Numerical study of Cosmic ray transport in compressible turbulence

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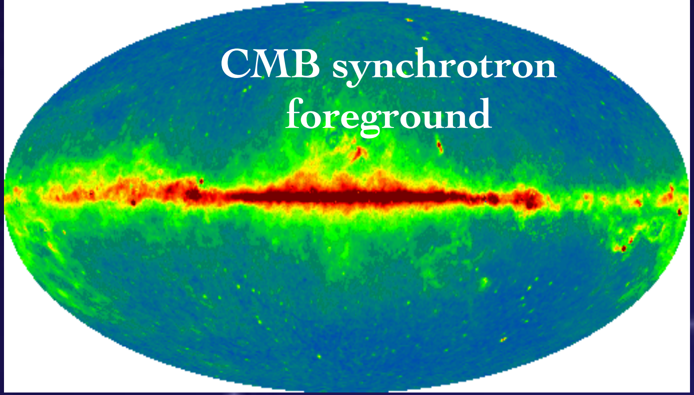
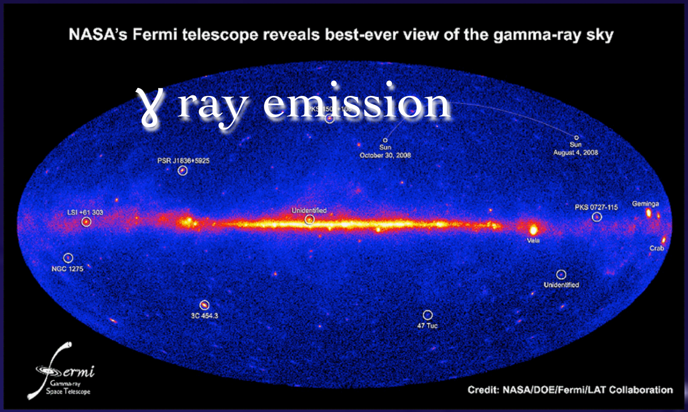
Importance: Cosmic Ray (CR) Propagation

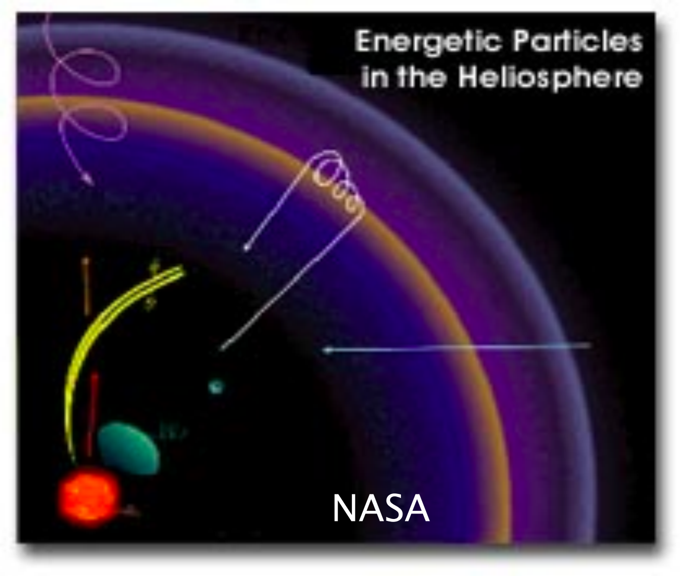


CMB synchrotron foreground

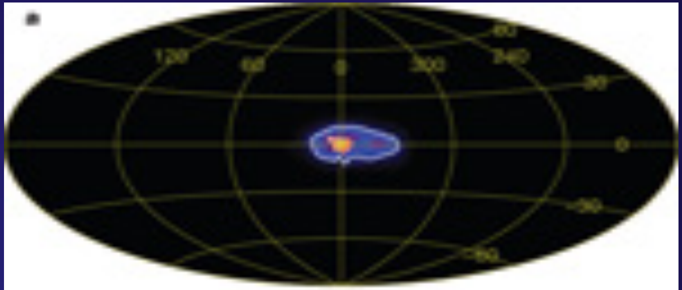


γ ray emission

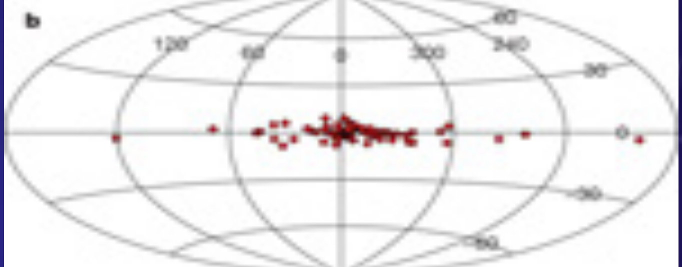
Credit: NASA/DOE/Fermi/LAT Collaboration



NASA



Diffuse Galactic 511 keV radiation

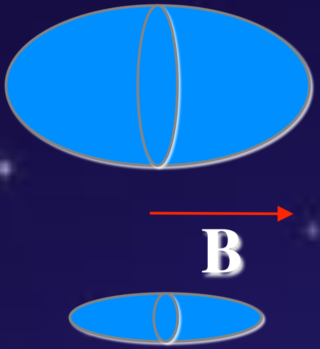


LONGSTANDING PROBLEMS OF COSMIC RAY RESEARCH

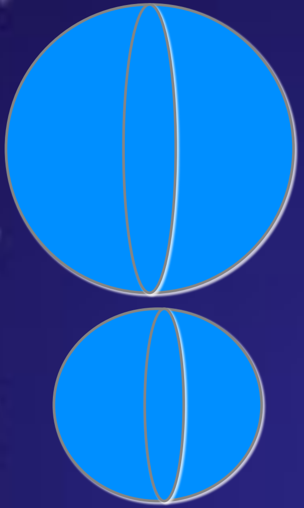
- Ad hoc turbulence models
- Inadequate description of the interactions between MHD perturbations and particles, e.g. 90 degree problem.
- Perpendicular Cosmic Ray (CR) transport

Numerically tested models for MHD turbulence

Alfven and slow modes (GS95)

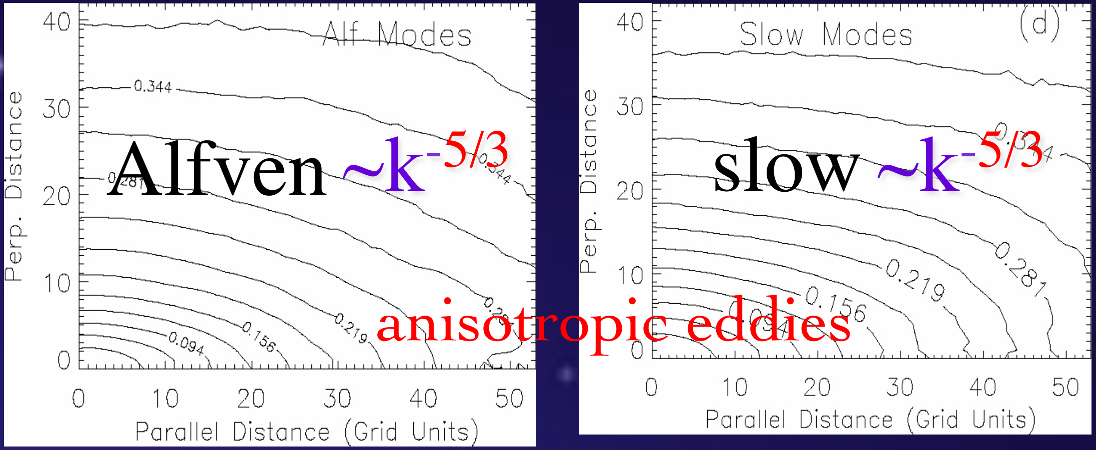


fast modes



Equal velocity correlation contour

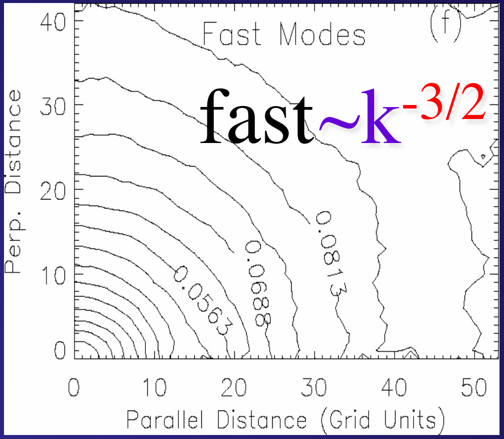
(Cho & Lazarian 02, Kowal & Lazarian 2010)



Alfven $\sim k^{-5/3}$

slow $\sim k^{-5/3}$

anisotropic eddies



fast $\sim k^{-3/2}$

Quasilinear theory is not adequate

- Long standing problem: 90 degree scattering

$K_{res} = \Omega/v_{||} \rightarrow \infty$, the scale is below the dissipation scale of turbulence → No scattering at 90°? → $\lambda_{||} \rightarrow \infty$?!

A key assumption in Quasilinear theory:

guiding center is unperturbed $Z_0 = v\mu t$;

→

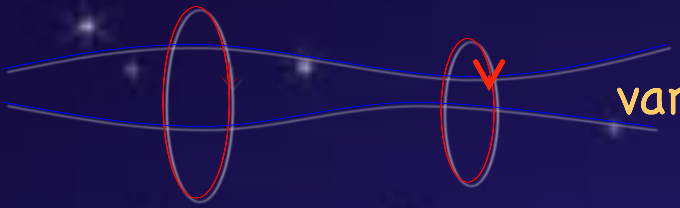
Nonlinear theory:

In reality, the guiding center is perturbed, especially on large scales,

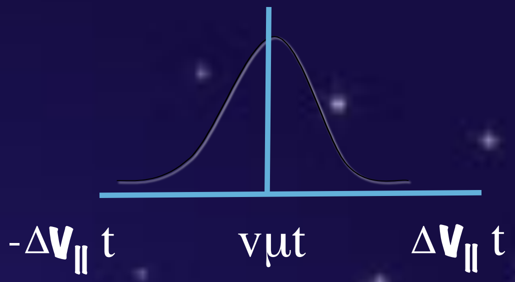
$z = (v\mu \pm \Delta v_{||})t.$

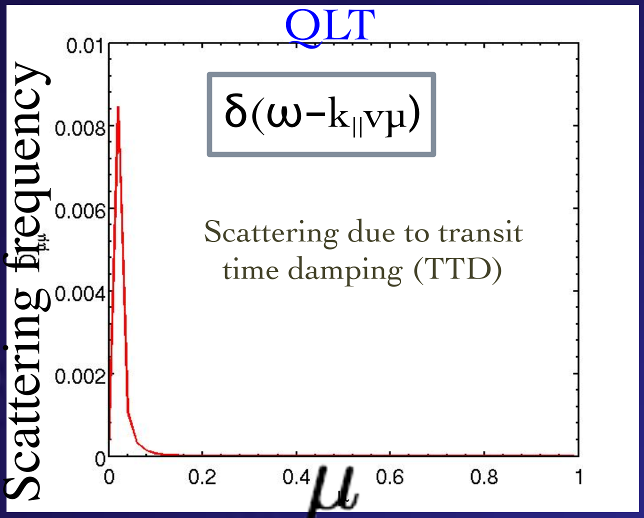
Nonlinear broadening of resonance solves the 90° problem!

- On large scale, unperturbed orbit assumption in QLT fails due to conservation of adiabatic invariant $v_\perp^2/B$ (Volk 75).



varying $v_\perp$ → varying $v_{||}$



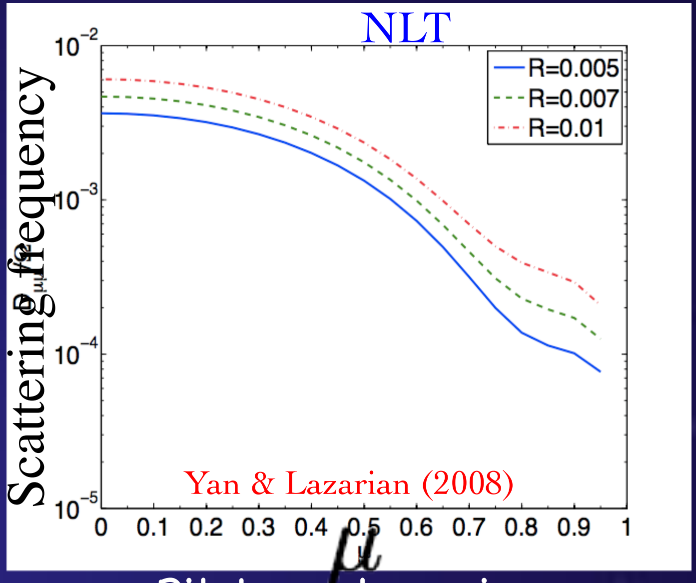


$\delta(\omega - k_{||} v\mu)$

Scattering due to transit time damping (TTD)

Pitch angle cosine

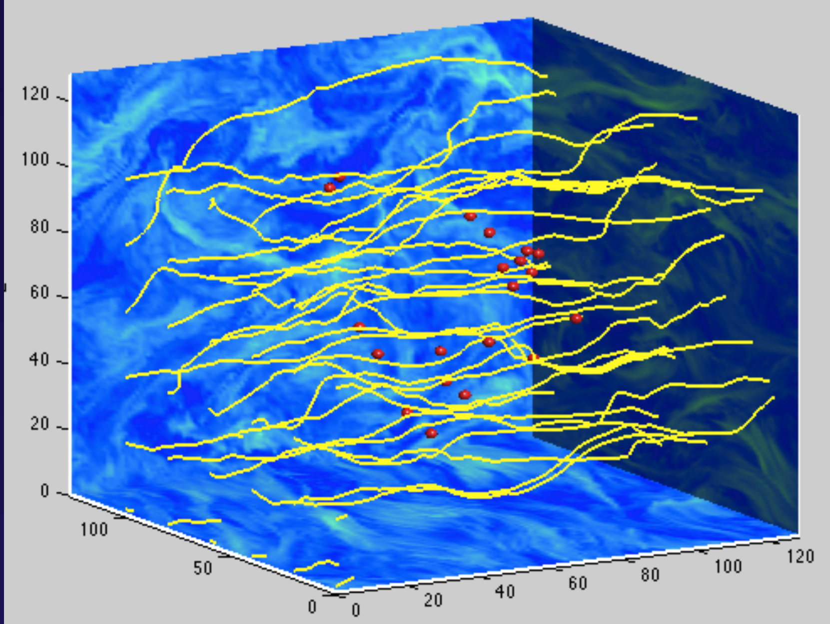
Broadened resonance →



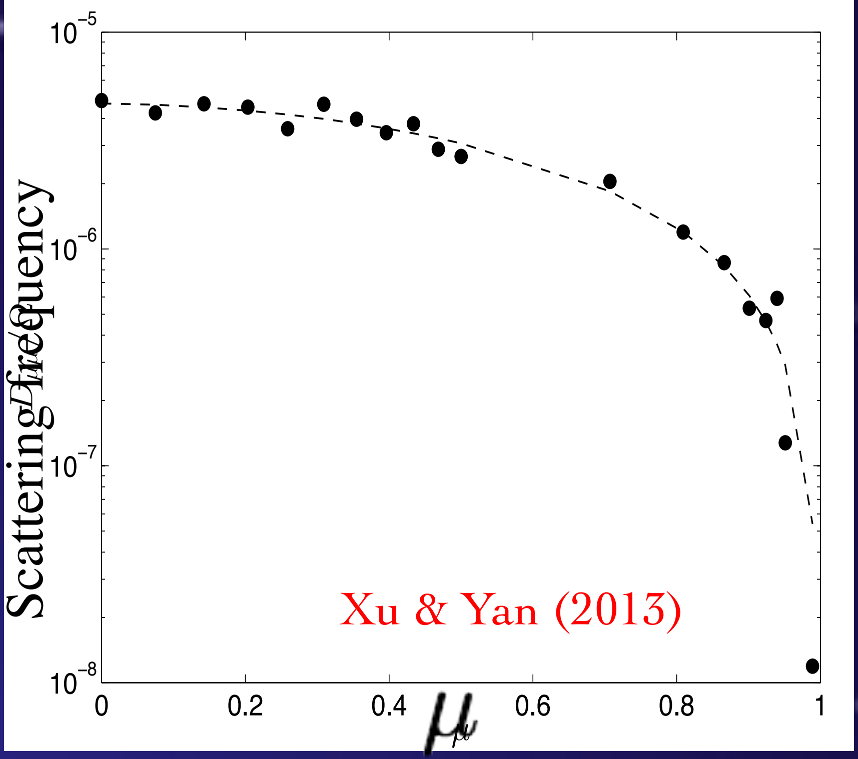
Yan & Lazarian (2008)

Pitch angle cosine

Comparison w. test particle simulation

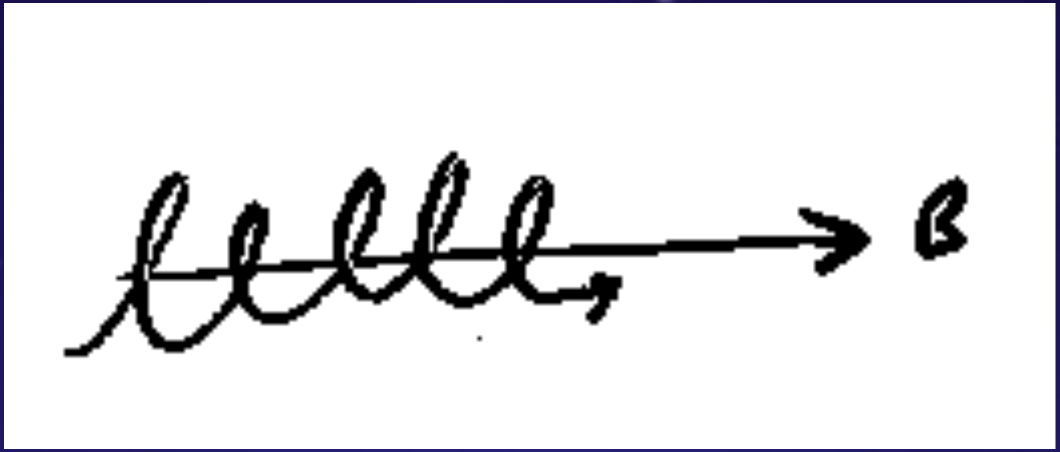


● Particle trajectory
— Magnetic field



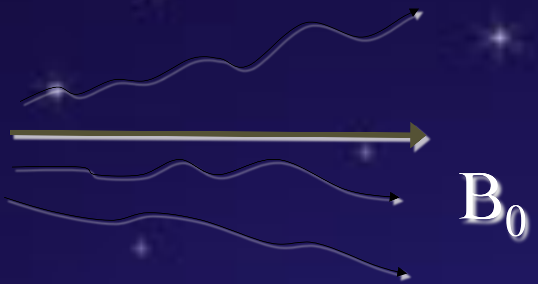
Transit time damping (resonant mirror) dominates scattering of most pitch angles *except* for small ones.

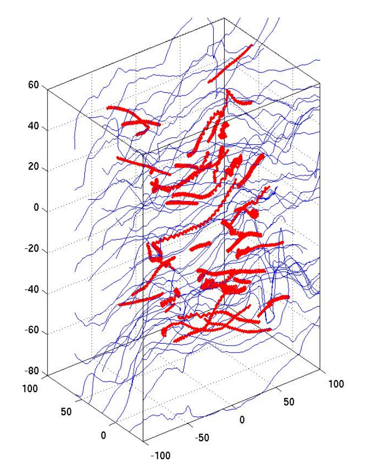
CROSS FIELD TRANSPORT IS IMPOSSIBLE WITHOUT δ B



Perpendicular transport

- Dominated by field line wandering.





Test particle simulations with realistic turbulence

– Particle trajectory

— Magnetic field

Intensive studies:

e.g., Jokipii & Parker 1969, Forman 74, Urch 77, Bieber & Matthaeus 97, Giacolone & Jokipii 99, Matthaeus et al 03

Is there subdiffusion ($\Delta x^2 \propto \Delta t^a$, $a<1$) ?

- Subdiffusion (or compound diffusion, Getmantsev 62, Lingenfelter et al 71, Fisk et al. 73, Webb et al 06) was observed in near-slab turbulence, which can occur on small scales due to instability.

$$\begin{array}{l} \Delta x^2 \propto \Delta z \\ \Delta z^2 \propto D_{||} \Delta t \end{array} \Rightarrow \boxed{\Delta x^2 \propto \sqrt{\Delta t}}$$

Diffusion is slow if particles retrace their trajectories.

What if we use the tested model of turbulence?

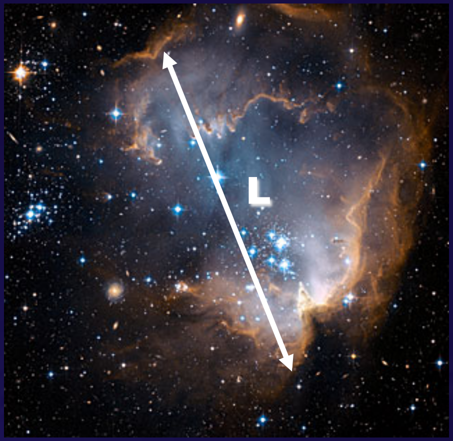
Subdiffusion is not typical!

- In turbulence, particles' trajectory become independent when field lines are separated by the smallest eddy size , $l_{\perp,min}$.

The separation between field lines has a Lyapanov type growth, provides Rechester-Rosembluth distance, $L_{RR} = l_{\|,min} \log(l_{\perp,min} / r_L)$ (Narayan & Medvedev 01, Lazarian 06)

- Subdiffusion only occurs below $l_{\perp,min}$. Beyond $l_{\perp,min}$, normal diffusion applies (Yan & Lazarian 2008).

Prediction for perpendicular transport ($\lambda_{||} > L$)



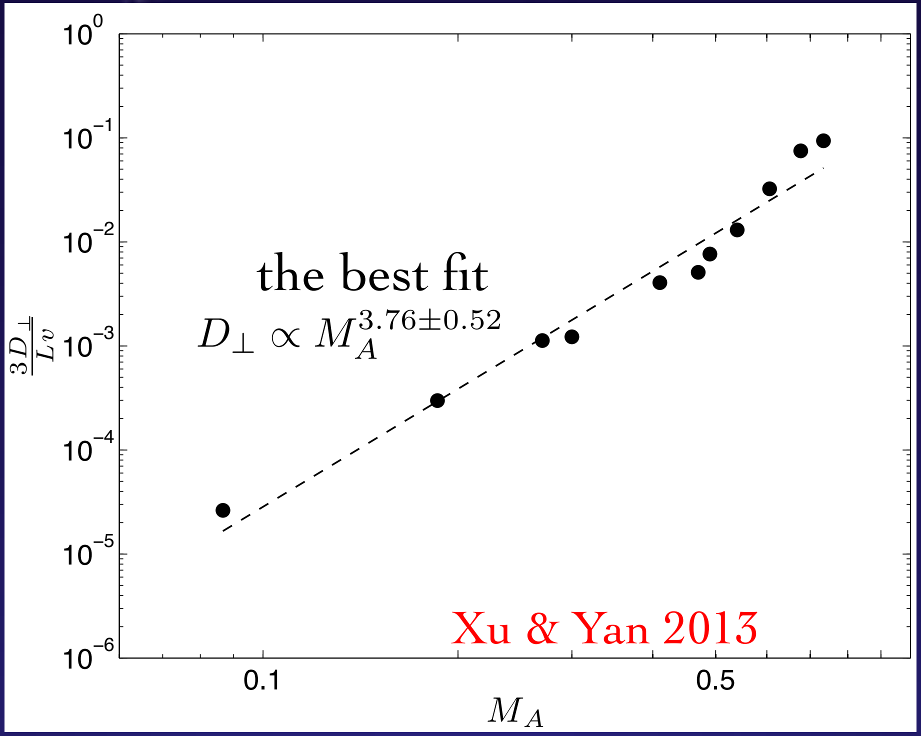
- $M_A < 1$, CRs free stream over distance L, thus $\Delta t = (R/L\, M_A^2)^2\, L/v_{||}$,

$D_\perp = R^2/\Delta t = 1/3 Lv\, M_A^4$ (Yan & Lazarian 2008)

(differs from the M_A^2 dependence in literature)

Perpendicular diffusion depends on $M_A \equiv \delta B/B_0$.

Numerical result for perpendicular diffusion ($\lambda_{\|} > L$)

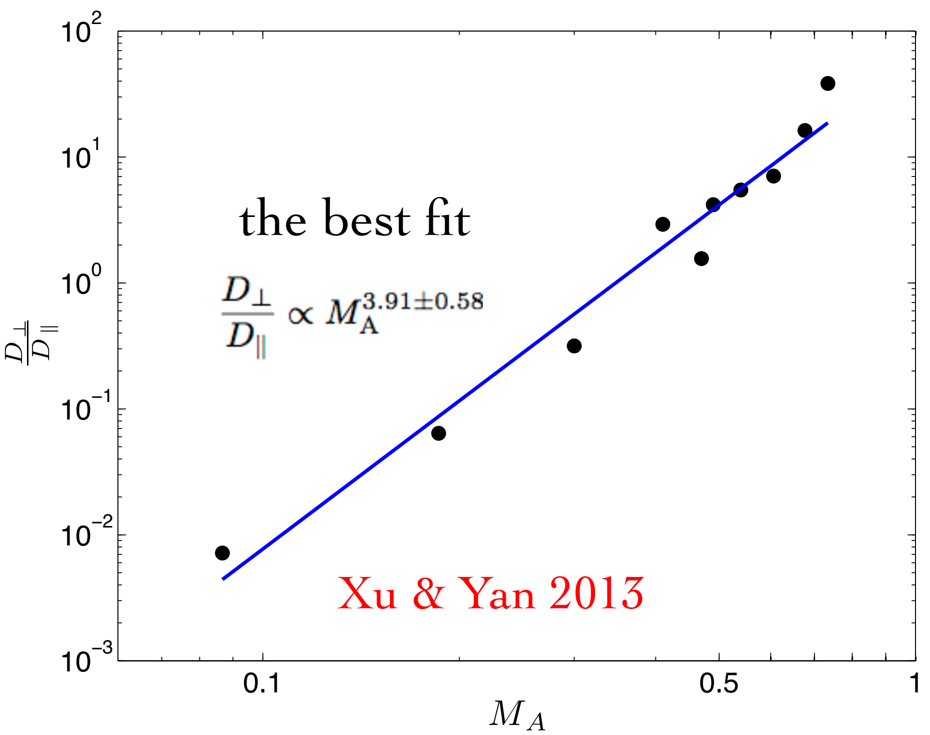


Cross field transport in 3D turbulence is in general a normal diffusion, which has a M_A^4 dependence!

Perpendicular diffusion ($\lambda_{\|} < L$)

- Prediction: $M_A < 1$, $D_\perp = D_{\|} M_A^4$ Yan & Lazarian (2008)
- Numerical result:

M_A^4 suppression compared to $D_{\|}$ is confirmed!

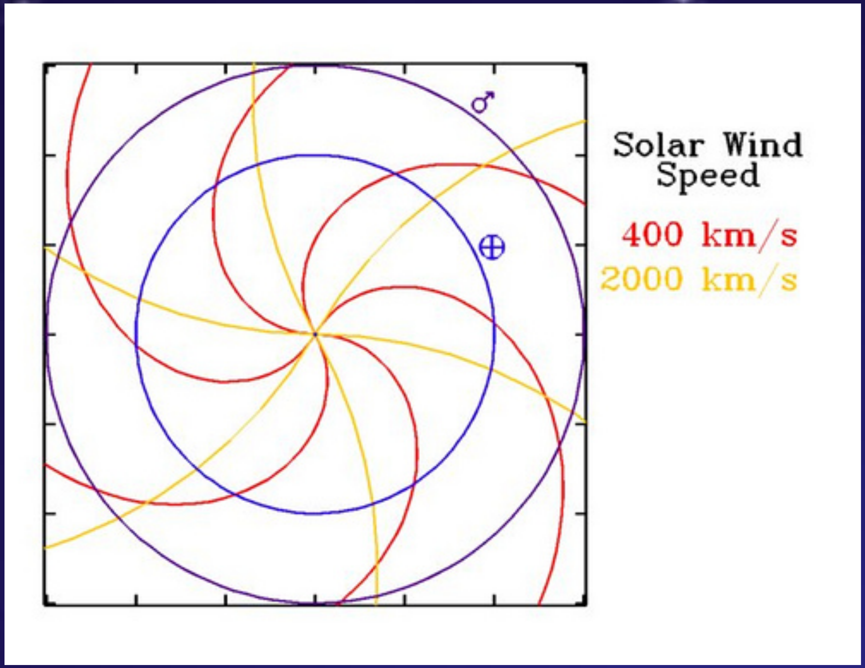


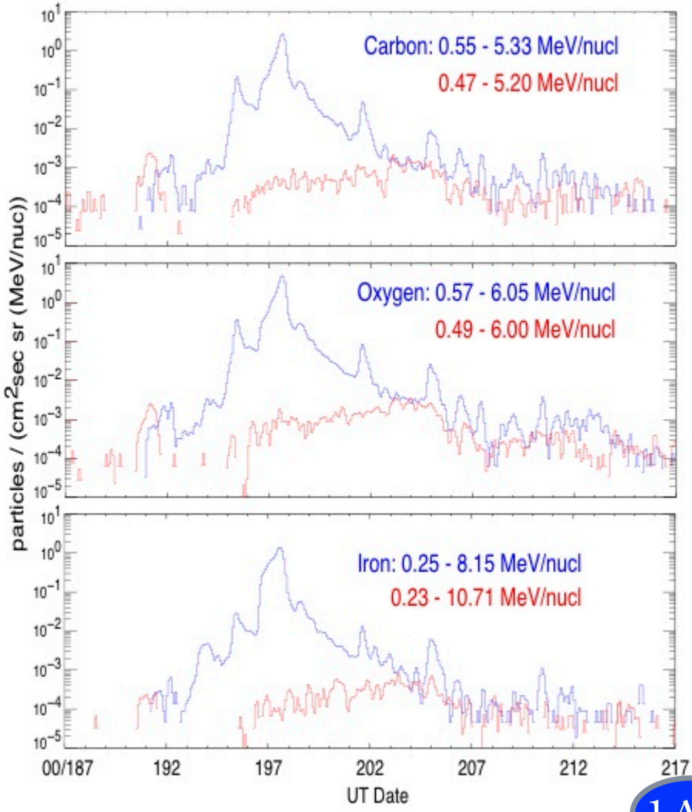
the best fit

$$\frac{D_\perp}{D_\|} \propto M_A^{3.91\pm0.58}$$

Xu & Yan 2013

CROSS FIELD TRANSPORT IN SOLAR WIND IS FAST!





1Au

Fig. 2. Heavy ion C, O, and Fe fluxes measured on both ACE (blue) and Ulysses (red) in the July 2000 event.

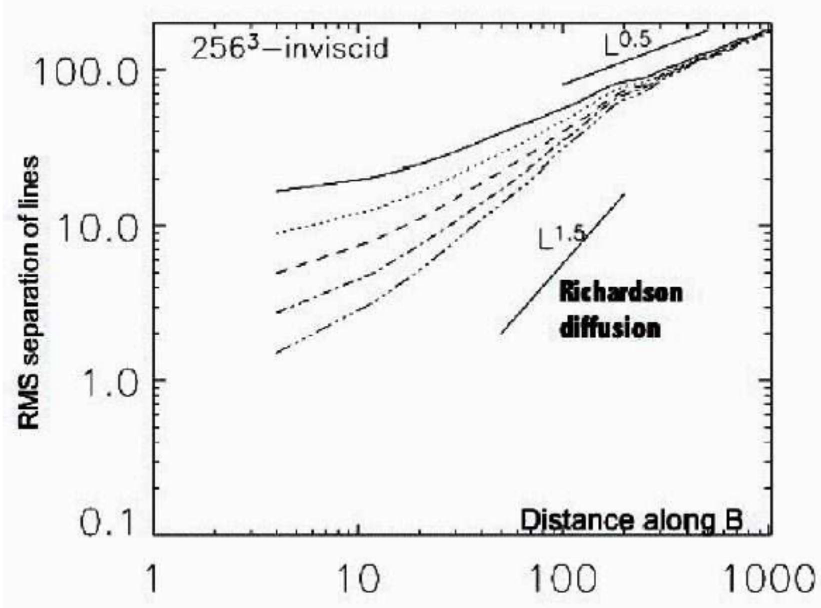
from Maclennan et al. (2001)

3.2AU

Field lines are superdiffusive on small scales

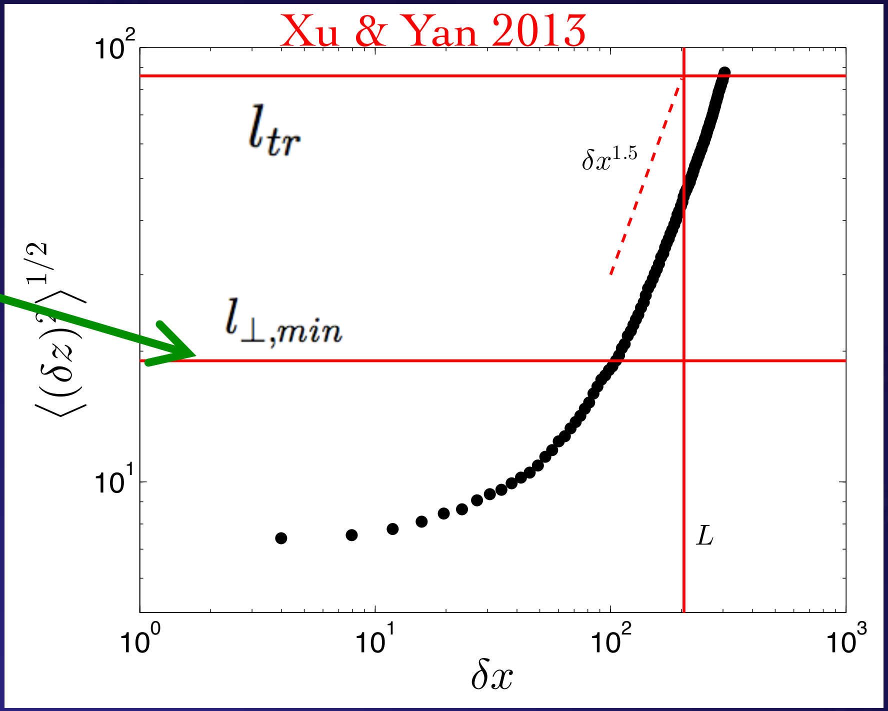
Richardson's Law

$$\langle|\mathbf{x}_1(t)-\mathbf{x}_2(t)|^2\rangle \sim t^3.$$



Lazarian, Cho & Vishniac (2004)

SUPERDIFFUSION OF CRS IS OBSERVED



Consistent with earlier theoretical predictions (Narayan & Medvedev 2001, Lazarian 2006 for thermal particles; Yan & Lazarian 2008 for CRs)

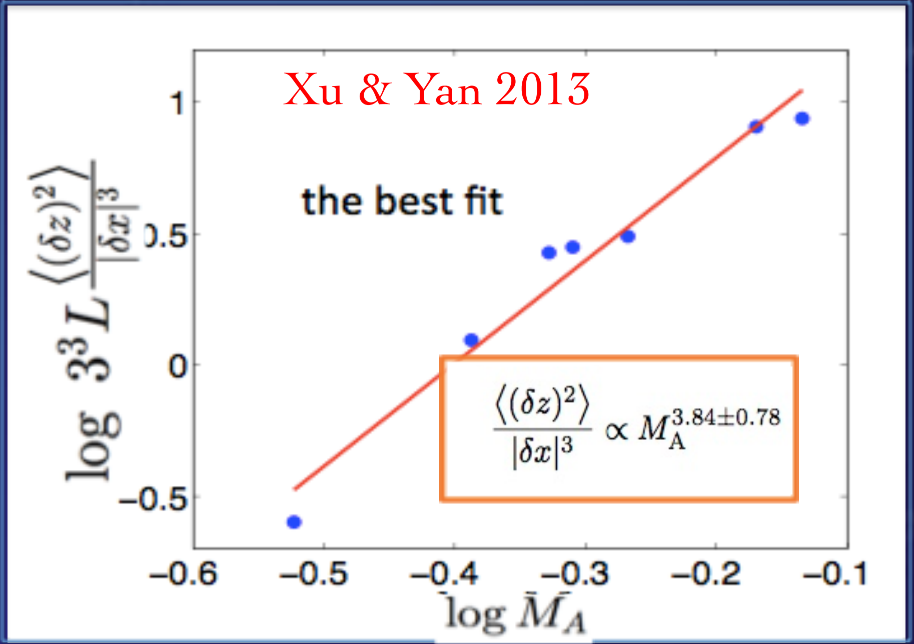
SUPERDIFFUSION HAS M_A^4 DEPENDENCE

- Theoretical prediction

  Lazarian & Vishniac 1999;
  Yan & Lazarian 2008

$$< (\delta z)^2 >= \frac{|\delta x|^3}{3^3 L} M_A^4$$

- Numerical result



Summary

- Changes in the MHD turbulence paradigm necessitates revision of particle's transport theories.
- CR scattering is dominated by broadened TTD (resonant mirror) interaction for most pitch angles but small pitch angles, *including 90 degree.*
- Subdiffusion does not apply.
- On large scales, CR perpendicular diffusion is suppressed by M_A^4 compared to parallel diffusion.
- On small scales, CR transport is *super-diffusive*, has a dependence of M_A^4 in sub-Alfvenic turbulence.
- Implications are wide, from thermal conduction in turbulent medium to turbulent reconnection.