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Perlin Noise as a Turbulence Model for Particle Transport

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[1] University of Helsinki

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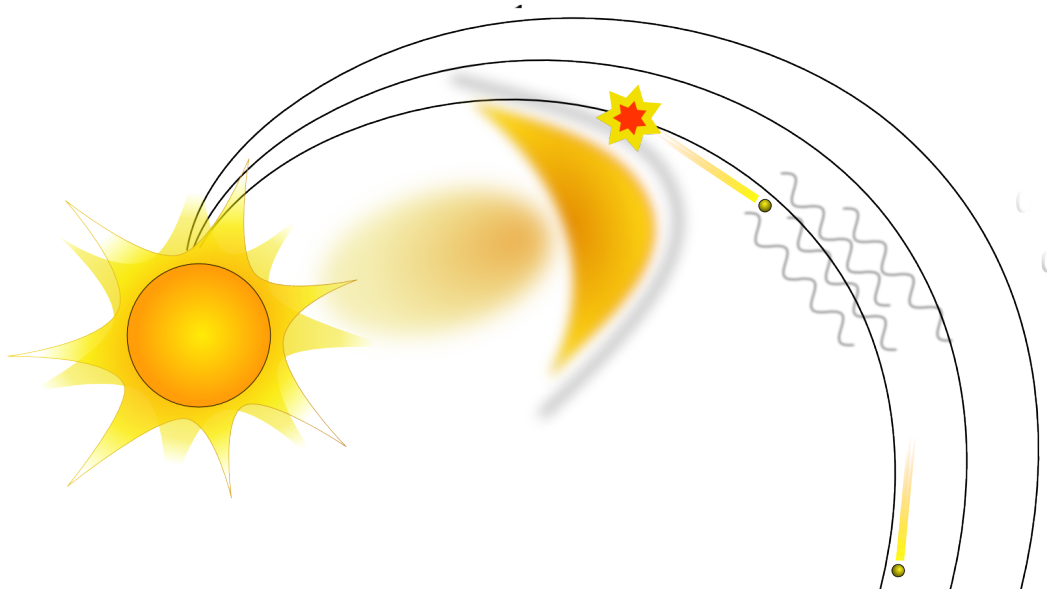
Particle Transport and Turbulence



- Particle transport in heliospheric and interstellar medium
- Test particle simulations
- Turbulent fields play important role.

Simulated Turbulence

- Getting turbulence right is difficult.
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- We'd rather have a way to construct quasi-turbulent fields.

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- Again bound to a grid.
- Time evolution?

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- Creating correct spectrum in Fourier space works.
- Again bound to a grid.
- Time evolution?
- We would like to have deterministic randomness with the correct spectrum without having to look at all space.

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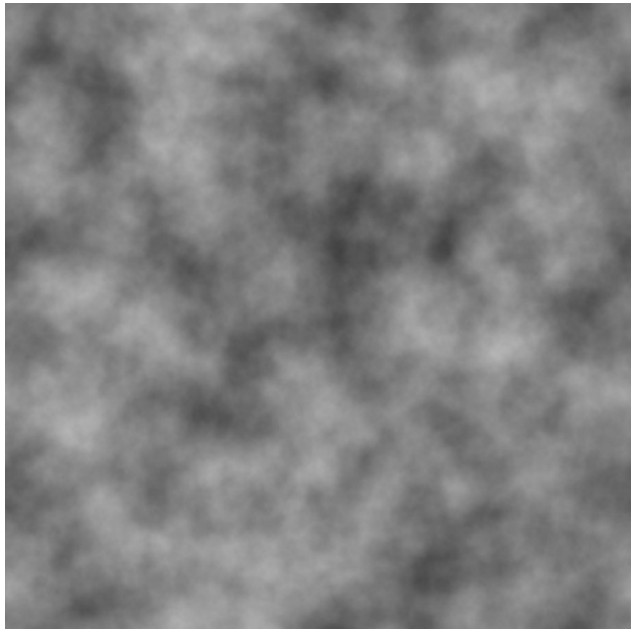
- Originally a cloud-texture generator.
- Structures of fixed scale.



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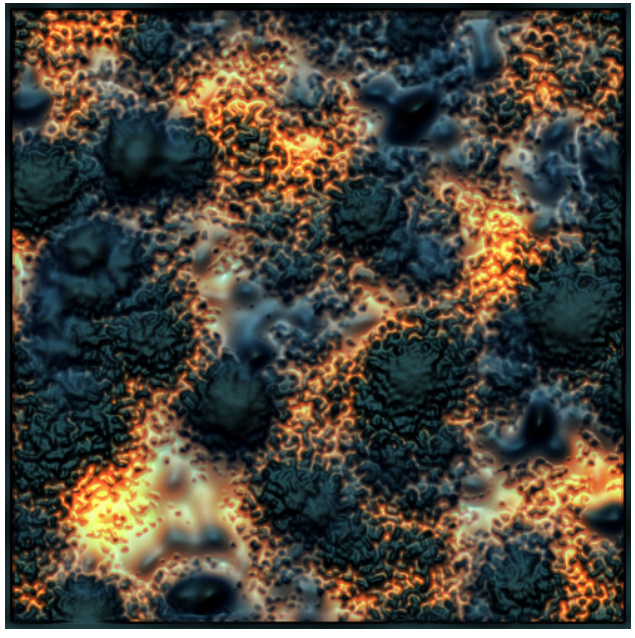

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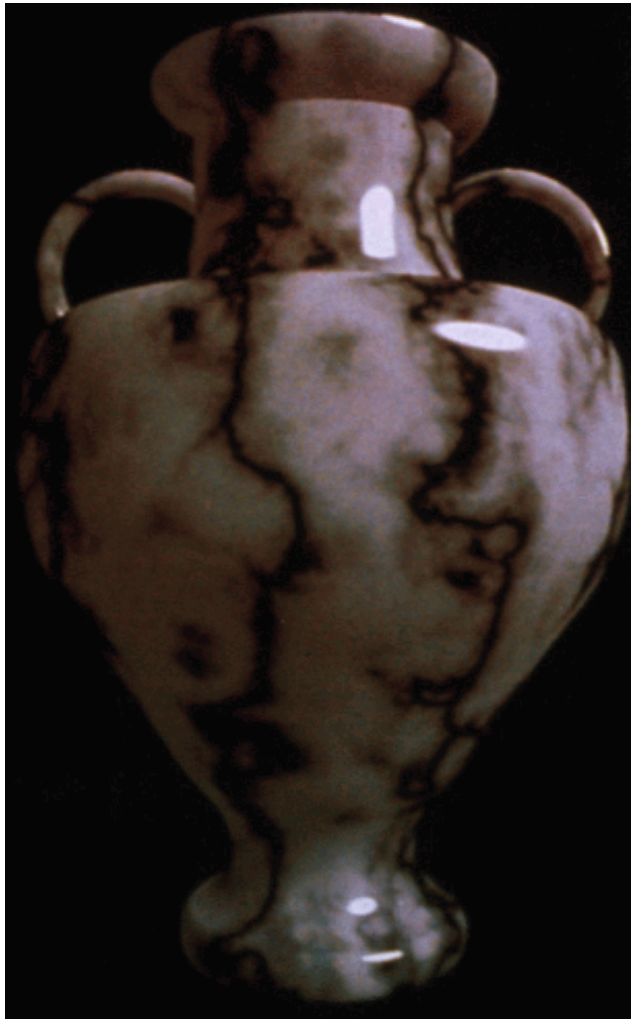

Perlin Noise - internals

Pick deterministic gradients on grid, interpolate using Hermite polynomial.

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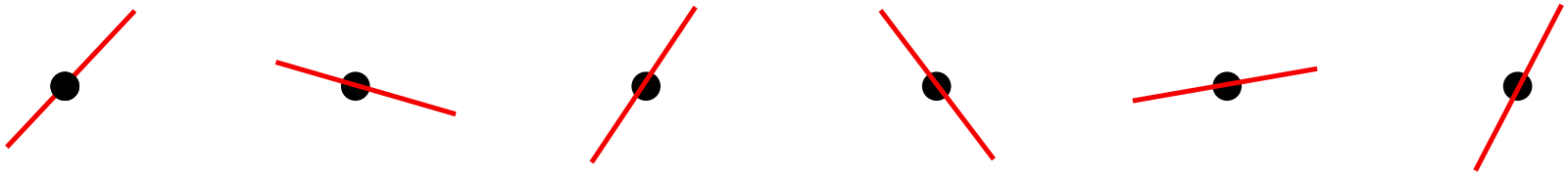

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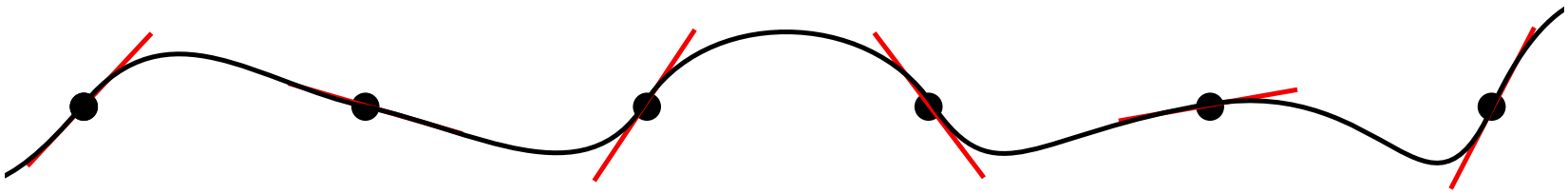

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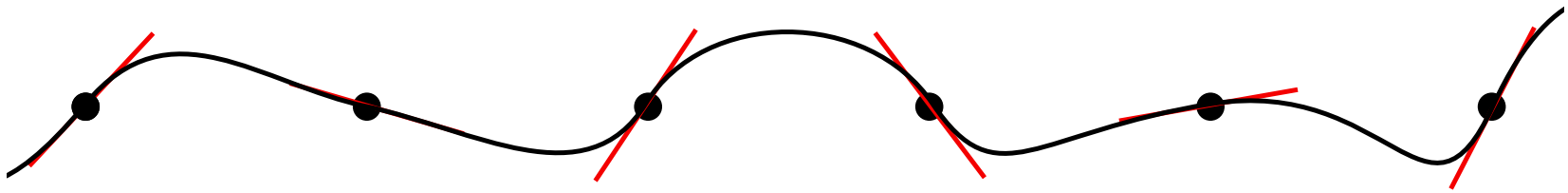

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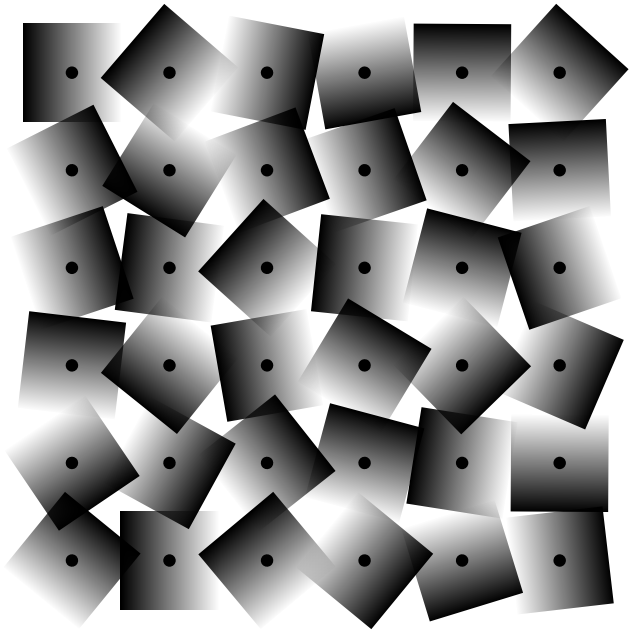

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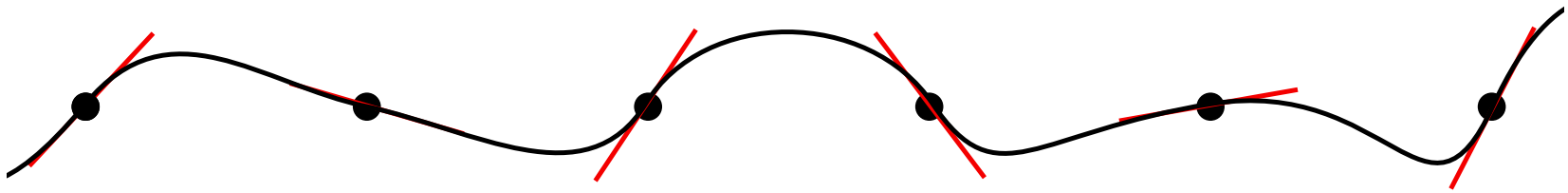

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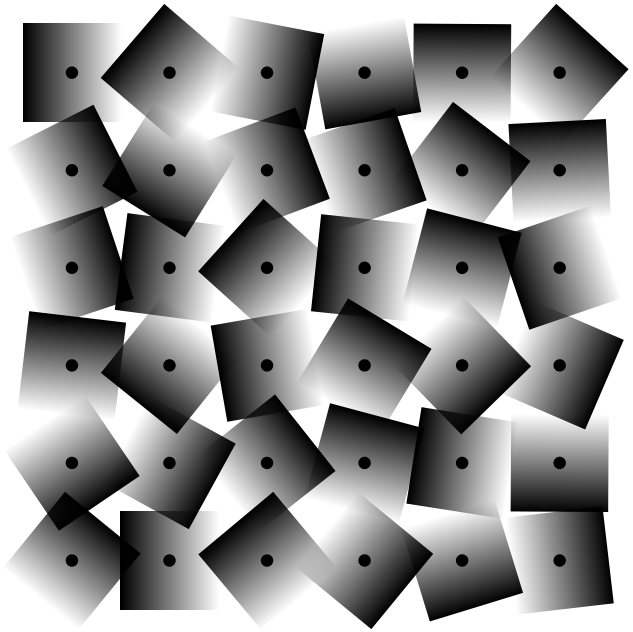
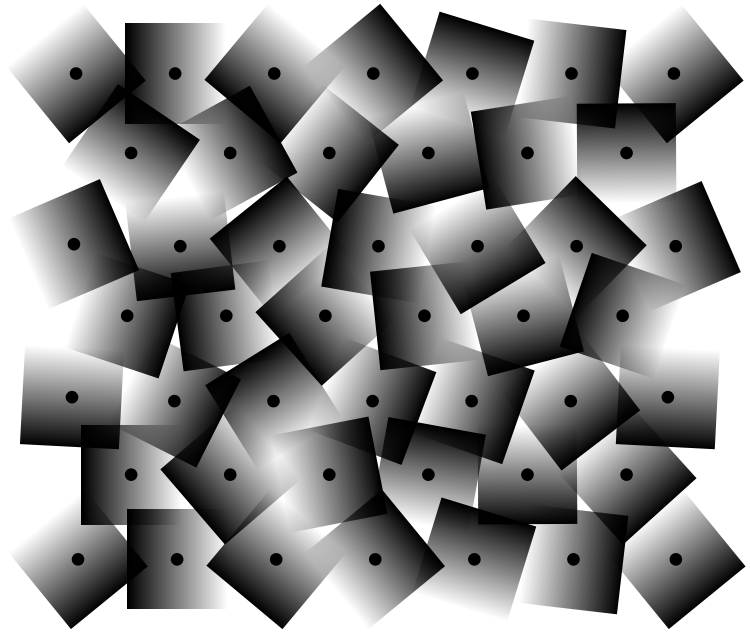


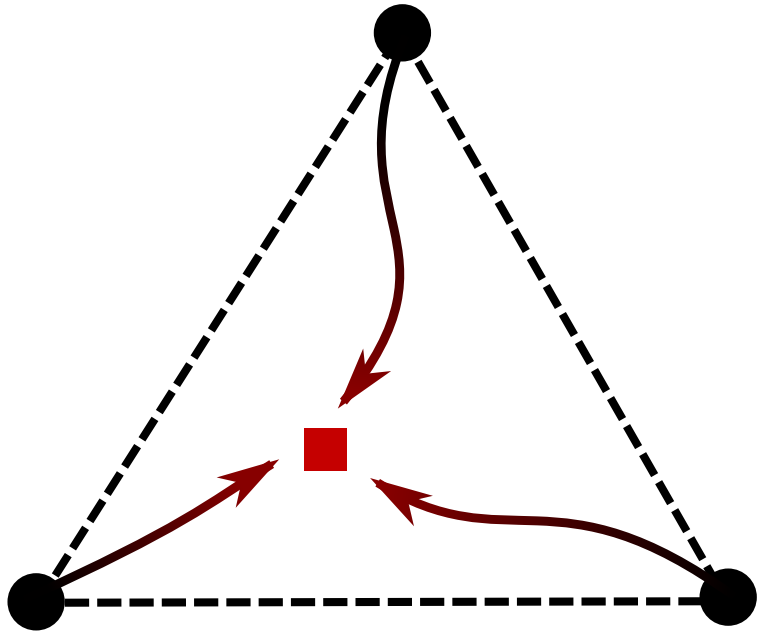
Simplex Noise (2001):



Implementation Details

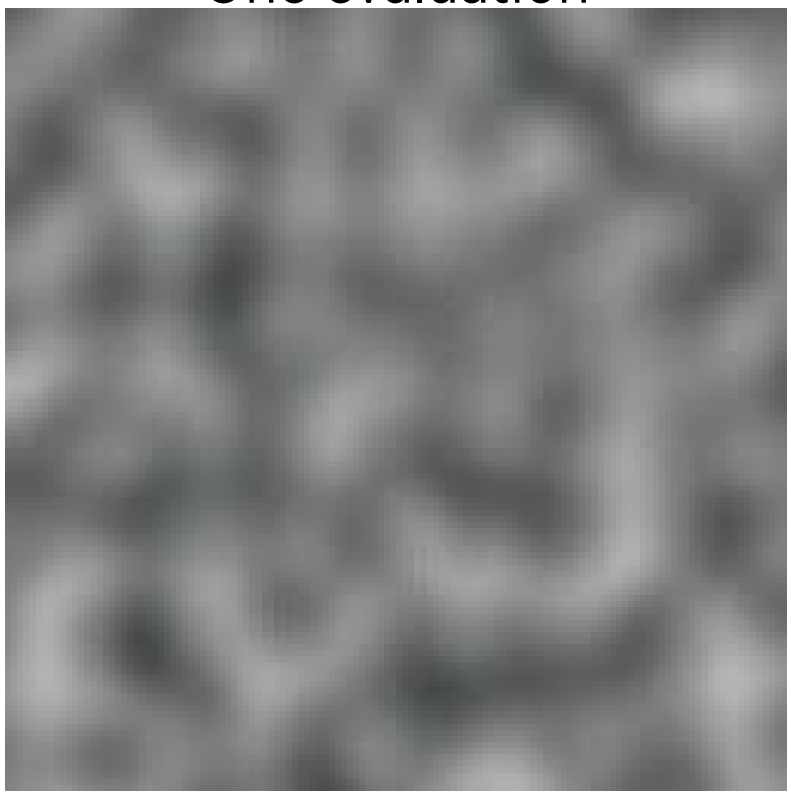
Advantage:

- Interpolation is completely local within one grid cell.
- Grid values are not stored, but calculated on-demand by a (cheap) hash function.
- Completely independent evaluation, trivially parallel.
- Not just suitable for GPUs - GPUs were designed for this.
- Spatial size is not inherently limited, except by floating point accuracy.



Noise Octaves

One evaluation



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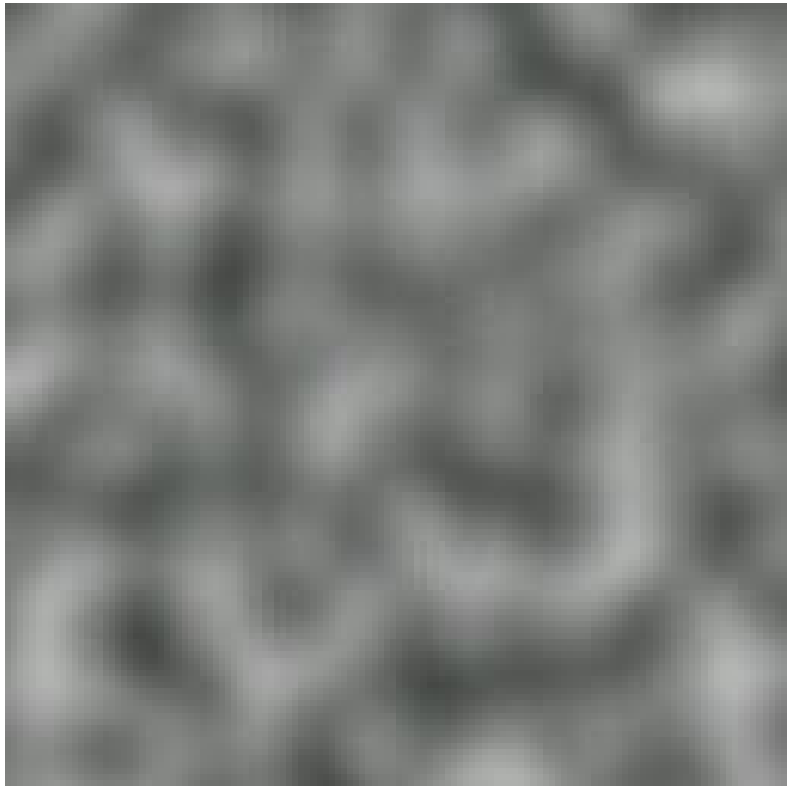
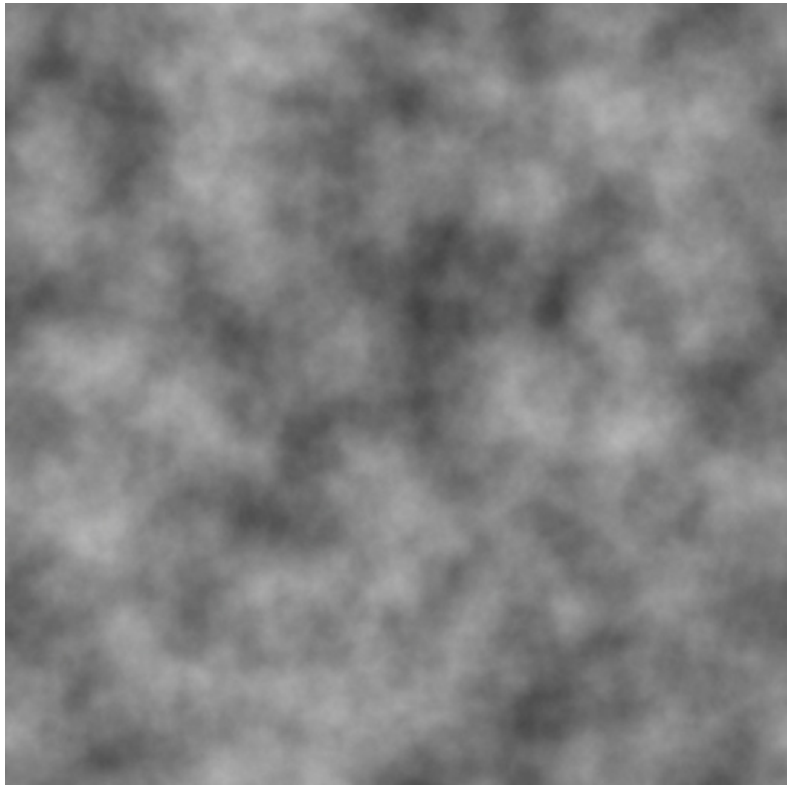
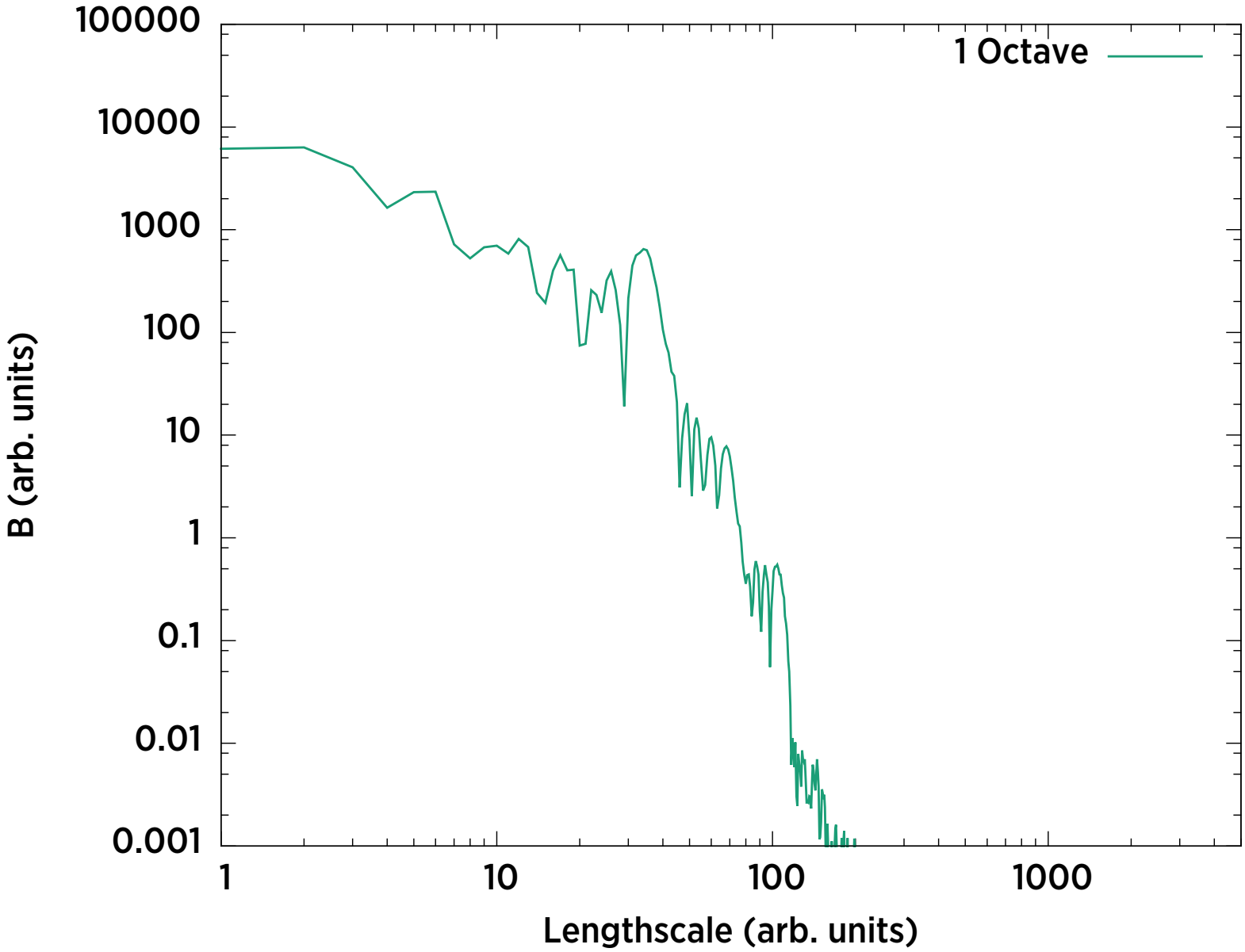


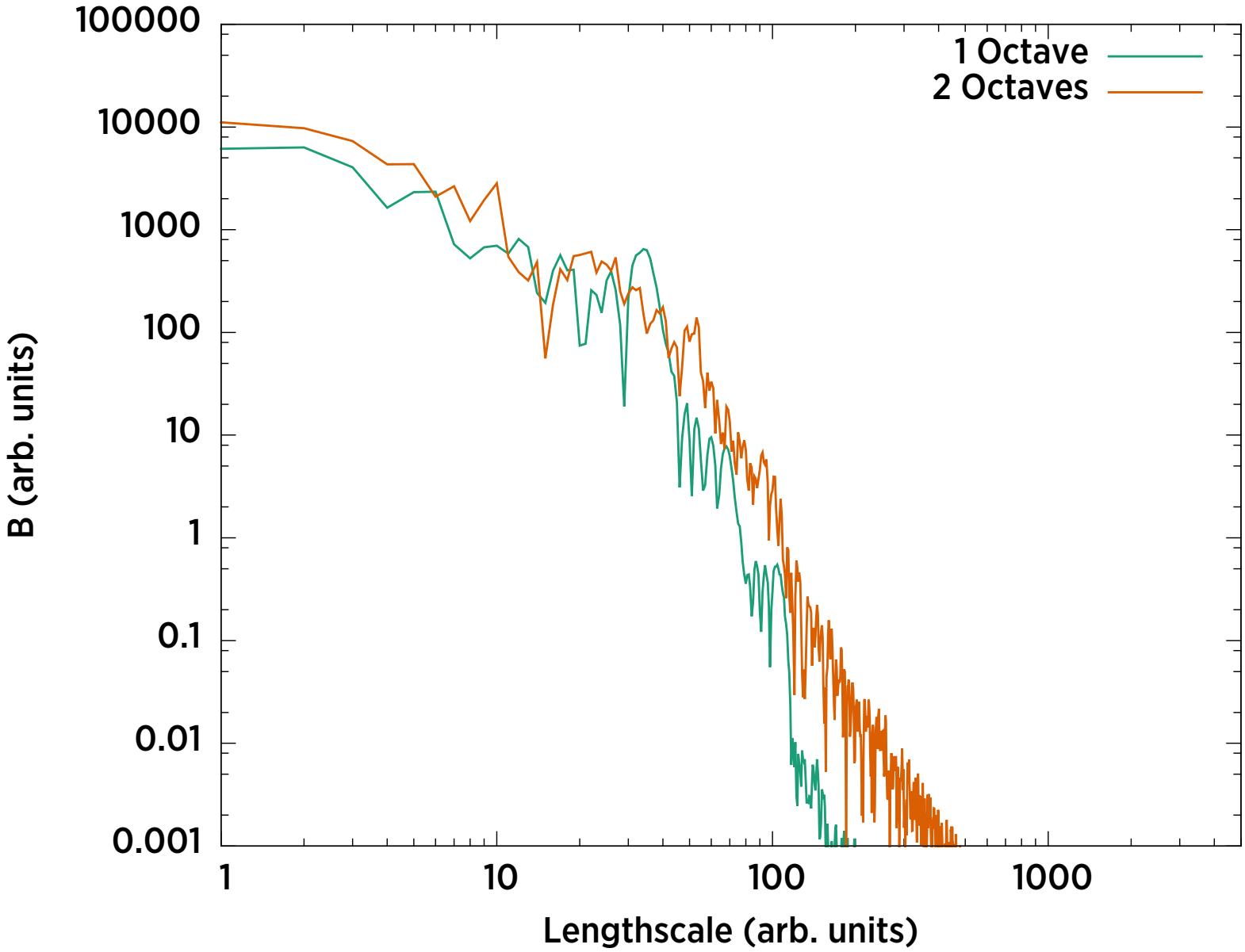
Multiple octaves, stacked



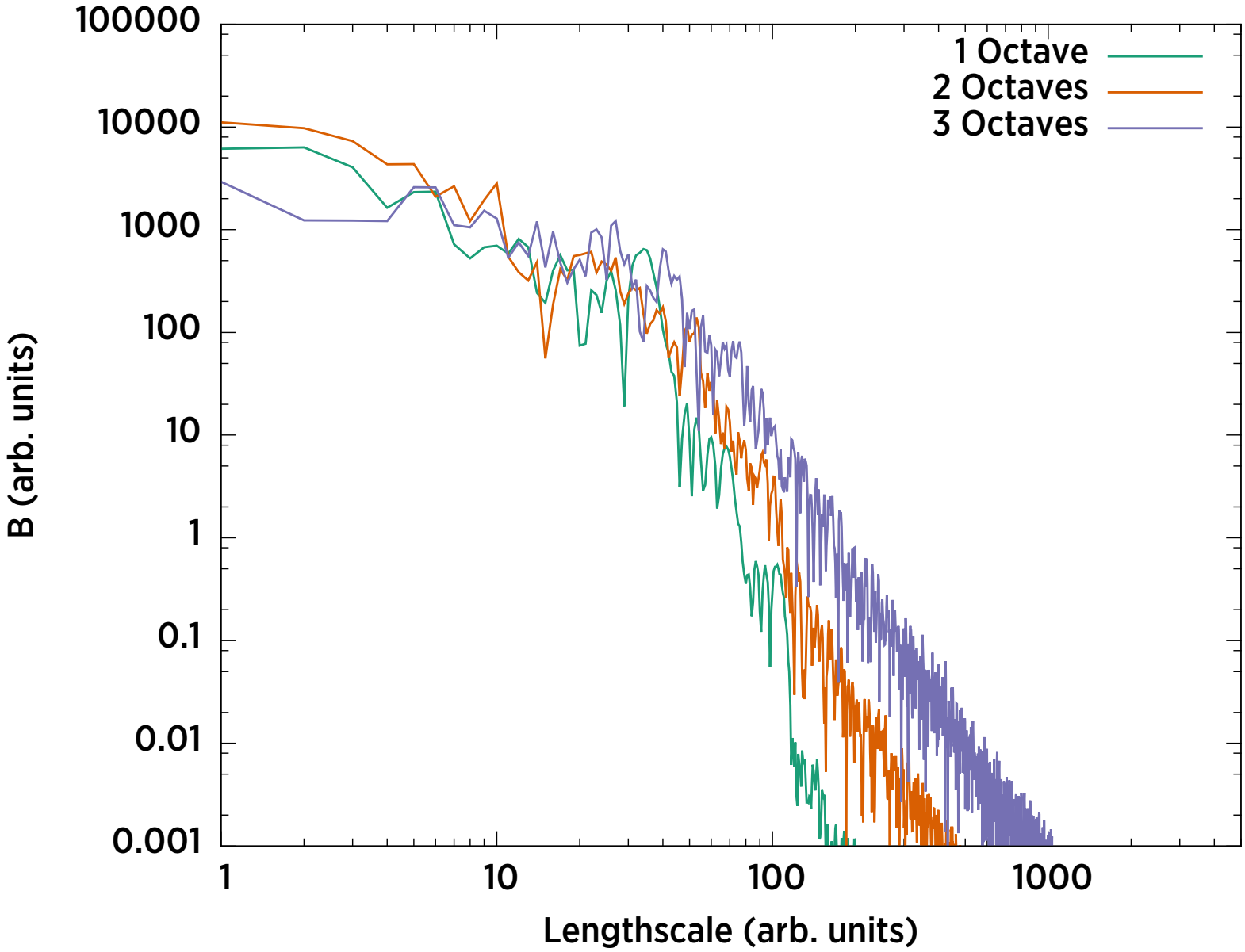
Spectrum


1 Octave
B (arb. units)
100000
10000
1000
100
10
1
0.1
0.01
0.001
1
10
100
1000
Lengthscale (arb. units)

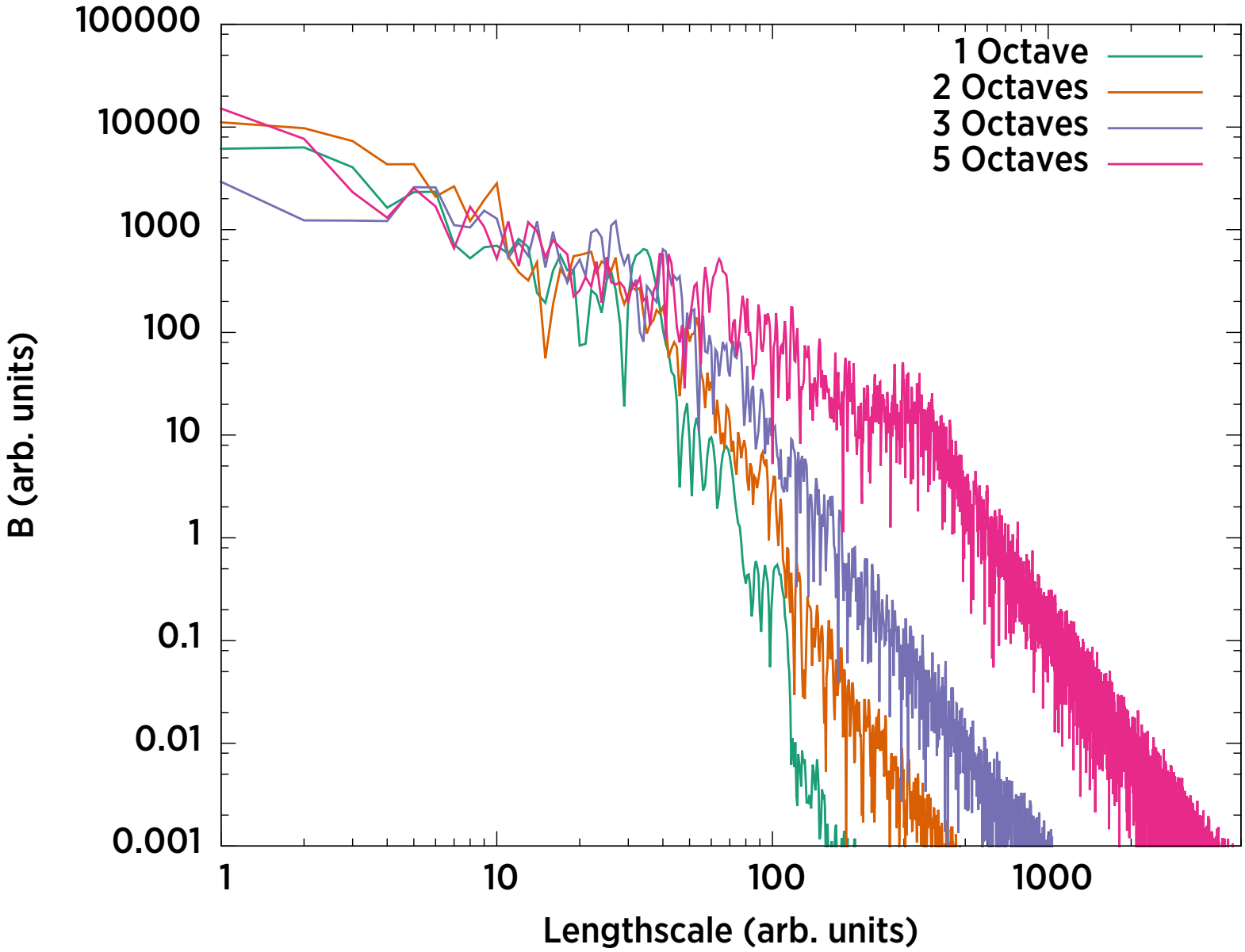
Spectrum


100000
10000
1000
100
10
1
0.1
0.01
0.001
B (arb. units)
1
10
100
1000
Lengthscale (arb. units)
1 Octave
2 Octaves

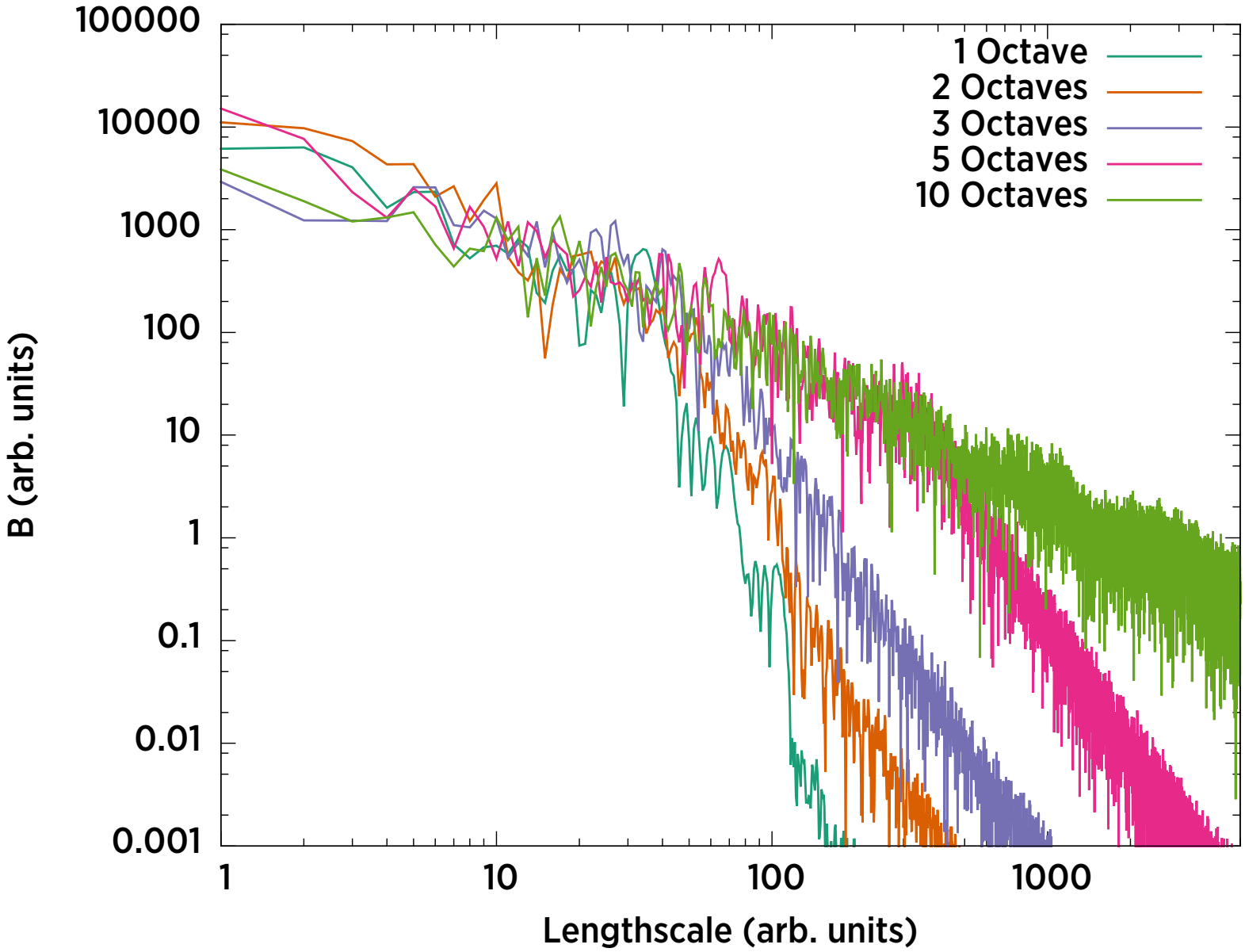
Spectrum


100000
10000
1000
100
10
1
0.1
0.01
0.001
B (arb. units)
1
10
100
1000
Lengthscale (arb. units)
1 Octave
2 Octaves
3 Octaves

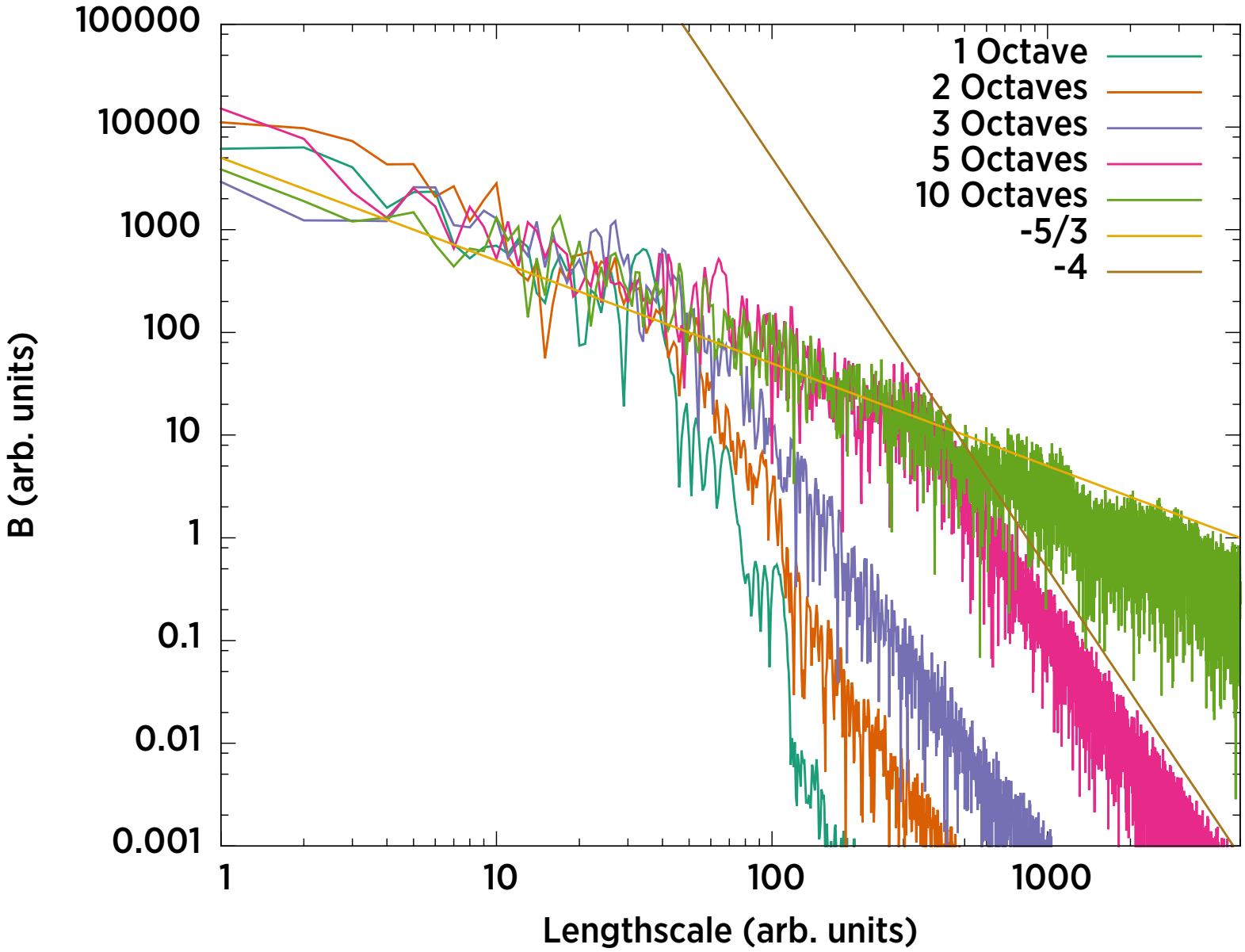
Spectrum


100000
10000
1000
100
10
1
0.1
0.01
0.001
B (arb. units)
1
10
100
1000
Lengthscale (arb. units)
1 Octave
2 Octaves
3 Octaves
5 Octaves

Spectrum


100000
10000
1000
100
10
1
0.1
0.01
0.001
B (arb. units)
1
10
100
1000
Lengthscale (arb. units)
1 Octave
2 Octaves
3 Octaves
5 Octaves
10 Octaves

Spectrum


100000
10000
1000
100
10
1
0.1
0.01
0.001
B (arb. units)
1
10
100
1000
Lengthscale (arb. units)
1 Octave
2 Octaves
3 Octaves
5 Octaves
10 Octaves
-5/3
-4

From Noise to Turbulent Fields

But these are still scalar noise functions! How do we get B-Fields from it?

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Dumb approach:

- 3 independent evaluations for 3 components
- Not divergence free

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Smarter:

- 3 independent evaluations for A
- $\vec{B} = \nabla \times \vec{A}$
- $\Rightarrow \nabla \cdot \vec{B} = \nabla \cdot (\nabla \times \vec{A}) = 0$

From Noise to Turbulent Fields

$$\vec{B}(\vec{x}) = \nabla \times \vec{A}(\vec{x})$$

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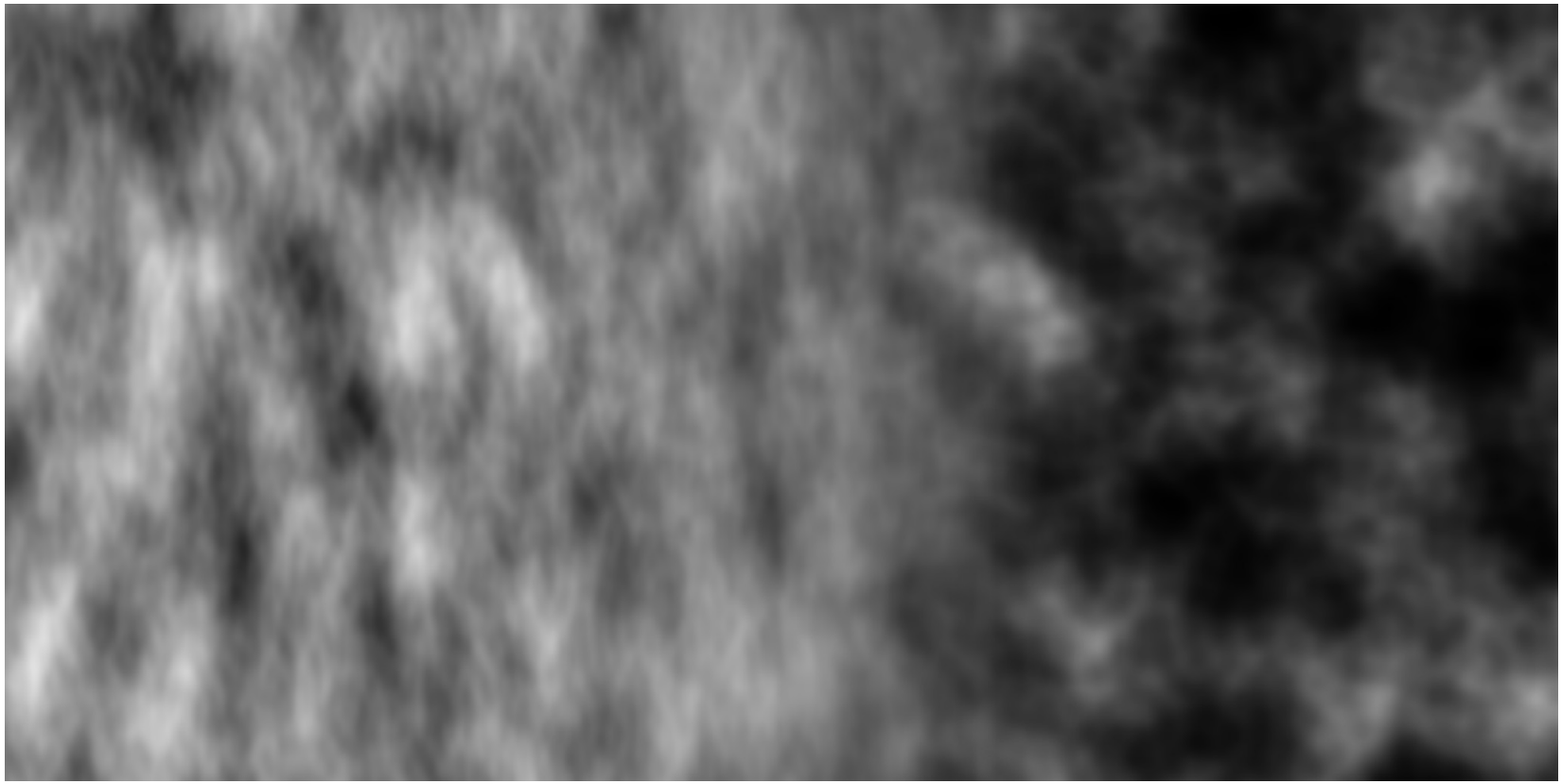


From Noise to Turbulent Fields

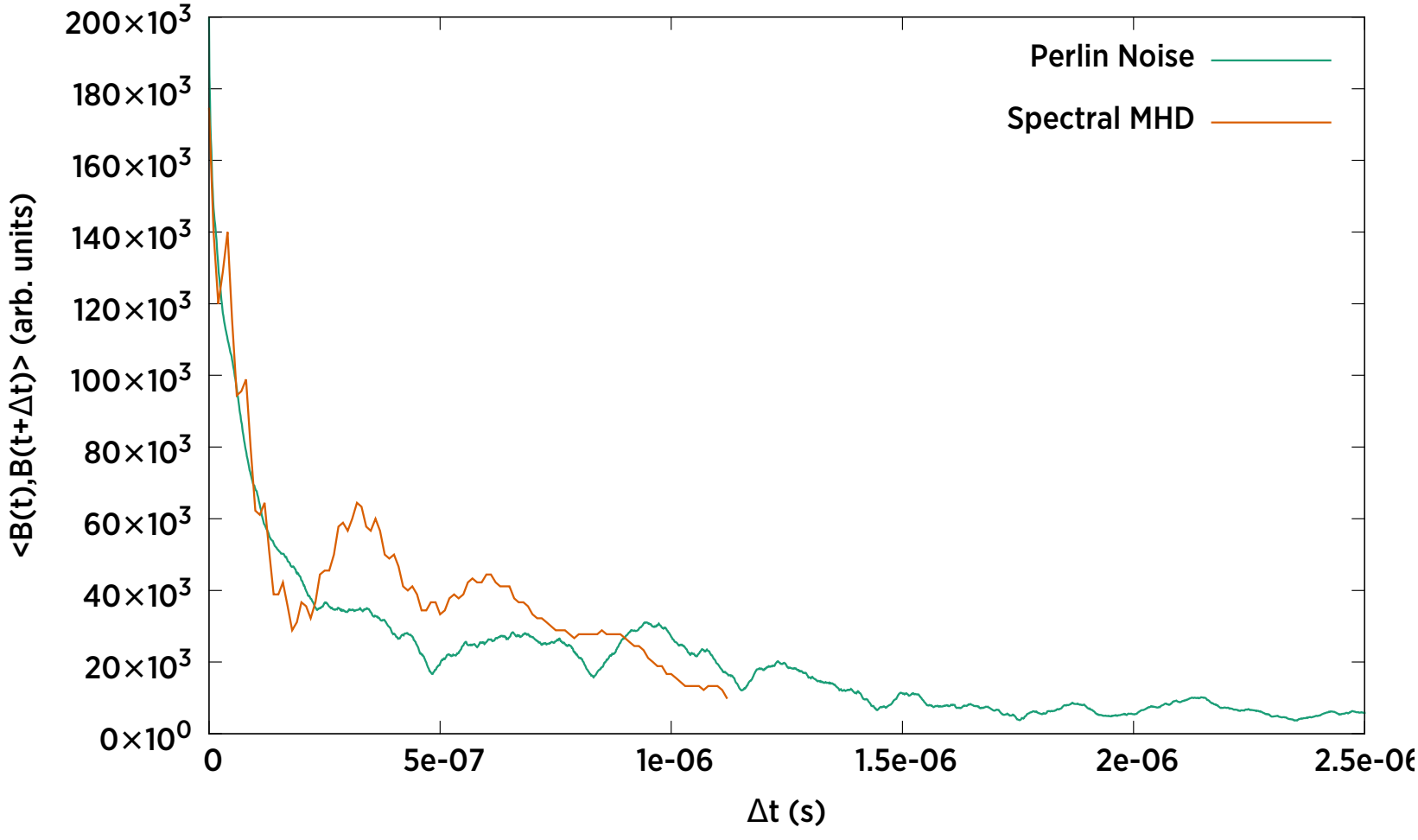
Weak Turbulence

- 3D noise advected along the background field.
- Or: One resting, one moving at $+v_A$, one moving at $-v_A$

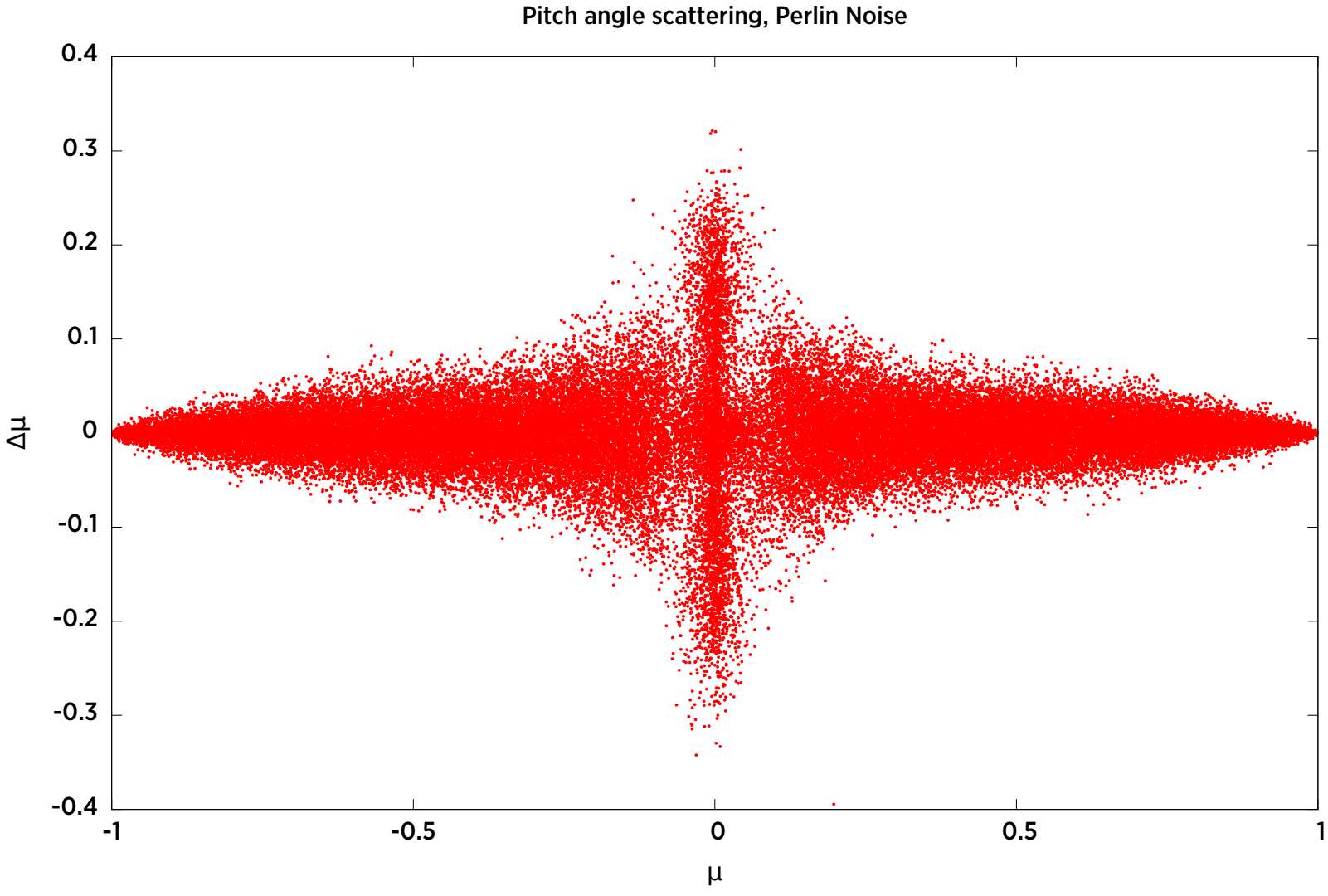
Strong Turbulence

- 4D noise, with 4th coordinate increasing with time.

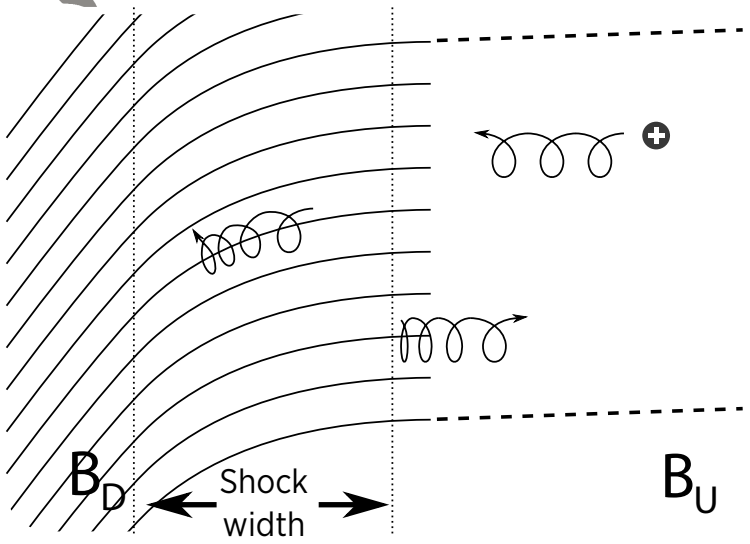
Field Autocorrelation



Pitch Angle Scattering



Application



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- Kinetic scale structures
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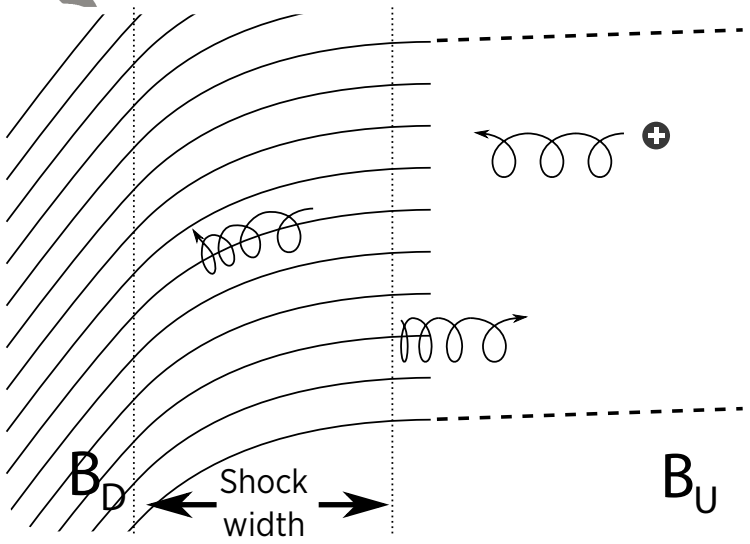

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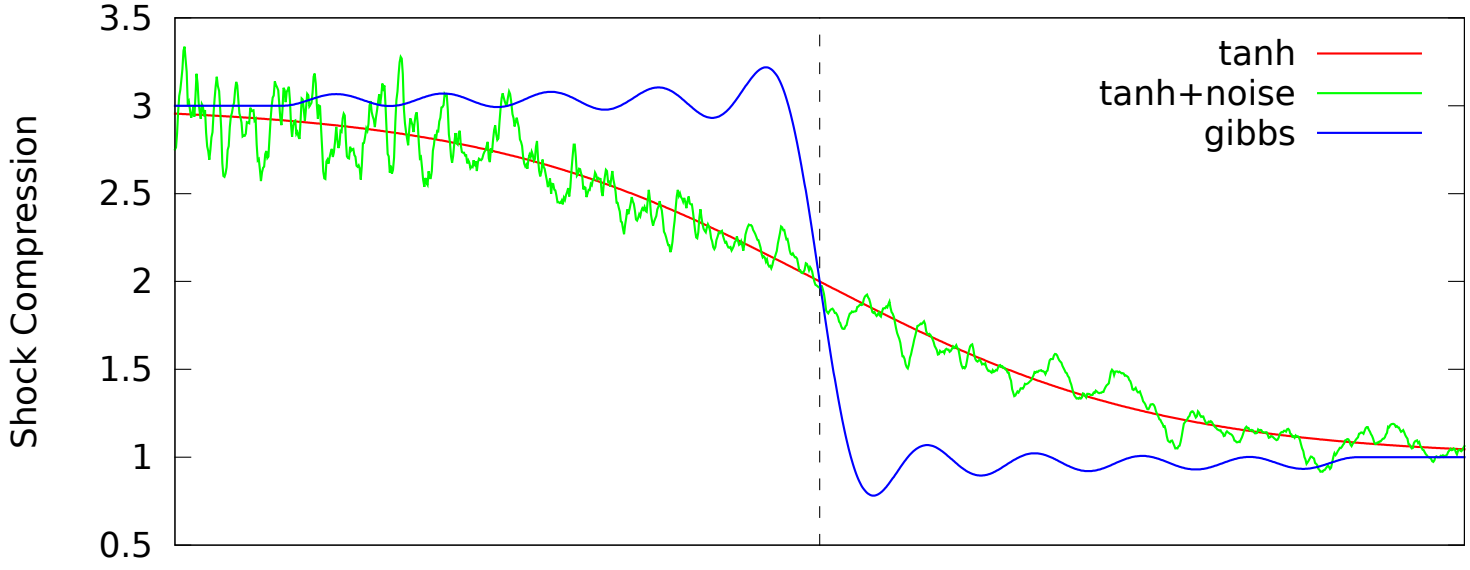
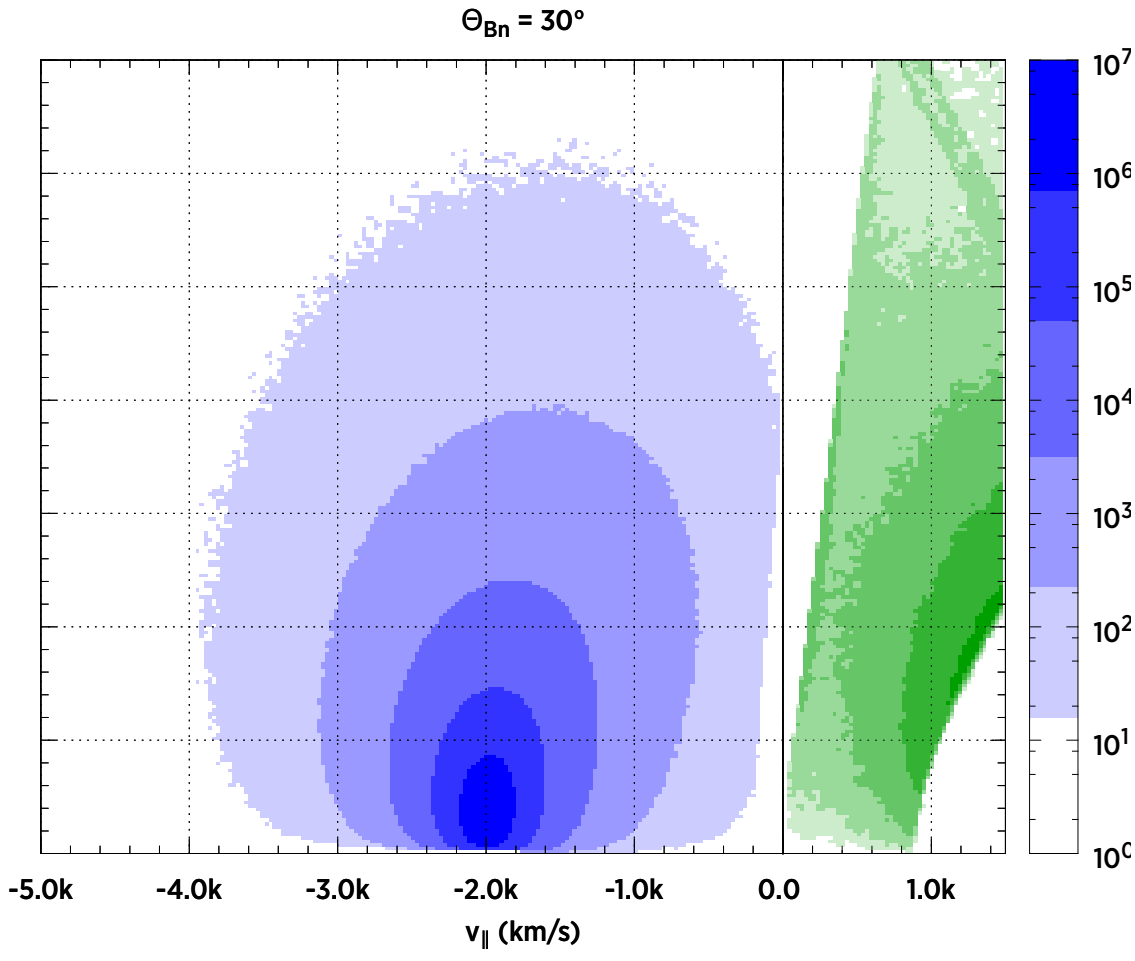


Preliminary results



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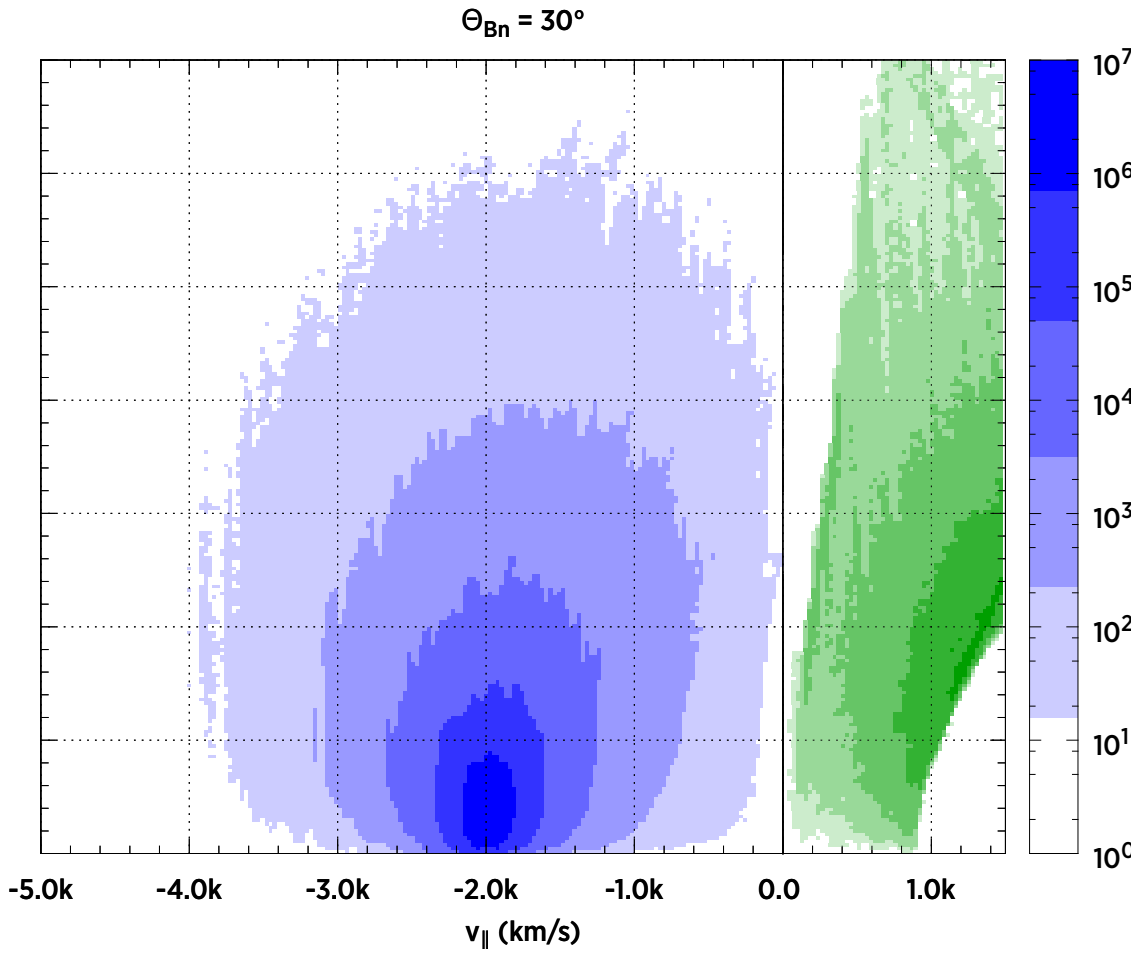

Conclusion

Perlin Noise

- Perlin Noise is a suitable way to create something like turbulence.
- Versatile and computationally inexpensive.

Outlook

- Further quantitative comparison to actual turbulence.
- Application to test-particle simulations of different scenarios.