Building a non-ideal numerical relativistic magnetohydrodynamics code for astrophysical applications.

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Abstract

We present a status report of the Ph.D. *Numerical Resistive Relativistic Magnetohydrodynamics: Applications to Relativistic Jets*, which is developed in the project *Computer Aided Modeling of Astrophysical Plasma*, CAMAP. The goal of the work is to probe the physics present in relativistic jets caused by accretion of plasma at hypercritical rates onto black holes.

Including resistive effects in relativistic magnetized plasmas is a challenging task, that a number of authors have recently tackled employing different methods ([4],[5]). From the numerical point of view, the difficulty in including non-ideal terms arises from the fact that, in the limit of very high plasma conductivity (i.e., close to the ideal MHD limit), the system of governing equations becomes stiff, and the standard explicit integrating methods produce instabilities that destroy the numerical solution. To deal with such a difficulty, we have extend the relativistic MHD (RMHD) code MR-GENESIS [1], to include a number of Implicit Explicit Runge-Kutta (IMEX-RK) numerical methods ([3], [6]).

To validate the implementation of the IMEX-RK schemes, several standard tests are presented in one and two spatial dimensions, covering different conductivity regimes.

EGLM Equations of RRMHD

$$
\begin{aligned}
\partial_t\psi &= -\nabla\cdot\mathbb{E} + q - \kappa\psi\\
\partial_t\phi &= -\nabla\cdot\mathbb{B} - \kappa\phi\\
\partial_t\mathbb{E} &= \nabla\times\mathbb{B} - \nabla\psi - \mathbb{J}\\
\partial_t\mathbb{B} &= -\nabla\times\mathbb{E} - \nabla\phi\\
\partial_t q &= -\nabla\cdot\mathbb{J}\\
\partial_t D &= -\nabla\cdot\mathbb{F}_D\\
\partial_t\tau &= -\nabla\cdot\mathbb{F}_\tau - \mathbb{B}\cdot\nabla\phi - \mathbb{E}\cdot\nabla\psi\\
\partial_t\mathbb{S} &= -\nabla\cdot\mathbf{F}_{\mathbb{S}} - (\nabla\cdot\mathbb{B})\mathbb{B}
\end{aligned}
$$

Stiff Term

Instead of combining the usual Maxwell equations with the fluid equations, we consider the extended field equations proposed by Dedner et al. [2]. In such equations the divergence constraint for the electromagnetic fields $\mathbb{E}, \mathbb{B}$ in the Maxwell equations has been coupled with the evolution equation for $\mathbb{E}$ and $\mathbb{B}$ by introducing new unknown functions ψ and ϕ .

References

[1] Aloy, M. A., et al, 1999, ApJS, 122, 151

[2] Dedner, A., et al, 2002, J. Comput. Phys., 175, 645

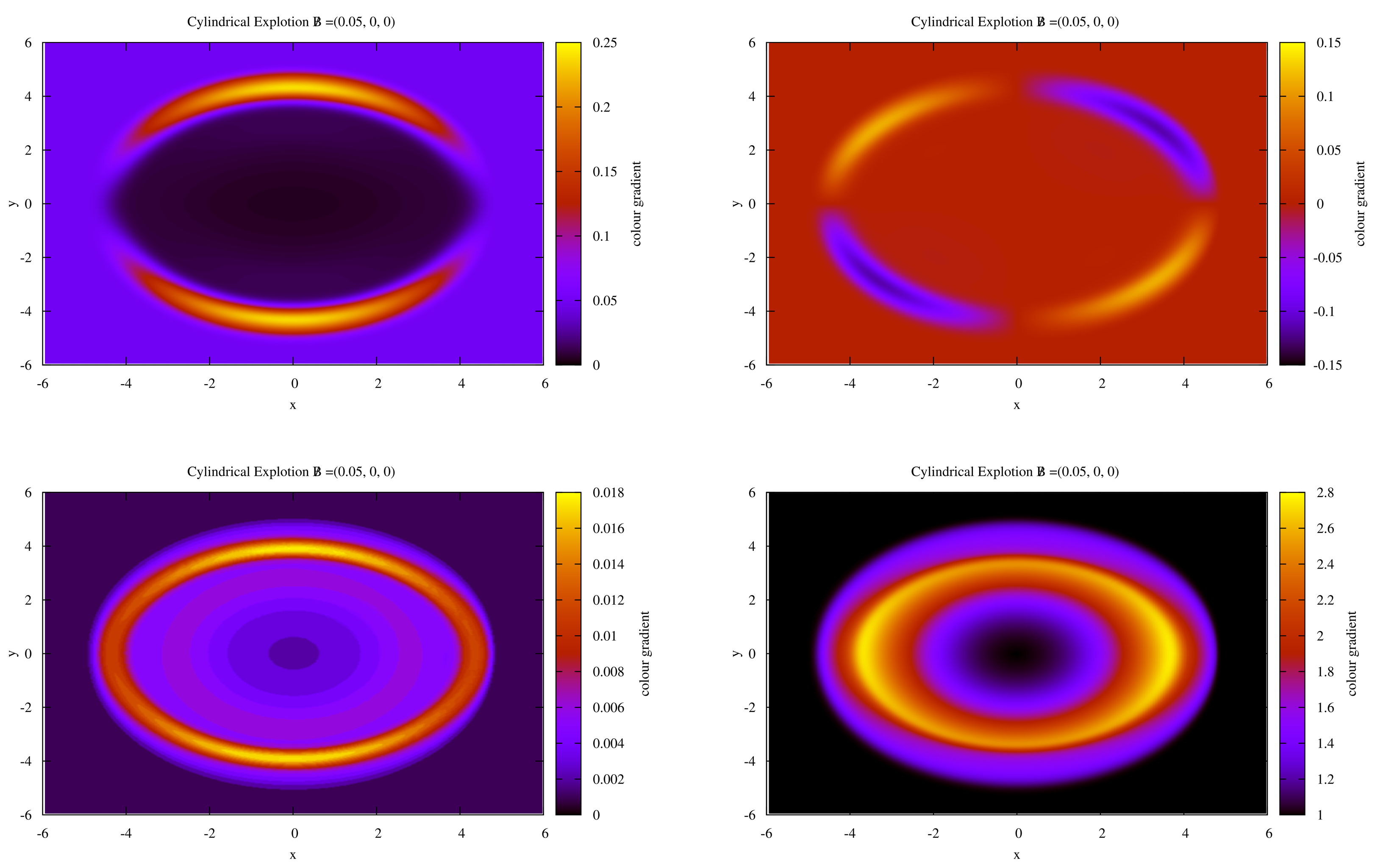
[3] Higueras I., et al, 2012, ASC Report No. 14/2012

[4] Komissarov, Serguei S., 2007, MNRAS, 382, 995

[5] Palenzuela, C., et al, 2009, MNRAS, 394, 1727

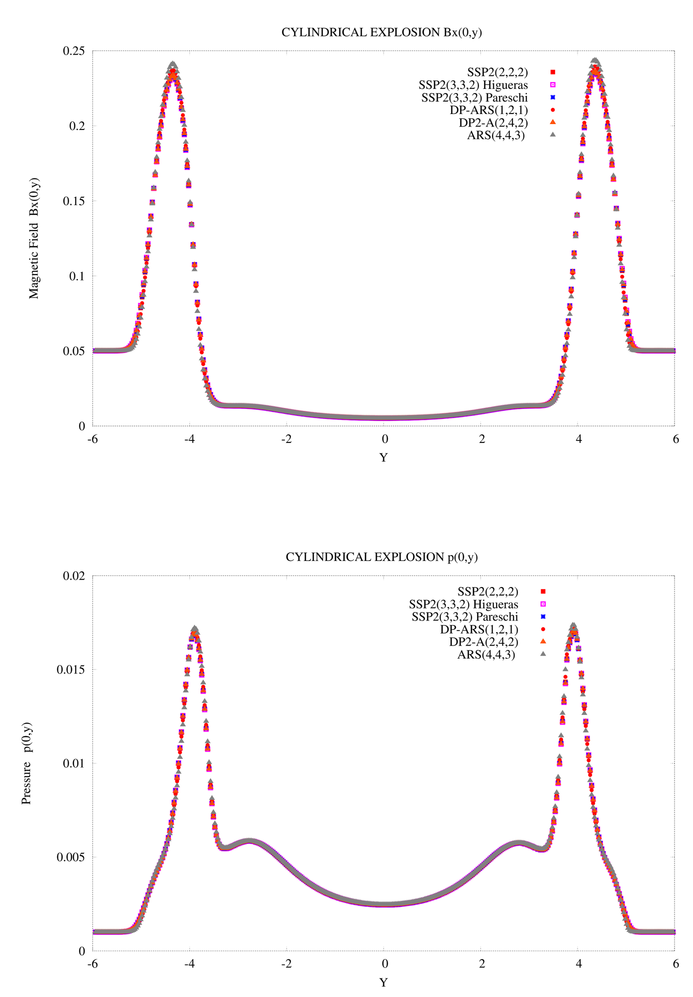
[6] Pareschi L., et al, 2005, J. Sci. Comput., 25, 112

Cylindrical Explosion Test



Top: Magnetic Field B_x (left) B_y (right).

Bottom: Pressure p (left) Lorentz factor W (right).



Top: Profile $B_x(0,y)$

Bottom: Profile $p(0,y)$

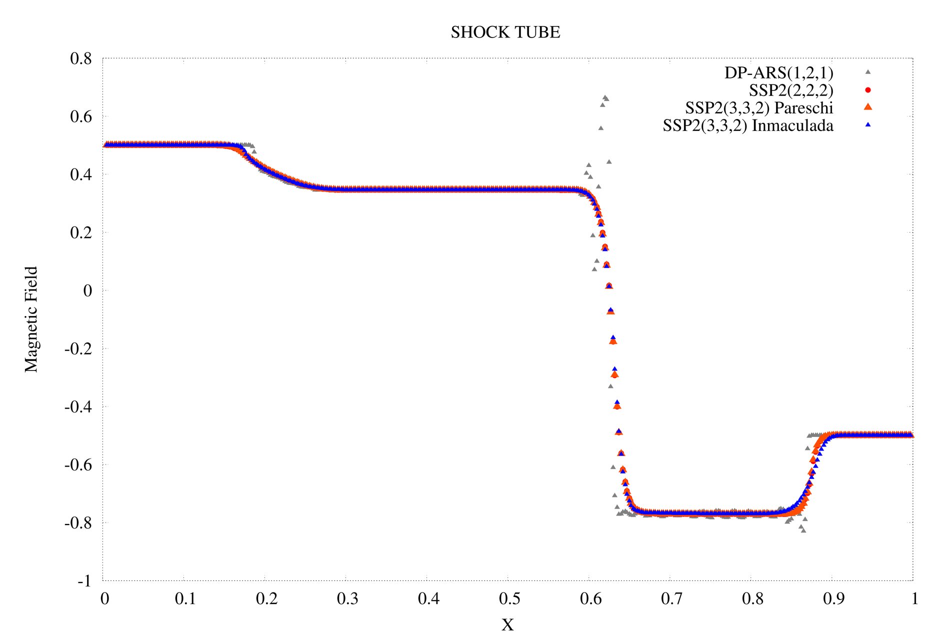
Initial Conditions: $B = (0.05, 0, 0)$, $\sigma = 1\times10^6$, resolution 400×400

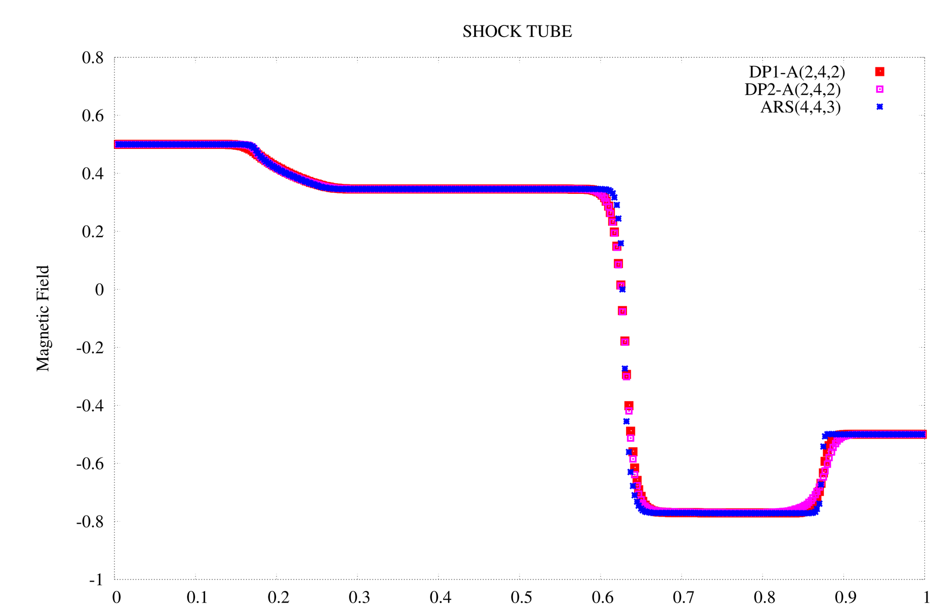
$$
\xrightarrow[\text{pressure}]{P}\begin{cases}1.0\\ 1.0 \quad e^{\alpha r}\\ 1\times10^{-3}\end{cases}
\qquad
\xrightarrow[\text{density}]{\rho}\begin{cases}1\times10^{-2} & \qquad r\le 0.8\\ 1\times10^{-2} \quad e^{\alpha r} & \qquad 0.8<r<0.8\\ 1\times10^{-3} & \qquad 1.0<r\end{cases}
$$

CFL Performance

Schemes	MinMod	MCL
SSP2(2,2,2)	0.3	NO
SSP2(3,3,2) [6]	0.4	NO
SSP2(3,3,2) [3]	0.4	0.3
DP-ARS(1,2,1)	0.3	NO
DP2-A(2,4,2)	0.4	NO
ARS(4,4,3)	0.1	NO

Shock Tube Problem Test





CFL Performance Shock Tube Test

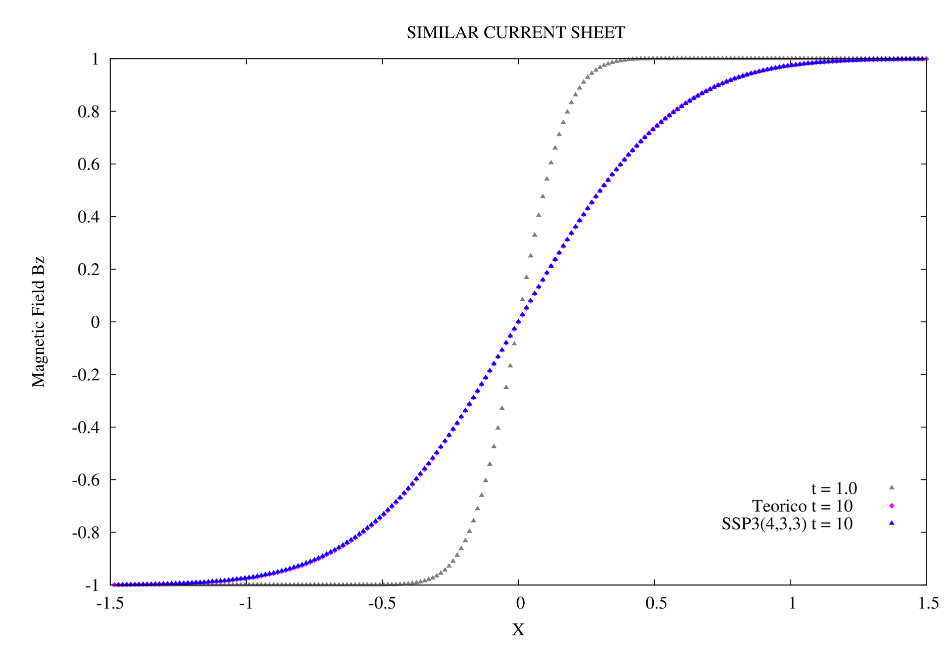
Schemes	sub-levels	MinMod	MCL
SSP2(2,2,2)	2	0.1	0.1
SSP2(3,3,2) [6]	3	0.2	0.1
SSP2(3,3,2) [3]	3	0.6	0.4
DP-ARS(1,2,1)	3	0.7	0.2
DP1-A(2,4,2)	4	0.1	NO
DP2-A(2,4,2)	4	0.5	0.4
ARS(4,4,3)	5	0.3	0.1

The initial left and right states are separated at $x = 0.5$ and are given by:

$$(\rho^L, p^L, B_y^L) = (1.0, 1.0, 0.5)$$
$$(\rho^R, p^R, B_y^R) = (0.125, 0.1, -0.5)$$

while all the other fields set to 0, the conductivity $\sigma = 10^6$ and resolution $\mathtt{nx} = 400$.

Similar Current Sheet



CFL Performance Similar Current Sheet

Schemes	sub-levels	MinMod	MCL
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SSP2(3,3,2) [6]	3	0.6	0.4
SSP2(3,3,2) [3]	3	0.6	0.4
DP-ARS(1,2,1)	3	0.7	0.6
DP1-A(2,4,2)	4	0.5	0.4
DP2-A(2,4,2)	4	0.6	0.4
ARS(4,4,3)	5	0.5	0.5

The initial magnetic field is given by the error function, $\mathrm{B} = \mathtt{erf}(\frac{1}{2}\sqrt{\frac{\sigma x^2}{t}})$, for $t = 1$, the conductivity $\sigma = 10^2$, the pressure and density by $p = 50$, $\rho = 1$, while all the other fields set to 0, with resolution $\mathtt{nx} = 400$.

Outlook

We will use the MR-GENESIS code, to study outflows associated to GRBs and explore the models aimed to explain the emission mechanisms of ultrarelativistic magnetized plasma and the dissipation of the magnetic energy of the flow due to resistivity, with particular interest in the processes of magnetic reconnection.