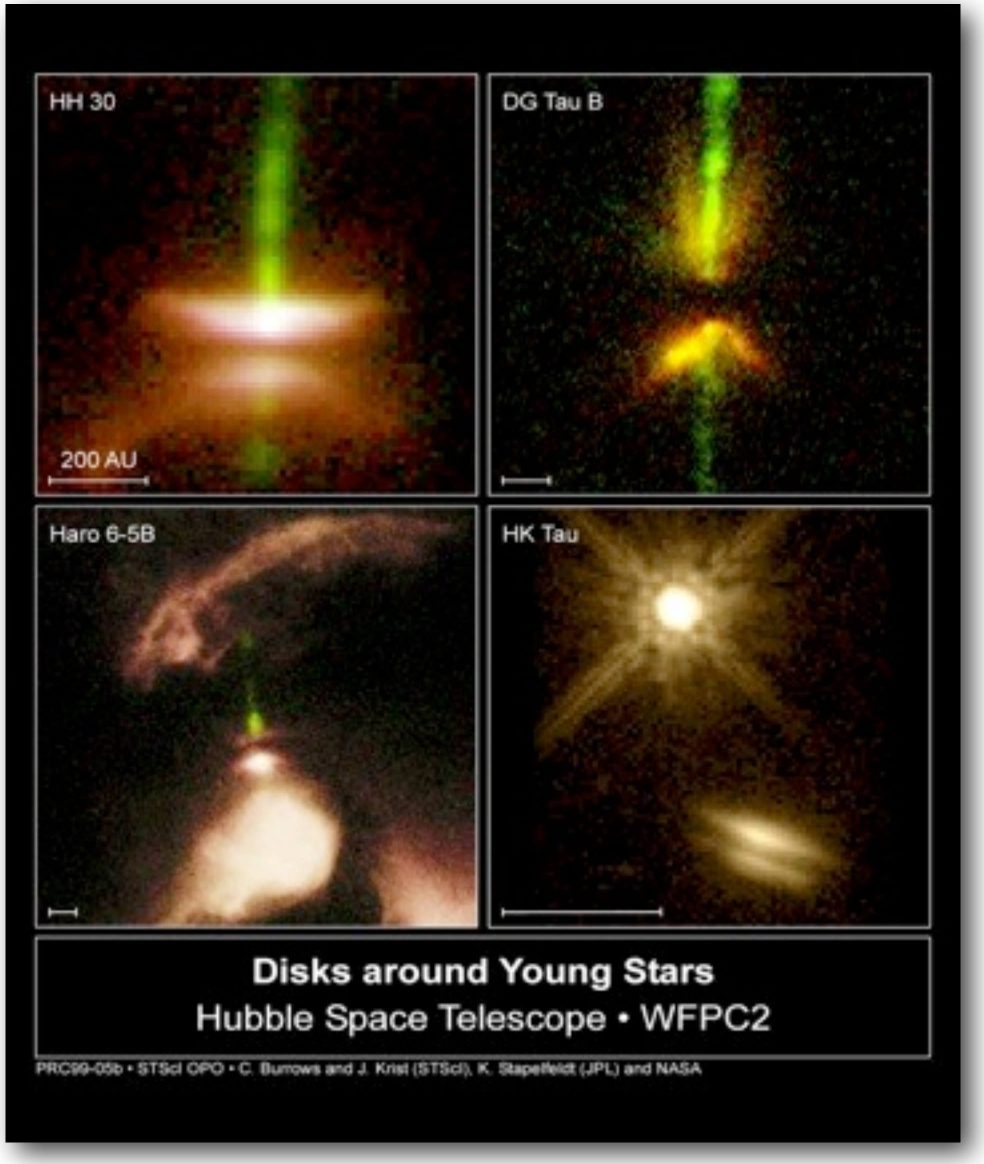
Hall effect in protoplanetary discs

Geoffroy Lesur (IPAG, Grenoble, France)

Matthew W. Kunz (Univ. of Princeton, USA)

An accretion problem...

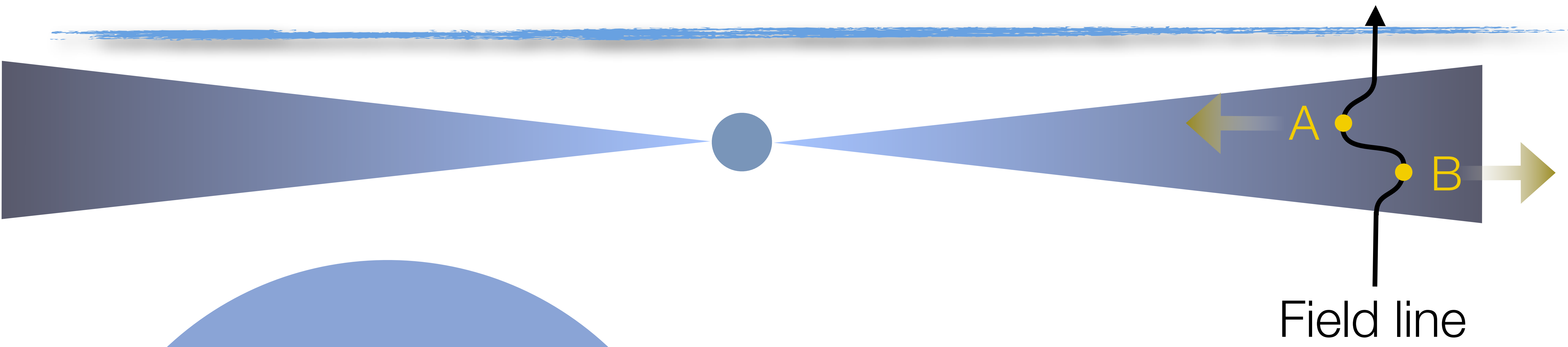
- Accretion discs are known to form around young stars and compact objects
- Gas can fall on the central object only if it looses angular momentum.
- One needs a way to transport angular momentum outward to have accretion:
  «angular momentum transport problem»

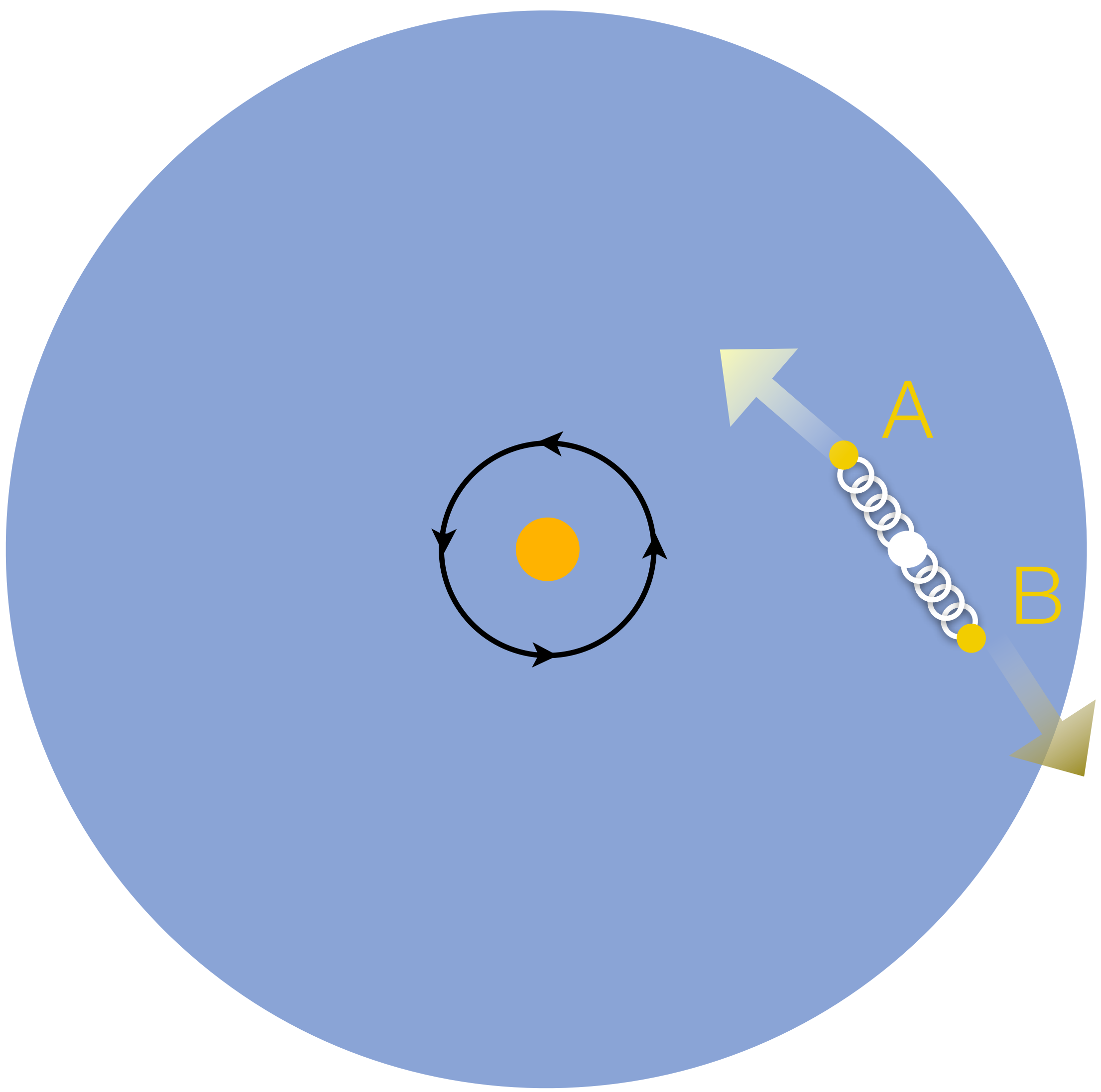


→ Turbulence produces a «turbulent viscosity»

$$\nu_t = \alpha c_s H$$

The magnetorotational instability (MRI)

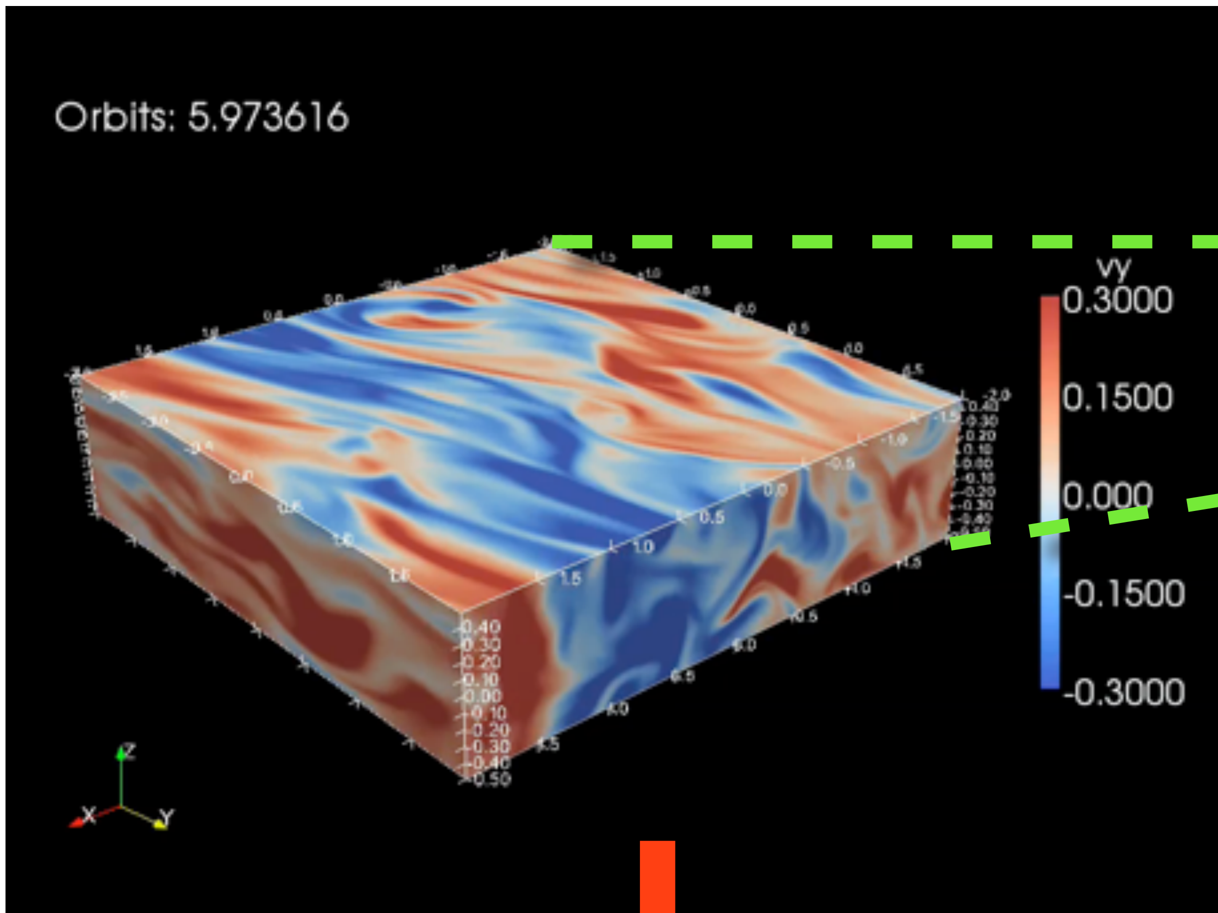


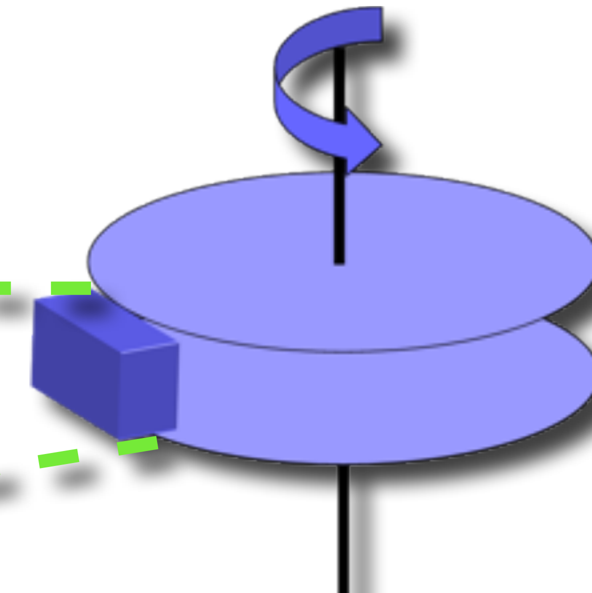


- MRI is an efficient mechanism which seems to produce $\alpha \sim 10^{-3}$—10^{-1}
- Need a relatively weak field (sub-thermal)
- Ideal MHD instability, modified by nonideal effects

Balbus & Hawley 1991, Balbus 2003

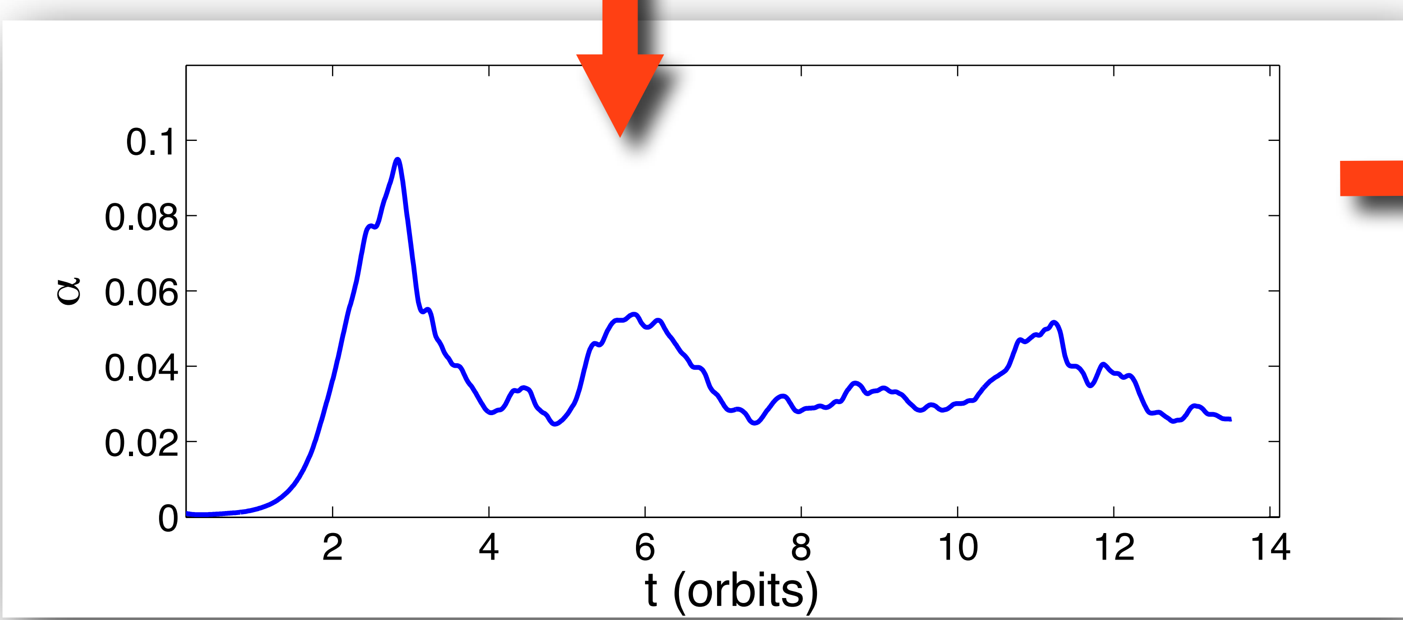
Simulation example





Simulation parameters: Re=1000, Pm=1, β=1000

3D map of v_y (azimuthal velocity)



It works!

Is it the end of the story?

Protoplanetary discs

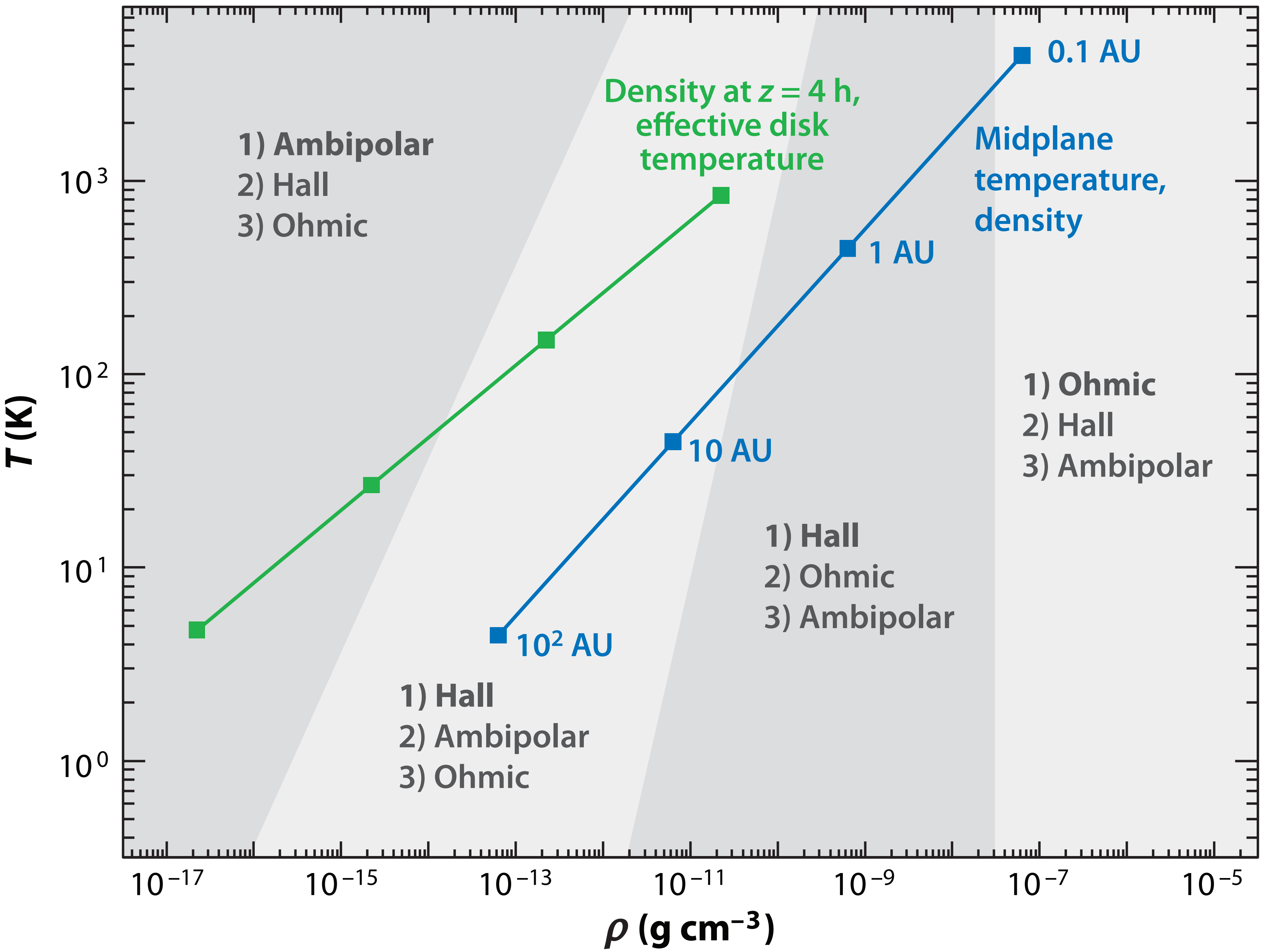
- Protoplanetary discs are far from being in the ideal MHD regime: very low ionisation fraction $\sim 10^{-13}$
- 3 non-ideal effects
  - Ohmic resistivity (electrons-neutrals collisions)
  - Hall effect (electrons-ions drift)
  - Ambipolar diffusion (electrons-neutral drift)

$$\frac{O}{H} = \left(\frac{n}{8\times 10^{17}\,\mathrm{cm}^{-3}}\right)^{1/2}\left(\frac{v_A}{c_s}\right)^{-1}$$

$$\frac{A}{H} = \left(\frac{n}{9\times 10^{12}\,\mathrm{cm}^{-3}}\right)^{-1/2}\left(\frac{T}{10^3\,\mathrm{K}}\right)^{1/2}\left(\frac{v_A}{c_s}\right)$$

→ Hall dominates for «intermediate» densities

Non-ideal protoplanetary discs



(Armitage 2011)

Hall effect basics

Fully ionised plasmas

- Equation of motion for electrons

$$\cancel{m_e \frac{d\boldsymbol{u}_e}{dt}} = -e(\boldsymbol{E} + \boldsymbol{u}_e \times \boldsymbol{B}) - \frac{1}{n_e}\boldsymbol{\nabla} P_e - \nu_{ei} m_e (\boldsymbol{u}_e - \boldsymbol{u}_i)$$

Long timescale compared to electrons gyro-frequency

- Introduce currents and average bulk velocity

$$\boldsymbol{J} = -e n_e (\boldsymbol{u}_e - \boldsymbol{u}_i) \qquad \boldsymbol{U} \sim \boldsymbol{u}_i$$

- Ohm's Law:

$$\boldsymbol{E} = -\boldsymbol{U} \times \boldsymbol{B} + \frac{1}{e n_e}\boldsymbol{J} \times \boldsymbol{B} - \cancel{\frac{1}{e n_e}\boldsymbol{\nabla} P_e} + \eta \boldsymbol{J}$$

Ideal MHD | Hall effect | Electron pressure | Ohmic resistivity

- Whistler waves:

$$\partial_t \delta\boldsymbol{b} = -\frac{c}{4\pi e n_e}\Big(\boldsymbol{k} \cdot \boldsymbol{B}_0\Big)\Big(\boldsymbol{k} \times \delta\boldsymbol{b}\Big)$$

MRI in the Hall regime
Linear stability analysis

- Introduce two dimensionless numbers

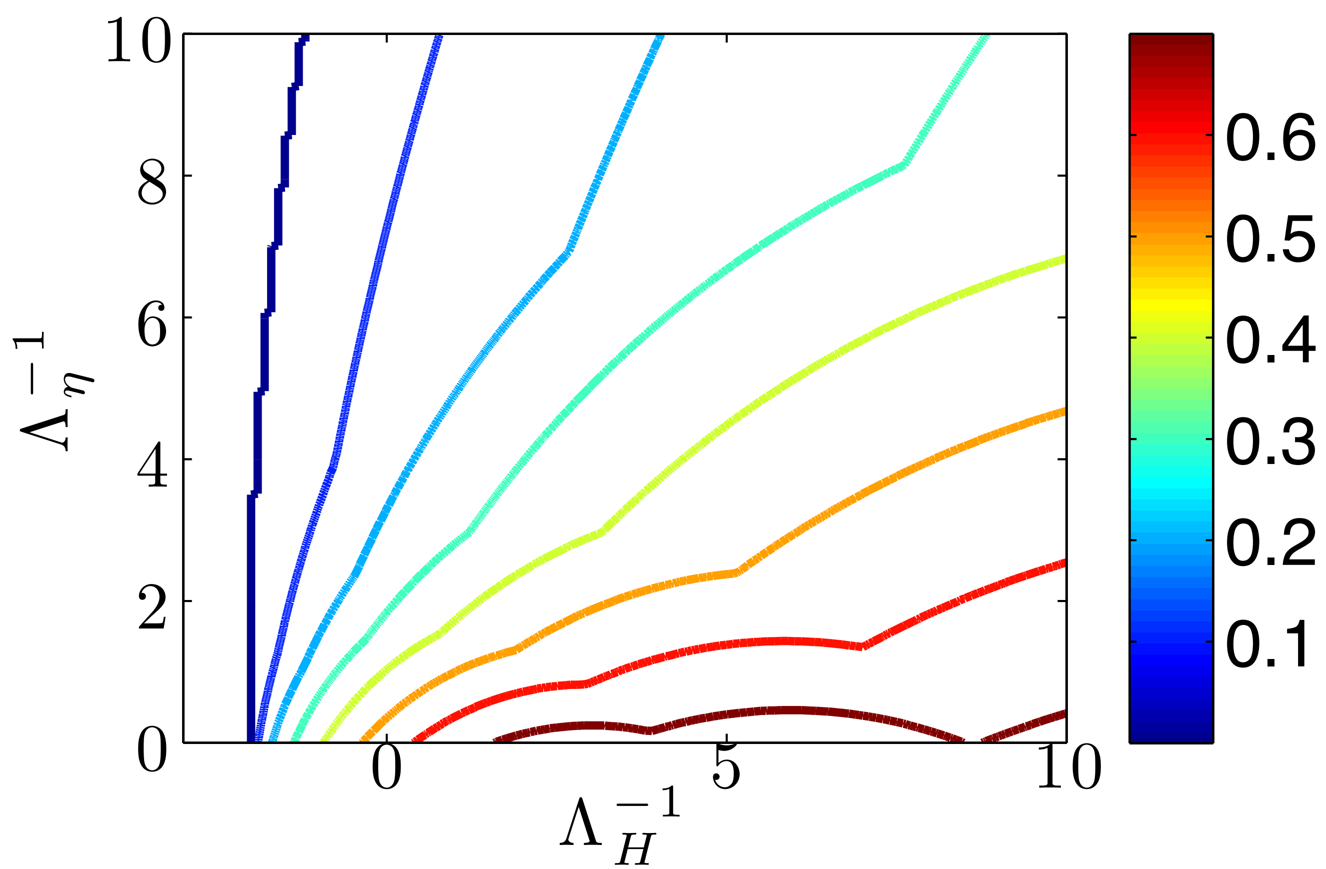
$$\Lambda_\eta = \frac{v_\mathrm{A}^2}{\eta\Omega}$$

Ohmic Elsasser number

$$\Lambda_\mathrm{H} = \frac{en_eB}{\rho c\Omega}$$

Hall Elsasser number

- Growth rate of the most unstable MRI mode



- MRI is more unstable with Hall and $\boldsymbol{\Omega} \cdot \boldsymbol{B} > 0$

Literature: Sano & Stone (2002)

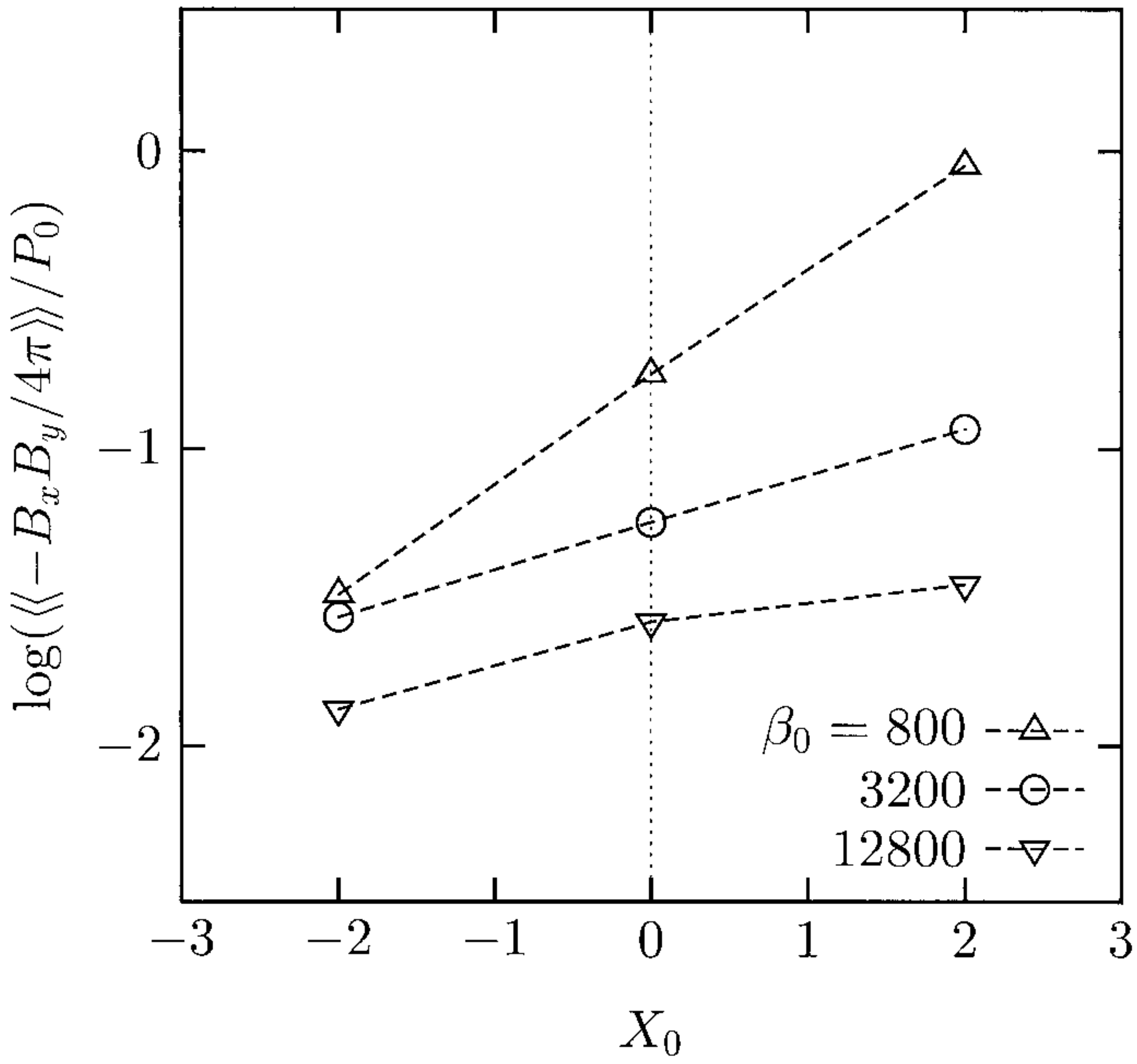


FIG. 5.—Saturation level of the Maxwell stress as a function of the Hall parameter X_0 for the models with $\beta_0 = 800$, 3200, and 12,800. The magnetic Reynolds number is $\mathrm{Re}_{M0} = 1$ for all the models.

→ Hall effect «does nothing»

Literature: Wardle & Salmeron 2012

Hall diffusion and the magnetorotational instability in protoplanetary discs

Mark Wardle[1★] and Raquel Salmeron[2]

[1]*Department of Physics & Astronomy and Research Centre for Astronomy, Astrophysics & Astrophotonics, Macquarie University, Sydney, NSW 2109, Australia*

[2]*Planetary Science Institute, Research School of Astronomy & Astrophysics and Research School of Earth Sciences, Australian National University, Canberra, ACT 2611, Australia*

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ABSTRACT

The destabilizing effect of Hall diffusion in a weakly ionized Keplerian disc allows the magnetorotational instability (MRI) to occur for much lower ionization levels than would otherwise be possible. However, simulations incorporating Hall and Ohm diffusion give the impression that the consequences of this for the non-linear saturated state are not as significant as suggested by the linear instability. Close inspection reveals that this is not actually the case as the simulations have not yet probed the Hall-dominated regime. Here we revisit the effect of Hall diffusion on the MRI and the implications for the extent of magnetohydrodynamic (MHD) turbulence in protoplanetary discs, where Hall diffusion dominates over a large range of radii.

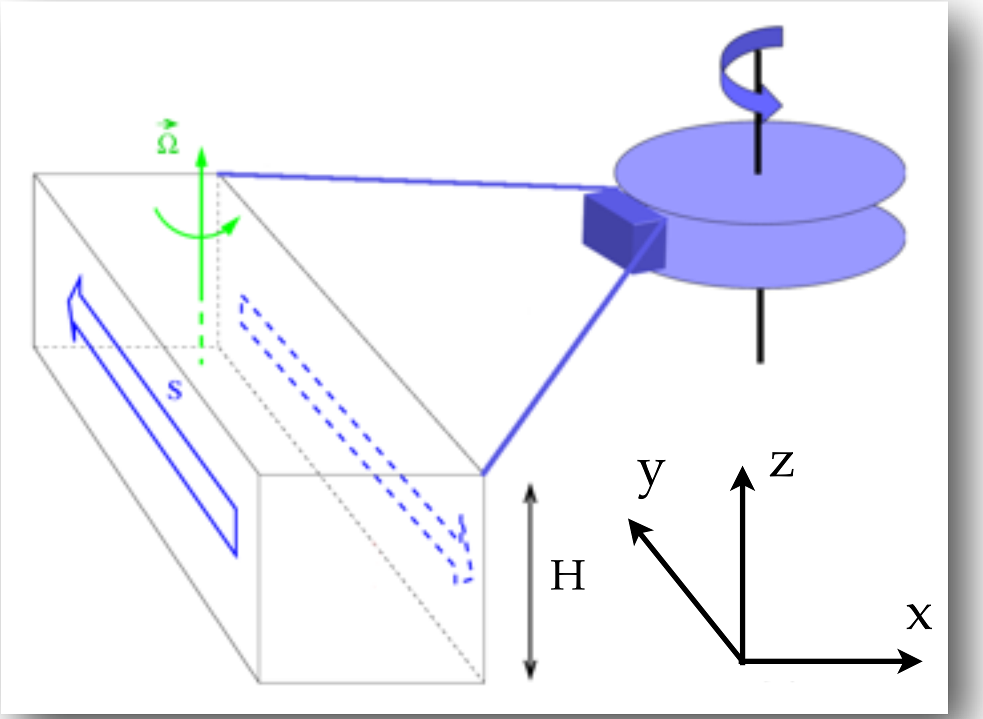
Wardle & Salmeron 2012

→ Simulations did not explore the right regime

The incompressible shearing box model

Separate the mean shear from the fluctuations:

$$\boldsymbol{u} = -q\Omega x \boldsymbol{e_y} + \boldsymbol{v}$$

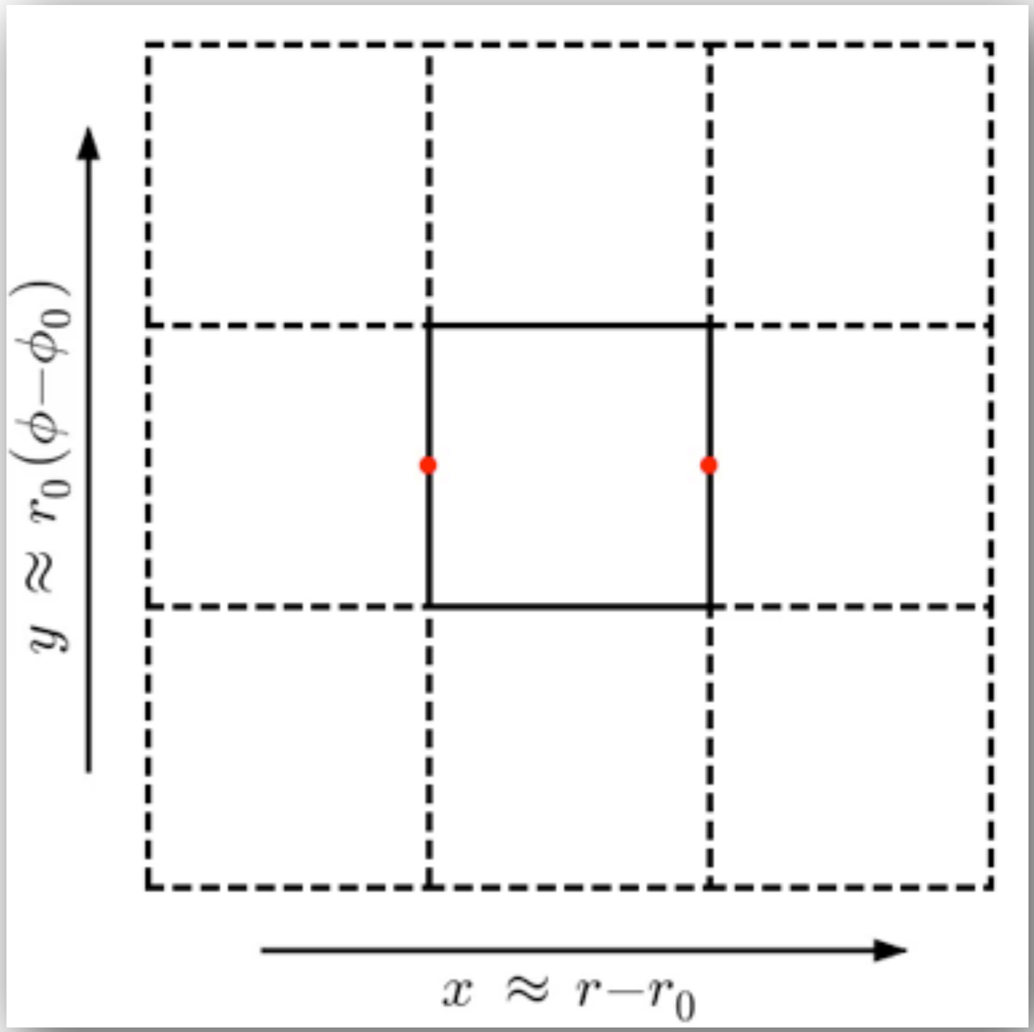


Shearing box equations:

$$\begin{aligned}
\boldsymbol{\nabla} \cdot \boldsymbol{v} &= 0 \\
\partial_t \boldsymbol{v} - q\Omega x \partial_y \boldsymbol{v} + \boldsymbol{v} \cdot \boldsymbol{\nabla} \boldsymbol{v} &= -\boldsymbol{\nabla} P + \boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B} - 2\boldsymbol{\Omega} \times \boldsymbol{v} \\
&\quad + q\Omega v_x \boldsymbol{e_y} + \nu \Delta \boldsymbol{v} \\
\partial_t \boldsymbol{B} - q\Omega x \partial_y \boldsymbol{B} &= \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B} - x_H \boldsymbol{J} \times \boldsymbol{B}) - q\Omega B_x \boldsymbol{e_y} + \eta \Delta \boldsymbol{B}
\end{aligned}$$

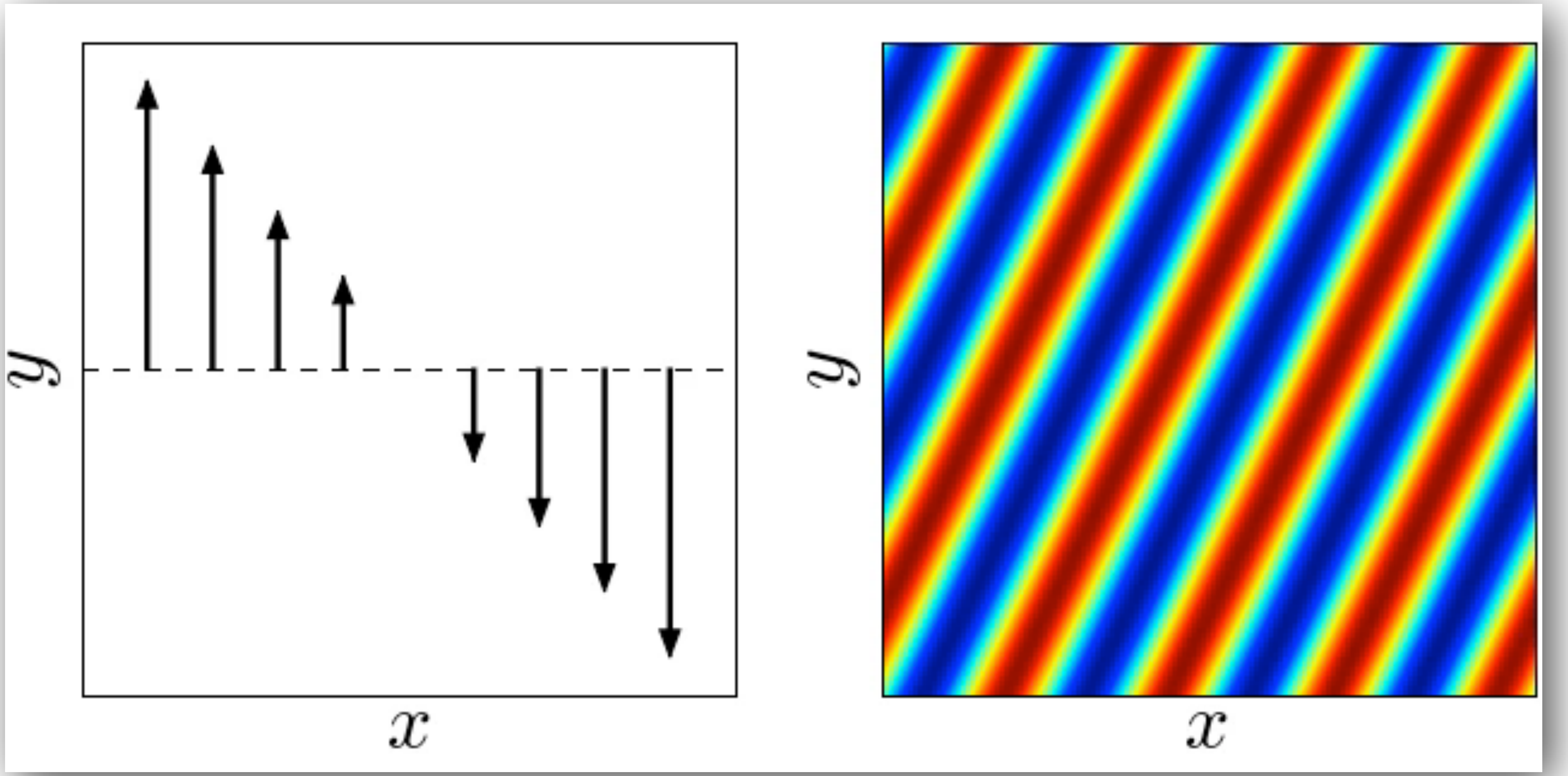
Boundary conditions

- Use shear-periodic boundary conditions= «shearing-sheet»
- Allows one to use a sheared Fourier Basis
- periodic in y and z (non stratified box)



Courtesy T. Heinemann

Spectral methods for shearing boxes
Shearing wave decomposition



Courtesy T. Heinemann

The Snoopy code

a spectral method for sheared flows

- MHD equations solved in a co-moving sheared frame
- Compute non linear terms using the pseudo spectral method
- 3rd order low storage Runge-Kutta integrator
- Non-ideal effects: Ohmic, Hall, ambipolar (coming soon), Braginskii
- Available online http://ipag.osug.fr/~glesur/snoopy.html

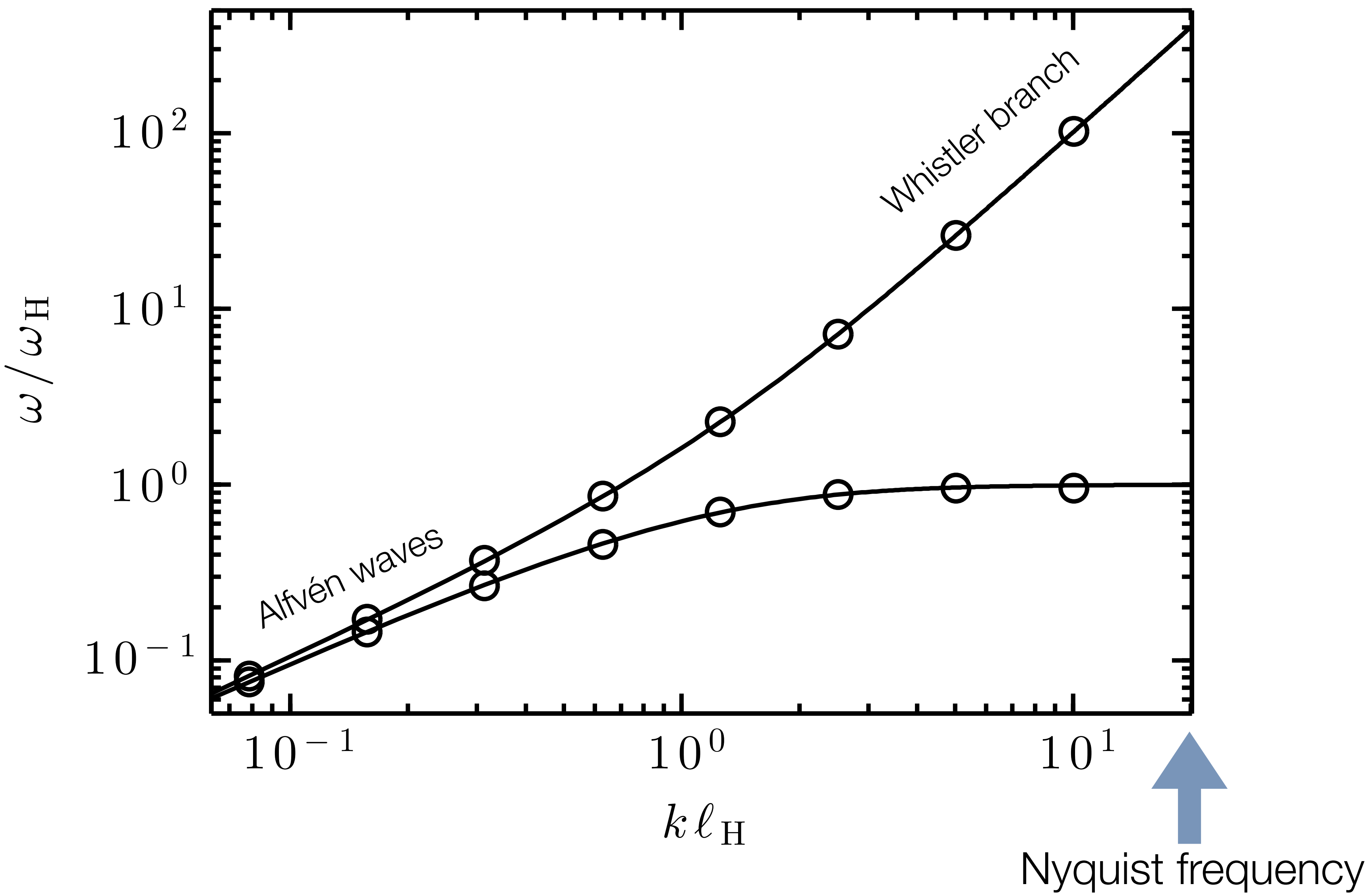
Advantages:

- Shearing waves are computed exactly (natural basis)
- Exponential convergence
- Magnetic flux conserved to machine precision
- Sheared frame & incompressible approximation: no CFL constrain due to the background sheared flow/sound speed.

Testing whistler waves with Snoopy

Falle (2003) «~~Explicit Hall-MHD codes are unconditionally unstable~~»

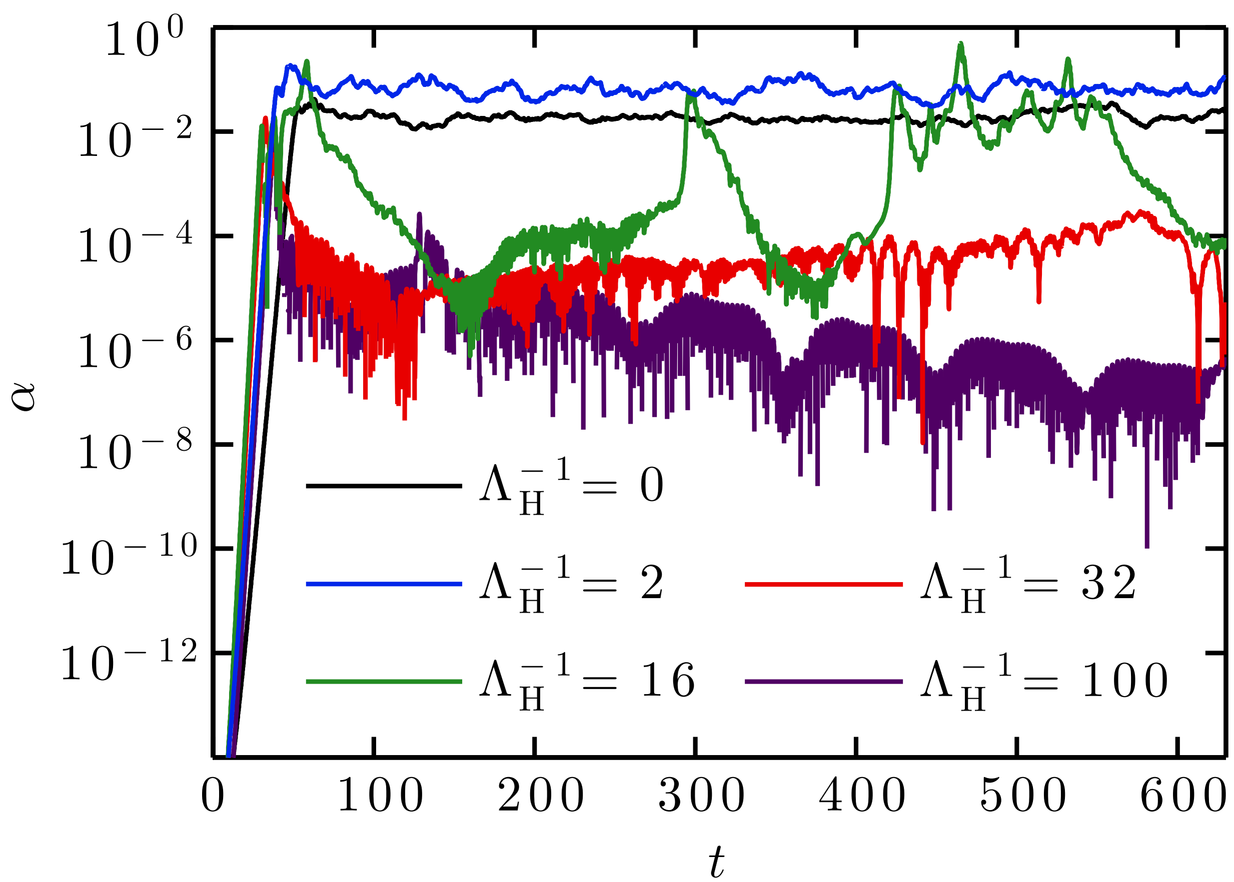
Kunz & Lesur (2013): stable for high order schemes

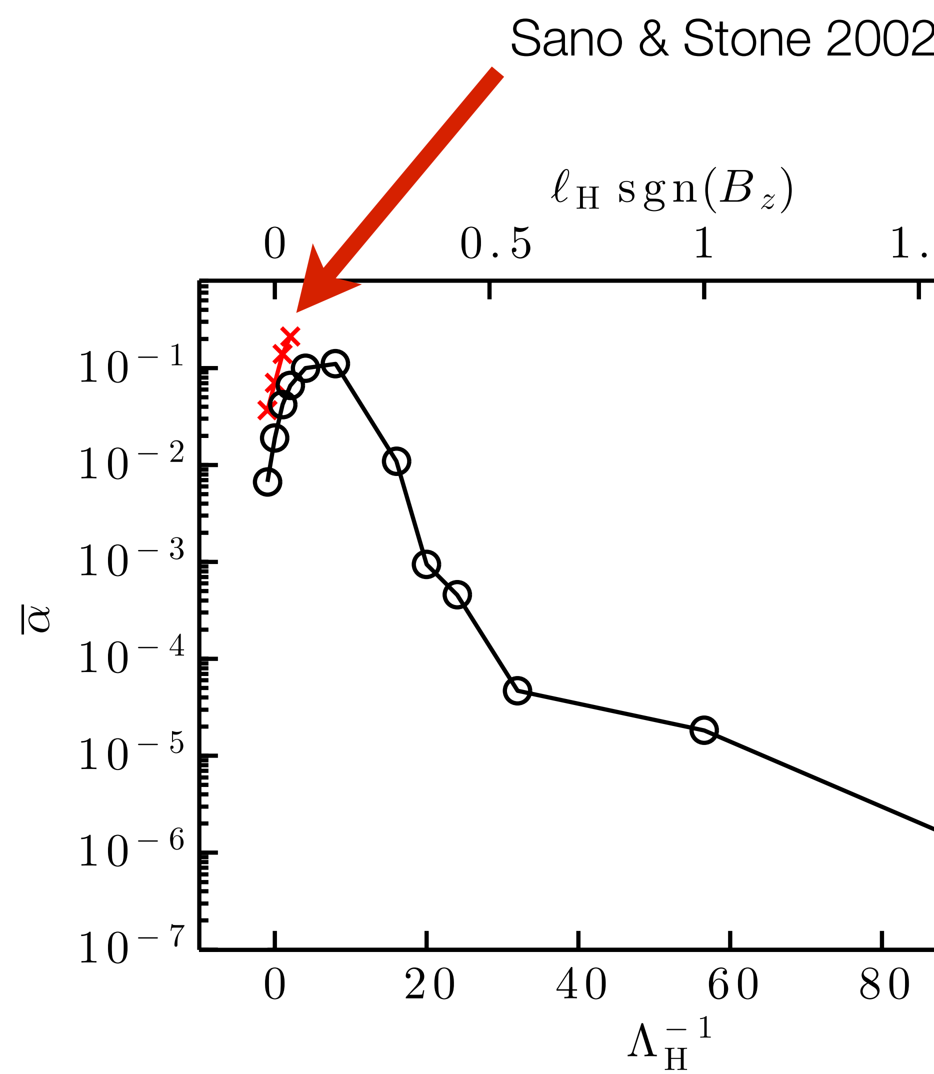


- Whistler waves are well captured down to the grid scale
- Stable explicit scheme (RK3)

Hall-MRI: turbulent viscosity

Does Hall-MRI look like «ideal» MRI?

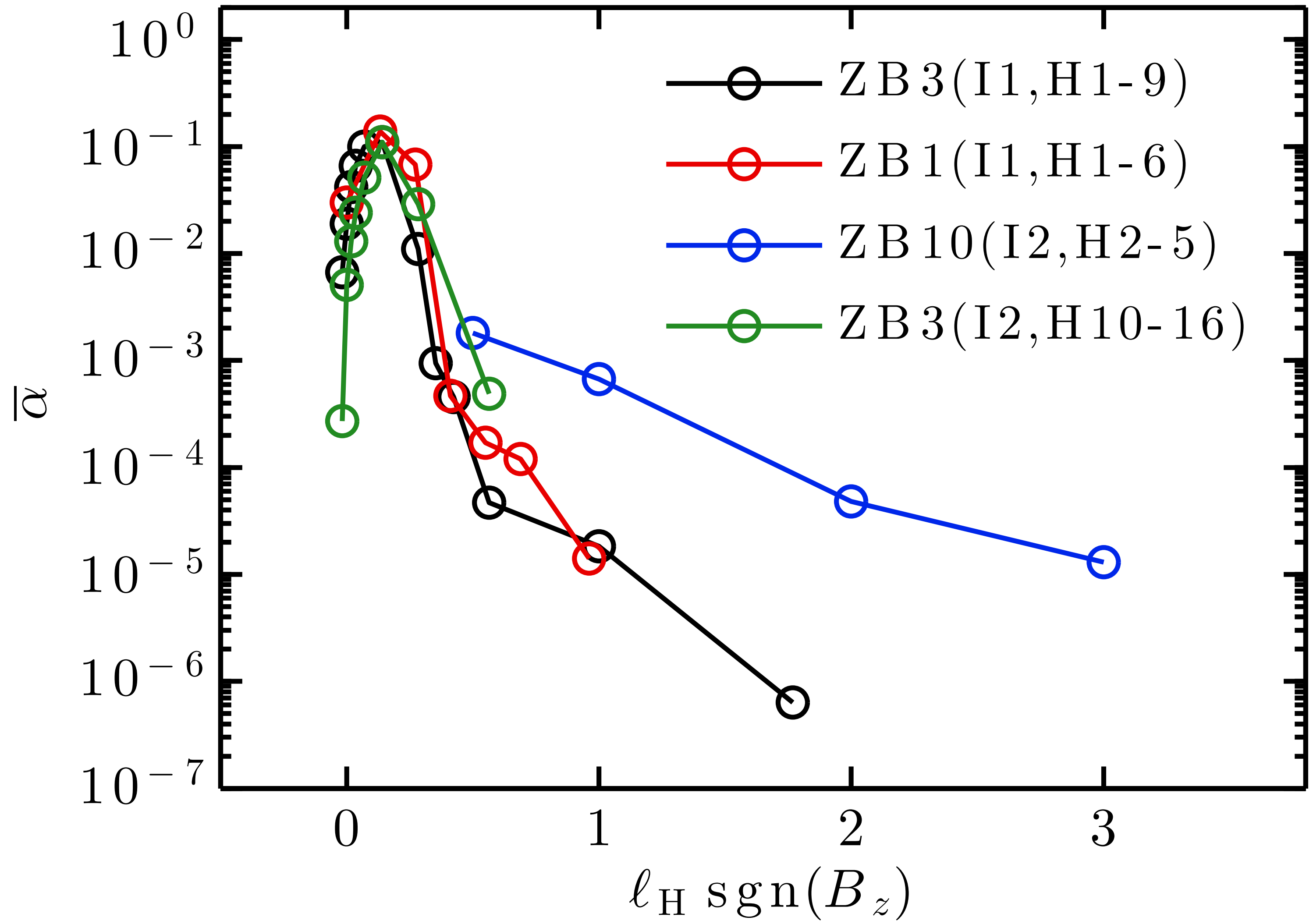




Although a powerful instability is present, Hall-MRI simulations have a very low level of turbulent transport

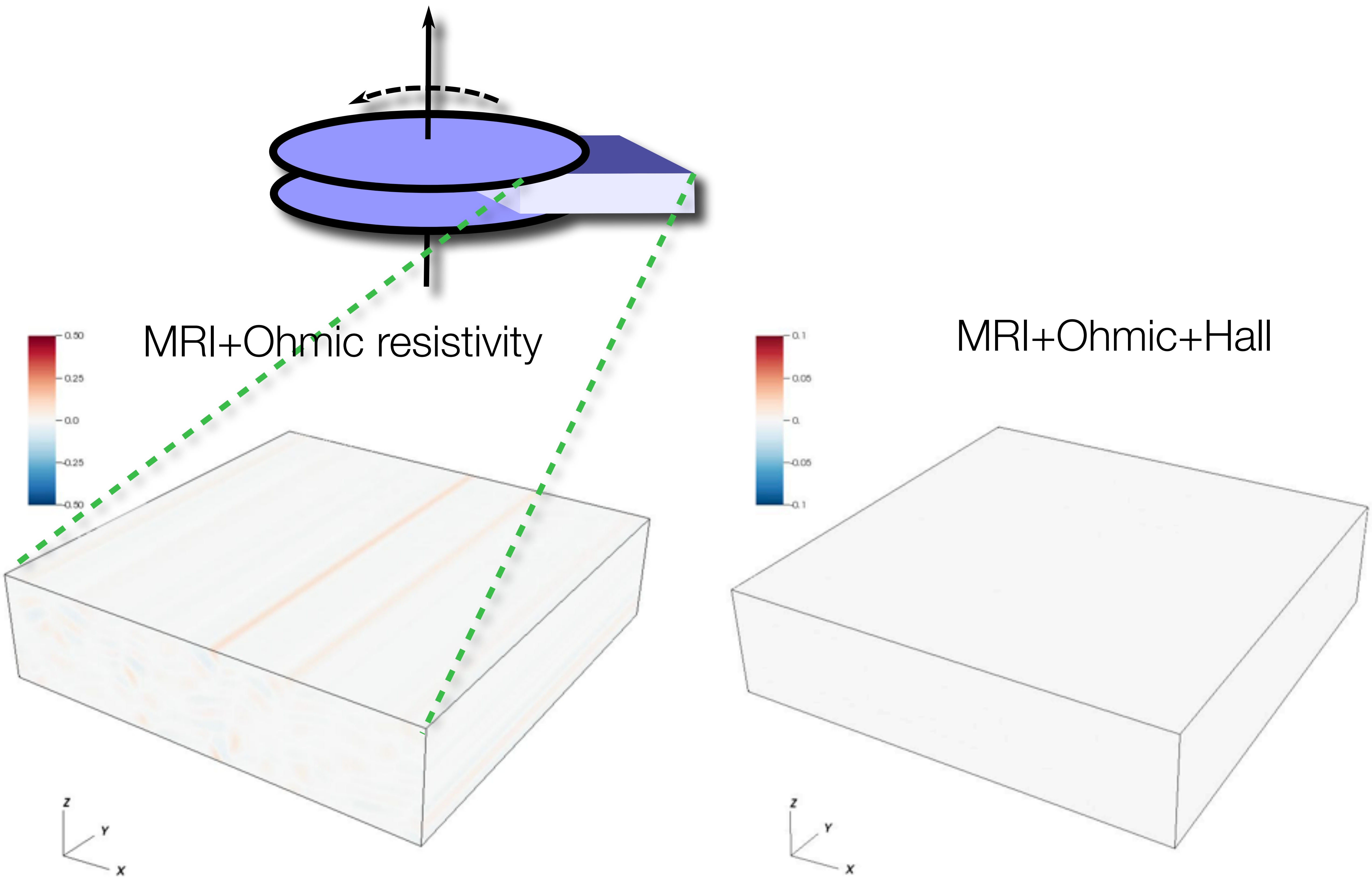
Hall-MRI: turbulent viscosity

Does Hall-MRI look like «ideal» MRI?

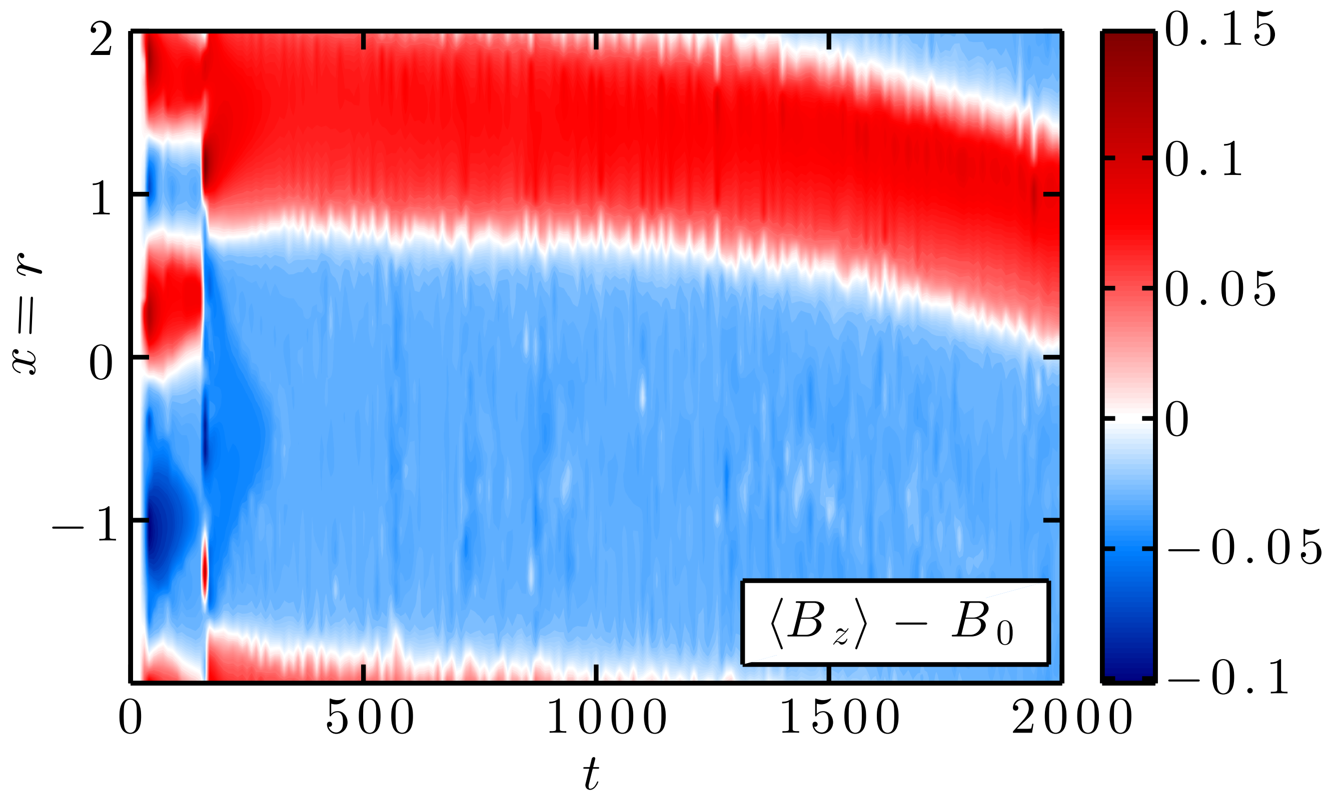


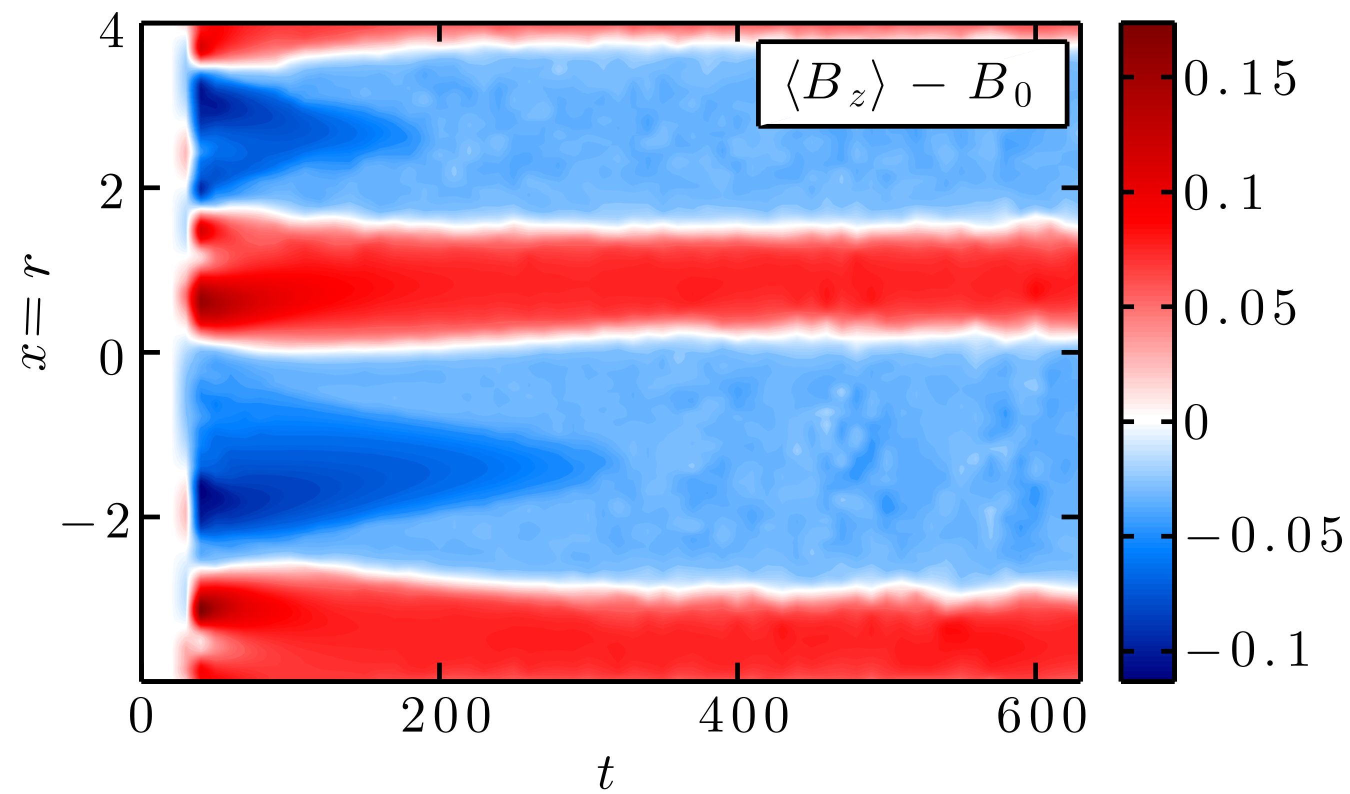
Transport is controlled by $\ell_{\rm H} \equiv \left(\frac{m_{\rm i} c^2}{4\pi e^2 n_{\rm i}}\right)^{1/2} \left(\frac{\rho}{\rho_{\rm i}}\right)^{1/2}$

Hall-MRI animation: Bz



Zonal field structures in Hall-dominated discs





→ Self Organisation!

Mean field model

Equations

- Consider the induction equation with Hall only

$$
\begin{aligned}
\partial_t \langle B_z \rangle &\sim -\frac{1}{e n_e} \partial_x \langle \boldsymbol{J} \times \boldsymbol{B} \rangle_y \\
&\sim -\frac{c}{4\pi e n_e} \partial_x \langle \boldsymbol{B} \cdot \boldsymbol{\nabla} \boldsymbol{B_y} \rangle \\
&\sim -\frac{c}{4\pi e n_e} \partial_x^2 \langle B_x B_y \rangle \\
&\sim \frac{c \Omega^2 H^2 \rho_0}{e n_e} \partial_x^2 \alpha(\langle B_z \rangle)
\end{aligned}
$$

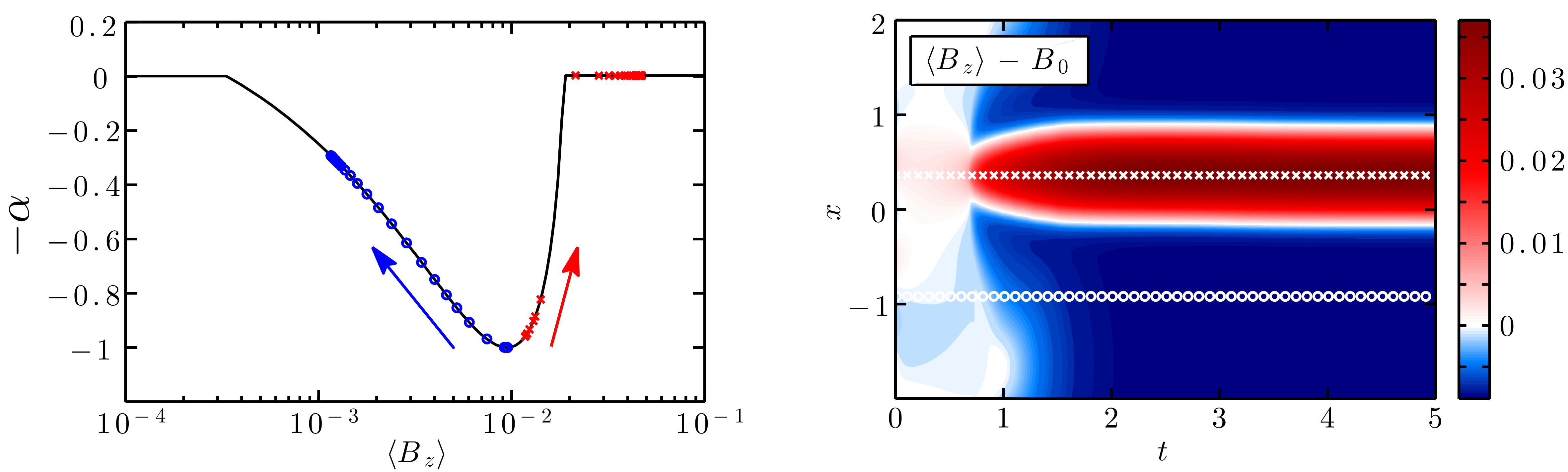
with $\langle \cdot \rangle = \iint dy\, dz$

- Assume: $\langle B_z \rangle = B_0 + \delta B_z$

$$
\partial_t \delta B_z = Q \left(\frac{\partial \alpha}{\partial B_z} \right)_{B_0} \partial_x^2 \delta B_z
$$

- Antidiffusive if $\left(\frac{\partial \alpha}{\partial B_z} \right)_{B_0} < 0\,!$

Mean field model
Application



Mean field model reproduces self-organisation behaviour

Conservation laws in Hall-MHD

Induction

$$\partial_t \boldsymbol{B} = \boldsymbol{\nabla} \times \left(\boldsymbol{v} \times \boldsymbol{B} - \frac{\boldsymbol{J} \times \boldsymbol{B}}{e n_e} \right)$$

Vorticity

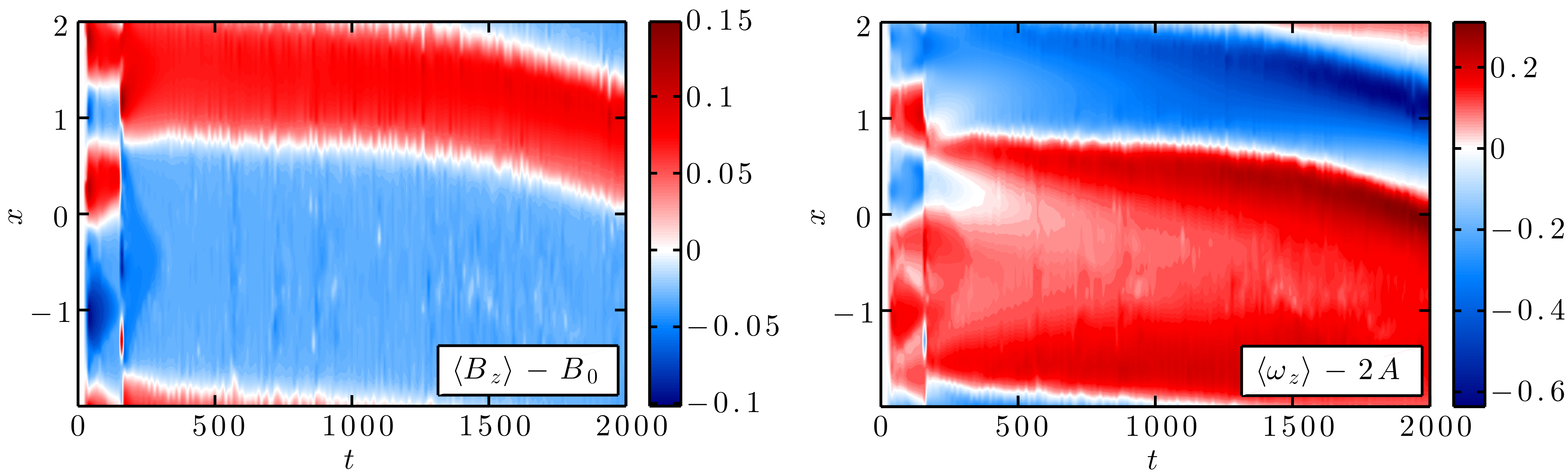
$$\partial_t \boldsymbol{\omega} = \boldsymbol{\nabla} \times \left(\boldsymbol{v} \times \boldsymbol{\omega} + \frac{\boldsymbol{J} \times \boldsymbol{B}}{c\rho} \right)$$

$$\boldsymbol{\omega}_C = \boldsymbol{\omega} + \frac{e \boldsymbol{B} n_e}{\rho c}$$

$$\partial_t \boldsymbol{\omega}_C = \boldsymbol{\nabla} \times \left(\boldsymbol{v} \times \boldsymbol{\omega}_C \right)$$

- Canonical vorticity behaves like magnetic field in ideal MHD
- Field line redistribution implies a redistribution of vorticity in the flow

Strong zonal flows in PP discs



- long-lived zonal flows are associated to Hall-MRI
- Good for planet formation?

Conclusions

- Hall MRI does not saturate like ideal MRI
  - Turbulent transport reduced by 2-3 orders of magnitude
  - Production of zonal fields
  - Mean field theory captures this behaviour
  - Zonal flows produced by zonal field regions: dust trapping regions?

- Open questions:
  - Stratification, compressibility?
  - Vertical ionisation profile?

References:

- Sano & Stone (2002), ApJ, 577, 534–553
- Wardle & Salmeron (2012), MNRAS, 422, 2737–2755
- Kunz & Lesur (2013), MNRAS, accepted