A three-dimensional Iroshnikov-Kraichnan phenomenology for MHD turbulence in a strong mean magnetic field

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Turbulence in Mean Magnetic Fields

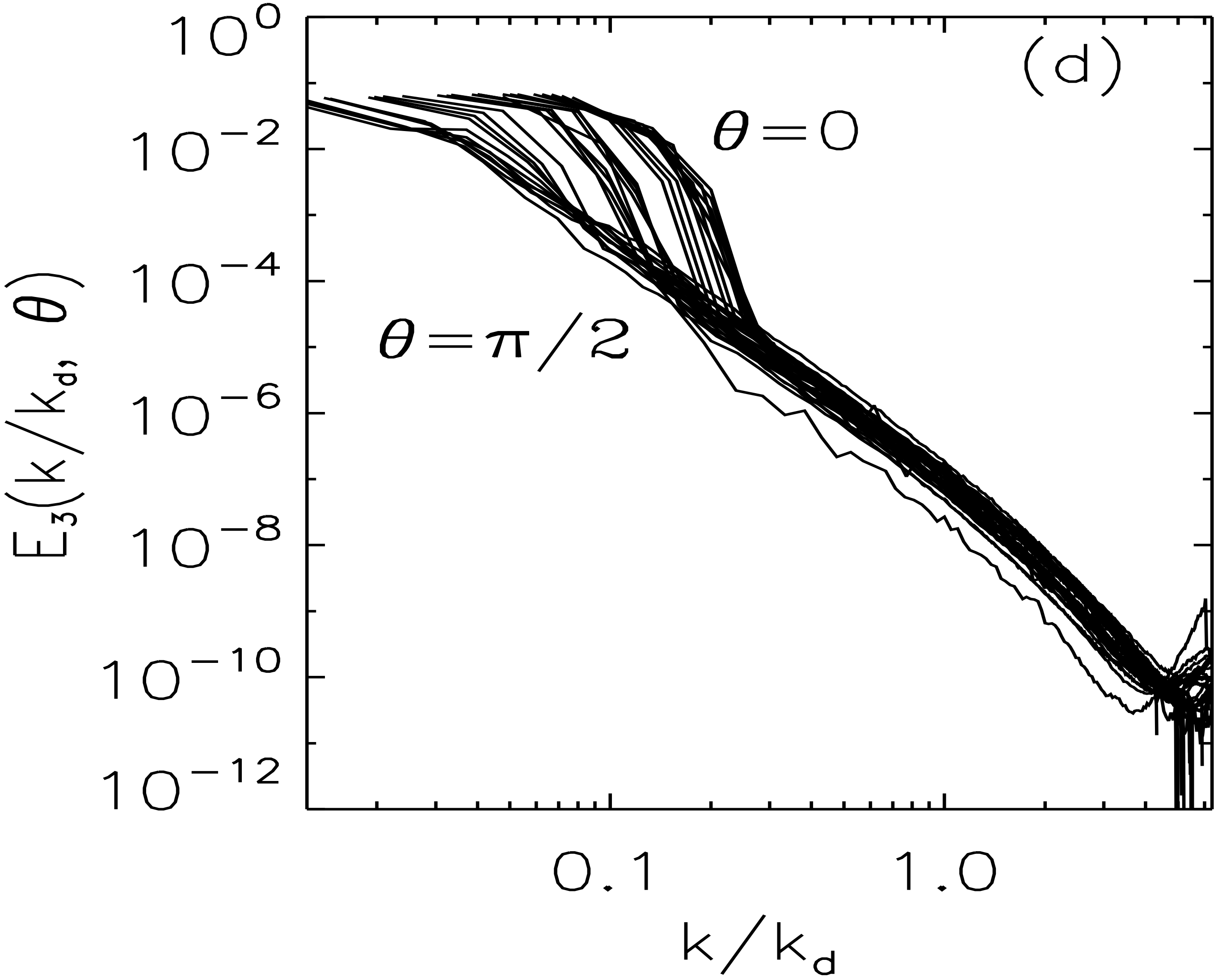
Presumably different turbulent regimes $\rightarrow$ constraints on

- **timescales** $\tau_{\|}^{\text{ac}}, \tau_{\perp}^{\text{ac}}$ (nonlinear time $\tau_{\text{NL}} = (k_\perp b_\perp)^{-1}$, Alfvén time $\tau_{\text{A}} = (k_\| B_0)^{-1}$)
- **Fourier space structure** of turbulent excitations/driving

Established phenomenologies

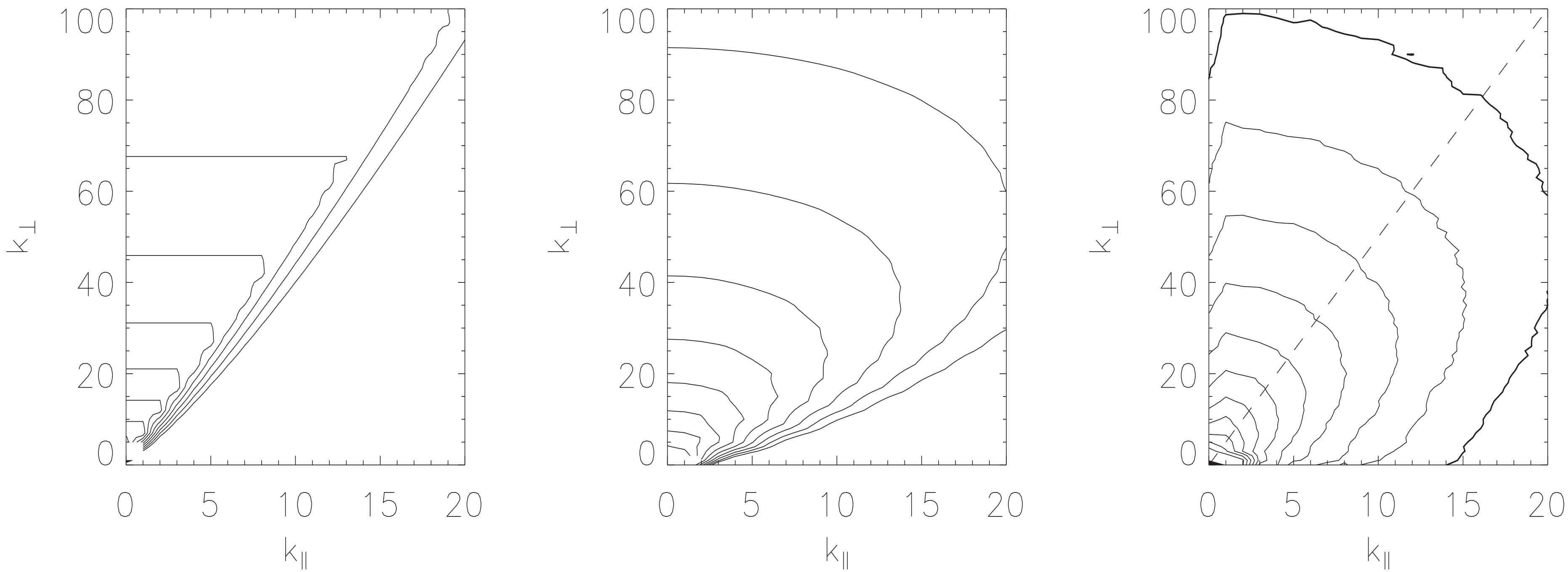
- **Strong regime** ($\tau_{\text{NL}} \sim \tau_{\text{A}}$): Goldreich-Sridhar (3D, $k_\perp^{-5/3}$), Boldyrev (3D, $k_\perp^{-3/2}$, ?)
- **Weak regime** ($\tau_{\text{NL}} \gg \tau_{\text{A}}$, $k_\perp \gg k_\|$): e.g. Galtier, Ng & Bhattacharjee (3D, $k_\perp^{-2}$)
- Iroshnikov-Kraichnan (2D, $k^{-3/2}$), weak turbulence variant, dwells only in 2D

Universality



$$E_3(k,\theta) = A(\theta)k^{-m-2} = A_0(k/k_\mathrm{d})^{-m-2}, \qquad A(\theta) \simeq k_\mathrm{d}(\theta)^{m+2}$$

Fourier Energy Distribution ($k_\parallel$-$k_\perp$ plane)



Left: Critical balance cone (local frame): $k_\parallel \sim k_\perp^{2/3}$

Middle: CB cone subject to fluctuations around mean direction $\sim \frac{b_\perp}{B_0} \simeq \frac{1}{5}$

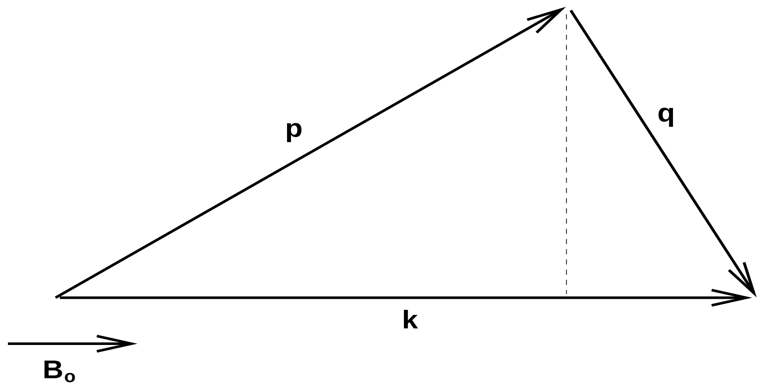
Right: DNS with isotropic large-scale driving

Extent along $k_\parallel$-axis apparently not explicable by reference-frame mapping

A Different Regime of MHD turbulence

- No standard weak Alfvén turbulence (no nonlinear transfer in parallel direction)
- No standard critically balanced turbulence (geometrically too restricted)
- Suspected reason: isotropic large-scale driving
- Possibility of extension of IK picture to three-dimensionality

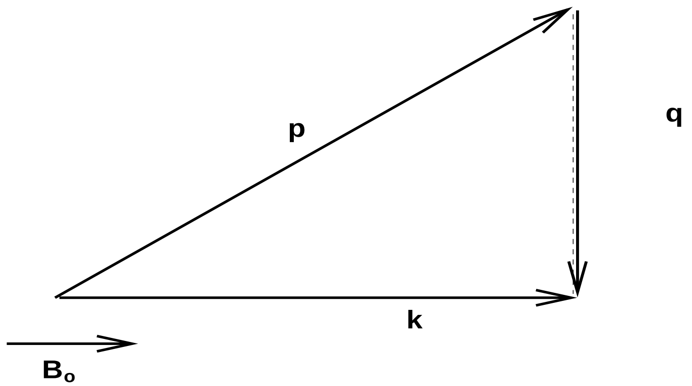
General Nonlinear Triad Interaction



Convolution constraint on three-mode interactions: $\mathbf{k} = \mathbf{p} + \mathbf{q}$

Example: finite $q_{||}$ allows nonlinear field-parallel transfer

Resonant Nonlinear Triad Interaction



Convolution constraint on three-mode interactions: $\mathbf{k} = \mathbf{p} + \mathbf{q}$

Weak turbulence

Resonance condition: $\omega(\mathbf{k}) = \omega(\mathbf{p}) + \omega(\mathbf{q})$

Alfvén waves: $\omega(\mathbf{k}) = \mathbf{k} \cdot \mathbf{B_0} = k_{||} B_0$

Resonance condition implies $q_{||} = 0$, i.e. no field-parallel cascade

Phase-mixing along $\mathbf{B_0}$ prevents structure formation perpendicular to $\mathbf{B_0}$

Causality

Generalization of GS-critical balance: $\tau_\perp^{\mathrm{ac}} \sim \tau_\parallel^{\mathrm{ac}} \sim \tau_{\mathrm{A}}$

Incompressible MHD ($B_0 \lesssim 2-3$): $\tau_{\mathrm{NL}_\perp} \sim \tau_{\mathrm{A}}$

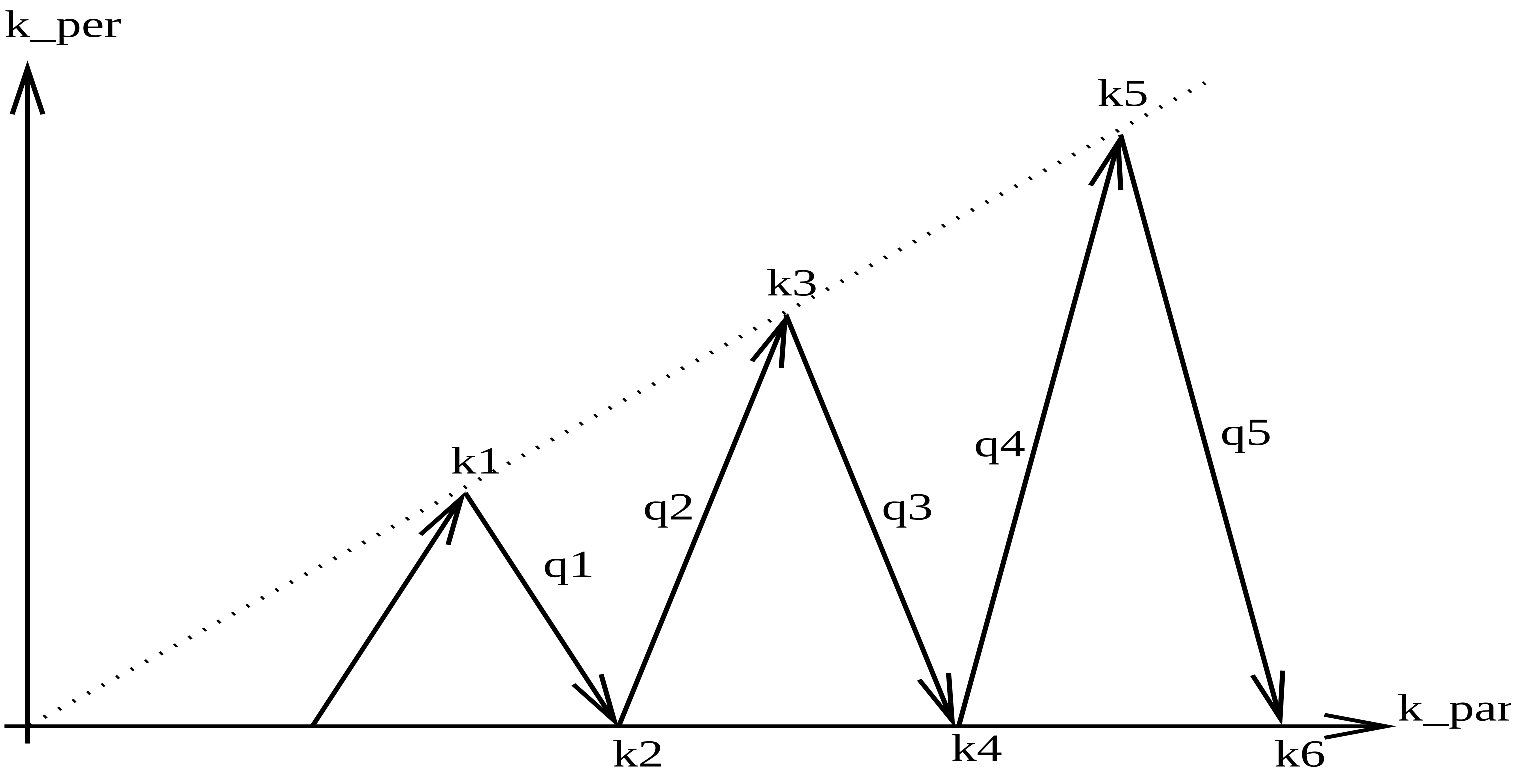
If transfer in planes perpendicular to $\mathbf{B_0}$ governed by IK cascade:

- $\tau_\perp^{\mathrm{ac}} \sim \tau_{\mathrm{A}_\perp} = (k_\perp b_{\mathrm{rms}_\perp})^{-1}$
- $\tau_{\mathrm{A}} < \tau_{\mathrm{A}_\perp} < \tau_{\mathrm{NL}}$

Relaxation of weak turbulence constraint ($\tau_{\mathrm{A}} \ll \tau_{\mathrm{NL}}$)
$\longrightarrow$ possibility of **quasi-resonant cascade**, allows small $q_\parallel \sim q_\perp \frac{b_{\mathrm{rms}_\perp}}{B_0}$

Ricochet Process

Realizes energy flow along directions oblique w.r.t. $\mathbf{B}_0$



Process based on two basic triads to transfer prolongations along two directions in Fourier space within region allowed by the quasi-resonance criterion.

Dependent on dominant perpendicular cascade process populating excitations within the CB region.

Start near Fourier origin requires externally excited fluctuations (e.g. isotropic large-scale forcing)

Nonlinear Energy Flux

Isotropic K41 flux:

$$F_{\text{K41}} \sim k v_k^3 \qquad (k^{-5/3})$$

Iroshnikov-Kraichnan flux:

$$F_{\text{IK}} \sim k b_k^2 b_q^2 / B_0 \qquad (k^{-3/2})$$

F_{IK} approximately reduced by factor $\frac{b_q}{B_0}$

comparison with quasi-resonant flux (triad counting)

Ensemble of triads reduced through quasi-resonance constraint by factor $\frac{b_{\text{rms}}}{B_0}$

Dissipative Regions

Estimating end of inertial range:

$$\tau_{\text{diss}} \sim \tau_{\text{flux}}$$

$$\tau_{\text{diss}}^{-1} \sim \nu k^2, \tau_{flux_{\parallel}} \sim \frac{ku_k^2}{b_{\text{rms}}}, \tau_{flux_{\perp}} \sim \frac{ku_k^2}{B_0} \text{ (IK)}$$

$$\frac{k_{\text{d}\parallel}}{k_{\text{d}\perp}} \sim \frac{b_{\text{rms}}}{B_0}$$

Found in numerical simulations (Grappin & Müller 2010)

Summary

- DNS of MHD turbulence with strong mean magnetic field, large-scale isotropic driving incompatible with standard theory
- Proposition of new cascade mechanism based on weak IK cascade if turbulent excitations outside critical balance region
- Ricochet mechanism allows for parallel and oblique nonlinear transport
- Three-dimensional extension of Iroshnikov-Kraichnan regime