MHD modeling of the kink “double-gradient” branch of the ballooning instability in the magnetotail

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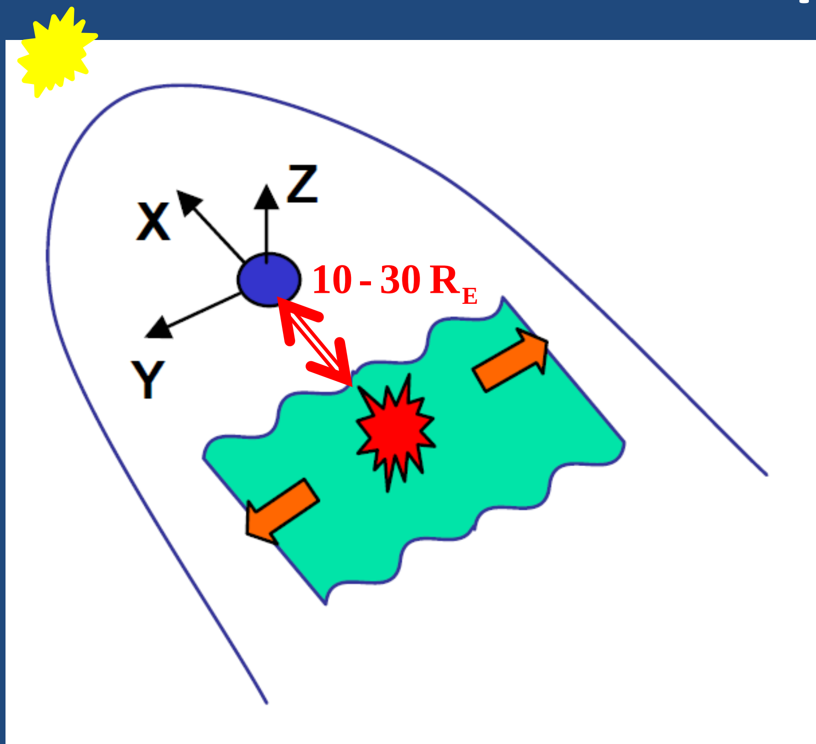
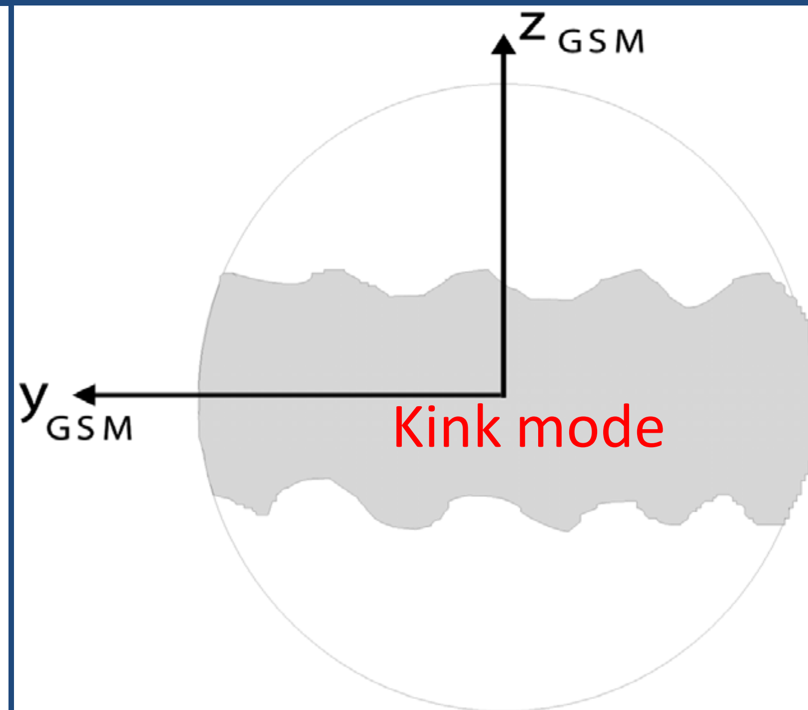
Astronum-2013

Introduction: Flapping oscillations



Sergeev et al. (2006), Ann. Geophys., 24, 2015–2024.

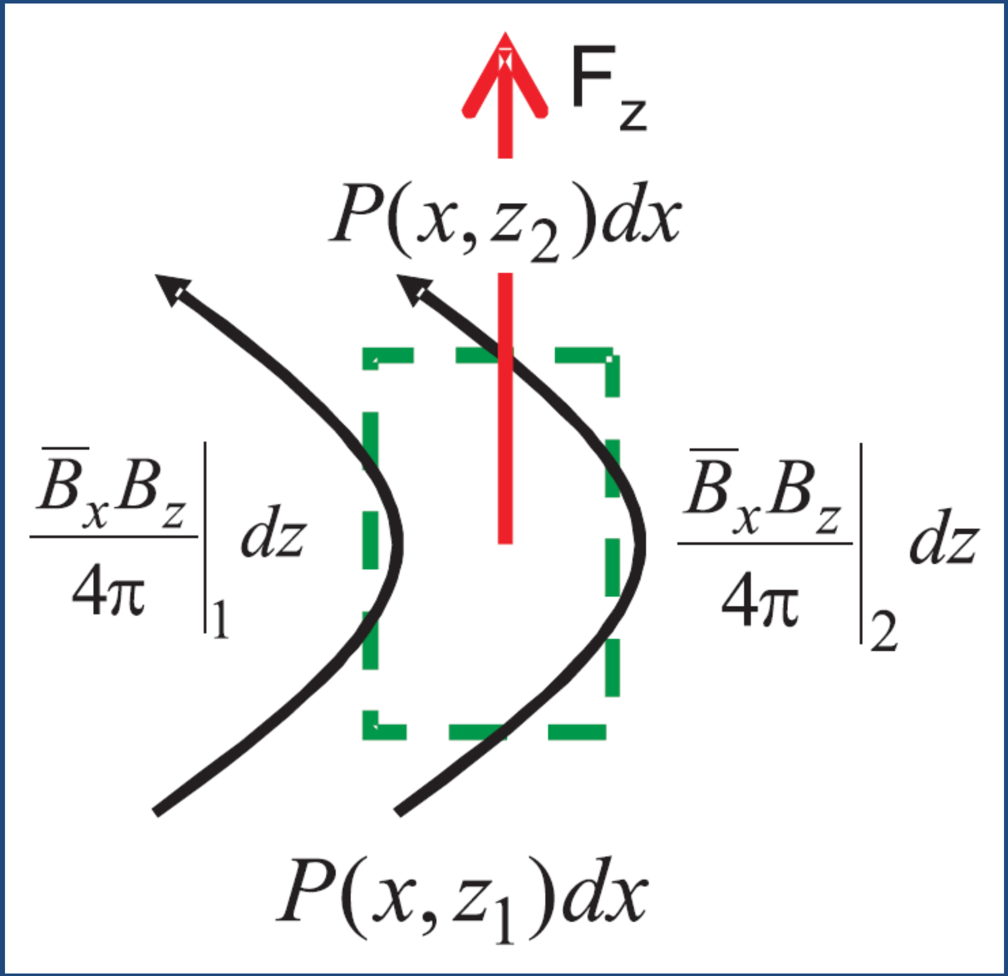


Golovchanskaya et al. (2006), J. Geophys. Res., 111, A11216.

$$T = 100 - 200\ s, \qquad V_g = 30 - 70\ km/s, \qquad \lambda = 2 - 5\ \mathrm{R}_E$$

Sergeev et al. (2003), Geophys. Res. Lett. ***30, 1327;*** *Runov et al. (2005), Ann. Geophys.* ***23, 1391;*** *Petrukovich et al. (2006), Ann. Geophys.* ***24, 1695.***

Introduction: Equilibrium



A plasma element at the center of the current sheet (CS)

In equilibrium state

$$\frac{\partial P}{\partial z} = \frac{1}{4\pi} B_x \frac{\partial B_z}{\partial x}$$

Displacement along the Z axis yields the restoring force

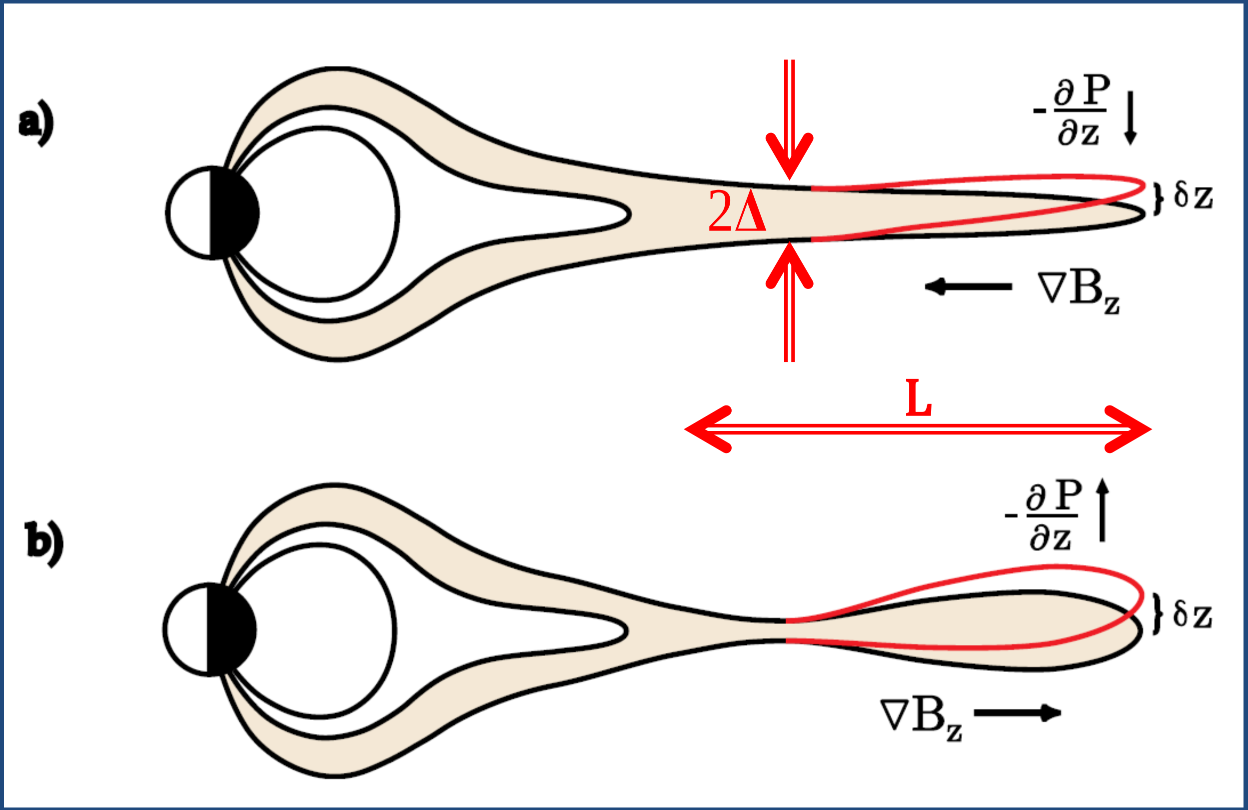
$$F_z = -\frac{1}{4\pi}\delta z \left(\frac{\partial B_x}{\partial z}\frac{\partial B_z}{\partial x} \right)_{z=0}$$

Equation of motion of the plasma element

$$\frac{\partial^2 \delta z}{\partial t^2} = -\omega_f^2 \delta z,$$

$$\boxed{\omega_f^2 = \left\langle \frac{1}{4\pi\rho}\frac{\partial B_x}{\partial z}\frac{\partial B_z}{\partial x} \right\rangle_{z=0}}$$

Introduction: (in)stability



$$\omega_f^2 > 0$$

Minimum of the total pressure in the center of the CS,
Stable situation,
Oscillations

$$\omega_f^2 < 0$$

Maximum of the total pressure in the center of the CS,
Unstable situation,
Wave growth

Features of the configuration:

$$\nu = \Delta / L << 1$$

$$\nu \sim 0.1$$

$$\varepsilon = \left(\frac{\partial B_z}{\partial x} \bigg/ \frac{\partial B_x}{\partial z} \right)_{z=0} << 1$$

$$\varepsilon \sim 0.01$$

$$\varepsilon << \nu$$

Introduction: Analytical solution of Erkaev et al., Ann. Geophys., 27, 417, 2009

System of ideal MHD equations

$$\rho \frac{d\mathbf{V}}{dt} + \nabla P = \frac{1}{4\pi\rho}(\mathbf{B}\cdot\nabla)\mathbf{B},$$

$$\frac{d\mathbf{B}}{dt} = (\mathbf{B}\cdot\nabla)\mathbf{V}, \qquad \frac{d\rho}{dt} = 0,$$

$$\nabla\cdot\mathbf{V} = 0, \qquad \nabla\cdot\mathbf{B} = 0.$$

Normalization

$$B^*, \quad \rho^*, \quad \Delta\&\mathrm{L}, \quad P^* = \frac{B^{*2}}{4\pi},$$

$$V_A = \frac{B^*}{\sqrt{4\pi\rho^*}}, \qquad t^* = V_A/\Delta$$

Simplifying assumptions

- incompressibility
- $\mathrm{B} = [\ \mathrm{B}_x(z),\ 0,\ \mathrm{B}_z(x)\ ]$
- perturbations are slow waves propagating in Y direction
- perturbations depend on Y and Z coordinates only, not on X

Introduction: Analytical solution

- Linearize the ideal MHD system
- Neglect small terms $\sim \varepsilon^2, \varepsilon\nu^2$

- background configuration
$$B_x = \tanh(z), \quad B_z = a + bx$$

- Substitute Fourier harmonics of perturbations $\sim \exp[i(\omega t - ky)]$

- Derive a second order ordinary differential equation for the amplitude of υ_z perturbation: $d^2 v_z / dz^2 + k^2 v_z \left(\omega_f^2 / \omega^2 - 1 \right) = 0$

- Obtain two modes of solution for $\omega(k)$

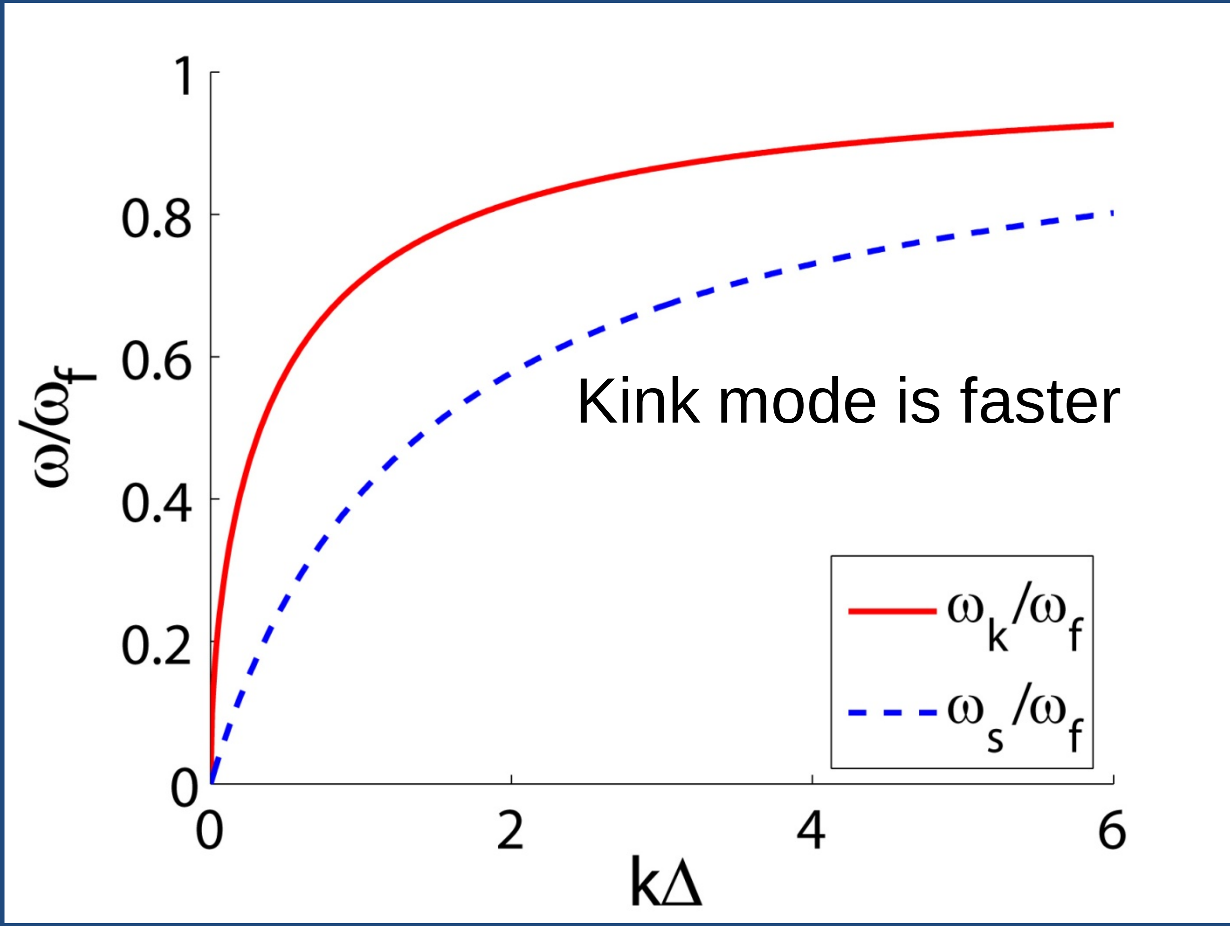
Even function $\upsilon_z(z)$ –
kink-like mode of the solution

$$\omega_k = \omega_f \sqrt{\frac{k\Delta}{k\Delta + 1}}$$

Odd function $\upsilon_z(z)$ –
sausage-like mode of the solution

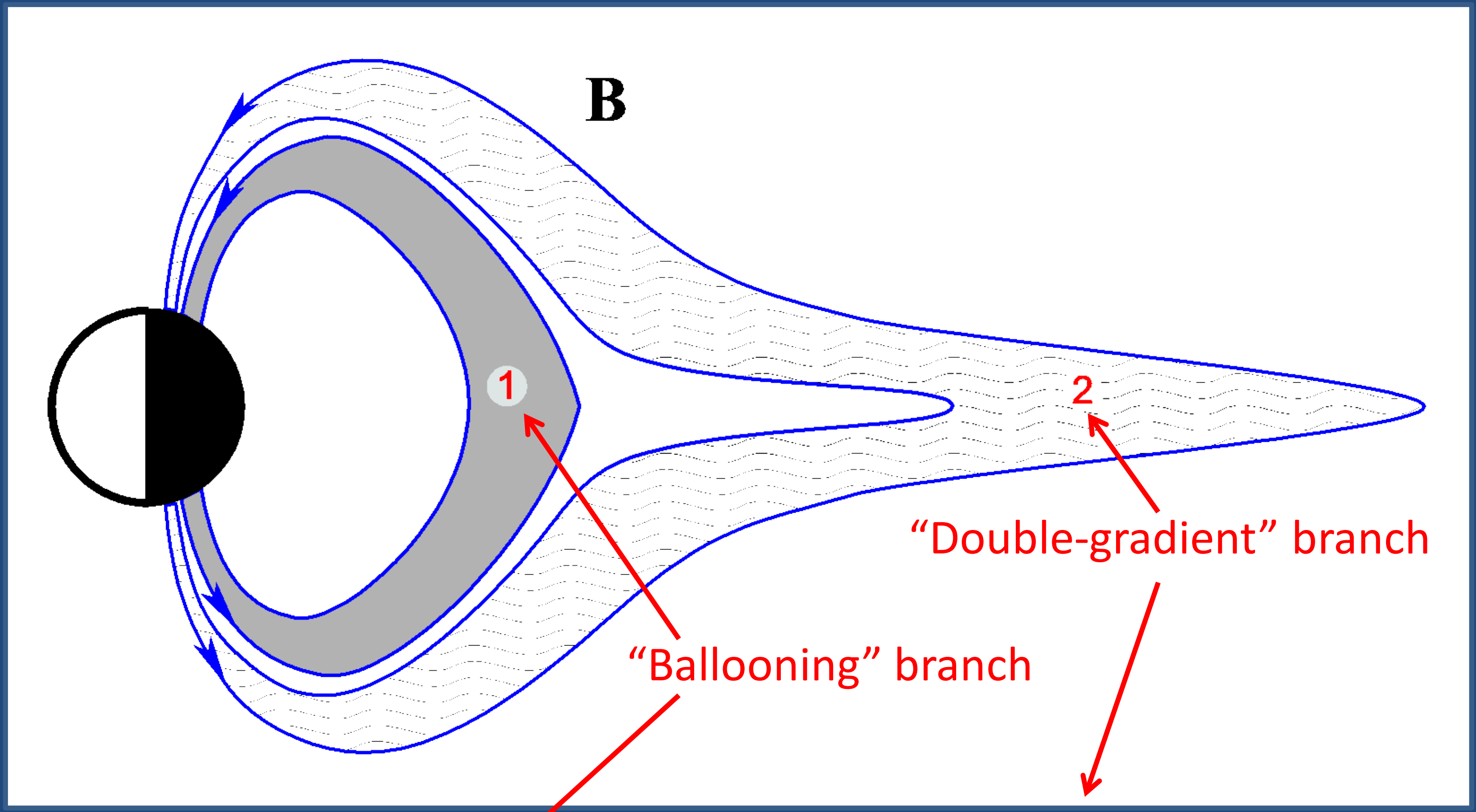
$$\omega_s = \frac{\omega_f k\Delta}{\sqrt{(k\Delta)^2 + 3k\Delta + 2}}$$

Introduction: Analytical solution



Dispersion curves of the double-gradient oscillations ($\mathrm{Im}[\omega] = 0$) / instability ($\mathrm{Re}[\omega] = 0$).

Two different magnetic configurations



1. $R_c \sim L >> \lambda$

2. $R_c < \lambda < L$

Curvature Radius

Wave Length

System Size

The “Ballooning” instability

Qualitatively: When plasma pressure decreases too sharply on R, plasma becomes unstable to the “ballooning” mode, which represents a locally swelling blobs. Some analogue to the Rayleigh-Taylor instability, where the curvature of the magnetic field replaces the gravitational force.

Mathematically: Consider a system of coupled equations for poloidal Alfvenic and SMS modes in a curvilinear magnetic field.

Result: The analytical dispersion relation for the small-scale, oblique-propagating ($\mathbf{k}\cdot\mathbf{B}\neq 0$) disturbances.

For our particular case:
$\boldsymbol{B} = (B_x, 0, B_z)$
$\boldsymbol{k} = (0, k_y, 0)$

The limiting $(k_y \to \infty)$ value of the “ballooning” growth rate:

$$\omega_b^2 = \frac{-2V_A^2\beta}{2+\kappa\beta}\frac{1}{R_c}\left[\frac{2+\kappa\beta}{2}\frac{\partial \ln(\mathrm{p})}{\partial x} - \frac{2\kappa}{R_c}\right]$$

κ – Polytropic index, β – Plasma parameter,
p – Plasma pressure, V_A – Alfvenic velocity.

DG and Ballooning growth rates

BALLOONING BRANCH

Mazur et al. (2012) [*Geomagnetism and Aeronomy, 52, 603–612*]

$$k \to \infty$$

$$\omega_b^2 = \frac{-2V_A^2\beta}{2+\kappa\beta}\frac{1}{R_c}\left[\frac{2+\kappa\beta}{2}\frac{\partial\ln(\mathrm{p})}{\partial x} - \frac{2\kappa}{R_c}\right]$$

DOUBLE-GRADIENT BRANCH

Erkaev et al. (2007) [Phys. Rev. Lett., 99, 235003]

$$k \sim 2\pi/L - 2\pi/R_c$$

$$\omega_f^2 = \frac{1}{4\pi\rho}\frac{\partial B_x}{\partial z}\frac{\partial B_z}{\partial x}$$

The unstable ballooning branch exists when $\omega_b^2 < 0$.
For equilibrium state this condition requires:

$$\beta \le \frac{2}{\kappa}\frac{1-\varepsilon}{1+\varepsilon},$$

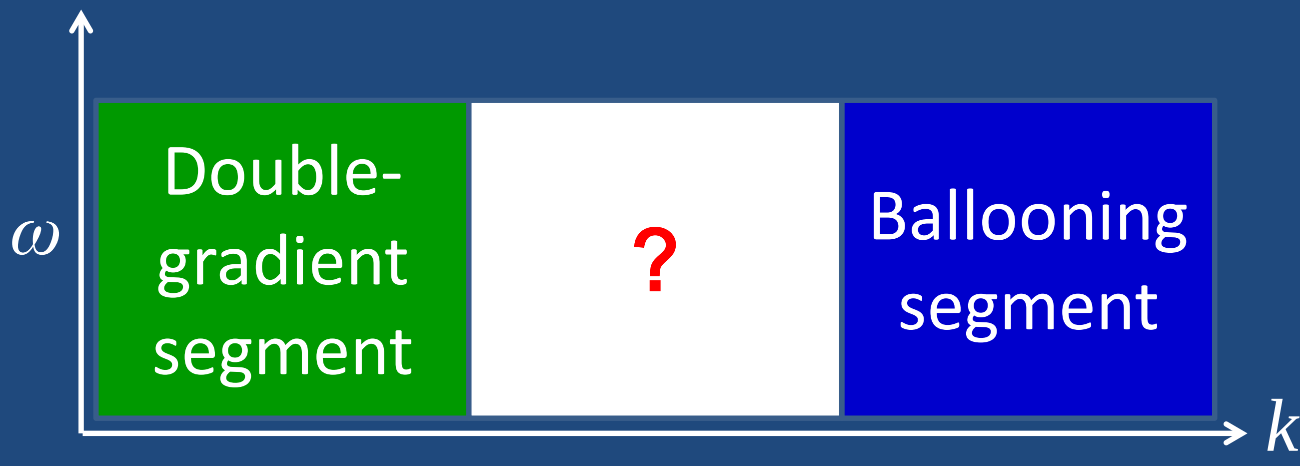
$$\varepsilon = \frac{\partial B_z}{\partial x}\Big/\frac{\partial B_x}{\partial z}.$$

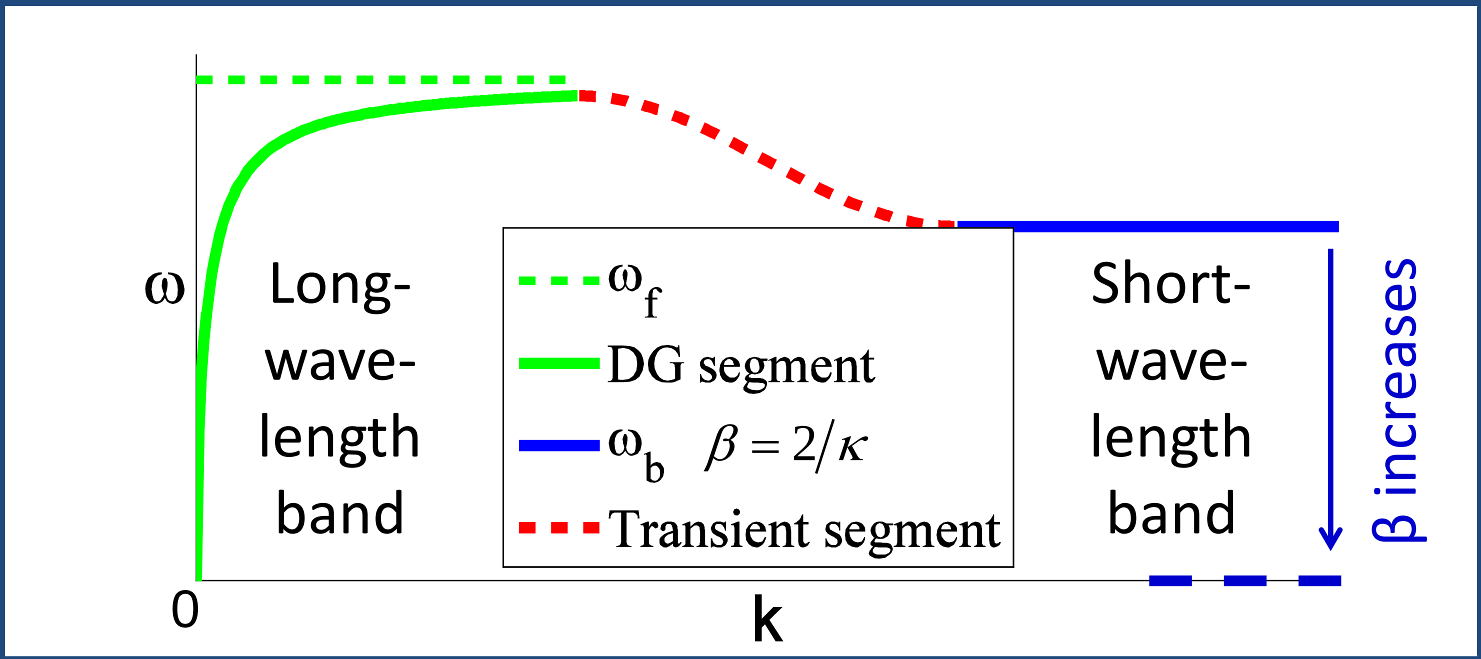
The common physical nature of these two branches (the Ampere force against the pressure gradient) is seen clearly in one particular case :

$$\beta = \frac{2}{\kappa}$$

$$\omega_b^2 = \frac{\omega_f^2}{2}$$

Generally: Ballooning





Aim

Analytical solution

of Erkaev et al. [Phys. Rev. Lett., 99, 235003, 2007] has

Advantages:

- Match observational data on flapping oscillations [Erkaev et al., 2007; Forsyth et al., Ann. Geophys., 27, 2457 – 2474, 2009]
- Simplicity, clearness

Disadvantages:

- Simplicity of the equations: quasy-1-D problem is solved
- Simplicity of the configuration

Isn't it excessively simple?

Aim:

Numerical examination of the double-gradient instability in the frame of linearized 2D / fully 3D ideal MHD to confirm / amend / disprove the Erkaev model.

2D simulations: Equations

Normalization: Δ, $B^* = B(0,z_{max})$, $\rho^* = \rho(0,0)$, $t^* = \Delta/V_A$, $V_A = B^*/(4\pi\rho^*)^{1/2}$, $p^* = B^{*2}/(4\pi)$.

Linearization:

$$\mathbf{U}_0 = (\rho_0, \mathbf{V}_0, \mathbf{B}_0, E_0), \quad \mathbf{U}_1 = (\rho_1, \mathbf{V}_1, \mathbf{B}_1, E_1),$$

$$\mathbf{U} = \mathbf{U}_0 + \mathbf{U}_1. \qquad E = \rho e + 0.5(\rho V^2 + B^2)$$

Perturbations:

$$\mathbf{U}_1(x,z,t;y) = \delta\mathbf{U}(x,z,t)\exp(iky)$$

Linearized system for the amplitudes:

$$\boxed{\frac{\partial(\delta\mathbf{U})}{\partial t} + \frac{\partial \mathbf{F}_x}{\partial x} + \frac{\partial \mathbf{F}_z}{\partial z} = \mathbf{S}.}$$

$$\{\mathbf{F}_x, \mathbf{F}_z, \mathbf{S}\} = \mathbf{f}\left[\mathbf{U}_0(x,z),\ \delta\mathbf{U}(x,z,t);\ k\right].$$

Korovinskiy et al. (2011), Adv. Space Res., 48, 1531–1536.

Solving this system for several fixed *k* <u>we obtain $\omega(k)$</u>

2D simulations: Method

$\gamma = Im[\omega]$. *Assume,* $\gamma_h = \gamma^* - A(h)$

Calculated value (γ_h) True value (γ^*) Scheme damping (A) Mesh step (h)

Grid $[h_x \times h_z] = 0.1 \times 0.025 \rightarrow$ *Lax-Friedrichs scheme one-step method I-order accuracy* $\rightarrow \gamma_h$

Grid $[h_x \times h_z] = 0.05 \times 0.0125 \rightarrow$ *Lax-Friedrichs scheme one-step method I-order accuracy* $\rightarrow \gamma_{h/2}$

The Richardson[1] *extrapolation:* $\gamma^* = 2\gamma_{h/2} - \gamma_h$ *II-order accuracy*

$\mathbf{BC}^{top}_{bottom} : \ \delta \mathbf{U} = 0$

$\mathbf{BC}^{left}_{right} : \ \partial \delta \mathbf{U} / \partial x = 0$

Seed perturbation

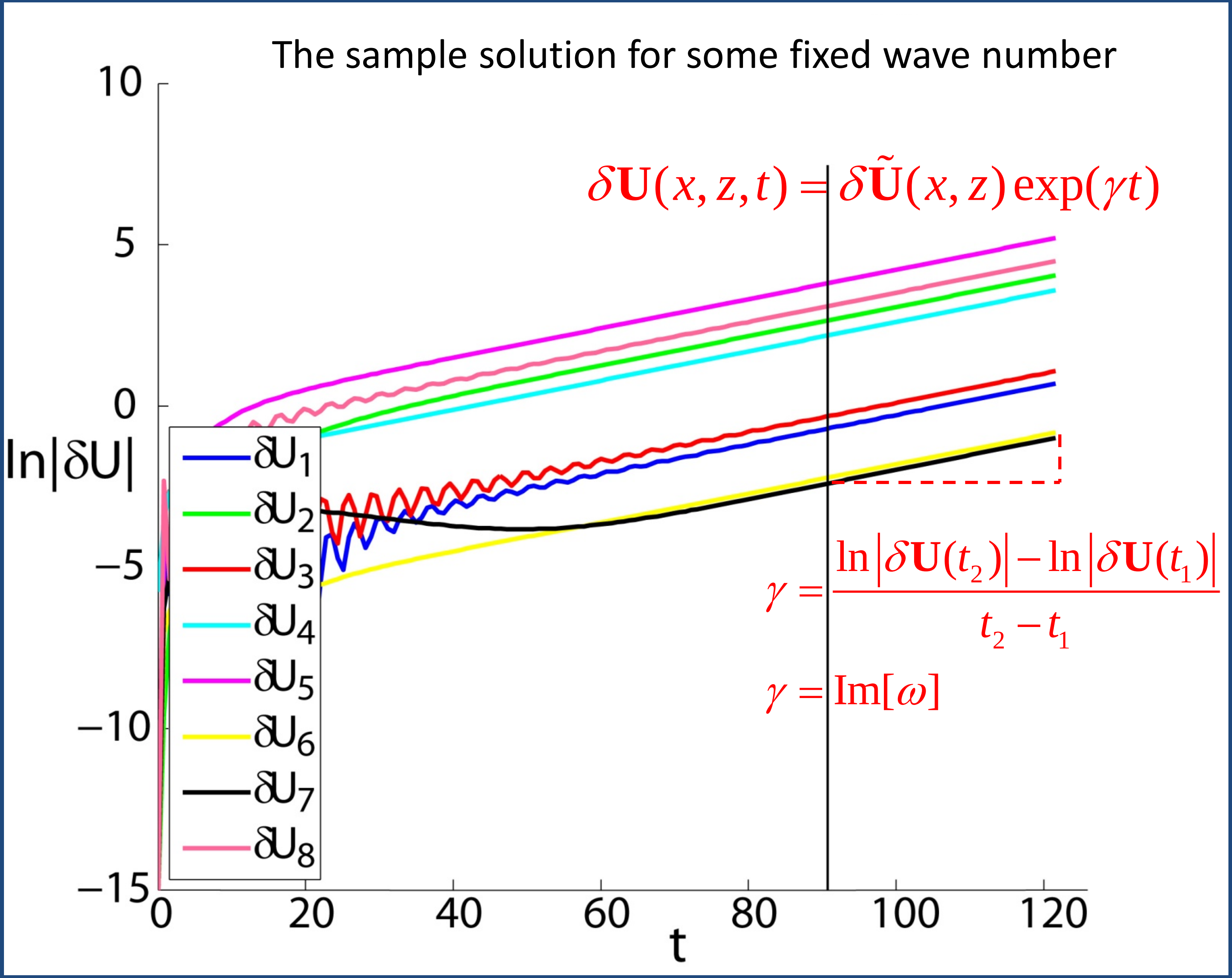
$\delta V_z = \exp(-z^2)$

Courant number

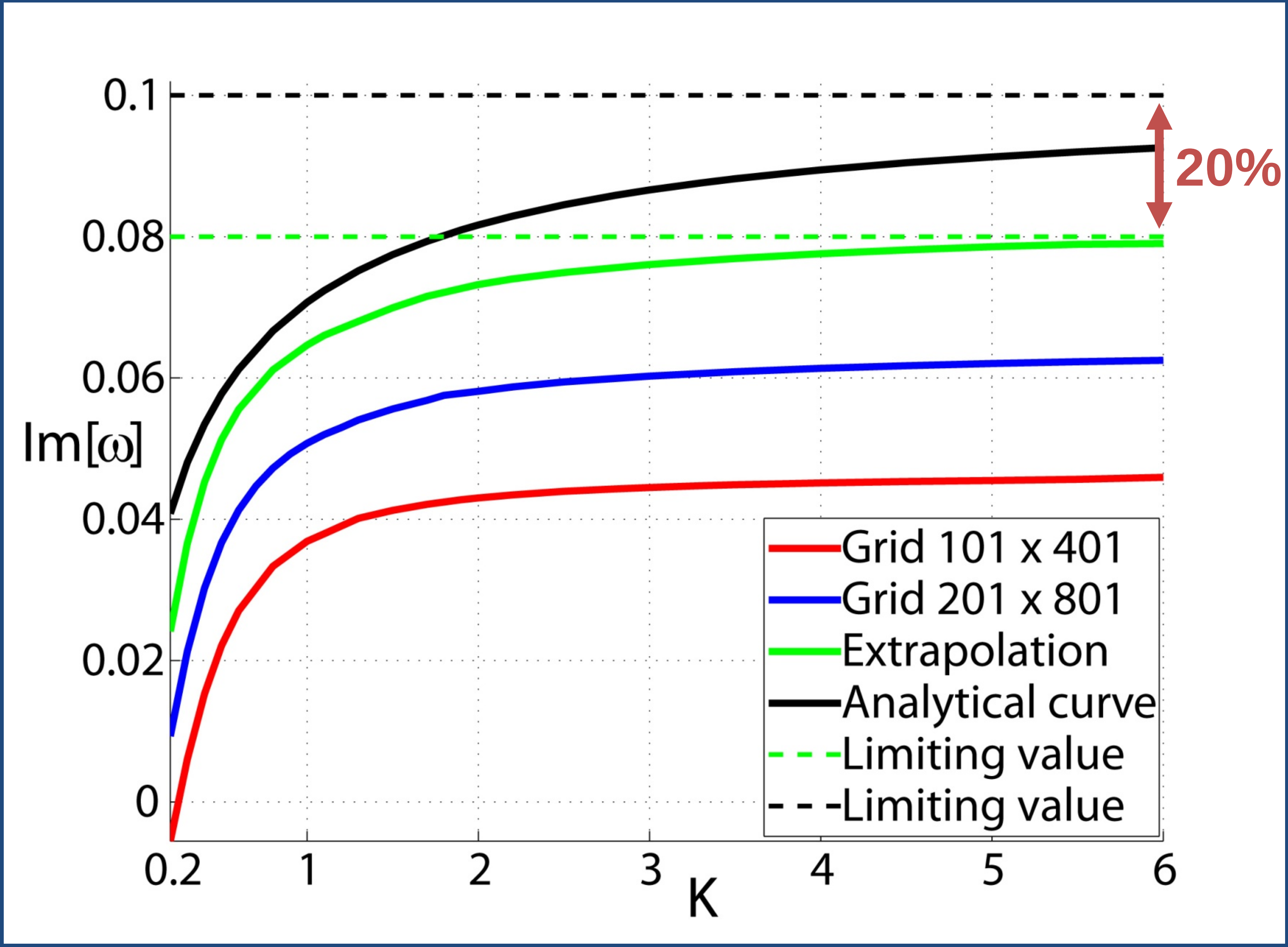
$C = 0.1$

[1]*Richardson, Phil. Trans. Royal Soc. Lond., A 210, 307 – 357, 1911.*

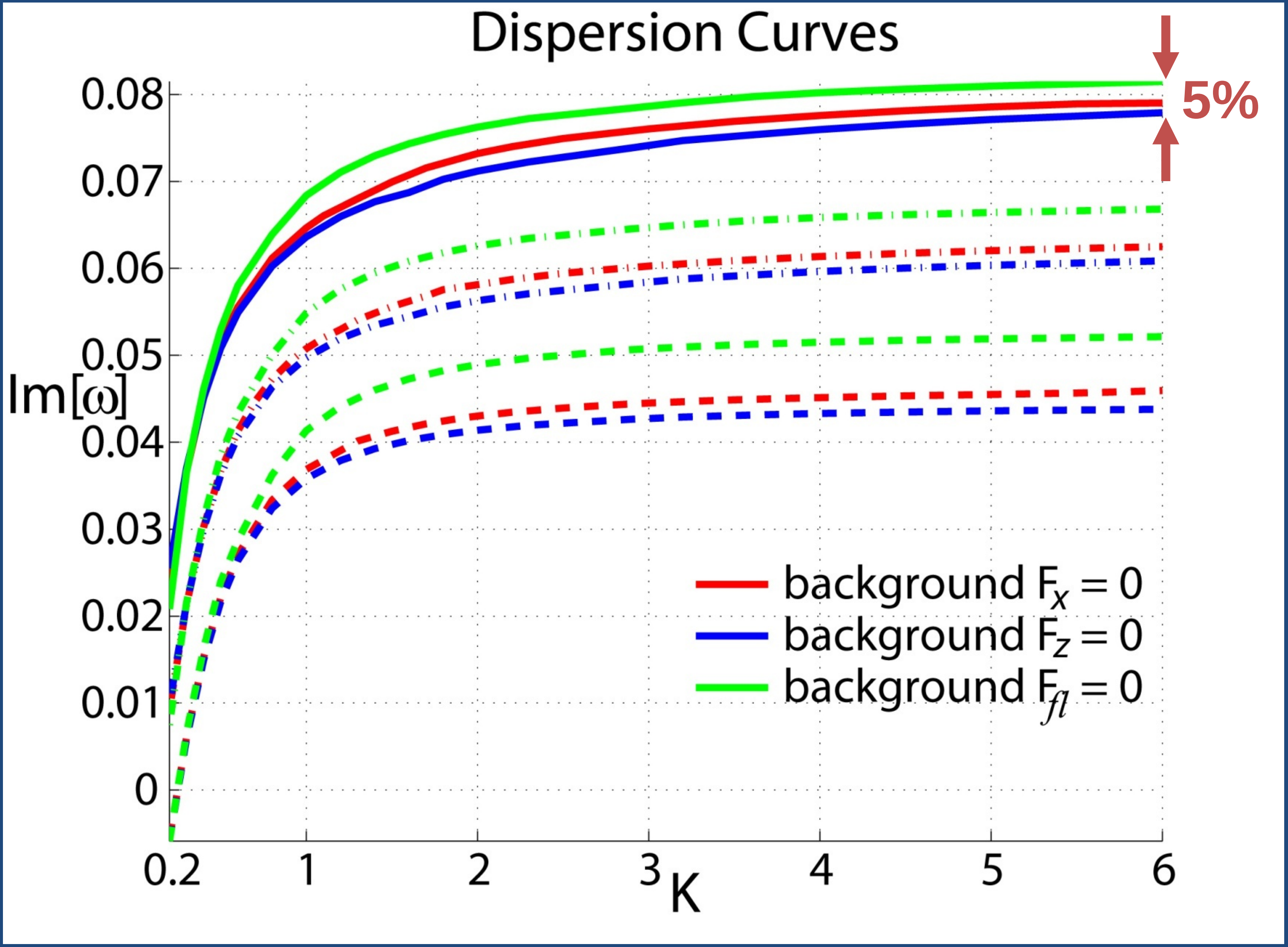
2D simulations: Growth rate



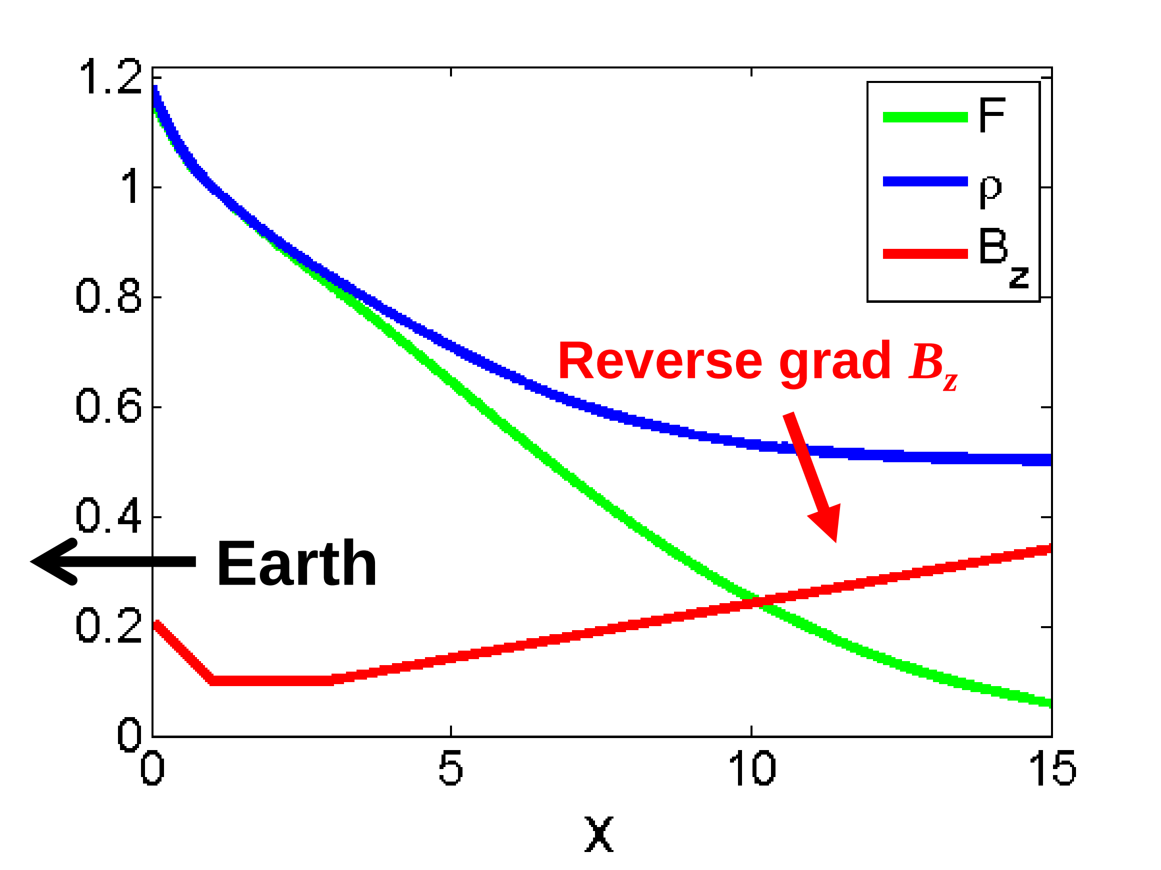
Erkaev's background: Dispersion curve



Dispersion curves for different p(x,z)



The Pritchett solution[1]: Profiles



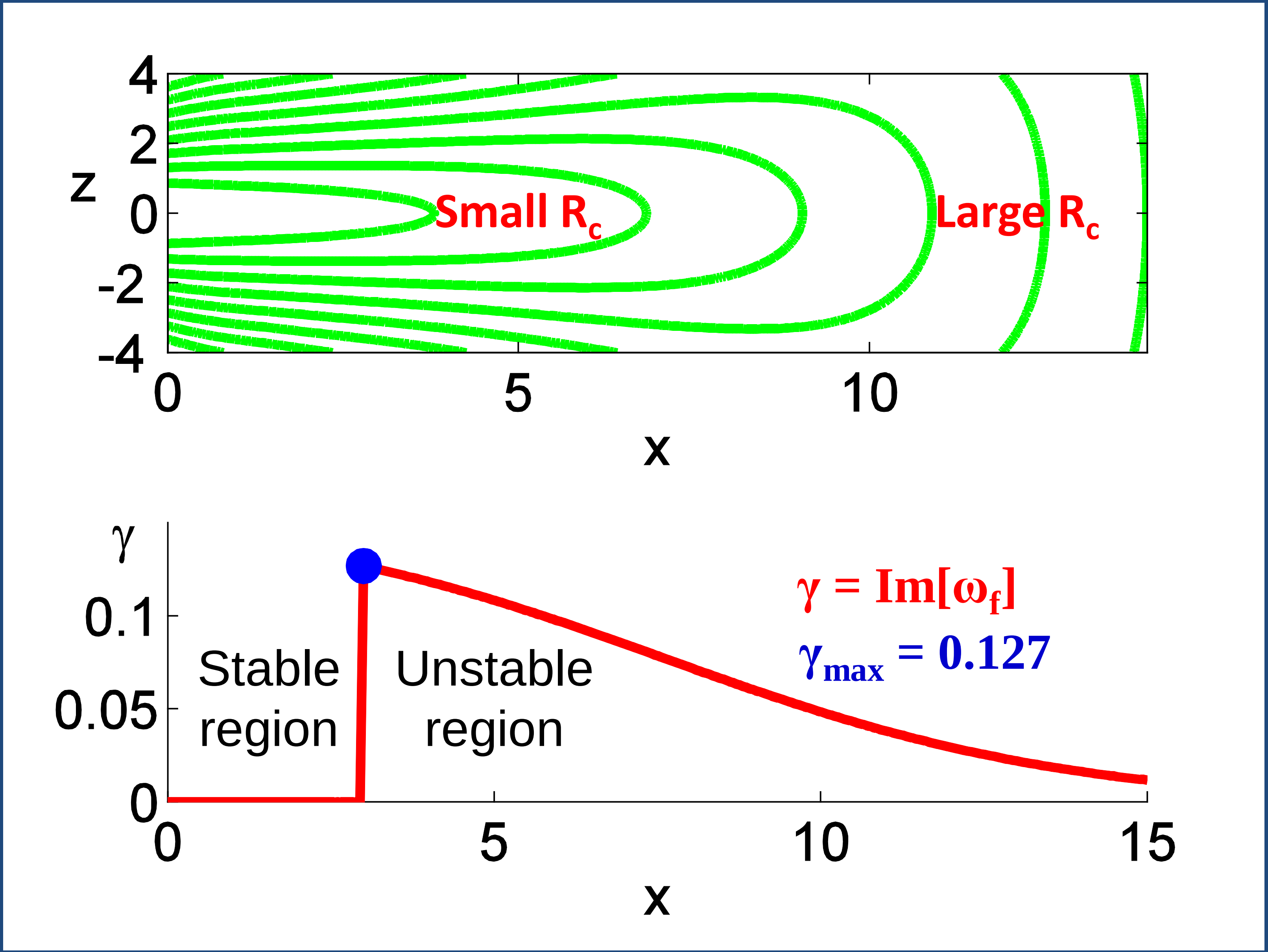
The Pritchett approximate solution of the Grad-Shafranov equation for the magnetic potential A (normalized units),

$$A_{0y} = \ln\left(\frac{\cosh[F(x)z]}{F(x)}\right)$$

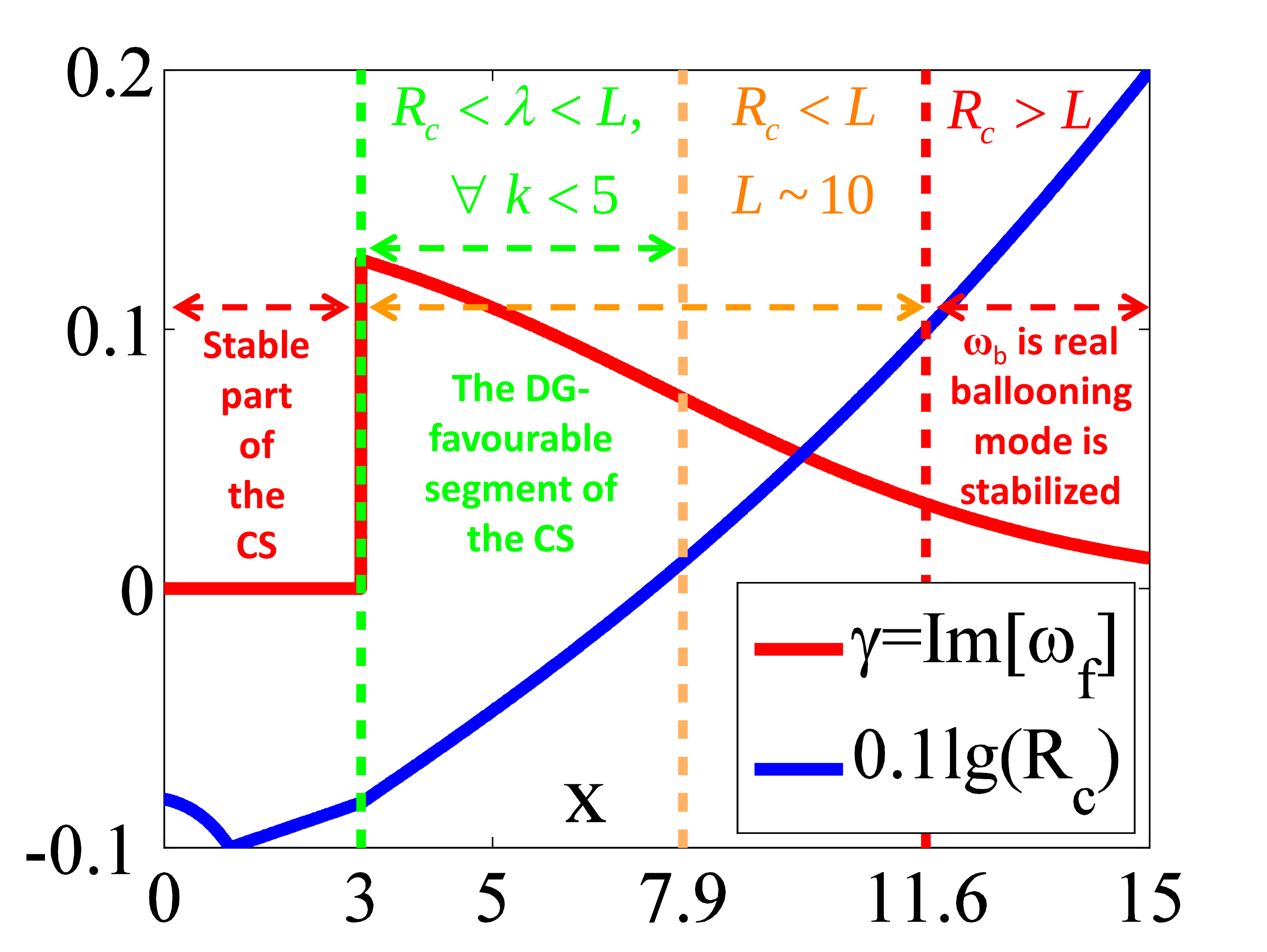
$$\rho = \frac{1}{2}\left[\exp(-2A_{0y}) + 1\right]$$

[1]*Pritchett and Coroniti, JGR,115, A06301, doi:10.1029/2009JA014752, 2010*

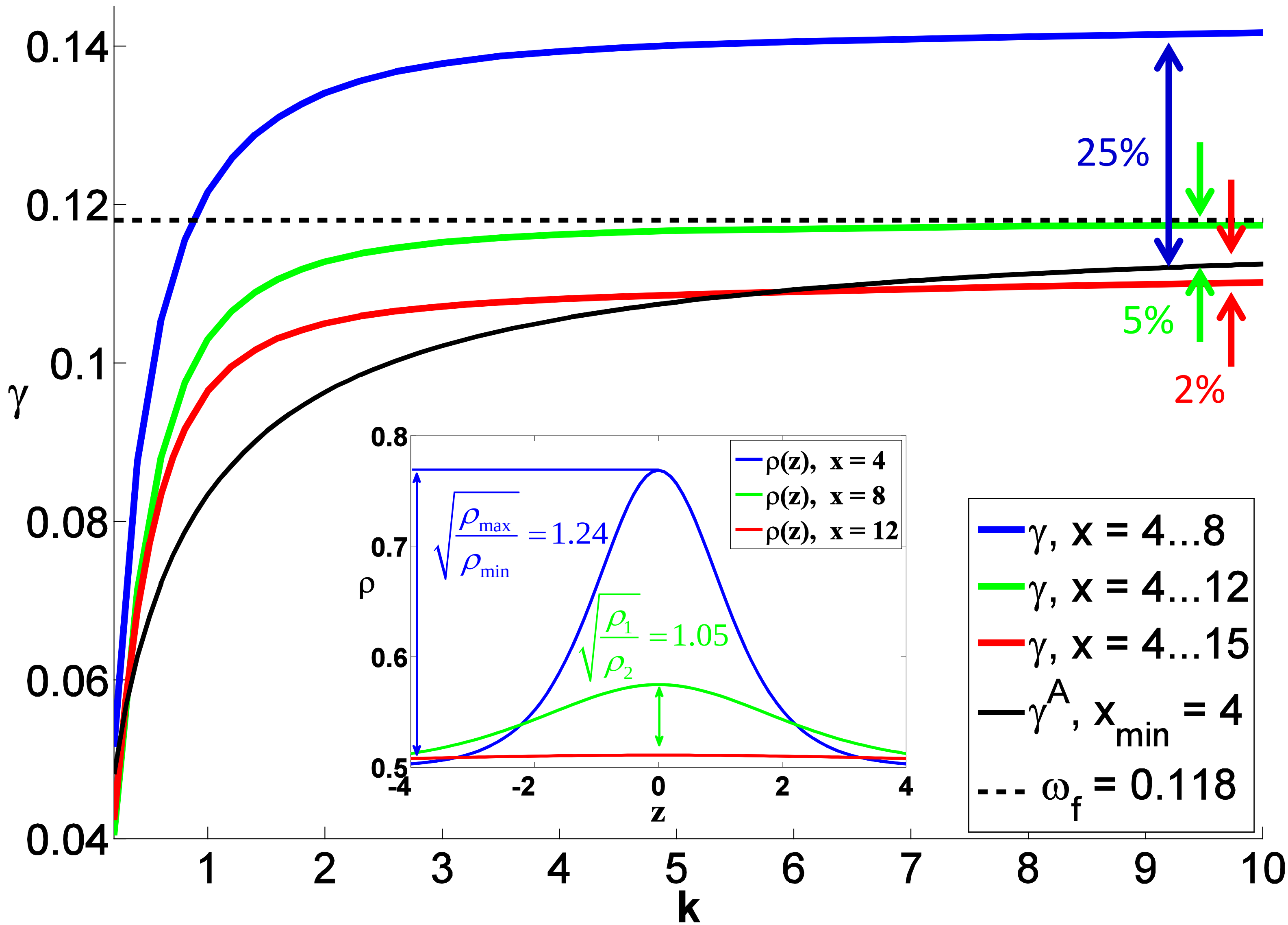
Magnetic configuration and ω_f



The configuration features

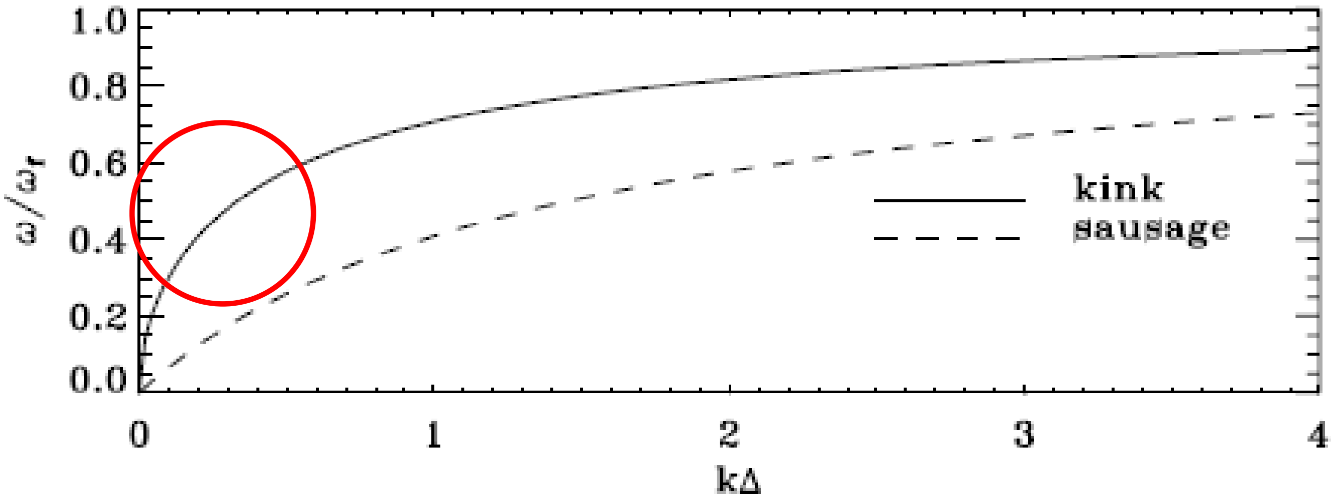


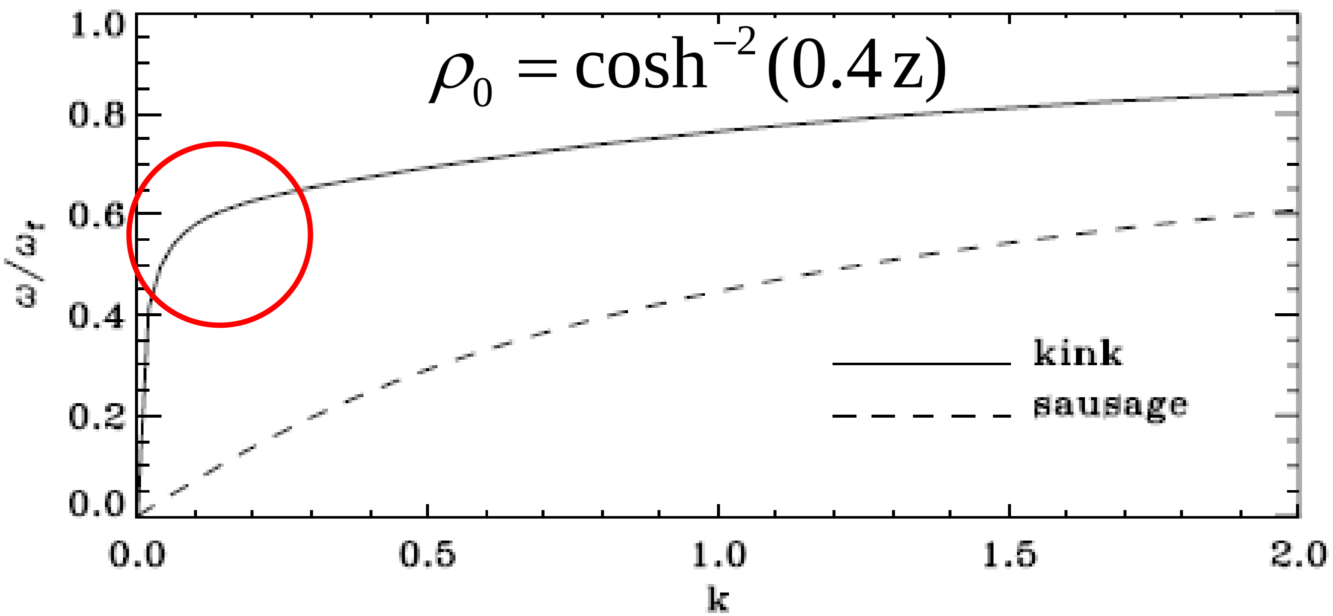
Disp. curves: DGI-favorable segment +



ρ_0(z) matters

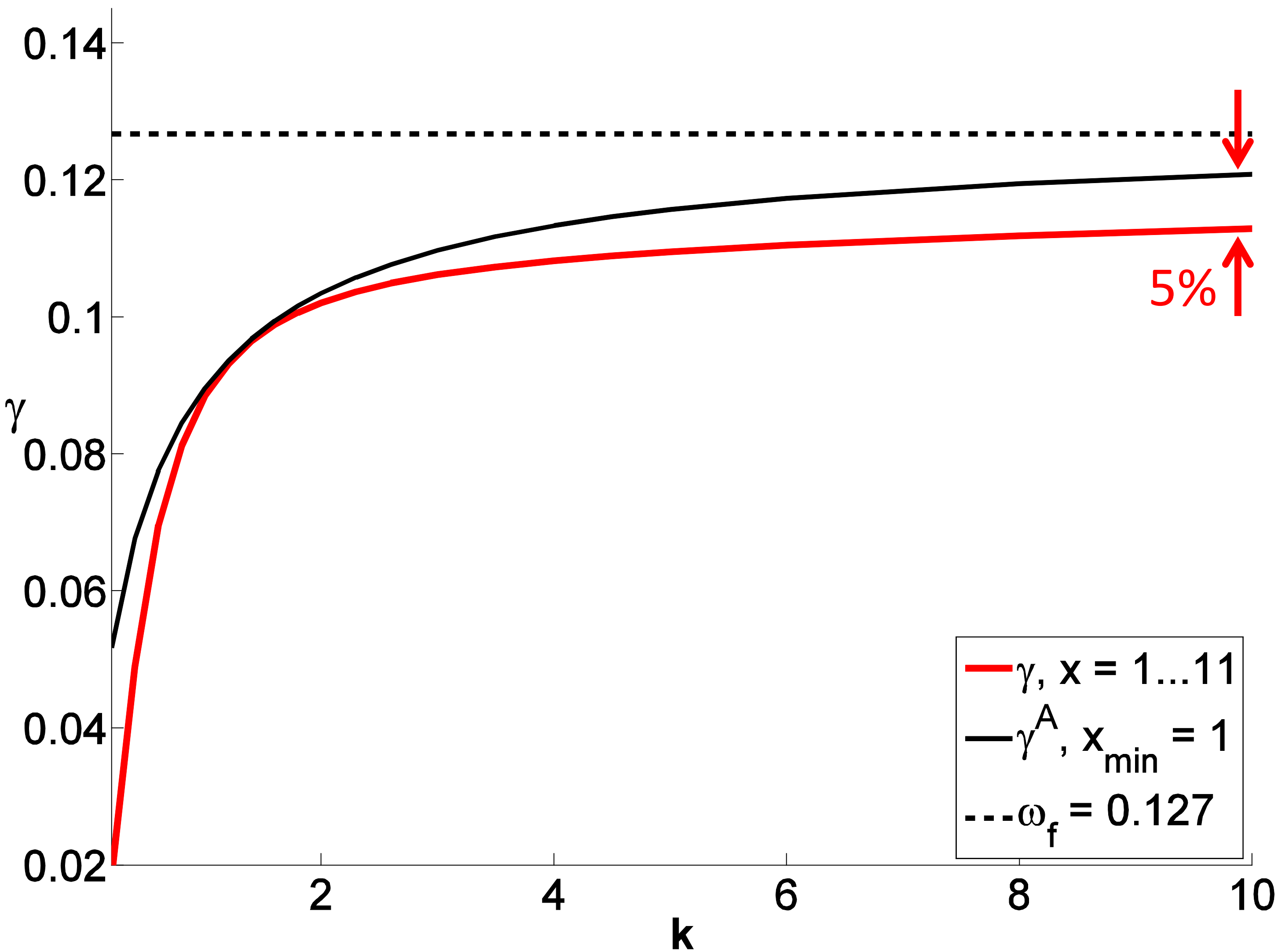
$\rho_0 = 1$



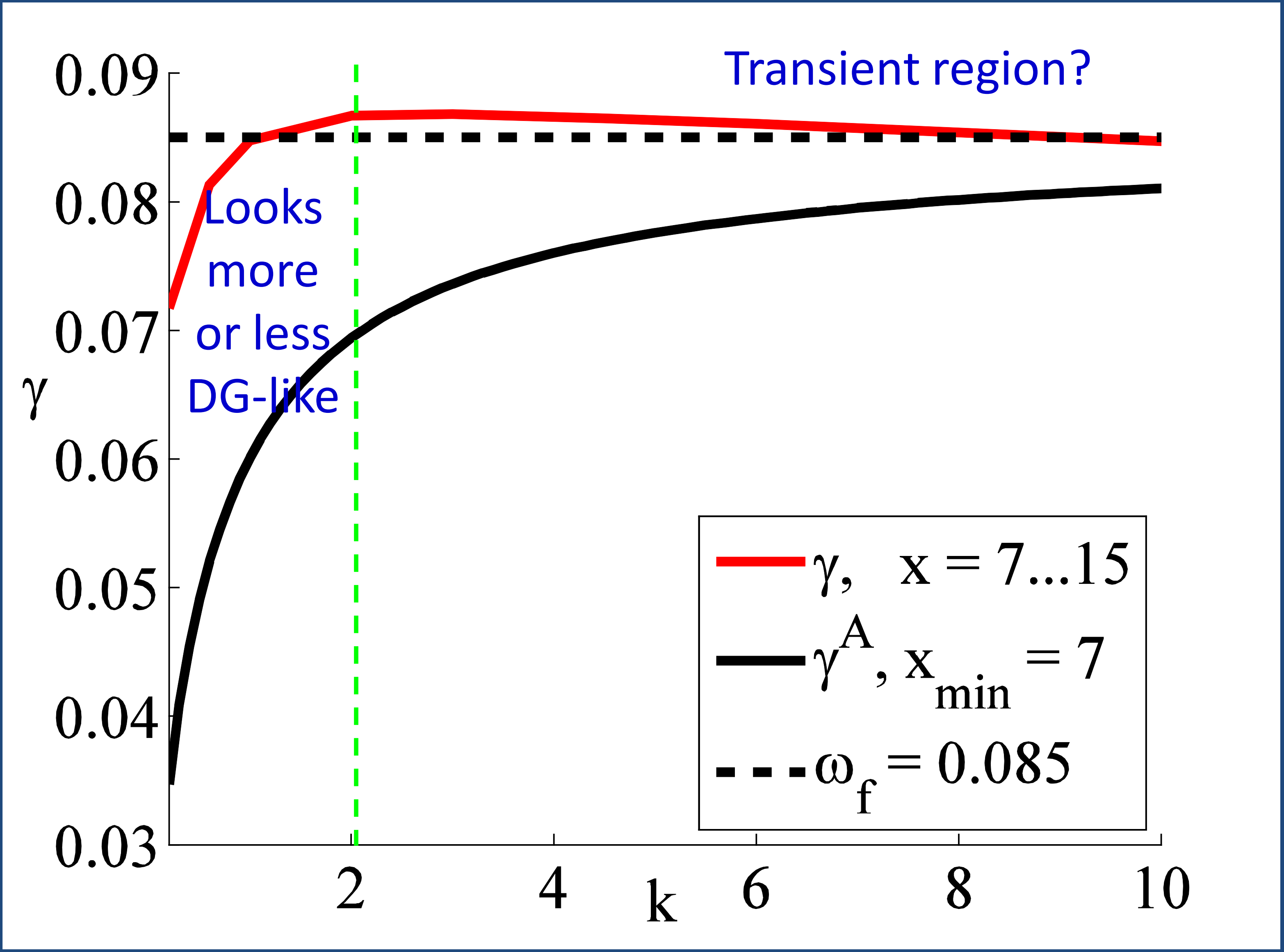


Erkaev et al., Ann. Geophys., 27, 417–425, 2009.

Disp. curves: stable segment +



Disp. Curves: Large-R_c region



3D MHD: background relaxation[1]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0,$$

$$\frac{\partial \rho \mathbf{V}}{\partial t} + \nabla \cdot (\mathbf{V} \otimes \rho \mathbf{V} - \mathbf{B} \otimes \mathbf{B}) + \nabla P = -\alpha \rho \mathbf{V},$$

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{V} \otimes \mathbf{B} - \mathbf{B} \otimes \mathbf{V}) = 0,$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (\mathbf{V} e + \mathbf{V} P - \mathbf{B} \otimes \mathbf{B} \cdot \mathbf{V}) = -\alpha \rho V^2,$$

$$e = \frac{p}{\kappa - 1} + \frac{\rho V^2}{2} + \frac{B^2}{2},$$

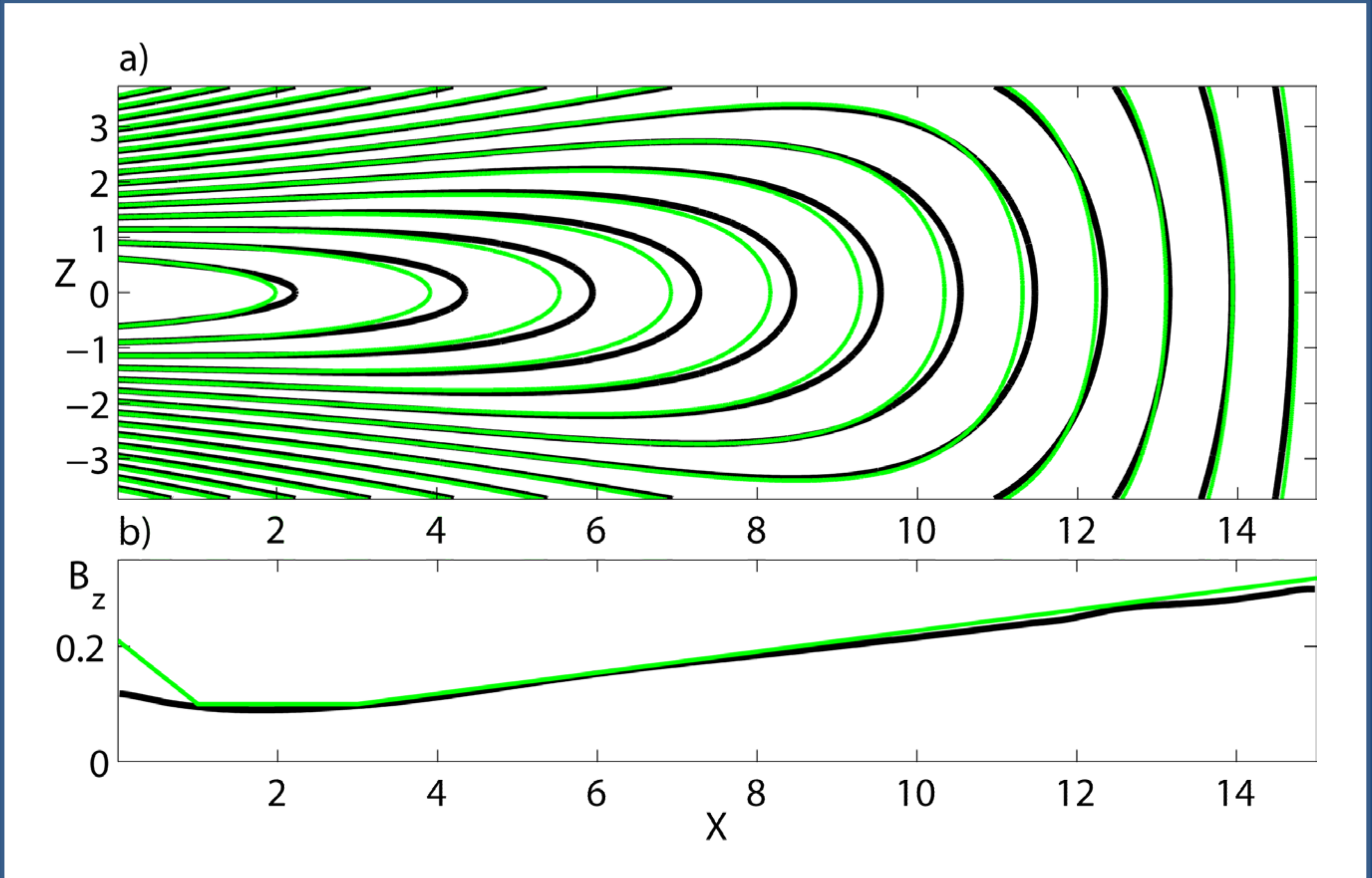
$$P = p + \frac{B^2}{2}.$$

2-dimensional friction MHD simulation is performed to minimize the net force $\nabla p - \mathbf{j} \times \mathbf{B}$

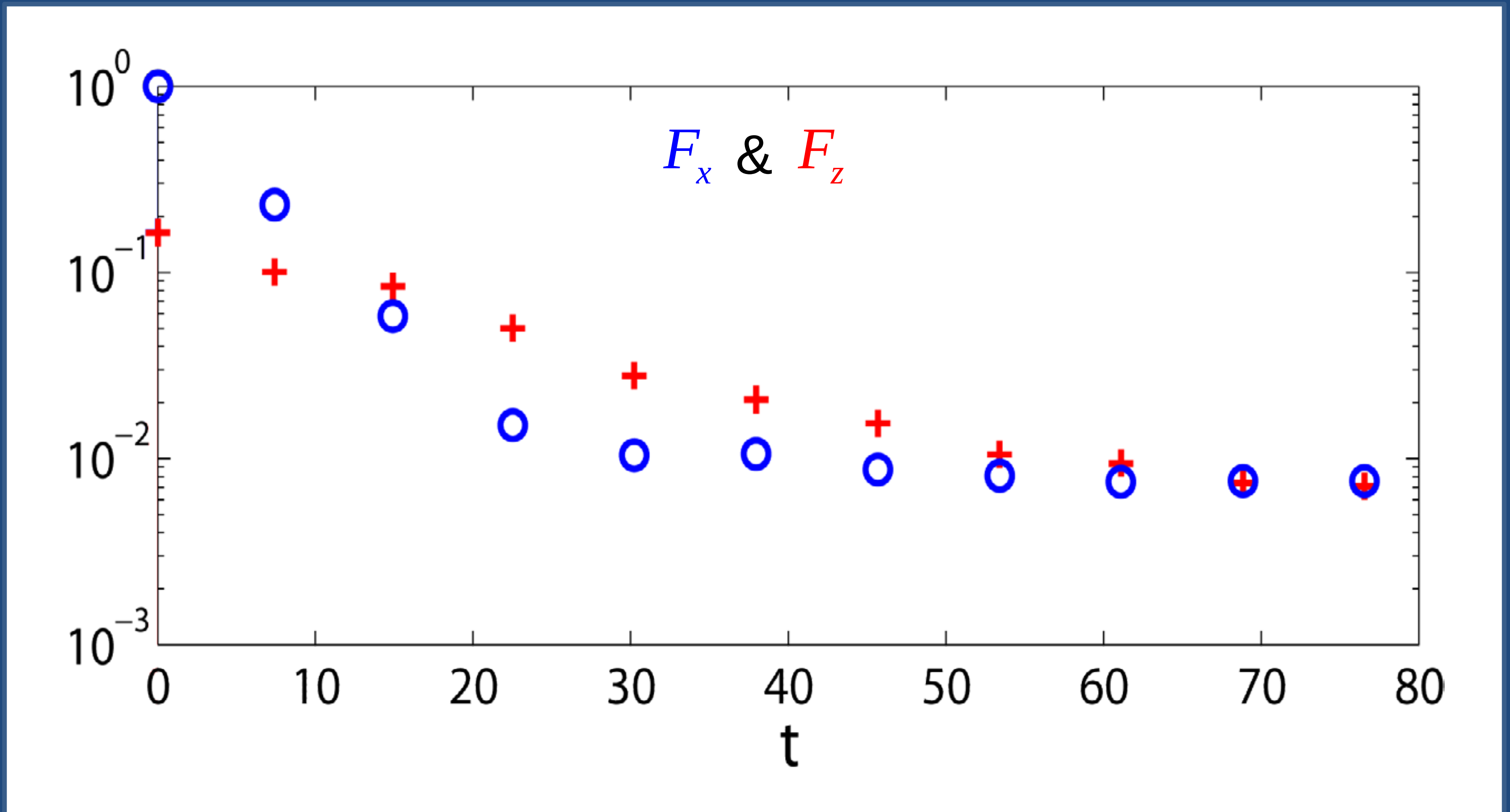
$$\alpha(t) = \begin{cases} 0.1 + 10\cos(\pi t/40), & t \le 20, \\ 0.1, & t \ge 20, \\ 0, & t > 80. \end{cases}$$

[1]Hesse&Birn (1993), *JGR, 98, 3973–3982, doi:10.1029/92JA02905*

Initial (green) and relaxed (black) background configurations



Relaxation efficiency



Total
Net
Force

$$F_k = \Delta x \Delta z \sum_{i,j} f_k(\mathrm{x}_i, \mathrm{z}_j),$$

$$f_k = [\mathbf{j} \times \mathbf{B} - \nabla p]_k .$$

$$k = \begin{cases} x, & \text{blue} \\ z, & \text{red} \end{cases}$$

Run parameters

$$L_x \times L_y \times L_z = 15 \times 7.5 \times 7.5$$

$$N_x \times N_y \times N_z = 384 \times 192 \times 192$$

BC in relaxation phase (2D in XZ plane) fix the magnetic flux entering domain

$$\partial / \partial \mathbf{n} \{ \rho, \mathbf{B}_\tau, p \} = 0,$$

$$\partial B_n / \partial t = 0, \quad \mathbf{V} = 0.$$

In the main phase the same BC are applied at Z-boundaries, and the Earthward X-boundary

Free BC are imposed at the tailword X-boundary and Y-boundaries

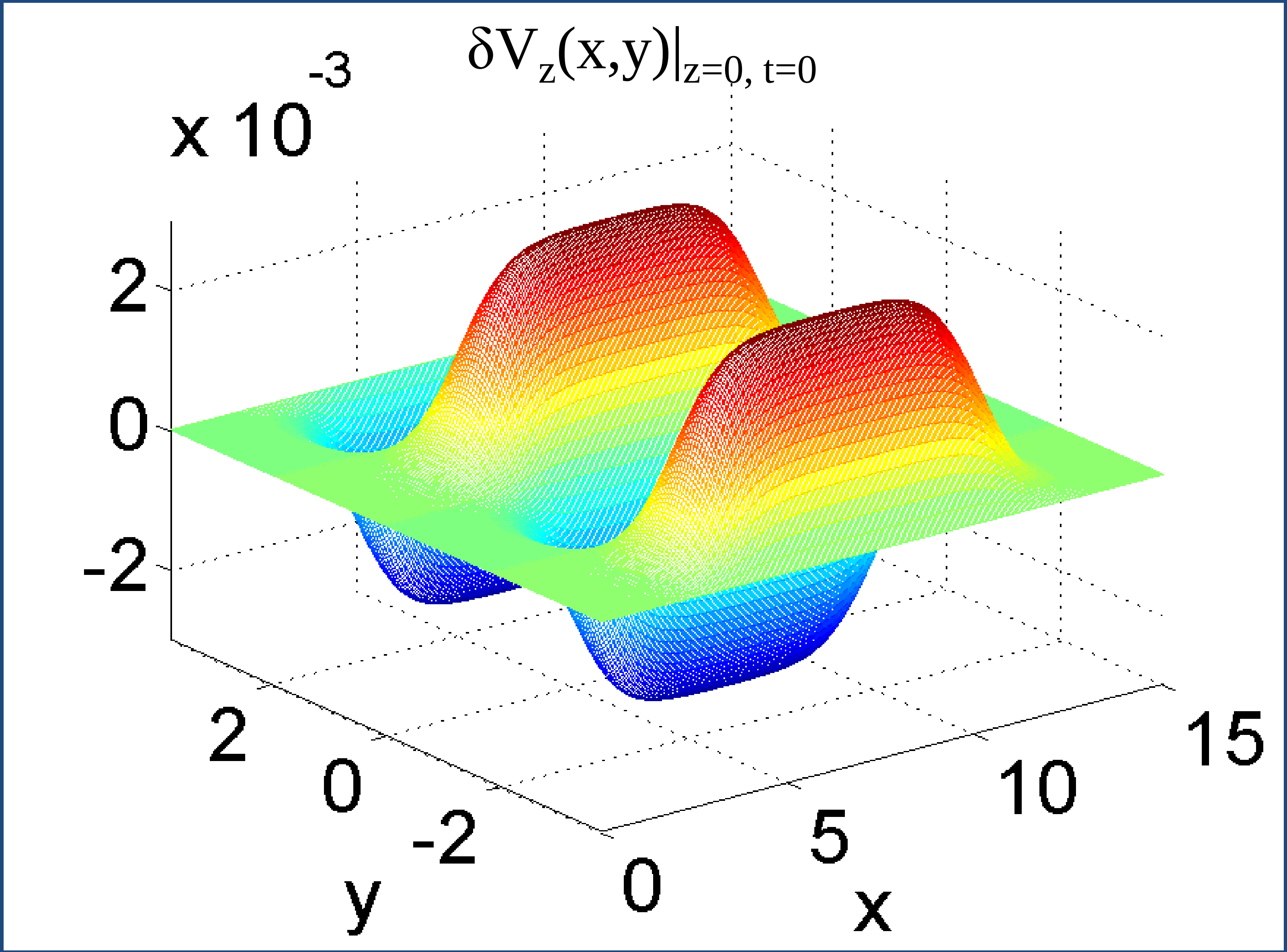
$$\partial / \partial \mathbf{n} \{ \rho, \mathbf{B}, \mathbf{V}, p \} = 0$$

The instability is seeded with a mode *m_y = 2 kick of V_z velocity:*

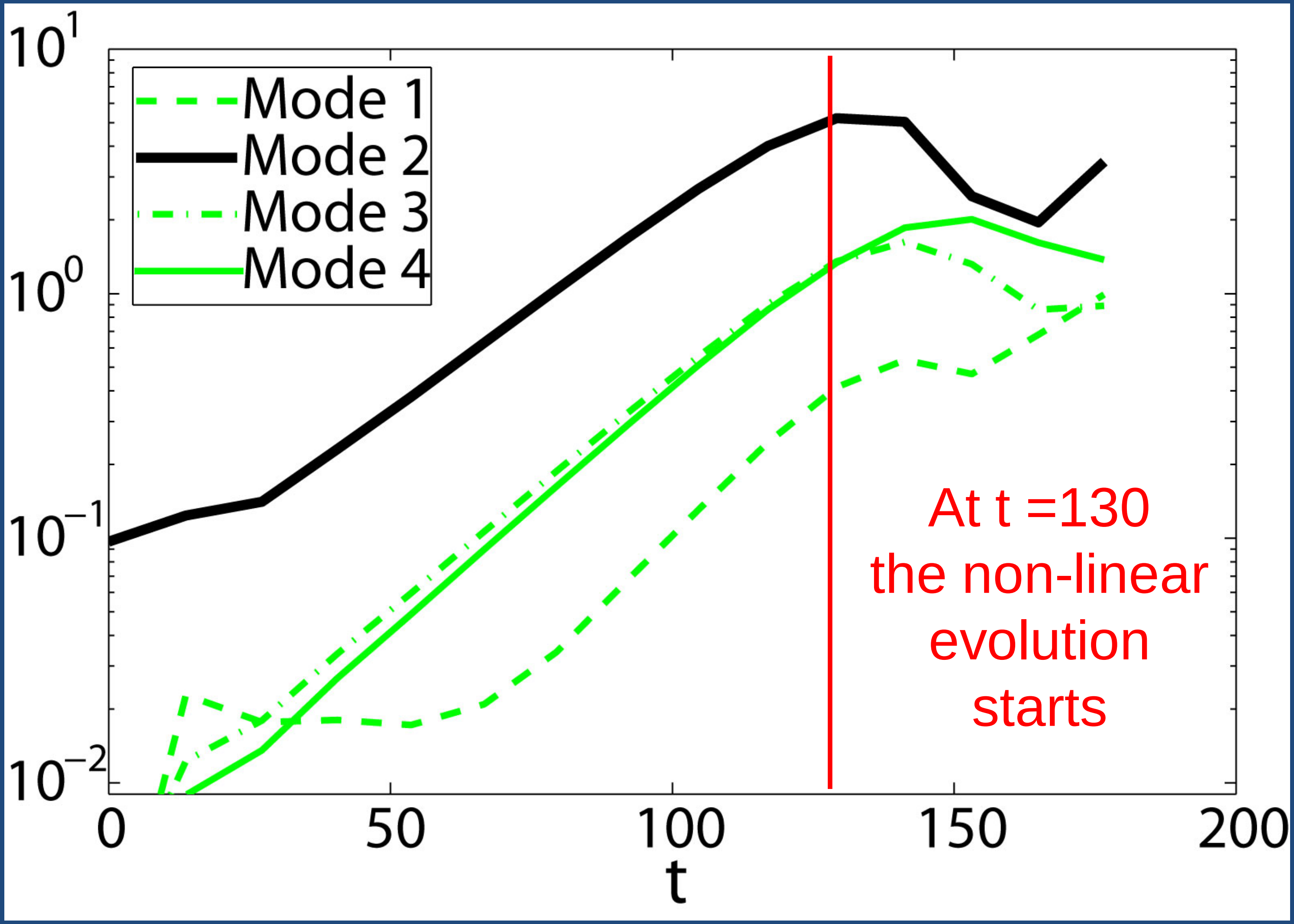
$$\delta V_z = 0.003 f(x) \sin\left(k_y y\right) \exp\left(-2z^2\right), \quad k_y = 2\pi m_y / L_y = 1.675$$

$$f(x) = 0.5\left[\tanh\left(x - L_x/4\right) - \tanh\left(x - 3L_x/4\right)\right].$$

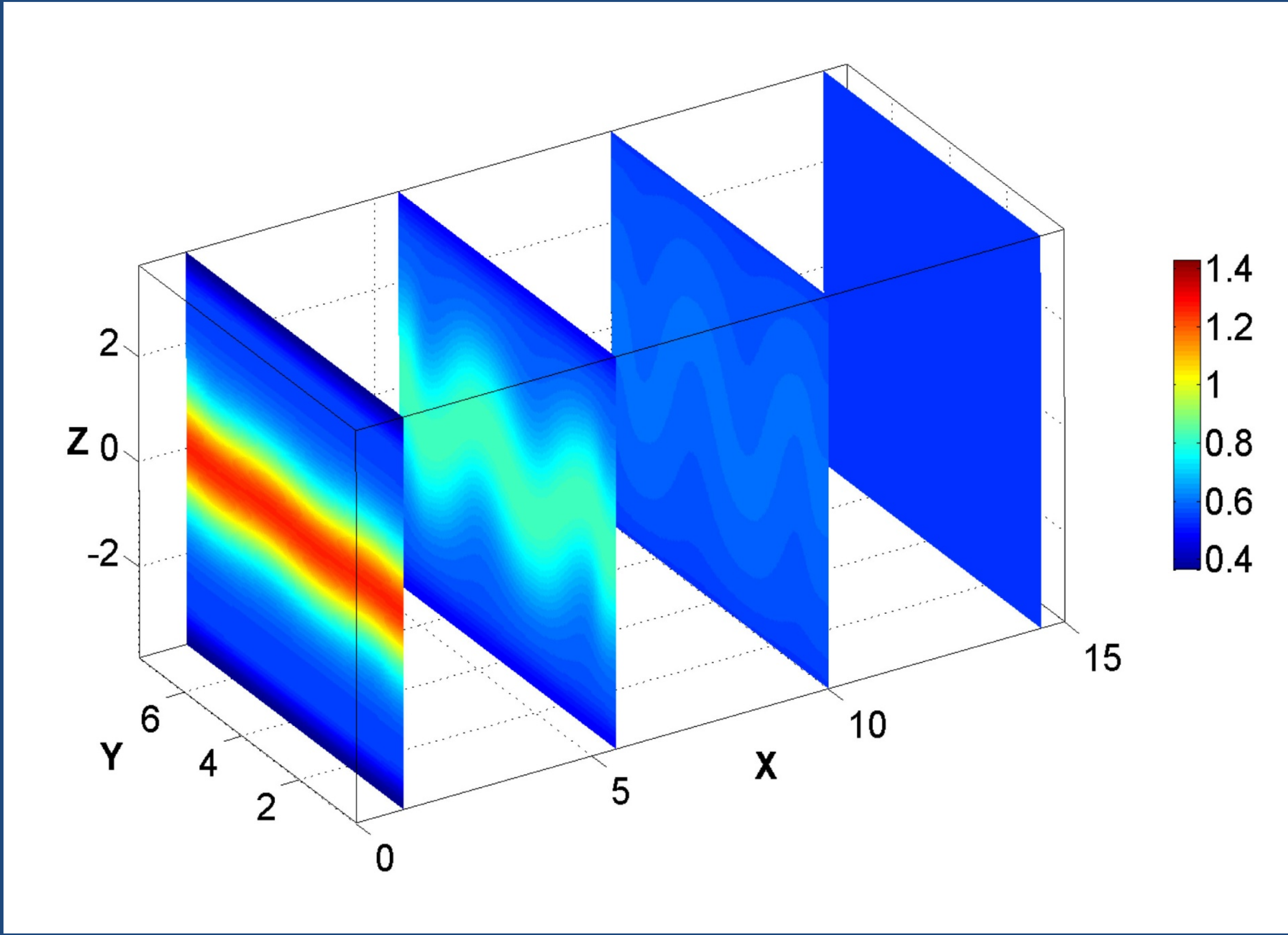
3D simulation: Seed perturbation



$|V_z|$ perturbation modes $m_y = \{1,2,3,4\}$
integrated over all *z and x ∈ [3.75, 11.25]*



ρ, t = 117, x-slices

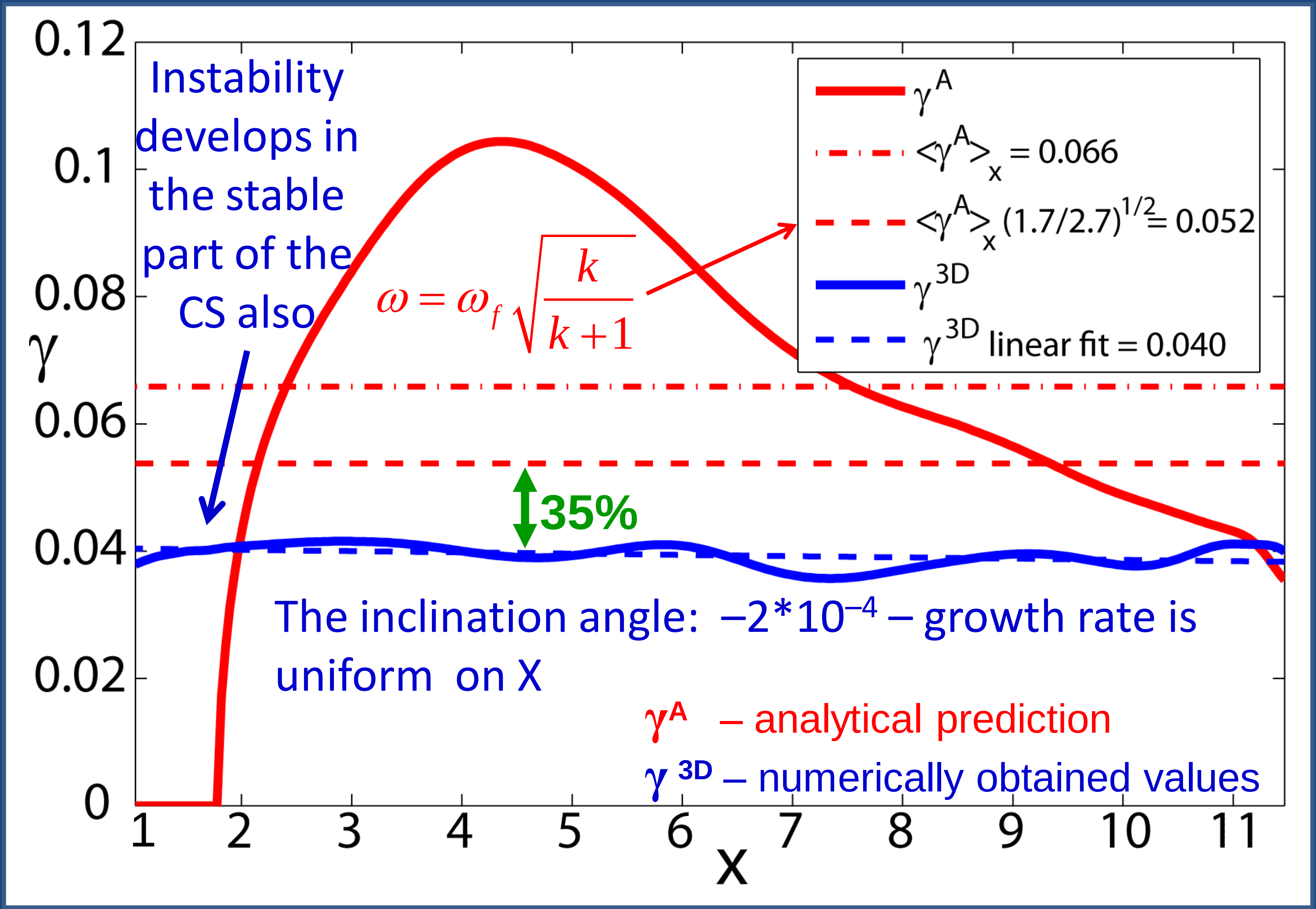


Estimation of γ: Method

- Use Fourier transform to find the amplitude of the mode 2 of the plasma density variation in Y direction, $A_2(x,z)$
- Calculate A_2 at different times t_1, t_2
- Calculate growth rate:

$$\gamma = \frac{1}{t_2 - t_1} \ln \frac{A_2(t_2)}{A_2(t_1)}$$

Growth rate by 3D simulation $|_{z = 0,\, t = 86}$



Conclusions[1]

Fully 3D MHD sim.	2D linearized MHD sim.
▪ DGI does not develop in the regions of too large R_c.	▪ ✓
▪ DGI can develop in the domains with mixed uniform / tailward-growing B_z.	▪ ✓
▪ The uniform / Earthward-growing-B_z regions produce strong stabilizing effect.	▪ ✓
▪ The growth rate is close to the analytical estimation averaged over the domain.	▪ The growth rate is close to the maximal analytical estimation in the domain.

[1]Korovinskiy et al. (2013), J. Geophys. Res., 118, 1146 – 1158.