3D Global MHD Simulations of the Solar Wind/Earth's Magnetosphere Interaction

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OUTLINE

- Overview of the COOLFluiD Framework
- Overview of the Global MHD Solver
- Results
- Conclusion

THE COOLFluiD FRAMEWORK

Component-based environment for multi-disciplinary research
(von Karman Institute for Fluid Dynamics, KU Leuven (CmPA) *et al.)*

Multiple 3D parallel solvers for unstructured grids

- Time Steppers: Runge Kutta-n, 1- & 3-point Backward Euler, Crank Nicholson
- Space Discretizations: **FV**, FE, RD, DG, Spectral FV/FD
- Linear System Solvers: PETSc, Trilinos, Pardiso, FlexMG, SAMG

Multiple physical models

- Compressible & incompressible flows
- Aerothermodynamics (thermo-chemical equilibrium / non-equilibrium)
- **Ideal MHD**, Inductively coupled plasma (ICP), Maxwell
- Aeroacoustics, LES, turbulence

❑ Parallel infrastructure for HPC where scalability is tested up to 1024 CPUs.

GLOBAL MHD SOLVER

- Ideal MHD equations
- Cell-center upwind Finite Volume schemes
- Total variation diminishing
- Implicit time stepping
- B_0+B_1 splitting
- divB=0 constraint satisfaction (hyperbolic divergence cleaning, 8-wave)
- Anisotropic mesh adaptation over unstructured grids

HYPERBOLIC DIVERGENCE CLEANING

$$\frac{\partial \vec{B}}{\partial t} + \nabla \cdot \left(\vec{v}\vec{B}^T - \vec{B}\vec{v}^T \right) + \nabla \psi = 0$$

$$D(\psi) + \nabla \cdot \vec{B} = 0$$

$D(\psi) = 0$ Elliptic correction (= projection scheme)

$D(\psi) = \frac{\psi}{c_p^2}, 0 < c_p < \infty$ Parabolic correction (dissipation of divB errors)

$D(\psi) = \frac{1}{c_h^2}\frac{\partial \psi}{\partial t}, \; 0 < c_h < \infty$ Hyperbolic correction (propagation of divB errors with c_h)

$D(\psi) = \frac{1}{c_h^2}\frac{\partial \psi}{\partial t} + \frac{\psi}{c_p^2}$ Mixed correction (both dissipation and propagation of divB errors)

HDC

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho\\ \rho\vec{v}\\ \vec{B}_1\\ E_1\\ \phi\end{pmatrix}+\nabla\cdot\begin{pmatrix}\rho\vec{v}\\ \rho\vec{v}\vec{v}+\bar{\mathbf{I}}\left(p+B_1^{\;2}/2+\vec{B}_1\cdot\vec{B}_0\right)-\left(\vec{B}_1\vec{B}_1+\vec{B}_0\vec{B}_1+\vec{B}_1\vec{B}_0\right)\\ \vec{v}\vec{B}-\vec{B}\vec{v}+\bar{\mathbf{I}}\phi\\ \left(E_1+p+B_1^{\;2}/2+\vec{B}_1\cdot\vec{B}_0\right)\vec{v}-\vec{B}\left(\vec{v}\cdot\vec{B}_1\right)\\ V_{ref}^2\vec{B}_1\end{pmatrix}=0$$

- Hyperbolic system of 9 equations with eigenvalues: v, $v \pm c_f$, $v \pm c_s$, $v \pm c_A$, $\pm V_{ref}$ (the same system as hyperbolic divergence cleaning with hyperbolic correction)
- ϕ is a scalar potential function, V_{ref} is a constant reference speed (e.g. freestream flow speed)
- div**B** = 0 condition is satisfied up to the machine accuracy level at convergence
- the system remains purely hyperbolic and can be solved with any conventional upwind finite volume numerical scheme suitable for hyperbolic PDE systems (e.g. TVD Rusanov scheme)

$B = B_0 + B_1$ SPLITTING

B_0 is a dipole field that represents the Earth's magnetic field.

$$\vec{B}_0 = \frac{1}{r^3}\left(3\left(\vec{m}\cdot\vec{n}_r\right)\vec{n}_r - \vec{m}\right)$$

It is a potential field that satisfies the following conditions:

$$\frac{\partial \vec{B}_0}{\partial t} = \nabla \cdot \vec{B}_0 = \nabla \times \vec{B}_0 = 0$$

B_1 is the variable magnetic field that is solved by the solver.

TVD RUSANOV SCHEME

$$\overline{\mathbf{F}}\left(\overline{\mathbf{U}}_L,\overline{\mathbf{U}}_R\right)=\frac{1}{2}\left[\overline{\mathbf{F}}\left(\overline{\mathbf{U}}_L\right)+\overline{\mathbf{F}}\left(\overline{\mathbf{U}}_R\right)\right]-\left|\lambda_{\max}\right|_{LR}\left(\overline{\mathbf{U}}_R-\overline{\mathbf{U}}_L\right)$$

- $\overline{\mathbf{U}}_L$ and $\overline{\mathbf{U}}_R$ are the solution variables that are linearly reconstructed at the cell and neighboring cell sides of a cell face, respectively.

- Barth and Jespersen and Venkatakrishnan are the limiters used.

IMPLICIT TIME STEPPING (1)

Unsteady implicit time-integration scheme for HDC

(Yalim et al. JCP Vol 230 pp. 6136-6154 2011)

Modified system of ideal MHD equations for HDC can be written as follows:

$$\bar{\mathbf{S}}\left(\bar{\mathbf{U}}\right) = \frac{d\bar{\mathbf{U}}}{dt} + \bar{\mathbf{R}}(\bar{\mathbf{U}}) = 0$$

Discretizing in time using the 2. order backward differentiation formula scheme, we obtain

$$\bar{\mathbf{S}}\left(\bar{\mathbf{U}}^{k+1}\right) = \frac{3\bar{\mathbf{U}}^{k+1} - 4\bar{\mathbf{U}}^{k} + \bar{\mathbf{U}}^{k-1}}{2\Delta t} + \bar{\mathbf{R}}\left(\bar{\mathbf{U}}^{k+1}\right) = 0$$

Linearizing the residual term around $\bar{\mathbf{U}}^{k}$, we get the following linear system:

$$\left[\frac{\partial \bar{\mathbf{S}}}{\partial \bar{\mathbf{U}}}\left(\bar{\mathbf{U}}^{k}\right)\right]\Delta\bar{\mathbf{U}}^{k} = -\bar{\mathbf{S}}\left(\bar{\mathbf{U}}^{k}\right)$$

$$\Delta\bar{\mathbf{U}}^{k} = \bar{\mathbf{U}}^{k+1} - \bar{\mathbf{U}}^{k}$$

Solve until $\left\|\Delta\bar{\mathbf{U}}^{k}\right\| < \sigma$ where σ is a very small positive number

IMPLICIT TIME STEPPING (2)

Divergence cleaning using implicit time integration for unsteady MHD

In order to satisfy the divB constraint during the transient, the equation for divergence cleaning is converged to steady state at each timestep by applying

$$\bar{\mathbf{S}}_{\phi}(\bar{\mathbf{U}}^{\mathbf{k+1}}) = \bar{\mathbf{R}}_{\phi}\left(\bar{\mathbf{U}}^{\mathbf{k+1}}\right)$$

Thus, a pure Newton iteration procedure is applied to the elliptic correction (= projection scheme). This equation is linear in **B** (even after discretization) and the divB constraint will be satisfied at each subiteration.

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ANISOTROPIC MESH ADAPTATION FOR UNSTRUCTURED GRIDS (1)

- Solar wind/planetary magnetosphere interaction applications contain very different geometric scales such as shock layers and large geometric distances.

- Therefore, solution-adaptive grid methods have received considerable attention focusing mainly on the AMR strategy (e.g. used in the AMRVAC code) using hierarchical refinement in the structured grid context.

- We investigate an alternative approach using fully unstructured grids composed of tetrahedra.

- These offer higher flexibility for anisotropic adaptation such as needed in shock layers (adaptation made in the direction normal to the shock).

ANISOTROPIC MESH ADAPTATION FOR UNSTRUCTURED GRIDS (2)

- The main advantage of anisotropic over isotropic grid adaptation is the substantially reduced number of cells in the adapted grid.

- The algorithm relies on grid remeshing based on the generation of a new grid in the Riemann space in which it is assumed that the cells are spaced uniformly.

- The final spacing of the new grid is a function of the metric field used for the definition of the Riemann space.

- This metric is obtained from the solution field by means of an error estimator based on a solution Hessian.

RESULTS

06/04/2000, 20/11/2003 and 05/04/2010 Magnetic Storm Simulations (Unsteady) including

- Search for the Presence of a Complex Planetary Bow Shock Structure

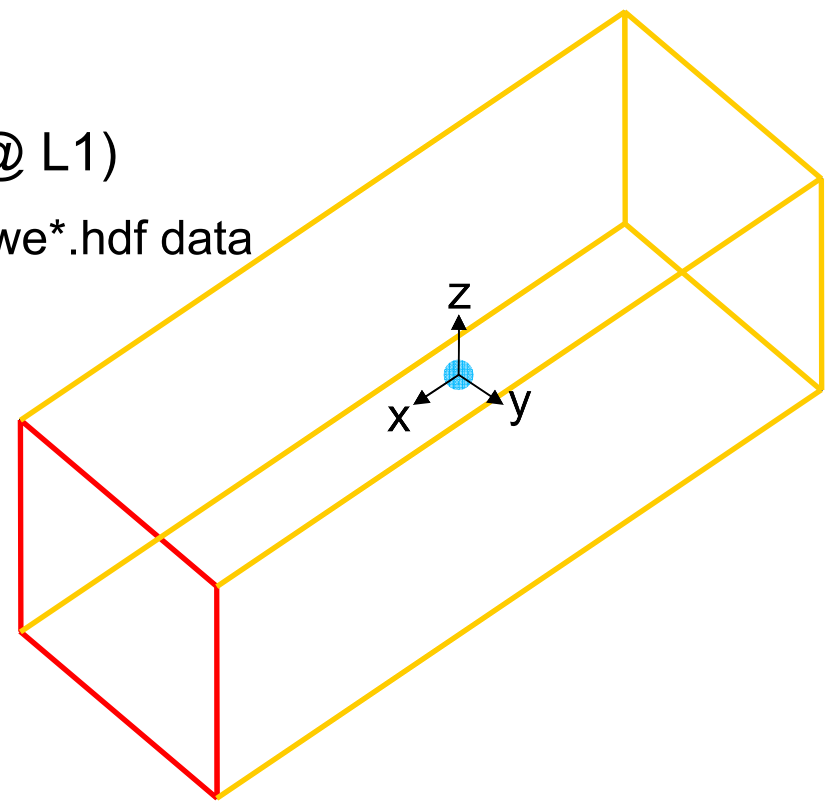
SIMULATION DOMAIN

• GSM coordinate system (x-axis points at the Sun, xz-plane contains the dipole ($\boldsymbol{B_0}$) axis where a fixed tilt angle of 11.94°is applied.)

Solar wind plasma parameters from ACE (@ L1)
(ρ, $\boldsymbol{v}$, $\boldsymbol{B_1}$, p)

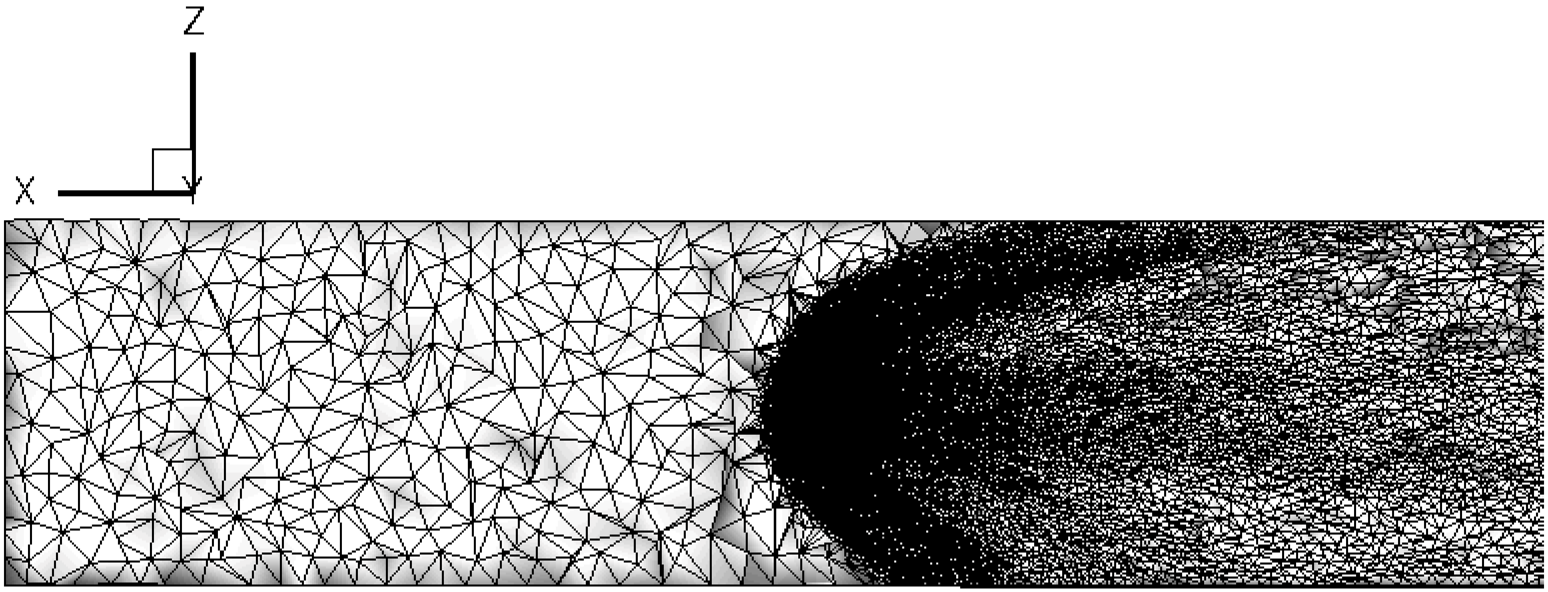
64 s average magswe*.hdf data

Outlet (simple extrapolation of parameters)

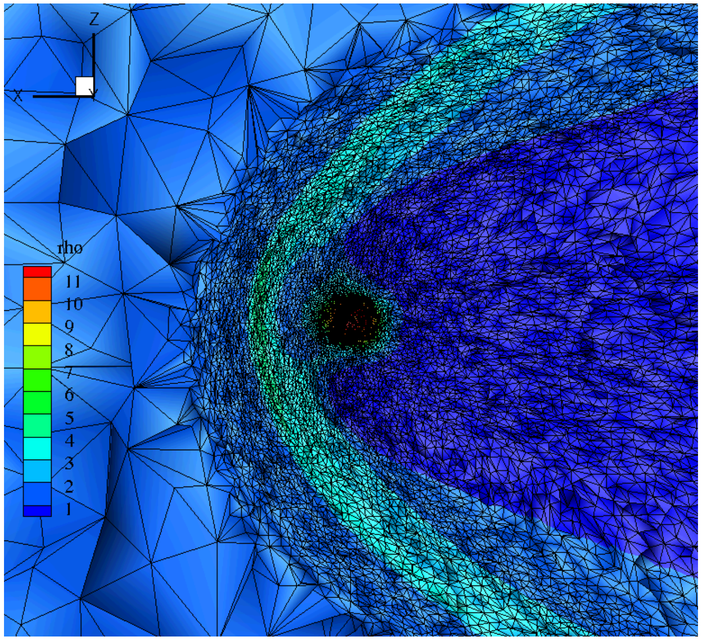


Inner boundary (@ 2.5R_E)
(Powell *et al.* 1999, Gombosi *et al.* 2002)

ρ is fixed to 56 AMU/cm^3;
$\boldsymbol{v}$ is 0;
$(B_1)_n$ is 0 and $(B_1)_t$ is extrapolated;
T is fixed to 35000 K.



Adapted Grid (# of nodes: 473067, # of tetrahedra: 2773426)



Density contours over the adapted grid in the vicinity of the Earth at t = 0

MAGNETIC STORM MOVIES

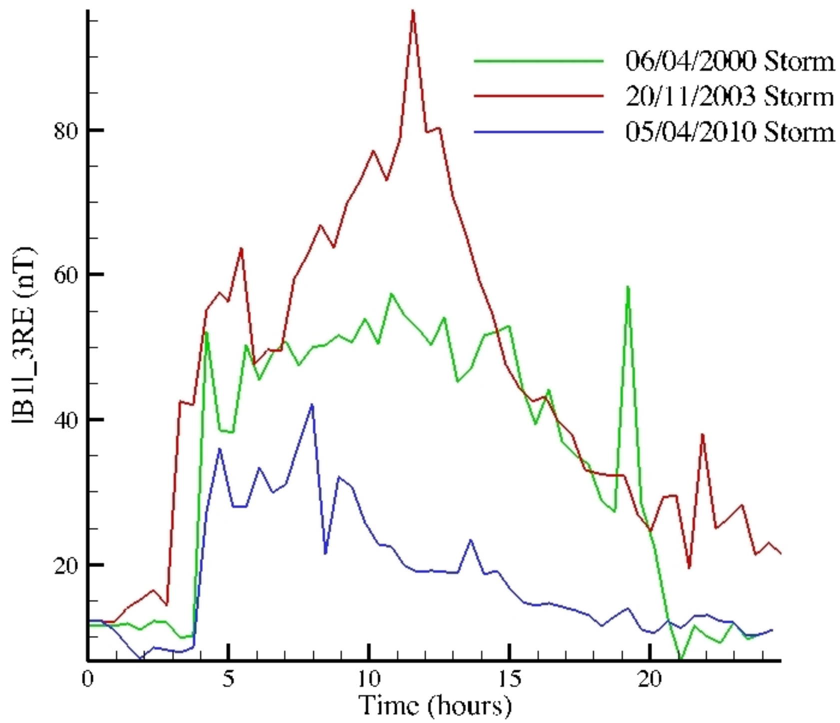
- 06/04/2000 storm → Dst_{min} : -288 nT
  20/11/2003 storm → Dst_{min} : -472 nT
  05/04/2010 storm → Dst_{min} : -81 nT

- **Proton number density movie**
- **Proton speed movie**
- **Proton pressure movie**
- **B_z movie**
- **Magnetic field lines movie**

A NUMERICAL MAGNETIC STORM INDEX

- We present a numerical magnetic storm index based on the average of the magnitude of the perturbation field, $\boldsymbol{B_1}$, inside a sphere of radius 3 Earth radii surrounding the Earth.

- For all the three storms of different strengths, the times of the initial jump and the peak of the storm index are close to the variations in the Dst index.



SEARCH FOR A COMPLEX PLANETARY BOW SHOCK STRUCTURE DURING THE 06/04/2000 MAGNETIC STORM

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NON-DIMENSIONAL PARAMETERS

Magnetic beta (i.e. ratio of hydrodynamic to magnetic pressures):

$$\beta = \frac{p}{B^2/2}$$

Mach number (e.g. in the shock normal direction):

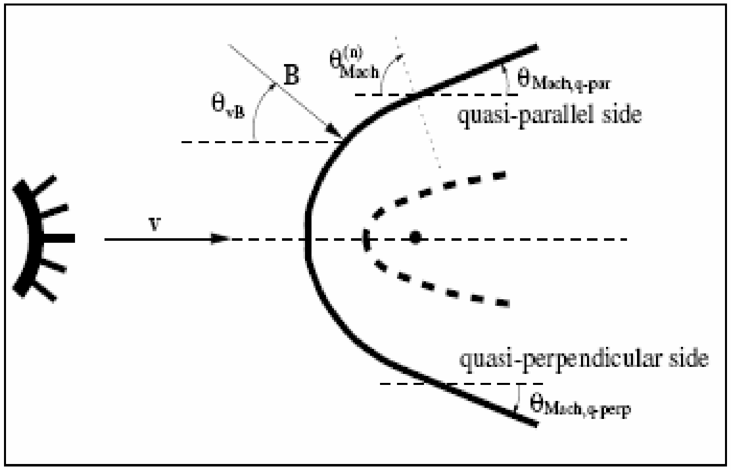
$$M_n = \frac{|v_n|}{c} \qquad \mathrm{c} \rightarrow \text{acoustic speed}$$

$$M_{f_n} = \frac{|v_n|}{c_{f_n}} \qquad \mathrm{c_f} \rightarrow \text{fast magnetosonic wave speed}$$

$$M_{A_n} = \frac{|v_n|}{c_{A_n}} \qquad \mathrm{c_A} \rightarrow \text{Alfven wave speed}$$

$$M_{s_n} = \frac{|v_n|}{c_{s_n}} \qquad \mathrm{c_s} \rightarrow \text{slow magnetosonic wave speed}$$

MAGNETICALLY DOMINATED REGIME (1)



Hans De Sterck, PhD Thesis, CmPA, KU Leuven, 1999

An upstream plasma flow (e.g. incoming solar wind) is said to be magnetically dominated when the following conditions are met::

$$(a)\ \beta < 2/\gamma$$

$$(b)\ 1 < M_{A_n} < \sqrt{\frac{\gamma(1-\beta)+1}{\gamma-1}} \quad \text{or} \quad \frac{1}{\cos\theta_{vB}}\sqrt{\frac{2}{\beta\gamma}} < M_n < \frac{1}{\cos\theta_{vB}}\sqrt{\frac{2\gamma(1-\beta)+2}{\beta\gamma(\gamma-1)}}$$

which can result in a complex dimpled planetary bow shock topology involving different types of intermediate and switch-on/off shocks.

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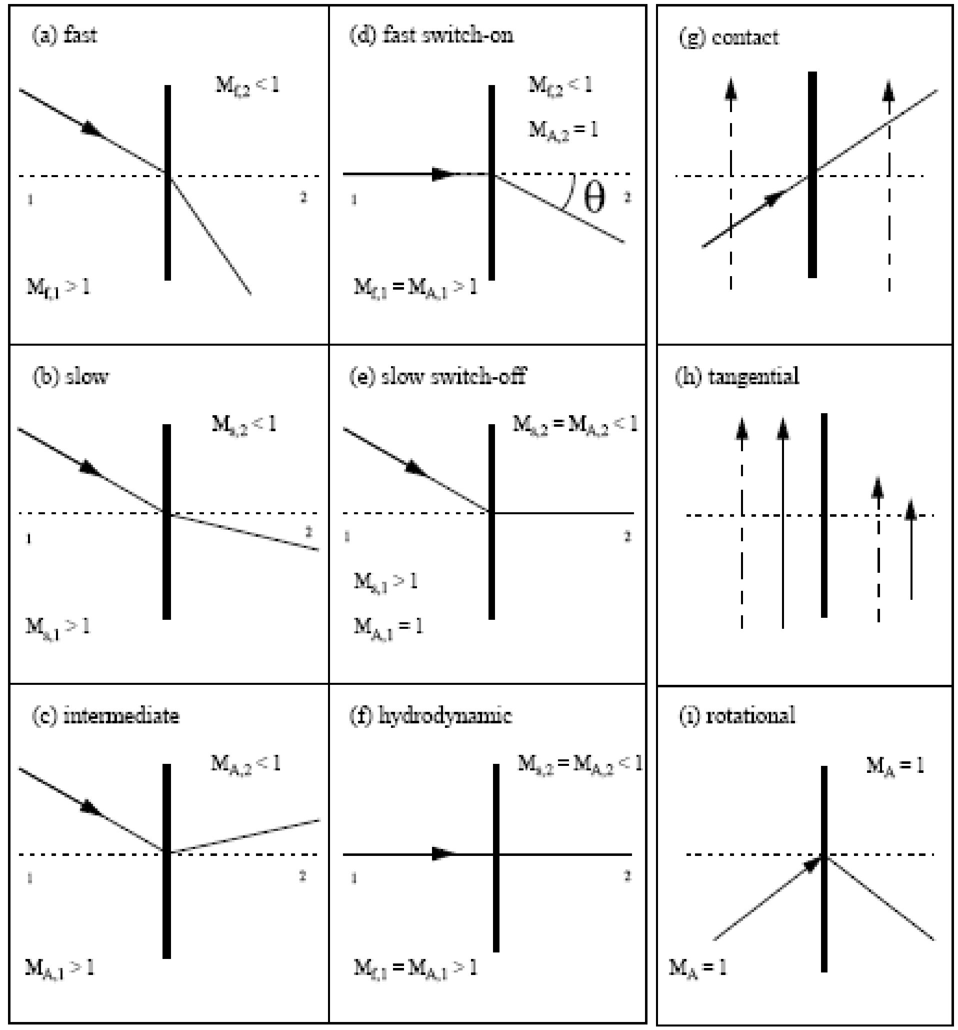
MAGNETICALLY DOMINATED REGIME (2)

1	$c_f^{(1)} < v_x^{(1)}$	$M_f^{(1)} > 1$	$M_A^{(1)} > 1$	$M_s^{(1)} > 1$
⇓				
2	$c_A^{(2)} < v_x^{(2)} < c_f^{(2)}$	$M_f^{(2)} < 1$	$M_A^{(2)} > 1$	$M_s^{(2)} > 1$
⇓				
3	$c_s^{(3)} < v_x^{(3)} < c_A^{(3)}$	$M_f^{(3)} < 1$	$M_A^{(3)} < 1$	$M_s^{(3)} > 1$
⇓				
4	$v_x^{(4)} < c_s^{(4)}$	$M_f^{(4)} < 1$	$M_A^{(4)} < 1$	$M_s^{(4)} < 1$



Hans De Sterck, PhD Thesis,
CmPA, KU Leuven, 1999

ON THE EXISTENCE OF COMPLEX PLANETARY BOW SHOCK STRUCTURE DURING A MAGNETIC STORM

• According to (De Sterck, PhD Thesis, 1999), a complex planetary bow shock structure is preferably to occur when:

- $\beta < 2/\gamma$ (in particular for low beta solar wind parameters);
- θ_{vB} is large in the solar wind;
- on the quasi-parallel side of the magnetosheath and not too far from the plane containing the incoming solar wind magnetic field;

and

- the solar wind parameters remain in the magnetically dominated regime for a sufficiently long time.

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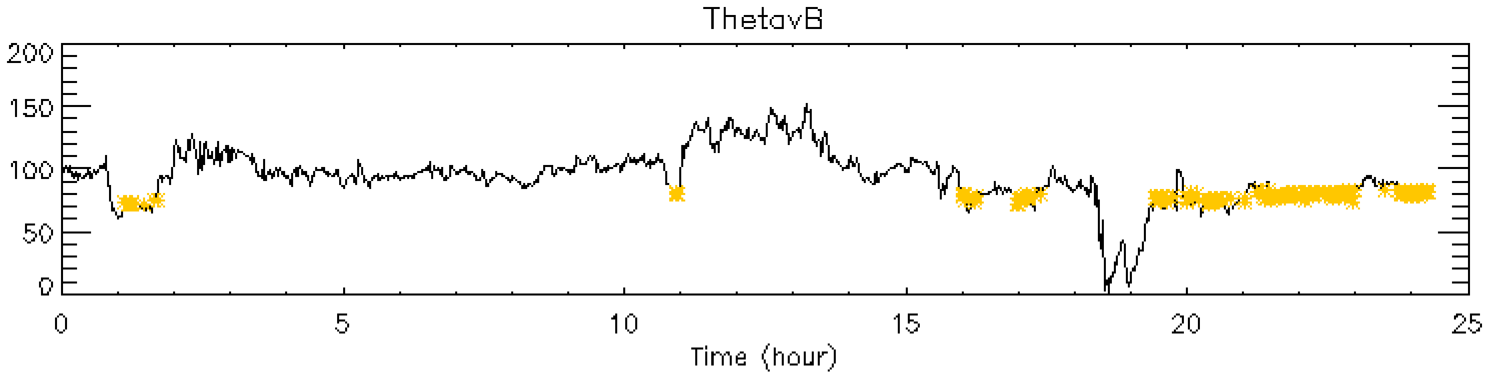
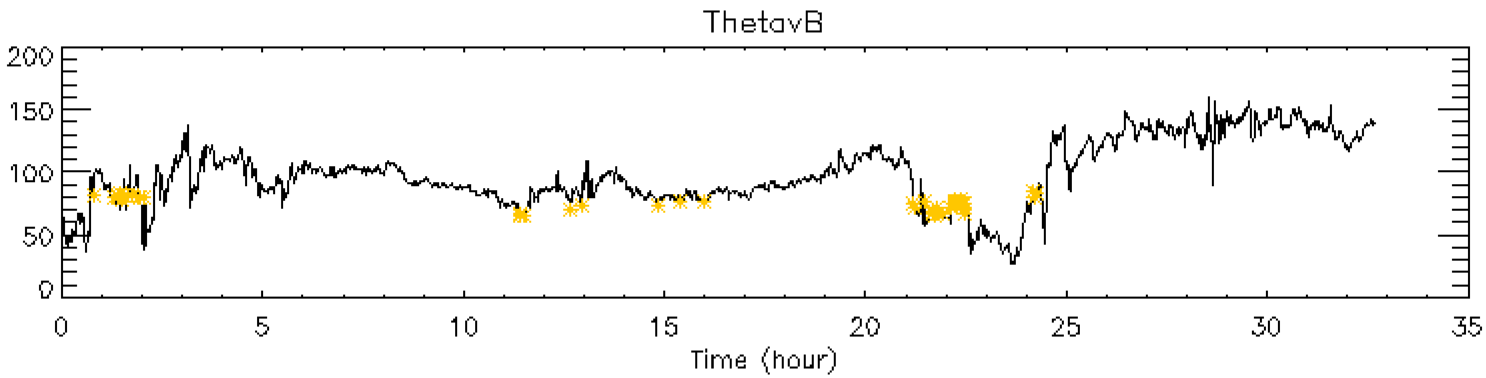
MAGNETICALLY DOMINATED REGIME DURING THE 06/04/2000 AND 20/11/2003 MAGNETIC STORMS



06/04/2000 Storm



20/11/2003 Storm

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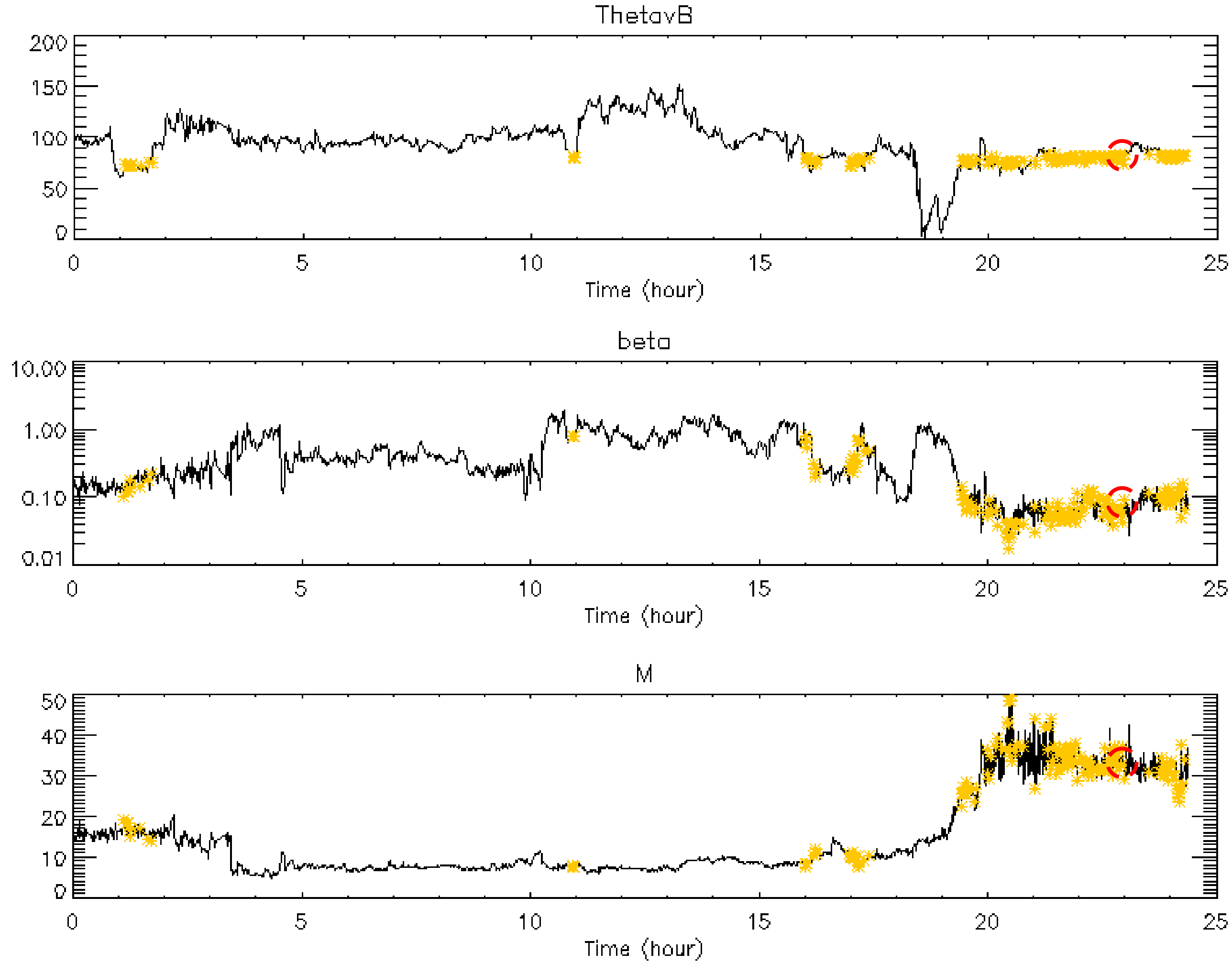
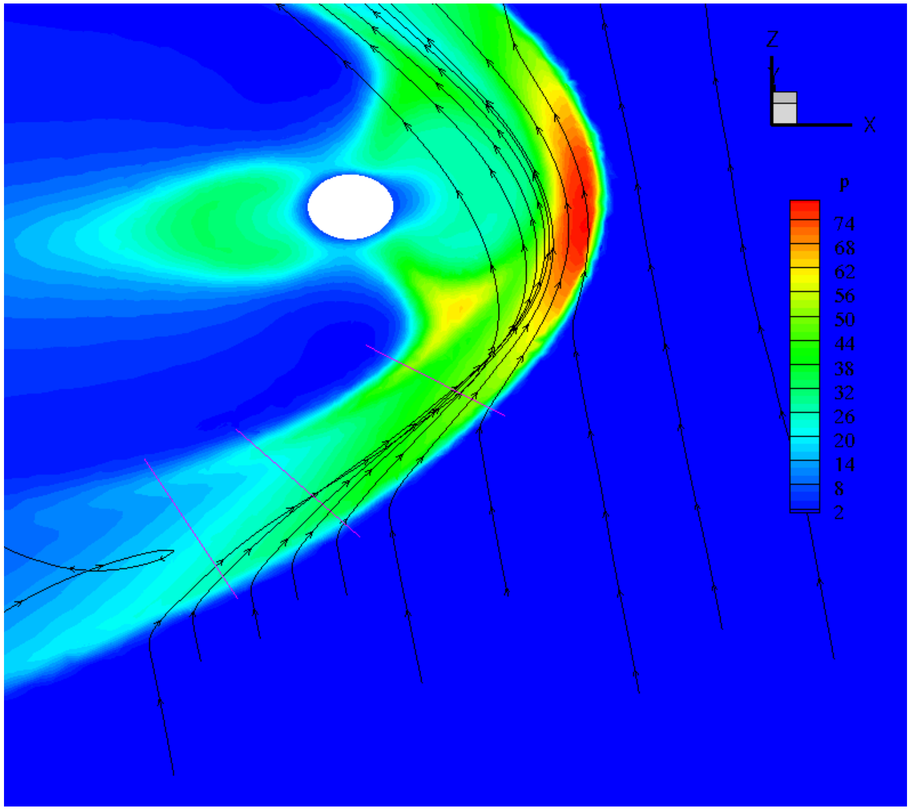
MAGNETICALLY DOMINATED REGIME DURING THE 06/04/2000 MAGNETIC STORM



ANALYSIS OF THE SELECTED PARAMETER REGIME (1)



- After ~40 minutes from the measurement taken by ACE, the related magnetically dominated solar wind plasma is thought to arrive at the vicinity of Earth's bow shock.
- Pressure contours with magnetic field lines are shown together with cuts for analysis.

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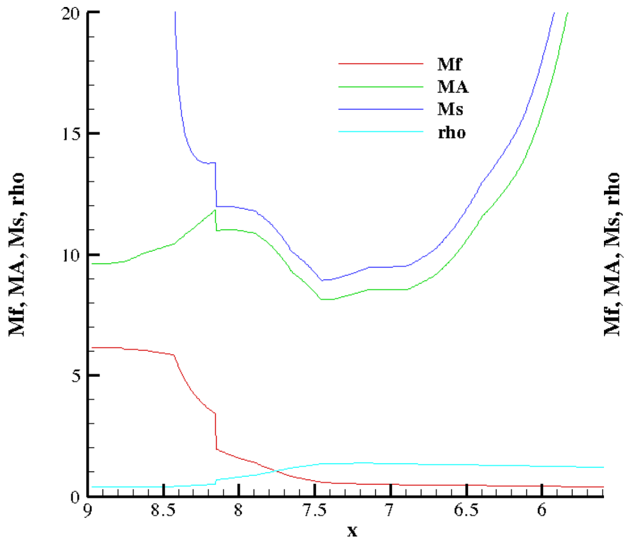
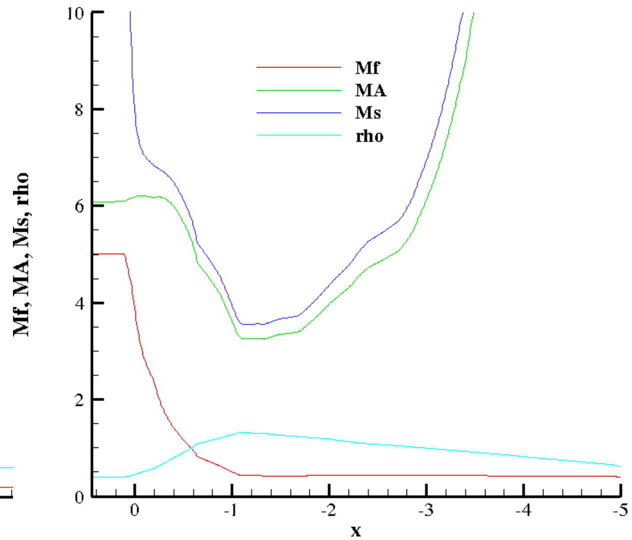
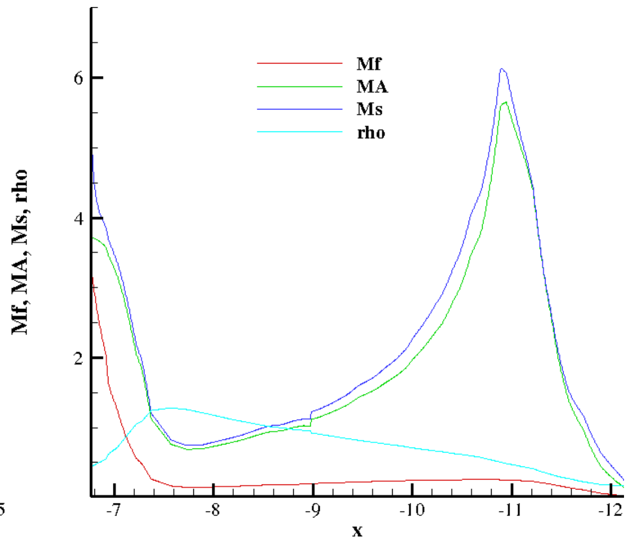
ANALYSIS OF THE SELECTED PARAMETER REGIME (2)



Right line



Middle line



Left line

- Along the bow shock while going from right to left, M_A and M_s values across the shock decrease from above to below 1 which is an indication of the presence of complex bow shock structure.

CONCLUSION

- Easy-to-configure Global MHD solver for a storm;
- Possibility of real-time storm simulation (with 120 processors);
- Possibility of coupling with different physical models;
- Introducing a numerical magnetic storm index;
- Presence of a complex bow shock structure during the 06/04/2000 magnetic storm.