COMPUTATION OF REALISTIC VECTOR POTENTIAL FOR LONG-TERM TRACKING

DE LA RECHERCHE À L'INDUSTRIE

ABELE SIMONA

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www.cea.fr

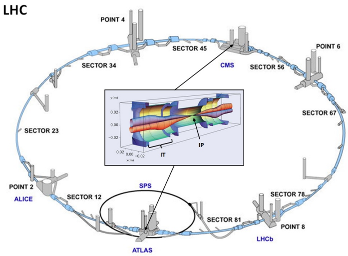
Outline

- introduction;
- theoretical aspect of the problem;
- implementation of the code;
- test;
- application to realistic quadrupole;
- general view of the interface with SixTrack[1];

[1] Tracking code used at the CERN

INTRODUCTION

In particle physics the most famous experimental structure is the Large Hadron Collider (LHC). It's the world's biggest and most powerful particle accelerator.



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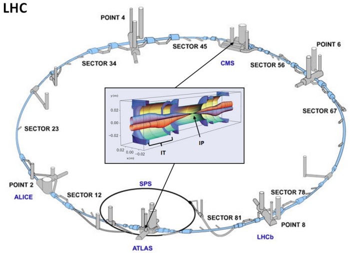

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HL-LHC: High Luminosity LHC

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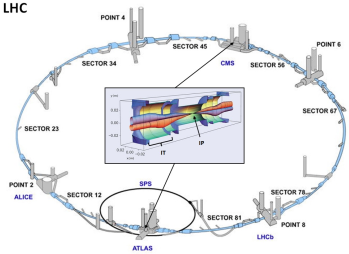

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Increase luminosity = Reduce size of the beam at the IP
$\Rightarrow$ Increase size of the beam in the last triplet, increase crossing angle
$\Rightarrow$ Bigger magnets mechanical aperture $\Rightarrow$ More non-linear effects

The magnetic field $\vec{B}$ in a quadrupole can be written as a Fourier series:

$$\vec{B}(\rho, \varphi, z) = \sum_m \vec{B}_m(\rho, z) sin(m\varphi) + \vec{A}_m(\rho, z) cos(m\varphi) \quad (1)$$

the coefficients of sinus and cosine are called respectively normal and skew *harmonics*.

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Magnetic Field Nonlinearities

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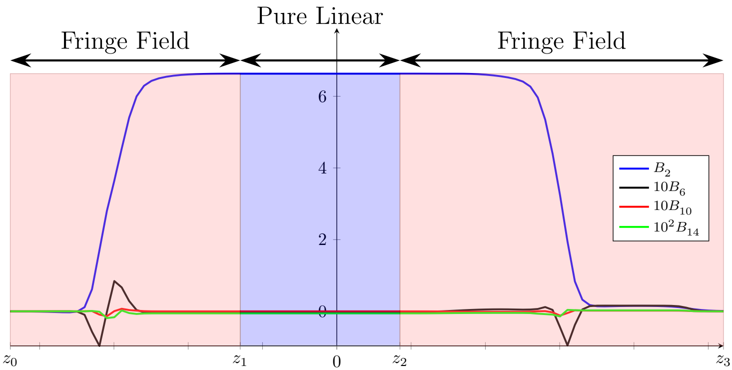
With *nonlinearities* we refers to the effects caused by the harmonics of order bigger then 2 and to the ones caused by the non-uniformity of the harmonics along z.

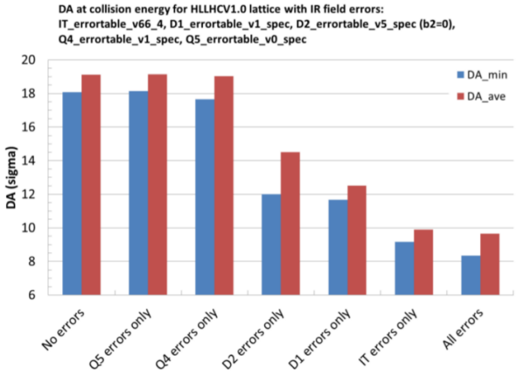
Fringe Field

The field at the sides of the quadrupole is called *Fringe Field* which adds significant non-linear contributions, as shown in the article of [AV Bogomyagkov et al.].



Inner Triplet errors

During the fourth HiLumi meeting M. Giovannozzi has shown that the error done in the final triplet ("Inner-Triplet", IT, a sequence of four quadupoles) before the interaction point has the biggest influence over the DA.



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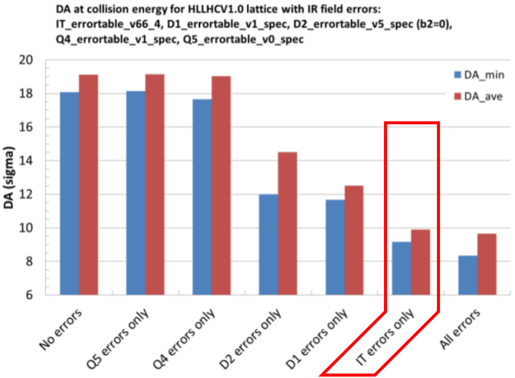

OBJECTIVES

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Therefore these are the objectives:

- Provide an accurate description of the magnetic vector potential starting from the harmonics of from the magnetic field;
- Provide it in a form that allows a fast tracking procedure, in particular in a polynomial form:

$$\vec{A}(x,y,z) = \sum_{i,j} \vec{a}_{i,j}(z) x^i y^j$$

THEORETICAL PROBLEM

Framework

Electromagnetic stationary field, no currents, no charges, vacuum.

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$$\text{Maxwell equations} \Rightarrow \begin{cases} \vec{\nabla} \cdot \vec{E} = 0 & \vec{\nabla} \times \vec{E} = 0 \\ \vec{\nabla} \cdot \vec{B} = 0 & \vec{\nabla} \times \vec{B} = 0 \end{cases}$$

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$\vec{B}$ can be expressed using a scalar potential $\vec{B} = \vec{\nabla}\psi$ or a vector potential $\vec{B} = \vec{\nabla} \times \vec{A}$

$$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \Delta\psi = 0$$

$$\vec{\nabla}\psi = \vec{B} \Rightarrow \vec{\nabla}\psi = \vec{\nabla} \times \vec{A}$$

Procedure

Magnetic Field

Procedure

Magnetic Field $\rightarrow$ Harmonics

Magnetic Field $\rightarrow$ Harmonics $\rightarrow$ Gradients

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Procedure

Magnetic Field → Harmonics → Gradients → Vector Potential

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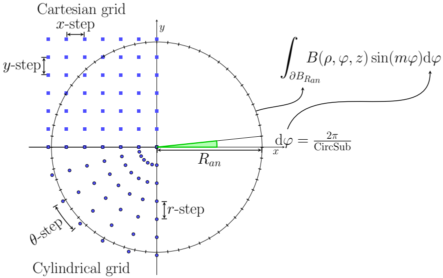
To compute the harmonics it's necessary to compute a Fourier Integral over a circumference.



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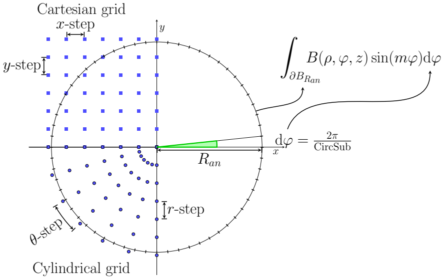

Depending on the grid an interpolation could be needed to provide the values of the field on the circle.

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- consider the magnetic scalar potential ψ expressed in cylindrical coordinates (as the structure of the quadrupole suggests);
- expand ψ in Fourier series on ϕ and in Fourier Transform on z we can obtain a general solution for $\Delta\psi = 0$ which involves I_m;

From this general solution and the harmonics at a specific radius it's possible to compute the generalized gradients using the following formula:

$$C_m^{[n]}(z) = \frac{i^n}{2^m m!} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{k^{m+n-1}}{I'_m(R_{an}k)} \widetilde{B}_m(R_{an}, k) e^{ikz} dk$$

Vector potential

Using the generalized gradients, the relation $\vec{\nabla} \times \vec{A} = \vec{\nabla}\psi$ and changing the coordinates from cylindrical to Cartesian the magnetic vector potential can finally be computed in the desired form:

$$A_x = \sum_{m=0}^{+\infty} \sum_{\ell=0}^{+\infty} \sum_{p=0:2:m} \sum_{q=0}^{\ell} \frac{1}{m} \frac{(-1)^{\ell} m!}{2^{2\ell} \ell! (\ell+m)!} C_m^{[2\ell+1]}(z) \binom{m}{p} \binom{\ell}{q} i^p \quad x^{m-p+2\ell-2q+1} \quad y^{p+2q}$$

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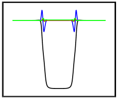
$$\vec{A} = \sum_{i,j} \vec{a}_{i,j}(z) \, x^i \, y^j$$

IMPLEMENTATION

Evolution of the code

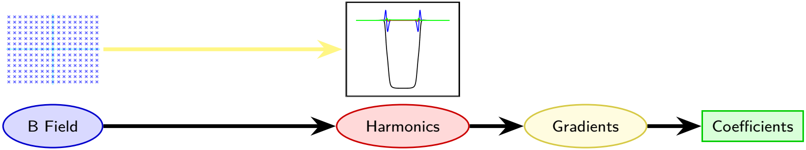


B Field → Harmonics → Gradients → Coefficients

Evolution of the code

C CODE

- Only Cartesian grid
- Hermite spline for interpolation
- Trapeze method for Fourier Integrals



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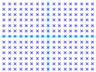
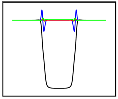
OCTAVE SCRIPT

- Filon Spline formula for Fourier Integrals





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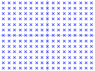

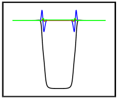

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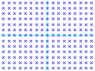
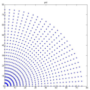




C++ CODE
- Configurable without recompilation
- Computational time reduced
- Optimized output file
- Modular structure

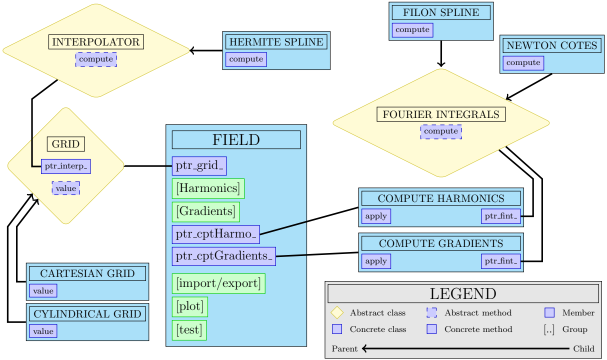
Fourier Integrals methods
- Filon Spline
- Newton-Cotes
- ...

Magnetic grid type
- Cartesian
- Cylindrical
- ...

Interpolation methods
- Hermite splines
- ...

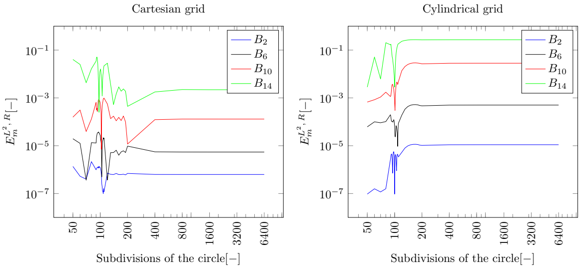
Structure of the new code

A modular structure allows to easily implement new methods and types of grid and to compare them at runtime.



TESTS

Reconstruction of the harmonics
Subdivisions of the circle and radius of analysis



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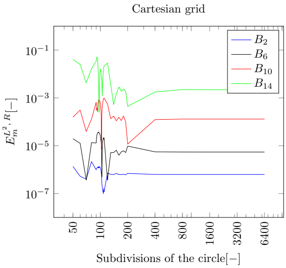
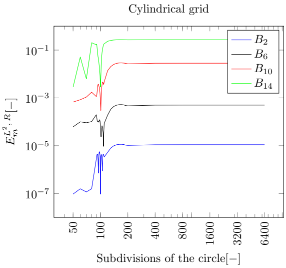
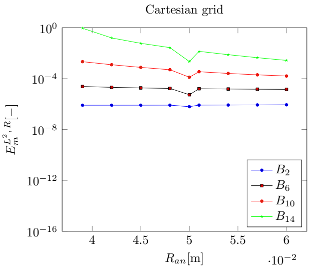
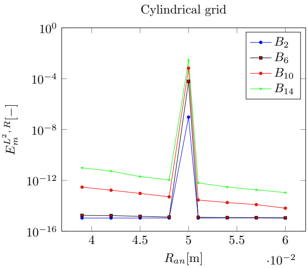
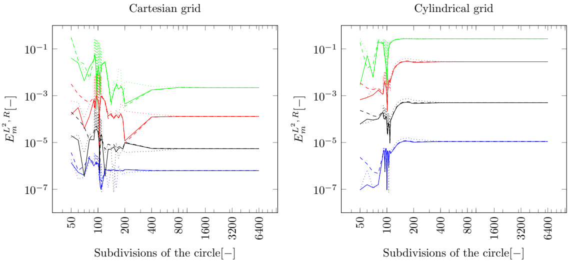




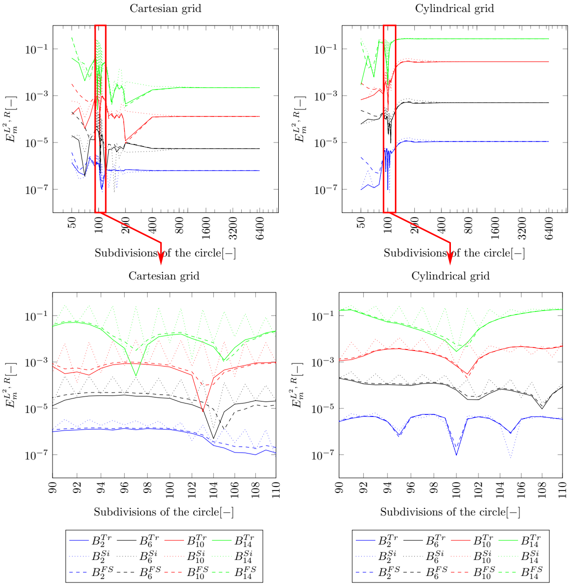





Cartesian grid
Cylindrical grid
10^{-1}
10^{-3}
10^{-5}
10^{-7}
$E_m^{L^2,R}[-]$
50
100
200
400
800
1600
3200
6400
Subdivisions of the circle[−]

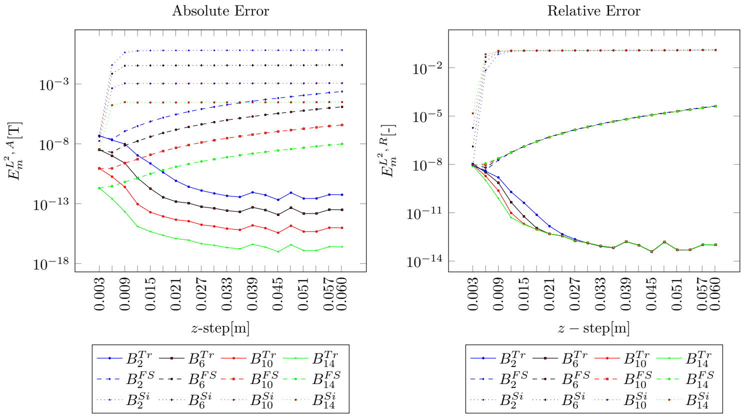
Reconstruction of the harmonics

Methods for Fourier Integrals



Reconstruction of the gradients
Methods for Fourier Integrals and z-step

$$C_m^{[n]}(z) = \frac{i^n}{2^m m!} \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \frac{k^{m+n-1}}{I'_m(R_{an} k)} \widetilde{B}_m(R_{an}, k) e^{ikz} dk$$



Reconstruction of the gradients
Angular frequency samples

Introduction
Objectives
Theoretical problem
Implementation
Tests
Applications to realistic quadrupole
SixTrack implementation
References

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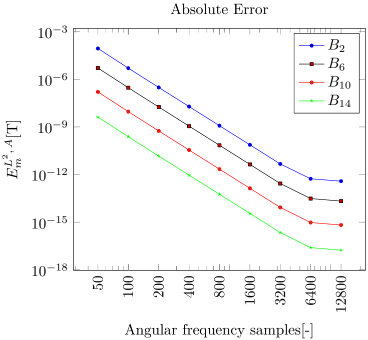
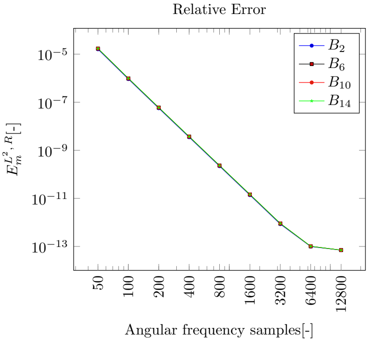
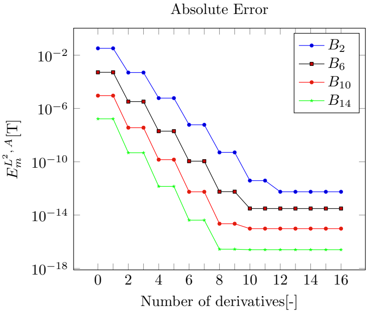


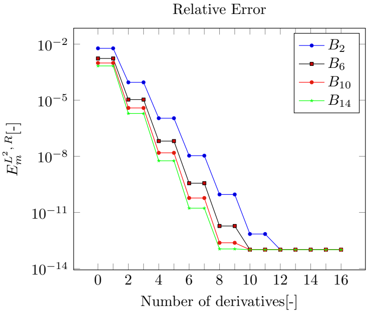


Reconstruction of the gradients
Number of derivatives

$$B_m = \sum_{\ell=0}^{ND} (-1)^\ell (m+2\ell) \frac{m!}{4^\ell \ell! (m+\ell)!} \rho^{m+2\ell-1} C_m^{[2\ell]}(z)$$

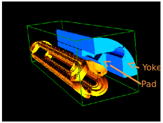
The analytical harmonics are built with 20 derivatives





Application to a realistic quadrupole (1)

Prototype for IT of HL-LHC



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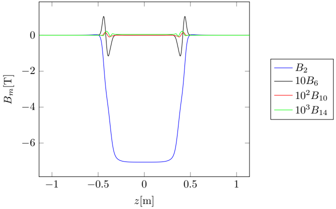

The magnetic field is provided on a Cartesian grid with a step of 0.003m (x, y, z). Both the harmonics and the gradients are computed.

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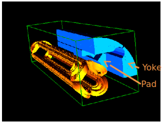

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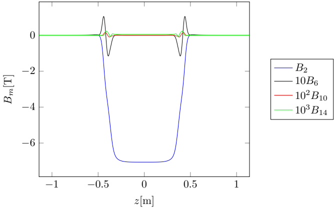

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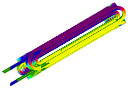
Harmonic	$E_m^{L^2}, A$	$E_m^{L^2}, R$	$E_m^{L^\infty}, A$	$E_m^{L^\infty}, R$
B_2	$2.65479 \cdot 10^{-6}$	$3.75942 \cdot 10^{-7}$	$2.7754 \cdot 10^{-5}$	$3.93021 \cdot 10^{-6}$
B_6	$2.93802 \cdot 10^{-7}$	$2.53589 \cdot 10^{-6}$	$3.20346 \cdot 10^{-6}$	$2.76499 \cdot 10^{-5}$
B_{10}	$1.11986 \cdot 10^{-7}$	$6.95825 \cdot 10^{-6}$	$1.272 \cdot 10^{-6}$	$7.90355 \cdot 10^{-5}$
B_{14}	$9.64632 \cdot 10^{-8}$	$4.33002 \cdot 10^{-5}$	$1.085 \cdot 10^{-6}$	$4.8703 \cdot 10^{-4}$

The maximum absolute value is comparable with the one obtained by [B. Dalena et al.].

* Courtesy of to S. Izquierdo Bermudez and E. Todesco.

Application to a realistic quadrupole (2)



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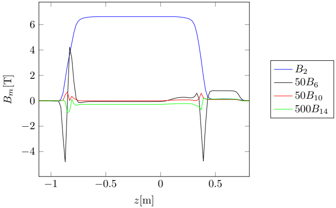
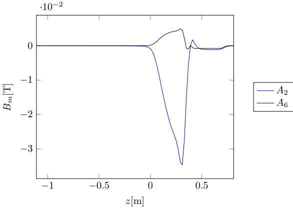
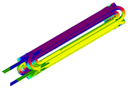




The normal and skew harmonics of a more detailed design, are provided with a z step of 0.02m. The presence of connectors on one side generates an asymmetry and skew harmonics.

Application to a realistic quadrupole (2)



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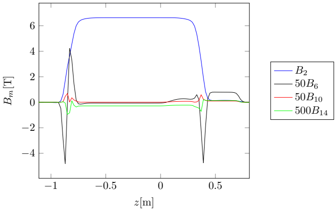

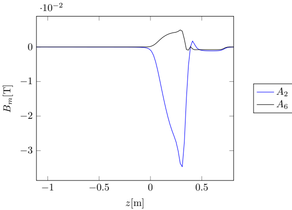

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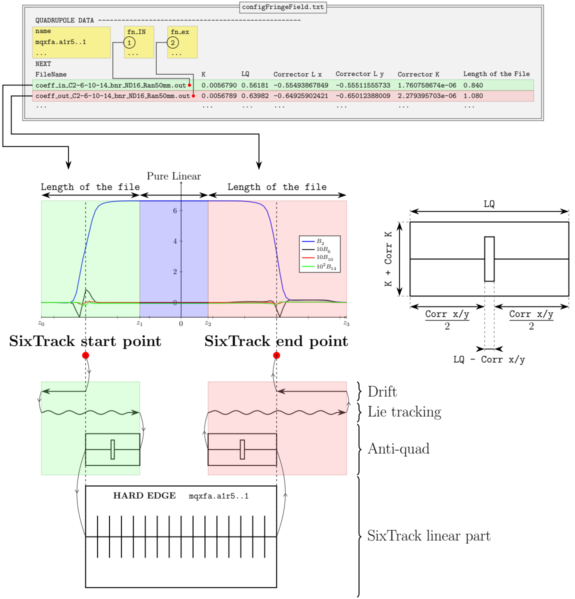
B_2	$2.73535 \cdot 10^{-7}$	$4.12566 \cdot 10^{-8}$	$1.3563 \cdot 10^{-6}$	$2.04567 \cdot 10^{-7}$
B_6	$7.65192 \cdot 10^{-9}$	$7.93646 \cdot 10^{-8}$	$3.61656 \cdot 10^{-8}$	$3.75105 \cdot 10^{-7}$
B_{10}	$8.69318 \cdot 10^{-11}$	$6.34951 \cdot 10^{-9}$	$4.31059 \cdot 10^{-10}$	$3.14846 \cdot 10^{-8}$
B_{14}	$5.47241 \cdot 10^{-12}$	$2.88597 \cdot 10^{-9}$	$2.37895 \cdot 10^{-11}$	$1.25458 \cdot 10^{-8}$
A_2	$1.60238 \cdot 10^{-9}$	$4.62285 \cdot 10^{-8}$	$1.09001 \cdot 10^{-8}$	$3.14467 \cdot 10^{-7}$
A_6	$7.92291 \cdot 10^{-11}$	$1.6105 \cdot 10^{-8}$	$4.52242 \cdot 10^{-10}$	$9.1928 \cdot 10^{-8}$

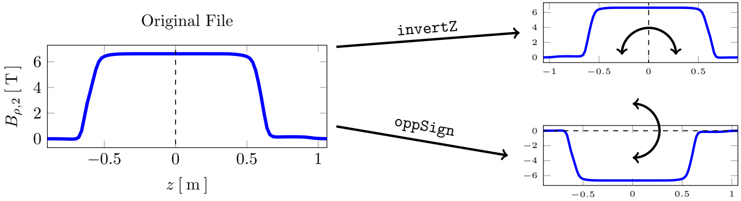
* Courtesy of S. Izquierdo Bermudez and E. Todesco.

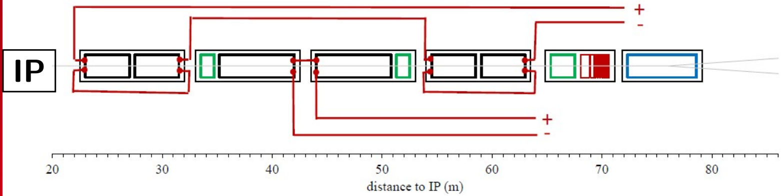
SixTrack IMPLEMENTATION

Procedure


configFringeField.txt
QUADRUPOLE DATA ---
name
mqxfa.a1r5..1
...
fn_IN
1
...
fn_ex
2
...
NEXT
FileName K LQ Corrector L x Corrector L y Corrector K Length of the File
coeff_in_C2-6-10-14_bnr_ND16_Ran50mm.out 0.0056790 0.56181 -0.55493867849 -0.55511555733 1.760758674e-06 0.840
coeff_out_C2-6-10-14_bnr_ND16_Ran50mm.out 0.0056789 0.63982 -0.64925902421 -0.65012388009 2.279395703e-06 1.080
... ...
Pure Linear
Length of the file
Length of the file
B_2
$10B_6$
$10B_{10}$
10^2B_{14}
z_0
z_1
0
z_2
z_3
SixTrack start point
SixTrack end point
LQ
K + Corr K
Corr x/y / 2
Corr x/y / 2
LQ - Corr x/y
HARD EDGE mqxfa.a1r5..1
Drift
Lie tracking
Anti-quad
SixTrack linear part

Different orientations





- theoretical derivation of the method to compute the vector potential;
- translation in C++ and improving of C code and octave scripts;
- systematic testing of the methods;
- in collaboration with Thomas Pugnat, implementation in SixTrack of the tracking code;

Perspective

- Harmonics: understand influence of the interpolating methods on various grids;
- Gradients: implement new stable methods of order higher than Trapeze that can also be used in the reconstruction of the harmonics;
- study error dependence on data noise;
- develop a symplectic integrator using an Hamiltonian without the paraxial approximation;
- study alternative symplectic integrators with respect to the one optimized by [T. Pugnat];
- derive a transfer map in the case of a dipole;

THANK YOU FOR YOUR ATTENTION

AV Bogomyagkov et al. "Analysis of the Non-Linear Fringe Effects of Large Aperture Triplets for the HL-LHC Project, IPAC 2013 (WEPEA049)". In: (2013).

B Dalena, O Gabouev, M Giovannozzi, R De Maria, RB Appleby, A Chancé, J Payet, and DR Brett. "Fringe fields modeling for the high luminosity LHC large aperture quadrupoles". In: (2014).

References II

Thomas Pugnat. “Calcul d’une "carte de transport" réaliste pour particules chargées”. In: (2015).