High-order schemes for non-ideal $3+1$ GRMHD: a study of the kinematic mean-field dynamo process in accreting tori around Kerr black holes

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Outline

Dynamo theory

Amplification of Magnetic Fields

Ordered magnetic fields on large scales are a fundamental part of many processes related to accretion disks:

- jets from protostellar systems, AGNs, and micro-quasars,
- Blandford-Znajek mechanism from Kerr Black Holes,
- MRI and turbulent viscosity.

A mean-field dynamo mechanism may do the job: *correlated* small-scale fluctuations in velocity and magnetic field can provide a mean electromotive force and amplify seed magnetic fields.

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Mean-field dynamo in classical MHD

Consider small-scale turbulent fluctuations in the fields $\boldsymbol{v}$ e $\boldsymbol{B}$:

$$\boldsymbol{v}(\boldsymbol{x},t) = \boldsymbol{v}_0(\boldsymbol{x},t) + \delta\boldsymbol{v}(\boldsymbol{x},t), \quad \boldsymbol{B}(\boldsymbol{x},t) = \boldsymbol{B}_0(\boldsymbol{x},t) + \delta\boldsymbol{B}(\boldsymbol{x},t)$$

[assumption of kinematical regime $\Rightarrow \boldsymbol{v}_0(\boldsymbol{x},t)$ fixed].

The resistive induction equation for the mean magnetic field reads:

$$\partial_t \boldsymbol{B}_0 = \nabla \times (\boldsymbol{v}_0 \times \boldsymbol{B}_0) + \eta_r \nabla^2 \boldsymbol{B}_0 + \nabla \times \boldsymbol{\mathcal{E}}$$

$$\boldsymbol{\mathcal{E}} = \langle \delta\boldsymbol{v} \times \delta\boldsymbol{b} \rangle \simeq \alpha \boldsymbol{B}_0 - \beta \nabla \times \boldsymbol{B}_0$$

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$$\partial_t \boldsymbol{B} = \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) + \eta \nabla^2 \boldsymbol{B} + \alpha \nabla \times \boldsymbol{B}$$

- Increase of the magnetic resistivity ($\eta = \eta_r + \beta$)
- Generation of magnetic field parallel to the current ($\boldsymbol{J} = \nabla \times \boldsymbol{B}$)
- Generalized Ohm's law ($\boldsymbol{E}' \equiv \boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = \eta \boldsymbol{J} - \alpha \boldsymbol{B}$)

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The $\alpha - \Omega$ dynamo

Let us decompose the magnetic field (and the other quantities) as

$$\boldsymbol{B} = \boldsymbol{B}_P + B_T \boldsymbol{e}_T, \quad \boldsymbol{B}_P = \boldsymbol{\nabla} \times (A_T \boldsymbol{e}_T).$$

The induction equation is split as

$$\begin{aligned} \partial_t B_T + (\boldsymbol{U}_P \cdot \boldsymbol{\nabla}) B_T &= (\boldsymbol{B}_P \cdot \boldsymbol{\nabla}) U_T + \eta \nabla^2 B_T \\ \partial_t A_T + (\boldsymbol{U}_P \cdot \boldsymbol{\nabla}) A_T &= \alpha B_T + \eta \nabla^2 A_T \end{aligned}$$

- Differential rotation $\Rightarrow$ Generation of toroidal field (Ω effect)
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$$\boldsymbol{B}_P \Rightarrow \boldsymbol{B}_T \Rightarrow \boldsymbol{B}_P \Rightarrow \dots$$

Exponentially growing $\alpha - \Omega$ dynamo modes, damped by resistivity.

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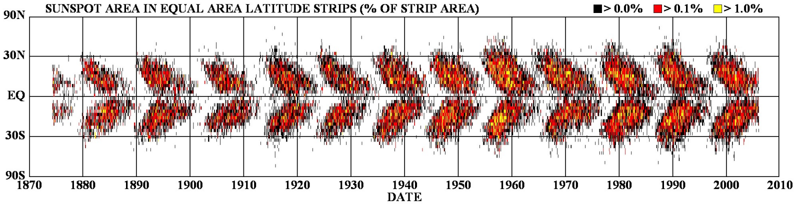
The solar cycle and the *butterfly diagram*

- Flux tubes of growing toroidal magnetic field emerge through the photosphere: formation of a Sun Spot
- Formation at latitudes $\pm 35°$
- Migration towards the equator
- Strong periodicity ($\sim$ 11 yr)

A possible mechanism at its basis is the $\alpha - \Omega$ dynamo.

DAILY SUNSPOT AREA AVERAGED OVER INDIVIDUAL SOLAR ROTATIONS



Non-ideal Ohm's law in $3+1$ GRMHD: formulation, methods, tests

The ECHO code for GRMHD

Numerical simulations will be performed with the *Eulerian Conservative High-Order* ECHO code (Del Zanna et al., 2007), a versatile tool for MHD and GRMHD combining shock-capturing properties and accuracy.

Main features (Londrillo & Del Zanna, 2000; 2004):

- conservative form, RK time-stepping,
- finite differences, high-order methods (CENO, WENO, MP...),
- simplified Riemann solvers: LLF, HLL, HLLC,
- F90, various geometries, different physical modules,
- Upwind Constrained Transport (UCT) based on 4-state B fluxes.

Upgrades:

- evolution of Einstein equations (Bucciantini & Del Zanna 2011),
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Conservative $3+1$ equations

In numerical GRMHD the spacetime metric is split in the $3+1$ form

$$ds^2 = -\alpha^2 dt^2 + \gamma_{ij}(dx^i + \beta^i dt)(dx^j + \beta^j dt),$$

where α and β^i are, respectively, the *lapse function* and *shift vector* ($\alpha = 1$, $\beta^i = 0$ in Newtonian MHD), and γ_{ij} is the 3-metric tensor.

The conservative equations in ECHO for *ideal* GRMHD are

$$\frac{\partial}{\partial t}\sqrt{\gamma}\begin{bmatrix} D \\ S_j \\ E \\ B^j \end{bmatrix} + \frac{\partial}{\partial x^i}\sqrt{\gamma}\begin{bmatrix} DV^i \\ \alpha S^i_j - \beta^i S_j \\ \alpha S^i - \beta^i E \\ V^i B^j - V^j B^i \end{bmatrix} = \sqrt{\gamma}\begin{bmatrix} 0 \\ \frac{1}{2}\alpha S^{ik}\partial_j\gamma_{ik} + S_i\partial_j\beta^i - E\partial_j\alpha \\ \alpha S^{ij}K_{ij} - S^j\partial_j\alpha \\ 0 \end{bmatrix}$$

where K_{ij} is the *extrinsic curvature* and $V^i = \alpha v^i - \beta^i$ is the *transport velocity*.

Let us now relax the ideal Ohm law (the same in MHD and GRMHD):

$$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B}.$$

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Covariant generalized Ohm's Law

Covariant Maxwell's equations:

$$\nabla_\mu F^{\mu\nu} = -I^\nu, \quad \nabla_\mu F^{*\mu\nu} = 0$$

$$F^{\mu\nu} = u^\mu e^\nu - u^\nu e^\mu + \epsilon^{\mu\nu\lambda\kappa} b_\lambda u_\kappa, \quad I^\mu = q_0 u^\mu + j^\mu$$

Fully covariant formulation for a resistive plasma with dynamo action:

$$\boxed{e^\mu = \eta j^\mu + \xi b^\mu}$$

$$(\xi \equiv -\alpha_{dyn}, \quad \eta = \eta_r + \beta_{dyn})$$

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$$\Gamma[\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} - (\boldsymbol{E} \cdot \boldsymbol{v})\boldsymbol{v}] = \eta(\boldsymbol{J} - q\boldsymbol{v}) + \xi\Gamma[\boldsymbol{B} - \boldsymbol{v} \times \boldsymbol{E} - (\boldsymbol{B} \cdot \boldsymbol{v})\boldsymbol{v}]$$

Classical limit ($|\boldsymbol{v}| \ll 1$, $|\boldsymbol{E}| \ll |\boldsymbol{B}|$):

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Implementation within $3+1$ GRMHD

Maxwell's equations in $3+1$ form are:

$$\gamma^{-1/2}\partial_t\left(\gamma^{1/2}\boldsymbol{B}\right)+\nabla\times(+\alpha\boldsymbol{E}+\boldsymbol{\beta}\times\boldsymbol{B})=0, \qquad (\nabla\cdot\boldsymbol{B}=0)$$

$$\gamma^{-1/2}\partial_t\left(\gamma^{1/2}\boldsymbol{E}\right)+\nabla\times(-\alpha\boldsymbol{B}+\boldsymbol{\beta}\times\boldsymbol{E})=-(\alpha\boldsymbol{J}-q\boldsymbol{\beta}), \qquad (\nabla\cdot\boldsymbol{E}=q)$$

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A stiff equation: terms $\propto\eta^{-1}$ can evolve on time scales $\tau_\eta\ll\tau_h$.

Letting $\eta=\xi=0$ we retrieve the ideal case:

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Stiff relaxation equations

System of hyperbolic ODEs + stiff relaxation equations:

$$\begin{aligned} \boldsymbol{X} = \gamma^{1/2}\boldsymbol{E}: \quad \partial_t \boldsymbol{X} &= \boldsymbol{Q}_X(\boldsymbol{X},\boldsymbol{Y}) + \boldsymbol{R}_X(\boldsymbol{X},\boldsymbol{Y}) \\ \boldsymbol{Y} = \gamma^{1/2}\boldsymbol{B}: \quad \partial_t \boldsymbol{Y} &= \boldsymbol{Q}_Y(\boldsymbol{X},\boldsymbol{Y}) \end{aligned}$$

- $\boldsymbol{Q}_{X,Y}$= non stiff terms, evolving as $\sim \tau_h$
- $\boldsymbol{R}_X$= stiff terms $\propto \eta^{-1}$, evolving as $\sim \tau_\eta \ll \tau_h$

We need appropriate techniques to evolve the system, even for $\xi = 0$:

- Splitting methods (Komissarov, 2007; Takamoto & Inoue, 2011; Takahashi et al., *subm.*).
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High-order IMEX schemes

High-order time integration via IMEX Runge-Kutta methods with Strong Stability Preserving properties (Pareschi & Russo, 2005):

- Phase 1: Explicit integration

$$\begin{aligned} \boldsymbol{X}_*^{(i)} &= \boldsymbol{X}^n + \Delta t \textstyle\sum_{j=1}^{i-1} \tilde{a}_{ij} \boldsymbol{Q}_X[\boldsymbol{X}^{(j)}, \boldsymbol{Y}^{(j)}] + \Delta t \textstyle\sum_{j=1}^{i-1} a_{ij} \boldsymbol{R}_X[\boldsymbol{X}^{(j)}, \boldsymbol{Y}^{(j)}] \\ \boldsymbol{Y}_*^{(i)} &= \boldsymbol{Y}^n + \Delta t \textstyle\sum_{j=1}^{i-1} \tilde{a}_{ij} \boldsymbol{Q}_Y[\boldsymbol{X}^{(j)}, \boldsymbol{Y}^{(j)}] \end{aligned}$$

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$$\begin{aligned} \boldsymbol{X}^{n+1} &= \boldsymbol{X}^n + \Delta t \textstyle\sum_{i=1}^{\nu} \tilde{\omega}_i \boldsymbol{Q}_X[\boldsymbol{X}^{(i)}, \boldsymbol{Y}^{(i)}] + \Delta t \textstyle\sum_{i=1}^{\nu} \omega_i \boldsymbol{R}_X[\boldsymbol{X}^{(i)}, \boldsymbol{Y}^{(i)}] \\ \boldsymbol{Y}^{n+1} &= \boldsymbol{Y}^n + \Delta t \textstyle\sum_{i=1}^{\nu} \tilde{\omega}_i \boldsymbol{Q}_Y[\boldsymbol{X}^{(i)}, \boldsymbol{Y}^{(i)}] \end{aligned}$$

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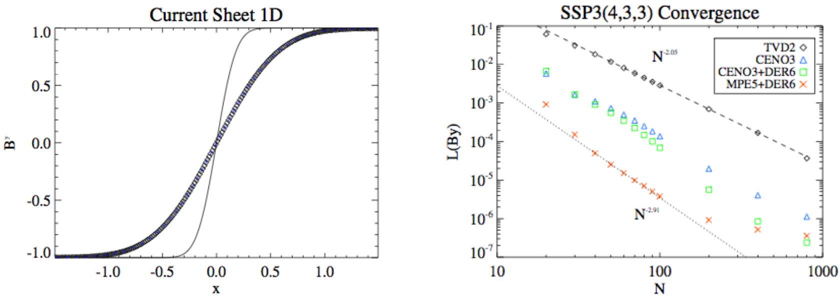
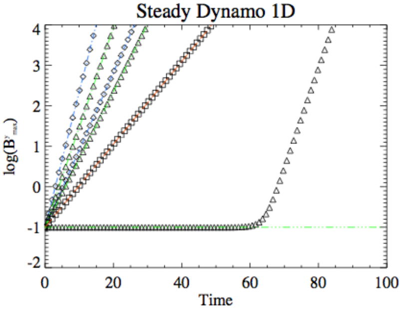
Numerical tests: 1D current sheet

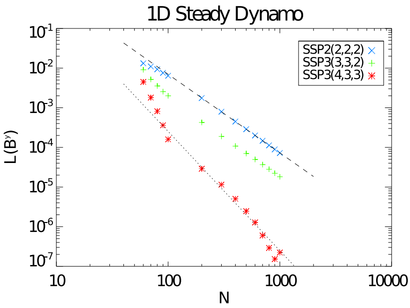


Diffusion equation:

$$p \gg B^2 \Rightarrow B^y(x,t) = B_0 \mathrm{erf}\left(\frac{x}{2\sqrt{\eta t}}\right) ; \quad [\eta = 0.01, t = 1, 10]$$

Numerical tests: 1D dynamo





Exponentially growing dynamo modes:

$$B^y(x,t) = B_0 \exp(\gamma t) \cos(kx), \quad \gamma = \frac{\sqrt{1 + 4\eta k(\xi - \eta k)} - 1}{2\eta}$$

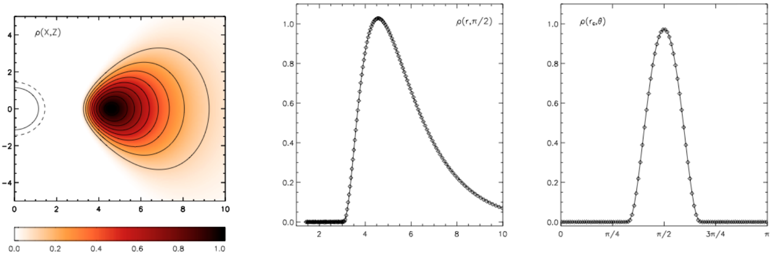
Kinematic dynamo in accretion tori

GRHD thick torus

Equilibrium: axisymmetric torus in Kerr metric and Boyer-Lindquist coordinates (Abramowicz et al., 1978; Font & Daigne, 2002).

Bernoulli equation for barotropic EOS $p \propto (\rho h)^\kappa$, differential rotation $\Omega = u^\phi / u_t$, constant specific angular momentum $l_0 = -u_\phi / u_t$:

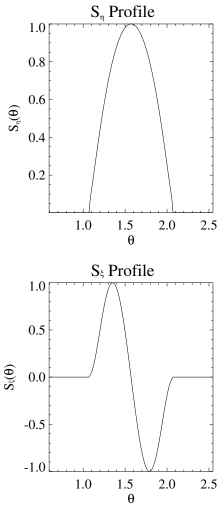
$$W - W_{\rm in} + \frac{\kappa}{\kappa - 1} \frac{p}{\rho h} = 0, \quad W = \ln |u_t| = \frac{1}{2} \ln \left(\frac{g_{t\phi}^2 - g_{rr} g_{\phi\phi}}{g_{\phi\phi} + 2 g_{t\phi} l_0 + g_{tt} l_0^2} \right)$$



Numerical set up

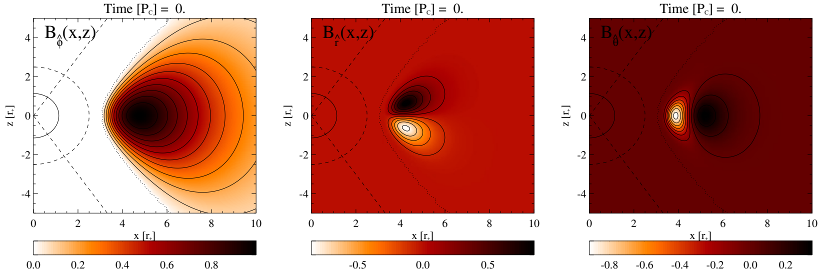
- Spatial interpolation: MPE5 + DER6.
- Time integration: IMEX-RK SSP3(4,3,3).
- Two-dimensional grid in (r, θ).
- Maximally rotating Kerr BH ($a = 0.99$).
- Ideal, corotating atmosphere.
- Spatially varying ξ and η.
- Different choices of dynamo numbers:

$$C_\xi = \frac{\xi R}{\eta} \geq 1, \qquad C_\Omega = \frac{\Delta\Omega R^2}{\eta} \gg 1$$



Initial seed magnetic field

- Initial seed magnetic field $B \sim 10^{-6}$ either toroidal or poloidal.
- Initial ideal electric field ($\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B}$).



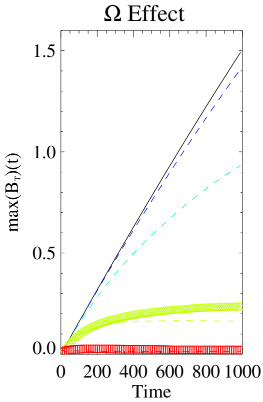
Study of the Ω effect vs resistivity

- No dynamo action ($\xi = 0$).
- Competition between ideal Ω effect and resistive η dissipation.
- Estimate of saturation toroidal field:

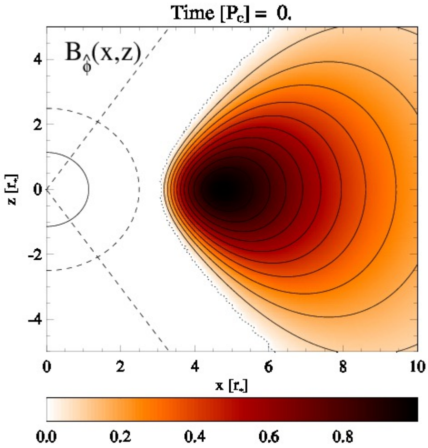
$$\Delta B \sim T \Delta\Omega |\boldsymbol{B}_P| \sim -T \frac{\eta}{R^2} B \Rightarrow$$

$$B_{sat} \sim \frac{\Delta\Omega R^2}{\eta} |\boldsymbol{B}_P| \equiv C_\Omega |\boldsymbol{B}_P|$$

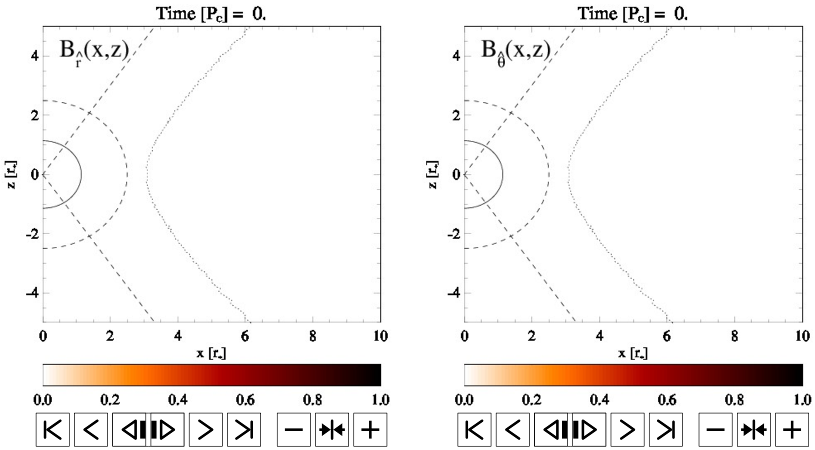
- Long-term runs ($P_c = 76.5$) confirm this ($\eta = 0, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}$).



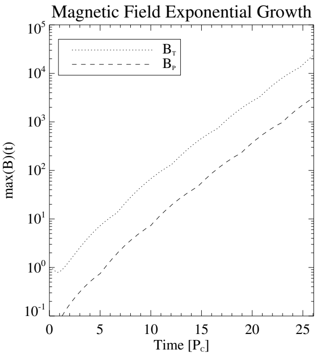
Evolution of the toroidal component

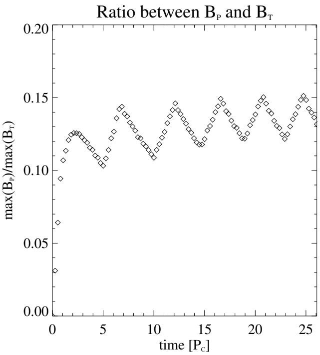


Evolution of the poloidal components

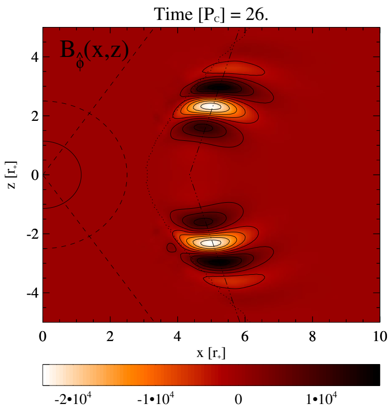


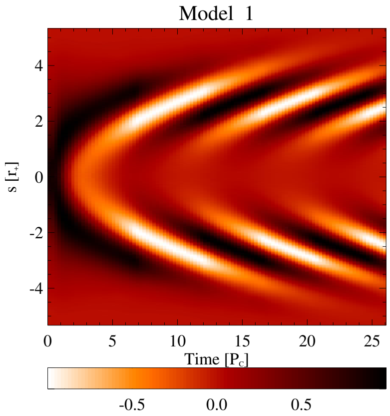
Growth rate and ratio B_P/B_T





Butterfly diagram





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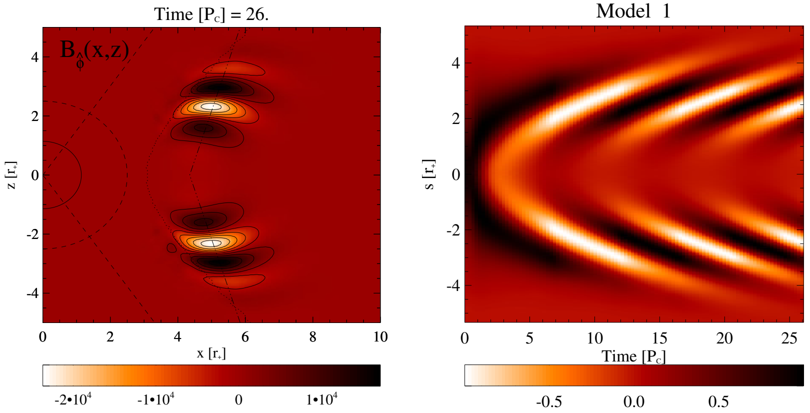
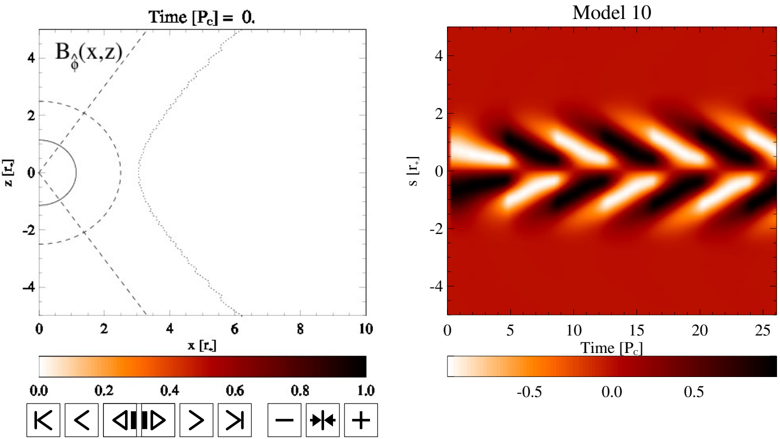


Migration reversal

The *solar analogy* is reproduced by choosing $\xi < 0 \Rightarrow \alpha_{dyn} > 0$:



Summary of results

	$\boldsymbol{B}_{\text{init}}$	C_ξ	C_Ω	Growth Rate	Period	s_{max}	$(B_P)_{max}/(B_T)_{max}$
Model 1	$\boldsymbol{B}_T$	5	400	0.39	8.43	2.41	0.13
Model 2	$\boldsymbol{B}_T$	-5	400	0.21	8.17	0.04	0.13
Model 3	$\boldsymbol{B}_T$	25	400	1.58	2.77	2.05	0.42
Model 4	$\boldsymbol{B}_T$	1	400	0.07	26.38	2.63	0.04
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Model 6	$\boldsymbol{B}_T$	5	2000	0.22	16.46	1.82	0.06
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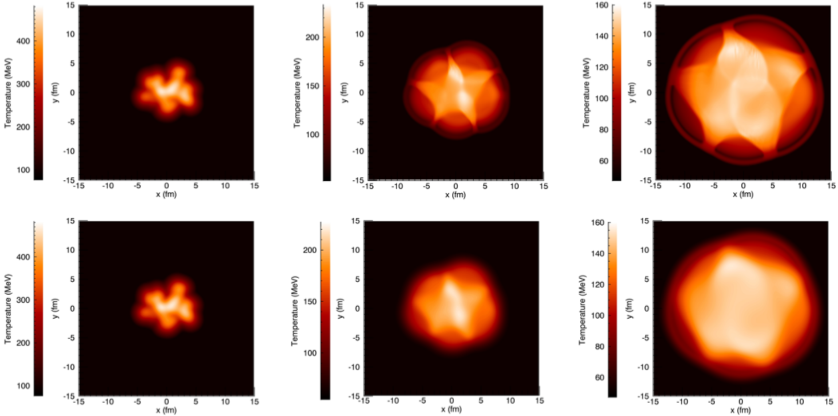
ECHO-QGP: viscous RHD for heavy-ion collisions



Thank you!