A New Numerical Scheme of Relativistic Magnetohydrodynamics with Dissipation and its Applications

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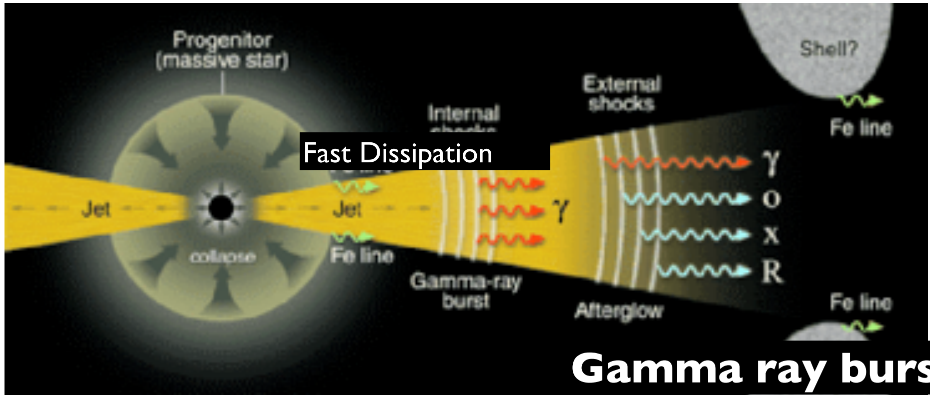
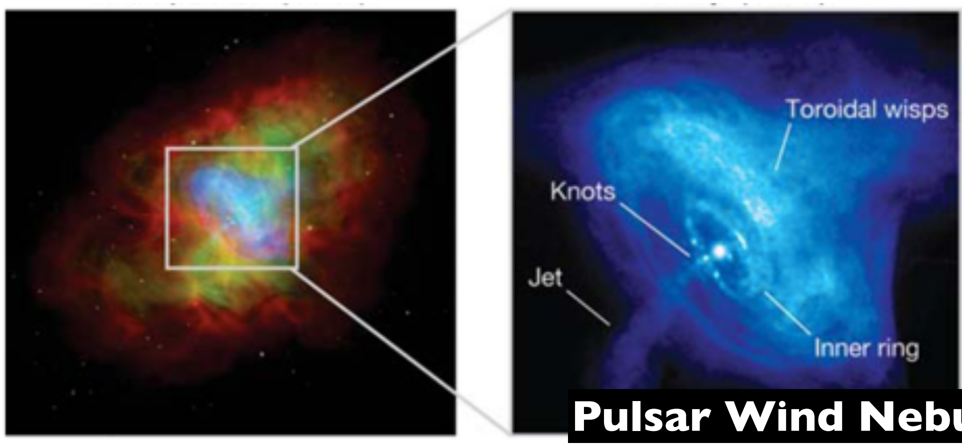
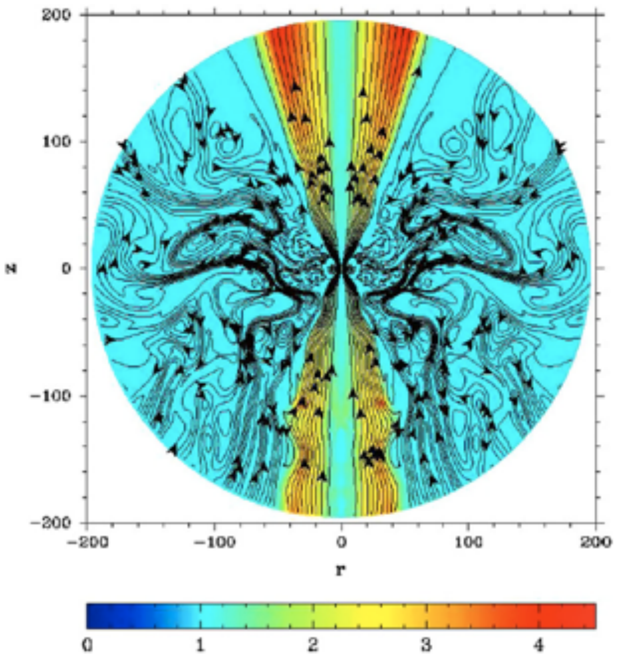
1.1. Poynting Dominated Plasma of Astrophysical Phenomena



Gamma ray burst



Pulsar Wind Nebula



Relativistic Jet

ref) M.V.Barkov & A.N.Baushev 2011
New Astronomy 16, 46-56

A New Method

ref) MT & S, Inutsuka., (2011), JCP, 230, 7002
MT & T, Inoue., (2011), ApJ, 735, 113
Y. Akamatsu, S. Inutsuka, C. Nonaka, MT, arXiv1302.1665

2.1. Difficulty of relativistic resistive MHD

in non-relativistic MHD,
resistivity can be considered as follows:

$$\frac{\partial}{\partial t}\mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times \left(\frac{\mathbf{J}}{\sigma}\right)$$

$$\mathbf{J} = \nabla \times \mathbf{B}$$

⟺ evolution of electric field E is neglected !

=> covariance of Maxwell equation is broken !!

2.2. Unphysical Mathematical Divergence

Dispersion relation of the parabolic energy equations is

$$\Gamma' + \alpha k'^2 = 0 \qquad (\alpha > 0)$$

Lorentz transformation into Lab frame:

$$\alpha\gamma^2\beta^2\Gamma^2 - (\gamma - 2i\alpha\gamma^2\beta k)\Gamma - \alpha\gamma^2 k^2 + i\gamma\beta k = 0$$

Solutions $\Gamma_\pm$ must satisfy the following conditions

$$\begin{cases} \Gamma_{R+} + \Gamma_{R-} = \dfrac{\rho + p}{\gamma\beta^2\eta} > 0 \\ \Gamma_{R+} \cdot \Gamma_{R-} = -\left(\Gamma_{I+} - \dfrac{k}{\beta}\right)^2 < 0 \end{cases}$$

One solution is always unstable !!

2.3. A Solution —Telegrapher Eq.—

Considering correction terms including time derivatives

$$\begin{cases} \partial_t Q + \nabla \cdot \mathbf{F} = 0, & \text{:Evolution of fluid} \\ \partial_t \mathbf{F} = -\dfrac{1}{\tau}(\mathbf{F} + \eta \nabla Q). & \text{:Evolution of dissipation} \end{cases}$$

The above equations reduce to

$$\partial_t^2 Q + \frac{1}{\tau}\partial_t Q - \frac{\eta}{\tau}\triangle Q = 0.$$

Telegrapher Equation ⇒ **Causal !!**

2.4. Basic equations of resistive RMHD

To satisfy causality,
evolution of electric field has to be considered !!

→ basic equations are:

$$\partial_t \begin{pmatrix} \gamma\rho \\ \rho h \gamma^2 v^i + (\mathbf{E}\times\mathbf{B})^i \\ \rho h\gamma^2 - p + \frac{1}{2}(E^2+B^2) \end{pmatrix} + \partial_j \begin{pmatrix} D v^i \\ m^i v^j + p g^{ij} - E^i E^j - B^i B^j + \frac{1}{2}(E^2+B^2) g^{ij} \\ \rho h \gamma^2 v^j + (\mathbf{E}\times\mathbf{B})^j \end{pmatrix} = 0$$

\+

Maxwell equations

2.5. Another Difficult Point

evolution equations of electric field

$$\partial_t \mathbf{E} = \nabla \times \mathbf{B} - \mathbf{J},$$
$$\mathbf{J} = \sigma\gamma[\mathbf{E} + \mathbf{v} \times \mathbf{B} - (\mathbf{E} \cdot \mathbf{v})\mathbf{v}] + q\mathbf{v}$$

highly stiff equations !!

difficult to solve ...

2.6. Piecewise Exact Solution Method

ref) Komissarov, (2007), MNRAS, 382, 995
T.Inoue & Inutsuka, (2008), ApJ, 687, 303
MT & T. Inoue., (2011), ApJ, 735, 113

$$\begin{cases} \partial_t \mathbf{E}_{\parallel} & + \quad \sigma\gamma \left[\mathbf{E}_{\parallel} - (\mathbf{E}\cdot\mathbf{v})\mathbf{v} \right] = 0, \\ \partial_t \mathbf{E}_{\perp} & + \quad \sigma\gamma \left[\mathbf{E}_{\perp} + \mathbf{v}\times\mathbf{B})\right] = 0, \end{cases}$$

stiff part of equations for electric field

$$\begin{cases} \mathbf{E}_{\parallel} & = \quad \mathbf{E}^0_{\parallel} \exp\left[-\frac{\sigma}{\gamma} t \right], \\ \mathbf{E}_{\perp} & = \quad \mathbf{E}^*_{\perp} + (\mathbf{E}^0_{\perp} - \mathbf{E}^*_{\perp}) \exp\left[-\sigma\gamma t \right], \end{cases}$$

:Formal solutions

Point:

First terms of right-hand side are independent of time since they are split from fluid equations.

⇒ Solvable using the formal solution

2.7. Numerical Scheme

ref) MT & S, Inutsuka., (2011), JCP, 230, 7002
MT & T, Inoue., (2011), ApJ, 735, 113

Split basic equations as follows:

Electromagnetohydrodynamics equations

→ fluid part + electromagnetic part

- fluid part = Riemann solver
- electromagnetic part

= method of characteristics

+ Piecewise Exact Solution (PES)

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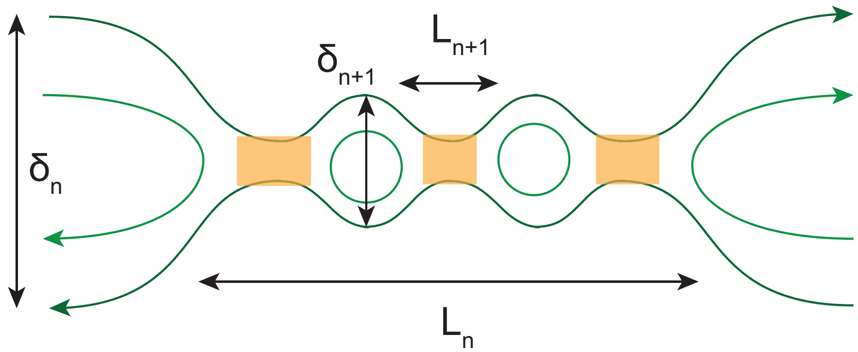
Applications

3.1. Fast Reconnection by Plasmoid-Chain

ref) Shibata & Tanuma, 2001, EPS, 53, 473
Uzdensky et al, 2010, PRL, 105, 235002

Sweet-Parker Reconnection
= very slow ... ($\tau_R \propto \sqrt{S}$)
($S = L\, c_A / \eta$)



⟺

If S reaches a critical value:

$S > S_c \sim 10^4$
(very long sheet)

⇒ Current sheet will be filled by a lot of plasmoids...

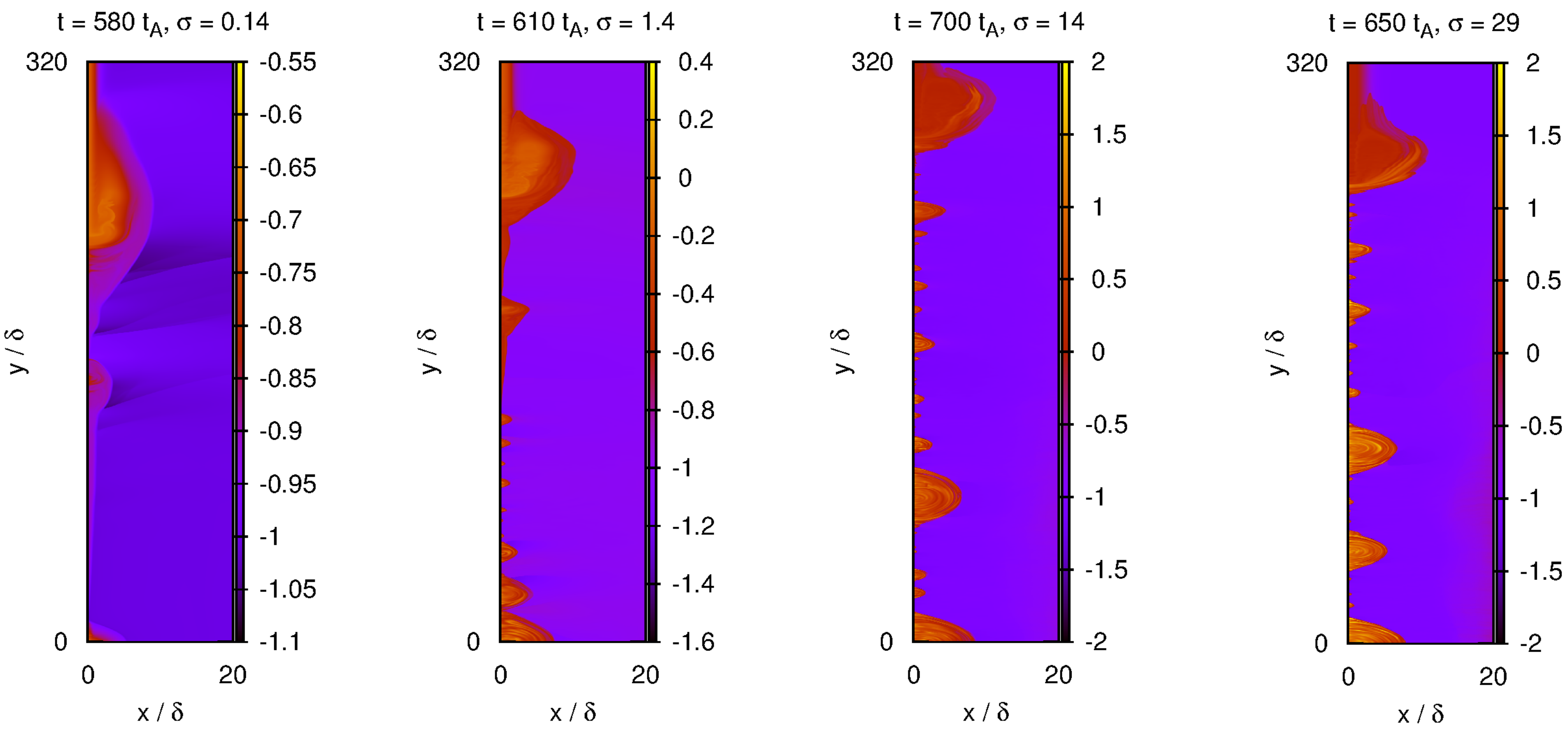
⇒ global reconnection rate becomes independent of S:
$v_{in} / c_A \sim 10^{-2}$ (non-relativistic cases)

(Plasmoid-Chain)

3.2. Relativistic Plasmoid-Chain

ref) MT & T, Inoue., (2011), ApJ, 735, 11
MT, (2013), submitting to ApJ

Pressure profiles

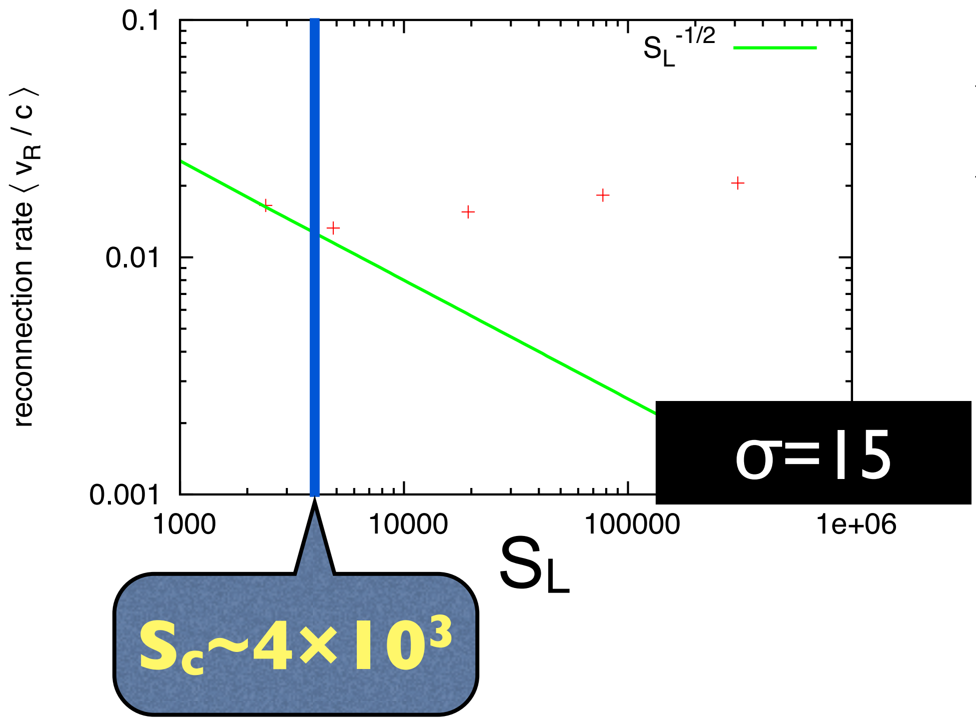


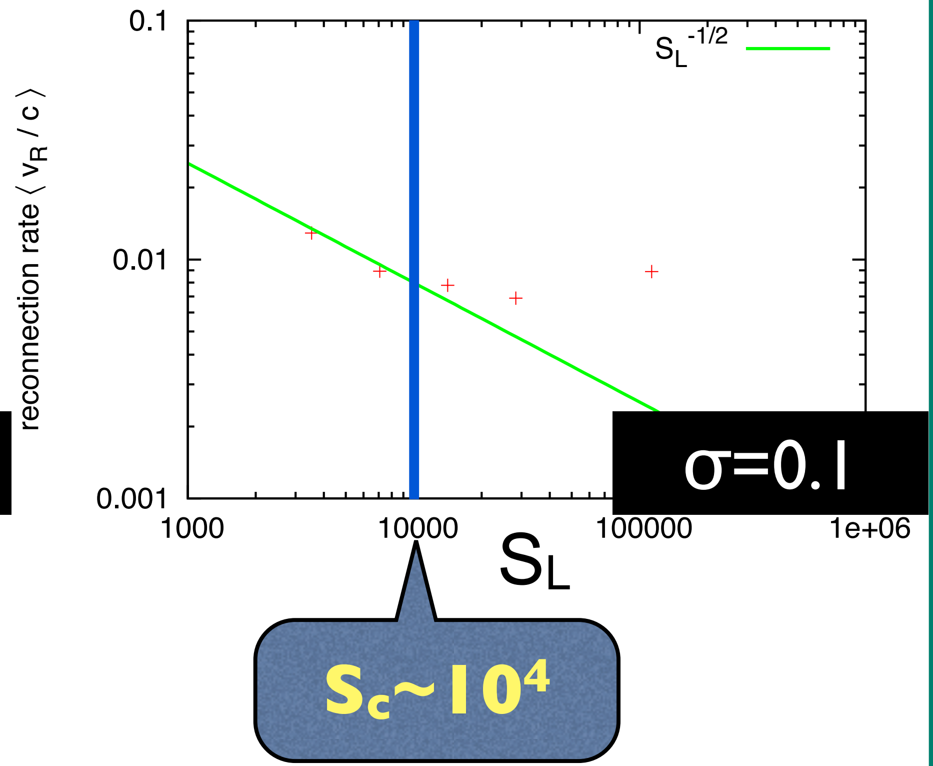
Weakly Magnetized:
σ < 1

Poynting-Dominated:
σ > 1

3.3. Lundquist Number Dependence

ref) MT, (2013), submitting to ApJ



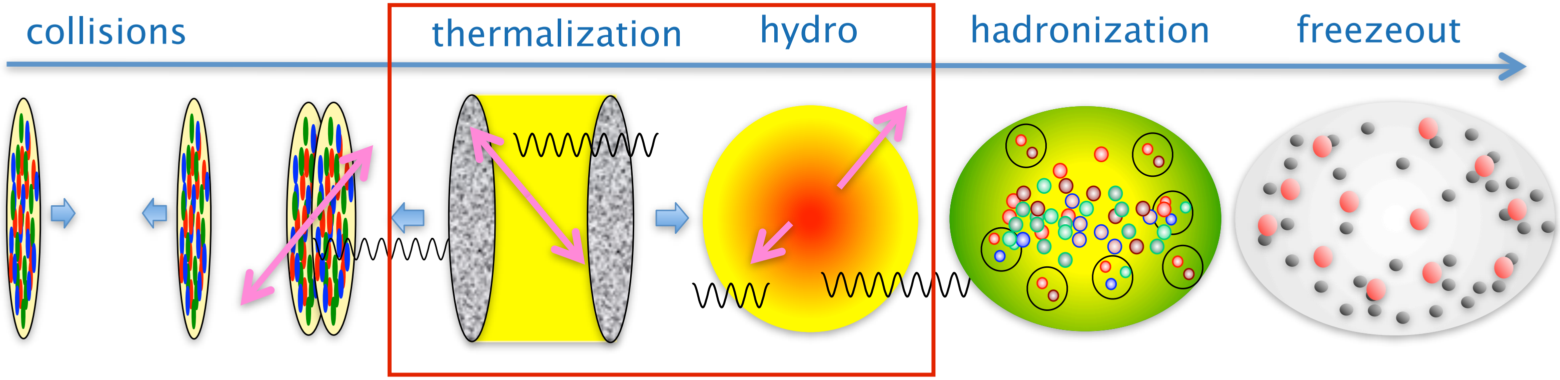


Reconnection Rate becomes independent of Lundquist number S_L

when S_L > $S_{L,C}$: critical value at which Plasmoid instability occurs

3.4. Application to Quark-Gluon Plasma (QGP)

- Heavy ion collision -> Generation of Quark-Gluon Plasma



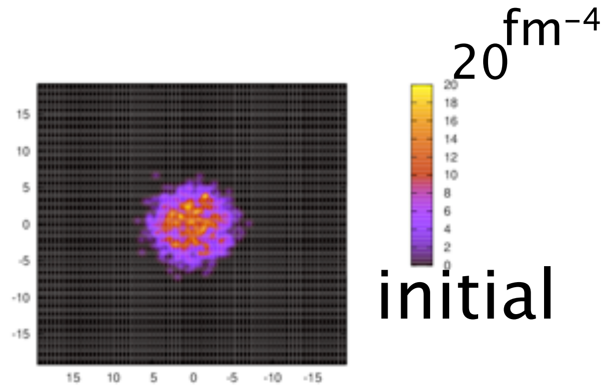
Hydrodynamic model with density fluctuation

New Scheme: **(COGNAC)**

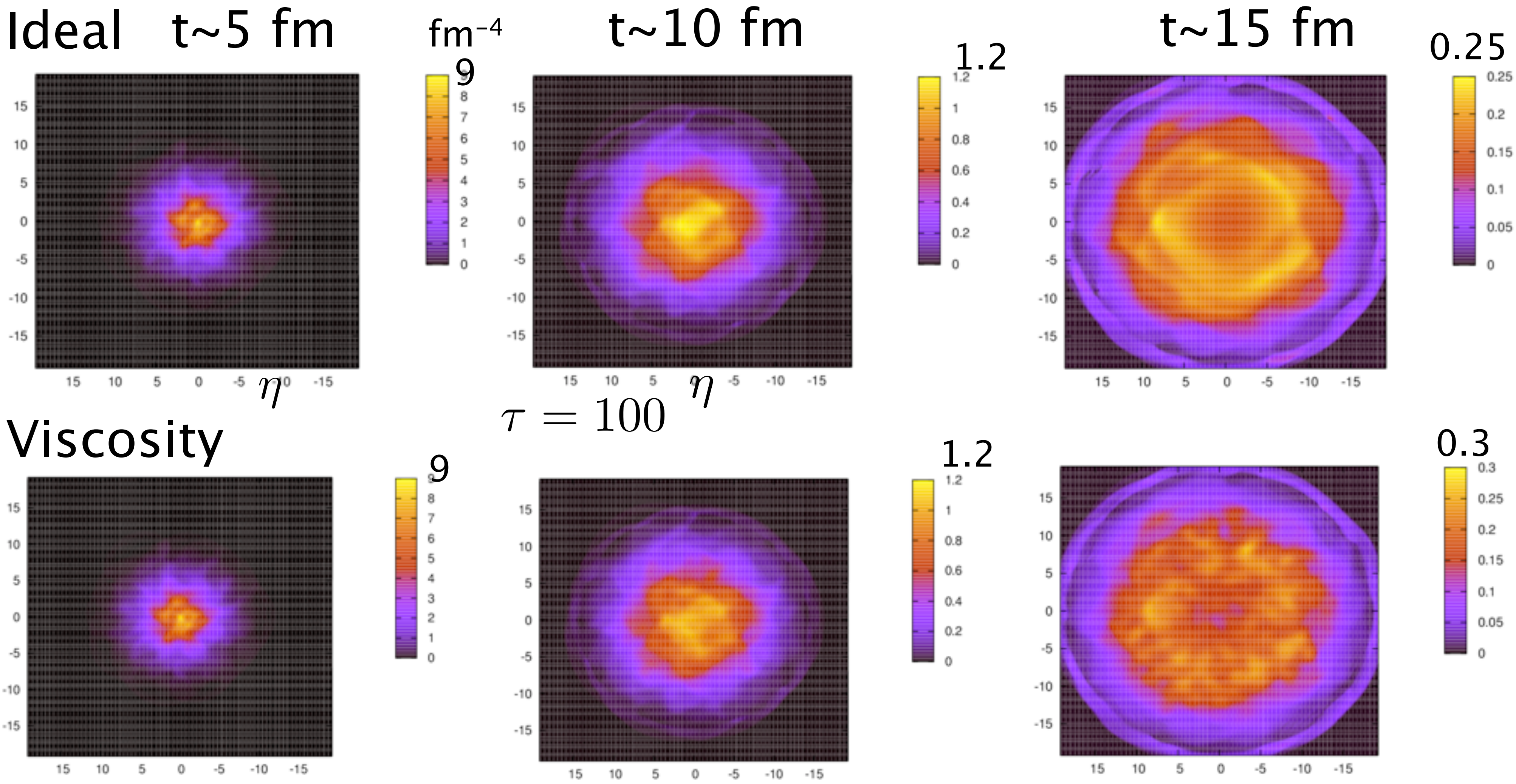
- ideal : full-Godunov
  (Exact Solution using QCD EoS)
- Dissipation: Piecewise-Exact Solution Method

ref) MT & S, Inutsuka., (2011), JCP, 230, 7002
Y. Akamatsu, S. Inutsuka, C. Nonaka, MT, arXiv1302.1665

Viscous Effect



Pressure distribution



Summary

- In the relativistic hydrodynamics case, it is very difficult to take into account the dissipation effects due to the covariance and existence of stiff-equations.
- We developed new numerical scheme of RMHD with dissipations.
- Using Piecewise Exact Solution, we can calculate the stiff relaxation equations very efficiently.
- Using this new scheme, we investigated the relativistic plasmoid-chain and found the magnetic reconnection rate becomes independent of the Lundquist number.
- We have recently developed a new dissipative RHD scheme using a QCD EoS and applied to QGP plasma.

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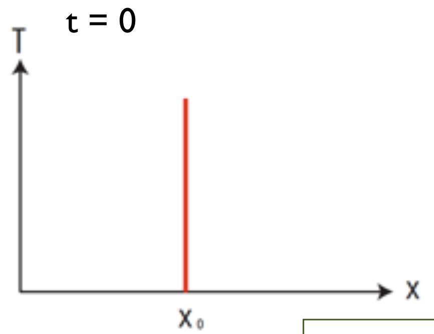
2.2. Acausality in dissipation theory

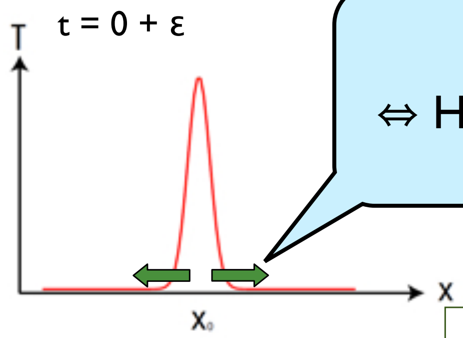
e.g.) energy equation (if relativistic extended heat flux is used)

$$nc_V \frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} = \nabla \cdot (\kappa \nabla T)$$: parabolic partial differential equation

→ **characteristic velocity is infinite**



$T = T_0\ \delta_D(x - x_0)$



$$T = \frac{1}{2\sqrt{\pi \chi t}} \exp\left[-\frac{(x - x_0)^2}{4\chi t}\right]$$

T ≠ 0 even at infinity!
⇔ Heat flows faster than light !!

Perturbations **grow unphysically** in dissipative RHD
because energy comes from acausal region unphysically!!

4. Causal and stable theory (Israel-Stewart theory)

ref) Israel & Stewart, 1979, Annals of Physics, 118, 341

Israel-Stewart theory = stable and causal relativistic dissipation theory

$$\partial_t \begin{pmatrix} D \\ m^i + q^0 u^i + q^i u^0 + \tau^{0i} + \gamma^{0i}\Pi \\ E + 2u^0 q^0 + \tau^{00} + \gamma^{00}\Pi \end{pmatrix} + \partial_j \begin{pmatrix} Dv^i \\ m^i v^j + pI^{ij} + u^i q^j + u^j q^i + \tau^{ij} + \gamma^{ij}\Pi \\ m^j + u^0 q^j + q^0 u^j + \tau^{0j} + \tau^{0j}\Pi \end{pmatrix} = 0,$$

$$\gamma \left(\partial_t + v^i \partial_i\right) \Pi = -\frac{1}{\tau_\Pi} \left(\Pi + \zeta \partial_\mu u^\mu\right) + I_0,$$

$$\gamma \left(\partial_t + v^i \partial_i\right) q^\mu = -\frac{1}{\tau_q} \left[q^\mu + \kappa \gamma^{\mu\nu} \left(\partial_\nu T - \frac{T}{\rho h} \partial_\nu p\right)\right] + I_1,$$

$$\gamma \left(\partial_t + v^i \partial_i\right) \tau^{\mu\nu} = -\frac{1}{\tau_\tau} \left[\tau^{\mu\nu} + \eta \gamma^{\mu\rho} \gamma^{\nu\sigma} \partial_{\langle\rho} u_{\sigma\rangle}\right] + I_2,$$

Features
- equations are hyperbolic and characteristic velocities are smaller than velocity of light (causal ⇒ stable)
- appearance of extremely short timescale (mean flight timescale) ⇒ difficult to resolve in time!!

3.4. Telegrapher Equation and Causality

Consider the following form of telegrapher equation

$$\begin{cases} \partial_t^2 u + 2\kappa \partial_t u - a^2 \partial_z^2 u = 0 \\ u(z, t=0) = f(z), \quad \partial_t u|_{t=0} = F(z) \end{cases}$$

Green function of the above equation is

$$u(z,t) = \frac{1}{2} e^{-\kappa t}[f(z-at) + f(z+at)] + \frac{e^{-\kappa t}}{2a} \int_{z-at}^{z+at} [F(x) + \kappa f(x)] I_0[2\sqrt{c(at+z-x)(at-z+x)}] dx + \sqrt{c}ate^{-\kappa t} \int_{z-at}^{z+at} \frac{f(x) I_1[2\sqrt{c(at+z-x)(at-z+x)}]}{\sqrt{(at+z-x)(at-z+x)}} dx$$

κ^2

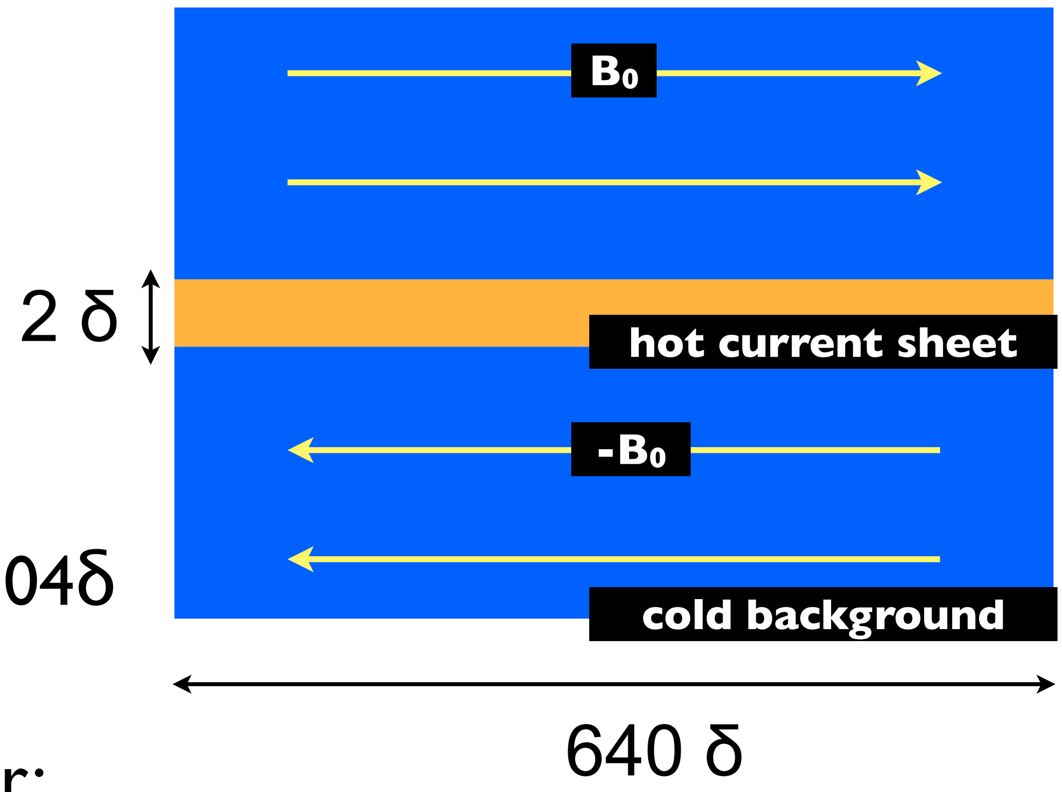
$4a$

Characteristics are always within the causal cone of ±a t

6. Numerical Setup

Initial condition:

- Harris current sheet
- cold upstream flow
  ($T \sim 0.1 mc^2$)
- hot current sheet
  ($T_{sheet} \sim mc^2$)
- mesh size: $\Delta \sim 0.02\delta$ - 0.04δ
- uniform resistivity
- Large Lundquist number:
  $S \sim 10^{3\text{-}5}$
- Poynting dominated upstream plasma:
  σ = 0.1, 1, 15, 30



$$\sigma \equiv \frac{[\mathbf{E} \times \mathbf{B} c/4\pi]}{\rho h c^2 \gamma^2 v}$$