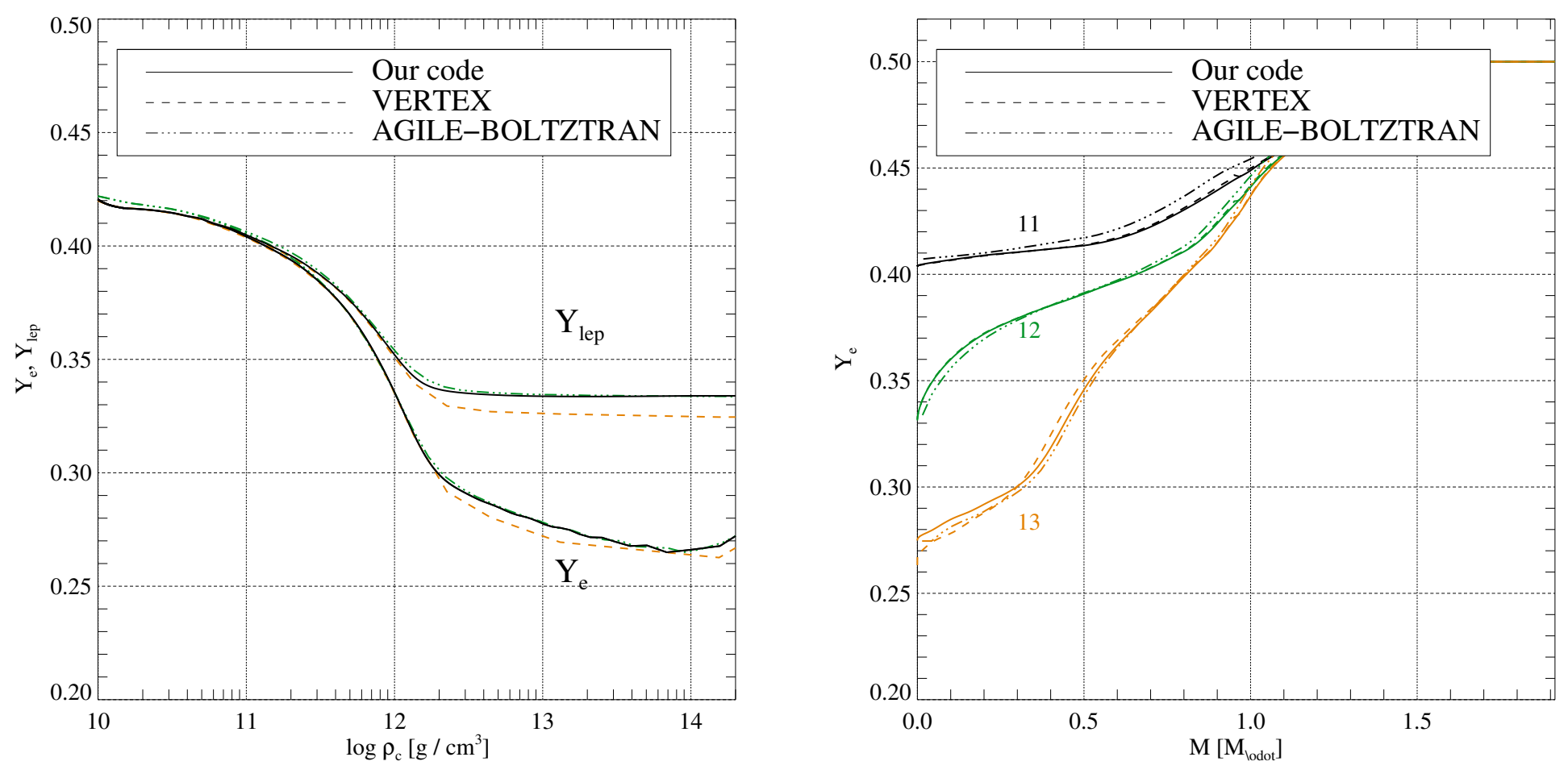
TESTS AND APPLICATIONS

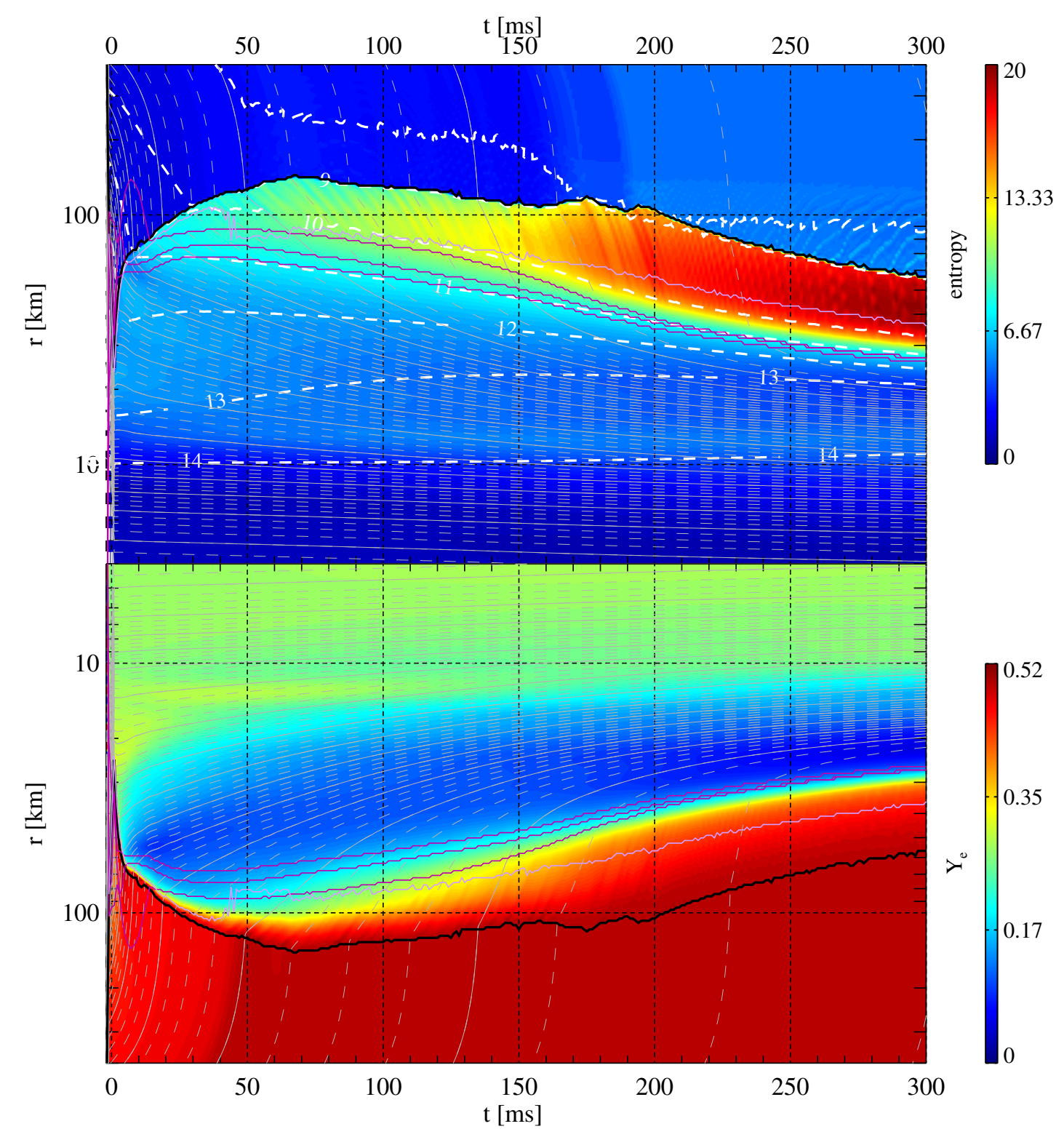
- we follow the collapse of the core of a star of $15\,M_\odot$ and compare to simulations using the Boltzmann codes AGILE-BOLTZTRAN and VERTEX [3]
- we assume spherical symmetry and use an approximation to GR gravity
- we include neutrinos of all flavours and used the maximum-entropy Eddington factor [1]

collapse phase: mostly ν_e are produced, and Y_e decreases until high densities lead to ν trapping



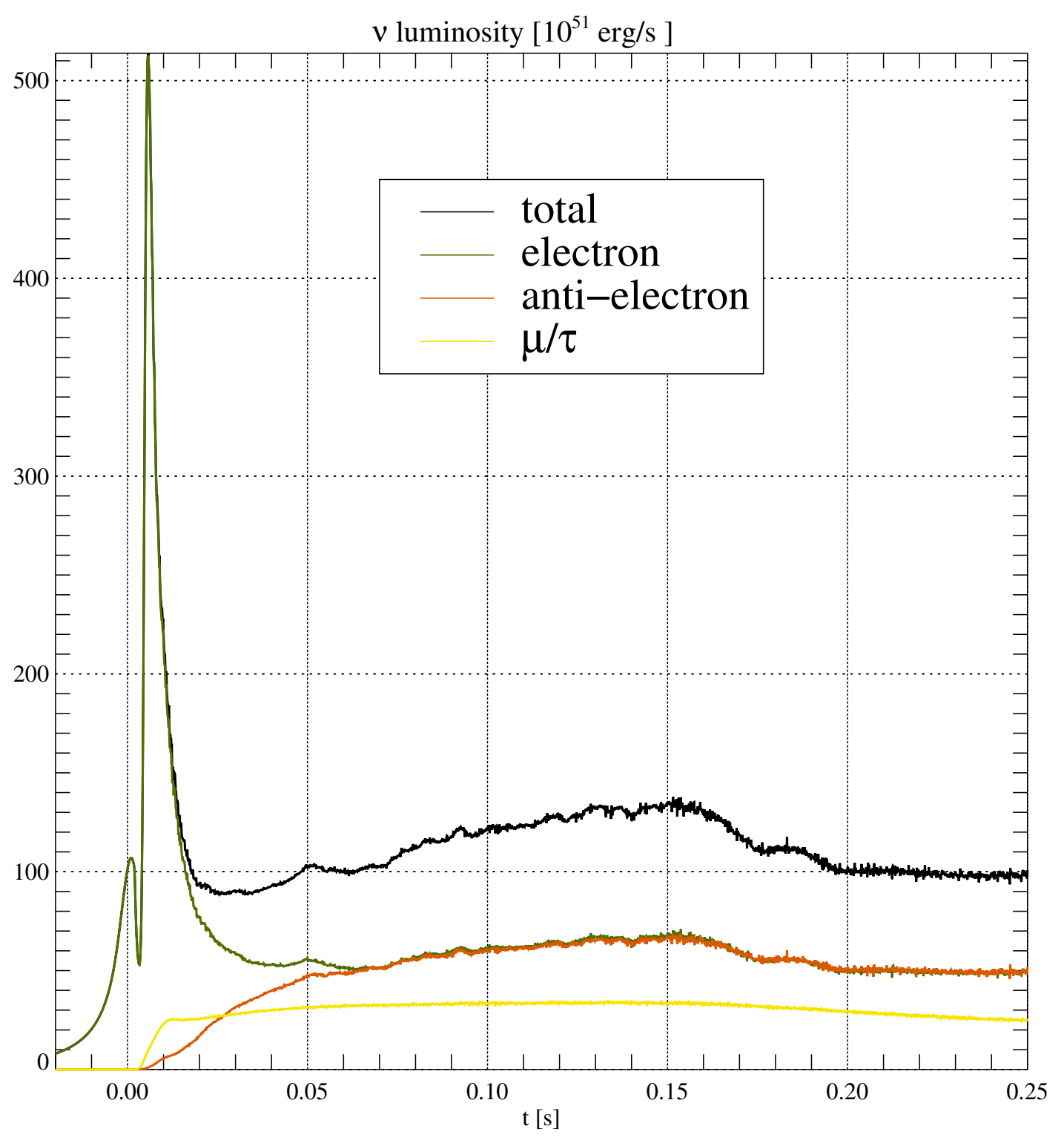
Left: central Y_e as a function of maximum density during collapse. Right: Y_e profiles at different times.

Post-bounce dynamics: the shock wave created at bounce stalls and retreats below 100 km; no explosion develops

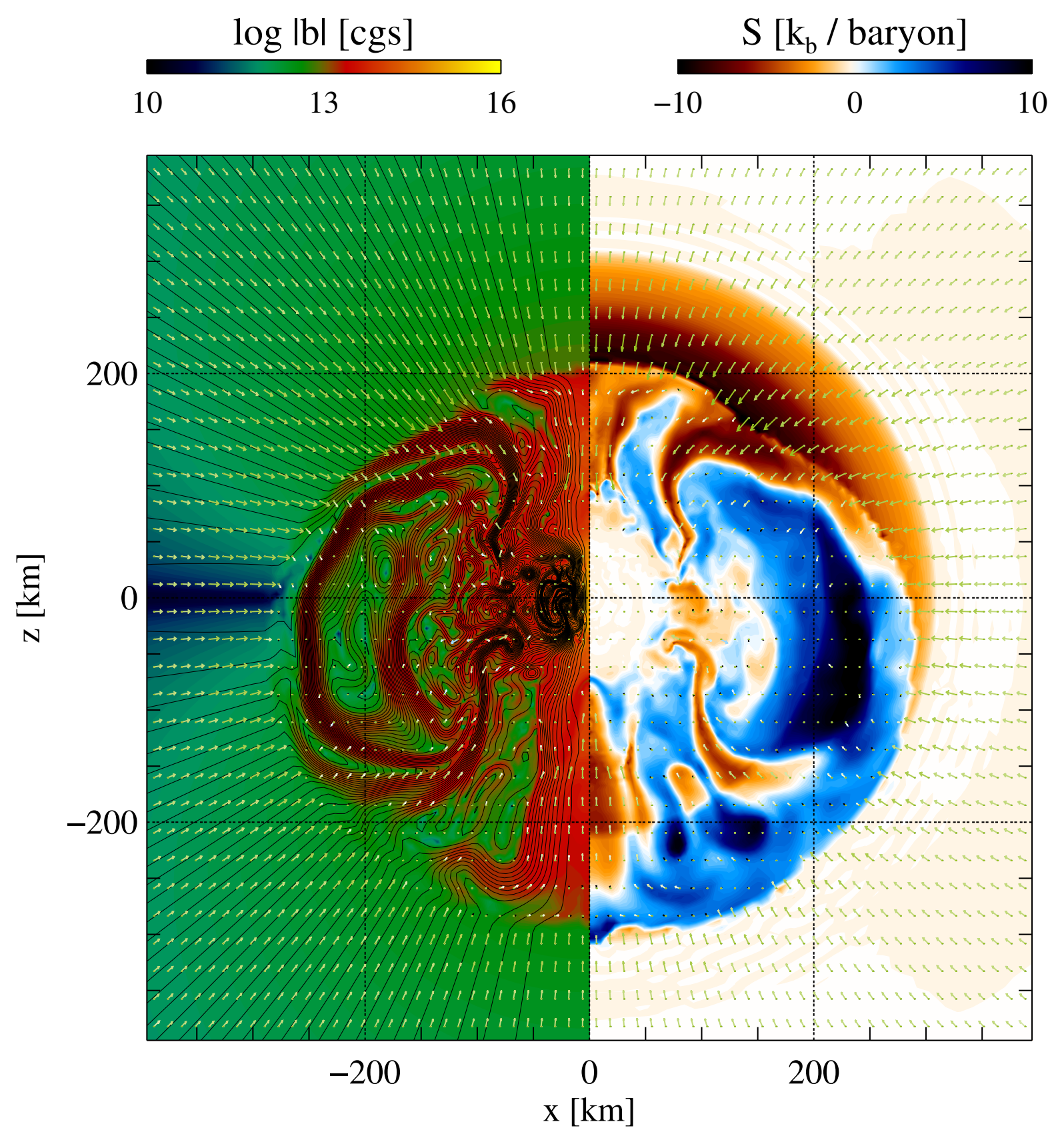


Y_e and entropy as a function of time and radius after the bounce; lines show the positions of mass elements, density contours, the shock and gain radii, and the neutrinospheres of the different flavours.

The neutrino luminosity as a function of time: the early bounce of the electron neutrinos and the later increase of the other flavours can be seen clearly.



Magnetic field and entropy contrast in a magnetised core collapse model.



- Our tests are in good agreement with the reference simulations in both dynamics and neutrino emission.
- The computational effort is considerably lower than that of Boltzmann solvers.

(Planned) applications are in the field of supernovae and gamma-ray bursts:
- multi-D study of magnetised collapse [4]
- accretion tori in mergers of neutron stars
- computation of Y_e profiles for GR simulations performed by others in our group