Charged particle transport in turbulent media

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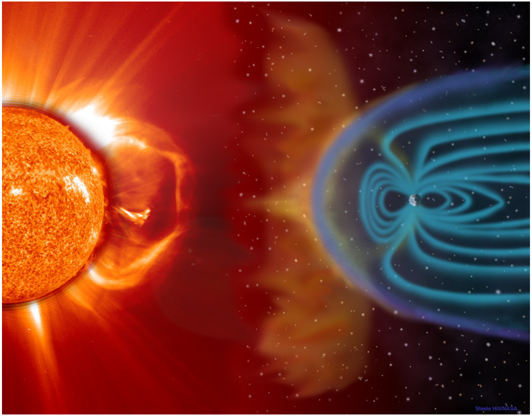
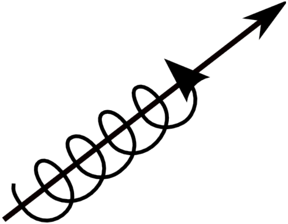
Motivation



- Particle transport in heliosphere and ISM
- What is the microphysics of transport?
- Turbulent magnetic fields $\Rightarrow$ charged particle scattering



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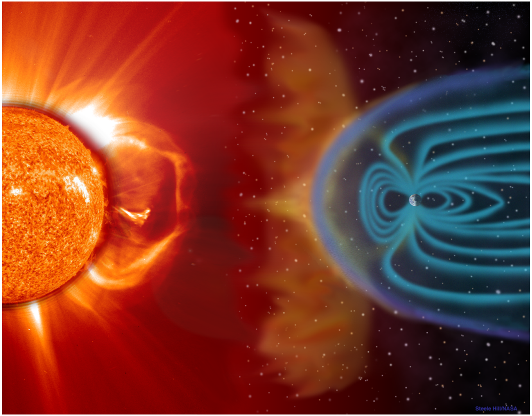

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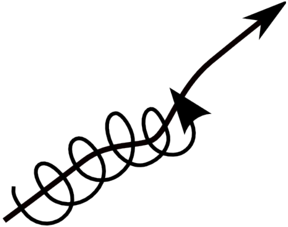


Fokker-Planck Equation

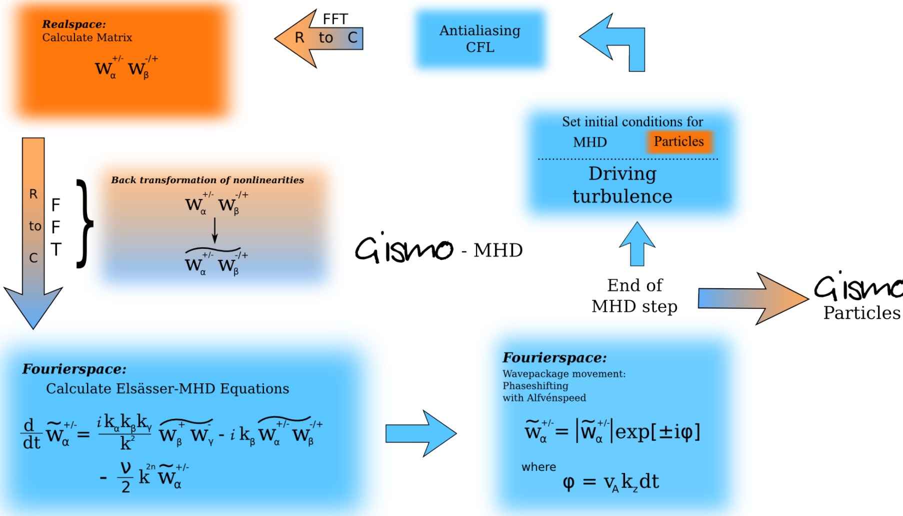
- Particle transport is described by the Fokker-Planck equation
- Vlasov equation in gyrocenter coordinates

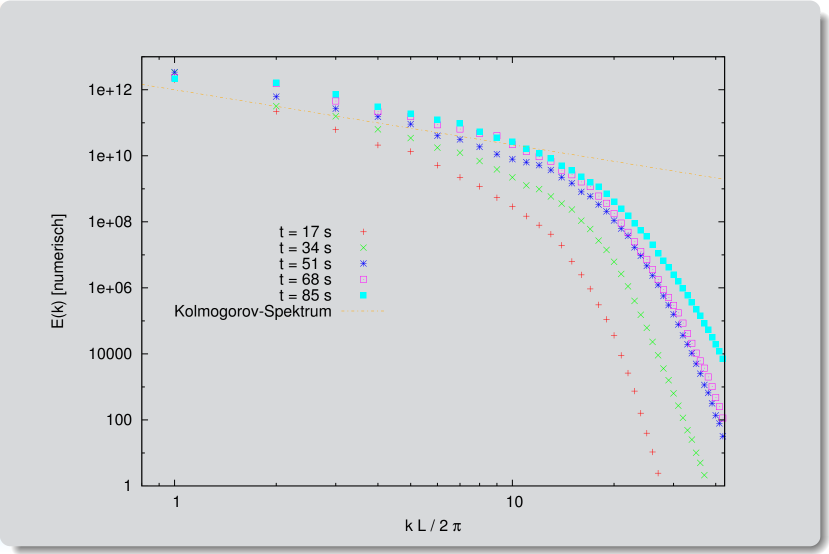
Fokker-Planck-Equation

$$\frac{\partial F_T}{\partial t} + v\mu \frac{\partial F_T}{\partial Z} - \epsilon\Omega \frac{\partial F_T}{\partial \phi} = S_T(X_\sigma, t) + \frac{1}{p^2} \frac{\partial}{\partial X_\sigma} \left(p^2 D_{X_\sigma X_\nu} \frac{\partial F_T}{\partial X_\nu} \right)$$

- Diffusion-convection equation
- Pitch angle diffusion coefficient $D_{\mu\mu}$ particularly important
- Mean free path $\lambda_{\|}$ derived from that

MHD Simulations





E(k) [numerisch]
1e+12
1e+10
1e+08
1e+06
10000
100
1
1
10
k L / 2 π
t = 17 s
t = 34 s
t = 51 s
t = 68 s
t = 85 s
Kolmogorov-Spektrum

Test particle Simulations

Interpolate **E** and **B** at the individual test particle position with 3D splines

$$S(x,y,z) = c_{j,k,l}\, x^j\, y^k\, z^l$$

Calculate Lorentz force for each test particle:

$$\mathbf{F} = q\,(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{B})$$

Numerical method:
Boris-Push

Gismo - Particles

Update to new particle velocities and positions

Boundary check:
periodic thread transfer

Gismo
MHD

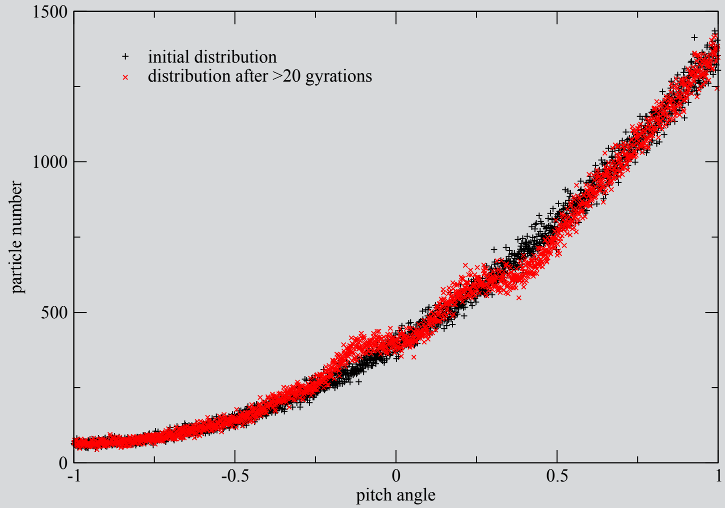
Calculate **E** and **B** :

$$\mathbf{B} = \frac{1}{2}(\mathbf{w}^- - \mathbf{w}^+) + v_A \mathbf{e}_z$$

$$\mathbf{u} = \frac{1}{2}(\mathbf{w}^- + \mathbf{w}^+)$$

$$\mathbf{E} = -\frac{1}{c}\mathbf{u} \times \mathbf{B}$$

Test particle Simulations

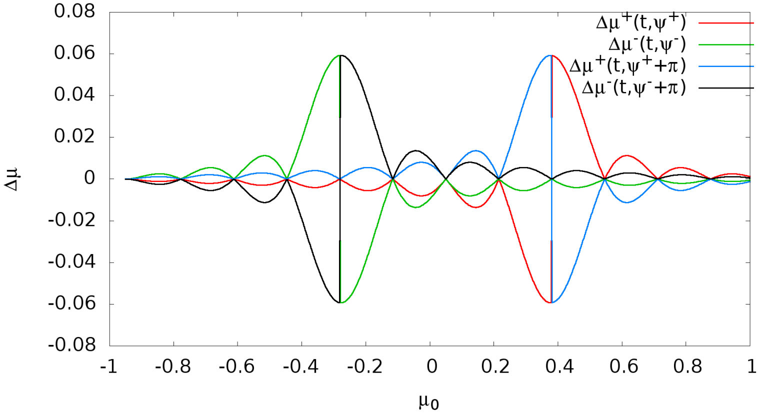


Gisv
MH

Testing the interaction of particles with a single wave

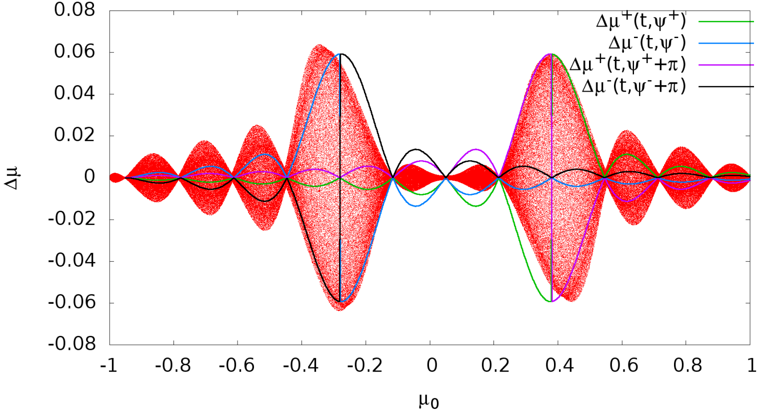
- Inject isotropic, monoenergetic particle distribution
- Assume background plasma with one Alfvén wave
- Plot $\Delta\mu(t)$ vs. μ_0

Simple wave-particle resonance



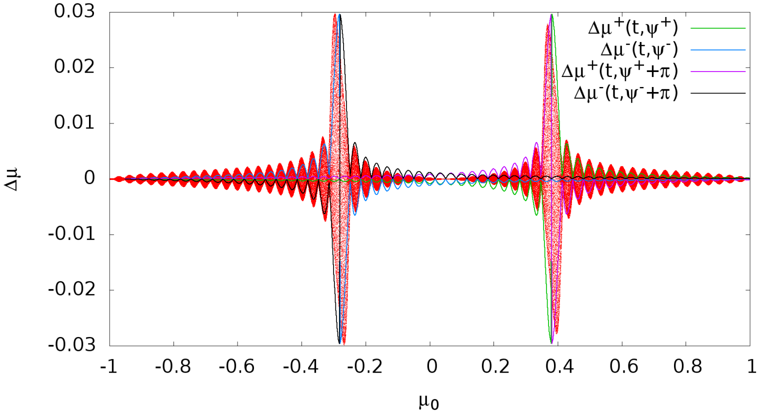
2 gyrations, wave amplitude $\delta B/B_0 = 0.01$, QLT prediction

Simple wave-particle resonance



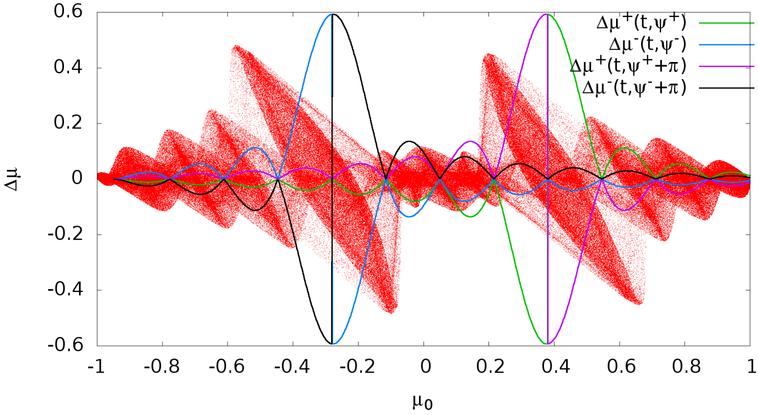
2 gyrations, wave amplitude $\delta B/B_0 = 0.01$

Simple wave-particle resonance



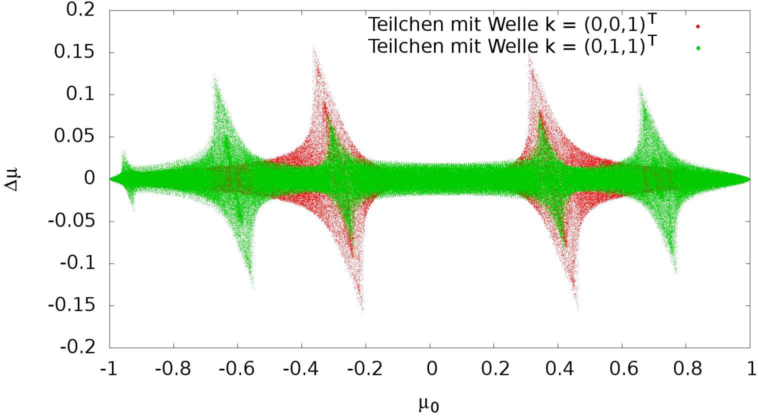
10 gyrations, wave amplitude $\delta B/B_0 = 0.001$

Simple wave-particle resonance



2 gyrations, wave amplitude $\delta B/B_0 = 0.1$

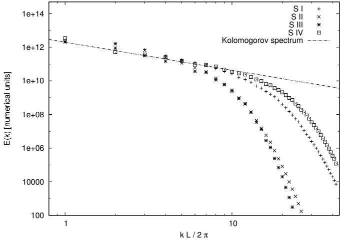
Simple wave-particle resonance



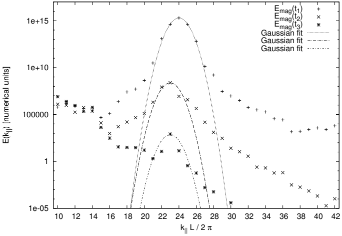
50 gyrations, wave amplitude $\delta B / B_0 = 0.001$

Testing the interaction of particles with turbulence

Undisturbed turbulence



Excited turbulence

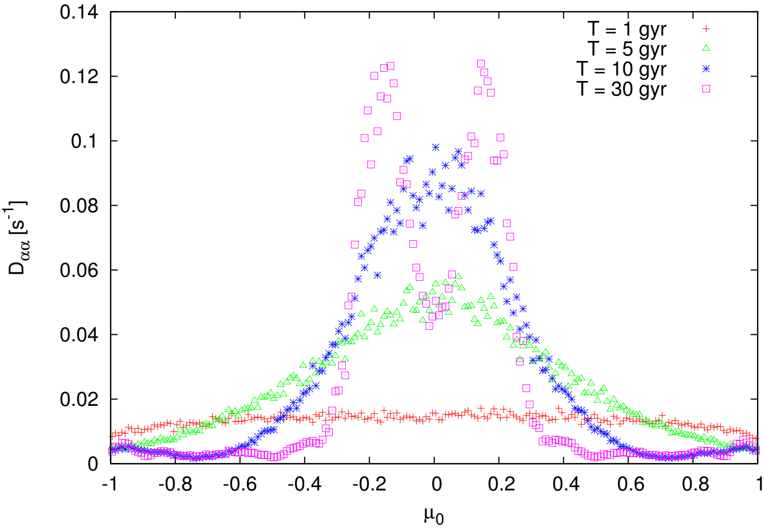


Not only $\mu_0 - \Delta\mu$ plot, but additional

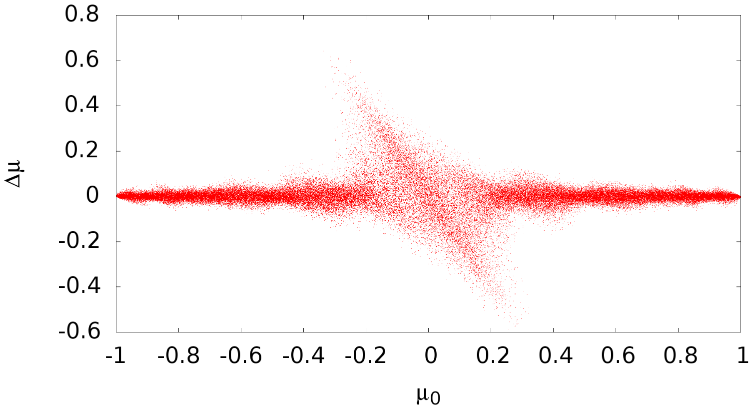
$$D_{\alpha\alpha} = \lim_{t \to \infty} \frac{\langle \alpha^2 \rangle}{2\Delta t}$$

scattering angle diffusion coefficient

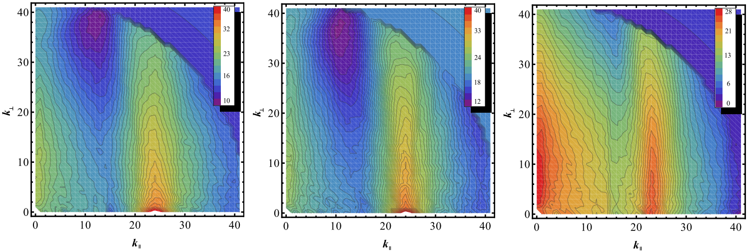
Scattering in MHD turbulence



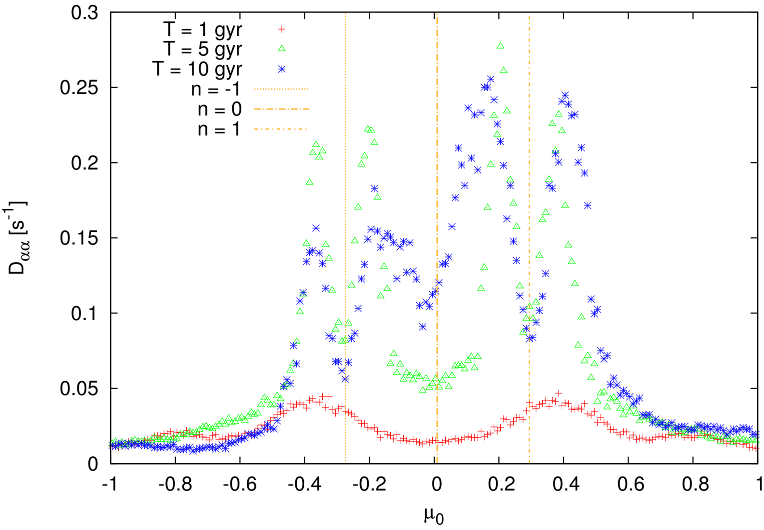
Scattering in MHD turbulence



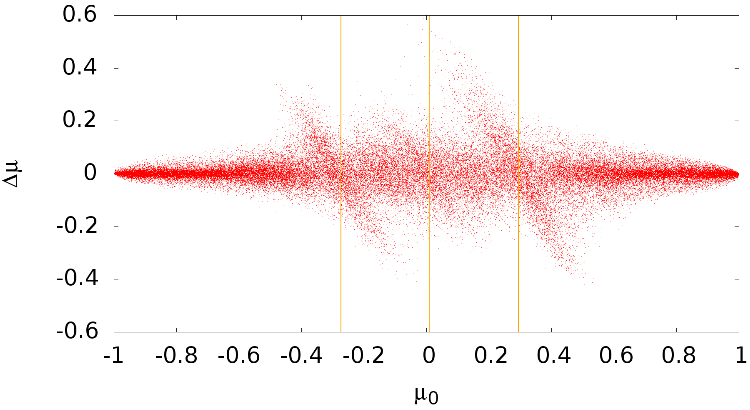
MHD excitation



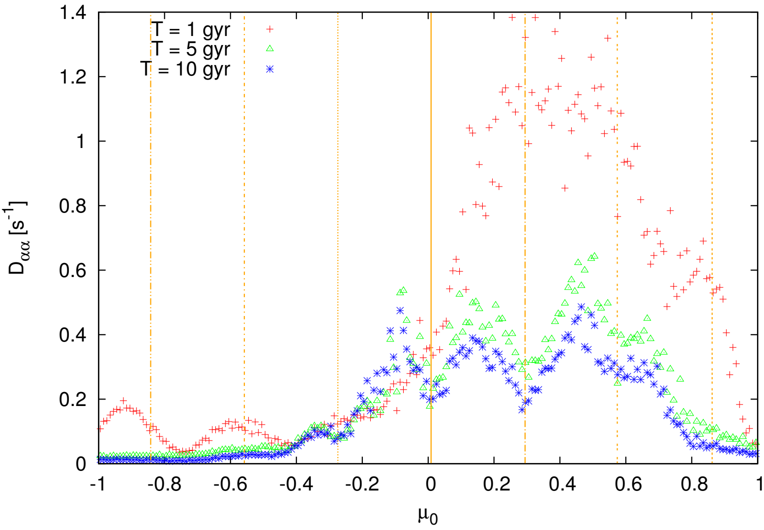
Scattering with excited modes I $k_{\parallel}$



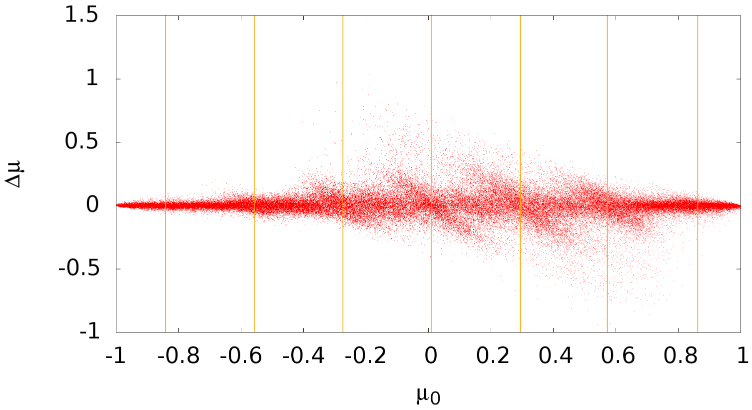
Scattering with excited modes I $k_{\|}$



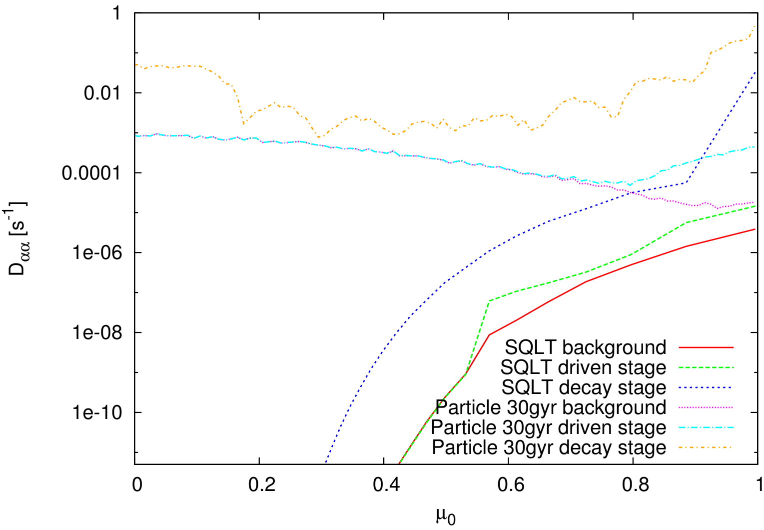
Scattering with excited modes II $k_\parallel$, $k_\perp$



Scattering with excited modes II $k_{\parallel}, k_{\perp}$



Quasilinear comparison



Turbulent transport

- Good agreement of testparticle and QLT results in $D_{\alpha\alpha}$
- SQLT misses Cherenkov resonance ($n = 0$)
- Limited spectrum yields resonance gap
- Finite simulation time results in broadened resonances

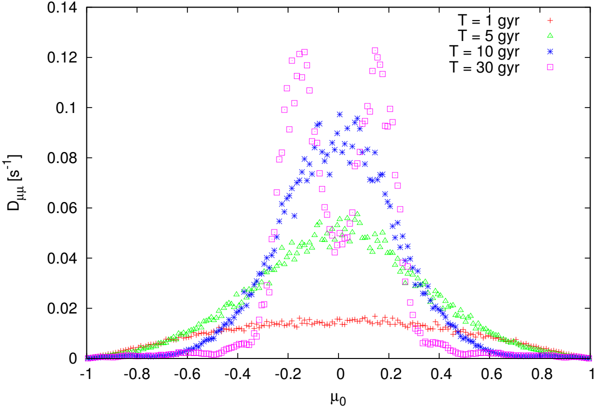
Derivation of coefficients

$\mu - \Delta\mu$ plots show the physics of scattering
For further use $D_{\mu\mu}$ or $D_{\alpha}\alpha$ is needed!
Determination is - as seen - flawed, especially for strong turbulence.

Running coefficient

Derivation of $D_{\mu\mu}$ via its definition:

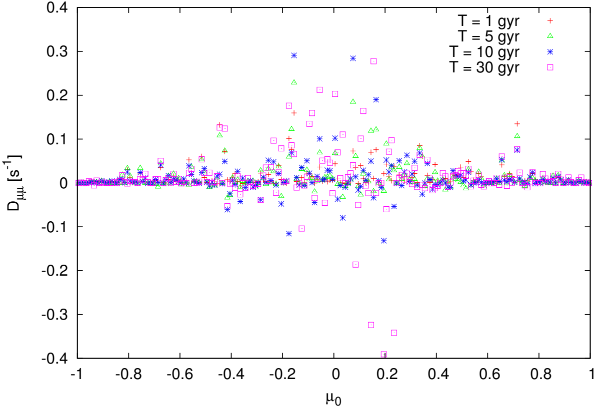
$$D_{\mu\mu} = \lim_{t\to\infty} \frac{(\Delta\mu)^2}{2\,\Delta t} \overset{t\gg t_0}{\approx} \frac{(\Delta\mu)^2}{2\,\Delta t}$$



Kubo-Formalismus

Integration along trajectories:

$$D_{\mu\mu} = \sum_T \frac{1}{N_T} \sum_{t_0=0}^{t} \Delta t \, \dot{\mu}(t_0)\dot{\mu}(t)$$



Discretisation of the diffusion equation for each μ^n:

$$\partial_t f = \frac{D_{\mu\mu}^{n+1} - D_{\mu\mu}^{n-1}}{2 \cdot \Delta\mu} \partial_\mu f + D_{\mu\mu}^n \partial_{\mu\mu} f$$

Tridiagonal matrix equation:

$$\begin{pmatrix} \partial_{\mu\mu} f^0 & \frac{\partial_\mu f^0}{2\Delta\mu} & 0 & 0 \\ -\frac{\partial_\mu f^1}{2\Delta\mu} & \partial_{\mu\mu} f^1 & \ddots & 0 \\ 0 & \ddots & \ddots & \frac{\partial_\mu f^{n-1}}{2\Delta\mu} \\ 0 & 0 & -\frac{\partial_\mu f^n}{2\Delta\mu} & \partial_{\mu\mu} f^n \end{pmatrix} \cdot \begin{pmatrix} D_{\mu\mu}^0 \\ D_{\mu\mu}^1 \\ \vdots \\ D_{\mu\mu}^n \end{pmatrix} = \begin{pmatrix} \partial_t f^0 \\ \partial_t f^1 \\ \vdots \\ \partial_t f^n \end{pmatrix}$$

Matrix inversion with standard methods!

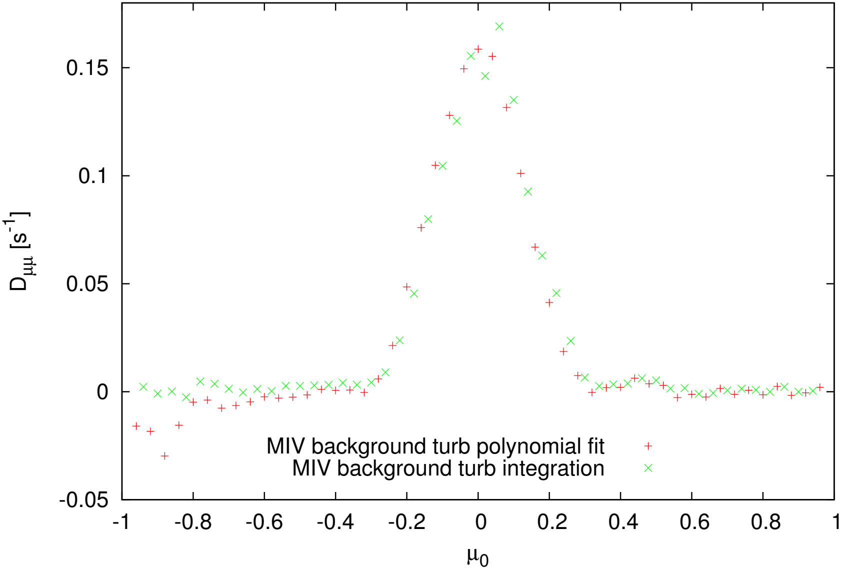
Integration method

- Ensemble averaging over several simulations would be statistically correct, but expensive
- Fitting or smoothing is usually required
- Integration of the diffusion equation over μ smoothes time derivatives

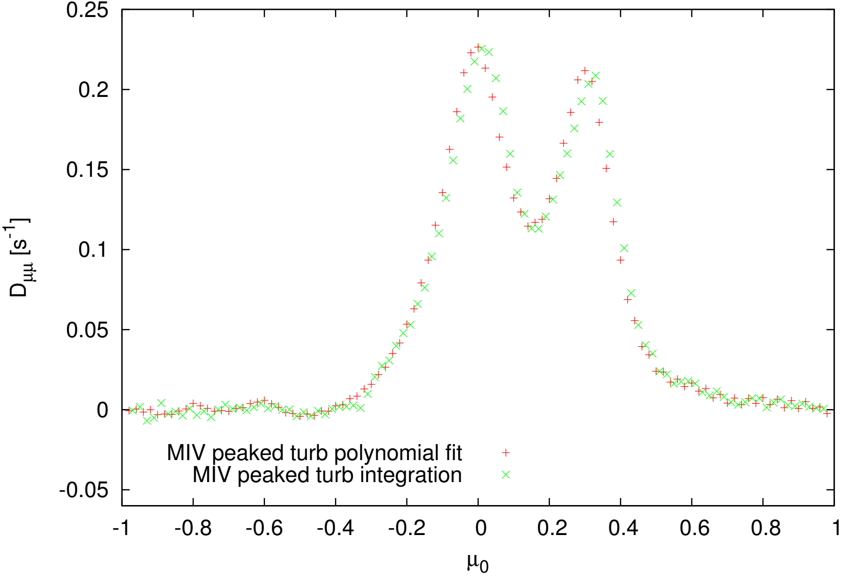
$$\int_{-1}^{\mu} \frac{\partial f_T(\mu, t)}{\partial t} \mathrm{d}\mu = D_{\mu\mu}(\mu) \frac{\partial f_T(\mu, t)}{\partial \mu} = -j_\mu(\mu)$$

- Diffusion coefficients are calculated via the integration of μ-stream

Results



0.15
0.1
0.05
0
-0.05
$D_{\mu\mu}$ [s^{-1}]
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
μ_0
MIV background turb polynomial fit
MIV background turb integration


0.25
0.2
0.15
0.1
0.05
0
-0.05
-1
-0.8
-0.6
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
$D_{\mu\mu}$ [s^{-1}]
μ_0
MIV peaked turb polynomial fit
MIV peaked turb integration

Wave-Particle Interaction with PiC

Physical Background:

magnetized plasma in heliosphere / solar wind

- thermal background plasma
- non thermal component of energetic particles
- resonant scattering of particles on plasma waves
- Fermi-II-acceleration

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magnetized plasma in heliosphere / solar wind

- thermal background plasma
- non thermal component of energetic particles
- resonant scattering of particles on plasma waves
- Fermi-II-acceleration

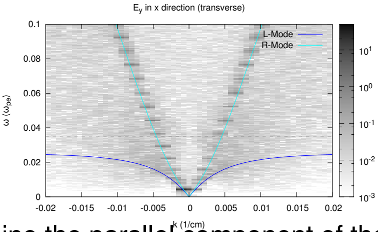
Numerical Setting:

- magnetized thermal background plasma
- one excited wave mode
- population of relativistic test particles

PiC simulations

Simulation Setup:

- excitation of low frequency wave (ideal case: *Alfvén wave*)
  $\rightarrow$ huge number of cells and timesteps required

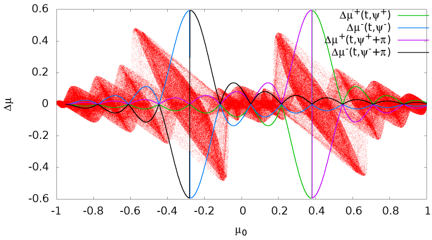


- use resonance condition to determine the parallel component of the test particles' velocities
  $\rightarrow$ parameters k_w, ω_w and Ω_i give constraints
  $\rightarrow$ resonant pitch angle μ_{res} is free
- initialize monoenergetic test particles ($|\boldsymbol{v}| = v_{||}$) with isotropic angular distribution
  $\rightarrow$ resonant scattering only for particles with $\mu = \mu_{res}$

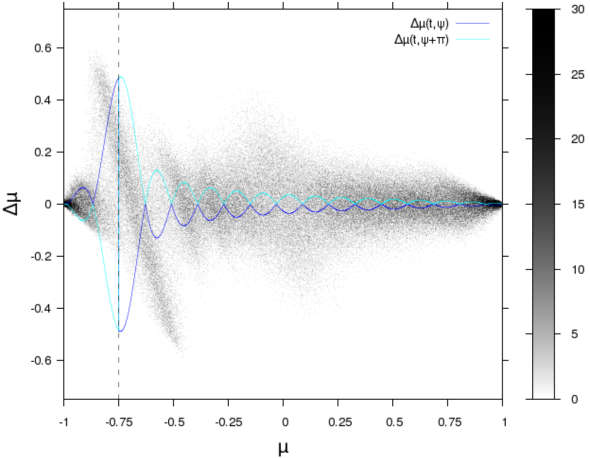
Pitch Angle Diffusion

Scatter Plots:

- peaks at $\pm\mu_{\text{res}}$
- left peak: lefthanded wave
- right peak: righthanded wave
- ballistic transport (smaller peaks)
- QLT approximation

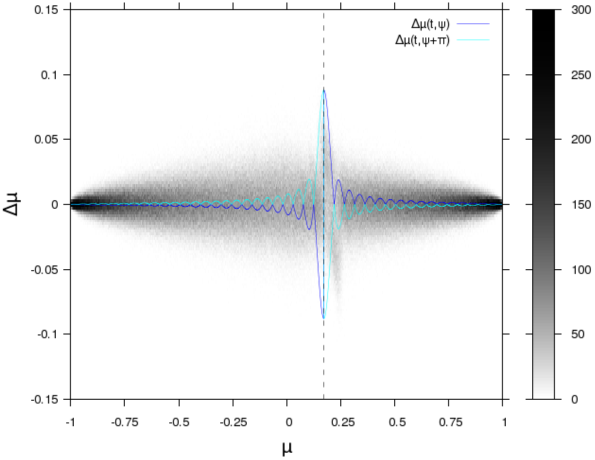


Pitch Angle Diffusion with PiC



lefthanded wave mode

Pitch Angle Diffusion with PiC



righthanded wave mode

Some Comments

- basic characteristics of resonant wave-particle interaction are found
- results are comparable to MHD simulations
- deviations from QLT due to
  $\rightarrow$ dispersive wave modes?
  $\rightarrow$ thermal broadening of test particle population?
  $\rightarrow$ 1D and 2D simulation effects?
- full 3D simulations are yet to come
  $\rightarrow$ waiting for computing time...