

# On gravity waves in the Sun

Allan Sacha Brun<sup>1</sup>, Lucie Alvan<sup>1</sup>, Antoine Strugarek<sup>1</sup>, Stéphane Mathis<sup>1</sup> & Rafael A. García<sup>1</sup>

Laboratoire AIM Paris-Saclay, CEA/Irfu Université Paris-Diderot CNRS/INSU, 91191 Gif-sur-Yvette, France

E-mail: [sacha.brun@cea.fr](mailto:sacha.brun@cea.fr)

**Abstract.** We briefly present our recent progress to model in 3-D the excitation and propagation of internal waves in the deep solar radiative interior. By modeling a rotating spherical convection zone on top of a radiative interior with a realistic seismically calibrated stable stratification (i.e solar-like Brunt-Väisälä frequency), we are able to generate a large spectrum of internal waves and modes thanks to the continuous pummeling of convective plumes. When comparing with an adiabatic oscillation code we find a good overall agreement and confirm that those waves are gravity waves.

## 1. Waves in the Sun

Various waves are present in the Sun, such as acoustic, gravity, Alfvén, and inertial waves to cite only a few. They can inform us on the internal structure and dynamics of our star as a function of depth and latitude with an accuracy that depends on their propagation properties and the time they spent traveling in any given region. Internal gravity waves (IGW) are essential for helio- and asteroseismology to probe the deep stably stratified interior of solar-like stars (e.g. Press 1981, Appourchaux 2010). Moreover, they are of great interest in the study of angular momentum transport over large distances and constitute an important ingredient in understanding the evolution of stellar rotation (Zahn et al. 1997; Charbonnel & Talon, 2005; Mathis 2009).

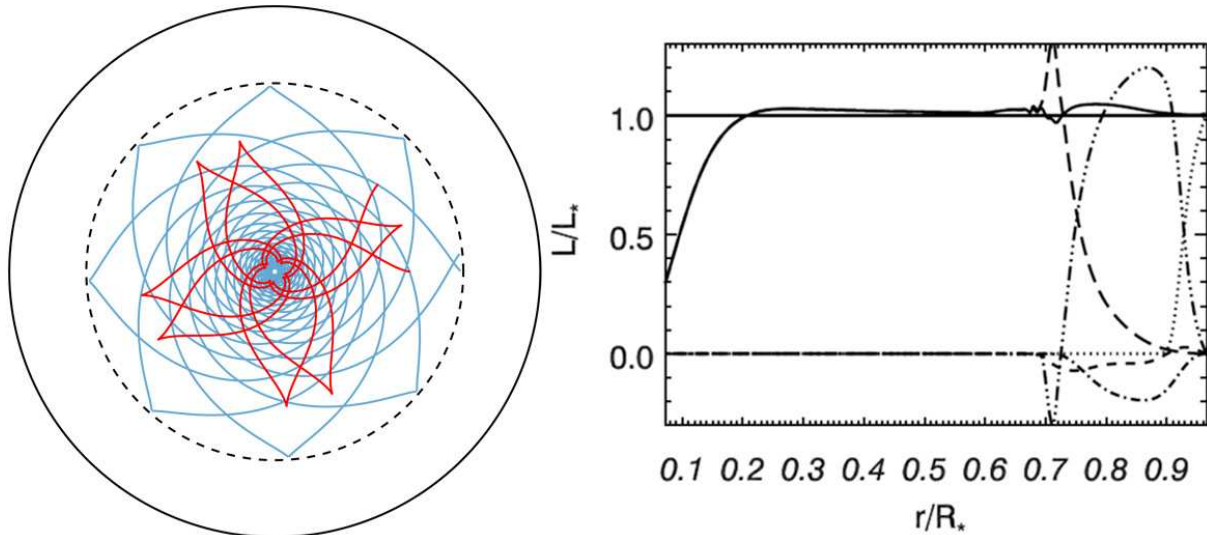
IGW propagate in the radiative zone under the condition  $\omega < N$  where  $\omega$  is the frequency of the waves and  $N$  the Brunt-Väisälä frequency, given by

$$N^2 = -g \left( \frac{1}{\bar{\rho}} \frac{\partial \bar{\rho}}{\partial r} - \frac{1}{\Gamma_1 \bar{p}} \frac{\partial \bar{p}}{\partial r} \right), \quad (1)$$

with  $g$  the gravity,  $\Gamma_1$  the adiabatic exponent,  $\bar{\rho}$  and  $\bar{p}$  the mean density, and pressure. The region where IGW propagate is visible in Figure 1 (left panel) where we have represented the path of propagation of two g-modes, using the linear dispersion relation (Christensen-Dalsgaard, 2003):

$$k_r^2 = \frac{\ell(\ell+1)}{r^2} \left( \frac{N^2}{\omega^2} - 1 \right), \quad (2)$$

where  $\ell$  is the spherical harmonic degree. Oscillations are trapped in the radiative zone between concentric spheres of radius  $r_0$  such as  $\omega = N(r_0)$ . They are thus difficult to observe at the surface of solar-like stars as they are evanescent in convective envelope. However, Garcia et al.



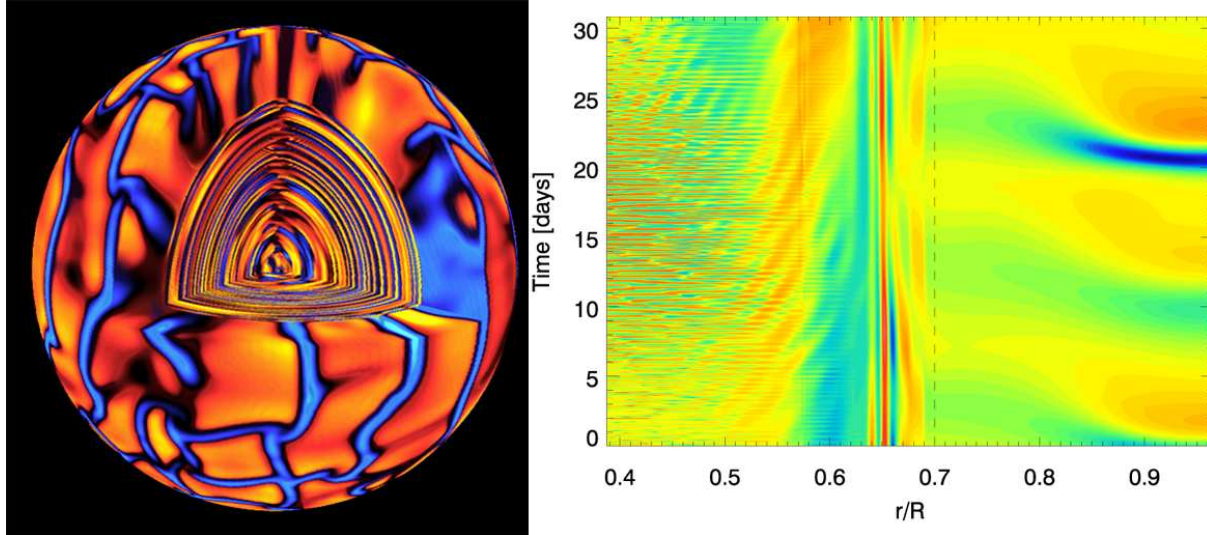
**Figure 1.** Left: Raytracing of 2 gravity modes (at 0.1 (blue line) and 0.3 (red)  $mHz$ ) in a stratified solar-like radiative interior. Right: Flux balance realized in a 3-D ASH simulation of a deep Sun (Brun et al. 2011). Radial fluxes have been converted to luminosity and normalized to the solar luminosity. We plot respectively the total (solid line), enthalpy (dash triple dots), radiative (long dash), unresolved (dot), viscous (dash), and kinetic (dash dot) luminosities.

(2007, 2008) have reported the detection at a 8.6 sigma level of the asymptotic period spacing of  $\ell = 1$  solar gravity modes in Golf/SoHO data. By doing the power spectrum of the periodogram of 11.5 yr of Golf/SoHO data, they have been able to show an excess of power at 24-25 min characteristic of IGW. One is thus lead to ask: how is a turbulent convective envelope filtering the IGW signal likely to be present in the solar radiative interior? Along with theoretical work (Gough 1985, Kumar et al. 1996, Belkacem et al. 2009), a direct comparison with numerical simulation could help us further characterizing such waves. This was recently done in 2-D by Rogers & Glatzmaier in 2005 and later work. We here propose to expand to 3-D their pioneering work.

## 2. 3-D model of a deep Sun

In order to study the generation of internal (possibly gravity) waves by convective motions in solar-like star, we present new results obtained with the anelastic spherical harmonic (ASH) code (Brun et al, 2004). Our model nonlinearly couples a rotating convective envelope to a stable radiative interior (Brun et al, 2011), assuming a realistic solar stratification from  $r = 0.07R_{\odot}$  up to  $0.97R_{\odot}$  (Brun et al. 2002). On Figure 1 (right panel) we show the radial energy balance, with the radiative luminosity (respectively the convective luminosity) carrying most of the energy for  $r < 0.7R = r_{bcz}$  (resp.  $r > 0.7R$ ). We note that the convective (enthalpy) flux is negative at the base of the convection zone  $r = r_{bcz}$ . This is due to the penetration of convective plumes into the top of the stable radiative zone. Such continuous pummeling generates internal waves that carry energy and angular momentum inward. A 3-D rendering of such waves is shown on Figure 2 (left panel), where we plot the radial velocity normalized to the local (at each radii) rms velocity. We note the strikingly different patterns observed in the convective envelope with respect to that realized in the deep stable interior. The convective flows, possess a large irregular structure, with down flow lanes delineating broad up flows. By contrast in the radiative interior we see almost concentric wave fronts of alternative signs.

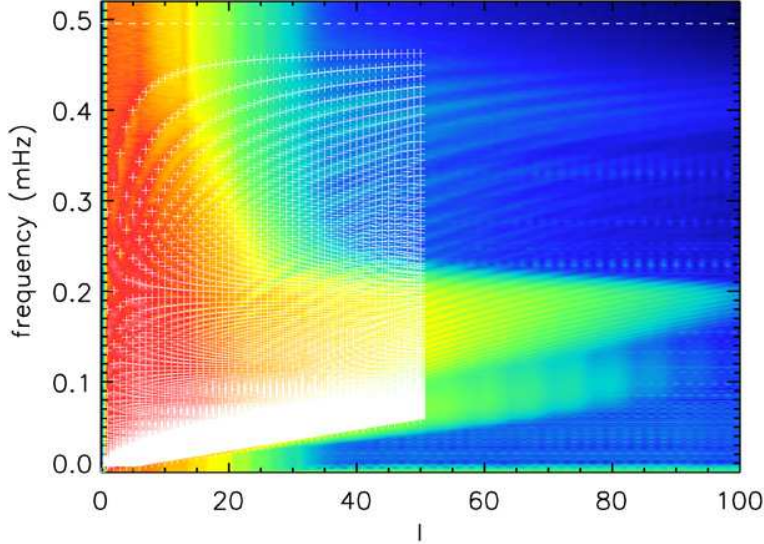
In order to confirm that such wave-like patterns correspond to IGW, we plot on Figure 2



**Figure 2.** Left: 3-D rendering of the model, showing both the convective envelope and the wave patterns realized in the deep radiative interior of a deep Sun simulations (Brun et al. 2011). Right: Time-radius plot showing a cut at fixed longitude in the equatorial plane of the normalized velocity.

(right panel) the normalized radial velocity as a function of radius and time for a fixed longitude in the equatorial plane. We see that near the base of the convection zone, three to four almost vertical modulations are present, likely due to the dynamical response of the top of the radiative interior to the continuous pummeling of downflow lanes. These are apparent in the convective envelope as tilted blue lanes that appear at the surface, propagate downward and hit the base of the convection zone. Their numbers (almost 4) over the 30 days covered by the figure, is a consequence of the Reynolds number and rotation rate used in the simulation. Indeed only about 12 of those downflow lanes coexist at any given time over the 360 degree of longitude of the model (6 of these downflow sheets aligned with the rotation axis can be seen on the left panel of Figure 2). A more turbulent model would possess a larger number and more complex downflow lanes (Miesch et al. 2008). The forward tilt of the plumes is due to the presence of a large scale differential rotation which near the equator is faster near the base of the convection zone than at the surface. Hence as the plumes sink at low latitudes, they are advected forward by the radial shear. At larger latitude, the tilt is in the opposite direction. Deeper in the radiative interior we note inclined wave fronts going from the lower left to the top right of the figure. This is an indication that the wavefront is moving outward. IGW are known to have an opposite sign between their vertical group and phase velocities. Since the energy is carried downward, such fronts correspond to phase velocity going outward, and hence are compatible with IGW.

To further analyze the properties of the waves excited in the radiative interior of our model, we have computed the energy power spectra at various depths and show on Figure 3 the spectra realized at  $r = 0.4R$  (see Alvan et al. 2012). A series of ridges is clearly apparent in the  $[0.1, 0.5]\mu\text{Hz}$  range. Their characteristic dispersion relation is clearly compatible with gravity modes, with all the ridges converging to a maximum corresponding to the Brunt-Väisälä (BV) frequency of that specific depth (we recall that BV frequency varies as a function of radius in the model). This is even more obvious after superimposing on the spectrum obtained with ASH, the frequencies computed with the *Adipls* code (Christensen-Dalsgaard 2011) using the background reference state structure used in the 3-D model. All the white crosses correspond to a gravity mode, and they clearly match very well the ridge-pattern seen in the power spectra. We are



**Figure 3.** Color contours: Power spectrum as a function of the spherical harmonic degree  $\ell$  of gravity mode frequencies at a depth of  $r = 0.4R$  in the 3-D nonlinear deep Sun ASH model. Superimposed with white crosses the gravity mode frequencies as a function of  $\ell$  computed with the *Adiphs* code (Christensen-Dalsgaard 2011). The Brunt-Väisälä frequency at that depth is also shown (horizontal white dash line). We clearly see the ridges characteristic of IGW.

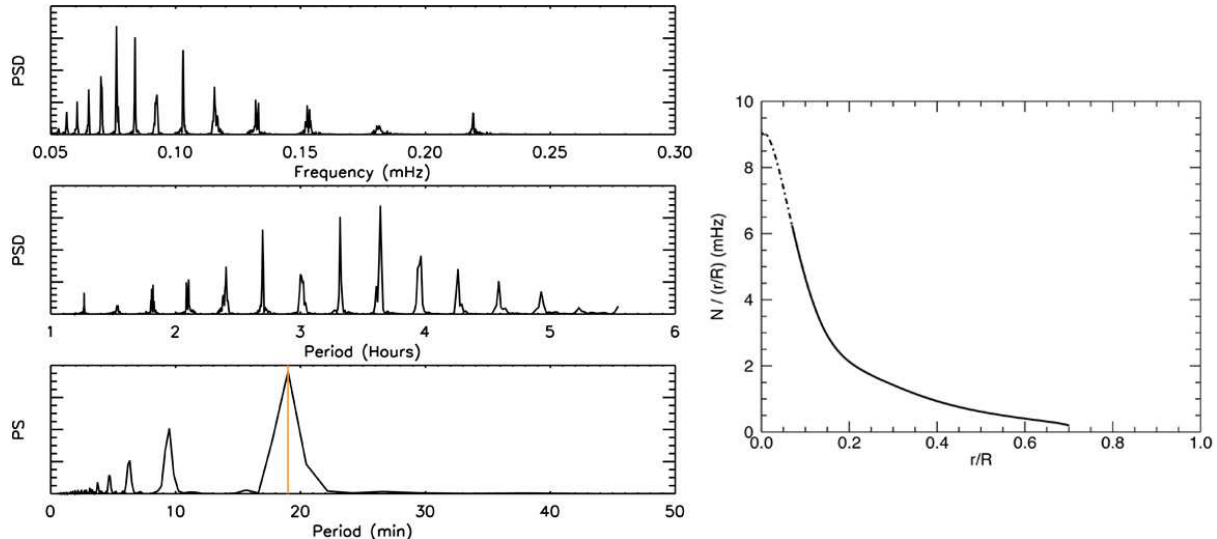
thus confident that we have excited successfully for the first time in a 3-D spherical geometry IGW by the direct nonlinear action of convective plumes.

Zooming on one of the low degree  $\ell$  mode, we plot on Figure 4 (left panel) the power spectra as a function of frequency and period. We clearly see that in the second panel, the peaks are spaced uniformly with respect to the period of the modes. Indeed it is well known that gravity modes have a nearly constant spacing, such that (e.g. Provost & Berthomieu 1986):

$$\Delta P_\ell \simeq \frac{\pi}{\sqrt{\ell(\ell+1)} \int_{r_1}^{r_2} N/r dr}. \quad (3)$$

We find the following values for respectively  $\Delta P_{1,2,3} \sim 44, 27, 19$  min, as is evident for the latter on the bottom left panel of Figure 4. Garcia et al. (2007) reported a value of about 24-25 min for  $\Delta P_1$  with a complex double peak structure of the signal and anticipated a  $\Delta P_3 \sim 11$  min, hence about 40% smaller than our value of 19 min.

The difference between the observed value and the computed one is certainly due to our truncated model, with the inner domain stopping at  $r = 0.07R$ . On Figure 4 (right panel), we plot the ratio  $N/r$  as a function of radius. The solid line corresponds to the part we model whereas the dash-dot line emphasizes the missing part. Whereas we just omit 10% of the resonant cavity for the IGW, the integrant used in formula 3 is actually modified by 37%, explaining the larger period spacing found in our model compared to observations. While it is very encouraging to see that we can apply similar techniques to our simulations and to real observations, we have yet to obtain such signal in the convective envelope as was reported in Garcia et al. (2007). Indeed Fig 4 is computed in the radiative zone of our deep Sun simulation, which is much easier, and show the interest of doing these computations. Looking at the eigenfunctions of the gravity modes excited in our simulations, we note that the high frequency, low degree  $\ell$  modes are the most likely to be visible (Kumar et al. 1996). We thus intend to generate longer 3-D time series



**Figure 4.** Left:  $\ell = 3$  mode power at  $r = 0.4R$  shown as a function of frequency (top) or period (middle); Power spectrum of the power spectrum for  $\ell = 3$  modes (bottom), with a clear peak around 19 min. Right: truncated gravity-wave cavity (Brunt-Väisälä frequency) due to the inner sphere of radius  $0.07R$ . This truncation is at the origin of the observed frequency/period shift.

than the 100 days currently available to be able to boost the signature of IGW near the surface, as well as to look for the most likely modes and, to study their stochastic variability.

### 3. Conclusions

We have demonstrated that we can now model in 3-D the excitation and propagation of internal gravity waves by turbulent convection and study their properties and nonlinear interactions. When comparing the power spectra obtained in our 3-D model with adiabatic computations performed with the *Adipls* code (Christensen-Dalsgaard 2011) we find an excellent overall agreement, provide we take into account the truncated radius of our reference background state. In order to make direct comparison with observation and guide the detection of IGW in the Sun, we must build a full deep Sun model going all the way down to the center. Further Brown et al. (2012) have demonstrated that all anelastic formulations of the MHD equations are not equivalent when modeling highly stratified radiative interior (such as the solar one) and IGW. Recent developments with the ASH code will allow us to improve our models by using a better anelastic description and by setting the inner boundary to  $r = 0$ . Understanding the influence of an inner magnetic field such as the one stored in the tachocline on IGW is also of great interest (Strugarek et al. 2011; Mathis & de Brye 2012). Finally having long time series will allow us to study the temporal variability of the mode excitation as well as boost their surface signature by applying data analysis developed for the real Sun and guided by our knowledge of the wave dynamics in the 3-D radiative interior. Please stay tuned.

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