The influence of frequency-dependent radiative transfer on the structures of radiative shocks

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Outline

Introduction to radiation hydrodynamics

I. Multigroup M1 model

II. Numerical method and tests

III. Multigroup radiative shocks

Conclusions and perspectives

The radiation hydrodynamics

Radiative transfer treatment: 2 solutions

1. diagnostic and interpretation tool

→ no feedback with hydrodynamics

fine transfer (atomic data, lines….)

2. dynamic effects of the radiation

→ global budget (energy – impulsion)

This is **radiation hydrodynamics**

Relevant applications for radiation hydrodynamics:

- In astrophysics
  - accretion shocks on massive object or in formation
  - stellar jets and flows
  - radiative winds of pulsating stars
  - supernovae explosions…
- In laboratory plasmas
  - physics of Inertial Confinement Fusion
  - radiative shocks

How to solve the transfer equation ?

$$\left(\frac{1}{c}\frac{\partial}{\partial t} + \mathbf{n}.\nabla\right) I(\mathbf{x},t,\mathbf{n},\nu) = \eta(\mathbf{x},t,\mathbf{n},\nu) - \chi(\mathbf{x},t,\mathbf{n},\nu) I(\mathbf{x},t,\mathbf{n},\nu)$$

- **Direct integration**
  - ✓ high cost (time – memory)
- **Monte-Carlo methods**
  - ✓ coupling with hydrodynamics not natural
  - ✓ high cost in optically thick regions
- **Moments models**
  - ✓ approximations of the physical model

$$\left\{\begin{array}{lclll} E_r(\mathbf{x},t,\nu) & = & \frac{1}{c} & \oint I(\mathbf{x},t;\mathbf{n},\nu) & d\omega \\ \mathbf{F}_r(\mathbf{x},t,\nu) & = & & \oint I(\mathbf{x},t;\mathbf{n},\nu)\mathbf{n} & d\omega \\ \mathbf{P}_r(\mathbf{x},t,\nu) & = & \frac{1}{c} & \oint I(\mathbf{x},t;\mathbf{n},\nu)\mathbf{n}\otimes\mathbf{n} & d\omega \end{array}\right.$$

Radiative energy
Radiative flux
Radiative pressure

The moments models

If LTE and no scattering:

$$\partial_t E_r^\nu + \nabla . F_r^\nu = \sigma^\nu c(4\pi B(\nu, T) - E_r^\nu)$$

$$\partial_t F_r^\nu + c^2 \nabla . P_r^\nu = -\sigma^\nu c F_r^\nu$$

Needs then a closure relation to the system:

$P_r^\nu = f(E_r^\nu, F_r^\nu)$

e.g. Diffusion : $F_r = -\frac{c}{\sigma}\nabla . P_r$ + $\mathbb{P}_r = \frac{1}{3} E_r \mathbb{I}$

$$\partial_t E_r^\nu + \nabla \cdot \lambda \frac{c}{3\sigma^\nu} \nabla E_r^\nu = \sigma^\nu c(4\pi B(\nu, T) - E_r^\nu)$$

rapid BUT - flux always colinear and proportional with the energy gradient

- ad-hoc flux limiter λ

The M1 model

(Levermore 1984 ; Dubroca & Feugeas 1999)

Planck function:

$$B(\nu,T) = \frac{2h\nu^3}{c^2}[\exp(\frac{h\nu}{kT}) - 1]^{-1}$$

M1 distribution function:

$$B(\nu,\vec{\Omega},T^*) = \frac{2h\nu^3}{c^2}[\exp(\frac{h\nu}{kT^*}(1 - \frac{2-\sqrt{4-3f^2}}{f^2}\vec{f}.\vec{\Omega})) - 1]^{-1}$$

with $T^* = \frac{2}{f}\left(-1+\sqrt{4-3f^2}\right)^{\frac{1}{4}}\sqrt{f^2-2+\sqrt{4-3f^2}}T_R$

and $f = \frac{F_r}{cE_r}$

- **Minimization of radiation entropy**
- **Lorentz transformation of a Planck function**

The M1 model

The closure relation for M1 model is:

$$\mathbf{P}_r = \mathbf{D} E_r$$

$$\mathbf{D} = \frac{1-\chi}{2}\mathbf{I} + \frac{3\chi-1}{2}\mathbf{n}\otimes\mathbf{n}$$

$$\chi = \frac{3+4f^2}{5+2\sqrt{4-3f^3}} \quad \text{with} \quad f = \frac{F_r}{cE_r}$$

General form assuming a privileged direction n

Advantages

- low cost
- take radiation anisotropies into account
- exact in both diffusive and free streaming limits
- can take (anisotropic) diffusion into account
- allow “proper” means over opacities

The radiation hydrodynamics

(Mihalas & Mihalas 1984)

In the comoving frame… at O(v/c)

$$
\begin{aligned}
\partial_t \rho &+ \partial_x(\rho u) &= &\; 0 \\
\partial_t(\rho u) &+ \partial_x(\rho u^2 + p) &= &\int_0^\infty (\sigma_\nu / c) F_\nu d\nu \\
\partial_t e &+ \partial_x(u(e+p)) &= &-\int_0^\infty \sigma_\nu (4\pi B - cE_\nu - uF_\nu/c)\; d\nu
\end{aligned}
$$

$$
\begin{aligned}
&\partial_t E_\nu + \partial_x F_\nu + \partial_x(uE_\nu) + P_\nu \partial_x u - \partial_x u\; \partial_\nu(\nu P_\nu) = \sigma_\nu(4\pi B - cE_\nu) \\
&\partial_t F_\nu + c^2 \partial_x P_\nu + \partial_x(uF_\nu) + F_\nu \partial_x u - \partial_x u\; \partial_\nu(\nu Q_\nu) = -\sigma_\nu c F_\nu
\end{aligned}
$$

advection

work by pressure
and flux

Doppler shift terms

interaction terms

The multigroup approach

The idea is to make some means over frequency ranges.

The multigroup method splits the frequency domain into domains (called groups) where the radiative variables are considered constant

$$E_g = \int_{\nu_{g-1/2}}^{\nu_{g+1/2}} E_\nu d\nu \qquad F_g = \int_{\nu_{g-1/2}}^{\nu_{g+1/2}} F_\nu d\nu \qquad P_g = \int_{\nu_{g-1/2}}^{\nu_{g+1/2}} P_\nu d\nu$$

Then, opacities weighted averages over these groups appear in the equations:

$$\sigma_{P\,g} = \frac{\int_{\nu_{g-1/2}}^{\nu_{g+1/2}} \sigma_\nu B(\nu,T) d\nu}{\int_{\nu_{g-1/2}}^{\nu_{g+1/2}} B(\nu,T) d\nu} \qquad \sigma_{E\,g} = \frac{\int_{\nu_{g-1/2}}^{\nu_{g+1/2}} \sigma_\nu E_\nu d\nu}{E_g} \qquad \sigma_{F\,g} = \frac{\int_{\nu_{g-1/2}}^{\nu_{g+1/2}} \sigma_\nu F_\nu d\nu}{F_g}$$

The comoving frame multigroup radiation hydrodynamics

(Turpault 2005 ; Vaytet et al. 2011)

$$
\begin{aligned}
\partial_t \rho &+ \partial_x(\rho u) &= 0 \\
\partial_t(\rho u) &+ \partial_x(\rho u^2 + p) &= \sum_{g=1}^{Ng} (\sigma_{Fg}/c) F_g \\
\partial_t e &+ \partial_x(u(e+p)) &= -\sum_{g=1}^{Ng} \Big(c(\sigma_{Pg}\Theta_g(T) - \sigma_{Eg}E_g) - (\sigma_{Fg}/c) u F_g \Big)
\end{aligned}
$$

$$
\partial_t E_g + \partial_x F_g + \partial_x(u E_g) + P_g \partial_x u - \partial_x u \int_{\nu_{g-1/2}}^{\nu_{g+1/2}} \partial_\nu(\nu P_\nu) d\nu = c(\sigma_{Pg}\Theta_g(T) - \sigma_{Eg}E_g)
$$

$$
\partial_t F_g + c^2 \partial_x P_g + \partial_x(u F_g) + F_g \partial_x u - \partial_x u \int_{\nu_{g-1/2}}^{\nu_{g+1/2}} \partial_\nu(\nu Q_\nu) d\nu = -\sigma_{Fg} c F_g
$$

↗ interaction of neighboring groups

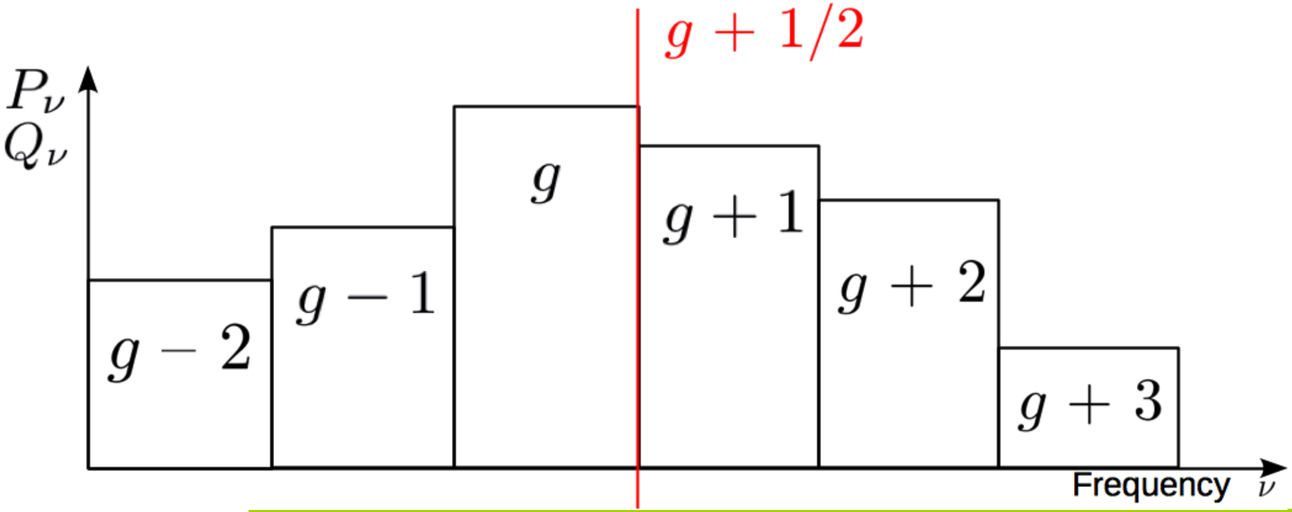
The Doppler shift terms

$$\partial_t E_g - \partial_x u \int_{\nu_{g-1/2}}^{\nu_{g+1/2}} \partial_\nu(\nu P_\nu) = 0 \qquad \partial_t F_g - \partial_x u \int_{\nu_{g-1/2}}^{\nu_{g+1/2}} \partial_\nu(\nu Q_\nu) = 0$$

They are treated with a finite volume scheme in the frequency domain

$$\frac{E_g^{n+1} - E_g^n}{\Delta t} - \partial_x u \left(\nu_{g+1/2} P_{g+1/2}^n - \nu_{g-1/2} P_{g-1/2}^n\right) = 0$$

$$\frac{F_g^{n+1} - F_g^n}{\Delta t} - \partial_x u \left(\nu_{g+1/2} Q_{g+1/2}^n - \nu_{g-1/2} Q_{g-1/2}^n\right) = 0$$



Upwind scheme:

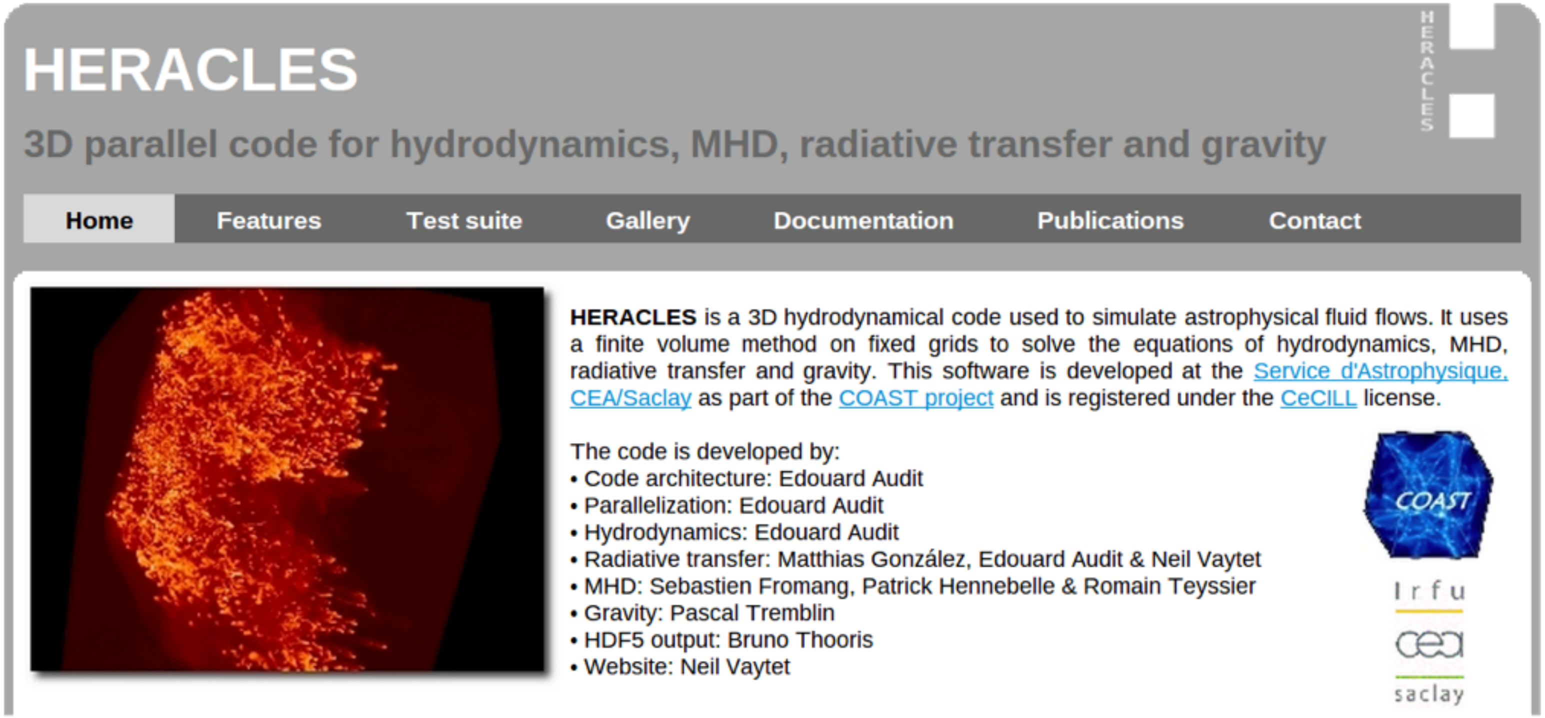
$$If\ \partial_x u \leq 0, P_{g+1/2} = \frac{P_g}{\Delta \nu_g}$$

$$If\ \partial_x u > 0, P_{g+1/2} = \frac{P_{g+1}}{\Delta \nu_{g+1}}$$

The HERACLES code

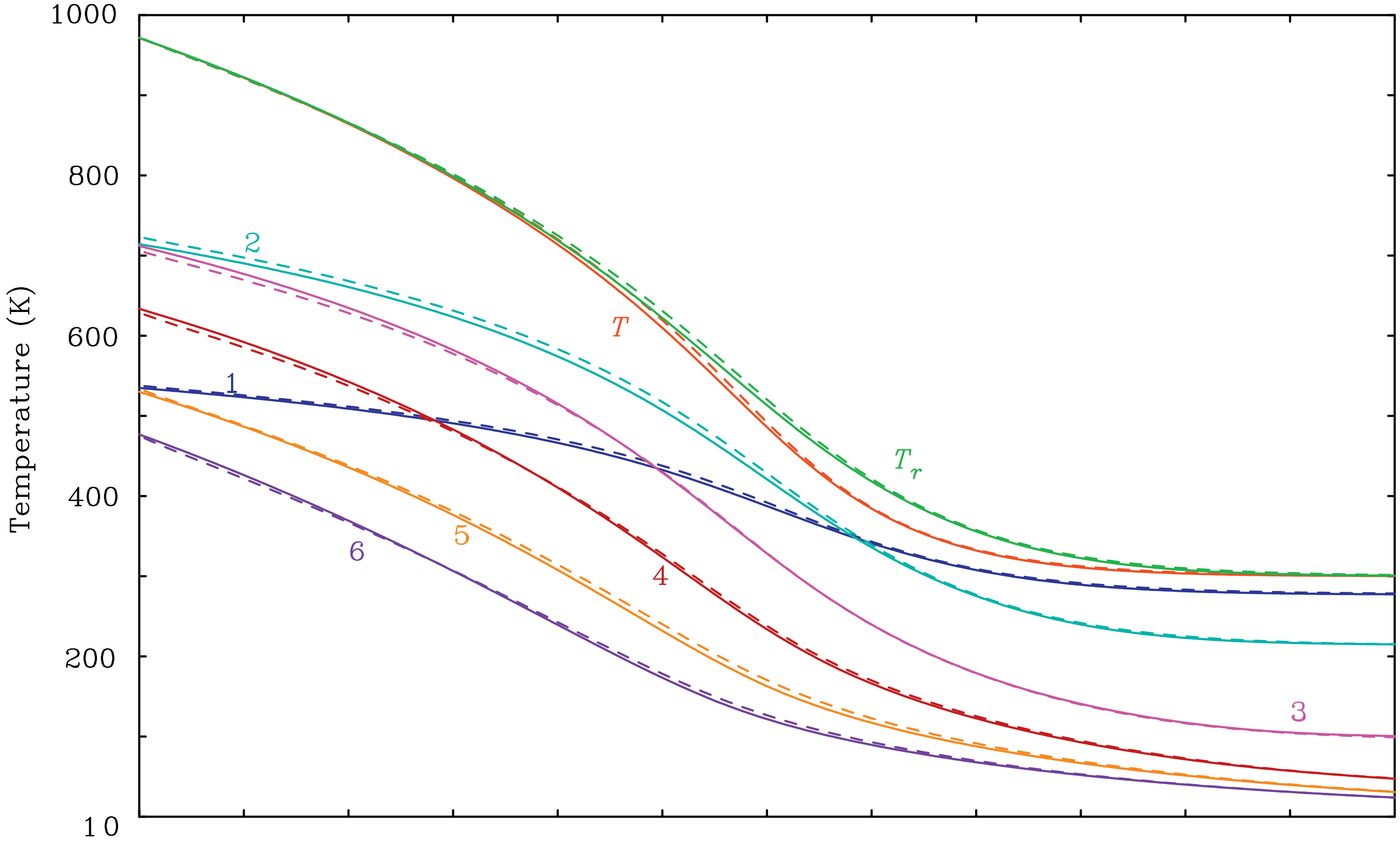
González *et al.* A&A 2007

- An Eulerian 3D MPI RMHD code
- Hydrodynamics: explicit, MUSCL-Hancock
- Implicit (Gauss-Seidel) multigroup M1 radiative transfer
- http://irfu.cea.fr/Projets/Site_heracles/

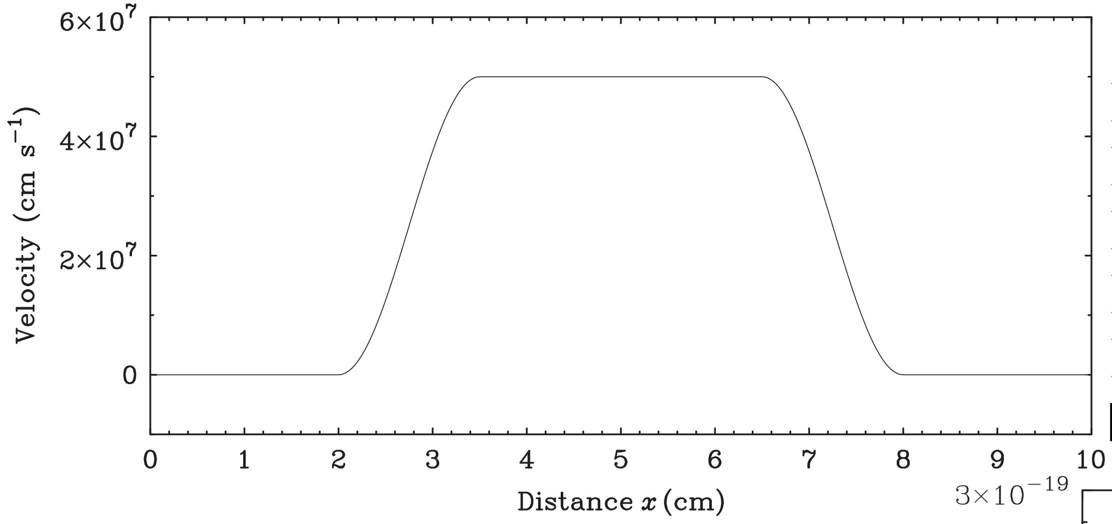


Numerical tests: Marshak wave

- T=1000K in domain with T=300K
- constant or frequency-, T-dependent opacities
- comparison with a kinetic model : error about 0.5%



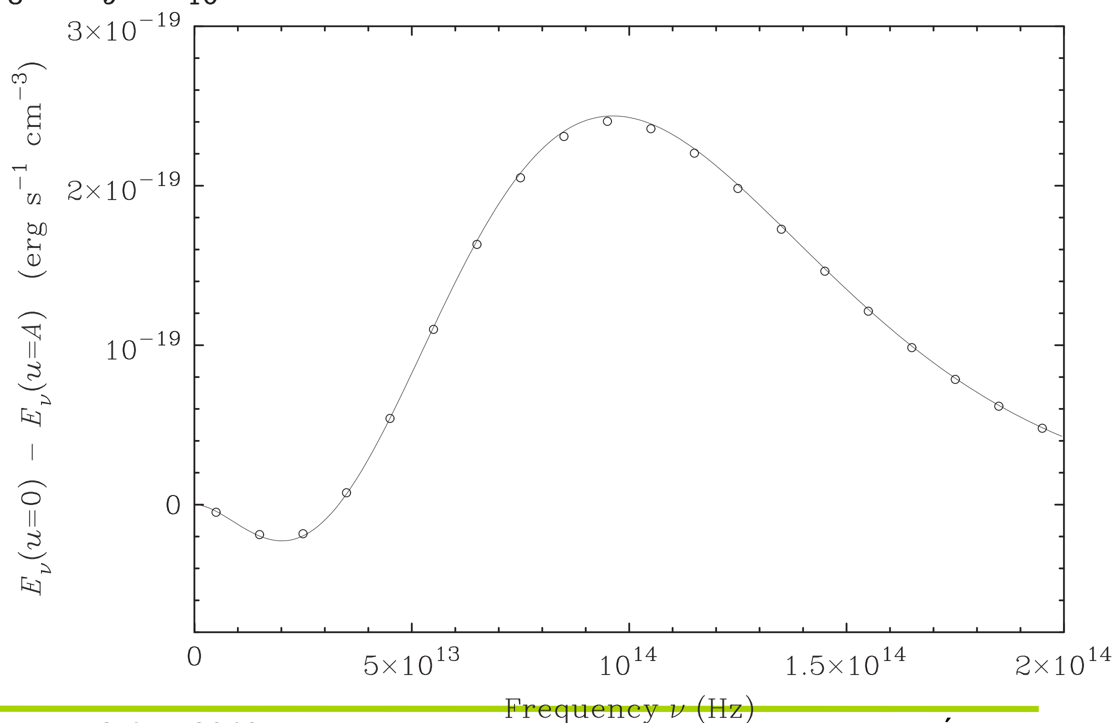
Numerical tests: Doppler terms



Test with frozen hydro

- Vacuum
- 10-20-40 groups
- velocity shape
- equilibrium
- radiation cast from the left, black body spectrum with a unit reduced flux

Difference between numerical and analytical solutions about 1%

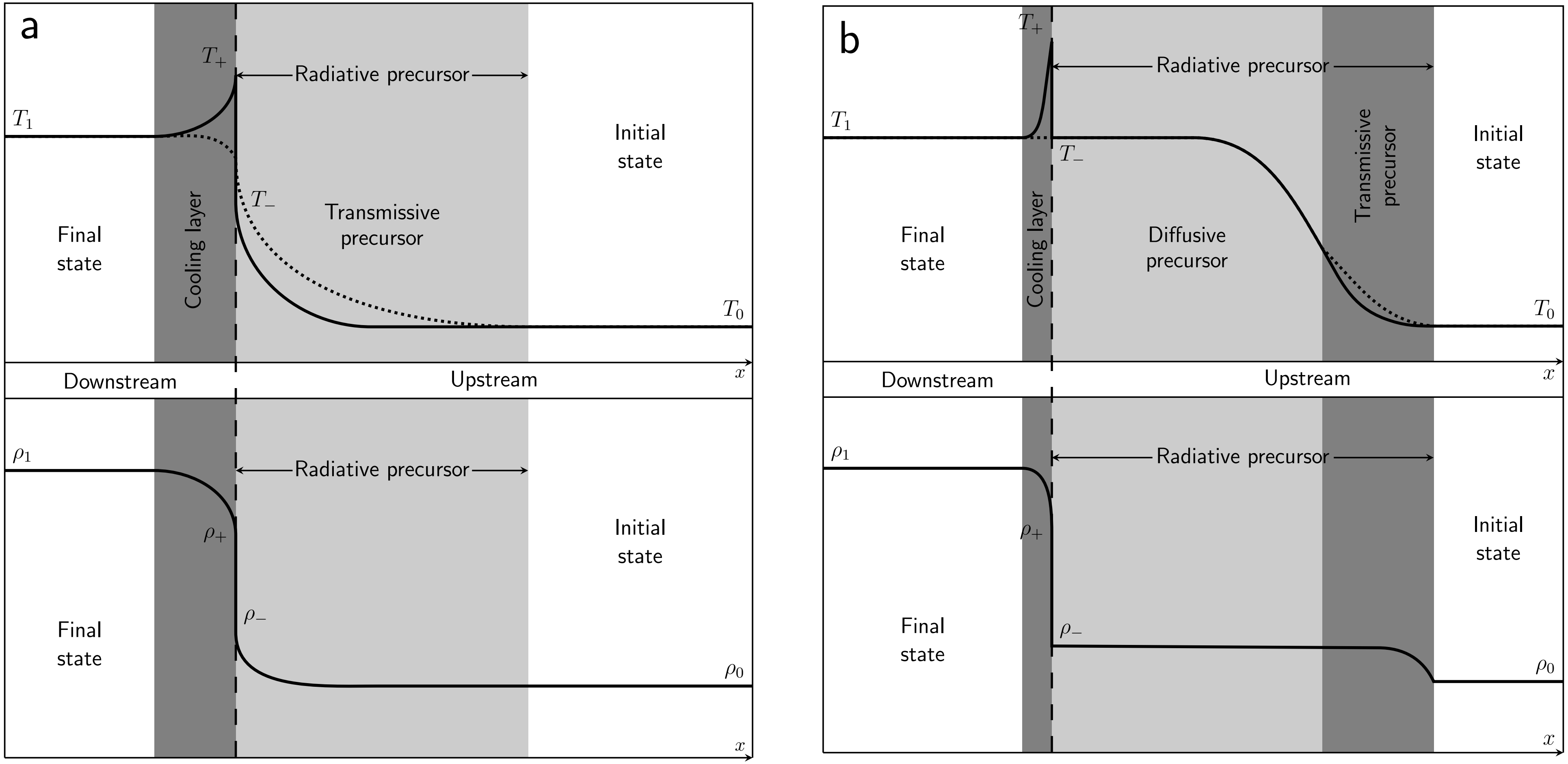


Radiative shocks

➢ They are found in astrophysics in a lot of situations: supernovae, stellar atmospheres, star formation, jets…

➢ They are reproduced on Earth on laser facilities: LMJ, OMEGA, Orion…

Radiative shocks: 2 categories



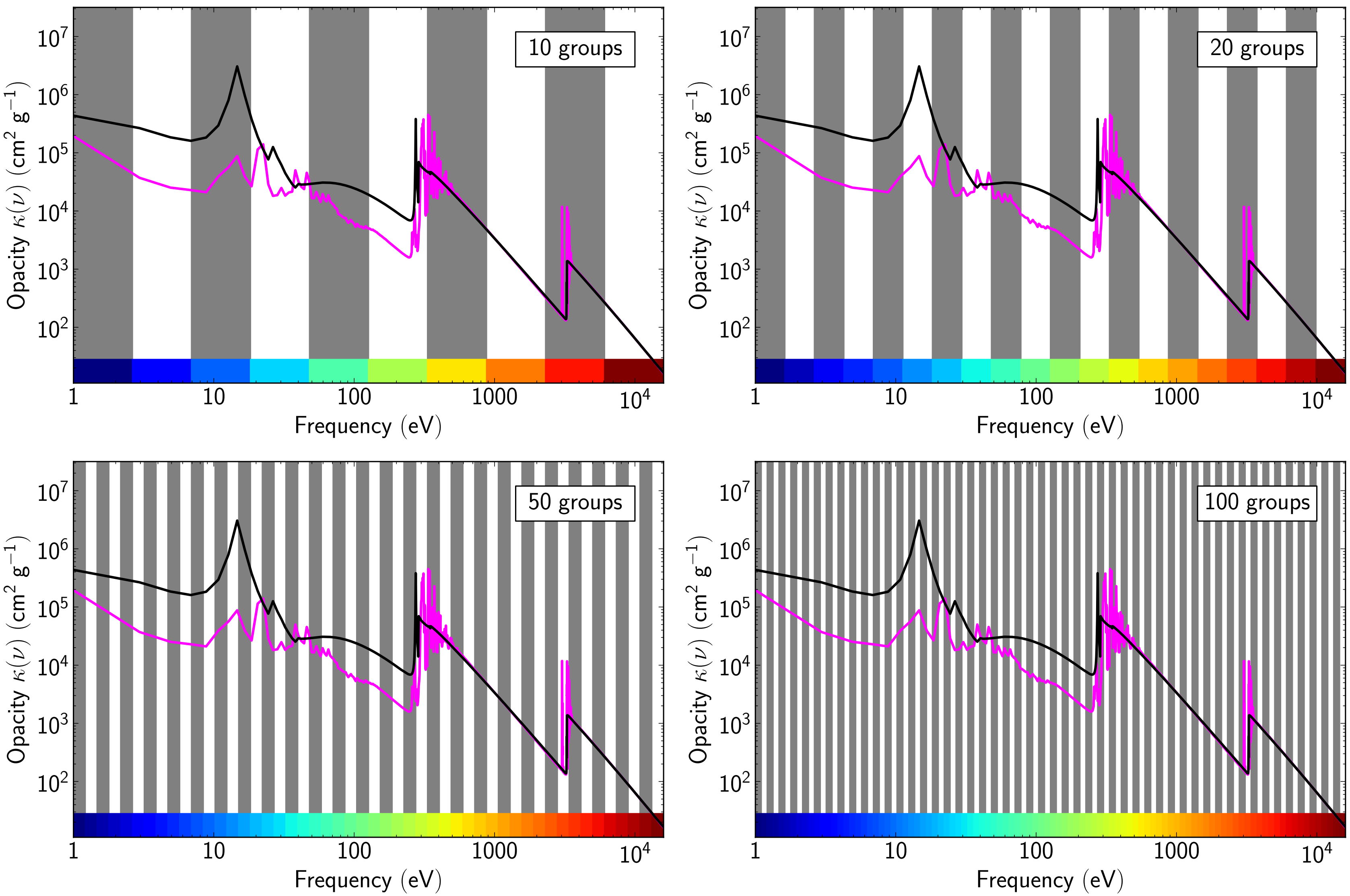
subcritical

supercritical

Radiative shocks: setup

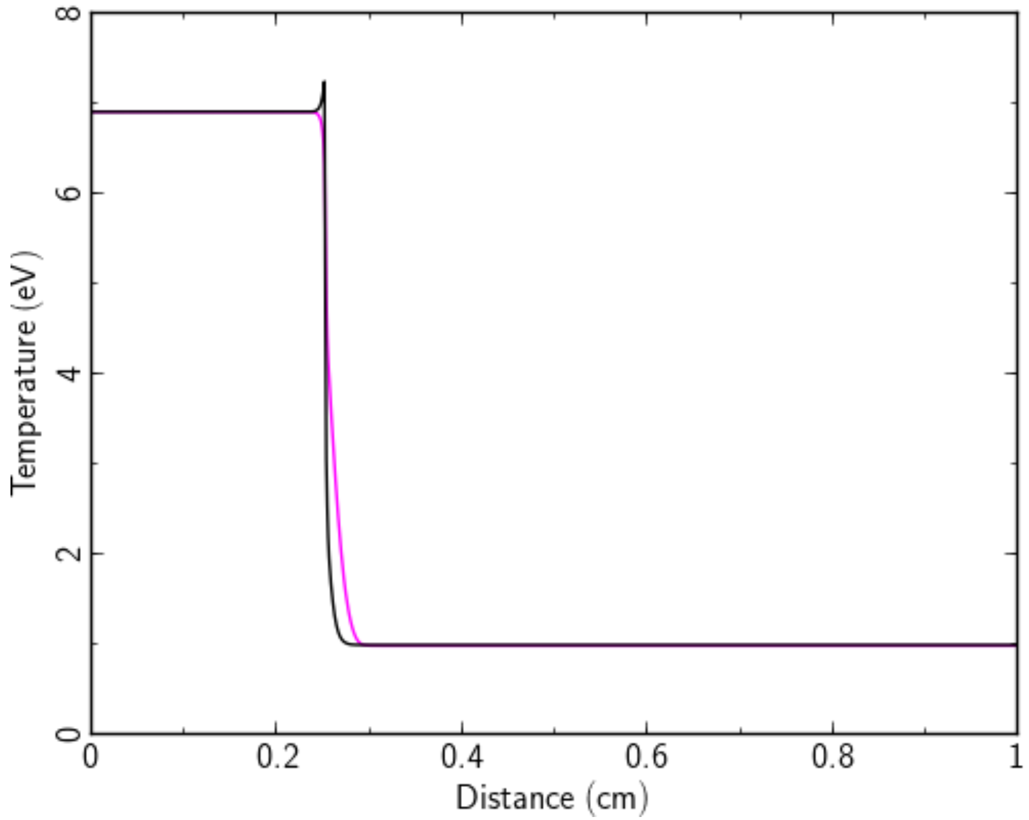
- Argon gas (opacities from ODALISC database)
- in the shock frame so that it is stationary
- pre-shock gas at $\rho = 10^{-3}$ g cm^{-3} , T = 1 eV, T_r=T
- u = 30 km/s (subcritical) or u = 100 km/s (supercritical)
- post-shock quantities computed with Rankine-Hugoniot relations
- we run the simulation until the stationary regime is obtained
- 1-5-10-20-50-100 groups

Argon opacities from ODALISC database

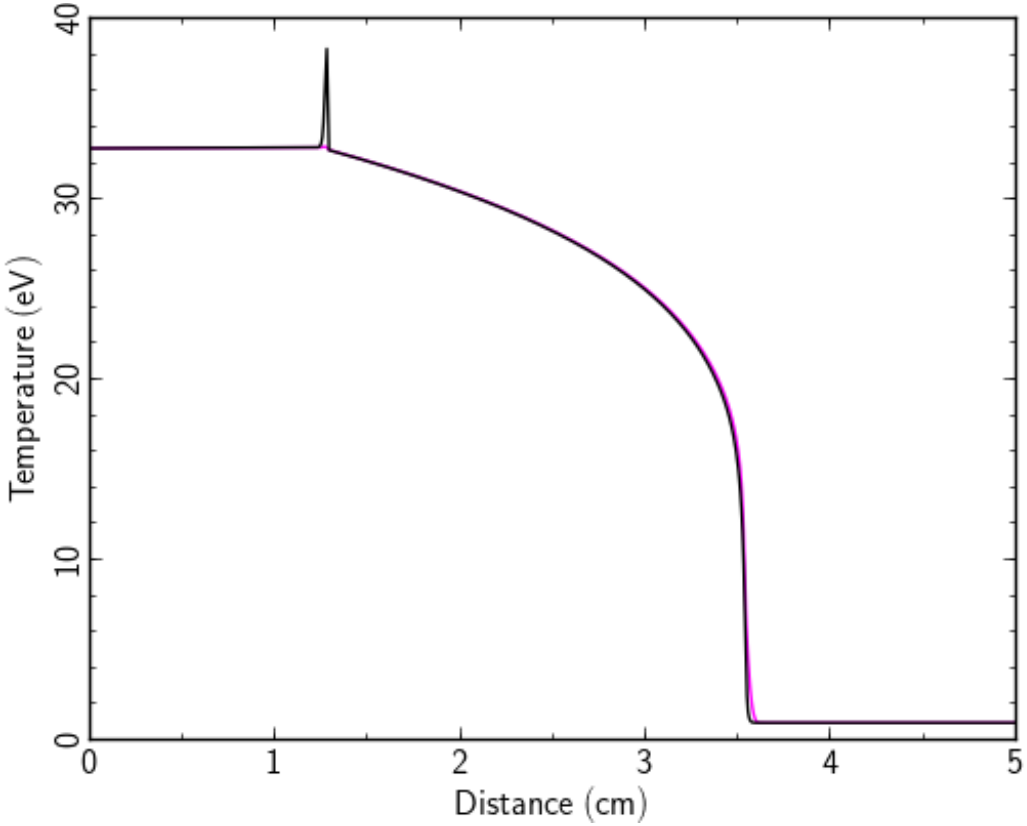


Argon opacities at initial conditions and post-shock conditions for u = 100 km/s

Temperature profiles: 1 group

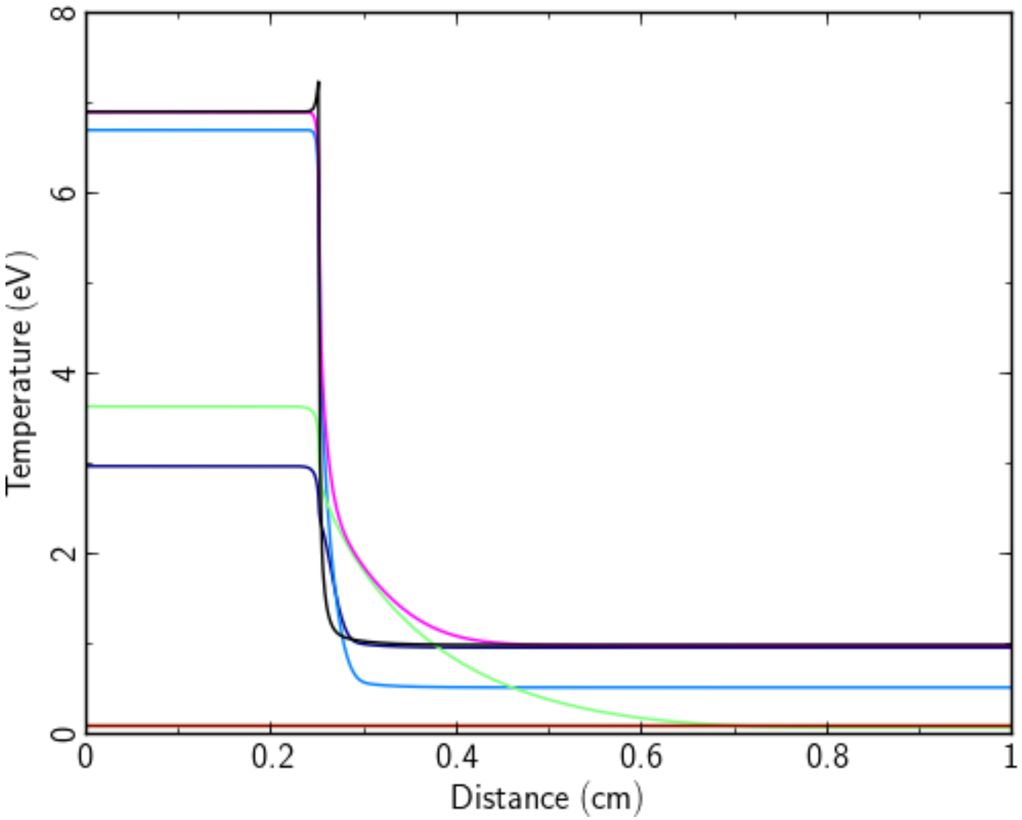


Subcritical

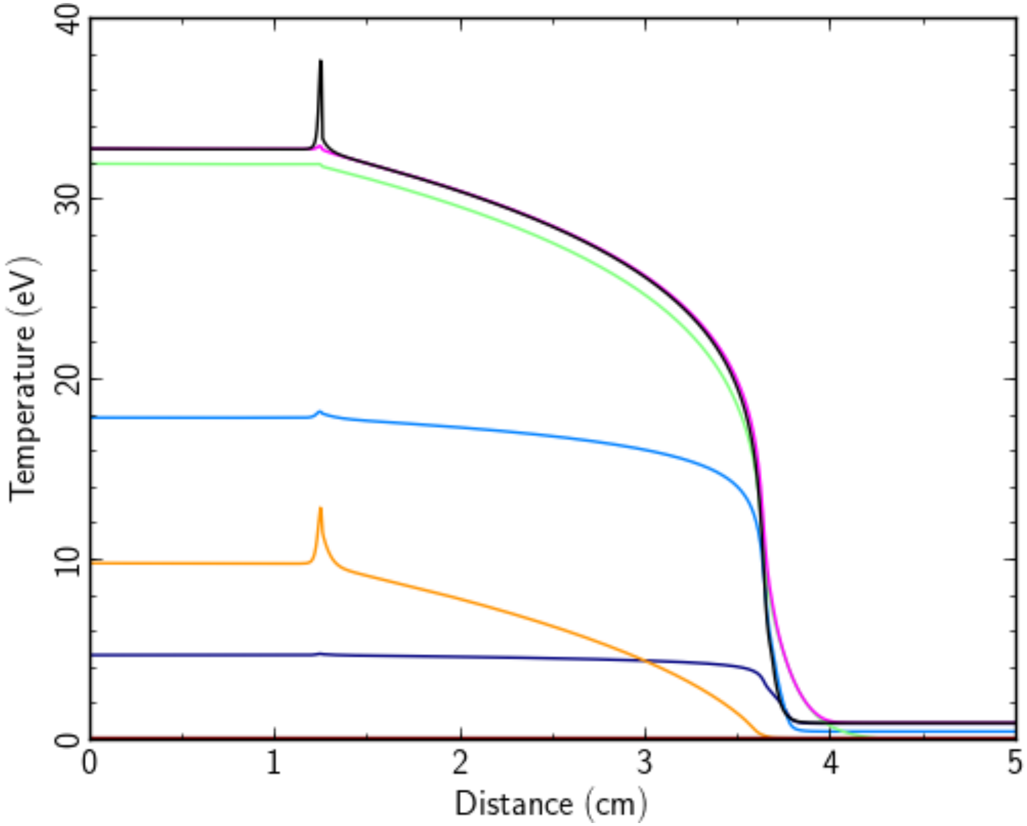


Supercritical

Temperature profiles: 5 groups

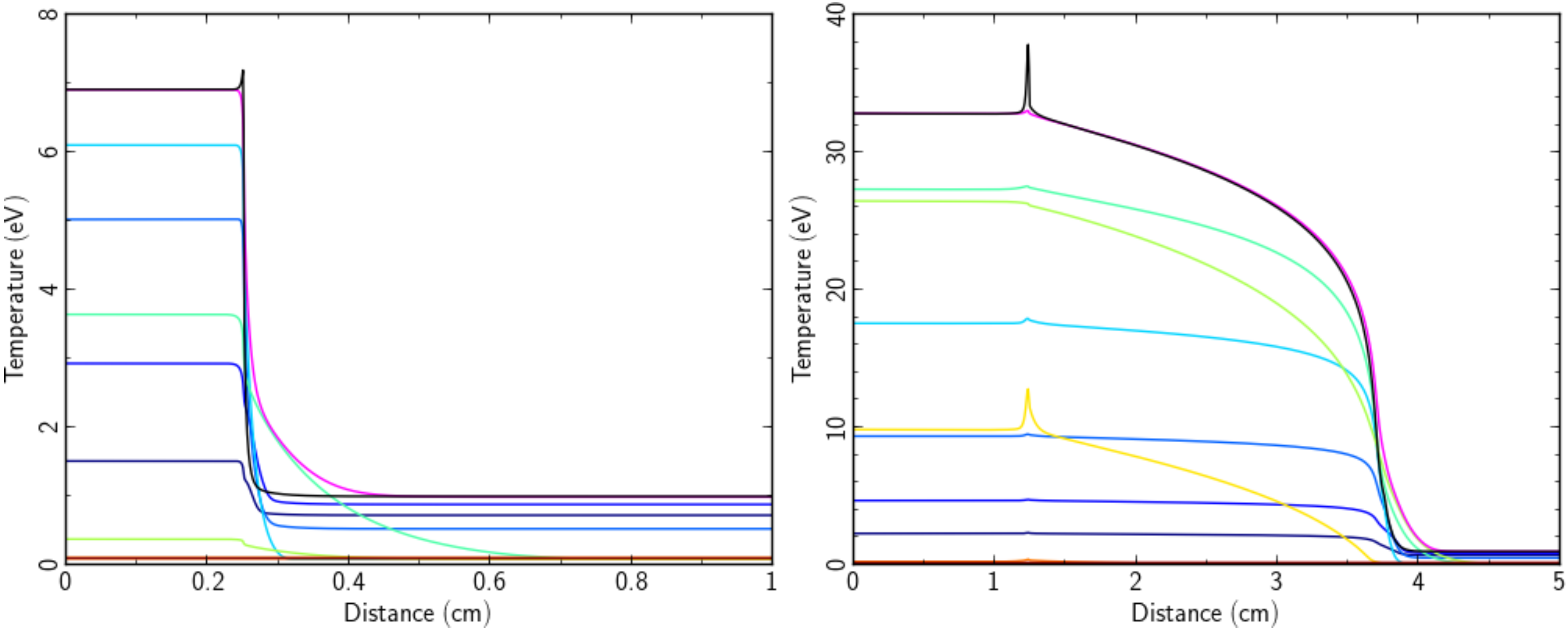


Subcritical



Supercritical

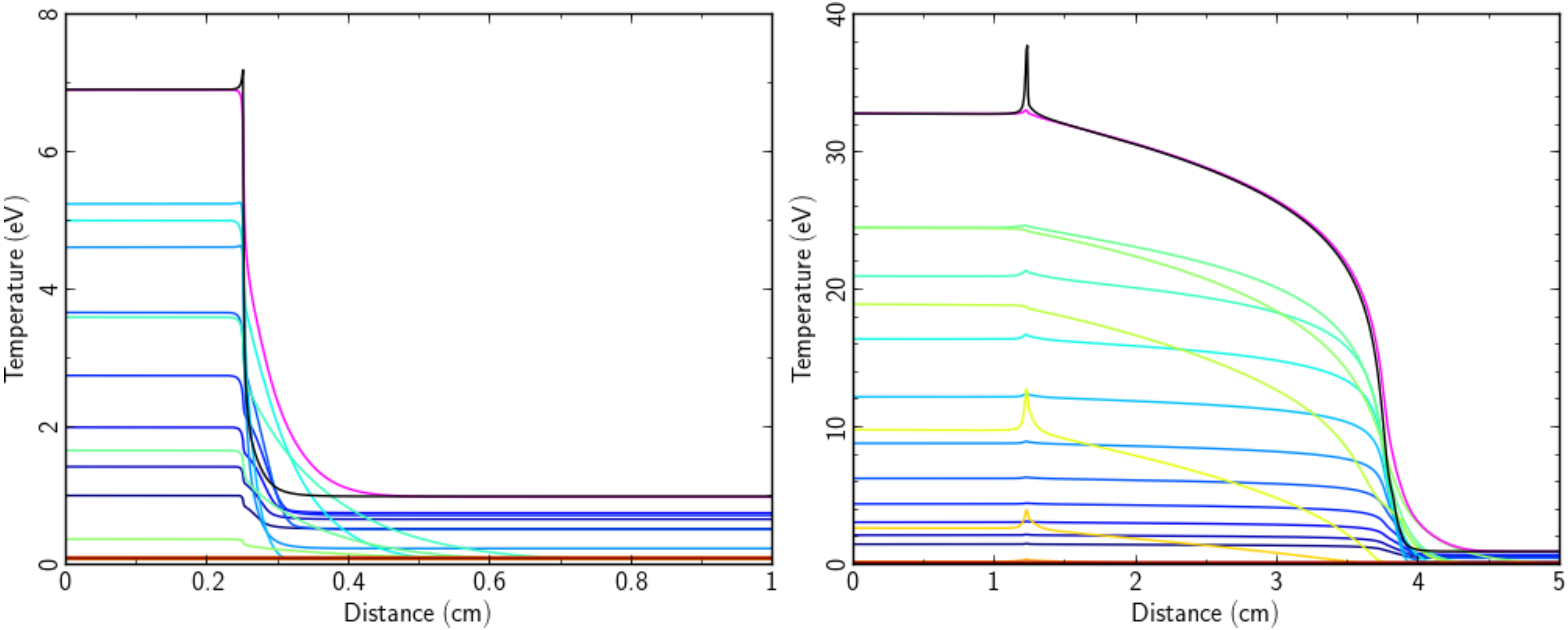
Temperature profiles: 10 groups



Subcritical

Supercritical

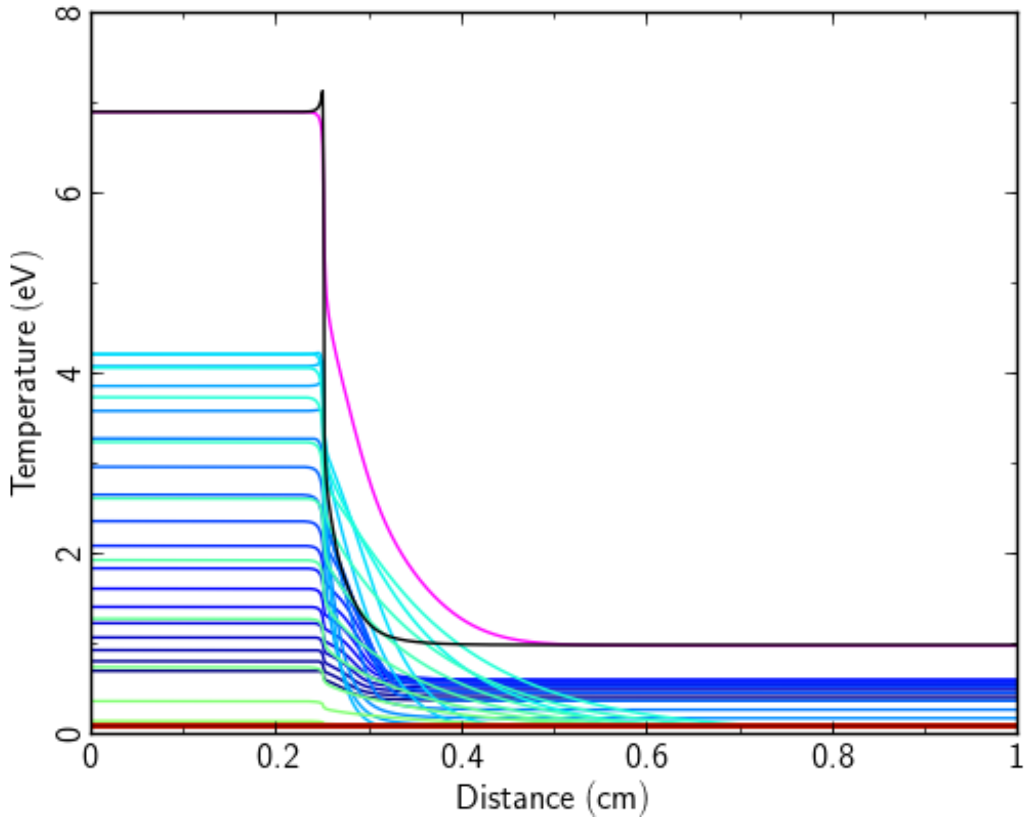
Temperature profiles: 20 groups



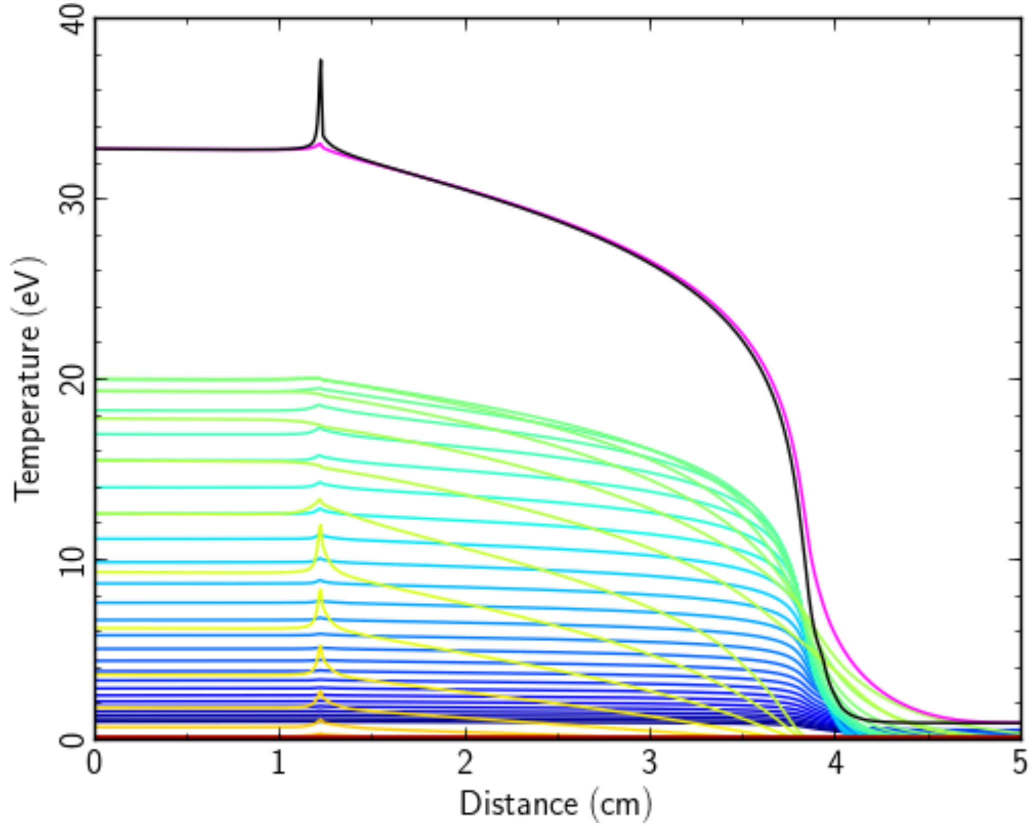
Subcritical

Supercritical

Temperature profiles: 50 groups

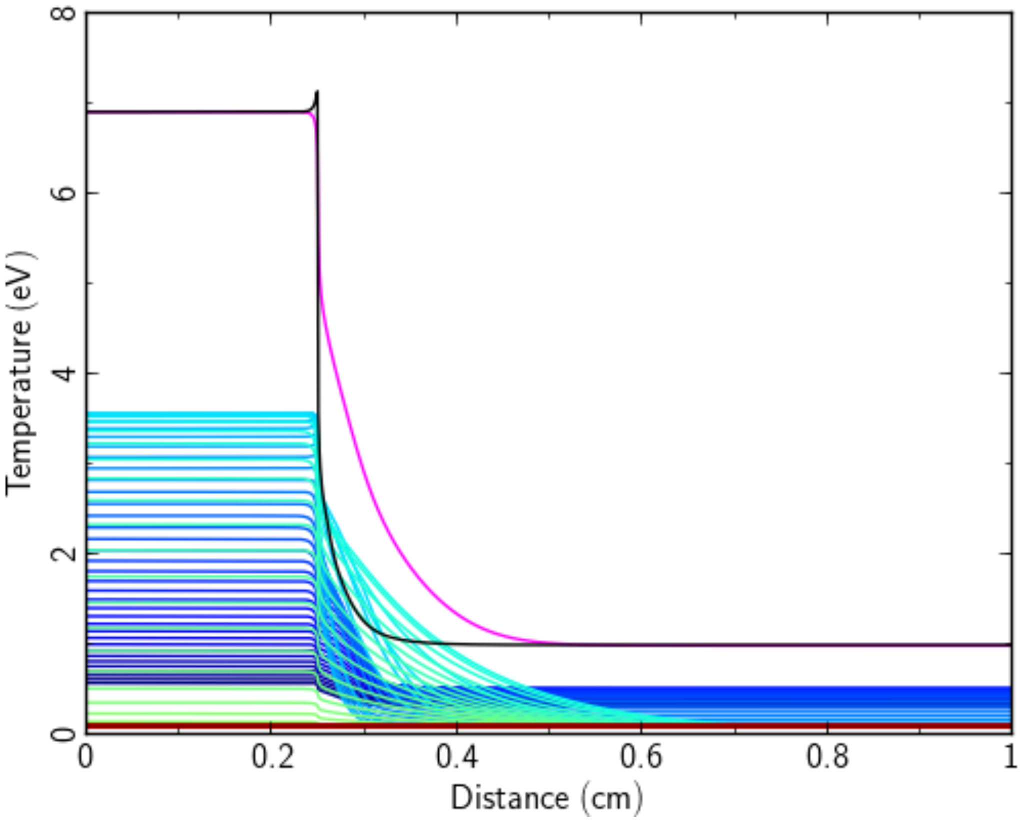


Subcritical

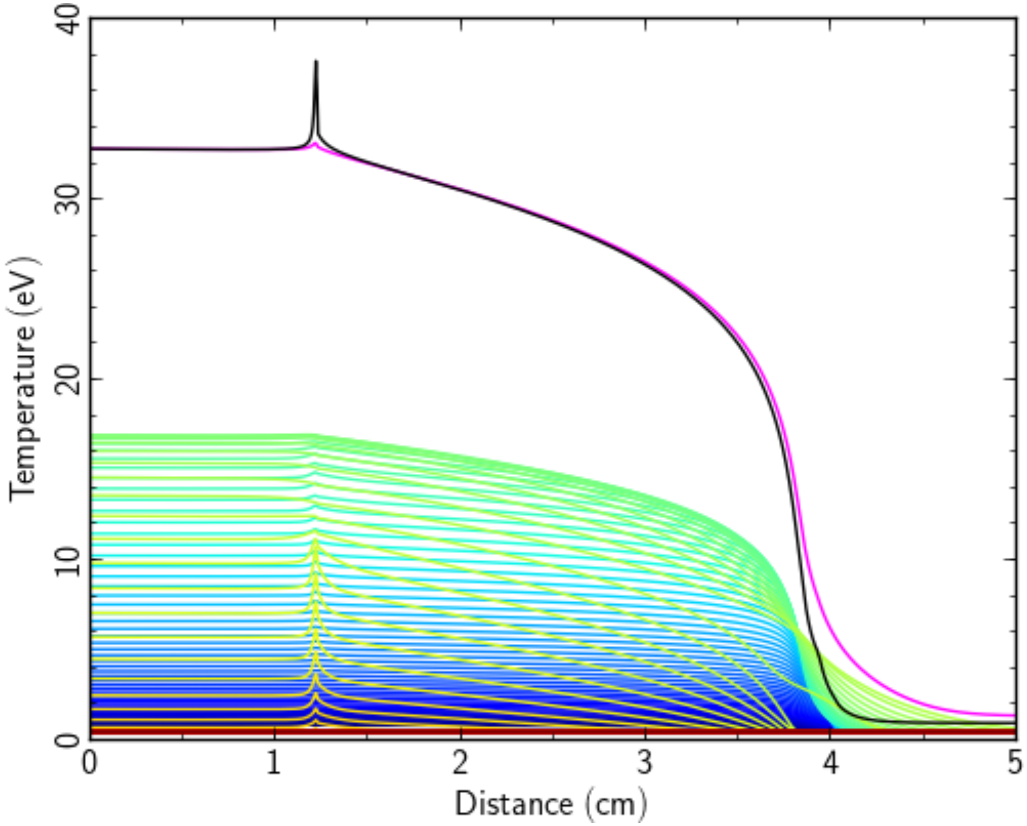


Supercritical

Temperature profiles: 100 groups



Subcritical

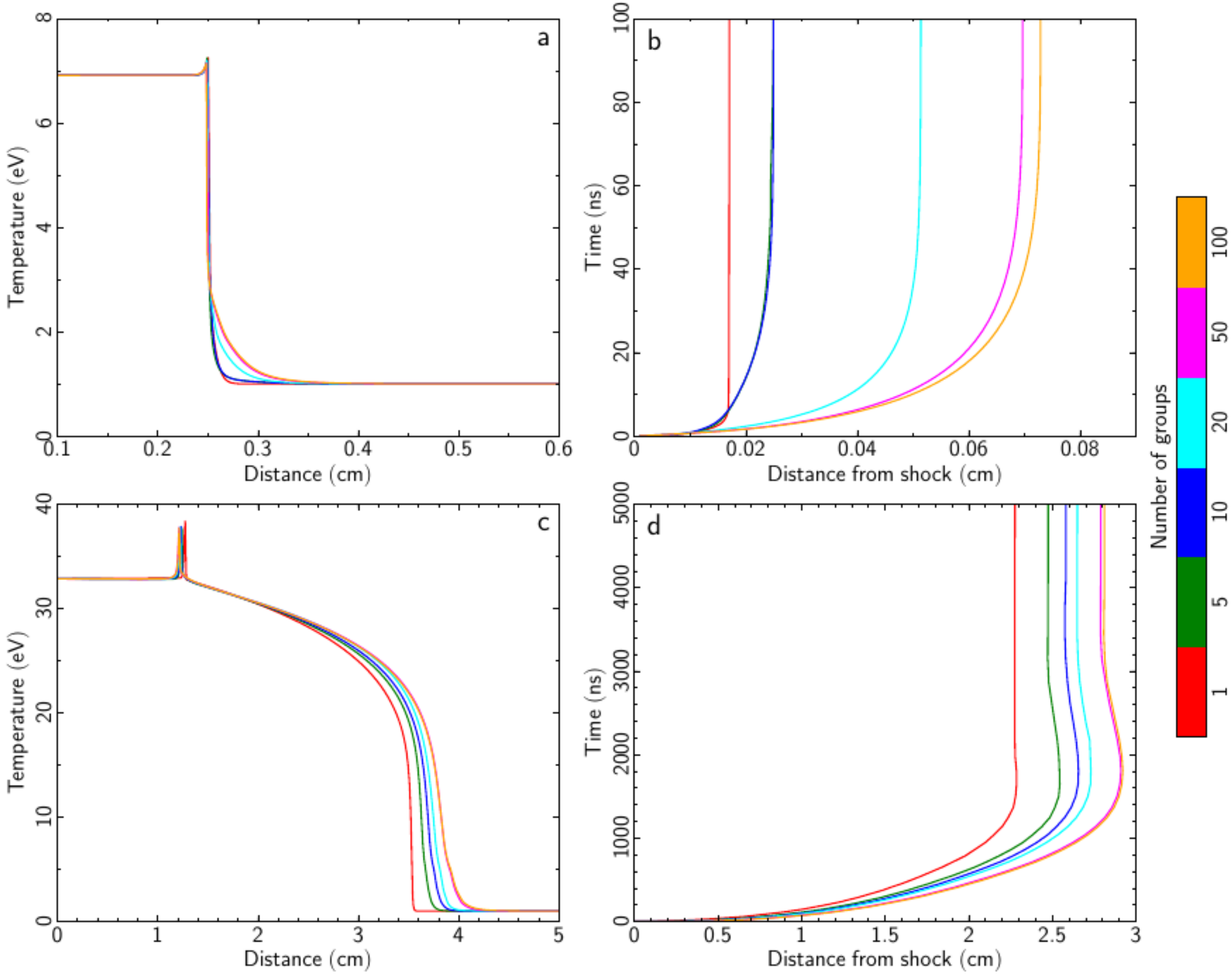


Supercritical

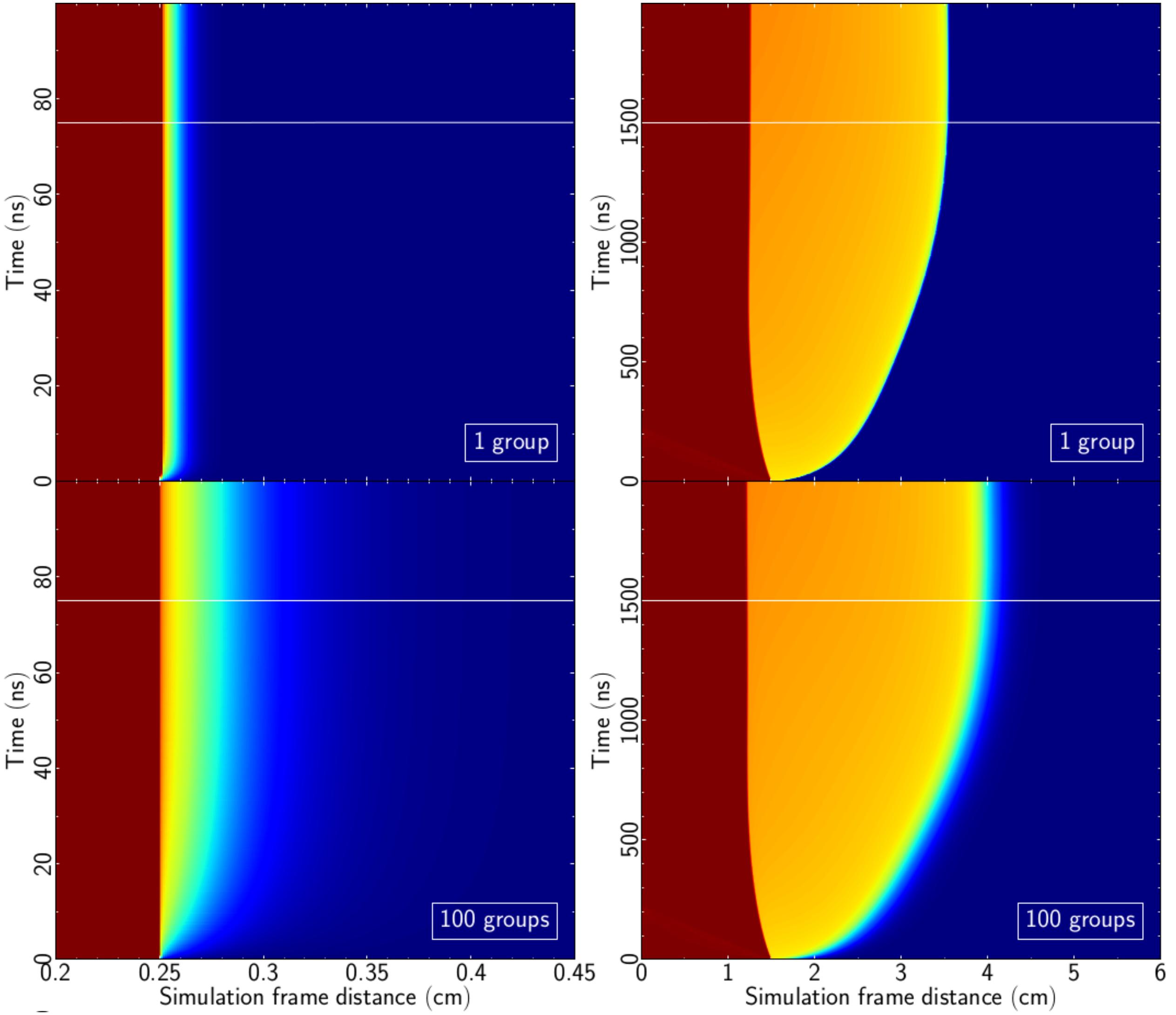
Precursor size

Subcritical

Supercritical



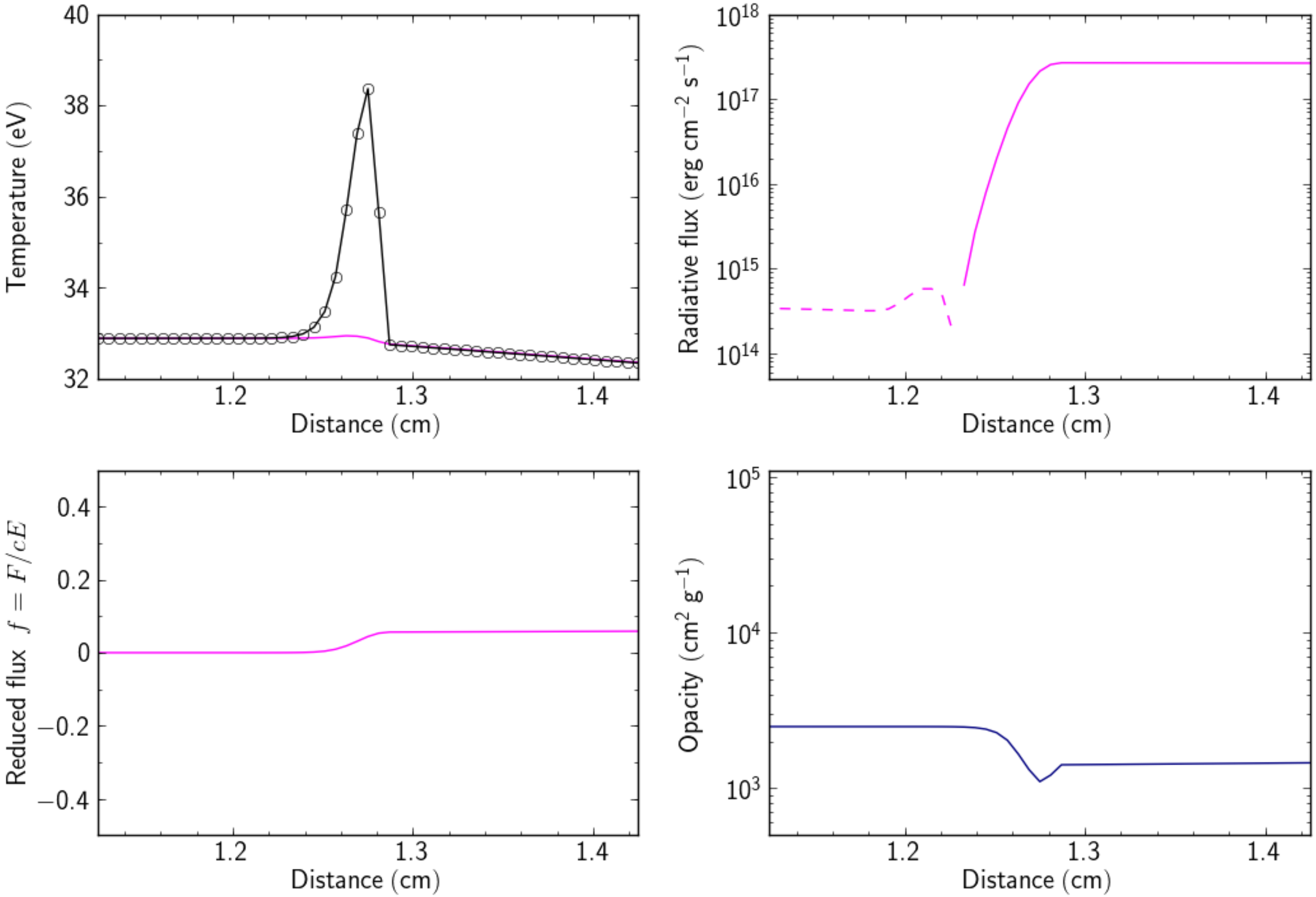
Electron densities



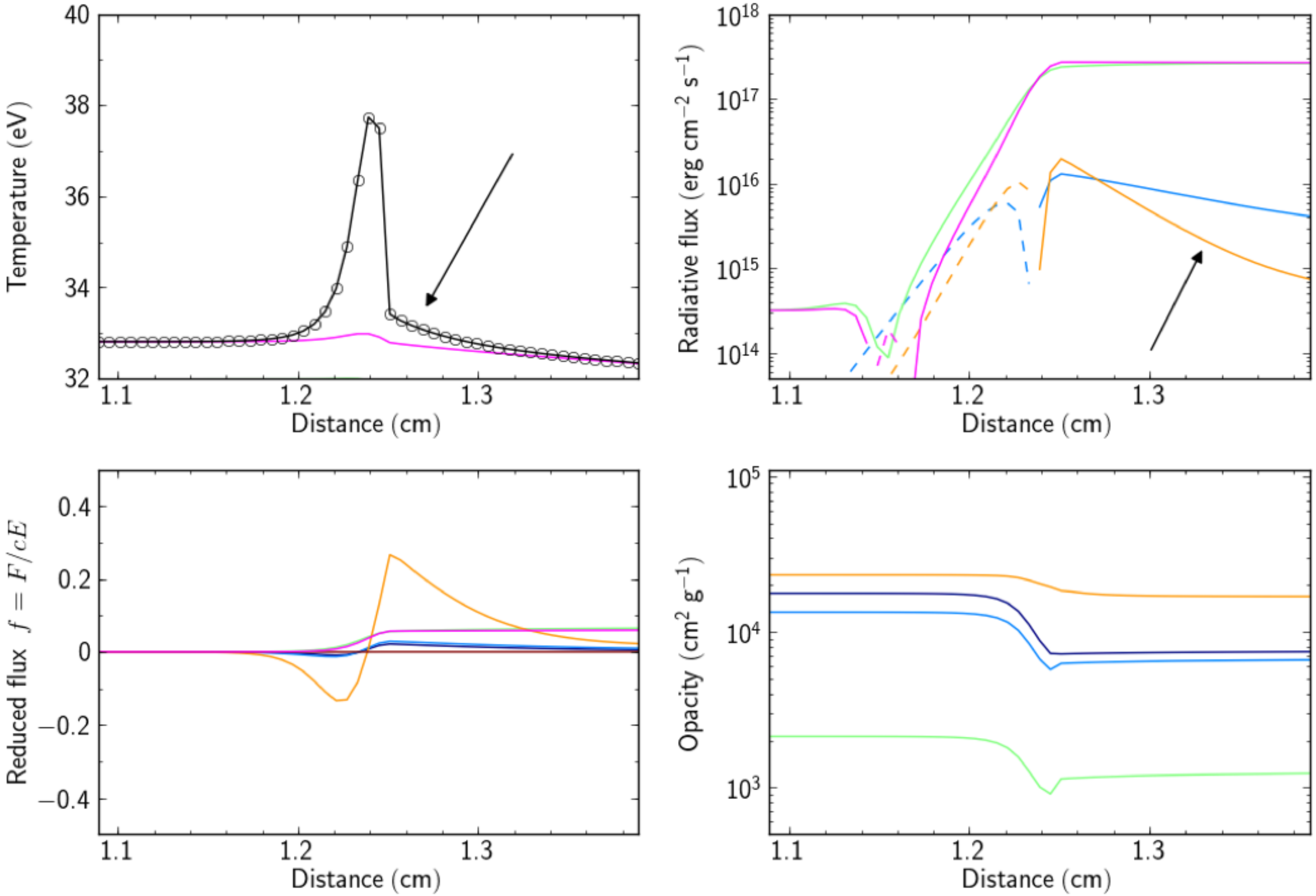
Subcritical

Supercritical

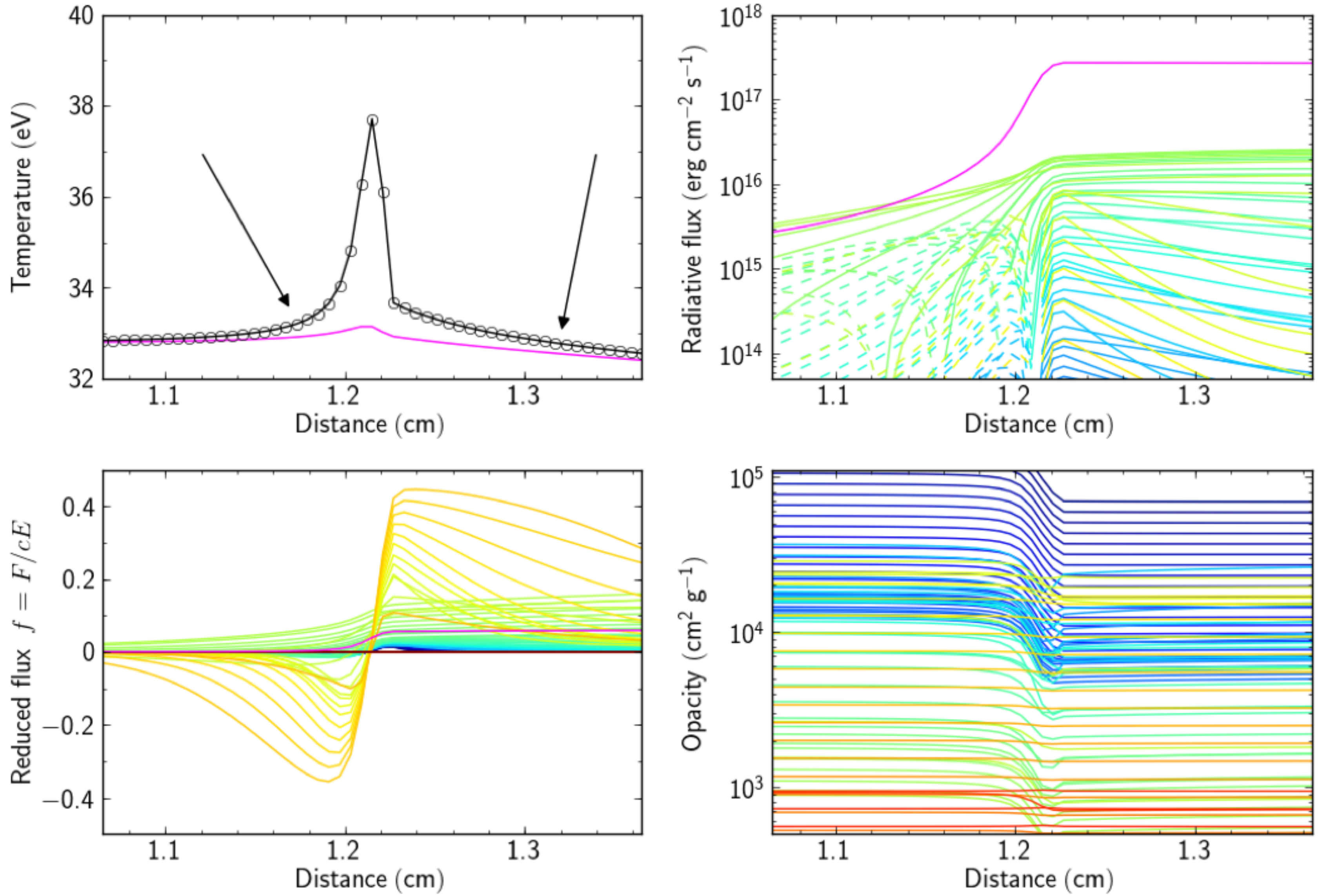
Zel'dovich spike: 1 group



Zel'dovich spike: 5 groups



Zel'dovich spike: 100 groups



Zel'dovich spike: explanation

The energy absorbed by the gas E^{abs} is proportional to $\int_0^\infty \kappa_\nu F_\nu d\nu = \sum_g \kappa_g F_g \Delta\nu_g$

Let consider 2 groups with

Group 1 :		Group 2 :	
E_1	$= E_0$	E_2	$= E_0/100$
κ_1	$= 0.1$	κ_2	$= 1.0$
f_1	$= 0.05$	f_2	$= 0.5$
F_1	$= 0.05cE_0$	F_2	$= 0.005cE_0$

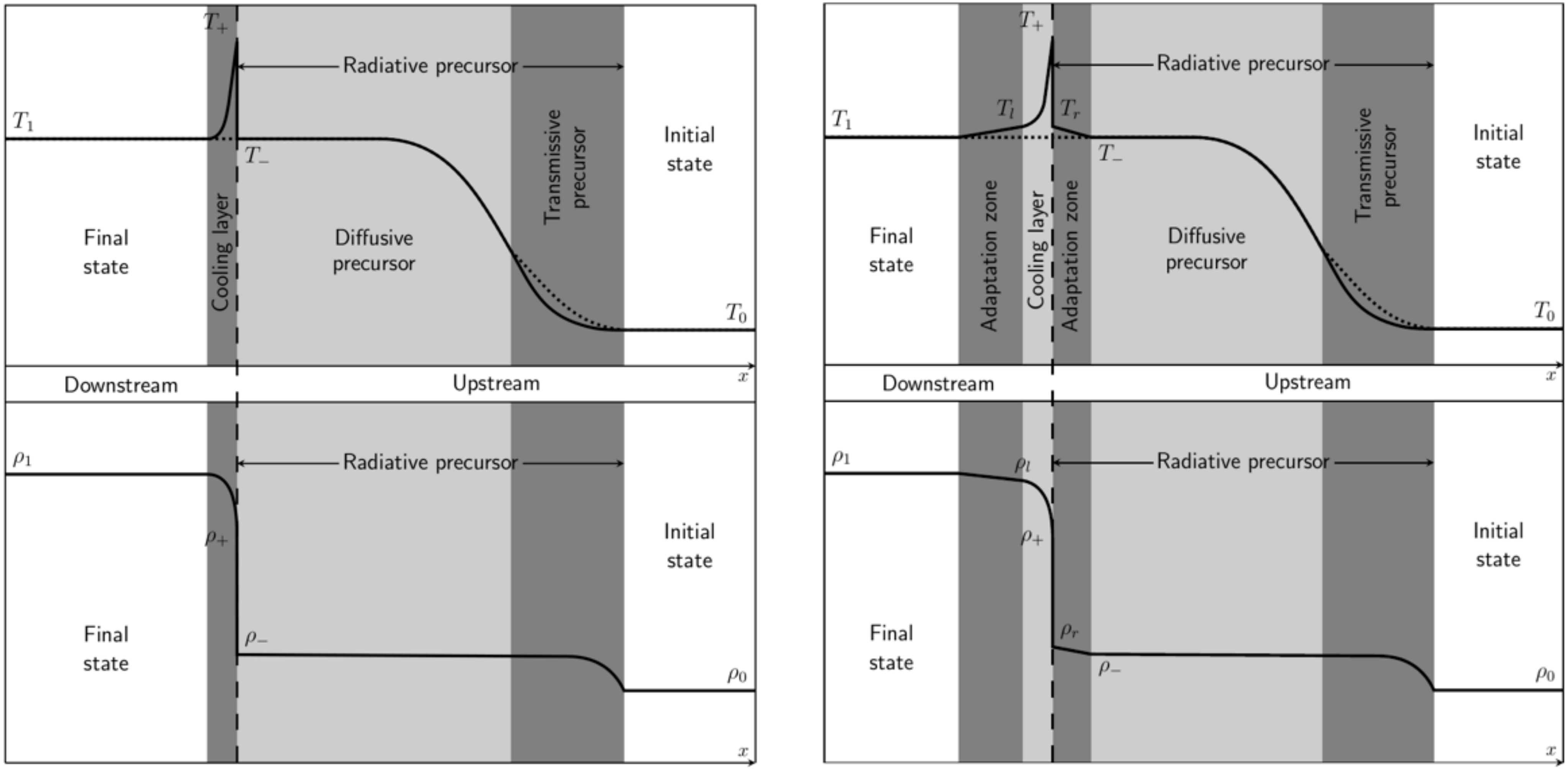
So

$$\begin{aligned} E^{\rm abs} &\propto [\kappa_1 F_1 + \kappa_2 F_2]\Delta\nu \\ &\propto [5\times 10^{-3} cE_0 + 5\times 10^{-3} cE_0]\Delta\nu \end{aligned}$$

which is twice the amount found by considering only the dominant group 1.

Including group 2 does not change the total radiative temperature, but does change the amount of energy absorbed by the gas, and the gas and radiative temperatures can hence be decoupled (cf. Drake 2007)

Adaptation zones



Summary and perspectives

➢ Summary

✓ development of M1 multigroup model

✓ influence of multigroup on the precursor size

✓ effects on electron densities detectable in experiments

✓ detection of adaptation zones

➢ Perspectives

✓ effects of different model of opacities

✓ development of multigroup M1 model in the AMR RAMSES code

✓ simulations of star formation (cf. N. Vaytet's talk on Thursday 12h15)